Name:

Group members: Last updated: 10/28/2025

ARE 336 – Problem set 3 – Emissions regulation

Due **November 7, 2025**

This problem set has 2 questions, for a total of 41 points (with the possibility to earn up to 2 bonus points).

1. Consider the case of Tar Heel Electric (THE), an energy plant which is situated across a valley from Wolfpack Winery and Vineyard (WWV). THE collects revenue by generating electricity through a process which pollutes the local air. Local air pollution decreases the enjoyment of visitors to WWV and can also affect the taste of its grapes.

   THE's marginal cost to produce one megawatt-hour (MWh) of electricity is defined by $MC_{THE}(q) = q$, where q is the number of MWhs produced (it costs \$1 for THE to produce 1 MWh). A team of scientisits studied THE's production process and determined that each MWh of electricity produced by THE creates a half-ton of SO_2 emissions that are released into the air. Thus, THE's emissions production can be modeled as $q(e) = \frac{1}{2}e$, where e is the number of tons of SO_2 emisisons.

   While THE's emissions are not good for our health (or WWV's profits), having electricity is great for our quality of life! Suppose that the marginal benefit of electricity is defined by $MB(q) = 12-2q$ where q is the number of MWhs. In this problem, assume that THE is the only supplier of electricity, so every q needs to be provided by THE. This marginal benefit curve is effectively the *demand for electricity* faced by THE.

   Because SO_2 is a "localized" pollutant, its effects are largely felt around THE's facility. WWV is the only entity nearby, so they are the only entity affected by the SO_2 emissions. Hence, the lost profits of WWV represent *the only* externality here.

   We will go through Coasian bargaining in the case of constant externality and proportional externality.

   (a) [1 point] State THE's marginal cost per unit of SO_2 emission?
   (**Hint:** That is, state $MC_{THE}(e)$.)

   Solution: Substituting $q = \frac{1}{2}e$ into $MC_{THE}(q)$ gives us

   $$MC_{THE}(e) = \frac{1}{2}e.$$

   (b) [1 point] State the public's marginal benefit per unit of SO_2 emission?
   (**Hint:** Recall that THE is the only possible producer of electricity in the market.)

   Solution: Substituting $q = \frac{1}{2}e$ into $MB(q)$ gives us

   $$MB(e) = 12 - 2 \cdot \frac{1}{2}e = 12 - e.$$

(c) [1 point] What does it mean (in words) to have a marginal benefit of a ton of SO_2 emissions??

> **Solution:** Because the public benefits from electricity, and electricity requires some SO_2 emissions in order to be produced, the public transitively receives *some* benefit from SO_2 emissions.

(d) Suppose that every ton of SO_2 emissions decreases WWV's profits by a constant $d(e) = 3$.

i. [2 points] Complete the following table:

e	$d(e)$	$MC_{THE}(e)$	$MSC(e)$	$MB(e)$
0				
2				
4				
6				
8				
10				
12				

> **Solution:**

e	$d(e)$	$MC_{THE}(e)$	$MSC(e)$	$MB(e)$
0	0	0	0	12
2	3	1	4	10
4	3	2	5	8
6	3	3	6	6
8	3	4	7	4
10	3	5	8	2
12	3	6	9	0

ii. [1 point] How many tons of SO_2 emissions will occur if THE only pays attention to its *private* marginal cost. Call this q_0.

> **Solution:** THE will increase its SO_2 emissions up to the point that marginal private cost equals marginal benefit. We see from the table that this happens with 8 tons of SO_2 emissions.

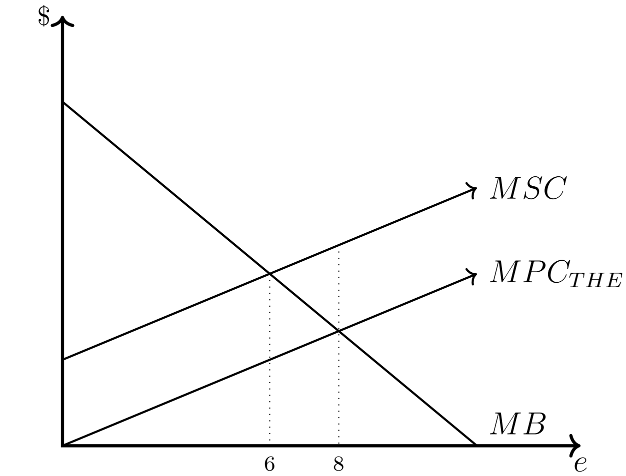
iii. [1 point] What is the *socially efficient* quantity of SO_2 emissions (in tons)? Call this q_1.

> **Solution:** The socially efficient level emissions equates the marginal social cost with the marginal benefit. We see from the table that this happens with 6 tons of SO_2 emissions.

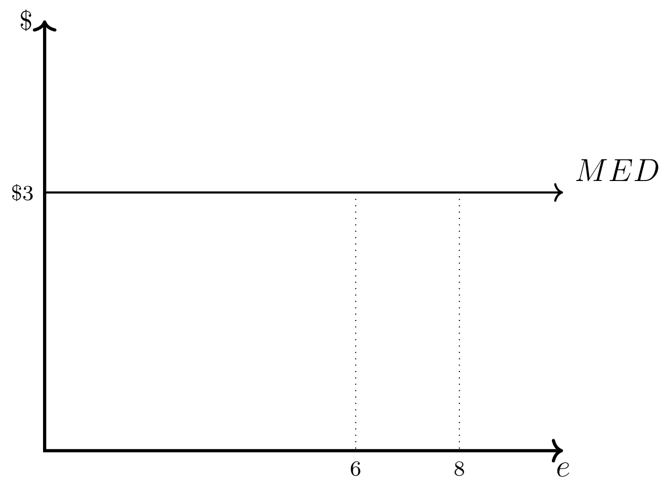
iv. [3 points] Draw a graph with SO_2 emissions (in tons) on the horizontal axis with dollars on the vertical axis. Your graph should have the axes labeled along with MPC_{THE}, MSC, MB. Label q_0 and q_1 using their *numeric* value.
In a separate graph, draw the marginal external damages function ($d(e)$) with emissions on the horizontal axis and dollars on the vertical axis. Again, label q_0 and q_1 using their

numeric value.

Solution:



The marginal external damage function looks like below:



We can clearly see that the total external damages at $q_0 = 8$ is $\$3 \cdot 8 = \$18$.

v. [2 points] At q_0, what are the *total lost profits* to WWV? At q_1, what are the *total lost profits* to WWV?
(**Hint:** You may think that the table you completed in part d.i is sufficient to answer this. However, this table increases in units of 2, so it does not completely cover damages for all emissions. You should refer to your graph in part d.iv—and use geometry—to calculate this.)

Solution: We can clearly see that the total external damages at $q_0 = 8$ is:

$$TED(q_0) = \$3 \cdot q_0 = \$3 \cdot 8 = \$24.$$

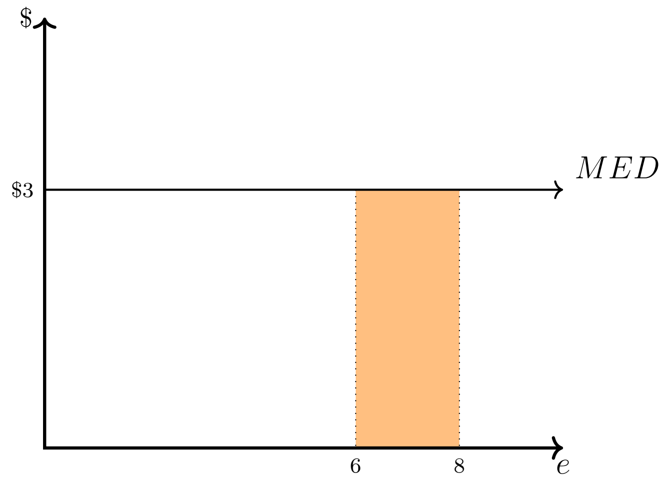
Similarly, we can see that the total external damages at $q_1 = 6$ is:

$$TED(q_1) = \$3 \cdot q_1 = \$3 \cdot 6 = \$18.$$

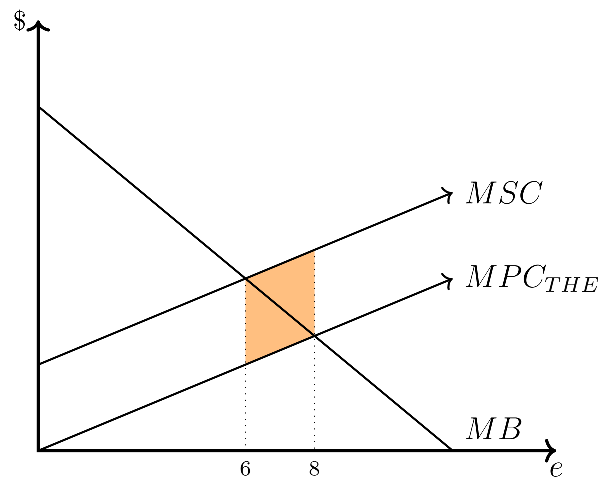
vi. [1 point] How much profit does WWV *get back* when THE reduces its emissions from q_0 to q_1?

(**Hint:** Your answer to part d.v gives you exactly what you need to answer this.)

Solution: Whe THE reduces its emissions from q_0 to q_1, WWV earns an additional \$6 in profit.



On the main graph:



vii. [1 point] Given your answer to part d.vi, how much would WWV be willing to pay THE in order for THE to produce q_1 tons of SO_2 emissions instead of q_0 tons.

Solution: WWV would be willing to pay up to \$6 to THE in order for them to produce q_1 tons of SO_2 instead of q_0.

(e) Suppose now that every ton of SO_2 emissions decreases WWV's profits at an increasing rate $d(e) = \frac{1}{2}e$.

i. [2 points] Complete the following table:

e	$d(e)$	$MC_{THE}(e)$	$MSC_{THE}(e)$	$MB(e)$
0				
2				
4				
6				
8				
10				
12				

Solution:

e	$d(e)$	$MC_{THE}(e)$	$MSC_{THE}(e)$	$MB(e)$
0	0	0	0	12
2	1	1	2	10
4	2	2	4	8
6	3	3	6	6
8	4	4	8	4
10	5	5	10	2
12	6	6	12	0

ii. [1 point] How many tons of SO_2 emissions will occur if THE only pays attention to its *private* marginal cost? Call this q_0.

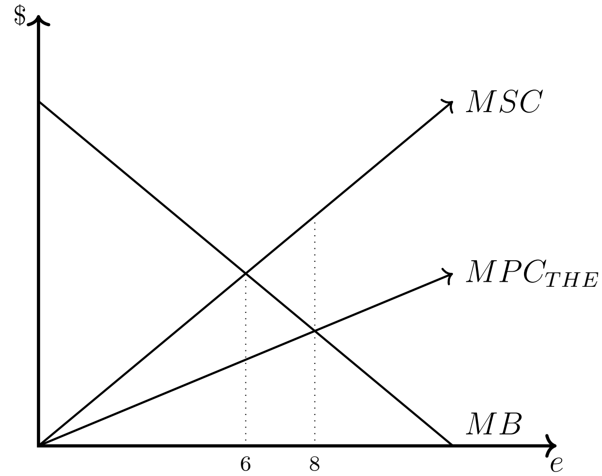
Solution: THE will increase is SO_2 emissions up to the point tha marginal private cost equals marginal benefit. We see from the table that this happens with 8 tons of SO_2 emissions.

iii. [1 point] What is the *socially efficient* quantity of SO_2 emissions (in tons)? Call this q_1.

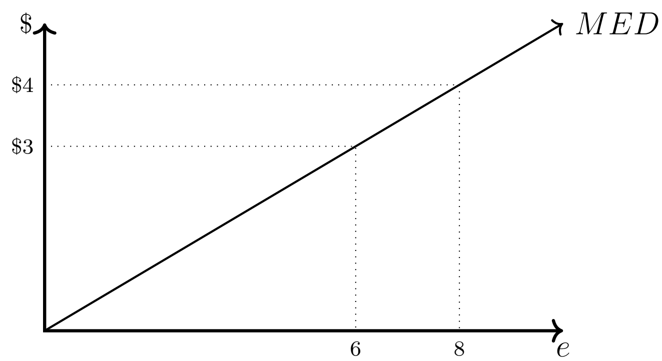
Solution: The socially efficient level of emissions equates the marginal social cost with the marginal benefit. We see from the table that this happens with 6 tons of SO_2 emissions.

iv. [3 points] Draw a graph with SO_2 emissions (in tons) on the horizontal axis with dollars on the vertical axis. Your graph should have the axes labeled, MPC_{THE}, MSC, MB. Label q_0 and q_1 using their *numeric* value.
In a separate graph, draw the marginal external damages function ($d(e)$) with emissions on the horizontal axis and dollars on the vertical axis. Again, label q_0 and q_1 using their *numeric* value.

Solution:



The marginal external damage function looks like below:



v. [2 points] At q_0, what are the *total lost profits* to WWV? At q_1, what are the *total lost profits* to WWV?

(**Hint:** You may think that the table you completed in part e.i is sufficient to answer this. However, this table increases in units of 2, so it does not completely cover damages for all emissions. You should refer to your graph in part e.iv to calculate this.)

Solution: We can clearly see that the total external damages at $q_0 = 8$ is:

$$TED(q_0) = \frac{1}{2}MED(q_0) \cdot q_0 = \frac{1}{2}\$4 \cdot 8 = \$16.$$

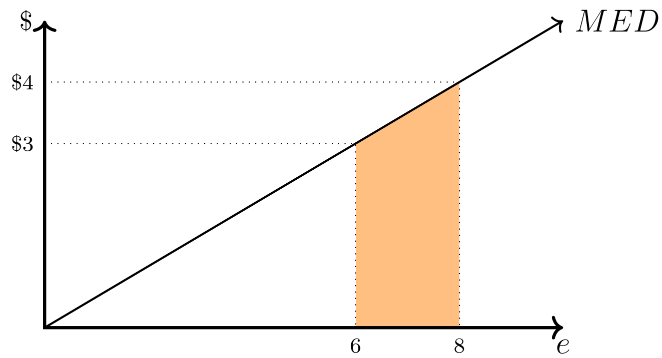
Similarly, we can see that the total external damages at $q_1 = 6$ is:

$$TED(q_1) = \frac{1}{2}MED(q_1) \cdot q_1 = \frac{1}{2}\$3 \cdot 6 = \$9.$$

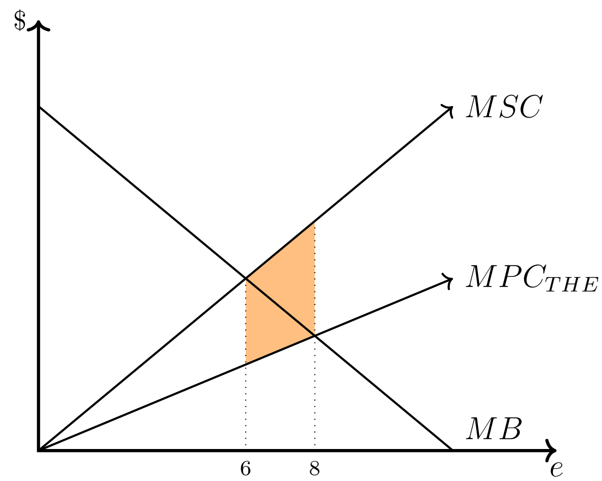
vi. [1 point] How much profit does WWV *get back* when THE reduces its emissions from q_0 to q_1?

(**Hint:** Your answer to part e.v gives you exactly what you need to answer this.)

Solution: When THE reduces its emissions from q_0 to q_1, WWV earns an extra \$7 in profit.



On the main graph:



vii. [1 point] Given your answer to part e.vi, how much would WWV be willing to pay THE in order for THE to produce q_1 tons of SO_2 emissions instead of q_0 tons.

Solution: WWV would be willing to pay up to \$7 to THE in order for them to producer q_1 tons of SO_2 instead of q_0.

2. Consider two firms, Apple (A) and Berkshire (B) with the following abatement costs:

$$C_A(a_A) = a_A^2$$
$$C_B(a_B) = 2a_B^2$$

where a_A represents the level of *abatement* of Apple and a_B represents the level of *abatement* of Berkshire. Each firm has a corresponding marginal abatement cost function:

$$MAC_A(a_A) = 2a_A$$
$$MAC_B(a_B) = 4a_B$$

Recall that a firm's level of abatement is defined relative to their BAU emissions (i.e., $a = \bar{e} - e$ where $\bar{e}$ is BAU emissions).

Apple's BAU emissions is $\bar{e}_A = 25$ while Berkshire's BAU emissions is $\bar{e}_B = 10$.

(a) [2 points] Derive the total abatement cost and marginal abatement costs for the firms as functions of *emissions*. For example, Apple's total abatement cost as a function of emissions is $C_A(e_A)$.

(**Hint:** You need to substitute something for a_A and a_B.)

Solution: Simple substitution gives us

$$C_A(e_A) = (25 - e_A)^2$$
$$MAC_A(e_A) = 50 - 2e_A$$
$$C_B(e_B) = 2(10 - e_B)^2$$
$$MAC_B(e_B) = 40 - 4e_B$$

(b) [3 points] The regulator sets the goal of allowing 15 tons of GHG emissions. She first tries a command and control approach where she gives each firm un-tradeable pollution permits—each firm is allowed to emit one ton of GHG for each permit.

Berkshire is "grandfathered" an amount of permits equal to its BAU emissions and gives, and Apple is given the remaining permits. State the number of permits Berkshire granted using the definition of grandfathering we discussed in class. What is the total cost of abatement in the market?

Solution: Berkshire is "grandfathered" 10 permits—grandfathering means it is granted its BAU emissions.

Berkshire faces no abatement costs because it is able to emit its BAU emissions. Because Berkshire isn't doing any abatement, Apple needs to do a lot of the heavy lifting. Apple is only allowed to emit 5 tons of GHG, so it needs to abate 20 tons. The total cost of this is

$$C_A(20) = 20^2 = \$400.$$

> Note that we could have equivalently used the $C_A(e_A) = (25 - e_A)^2$ specification using $e_A = 5$.

(c) The regulator changes her mind and wants to try using cap-and-trade to reach her 15 tons goal.
First she splits the permits evenly between the two firms so that Apple gets 7.5 permits and Berkshire gets 7.5 permits.

i. [1 point] What would the total abatement cost be at the initial permit allocation (i.e., before any trading occurs)?

> **Solution:** Apple needs to abate 18.5 tons of GHG emissions and Berkshire needs to abate 2.5 tons. The total abatement is
>
> $$\begin{aligned} C_A(18.5) + C_B(2.5) &= 18.5^2 + 2 \cdot 2.5^2 \\ &= \$342.25 + 2 \cdot \$6.25 \\ &= \$354.75 \end{aligned}$$

ii. [3 points] Complete the following statements. Your statements should reflect the state of the world *after* the regulator gives them their allocated permits and *before* any trading occurs.

- Apple will buy a permit if it is less than **$35**.
- Berkshire will sell a permit at any price greater than **$10**.
- Given this, **Apple** will purchase permits from **Berkshire**.

> **Solution:** Apple will buy a permit at any price less than its marginal abatement cost at its permit allocation, and Berkshire will sell a permit at any price greater than its marginal abatement cost at its permit allocation.
> We can succinctly define the set of acceptable prices for both firms below where p: means "p such that"
>
> $$\begin{aligned} \{p_A\} &= \{p \colon p \leq MAC_A(7.5)\} \\ \{p_B\} &= \{p \colon p \geq MAC_B(7.5)\} \end{aligned}$$
>
> Note that in the two-firm case, we can make some statement like "the firm with the lower marginal abatement cost at the initial permit allocations is the demander of permits."

iii. [2 points] Using the marginal abatement cost curves you calculated in part a, derive a "demand for permits" for each firm that's a function of price. Which firm is more price sensitive?

(**Hint:** This is similar to our trick for turning an inverse demand function into a demand function.)

(**Hint:** The more price sensitive firm has a steeper slope.)

Solution: First write the identity $p = MAC_A(e_A)$, and "rewrite" e_A to be a function of p:

$$\begin{aligned} p &= MAC_A(e_A) \\ p &= 50 - 2e_A(p) \\ 2e_A(p) &= 50 - p \\ e_A(p) &= 25 - \frac{p}{2} \end{aligned}$$

and doing the same for Berkshire gives us

$$\begin{aligned} p &= MAC_B(e_B) \\ p &= 40 - 4e_B(p) \\ 4e_B(p) &= 40 - p \\ e_B(p) &= 10 - \frac{p}{4} \end{aligned}$$

iv. [3 points] Use the market clearing condition to determine the price that satisfies the equimarginal principle. Call this p^*.
Using the demand functions you found in part iii and the p^* you just found, determine the amount of emissions both firms will emit. Call these e_A^* and e_B^*.
Draw a graph with emissions on the horizontal axis and price on the vertical axis. Show $e_A(p)$, $e_B(p)$, p^*, e_A^*, and e_B^* on your graph—use the numeric values for all of these.
(**Hint:** Round any numbers to two decimal points.)

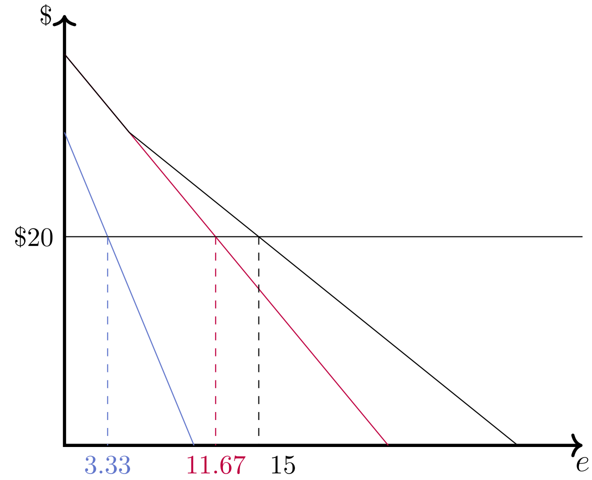
v. [1 bonus point] Represent the aggregate demand on the graph.

Solution: First we need to find the market clearing price by setting the horizontal summation of emissions equal to p:

$$\begin{aligned} e_A(p^*) + e_B(p^*) &= 15 \\ 25 - \frac{p^*}{2} + 10 - \frac{p^*}{4} &= 15 \\ \frac{3}{4}p^* &= 20 \\ p^* &= 26.67 \end{aligned}$$

Thus, we can find the optimal distribution of emission permits:

$$\begin{aligned} e_A^* &= e_A(p^*) = 25 - \frac{26.67}{2} = 11.67 \\ e_B^* &= e_B(p^*) = 10 - \frac{26.67}{4} = 3.33 \end{aligned}$$



vi. [2 points] Finally, calculate the total abatement cost at e_A^* and e_B^*. Ignore the cost of purchasing permits. Is this abatement cost bigger or smaller than the costs you calculated in parts b and c.i?

Solution: The total abatement cost:

$$\begin{aligned} C_A(e_A^*) + C_B(e_B^*) &= 11.67^2 + 2 \cdot 3.33^2 \\ &= \$136.19 + \$11.09 \\ &= \$147.28 \end{aligned}$$

This is *much* smaller than the cost in either of the other emissions levels.

vii. [1 bonus point] Explain why we can safely ignore the cost of purchasing permits when calculating total abatement cost.

Solution: We can ignore the cost of purchasing permits when calculating total abatement cost because it is simply a transfer from one firm to another. So total cost of compliance is unaffected by this transfer—the only cost incurred is the abatement cost.