

RECURSIVE ALGORITHM

--- THUY NGUYEN ---

CS 131

11/30/2021

What is recursive algorithm?

- **Algorithm:** a sequence of steps for solving a problem.
- **Recursive algorithm:** an algorithm that breaks problem into smaller subproblems and applies the same algorithm to solve those subproblems.

Recursive process of painting the wall:



Paint wall:

- Paint the left side:
Paint the upper left
Paint the lower left
- Paint the right side:
Paint the upper right
Paint the lower right

Recursive function

- Recursive function uses an if-else statement:
 - The if branch specifies value of function at the initial value (basis step)
 - The else branch has recursive calls (recursive step)
- Recursive function will call itself repeatedly, minimizing the problem size each time.
- In order to terminate, the instant of problem must eventually be reduced to the initial case for which the solution is known.

Give recursive algorithm for computing $n!$ with n is a non-negative integer:

Procedure factorial (n : non-negative int)

if $n=0$ then return 1 $\longrightarrow 0! = 1$

else return $n * \text{factorial}(n-1)$

{output is $n!$ }

$$\begin{aligned} n! &= n(n-1)! \\ &= n(n-1)(n-2)! \\ &= \dots \end{aligned}$$

$$\begin{aligned} n! &= n(n-1)! \\ (n-1)! &= (n-1)(n-2)! \end{aligned}$$

...

\Rightarrow Factorial function calls itself repeatedly until reaching $\text{factorial}(n-n)$, which is $\text{factorial}(0) = 0! = 1$



Before writing a recursive function, a programmer should determine:

1. Whether the problem have a recursive solution
2. Whether the recursive solution is better than a non-recursive solution

Non-recursive function

```
long factorial(int n) {  
    int fac=1;  
    for (int i = n; i > 1; i--)  
        fac = fac * i;  
    return fac;  
}
```

Recursive function

```
long factorial(int n){  
    if (n == 0)  
        return 1;  
    else  
        return n * factorial(n - 1);  
}
```

=> In the example of computing $n!$, non-recursive function is easier to understand

Give recursive algorithm for computing a^n (a is nonzero real number, n is non-negative integer):

Procedure power (a : nonzero real number, n : non-negative integer)

if $n=0$ then return 1 $\longrightarrow a^0 = 1$ (base case)

else return $a * \text{power}(a, n-1)$ $\longrightarrow a^n = a.a^{(n-1)}$

{output is a^n } \longrightarrow

$$\begin{aligned} a^n &= a.a^{(n-1)} \\ &= a.a.a^{(n-2)} \\ &= a.a.a....a^{(n-n)} \\ &= a.a.a....a^0 \\ &= a.a.a....1 \text{ (recursive step stops at } a^0 = 1, \\ &\text{which is the base case)} \end{aligned}$$

Give recursive algorithm for computing a^n (a is nonzero real number, n is non-negative integer):

- Ex: $a=2, n=4$

$$2^4 = 2.2^3$$

$$= 2.2.2^2$$

$$= 2.2.2.2^1$$

$$= 2.2.2.2.2^0$$

$$= 16$$

Fibonacci sequence (0, 1, 1, 2, 3, 5, 8, 13, 21, 34, etc.) :

1. $a_1 = 0, a_2 = 1$

2. $a_n = a_{n-1} + a_{n-2}$

Computing the term a_n in the Fibonacci sequence:

Procedure ComputeFibonacci ($a_1 = 0$, $a_2 = 1$, n : non-negative integer)

if $n = 1$ then return a_1

else if $n = 2$ then return a_2

else $a_n = \text{ComputeFibonacci}(a_2, a_1 + a_2, n - 1)$

```
#include <iostream>
using namespace std;
```

```
void ComputeFibonacci(int a1, int a2, int n) {
    if (n == 1) { cout << "=> Value of the desired term is " << a1 << endl; }

    else if (n == 2) { cout << "=> Value of the desired term is " << a2 << endl; }

    else {
        cout << a1 << " + " << a2 << " = " << a1 + a2 << endl;
        ComputeFibonacci(a2, a1 + a2, n - 1);
    }
}
```

```
int main() {
    int termN;
    char repeat;
    cout << "This program finds the nth term in the Fibonacci sequence by calculating step-by-step" << endl;

    do {
        termN = 0; //assign initial value for termN to activate the while loop

        while (termN < 1) {
            cout << "\n which term do you want to find? ";
            cin >> termN; //user need to enter the valid termN (>=1)
        }

        cout << "0" << endl << "1" << endl; //first term is 0, second term is 1
        ComputeFibonacci(0, 1, termN);

        repeat = 'o'; //assign initial value for repeat to activate the while loop

        while (toupper(repeat) != 'Y' && toupper(repeat) != 'N') {
            cout << "Do you want to find another term? (Y/N)";
            cin >> repeat;
        }
    } while (toupper(repeat) == 'Y');
    return 0;
}
```



Thank you!

- Thuy Nguyen -