Name:

Group members: Last updated: 09/17/2025

ARE 336 – Problem set 2 – Externalities and public/common goods

Due **October 3, 2025**

This problem set has 4 questions, for a total of 26 points (with the possibility to earn up to 2 bonus points).

1. Many outdoor recreation activities take place on private land with the owner's approval. Organizations like the Carolina Climbers Coalition (CCC) work with landowners to keep these avenues of communication open to benefit us all. But what happens when a landowner doesn't want climbers on her land? Sometimes, like the Maibauer bouldering area, the CCC will raise funds and purchase this land.

   This sounds a lot like Coasian bargaining! This raises the question: How did the CCC know that this was a welfare-enhancing purchase? In this question, we will go through a stylized model of the CCC's decision process.

   Suppose that the owner of the Maibauer area has built a house on the land that she rents for passive income. This rental revenue reliably earns hear \$6,000 each year, but she needs to spend \$1,000 in upkeep. From the owner's perspective, as long as the property is earning a profit, she'll keep it; she has no familial ties to the land and no value for the land outside of its profitability. Assuming a discount rate of 5%, the net present value of the the land is

   $$NPV = \sum_{t=0}^{\infty} \frac{1}{(1.05)^t} \cdot \$5,000 = \$105,000.$$

   This means that she is *just as happy* keeping the land as she is getting a \$105,000 cash payment right now.

   Now consider the CCC. They want to make an offer to the land owner, but they need to raise revenue and they only want to do this if they know that it will provide a net benefit to climbers. For simplicity, assume there are only two types of relevant climbers: locals and out-of-towners. Naturally, the locals stand to benefit much more than the out-of-towners if the CCC buys the land, but the out-of-towners will still benefit. Suppose there are $N_L = 2,000$ local climbers and $N_O = 4,000$ out-of-town climbers.

   (a) [1 point] Normally we talk about "marginal" benefits, but this time we are just talking about benefits. Why is this?

   (b) [2 points] Suppose that everybody knows exactly how good the bouldering is on the Maibauer property. Under this assumption, locals are willing to pay \$50 and out-of-towners are willing to pay \$10 to secure the land. If everybody made a donation to the CCC of an amount equal to their willingness to pay for access to the Maibauer boulders, how much revenue could the CCC collect? Is this enough to fund the purchase of the Maibauer land?

   (c) Now suppose that the CCC isn't allowed to see the land before purchase and the quality of the bouldering (either "great" or "bad") is not known. Formally, suppose

that the boulders are "great" with a 50% probability; hence, they are "bad" with a 50% probability. Suppose that, if the boulders are great, the locals value access to them at \$50 and the out-of-towners value the access at \$10. However, if the boulders are bad, the locals value having access to the boulders at \$25 and the out-of-towners value the access at \$1.

i. [1 point] Complete the following payoff matrix:

Boulder quality	Climbers: Locals	Climbers: Out of towners
Good		
Bad		

(**Hint:** The payoff matrix represents the payoffs in the different states of the world. There is no expected values or probability in this.)

ii. [1 point] What is the expected benefit to each type of climber of purchasing the property?

(**Hint:** A random variable X which can take on the value x_1 with probability p and can take on the value x_2 with probability $1-p$ has an expected value $\mathbb{E}[X] = p \cdot x_1 + (1-p) \cdot x_2$.

iii. [1 point] Assume that each climber makes a contribution to CCC equal to his expected benefit. Fill out the following payoff matrix:

Boulder quality	Climbers: Locals	Climbers: Out of towners
Good		
Bad		

What do you notice about the payoffs?

(**Hint:** Fill out the "net benefits" in the cells, assuming that the true quality of the boulders is labeled in the rows.)

iv. [1 point] Suppose that the CCC has to collect its funding before it can make a move to purchase the land. Thus everybody makes their contribution based on their *expected benefit* of having access to the land. How much revenue will the CCC raise? Is this enough to purchase the Maibauer property?

Question 1: 7 points

2. Mimi-Radio is a public radio station broadcasting to the Triangle. It is freely broadcast on public FM airwaves. Assume that there are two types of consumers: Big Fans (BF) and Moderate Fans (MF). Their marginal benefit curves are as follows:

$$MB_{BF} = 10 - \frac{1}{2}q$$
$$MB_{MF} = 10 - \frac{3}{2}q$$

where q is the hours of programming listened to. The marginal cost of providing programming is a constant \$10 per hour.

For simplicity, assume that the relative size of the two groups are normalized to be 1.

(a) [1 point] Is Mimi-Radio a public good? Explain.

(b) [2 points] Draw the marginal benefit curves of both types of consumers and the marginal cost of radio provision on a graph with "hours" on the horizontal axis. Do a *vertical summation* of the two marginal benefit curves to create the social marginal benefit curve.

Assume that the two types of consumers have to supply all of their own Mimi-Radio (i.e., they have to pay the marginal cost of each q). How many hours of programming would the Big Fans, acting in their own self-interest, provide (if they had to purchase hours of Mimi-Radio)? What about the Moderate Fans?

(c) [2 points] What is the *efficient quantity* of radio programming?

(d) [1 point] Without intervention, why might we expect the provision of radio to be less than socially optimal?

Question 2: 6 points

3. Mountaineer Village (MV) is an apartment complex in Boone, North Carolina with 3,000 single-occupant units (each fully rented). MV is thinking of installing a swimming pool. If it does, it will add an additional charge to each unit's rent. Building and operating the pool will cost MV $5,000 per day.

   The occupants of the units have different valuations of the pool. These valuations represent the daily rent increase they are willing to accept for the pool. The breakdown of their willingness-to-pay is as follows:

No. individuals	WTP per day
1,000	$3
1,000	$2
1,000	$1

   Suppose also that the intended pool is large enough so that whatever number of occupants come on any day will not affect what people are willing to pay for the pool (i.e., there is no congestion externality). Also assume that only MV occupants will be allowed in the pool (i.e., no outsiders can come).

   (a) [1 point] Is this a situation of rivalry or non-rivalry?
   (**Hint:** Focus only on this model, not any prior expectations you may have about congestion.)

   (b) [1 point] Is this a situation of excludability or non-excludability?

   (c) [1 point] Would building the pool be an efficient use of resources?
   (**Hint:** Remember that "efficiency" relies on the benefits and costs of something, not the actual funding scheme.)

   (d) Consider four possible prices for admission to the pool: $3, $2, $1, $0.
   (**Hint:** These are prices to enter the pool – the occupants pay this everytime they visit the pool.)

      i. [1 point] What number of units will pay at each of these prices? Plot these points on a graph with "visitors" on the horizontal axis and "price" on the vertical axis.
      (**Hint:** These are individual pairs of price and quantity, do not connect them as a demand function.)

      ii. [1 point] Will any of these prices cover the cost of providing the pool?

      iii. [1 point] Which of these prices will lead to an efficient allocation of resources?

   (e) [1 point] Is there any pricing scheme for admission to this pool that would both cover the pool's cost *and* achieve the efficient number of visitors?
   (**Hint:** There are two possible answers here. Either one is sufficient as long as your answer is well-reasoned.)

   (f) [2 points] Suppose that this pool has a capacity of only 2,000 individuals per day. If more than 2,000 individuals come, the willingness to pay of each individual falls to $0.50 per day. Now what is the efficient pricing scheme for the pool?

   (g) [1 bonus point] Suppose that MV can add an additional charge to its residents, but it has to charge everyone the same amount. Would a uniform rent increase of $1.67 and making the pool open to all renters be an efficient outcome?

(h) [1 bonus point] Suppose again that MV can add an additional charge to its residents, but it has to charge each unit the same amount. Would all renters be better off under a uniform rent increase of \$1.67, relative to the scenario where there is no pool at all?

Question 3: 9 points

4. Assume that the constant marginal cost of providing a public pool is \$2. The enjoyment of pool users falls with more people at the pool such that the utility of individual i of visiting the pool is

$$u_i = 11 - Q$$

where Q is the number of people in the pool.

   (a) [2 points] How many people, acting in their own self-interest, will enter the pool? What is the total social net benefit at this number?
   (**Hint:** This is the "open access" number.)

   (b) [2 points] What would be the socially optimal number of people in the pool? What is the social net benefit at this number?
   (**Hint:** This is the "efficient" number.)

Question 4: 4 points