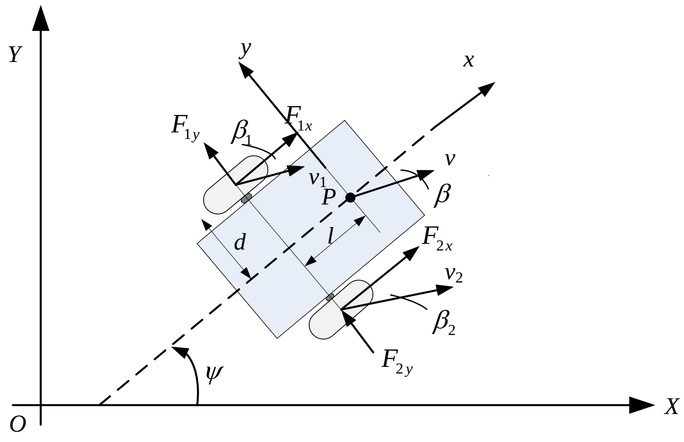

Parms.	Specific (Constants)	Parms.	Specific (Variables)
m	mass	v	velocity
I_z	Inertia around z	β	Sideslip angle of rover at center of mass
d	Distance between wheel and centerline	ψ	Yaw angle of rover
l	Distance between wheel and center of mass	F_{1x}, F_{2x}	Tractive force of left and rear wheel
α	Slope angle	F_{1y}, F_{2y}	Lateral force of left and rear wheel
c_1, c_2	Cornering Stiffness of left and rear wheel	β_1, β_2	Sideslip angel of left and rear wheel

Dynamic model,

$$\begin{cases} m\left(\dot{v}_x + v_y\dot{\psi}\right) + mg\sin\alpha\sin\psi = F_{1x} + F_{2x} \\ m\left(v_x\dot{\psi} - \dot{v}_y\right) + mg\sin\alpha\cos\psi = F_{1y} + F_{2y} \\ I_Z\ddot{\psi} = d\left(F_{2x} - F_{1x}\right) - l\left(F_{1y} + F_{2y}\right) \end{cases} \tag{1}$$

Kinematic model,

$$\begin{cases} v\cos\beta = v_1\cos\beta_1 + d\dot{\psi} = v_2\cos\beta_2 - d\dot{\psi} \\ v\sin\beta = v_1\sin\beta_1 - l\dot{\psi} = v_2\sin\beta_2 - l\dot{\psi} \end{cases} \tag{2}$$

and,

$$\begin{cases} v_x = v\cos\beta = v \\ v_y = v\sin\beta = v\beta \\ \dot{v}_x = \dot{v}\cos\beta - v\dot{\beta}\sin\beta = \dot{v} - v\dot{\beta}\beta \\ \dot{v}_y = \dot{v}\sin\beta + v\dot{\beta}\cos\beta = \dot{v}\beta + v\dot{\beta} \end{cases} \tag{3}$$

The sideslip angle of each wheel using Eq. (2),

$$\begin{cases} \beta_1 = \dfrac{v\sin\beta + l\dot{\psi}}{v\cos\beta - d\dot{\psi}} \approx \dfrac{v\beta + l\dot{\psi}}{v} = \beta + \dfrac{l}{v}\dot{\psi} \\ \beta_2 = \dfrac{v\sin\beta + l\dot{\psi}}{v\cos\beta + d\dot{\psi}} \approx \dfrac{v\beta + l\dot{\psi}}{v} = \beta + \dfrac{l}{v}\dot{\psi} \end{cases} \qquad (\beta_1 \approx \beta_2) \tag{4}$$

The lateral force of wheel,

$$\begin{cases} F_{1y} = c_1\left(\beta + \dfrac{l}{v}\dot{\psi}\right) \\ F_{2y} = c_2\left(\beta + \dfrac{l}{v}\dot{\psi}\right) \end{cases} \tag{5}$$

Final dynamic system using (1), (3), and (5),

$$\begin{cases} m\left(\dot{v} - v\dot{\beta}\beta + v\beta\dot{\psi}\right) + mg\sin\alpha\sin\psi = F_{1x} + F_{2x} \\ m\left(v\dot{\psi} - \dot{v}\beta - v\dot{\beta}\right) + mg\sin\alpha\cos\psi = \left(c_1 + c_2\right)\left(\beta + \dfrac{l}{v}\dot{\psi}\right) \\ I_Z\ddot{\psi} = d\left(F_{2x} - F_{1x}\right) - l\left(c_1 + c_2\right)\left(\beta + \dfrac{l}{v}\dot{\psi}\right) \end{cases} \tag{6}$$

The input is defined as,

$$u \triangleq \begin{bmatrix} F_{1x} \\ F_{2x} \end{bmatrix}$$

The state is defines as,

$$\xi \triangleq \begin{bmatrix} \beta \\ \psi \\ \dot{\psi} \end{bmatrix}$$

The output is defined as,

$$\dot{\eta} = \begin{bmatrix} \dot{X} \\ \dot{Y} \end{bmatrix} = \begin{bmatrix} v\cos\left(\psi - \beta\right) \\ v\sin\left(\psi - \beta\right) \end{bmatrix}$$

Values

m	24 kg	d	0.18 m
I_Z	11.89 kg m^2	l	0.16 m
c_1, c_2	513.6 N/rad (1000~200)	α	15°

With considering saturation of tractive force, and $\left|F_{ix}\right| < F_M$,

$$F_{ix} = \begin{cases} -F_M & F_{ix} < -F_M \\ F_{ix} & -F_M < F_{ix} < F_M \\ F_M & F_{ix} < F_M \end{cases}$$

Performance index:

Reaching time: t

Tracking error: $\left\| \left(X - X_d\right)^2 + \left(Y - Y_d\right)^2 \right\|_2$

Energy consumption: $\int_0^t \left(F_{1x}\left(\tau\right) + F_{2x}\left(\tau\right)\right) \mathrm{d}\tau$

Eq. (6) can be transformed as,

$$\begin{cases} \dot{v} = \frac{1}{m}\left(F_{1x} + F_{2x}\right) + \beta g \sin\alpha \cos\psi - g \sin\alpha \sin\psi - \frac{\left(c_1 + c_2\right)}{m}\beta\left(\beta + \frac{l}{v}\dot{\psi}\right) \\ \dot{\beta} = -\frac{\left(c_1 + c_2\right)}{mv}\beta + \left(1 - \frac{\left(c_1 + c_2\right)l}{mv^2}\right)\dot{\psi} + \frac{1}{v} g \sin\alpha \cos\psi + \frac{1}{v}\beta g \sin\alpha \sin\psi - \frac{\beta}{mv}\left(F_{1x} + F_{2x}\right) \\ \ddot{\psi} = -\frac{l\left(c_1 + c_2\right)}{I_Z}\beta - \frac{l^2\left(c_1 + c_2\right)}{vI_Z}\dot{\psi} + \frac{d}{I_Z}\left(F_{2x} - F_{1x}\right) \end{cases} \quad (7)$$