Matrices in Computer Science

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What is a matrix?

A matrix is simply a collection of rows and columns, with values such as numbers in each element of the matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

2 x 3 matrix

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}.$$

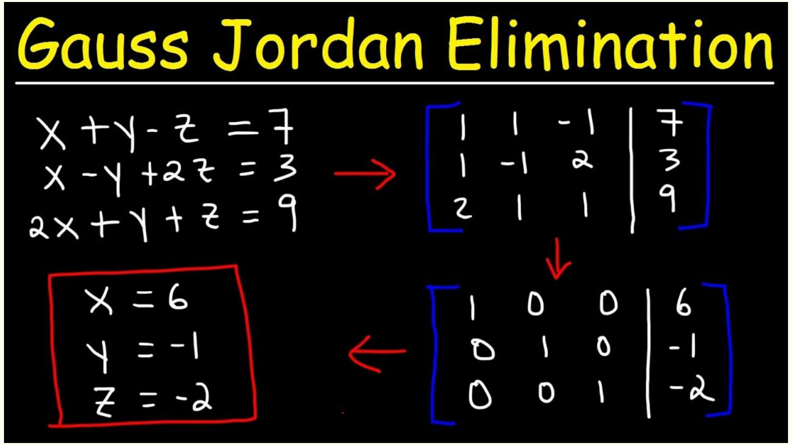
3 x 2 matrix

In Chapter 2 of our CS131 class, we were given a brief introduction to matrices. However, there are so many more things to learn about matrices, and how they can be applied to different tasks.

Operations of Matrices

Matrices are extremely useful, because there are many operations and algorithms that can be performed on a matrix

- Matrix Addition
- Matrix Subtraction
- Matrix Multiplication
- Scalar Multiplication
- Transpose Matrix
- Inverse of Matrix
- Gauss-Jordan Elimination



Various Applications of Matrices

- Represent Relations (i.e. Networks)
- Machine Learning (i.e. Image Recognition)
- Data Structures
- Cryptography
- Graphics

Cryptography using Matrices

- In CS131, we learned to encrypt messages using modular arithmetic
- I will show you how to encrypt and decrypt a message using a matrix

[13	5	5]	[20	0	13]	[5	0	13]	[15	14	4]	[1	25	0]
M	*E*	*E*	*T*	_	*M*	*E*	_	*M*	*O*	*N*	*D*	*A*	*Y*	_

$$A = \begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix}$$

3 x 3 encryption matrix

$$A^{-1} = \begin{bmatrix} -1 & -10 & -8 \\ -1 & -6 & -5 \\ 0 & -1 & -1 \end{bmatrix}$$

3 x 3 decryption matrix (inverse)

Encryption with Matrices (continued)

Uncoded Row Matrix	Encoding Matrix A	Coded Row Matrix

$$[13 \quad 5 \quad 5]\begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix} = [13 \quad -26 \quad 21]$$

$$[20 \quad 0 \quad 13]\begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix} = [33 \quad -53 \quad -12]$$

$$[5 \quad 0 \quad 13]\begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix} = [18 \quad -23 \quad -42]$$

$$[15 \quad 14 \quad 4]\begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix} = [5 \quad -20 \quad 56]$$

$$[1 \quad 25 \quad 0]\begin{bmatrix} 1 & -2 & 2 \\ -1 & 1 & 3 \\ 1 & -1 & -4 \end{bmatrix} = [-24 \quad 23 \quad 77]$$

Coded Row Matrix	Decoding Matrix A^{-1}	Decoded Row Matrix

$$[13 \quad -26 \quad 21]\begin{bmatrix} -1 & -10 & -8 \\ -1 & -6 & -5 \\ 0 & -1 & -1 \end{bmatrix} = [13 \quad 5 \quad 5]$$

$$[33 \quad -53 \quad -12]\begin{bmatrix} -1 & -10 & -8 \\ -1 & -6 & -5 \\ 0 & -1 & -1 \end{bmatrix} = [20 \quad 0 \quad 13]$$

$$[18 \quad -23 \quad -42]\begin{bmatrix} -1 & -10 & -8 \\ -1 & -6 & -5 \\ 0 & -1 & -1 \end{bmatrix} = [5 \quad 0 \quad 13]$$

$$[5 \quad -20 \quad 56]\begin{bmatrix} -1 & -10 & -8 \\ -1 & -6 & -5 \\ 0 & -1 & -1 \end{bmatrix} = [15 \quad 14 \quad 4]$$

$$[-24 \quad 23 \quad 77]\begin{bmatrix} -1 & -10 & -8 \\ -1 & -6 & -5 \\ 0 & -1 & -1 \end{bmatrix} = [1 \quad 25 \quad 0]$$

You can use any encryption matrix, as long as it has an inverse

$$\mathbf{A} = \begin{bmatrix} 3 & 5 & 7 & 2 \\ 1 & 4 & 7 & 2 \\ 6 & 3 & 9 & 17 \\ 13 & 5 & 4 & 16 \end{bmatrix} \qquad \mathbf{A}^{-1} = \begin{bmatrix} 15 & 21 & 0 & 15 \\ 23 & 9 & 0 & 22 \\ 15 & 16 & 18 & 3 \\ 24 & 7 & 15 & 3 \end{bmatrix}$$

Manipulating Images using Matrices

0	2	15	0	0	11	10	0	0	0	0	9	9	0	0	0
0	0	0	4	60	157	236	255	255	177	95	61	32	0	0	29
0	10	16	119	238	255	244	245	243	250	249	255	222	103	10	0
0	14	170	255	255	244	254	255	253	245	255	249	253	251	124	1
2	98	255	228	255	251	254	211	141	116	122	215	251	238	255	49
13	217	243	255	155	33	226	52	2	0	10	13	232	255	255	36
16	229	252	254	49	12	0	0	7	7	0	70	237	252	235	62
6	141	245	255	212	25	11	9	3	0	115	236	243	255	137	0
0	87	252	250	248	215	60	0	1	121	252	255	248	144	6	0
0	13	113	255	255	245	255	182	181	248	252	242	208	36	0	19
1	0	5	117	251	255	241	255	247	255	241	162	17	0	7	0
0	0	0	4	58	251	255	246	254	253	255	120	11	0	1	0
0	0	4	97	255	255	255	248	252	255	244	255	182	10	0	4
0	22	206	252	246	251	241	100	24	113	255	245	255	194	9	0
0	111	255	242	255	158	24	0	0	6	39	255	232	230	56	0
0	218	251	250	137	7	11	0	0	0	2	62	255	250	125	3
0	173	255	255	101	9	20	0	13	3	13	182	251	245	61	0
0	107	251	241	255	230	98	55	19	118	217	248	253	255	52	4
0	18	146	250	255	247	255	255	255	249	255	240	255	129	0	5
0	0	23	113	215	255	250	248	255	255	248	248	118	14	12	0
0	0	6	1	0	52	153	233	255	252	147	37	0	0	4	1
0	0	5	5	0	0	0	0	0	14	1	0	6	6	0	0

Manipulating Images using Matrices

- In an image, the values of all the pixels can be stored in a matrix
- The image can be manipulated using operations of matrices
- For example: to rotate an image you would transpose the matrix
    - The rows of the matrix become the columns

The transpose of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$ is the matrix $\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$.

- Another example: you can invert the color of an image by performing scalar multiplication on the matrix
    - Multiply every element of the matrix by -1
- Image filters

$$3\begin{bmatrix} 1 & 2 & 4 \\ -3 & 0 & -1 \\ 2 & 1 & 2 \end{bmatrix} = \begin{bmatrix} 3(1) & 3(2) & 3(4) \\ 3(-3) & 3(0) & 3(-1) \\ 3(2) & 3(1) & 3(2) \end{bmatrix} = \begin{bmatrix} 3 & 6 & 12 \\ -9 & 0 & -3 \\ 6 & 3 & 6 \end{bmatrix}$$

Now I will demonstrate these two concepts with my code