Name:

Group members:

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ARE 336 – Problem set 3 – Emissions regulation

Due **November 7, 2025**

This problem set has 2 questions, for a total of 41 points (with the possibility to earn up to 2 bonus points).

1. Consider the case of Tar Heel Electric (THE), an energy plant which is situated across a valley from Wolfpack Winery and Vineyard (WWV). THE collects revenue by generating electricity through a process which pollutes the local air. Local air pollution decreases the enjoyment of visitors to WWV and can also affect the taste of its grapes.

   THE's marginal cost to produce one megawatt-hour (MWh) of electricity is defined by $MC_{THE}(q) = q$, where q is the number of MWhs produced (it costs \$1 for THE to produce 1 MWh). A team of scientisits studied THE's production process and determined that each MWh of electricity produced by THE creates a half-ton of SO_2 emissions that are released into the air. Thus, THE's emissions production can be modeled as $q(e) = \frac{1}{2}e$, where e is the number of tons of SO_2 emisisons.

   While THE's emissions are not good for our health (or WWV's profits), having electricity is great for our quality of life! Suppose that the marginal benefit of electricity is defined by $MB(q) = 12 - 2q$ where q is the number of MWhs. In this problem, assume that THE is the only supplier of electricity, so every q needs to be provided by THE. This marginal benefit curve is effectively the *demand for electricity* faced by THE.

   Because SO_2 is a "localized" pollutant, its effects are largely felt around THE's facility. WWV is the only entity nearby, so they are the only entity affected by the SO_2 emissions. Hence, the lost profits of WWV represent *the only* externality here.

   We will go through Coasian bargaining in the case of constant externality and proportional externality.

   (a) [1 point] State THE's marginal cost per unit of SO_2 emission?
   (**Hint:** That is, state $MC_{THE}(e)$.)

   (b) [1 point] State the public's marginal benefit per unit of SO_2 emission?
   (**Hint:** Recall that THE is the only possible producer of electricity in the market.)

   (c) [1 point] What does it mean (in words) to have a marginal benefit of a ton of SO_2 emissions??

   (d) Suppose that every ton of SO_2 emissions decreases WWV's profits by a constant $d(e) = 3$.

      i. [2 points] Complete the following table:

e	$d(e)$	$MC_{THE}(e)$	$MSC(e)$	$MB(e)$
0				
2				
4				
6				
8				
10				
12				

ii. [1 point] How many tons of SO_2 emissions will occur if THE only pays attention to its *private* marginal cost. Call this q_0.

iii. [1 point] What is the *socially efficient* quantity of SO_2 emissions (in tons)? Call this q_1.

iv. [3 points] Draw a graph with SO_2 emissions (in tons) on the horizontal axis with dollars on the vertical axis. Your graph should have the axes labeled along with MPC_{THE}, MSC, MB. Label q_0 and q_1 using their *numeric* value.
In a separate graph, draw the marginal external damages function ($d(e)$) with emissions on the horizontal axis and dollars on the vertical axis. Again, label q_0 and q_1 using their *numeric* value.

v. [2 points] At q_0, what are the *total lost profits* to WWV? At q_1, what are the *total lost profits* to WWV?
(**Hint:** You may think that the table you completed in part d.i is sufficient to answer this. However, this table increases in units of 2, so it does not completely cover damages for all emissions. You should refer to your graph in part d.iv—and use geometry—to calculate this.)

vi. [1 point] How much profit does WWV *get back* when THE reduces its emissions from q_0 to q_1?
(**Hint:** Your answer to part d.v gives you exactly what you need to answer this.)

vii. [1 point] Given your answer to part d.vi, how much would WWV be willing to pay THE in order for THE to produce q_1 tons of SO_2 emissions instead of q_0 tons.

(e) Suppose now that every ton of SO_2 emissions decreases WWV's profits at an increasing rate $d(e) = \frac{1}{2}e$.

i. [2 points] Complete the following table:

e	$d(e)$	$MC_{THE}(e)$	$MSC_{THE}(e)$	$MB(e)$
0				
2				
4				
6				
8				
10				
12				

ii. [1 point] How many tons of SO_2 emissions will occur if THE only pays attention to its *private* marginal cost? Call this q_0.

iii. [1 point] What is the *socially efficient* quantity of SO_2 emissions (in tons)? Call this q_1.

iv. [3 points] Draw a graph with SO_2 emissions (in tons) on the horizontal axis with dollars

on the vertical axis. Your graph should have the axes labeled, MPC_{THE}, MSC, MB. Label q_0 and q_1 using their *numeric* value.

In a separate graph, draw the marginal external damages function ($d(e)$) with emissions on the horizontal axis and dollars on the vertical axis. Again, label q_0 and q_1 using their *numeric* value.

v. [2 points] At q_0, what are the *total lost profits* to WWV? At q_1, what are the *total lost profits* to WWV?

(**Hint:** You may think that the table you completed in part e.i is sufficient to answer this. However, this table increases in units of 2, so it does not completely cover damages for all emissions. You should refer to your graph in part e.iv to calculate this.)

vi. [1 point] How much profit does WWV *get back* when THE reduces its emissions from q_0 to q_1?

(**Hint:** Your answer to part e.v gives you exactly what you need to answer this.)

vii. [1 point] Given your answer to part e.vi, how much would WWV be willing to pay THE in order for THE to produce q_1 tons of SO_2 emissions instead of q_0 tons.

2. Consider two firms, Apple (A) and Berkshire (B) with the following abatement costs:

$$C_A(a_A) = a_A^2$$
$$C_B(a_B) = 2a_B^2$$

where a_A represents the level of *abatement* of Apple and a_B represents the level of *abatement* of Berkshire. Each firm has a corresponding marginal abatement cost function:

$$MAC_A(a_A) = 2a_A$$
$$MAC_B(a_B) = 4a_B$$

Recall that a firm's level of abatement is defined relative to their BAU emissions (i.e., $a = \bar{e} - e$ where $\bar{e}$ is BAU emissions).

Apple's BAU emissions is $\bar{e}_A = 25$ while Berkshire's BAU emissions is $\bar{e}_B = 10$.

(a) [2 points] Derive the total abatement cost and marginal abatement costs for the firms as functions of *emissions*. For example, Apple's total abatement cost as a function of emissions is $C_A(e_A)$.
(**Hint:** You need to substitute something for a_A and a_B.)

(b) [3 points] The regulator sets the goal of allowing 15 tons of GHG emissions. She first tries a command and control approach where she gives each firm un-tradeable pollution permits—each firm is allowed to emit one ton of GHG for each permit.
Berkshire is "grandfathered" an amount of permits equal to its BAU emissions and gives, and Apple is given the remaining permits. State the number of permits Berkshire granted using the definition of grandfathering we discussed in class. What is the total cost of abatement in the market?

(c) The regulator changes her mind and wants to try using cap-and-trade to reach her 15 tons goal.
First she splits the permits evenly between the two firms so that Apple gets 7.5 permits and Berkshire gets 7.5 permits.

i. [1 point] What would the total abatement cost be at the initial permit allocation (i.e., before any trading occurs)?

ii. [3 points] Complete the following statements. Your statements should reflect the state of the world *after* the regulator gives them their allocated permits and *before* any trading occurs.
- Apple will buy a permit if it is less than _____.
- Berkshire will sell a permit at any price greater than _____.
- Given this, _____ will purchase permits from _____.

iii. [2 points] Using the marginal abatement cost curves you calculated in part a, derive a "demand for permits" for each firm that's a function of price. Which firm is more price sensitive?
(**Hint:** This is similar to our trick for turning an inverse demand function into a demand function.)
(**Hint:** The more price sensitive firm has a steeper slope.)

iv. [3 points] Use the market clearing condition to determine the price that satisfies the equimarginal principle. Call this p^*.
Using the demand functions you found in part iii and the p^* you just found, determine the amount of emissions both firms will emit. Call these e_A^* and e_B^*.
Draw a graph with emissions on the horizontal axis and price on the vertical axis. Show $e_A(p)$, $e_B(p)$, p^*, e_A^*, and e_B^* on your graph—use the numeric values for all of these.
(**Hint:** Round any numbers to two decimal points.)

v. [1 bonus point] Represent the aggregate demand on the graph.

vi. [2 points] Finally, calculate the total abatement cost at e_A^* and e_B^*. Ignore the cost of purchasing permits. Is this abatement cost bigger or smaller than the costs you calculated in parts b and c.i?

vii. [1 bonus point] Explain why we can safely ignore the cost of purchasing permits when calculating total abatement cost.