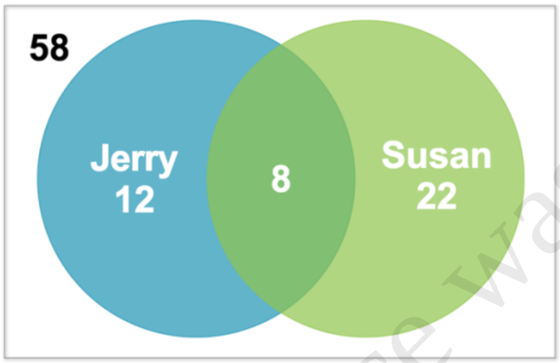
CS-513-A Homework 1 - Probability

1.1
Jerry and Susan have a joint bank account.
Jerry goes to the bank 20% of the days.
Susan goes there 30% of the days.
Together they are at the bank 8% of the days.

We can create their Venn diagram as follows:



Let 'J' denote the set for Jerry and 'S' denote the set for Susan.

a. Susan was at the bank last Monday. What's the probability that Jerry was there too?

$$P(J|S) = \frac{P(J \cap S)}{P(S)} = \frac{8}{30} = 26.67\%$$

b. Last Friday, Susan wasn't at the bank. What's the probability that Jerry was there?

$$P(J|S') = \frac{P(J - S)}{P(S')} = \frac{12}{70} = 17.14\%$$

c. Last Wednesday at least one of them was at the bank. What is the probability that both of them were there?

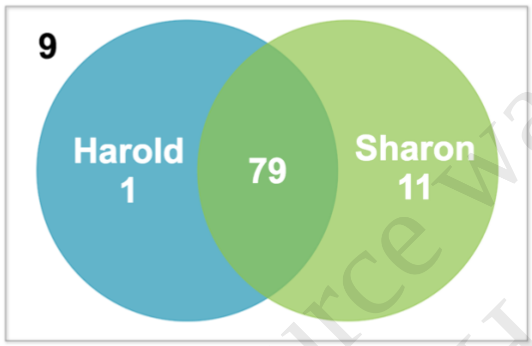
$$\frac{P(J \cap S)}{P(J)\ \cup\ P(S)} = \frac{8}{42} = 19.05\%$$

1.2
Harold and Sharon are studying for a test.
Harold's chances of getting a "B" are 80%.
Sharon's chances of getting a "B" are 90%.
The probability of at least one of them getting a "B" is 91%.

Let ‘H’ denote the set for Harold’s chances of getting a “B” and ‘S’ denote the set for Sharon’s chances of getting a “B”.
P(H) = 80%
P(S) = 90%
P(H ∪ S) = 91%

Now,
P(H ∪ S) = P(H) + P(S) - P(H ∩ S)
P(H ∩ S) = P(H) + P(S) - P(H ∪ S) = 80 + 90 – 91 = 79%

We can create their Venn diagram as follows:



a. **What is the probability that only Harold gets a “B”?**

$$P(H - S) = P(H) - P(H \cap S) = 80 - 79 = 1\%$$

b. **What is the probability that only Sharon gets a “B”?**

$$P(S - H) = P(S) - P(H \cap S) = 90 - 79 = 11\%$$

c. **What is the probability that both won’t get a “B”?**

$$P(H \cup S)' = 100 - P(H \cup S) = 100 - 91 = 9\%$$

1.3
Jerry and Susan have a joint bank account.
Jerry goes to the bank 20% of the days.
Susan goes there 30% of the days.
Together they are at the bank 8% of the days.
Are the events “Jerry is at the bank” and “Susan is at the bank” independent?

Let ‘J’ denote the set for Jerry and ‘S’ denote the set for Susan.
If the two events, ‘J’ and ‘S’ are independent then, $P(J|S) = P(J)$ and $P(S|J) = P(S)$
Thus,

$$P(J|S) = \frac{P(J \cap S)}{P(S)} \rightarrow P(J \cap S) = P(J|S)P(S) = P(J)P(S)$$

This implies that if two events ‘J’ and ‘S’ are independent then $P(J \cap S) = P(J)P(S)$

Using this,
LHS = $P(J \cap S) = 8\%$
RHS = $P(J)P(S)$ = 20% * 30% = 6%
Since, LHS ≠ RHS, we can say that the events “Jerry is at the bank” and “Susan is at the bank” are not independent.

1.4
You roll 2 dice.

We can draw the probability table of rolling 2 dice as:

2	3	4	5	6	7
3	4	5	6	7	8
4	5	6	7	8	9
5	6	7	8	9	10
6	7	8	9	10	11
7	8	9	10	11	12

a. **Are the events “the sum is 6” and “the second die shows 5” independent?**

Let, event ‘A’ correspond to “sum is 6” and event ‘B’ correspond to “second die shows 5” events.

$P(A) = \frac{5}{36}$ and $P(B) = \frac{6}{36} = \frac{1}{6}$ $, P(A)P(B) = \frac{5}{36} * \frac{1}{6} = \frac{5}{216}$

$P(A \cap B) = \frac{1}{36}$

Thus, $P(A \cap B) \neq P(A)P(B)$ which means that they are dependent events.

b. **Are the events “the sum is 7” and “the first die shows 5” independent?**

Let, event ‘A’ correspond to “sum is 7” and event ‘B’ correspond to “first die shows 5” events.

$P(A) = \frac{6}{36} = \frac{1}{6}$ and $P(B) = \frac{6}{36} = \frac{1}{6}$ $, P(A)P(B) = \frac{1}{6} * \frac{1}{6} = \frac{1}{36}$

$P(A \cap B) = \frac{1}{36}$

Thus, $P(A \cap B) = P(A)P(B)$ which means that they are independent events.

1.5
An oil company is considering drilling in either TX, AK and NJ. The company may operate in only one state. There is 60% chance the company will choose TX and 10% chance – NJ. There is 30% chance of finding oil in TX, 20% - in AK, and 10% - in NJ.

Since drilling in either TX, AK and NJ are disjoint sets and composite to the universe set here.
$P(TX) = 60\%, P(AK) = 30\%, P(NJ) = 10\%$
$P(Oil|TX) = 30\%, P(Oil|AK) = 20\%, P(Oil|NJ) = 10\%$

1. **What's the probability of finding oil?**

   Using the law of total probability for disjoint sets
   $P(Oil) = P(Oil|TX)P(TX) + P(Oil|AK)P(AK) + P(Oil|NJ)P(NJ)$
   $P(Oil) = 30\% * 60\% + 20\% * 30\% + 10\% * 10\%$
   $P(Oil) = 18\% + 6\% + 1\%$
   $P(Oil) = 25\%$

2. **The company decided to drill and found oil. What is the probability that they drilled in TX?**

   Using conditional probability,
   $P(TX|Oil)P(Oil) = P(Oil|TX)P(TX)$
   $$P(TX|Oil) = \frac{P(Oil|TX)P(TX)}{P(Oil)} = \frac{30\% * 60\%}{25\%} = \frac{18\%}{25\%} = 72\%$$

1.6
The following shows the survival status of individual passengers on the Titanic. Use this information to answer the questions here.

Survived		Cabin				
		1st	2nd	3rd	Crew	Sub Total
Age	Adult	197	94	151	212	654
	Child	6	24	27	-	57
	Sub Total	203	118	178	212	711

Not Survived		Cabin				
		1st	2nd	3rd	Crew	Sub Total
Age	Adult	122	167	476	673	1438
	Child	-	-	52	-	52
	Sub Total	122	167	528	673	1490

Total		Cabin				
		1st	2nd	3rd	Crew	Sub Total
Age	Adult	319	261	627	885	2092
	Child	6	24	79	-	109

	Sub Total	325	285	706	885	2201

There can be 2 scenarios here:

a. If crew members are included in passengers for a flight.
b. If crew members are not included in passengers for a flight.

Dividing my answer set into 2 parts:

a. <u>If crew members are included in 'passengers' for a flight.</u>

1. **What is the probability that a passenger did not survive?**

$$P(NS) = \frac{1490}{2201} = 67.69\%$$

2. **What is the probability that a passenger was staying in the first class?**

$$P(1st) = \frac{325}{2201} = 14.76\%$$

3. **Given that a passenger survived, what is the probability that the passenger was staying in the first class?**

$$P(1st|S) = \frac{P(1st \cap S)}{P(S)} = \frac{203}{711} = 28.55\%$$

4. **Are survival and staying in the first class independent?**

$$P(S) = \frac{711}{2201} \; and \; P(1st) = \frac{325}{2201} \quad , P(S)P(1st) = \frac{711}{2201} * \frac{325}{2201} = 4.77\%$$

$$P(S \cap \text{ 1st}) = \frac{203}{2201} = 9.22\%$$

Thus, $P(S \cap \text{ 1st}) \neq P(S)P(1st)$ which means that they are not independent events.

5. **Given that a passenger survived, what is the probability that the passenger was staying in the first class and the passenger was a child?**

$$P(1st \cap C|S) = \frac{P(1st \cap C \cap S)}{P(S)} = \frac{6}{711} = 0.84\%$$

6. **Given that a passenger survived, what is the probability that the passenger was an adult?**

$$P(A|S) = \frac{P(A \cap S)}{P(S)} = \frac{654}{711} = 91.98\%$$

7. **Given that a passenger survived, are age and staying in the first class independent?**

$$P(A \cup C|S) = \frac{711}{711} \; and \; P(1st|S) = \frac{203}{711}$$

$$P(A \cup C|S)P(1st|S) = \frac{711}{711} * \frac{203}{711} = 28.55\%$$

$$P\big((A \cup C|S) \cap (1st|S)\big) = \frac{P\big(\big((A \cup C) \cap\ 1st\big)|S\big)}{P(S)} = \frac{203}{711} = 28.55\%$$

Thus, $P\big((A \cup C|S) \cap (1st|S)\big) = P(A \cup C|S)P(1st|S)$, the two events are independent.

b. ***<u>If crew members are not included in 'passengers' for a flight.</u>***

1. **What is the probability that a passenger did not survive?**

$$P(NS) = \frac{1490 - 673}{2201 - 885} = \frac{817}{1316} = 62.08\%$$

2. **What is the probability that a passenger was staying in the first class?**

$$P(1st) = \frac{325}{2201 - 885} = \frac{325}{1316} = 24.69\%$$

3. **Given that a passenger survived, what is the probability that the passenger was staying in the first class?**

$$P(1st|S) = \frac{P(1st \cap S)}{P(S)} = \frac{203}{711 - 212} = \frac{203}{499} = 40.68\%$$

4. **Are survival and staying in the first class independent?**

$P(S) = \dfrac{711}{2201}$ $and\ P(1st) = \dfrac{325}{2201}$ $, P(S)P(1st) = \dfrac{711}{2201} * \dfrac{325}{2201} = 4.77\%$

$P(S \cap\ \text{1st}) = \dfrac{203}{2201} = 9.22\%$

Thus, $P(S \cap\ \text{1st}) \neq P(S)P(1st)$ which means that they are not independent events.

5. **Given that a passenger survived, what is the probability that the passenger was staying in the first class and the passenger was a child?**

$$P(1st\ \cap\ C|S) = \frac{P(1st \cap\ C \cap\ S)}{P(S)} = \frac{6}{711 - 212} = \frac{6}{499} = 1.20\%$$

6. **Given that a passenger survived, what is the probability that the passenger was an adult?**

$$P(A|S) = \frac{P(A \cap\ S)}{P(S)} = \frac{654 - 212}{711 - 212} = \frac{442}{499} = 88.58\%$$

7. **Given that a passenger survived, are age and staying in the first class independent?**

$P(A \cup C|S) = \dfrac{711 - 212}{711 - 212} = 1\ and\ P(1st|S) = \dfrac{203}{711 - 212} = \dfrac{203}{499}$

$P(A \cup C|S)P(1st|S) = 1 * \dfrac{203}{499} = 40.68\%$

$$P\big((A \cup C|S) \cap (1st|S)\big) = \frac{P\big(\big((A \cup C) \cap\ 1st\big)|S\big)}{P(S)} = \frac{203}{711 - 212} = \frac{203}{499} = 40.68\%$$

Thus, $P((A \cup C|S) \cap (1st|S)) = P(A \cup C|S)P(1st|S)$, the two events are independent.

1.7

Replace the missing values below (?), assuming independence between age and cabin class.

Total:

Total		Cabin				
		1st	2nd	3rd	Crew	Sub Total
Age	Adult	308.9050432	270.8859609	671.0368015	841.1721945	2092
	Child	16.09495684	14.11403907	34.96319855	43.82780554	109
	Sub Total	325	285	706	885	2201

This is by using the relation between joint and marginal probability, but since an individual cannot be represented as fractions, rounding this up maintain the mathematical relations between the marginal probabilities given.

Total		Cabin				
		1st	2nd	3rd	Crew	Sub Total
Age	Adult	309	271	671	841	2092
	Child	16	14	35	44	109
	Sub Total	325	285	706	885	2201

Replace the missing values below (?), assuming independence between age and cabin class given survival status (conditional independence).

Survived:

Survived		Cabin				
		1st	2nd	3rd	Crew	Sub Total
Age	Adult	186.7257384	108.5400844	163.7299578	195.0042194	654
	Child	16.2742616	9.459915612	14.27004219	16.99578059	57
	Sub Total	203	118	178	212	711

This is by using the relation between joint and marginal probability, but since an individual cannot be represented as fractions, rounding this up maintain the mathematical relations between the marginal probabilities given.

Survived		Cabin				
		1st	2nd	3rd	Crew	Sub Total
Age	Adult	187	108	164	195	654

	Child	16	10	14	17	57
	Sub Total	203	118	178	212	711

Not Survived:

Not Survived		Cabin				
		1st	2nd	3rd	Crew	Sub Total
Age	Adult	117.7422819	161.1718121	509.5731544	649.5127517	1438
	Child	4.257718121	5.828187919	18.42684564	23.48724832	52
	Sub Total	122	167	528	673	1490

This is by using the relation between joint and marginal probability, but since an individual cannot be represented as fractions, rounding this up maintain the mathematical relations between the marginal probabilities given.

Not Survived		Cabin				
		1st	2nd	3rd	Crew	Sub Total
Age	Adult	118	161	510	649	1438
	Child	4	6	18	24	52
	Sub Total	122	167	528	673	1490