CS131

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Discrete Probability in Blackjack

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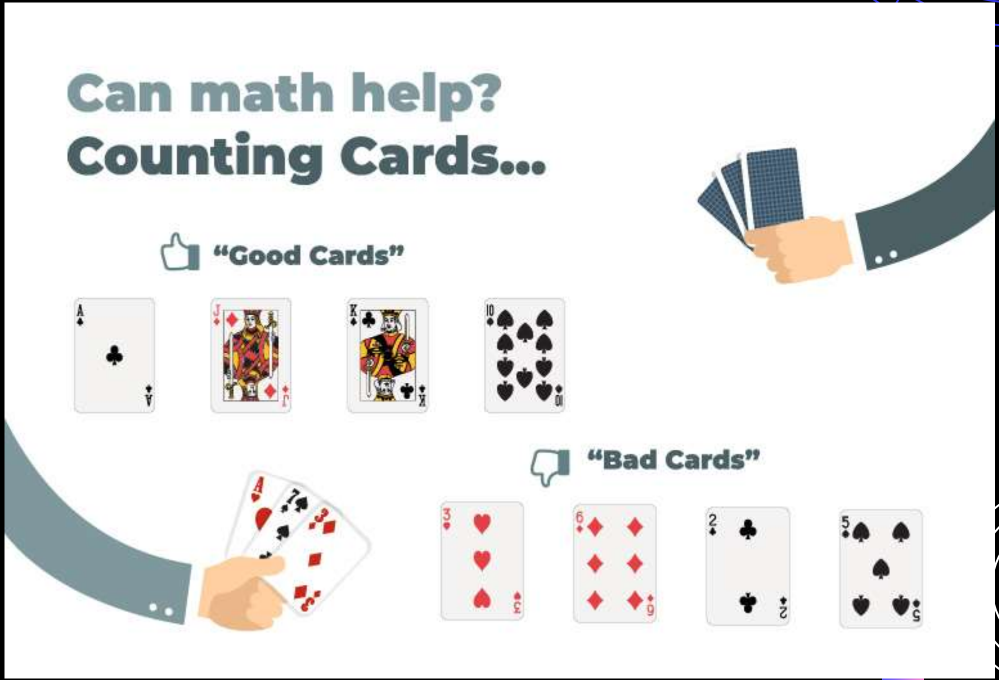
01 Introduction

- The game's outcome can be predicted to some extent only if we **apply the basic strategy correctly** and have the needed skills and discipline.
- However, there is **no guarantee** about the outcome of a particular hand.



02

Basic Concepts-

Permutation & Combination

Definition of Permutation and Combination

Permutation & Combination

A **permutation** of a set of distinct objects is an **ordered** arrangement of these objects. We also are interested in ordered arrangements of some of the elements of a set. An ordered arrangement of r elements of a set is called an **r-permutation.**

If n is a positive integer and r is an integer with $1 \le r \le n$, then there are

$$P(n, r) = n(n-1)(n-2)\cdots(n-r+1)$$

r-permutations of a set with n distinct elements.

An **r-combination** of elements of a set is an **unordered** selection of r elements from the set. Thus, an r-combination is simply a subset of the set with r elements.

The number of r-combinations of a set with n elements, where n is a nonnegative integer and r is an integer with $0 \le r \le n$, equals

$$C(n, r) = \frac{n!}{r!\,(n-r)!}.$$

03

Basic Concepts- Probabilities

Addition, Complementation, and Multiplication Laws

Probability of An Event

If S is a finite nonempty sample space of equally likely outcomes, and E is an event, that is, a subset of S, then the *probability* of E is $p(E) = \frac{|E|}{|S|}$.

Example

- A deck has a **total of 52 cards**. The probability of **drawing a specific card**, let say Queen of Hearts is 1/52 = 0.0192 = 1.92%.
- There are **four Queens**, hence there is a higher probability of pulling out one of them: 4/52 = 0.7369 = 7.369%

Addition/Unions of Events

- The likelihood of one out of two, three, four, etc. outcomes occurring is a result of the **total** of the events' individual probabilities **minus** the likelihood of **all of them happening.**

- Example: The likelihood of **getting a deuce or a card of the clubs suit** is equal to [(4 / 52) + (13 / 52)] – 1 / 52 = 17 / 52 – 1 / 52 = 16 / 52 = 0.3076, or 30.76%.

Complementation

Under the law of complementation, the likelihood of **something not happening** equals 1 minus the likelihood of the same thing happening.

Let E be an event in a sample space S. The probability of the event $\overline{E} = S - E$, the complementary event of E, is given by

$$p(\overline{E}) = 1 - p(E).$$

There are 13 cards of each suit in a single deck. Therefore, the **likelihood of you pulling out any card of the clubs suit** is equal to 13 / 52 x 100 = 0.25, or 25%. Respectively, the **probability against pulling clubs** is 1 – 0.25 = 0.75, or 75%.

Multiplication

For games of dependent trials like blackjack, it dictates that the likelihood of all possible events from a fixed set of outcomes is the result of their **conditional probabilities.**

Let E and F be events with $p(F) > 0$. The *conditional probability* of E given F, denoted by $p(E \mid F)$, is defined as

$$p(E \mid F) = \frac{p(E \cap F)}{p(F)}.$$

The likelihood of an outcome occurring is **influenced** by the outcomes that occurred before it.

Application

Real situations

Drawing a Blackjack

- Blackjack contains **an ace** and **another face card.**
- The number of ways to draw **2 random cards** from a standard deck of **52 cards** are : C(52,2) = 1326 ways.
- There are **4 aces** in a standard deck of 52 cards so the probability that the first card is an ace is 4/52.
- There are **16 face cards** in a standard deck of 52 cards. The probability that the second card is a 10-point card is 16/51.
- The probability of drawing a blackjack with **an ace appeared first** is P = (4/52)*(16/51) = 16/663.
- Multiply the probability value by 2 because we can draw a **face card before** an ace: P = 2*(16/663) = 0.0482 = 4.82%.

Playing with Multidecks

Probability of Blackjack

The probability of gaining a blackjack from **1 deck** is 2*(4/52)*(16/51)=0.0482=4.82%.

The probability of gaining a blackjack from **8 decks** is 2*(32/416)(128/415)=0.0474=4.74%

Decks	Probability
1	4.82%
2	4.78%
3	4.76%
4	4.75%
5	4.75%
6	4.74%
7	4.74%
8	4.74%

Winning-Losing Streaks

We can calculate the probability of losing consecutive rounds. Usually, a losing streak that lasts for more than 4 rounds is rare but it does not mean that it won't happen.

When ignoring the pushes, the probabilities of **winning are 47.4%** and the probabilities of **losing amount to 52.6%.**

The probability of winning **2 hands in a row** is $0.474^2=0.2246=22.46\%$. On the contrary, the chance of losing 2 hands in a row equals $0.526^2=0.2766=27.66\%$

Length of **Streak**	Probability of **Winning**	Probability of **Losing**
2	22.47%	27.66%
3	10.65%	14.55%
4	5.05%	7.65%
5	2.39%	4.02%
6	1.13%	2.12%

Winning-Losing Streaks

Dealer's Up Card

Dealer's Up Card	*Chance of getting bust*
2	35.30%
3	37.56%
4	40.28%
5	42.89%
6	42.08%
7	25.99%
8	23.86%
9	23.34%
10	21.43%
A	11.65%

We can look at the **probability that a dealer will bust** based on her up card to make our decisions.

- There's a **big jump** in the probability of a dealer busting between the numbers **six and seven**.
- Players generally **stand more** often when the dealer has a six or lower showing.

- Probability skill **guides** the players through their gambling decisions.
- Knowing probability can help the players **estimate the odds of winning** a particular event.
- Based on the mathematical properties of probability, odds of winning, and size of bet, player can participate **how risky a bet is** and **percentage of winning that bet**.
- Lower your **loss.**

Final Notes

Thanks!

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