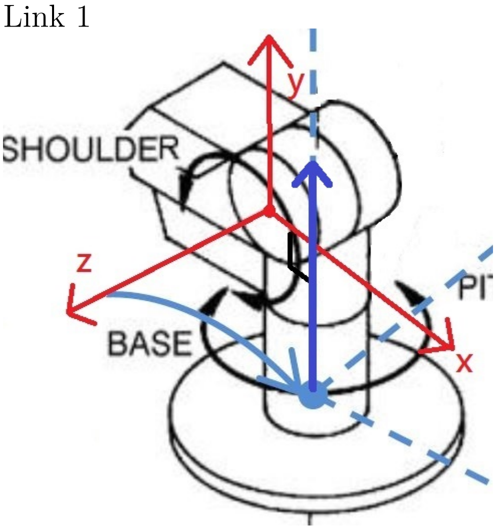
Robotics : Assignment 2

B03902062 資工三　董文捷

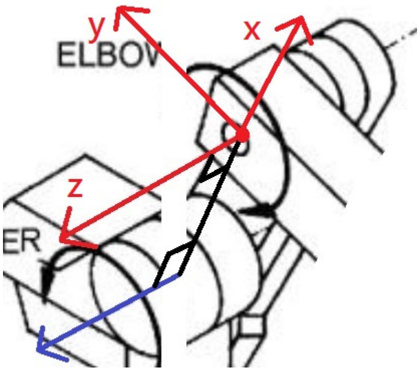
PART A

(1) **According to ER-7 arm, draw the link coordinate diagram using D-H convention.**

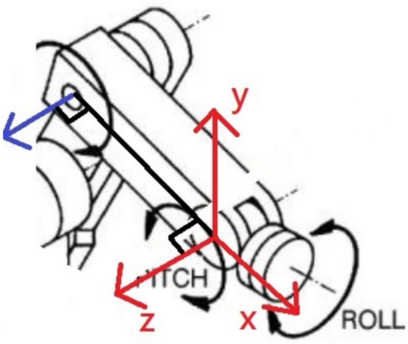
Link 1



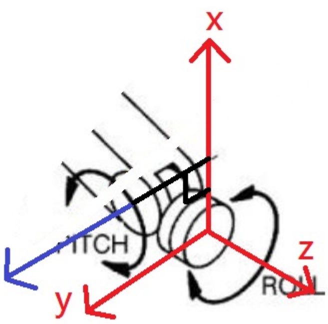
Link 2



Link 3



Link 4



(2) **According to the 2 joints shown in Fig 1, please define the four D-H parameters.**

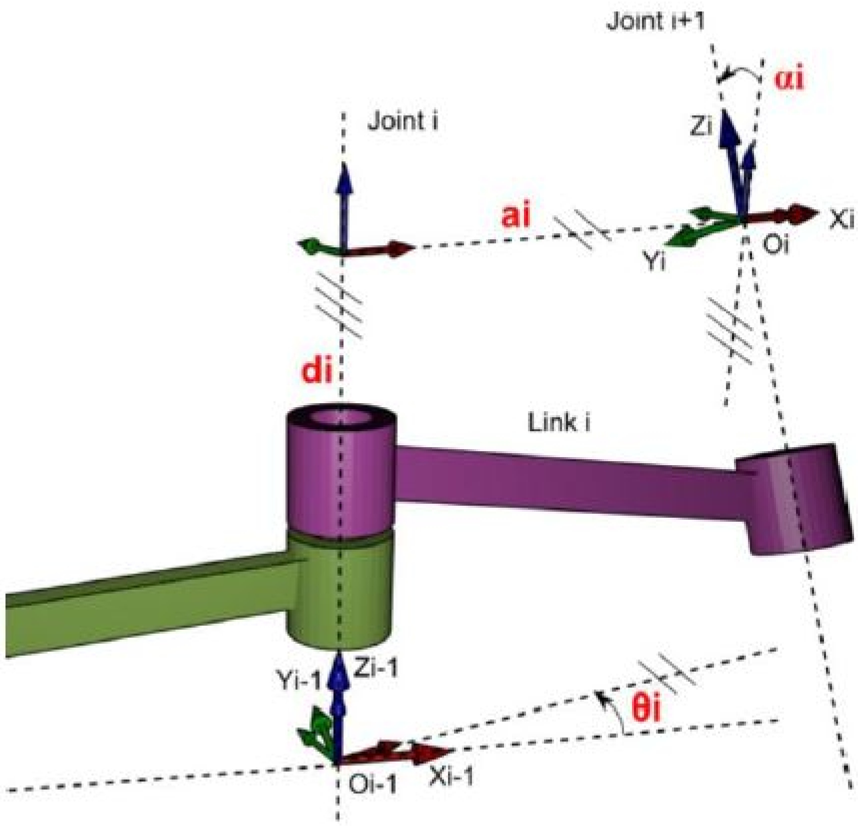


Fig 1 The D-H representation of joint i and joint i+1

- α_i = the angle between z_{i-1} and z_i measured about common normal x_i
- a_i = the distance from z_{i-1} to z_i measured along common normal x_i
- d_i = the distance from x_{i-1} to x_i measured along z_{i-1}
- θ_i = the angle between x_{i-1} and x_i measured about z_{i-1}

(3) **Find the kinematics parameters of ER-7 and fill the table below:**

Joint	θ_i (rad)	d_i (mm)	a_i (mm)	α_i (rad)
1	θ_1	358.5	50	$\frac{\pi}{2}$
2	θ_2	-35	300	0
3	θ_3	0	350	0
4	θ_4	0	0	$\frac{\pi}{2}$
5	θ_5	251	0	0

PART B

The transformation matrix from joint $i-1$ to joint i based on the *DH convention* is given by

$$^{i-1}T_i = \begin{bmatrix} \cos\theta_i & -\cos\alpha_i \sin\theta_i & \sin\alpha_i \sin\theta_i & a_i \cos\theta_i \\ \sin\theta_i & \cos\alpha_i \cos\theta_i & -\sin\alpha_i \cos\theta_i & a_i \sin\theta_i \\ 0 & \sin\alpha_i & \cos\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

By taking DH parameters found in **PART A(3)**, we can get

$$^0T_1 = \begin{bmatrix} \cos\theta_1 & -\cos\frac{\pi}{2}\sin\theta_1 & \sin\frac{\pi}{2}\sin\theta_1 & 50\cos\theta_1 \\ \sin\theta_1 & \cos\frac{\pi}{2}\cos\theta_1 & -\sin\frac{\pi}{2}\cos\theta_1 & 50\sin\theta_1 \\ 0 & \sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 358.5 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_1 & 0 & \sin\theta_1 & 50\cos\theta_1 \\ \sin\theta_1 & 0 & -\cos\theta_1 & 50\sin\theta_1 \\ 0 & 1 & 0 & 358.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^1T_2 = \begin{bmatrix} \cos\theta_2 & -\cos 0\sin\theta_2 & \sin 0\sin\theta_2 & 300\cos\theta_2 \\ \sin\theta_2 & \cos 0\cos\theta_2 & -\sin 0\cos\theta_2 & 300\sin\theta_2 \\ 0 & \sin 0 & \cos 0 & -35 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_2 & -\sin\theta_2 & 0 & 300\cos\theta_2 \\ \sin\theta_2 & \cos\theta_2 & 0 & 300\sin\theta_2 \\ 0 & 0 & 1 & -35 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^2T_3 = \begin{bmatrix} \cos\theta_3 & -\cos 0\sin\theta_3 & \sin 0\sin\theta_3 & 350\cos\theta_3 \\ \sin\theta_3 & \cos 0\cos\theta_3 & -\sin 0\cos\theta_3 & 350\sin\theta_3 \\ 0 & \sin 0 & \cos 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 & 0 & 350\cos\theta_3 \\ \sin\theta_3 & \cos\theta_3 & 0 & 350\sin\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^3T_4 = \begin{bmatrix} \cos\theta_4 & -\cos\frac{\pi}{2}\sin\theta_4 & \sin\frac{\pi}{2}\sin\theta_4 & 0\cos\theta_4 \\ \sin\theta_4 & \cos\frac{\pi}{2}\cos\theta_4 & -\sin\frac{\pi}{2}\cos\theta_4 & 0\sin\theta_4 \\ 0 & \sin\frac{\pi}{2} & \cos\frac{\pi}{2} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_4 & 0 & \sin\theta_4 & 0 \\ \sin\theta_4 & 0 & -\cos\theta_4 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$^4T_5 = \begin{bmatrix} \cos\theta_5 & -\cos 0\sin\theta_5 & \sin 0\sin\theta_5 & 0\cos\theta_5 \\ \sin\theta_5 & \cos 0\cos\theta_5 & -\sin 0\cos\theta_5 & 0\sin\theta_5 \\ 0 & \sin 0 & \cos 0 & 251 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \cos\theta_5 & -\sin\theta_5 & 0 & 0 \\ \sin\theta_5 & \cos\theta_5 & 0 & 0 \\ 0 & 0 & 1 & 251 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

PART C

(1) Multiplying these matrices to form a transformation matrix from the base frame to the gripper tip :

$$
{}^3T_5 = {}^3T_4 \times {}^4T_5 = \begin{bmatrix} c_4c_5 & -c_4s_5 & s_4 & 251s_4 \\ c_5s_4 & -s_4s_5 & -c_4 & -251c_4 \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^2T_5 = {}^2T_3 \times {}^3T_5 = \begin{bmatrix} c_{34}c_5 & -c_{34}s_5 & s_{34} & 350c_3 + 251s_{34} \\ s_{34}c_5 & -s_{34}s_5 & -c_{34} & 350s_3 - 251c_{34} \\ s_5 & c_5 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^1T_5 = {}^1T_2 \times {}^2T_5 = \begin{bmatrix} c_{234}c_5 & -c_{234}s_5 & s_{234} & 300c_2 + 350c_{23} + 251s_{234} \\ s_{234}c_5 & -s_{234}s_5 & -c_{234} & 300s_2 + 350s_{23} - 251c_{234} \\ s_5 & c_5 & 0 & -35 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^0T_5 = {}^0T_1 \times {}^1T_5 =
$$
$$
\begin{bmatrix} s_1s_5 + c_1c_{234}c_5 & s_1c_5 - c_1c_{234}s_5 & c_1s_{234} & -35s_1 + c_1(50 + 300c_2 + 350c_{23} + 251s_{234}) \\ s_1c_{234}c_5 - c_1s_5 & -c_1c_5 - c_{234}s_1s_5 & s_1s_{234} & 35c_1 + s_1(50 + 300c_2 + 350c_{23} + 251s_{234}) \\ s_{234}c_5 & -s_{234}s_5 & -c_{234} & 300s2 + 350s_{23} - 251c_{234} + 358.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

For $Z - Y - X$ Euler angles, the rotation matrix is

$$
{}^AR_B = \begin{bmatrix} \cos\phi\cos\theta & \cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi & \cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi \\ \sin\phi\cos\theta & \sin\phi\sin\theta\sin\psi + \cos\phi\cos\psi & \sin\phi\sin\theta\cos\psi - \cos\phi\sin\psi \\ -\sin\theta & \cos\theta\sin\psi & \cos\theta\cos\psi \end{bmatrix}
$$

$$
{}^0T_5 = P = \begin{bmatrix} c_\phi c_\theta & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi & x \\ s_\phi c_\theta & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & y \\ -s_\theta & c_\theta s_\psi & c_\theta c_\psi & z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

For inverse kinematics,

$$
[{}^0T_1]^{-1}P = {}^1T_0{}^0T_5 = {}^1T_5
$$

$$
\begin{bmatrix} c_1 & s_1 & 0 & -50 \\ 0 & 0 & 1 & -358.5 \\ s_1 & -c_1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} c_\phi c_\theta & c_\phi s_\theta s_\psi - s_\phi c_\psi & c_\phi s_\theta c_\psi + s_\phi s_\psi & x \\ s_\phi c_\theta & s_\phi s_\theta s_\psi + c_\phi c_\psi & s_\phi s_\theta c_\psi - c_\phi s_\psi & y \\ -s_\theta & c_\theta s_\psi & c_\theta c_\psi & z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
= \begin{bmatrix} c_{234}c_5 & -c_{234}s_5 & s_{234} & 300c_2 + 350c_{23} + 251s_{234} \\ s_{234}c_5 & -s_{234}s_5 & -c_{234} & 300s_2 + 350s_{23} - 251c_{234} \\ s_5 & c_5 & 0 & -35 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

The $(3,3)$ element leads to the equation $s_1 x - c_1 y = -35$
Let $\rho = \sqrt{x^2+y^2}$ and $\alpha = Atan2(y,x)$
We can get $\sin(\theta_1 - \alpha) = -\frac{35}{\rho}$
$\theta_1 = Atan2(y,x) + Atan2(-35, \pm\sqrt{x^2+y^2-(-35)^2})$

The $(3,1)$ element leads to the equation $s_1 c_\phi c_\theta - c_1 s_\phi c_\theta = s_5$
The $(3,2)$ element leads to the equation
$s_1(c_\phi s_\theta s_\psi - s_\phi c_\psi) - c_1(s_\phi s_\theta s_\psi + c_\phi c_\psi) = c_5$
ϕ, θ, ψ, and θ_1 are all known, so
$\theta_5 = Atan2(s_1 c_\phi c_\theta - c_1 s_\phi c_\theta,\ s_1(c_\phi s_\theta s_\psi - s_\phi c_\psi) - c_1(s_\phi s_\theta s_\psi + c_\phi c_\psi))$

The $(1,1)$ element leads to the equation $c_1 c_\phi c_\theta + s_1 s_\phi s_\theta = c_{234} c_5$
The $(2,1)$ element leads to the equation $-s_\theta = s_{234} c_5$
ϕ, θ, θ_1, and θ_5 are all known, so
$\theta_2 + \theta_3 + \theta_4 = Atan2(-s_\theta, c_1 c_\phi c_\theta + s_1 s_\phi s_\theta)$

The $(1,4)$ element leads to the equation
$c_1 x + s_1 y - 50 = 300 c_2 + 350 c_{23} + 251 s_{234}$
The $(2,4)$ element leads to the equation
$z - 358.5 = 300 s_2 + 350 s_{23} - 251 c_{234}$
Put unknown parameters on the left side
$300 c_2 + 350 c_{23} = c_1 x + s_1 y - 50 - 251 s_{234}$
$300 s_2 + 350 s_{23} = z - 358.5 + 251 c_{234}$
Let $c_1 x + s_1 y - 50 - 251 s_{234} = A$, $z - 358.5 + 251 c_{234} = B$
Square the two equations and add them together, we get
$300^2 + 350^2 + 2 \times 300 \times 350 \times c_3 = A^2 + B^2$
$c_3 = \frac{A^2+B^2-212500}{210000}$
So $\theta_3 = Atan2(\pm\sqrt{1-(\frac{A^2+B^2-212500}{210000})^2},\ \frac{A^2+B^2-212500}{210000})$

Since θ_3 is known, we can write c_{23} as $c_2 c_3 - s_2 s_3$ and s_{23} as $s_2 c_3 + c_2 s_3$ Then the above equations become
$300 c_2 + 350(c_2 c_3 - s_2 s_3) = A$
$300 s_2 + 350(s_2 c_3 + c_2 s_3) = B$
Multiply first equation by $(300 + 350 c_3)$ and second equation by $350 s_3$ and add them together, we get
$c_2 = \frac{(300+350c_3)A + 350 s_3 B}{(300+350c_3)^2 + (350 s_3)^2}$
So $\theta_2 = Atan2(\pm\sqrt{1-(\frac{(300+350c_3)A+350s_3B}{(300+350c_3)^2+(350s_3)^2}},\ \frac{(300+350c_3)A+350s_3B}{(300+350c_3)^2+(350s_3)^2})$

Finally, θ_4 can be calculated by $\theta_4 = (\theta_2 + \theta_3 + \theta_4) - \theta_2 - \theta_3$

(2) For convenience, only find one possible solution.

(a) $(x, y, z, \phi, \theta, \psi) = (400, 100, 0, \frac{\pi}{4}, 0, \pi)$

- $\theta_1 = Atan2(100, 400) + Atan2(-35, \pm\sqrt{400^2 + 100^2 - (-35)^2}) = 0.2448$
- $\theta_5 = Atan2(\sin\theta_1 \cos\frac{\pi}{4}\cos 0 - \cos\theta_1 \sin\frac{\pi}{4}\cos 0,\ \ \sin\theta_1(\cos\frac{\pi}{4}\sin 0 \sin\pi - \sin\frac{\pi}{4}\cos\pi) - \cos\theta_1(\sin\frac{\pi}{4}\sin 0\sin\pi + \cos\frac{\pi}{4}\cos\pi)) = -0.5406$
- $\theta_2 + \theta_3 + \theta_4 = Atan2(-\sin 0, \cos\theta_1\cos\frac{\pi}{4}\cos 0 + \sin\theta_1\sin\frac{\pi}{4}\sin 0) = 0.0$
- $A = \cos\theta_1 x + \sin\theta_1 y - 50 - 251\sin(\theta_2 + \theta_3 + \theta_4) = 362.31$
- $B = z - 358.5 + 251\cos(\theta_2 + \theta_3 + \theta_4) = -107.5$
- $\theta_3 = Atan2(\pm\sqrt{1 - (\frac{362.31^2 + (-107.5)^2 - 212500}{210000})^2},\ \frac{362.31^2 + (-107.5)^2 - 212500}{210000})$ $= 1.909$
- θ_2 $= Atan2(\pm\sqrt{1 - (\frac{(300+350\cos\theta_3)362.31 + 350\sin\theta_3(-107.5)}{(300+350\cos\theta_3)^2 + (350\sin\theta_3)^2})^2},\ \frac{(300+350\cos\theta_3)362.31 + 350\sin\theta_3(-107.5)}{(300+350\cos\theta_3)^2 + (350\sin\theta_3)^2})$ $= 1.3511$
- $\theta_4 = 0 - 1.3511 - 1.909 = -3.2601$

(b) $(x, y, z, \phi, \theta, \psi) = (400, 120, 100, \frac{\pi}{4}, 0, \pi)$

- $\theta_1 = Atan2(120, 400) + Atan2(-35, \pm\sqrt{400^2 + 120^2 - (-35)^2}) = 0.2075$
- $\theta_5 = Atan2(\sin\theta_1 \cos\frac{\pi}{4}\cos 0 - \cos\theta_1 \sin\frac{\pi}{4}\cos 0,\ \ \sin\theta_1(\cos\frac{\pi}{4}\sin 0 \sin\pi - \sin\frac{\pi}{4}\cos\pi) - \cos\theta_1(\sin\frac{\pi}{4}\sin 0\sin\pi + \cos\frac{\pi}{4}\cos\pi)) = -0.5778$
- $\theta_2 + \theta_3 + \theta_4 = Atan2(-\sin 0, \cos\theta_1\cos\frac{\pi}{4}\cos 0 + \sin\theta_1\sin\frac{\pi}{4}\sin 0) = 0.0$
- $A = \cos\theta_1 x + \sin\theta_1 y - 50 - 251\sin(\theta_2 + \theta_3 + \theta_4) = 366.14$
- $B = z - 358.5 + 251\cos(\theta_2 + \theta_3 + \theta_4) = -7.5$
- $\theta_3 = Atan2(\pm\sqrt{1 - (\frac{366.14^2 + (-7.5)^2 - 212500}{210000})^2},\ \frac{366.14^2 + (-7.5)^2 - 212500}{210000})$ $= 1.9533$
- θ_2 $= Atan2(\pm\sqrt{1 - (\frac{(300+350\cos\theta_3)366.14 + 350\sin\theta_3(-7.5)}{(300+350\cos\theta_3)^2 + (350\sin\theta_3)^2})^2},\ \frac{(300+350\cos\theta_3)366.14 + 350\sin\theta_3(-7.5)}{(300+350\cos\theta_3)^2 + (350\sin\theta_3)^2})$ $= 1.1105$
- $\theta_4 = 0 - 1.1105 - 1.9533 = -3.0638$

(c) $(x, y, z, \phi, \theta, \psi) = (400, -100, 120, -\frac{\pi}{4}, 0, \pi)$

- $\theta_1 = Atan2(-100, 400) + Atan2(-35, \pm\sqrt{400^2 + (-100)^2 - (-35)^2}) = -0.3300$
- $\theta_5 = Atan2(\sin\theta_1 \cos-\frac{\pi}{4}\cos 0 - \cos\theta_1 \sin-\frac{\pi}{4}\cos 0,\ \ \sin\theta_1(\cos-\frac{\pi}{4}\sin 0 \sin\pi - \sin-\frac{\pi}{4}\cos\pi) - \cos\theta_1(\sin-\frac{\pi}{4}\sin 0\sin\pi + \cos-\frac{\pi}{4}\cos\pi)) = 0.4554$
- $\theta_2 + \theta_3 + \theta_4 = Atan2(-\sin 0, \cos\theta_1\cos-\frac{\pi}{4}\cos 0 + \sin\theta_1\sin-\frac{\pi}{4}\sin 0) = 0.0$
- $A = \cos\theta_1 x + \sin\theta_1 y - 50 - 251\sin(\theta_2 + \theta_3 + \theta_4) = 360.82$
- $B = z - 358.5 + 251\cos(\theta_2 + \theta_3 + \theta_4) = 12.5$

- $\theta_3 = Atan2(\pm\sqrt{1-(\frac{360.82^2+12.5^2-212500}{210000})^2},\ \frac{360.82^2+12.5^2-212500}{210000})$
$= 1.9727$
- θ_2
$= Atan2(\pm\sqrt{1-(\frac{(300+350\cos\theta_3)360.82+350\sin\theta_3 12.5}{(300+350\cos\theta_3)^2+(350\sin\theta_3)^2})^2},\ \frac{(300+350\cos\theta_3)360.82+350\sin\theta_3 12.5}{(300+350\cos\theta_3)^2+(350\sin\theta_3)^2})$
$= 1.0675$
- $\theta_4 = 0 - 1.0675 - 1.9727 = -3.0402$