Name:

Group members: Last updated: 12/07/2025

ARE 336 – Problem set 4 – Benefit-cost/Nonmarket valuation/Uncertainty

Due **December 2, 2025**

This problem requires you to complete 3 questions, for a total of 45 points.

Note that you are not required to turn in this problem set if you are submitting a class project. If you choose to do this problem set instead of a class project, this problem set's grade will be your class project grade. If you choose to do both this problem set and a class project, 5 points will be added to either your midterm or your final exam grade (whichever is lower).

1. Assume you live 100 miles from a beach that is free to enter. AAA tells you that the cost per mile of driving is \$0.25. It takes you 1 hour to drive there from home, and your wage rate is \$30 per hour. Your demand for trips to this beach (q) is

$$d_0(p) = 100 - p$$

where p is price per trip.

(a) [2 points] Assuming that your opportunity cost of time is one third of your wage rate, what is the total travel cost to visit the beach? Call this p^*.

(**Hint:** Ignore the opportunity cost of time spent on the beach.)

(**Hint:** The opportunity cost of driving should include *round trip* time and mileage.)

Solution: The opportunity cost of time is one third of the wage rate, so

$$2 \cdot \underbrace{\frac{1}{3} \cdot \$30}_{\text{one-way}} = \$20$$

per hour. The total driving cost is

$$2 \cdot \underbrace{\$0.25 \cdot 100}_{\text{one-way}} = \$50.$$

Hence the total travel cost is

$$p^* = \$20 + \$20 = \$70.$$

(b) [1 point] At the price p^*, how many trips to the beach will you take? Call this q_0.

Solution: The number of trips taken is

$$q_0 = 100 - p^* = 100 - 70 = 30.$$

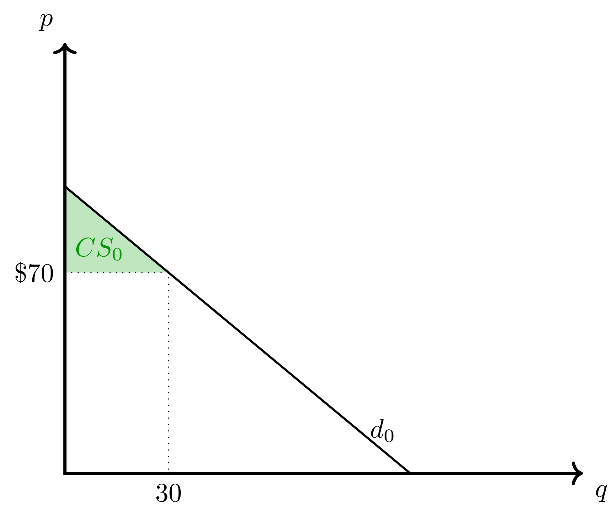
(c) [2 points] How much consumer surplus will you receive? Call this CS_0.

Solution: The consumer surplus is the area below the demand curve and above the price. This is a triangle.

$$\begin{aligned} CS_0 &= \frac{1}{2} \cdot (\bar{p} - p^*) \cdot d_0(p^*) \\ &= \frac{1}{2} \cdot (\$100 - \$70) \cdot 30 \\ &= \$450 \end{aligned}$$

(d) [2 points] Draw your demand for trips with trips on the horizontal axis and price on the vertical axis. Label p^* and q_0 by their numeric value. Label the consumer surplus region CS_0.

Solution: Observe:



(e) The government invests in a beach nourishment project which doubles the width of the beach. This project increases your demand for trips to the beach. Your new demand is

$$d_1(p) = 120 - p.$$

i. [1 point] Does your travel cost change?

Solution: No.

ii. [1 point] How many trips will you take now? Call this q_1.

Solution: The new number of trips taken is

$$q_1 = d_1(p^*) = 120 - p^* = 50.$$

iii. [2 points] How much consumer surplus will you receive now? Call this CS_1.

Solution: The consumer surplus is the area below the new demand curve and above the price. This is a triangle.

$$\begin{aligned} CS_1 &= \frac{1}{2} \cdot (\bar{p}_1 - p^*) \cdot d_1(p^*) \\ &= \frac{1}{2} \cdot (\$120 - \$70) \cdot 50 \\ &= \$1,250 \end{aligned}$$

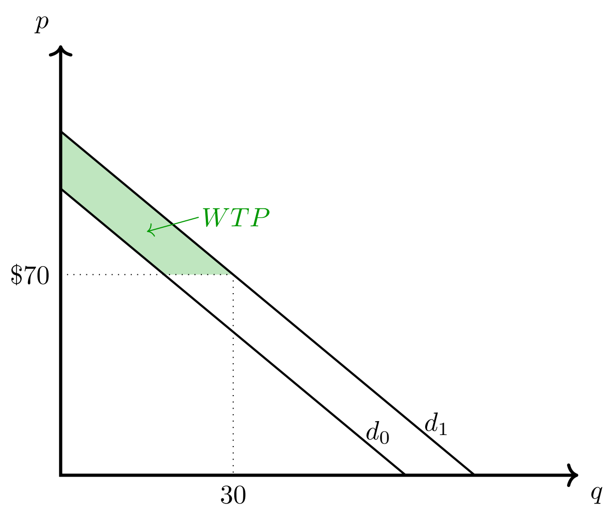
iv. [3 points] How much would you be willing to pay for the beach nourishment project?

Solution: You would be willing to pay up to the amount of extra consumer surplus you receive from the nourishment project. That is,

$$WTP = CS_1 - CS_0 = \$1,250 - \$450 = \$800.$$

v. [2 points] Draw a graph with trips (q) on the horizontal axis, price (p) on the vertical axis, the original demand (d_0), and the new demand (d_1). Label the section that equals your willingness to pay for the nourishment project as WTP.

Solution: Observe:



Question 1: 16 points

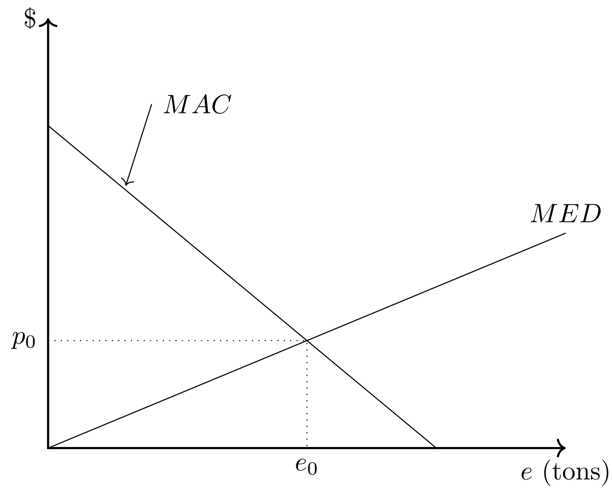
2. Consider the market for emissions abatement for a single, representative firm. There is a marginal external damages function that is known with certainty:

$$MED(e) = \frac{e}{2}$$

where e represents 1 ton of emissions. Initially, the regulator believes that the marginal abatement cost for the firm is

$$MAC(e) = 15 - e$$

where e again represents 1 ton of emissions. This market is represented in the figure below. In the figure, e_0 represents the socially optimal level of emissions based on the *expected marginal abatement cost* function MAC, and p_0 is the corresponding value of the marginal external damages.



(**Hint:** Round any cacluations to the nearest hundreth [i.e., two decimal points].)

(a) [2 points] The regulator uses this expected marginal abatement cost curve in calculating the socially optimal quantity of emissions (in tons), e_0. What is the value of e_0, and what is the value of p_0.

Solution: We find e_0 by finding the level of emissions that equates MAC with MED.

$$\begin{aligned} MAC(e_0) &= MED(e_0) \\ 15 - e_0 &= \frac{e_0}{2} \\ \frac{3}{2}e_0 &= 15 \\ e_0 &= 10 \end{aligned}$$

and to find p_0, we find the value of the marginal external damages at e_0 tons of emissions.

$$p_0 = \frac{e_0}{2} = \frac{10}{2} = 5.$$

(b) [3 points] Suppose the regulator sets a tax at what she thinks is the socially optimal price. That is, $\tau = p_0$. What is the dollar value in terms of total avoided damages of this tax? What is the total abatement cost incurred by the representative firm? Is the net benefit of this tax positive or negative?

(**Hint:** The BAU emissions with this MAC is 15.)

Solution: The avoided damages by the tax is the area of MED between BAU emissions and e_0. This is the sum of a rectangle with height p_0 and width $15 - e_0$, and a triangle with height $MED(15) - p_0$ and width $15 - e_0$. That is, the avoided damages are

$$\begin{aligned} B &= p_0 \cdot (15 - e_0) + \frac{1}{2} \cdot [MED(15) - 5] \cdot (15 - e_0) \\ &= 5 \cdot (15 - 10) + \frac{1}{2} \cdot [7.5 - 5] \cdot (15 - 10) \\ &= \$31.25 \end{aligned}$$

The total abatement cost is the area below the MAC between BAU emissions and e_0. That is,

$$\begin{aligned} C &= \frac{1}{2} \cdot p_0 \cdot (15 - e_0) \\ &= \frac{1}{2} \cdot 5 \cdot (15 - 10) \\ &= \$12.5 \end{aligned}$$

The net benefits are

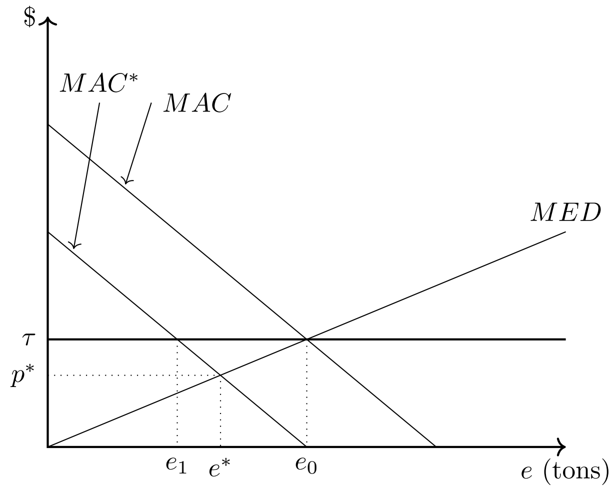
$$NB = B - C = \$31.25 - \$12.5 = \$18.75 > 0,$$

so they are positive.

(c) It turns out that the regulator was incorrect in her estimate of the marginal abatement cost. The true marginal abatement cost is

$$MAC^*(e) = 10 - e,$$

but it is too late to change the policy, so the tax of $\tau = p_0$ remains in place. This scenario is shown in the figure below. In the figure, e_0 is the same as before, the BAU emissions quantity of the true marginal abatement cost function is 10, e_1 is the quantity of emissions that will occur in the presence of a tax $\tau = p_0$, and, finally, p^* and e^* are the true socially optimal tax and emissions quantity.



i. [2 points] The original tax (which the regulator *thought* was optimal) is higher than the true socially optimal tax. Explain the intuition behind this, using the relationship between MAC^* and MAC in your argument.

> **Solution:** The abatement cost faced by the firm is a factor of social surplus. So, when the regulator believed that abatement was a higher burden on the firm, she placed a more stringent tax because it was expected that the firm would need a stronger incentive to reduce its emissions.

ii. [1 point] Because $e_1 < e^*$, we have an *underprovision* of emissions when the tax is in place. What does it mean to have less-than-optimal emissions?
(**Hint:** You may want to *assume* that the representative firm produces something like electricity. If you do, state this assumption.)

> **Solution:** Assume that the firm produces electricity. Having less-than-optimal emissions means that society would benefit from a little more emissions if those emissions are created by supplying additional electricity.

iii. [3 points] Calculate e_1, p^*, and e^*.

> **Solution:** e_1 occurs when the true marginal abatement cost is equal to τ.
>
> $$\begin{aligned} MAC^*(e_1) &= \tau \\ 10 - e_1 &= 5 \\ e_1 &= 5 \end{aligned}$$
>
> e^* is the quantity of emissions where the true marginal abatement cost equals the

marginal external damages.

$$\begin{aligned} MAC^*(e^*) &= MED(e^*) \\ 10 - e^* &= \frac{e^*}{2} \\ \frac{3}{2}e^* &= 10 \\ e^* &= 6.67 \end{aligned}$$

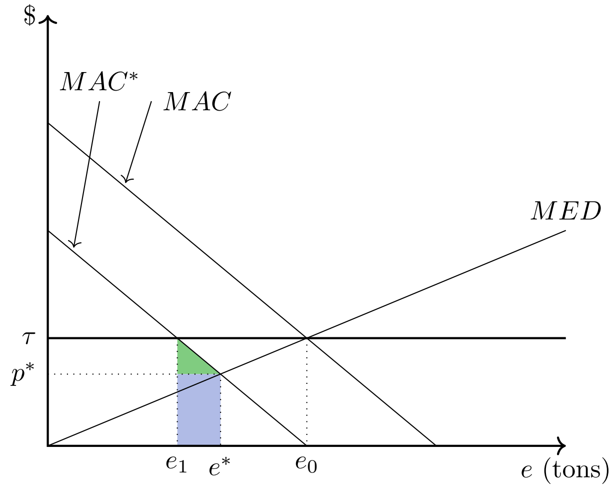
and p^* is the value of the marginal external damages at e^*.

$$\begin{aligned} p^* &= MED(e^*) \\ p^* &= \frac{e^*}{2} \\ p^* &= \$3.33 \end{aligned}$$

iv. [3 points] The firm can hire lobbyists to sway the regulator. Ignoring any externalities, how much money would the firm be willing to spend on lobbyists in order to decrease the tax from τ to p^*?

Solution: The benefit of lobbying is getting the ability to produce more emissions. Naturally, the maximum amount the firm is willing to pay for lobbying is the amount of cost-savings it can earn by increasing emissions from e_1 to e^*. This is the sum of a rectangle with a height of p^* and a width of $e^* - e_1$, and a triangle with a height of $\tau - p^*$ and a width of $e^* - e_1$.

$$\begin{aligned} AC &= p^* \cdot (e^* - e_1) + \frac{1}{2} \cdot (\tau - p^*) \cdot (e^* - e_1) \\ &= \underbrace{\$3.33 \cdot (6.67 - 5)}_{\text{cost savings transfer to tax}} + \underbrace{\frac{1}{2} \cdot (\$5 - \$3.33) \cdot (6.67 - 5)}_{\text{total tax savings}} \\ &\approx \$5.56 + \$1.39 \\ &= \$6.96 \end{aligned}$$

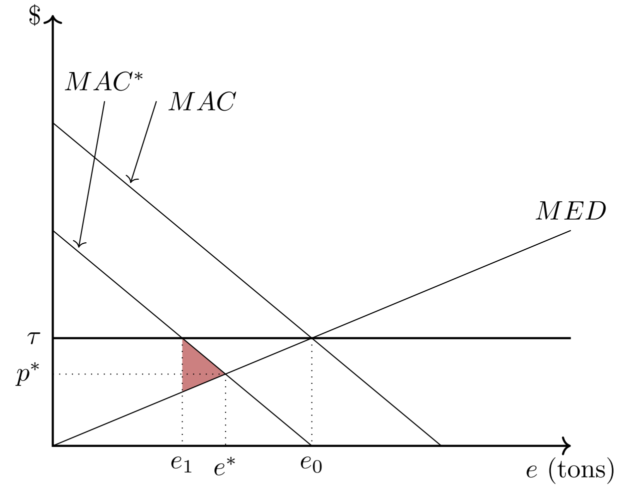


Truly, the firm would be willing to pay \$1.39, because the other cost savings are just a transfer. However, either \$6.96 or \$1.39 are correct.

v. [2 points] What is the deadweight loss of having $\tau = p_0$ given that the true marginal abatement cost is MAC^*?

Solution: The deadweight loss occurs because the abatement cost is lower than expected. When the regulator thought the marginal abatement cost was going to be higher, she placed a tax that was too high. As such, we could have decreased emissions at a cheaper than expected price. The deadweight loss is the net benefit of decreasing emissions from e_1 to e^*. This is a triangle with base $\tau - MED(e_1)$ and height $e^* - e_1$.

$$\begin{aligned} DWL &= \frac{1}{2} \cdot (\tau - MED(e_1)) \cdot (e^* - e_1) \\ &= \frac{1}{2} \cdot \left(5 - \frac{5}{2}\right) \cdot (6.67 - 5) \\ &= \$2.09 \end{aligned}$$



Question 2: 16 points

Choose either question 3 or question 4.

3. Sugar Mountain ("Sugar") is a ski resort in Banner Elk, NC. Banner Els is considering investing in an upgrade of Sugar's snow cannons believing that this will entice more visitors during the ski season, improving crucial tourism revenue. The cost of the snow cannon upgrades is a one-time \$50,000—this would be paid entirely by the town of Banner Elk. If Banner Elk does upgrade the snow cannons, it will increase tourism revenue by a constant \$5,000 every year that the cannons are in place. The new snow cannons will last for 20 years, but each year they have a probability $p = 0.2$ of breaking. If the snow cannons break, Banner Elk has agreed to pay the constant \$5,500 in order to fix them. Assume that the repair happens instantly, so that the snow cannons will still provide the \$5,000 in tourism revenue, but Banner Elk will net -\$500 in for the year.

   (a) [1 point] Fill in the following payoff matrix with the annual payoffs to Banner Elk, where the row determines whether the town invests in the upgrade and the columns determine whether the snow cannons break.

		Break?	
		Yes	No
Upgrades snow cannons?	Yes		
	No		

   Solution:

		Break?	
		Yes	No
Upgrades snow cannons?	Yes	-\$500	\$5,000
	No	0	0

   (b) [2 points] What is the *expected* annual benefit to Sugar of installing the snow cannons? (**Hint:** Calculate the expected benefit using the same process that you used in the second problem set.)

   Solution: The expected net benefit in year t is

$$\mathbb{E}[B_t] = 0.8 \cdot \$5,000 - 0.2 \cdot \$500 = \$3,900.$$

   (c) [2 points] Assume that the future is just as valuable as today (i.e., the discount rate is zero). Then what is the expected lifetime benefit of investing in the snow cannon upgrades? Does this cover the cost?

(**Hint:** Note that the benefits start immediately, so the expected benefits over 20 years is $\sum_{t=0}^{20} B = 21 \cdot B$ where B is the annual expected net benefit.)

Solution: Since there is no discounting, the lifetime expected benefits is the sum of the expected benefits received every year. That is

$$\begin{aligned}\mathbb{E}[B] &= \sum_{t=0}^{20} \mathbb{E}[B_t] \\ &= 21 \cdot \$3,900 \\ &= \$81,900\end{aligned}$$

Given that the cost is $50,000, the lifetime net benefits are

$$\$81,900 - \$50,000 = \$31,900 > 0.$$

(d) [2 points] Now calculate the expected lifetime benefit with a discount rate of $r = 0.1$. Is this a good investment?
$\left(\textbf{Hint: } \text{Note that } \sum_{t=0}^{T} \delta^t = \dfrac{1-\delta^{T+1}}{1-\delta} \text{ for any } 0 < \delta < 1\right)$.

Solution: The discount factor for year t is $\left(\frac{1}{1+r}\right)^t$, so the lifetime benefits are

$$\begin{aligned}\mathbb{E}[B] &= \sum_{t=0}^{20} \left(\frac{1}{1+r}\right)^t \$3,900 \\ &= \frac{1-\left(\frac{1}{1.1}\right)^{21}}{1-\left(\frac{1}{1.1}\right)} \cdot \$3,900 \\ &\approx \$37,102.90\end{aligned}$$

Given that the cost is $50,000, the expected net benefits are

$$\$37,102.90 - \$50,000 = -\$12,897.10 < 0.$$

(e) [2 points] Calculate the expected lifetime benefit with a discount rate of $r = 0.05$. Is this a good investment?

> **Solution:** The lifetime benefits are
>
> $$\begin{aligned}\mathbb{E}[B_t] &= \sum_{t=0}^{20}\left(\frac{1}{1.05}\right)^t \cdot \$3,900 \\ &= \frac{1-\left(\frac{1}{1.05}\right)^{21}}{1-\left(\frac{1}{1.05}\right)} \cdot \$3,900 \\ &\approx \$52,502.62\end{aligned}$$
>
> Given that the cost is \$50,000, the expected net benefits are
>
> $$\$52,502.62 - \$50,000 = \$2,502.62 > 0$$

(f) [2 points] Suppose that instead of upgrading the snow cannons, Banner Elk can build a visitor center that will increase tourism revenues by \$4,000 each year. The visitor center costs \$25,000; it will last forever and is guaranteed to never break down. What is the total lifetime benefits assuming a discount rate of $r = 0.05$? Do the benefits outweigh the costs?
(**Hint:** Note that $\sum_{t=0}^{\infty}\delta^t = \frac{1}{1-\delta}$ for any $0 < \delta < 1$.)

> **Solution:** The lifetime benefit of this policy is
>
> $$\begin{aligned}B &= \sum_{t=0}^{20}\left(\frac{1}{1.05}\right)^t \cdot \$4,000 \\ &= \frac{1-\left(\frac{1}{1.05}\right)^{21}}{1-\left(\frac{1}{1.05}\right)} \cdot \$4,000 \\ &\approx \$53,848.84\end{aligned}$$
>
> Because the cost is \$25,000, the net benefits of this project are
>
> $$\$53,848.84 - \$25,000 = \$28,848.84$$

(g) [2 points] Calculate the benefit-cost ratios using the expected lifetime benefits calculated in part e and part f, using the correct up front cost. Which upgrade should Sugar make?

> **Solution:** The benefit-cost ratio of part e is
>
> $$\frac{\$2,502.62}{\$50,000} \approx 1.05$$
>
> and the benefit-cost ratio of part f is
>
> $$\frac{\$53,848.84}{\$25,000} \approx 2.15$$

Clearly the benefit-cost ratio for upgrading the lifts provides the most benefit for each dollar of cost. That is the better project.

Question 3: 13 points

4. Suppose that your demand for drinking water is

$$q(p) = 100 - 10p$$

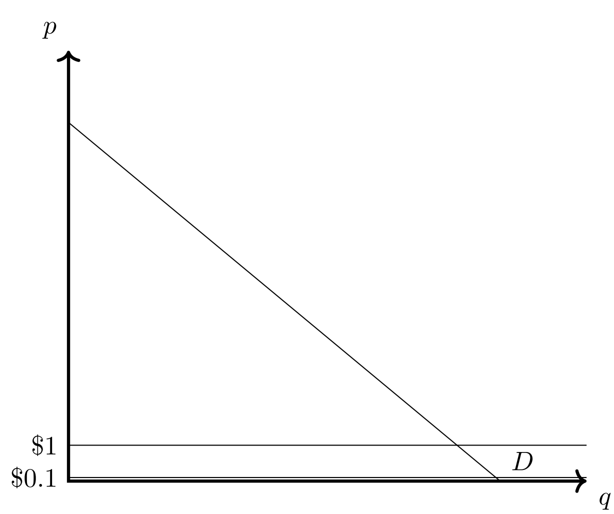
where q is gallons of water and p is the price per gallon. You have two sources of drinking water: (i) you may pump groundwater at \$0.10/gallon, or (ii) you may purchase bottled water at \$1/gallon. Bottled water and groundwater are perfect substitutes.

(a) [1 point] What is your inverse demand for water?

Solution:

$$\begin{aligned} q &= 100 - 10p(q) \\ 10p(q) &= 100 - q \\ p(q) &= 10 - 0.1q \end{aligned}$$

(b) [2 points] Draw your demand for water with gallons (q) on the horizontal axis and price (p) on the vertical axis. Draw horizontal lines at the prices of ground water ($p = \$0.1$) and bottled water ($p = \$1$).



Solution:

(c) [1 point] How much of each type of water do you consume?
(**Hint:** You may consume zero of one of these.)

Solution: You consume only groundwater, so $p = 0.1$. Total consumption is

$$q(0.1) = 100 - 10 \cdot 0.1 = 99$$

gallons.

(d) [2 points] How much consumer surplus do you receive, given your answer to part c?

Solution:

$$\begin{aligned} CS_0 &= \frac{1}{2} \cdot (10 - 0.1) \cdot 99 \\ &= \frac{1}{2} \cdot 9.9 \cdot 99 \\ &= 490.05 \end{aligned}$$

(e) Your well was contaminated! Now the groundwater is unsafe to drink and you must consume only bottled water.

i. [1 point] How many gallons of bottled water will you consume?

Solution: Now the price is \$1, so we consume

$$q(1) = 100 - 10 \cdot 1 = 90$$

gallons of water.

ii. [2 points] How much consumer surplus do you receive now?

Solution:

$$\begin{aligned} CS_1 &= \frac{1}{2} \cdot (10 - 1) \cdot 90 \\ &= \frac{1}{2} \cdot 9 \cdot 90 \\ &= 405 \end{aligned}$$

iii. [3 points] How much would you be willing to pay to cleanup the contaminated well? (**Hint:** Assume that the cleanup removes all contamination and the water will be perfectly safe to drink.)

Solution: The willingness to pay is equal to the amount of consumer surplus lost by the well's contamination. Hence, the willingness to pay for a well cleanup is

$$WTP = \Delta CS = 490.05 - 405 = \$85.5$$

(f) [1 point] What kind of non-market valuation technique is this called?

Solution: This is "defensive expenditures."

Question 4: 13 points