Name:

Group members:

Last updated: 09/02/2025

ARE 336 – Problem set 1 – Microeconomics review

Due **September 5**

This problem set has 3 questions, for a total of 100 points (with the possibility to earn up to 20 bonus points).

1. Consider the demand functions $D_1(p) = 10 - \frac{p}{250}$ and $D_2(p) = 15 - \frac{p}{100}$.
   (a) [3 points] Find the inverse demand functions.
   (b) [1 ½ points] Calculate the "choke price."
      i. [1 ½ points] State, in words, what the choke price is.
   (c) [3 points] Calculate the maximum quantity demanded for both demands.
   (**Hint:** The maximum quantity demanded occurs when the good is given away for free; when a good is free, it means $p = 0$.)
   (d) Do the following for the prices $p = \$100, \$500, \$1000, \$1500, \$2000, \$2500$:
      i. [4 points] Calculate quantity demanded. What do you notice about the relationship between quantities demanded for the two demand functions?
      ii. [4 points] Calculate consumer surplus. What do you notice about the relationship between consumer surplus for the two demand functions?
   (**Hint:** Consumer surplus is the "whole triangle.")
   (e) [3 points] Graph these demand functions on a figure with "Price" on the vertical axis and "Quantity" on the horizontal axis.

Question 1: 20 points

Solution:

(a) The inverse demands are found by "solving" for p. For D_1:

$$
\begin{aligned}
q &= 10 - \frac{P_1(q)}{250} && \text{relabel } D_1(p) \text{ and } p \text{ to } q \text{ and } P_1(q) \\
\frac{P_1(q)}{250} &= 10 - q && \text{move } P_1(q) \text{ to the left side} \\
P_1(q) &= 2500 - 250q && \text{multiply by 250}
\end{aligned}
$$

Following the same process, the inverse for $D_2(p)$ is

$$P_2(q) = 1500 - 100q.$$

(b) To find the choke prices, solve the inverse demands for $q = 0$. Call these $\overline{p}_1$ and $\overline{p}_2$.

$$
\begin{aligned}
\overline{p}_1 &= P_1(0) = 2500 - 250 \cdot 0 = \$2,500 \\
\overline{p}_2 &= P_2(0) = 1500 - 100 \cdot 0 = \$1,500
\end{aligned}
$$

i. The choke price is the maximum price for which there is any quantity demanded for a good. Above this price, everybody leaves the market.

(c) To find the maximum quantity demanded, use the demand functions with $p = 0$. Call these $\overline{q}_1$ and $\overline{q}_2$:

$$\overline{q}_1 = D_1(0) = 10 - \frac{0}{250} = 10$$
$$\overline{q}_2 = D_2(0) = 15 - \frac{0}{100} = 15$$

(d) i. The price and quantities are in the table below:

p	$D_1(p)$	$D_2(p)$
\$100	9.6	14
\$500	8	10
\$1,000	6	5
\$1,500	4	0
\$2,000	2	0
\$2,500	0	0

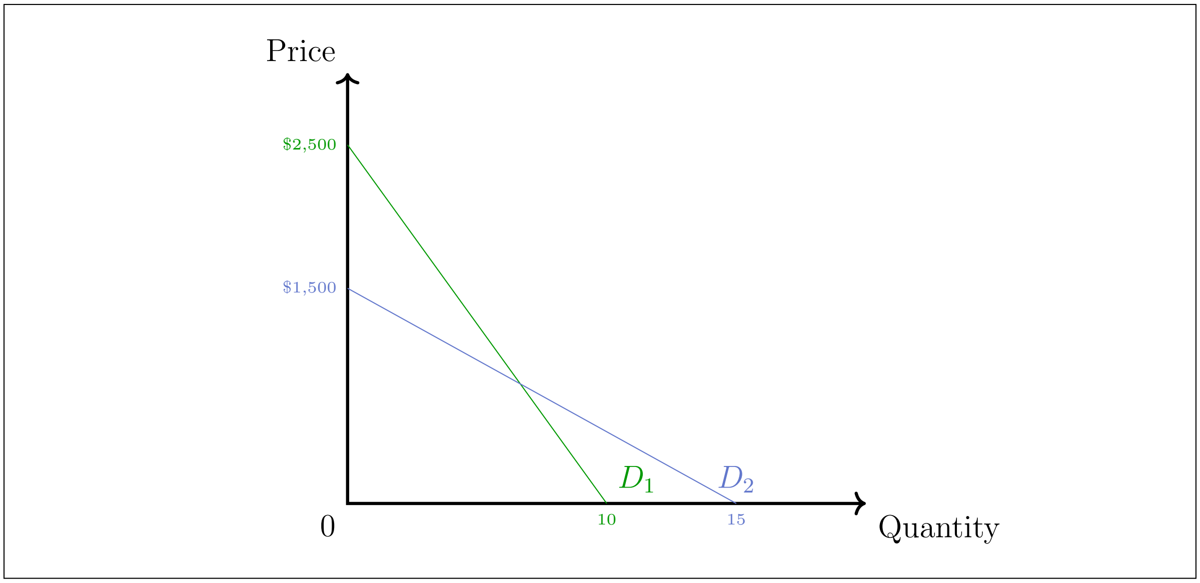
ii. The consumer surplus is given by the general formula

$$CS(p) = \frac{1}{2}\left(\overline{p}_i - p\right) \cdot D_i(p)$$

where i is either 1 or 2. The consumer surpluses are in the table below:

p	$CS_1(p)$	$CS_2(p)$
\$100	\$11,520	\$9,800
\$500	\$8,000	\$5,000
\$1,000	\$4,500	\$1,250
\$1,500	\$2,000	\$0
\$2,000	\$500	\$0
\$2,500	\$0	\$0

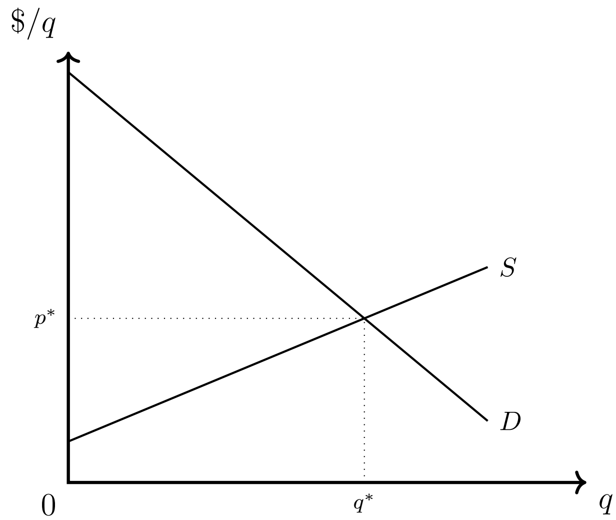
(e) The demands are shown in this figure:


Price
$2,500
$1,500
D_1
D_2
0
10
15
Quantity

2. Some policymakers promote taxes as a way to dis-incentivize undesireable behavior. In 2015, the city of Berkeley, CA imposed a tax on all sugar-sweetened beverages. In this exercise, you will work through the impacts of a buyer-side sales tax. Suppose we are given the inverse demand and supply functions for soda in Berkeley *before* the tax is implemented:

$$P(q) = 20 - q$$
$$S(q) = 2 + \frac{1}{2}q$$

where q is cans of soda. The market is shown graphically below:



The original equilibrium price is p^*. At this price, q^* cans of soda are in the market.

Now assume that a tax of $2 is placed on the purchasers. We will use intuition and logic to work through the impacts of this tax on the market.

(a) [2 points] Use the equilibrium condition $S(q^*) = P(q^*)$ to find the equilibrium quantity. Calculate the inverse demand at this quantity to get the equilibrium price. (that is, $p^* = P(q^*)$).

(b) [2 points] Calculate the choke price of the pre-tax inverse demand function.

(c) [4 points] Note that, with the tax in place, the market price of soda will have to decrease by $2 in order for the equilibrium quantity to be demanded. In order to continue consuming the equilibrium quantity, what would the market price need to be? Mark this point on your graph, labeled a.

(**Hint:** The market price is the price *not including* the tax. So if the consumers face a market price of $3, they will pay $4 in total).

(**Hint:** This point a should be on the dotted line connecting q^* to the original equilibrium, but the price will be $p^* - \$2$ instead of p^*.)

(d) [4 points] By similar reasoning, the choke price for the inverse demand function with the tax should be decreased by $2 as well. Mark this point on your graph, labeled b.

(**Hint:** The quantity demanded will still be 0, but the price will be \$2 less than the original choke price.)

(e) [2 points] Connect points a and b to create the hypothetical tax-inclusive demand curve. Label this demand curve D_{tax}.

(f) [4 points] Find the new equilibrium *quantity*, q^{**}, at the intersection of the hypothetical demand and supply.

(**Hint:** Calculate q^{**} using the condition $D_{\text{tax}}(q^{**}) = S(q^{**})$).

(g) [4 points] Now we need to determine what the true market price is, and what the effective price for the buyers is. We know that the new equilibrium quantity is q^{**}. Calculate the price that supplies this amount using the supply function. Label this price p_m^{**}.

(**Hint:** i.e., $p_m^{**} = S(q^{**})$).

(h) [4 points] You just calculated the market price, p_m^{**}. This price reflects the "sticker price" of the soda. The suppliers are selling their good for this price, so their profit is the triangle between p_m^{**} and S. Label this triangle PS on your graph. What is the value of the producer surplus?

(i) [4 points] To calculate consumer surplus, we need to understand what consumers are willing to pay at the new equilibrium quantity. Use the original inverse demand curve to find this price, and label it p_b^{**}.

(**Hint:** i.e., $p_b^{**} = P(q^{**})$).

(j) [4 points] The consumer surplus is the area below D and above p_b^{**}. Label this triangle CS on your graph. What is the value of the consumer surplus?

(k) [3 points] The "tax wedge" is the rectangle beteween p_b^{**} and p_m^{**} on the vertical axis and between 0 and q^{**} on the horizontal axis. Label this rectangle Tax on your graph. What is the value of total tax revenue?

(l) [3 points] The deadweight loss is the total surplus that was being enjoyed in the pre-tax equilibrium, but is neither collected as tax revenue nor enjoyed as surplus after the tax goes into effect. Label this triangle DWL on your graph. What is the value of the deadweight loss?

Solution:

(a) This is done algebraicly, where q^* is defined as the quantity satisfying

$$\begin{aligned} S(q^*) &= P(q^*) \\ 2 + \frac{1}{2}q^* &= 20 - q^* \\ \frac{3}{2}q^* &= 18 \\ q^* &= 12 \end{aligned}$$

Then using the inverse demand, the equilibrium price is

$$p^* = P(q^*) = 20 - q^* = 20 - 12 = \$18.$$

Note we could do this using the supply curve as well:

$$p^* = S(q^*) = 2 + \frac{1}{2}q^* = 2 + \frac{1}{2} \cdot 12 = \$8.$$

(b) The choke price is $P(0)$, so

$$\overline{p} = P(0) = 20 - 0 = \$20.$$

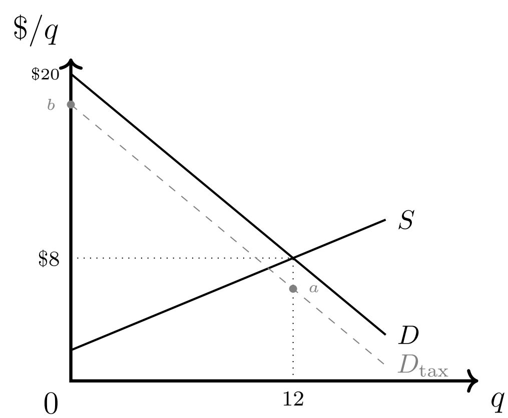
(c) Since the original equilibrium price is \$8, this means the market price should be \$6.

(d) Since the original choke price is \$20, this means the new choke price should be \$18.

(e) The inverse demand in the taxed world is

$$P_{\text{tax}}(q) = P(q) - \$2 = 18 - q.$$

See graph below:



(f) We use the equilibrium condition

$$\begin{aligned} D_{\text{tax}}(q^{**}) &= S(q^{**}) \\ 18 - q^{**} &= 2 + \frac{1}{2}q^{**} \\ \frac{3}{2}q^{**} &= 16 \\ q^{**} &= \frac{32}{3} \approx 10.67 \end{aligned}$$

(g) The price needed to supply q^{**} cans of soda is

$$p_m^{**} = S(q^{**}) = 2 + \frac{1}{2}q^{**} = 2 + \frac{1}{2} \cdot \frac{32}{3} = 2 + \frac{16}{3} = \frac{22}{3} \approx 7.33$$

(h) The producer surplus is

$$\begin{aligned} PS &= \frac{1}{2} \cdot (p_m^{**} - 2) \cdot q^{**} \\ &= \frac{1}{2}\left(\frac{22}{3} - 2\right) \cdot \frac{32}{3} \\ &= \frac{1}{2} \cdot \frac{16}{3} \cdot \frac{32}{3} \\ &= \frac{256}{9} \approx \$28.44 \end{aligned}$$

(i) The original inverse demand tells us what the consumers are willing to pay for the new equilibrium quantity:

$$p_b^{**} = 20 - \frac{32}{3} = \frac{28}{3} \approx \$9.33.$$

Note also that the price that consumer's pay is equal to the market price plus the tax:

$$p_b^{**} = p_m^{**} + \$2 = \frac{22}{3} + 2 = \frac{28}{3} \approx \$9.33.$$

(j) The consumer surplus is

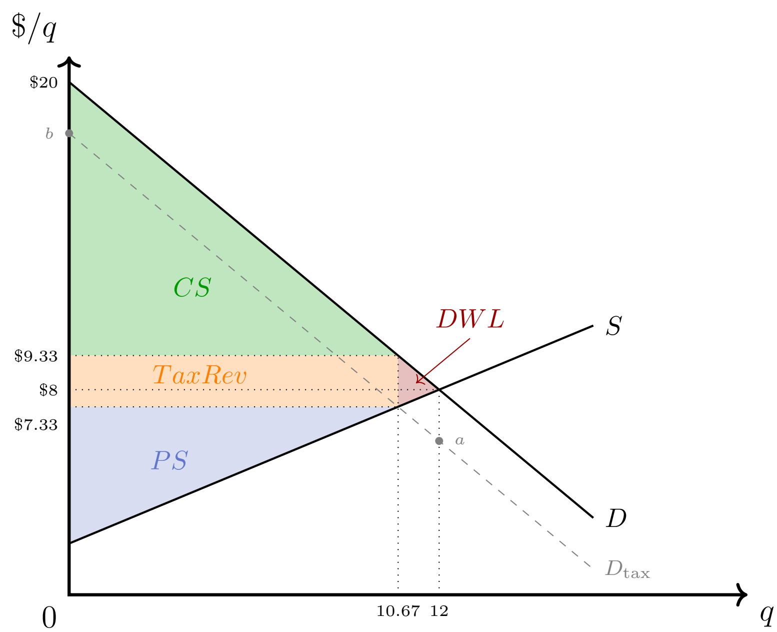
$$\begin{aligned} CS &= \frac{1}{2} \cdot (20 - p_b^{**}) \cdot q^{**} \\ &= \frac{1}{2} \cdot \left(20 - \frac{28}{3}\right) \cdot \frac{32}{3} \\ &= \frac{1}{2} \cdot \frac{32}{3} \cdot \frac{32}{3} \\ &= \frac{1024}{18} \\ &= \frac{512}{9} \approx \$56.89 \end{aligned}$$

(k) The tax revenue is simply the price difference times the quantity:

$$TaxRev = (p_b^{**} - p_m^{**}) \cdot q^{**} = \left(\frac{28}{3} - \frac{22}{3}\right) \cdot \frac{32}{3} = \frac{64}{3} \approx \$21.33.$$

(l) The deadweight loss is the total surplus that was generated before, that is no longer generated. This is the triangle with height $q^* - q^{**}$ and with base $p_b^{**} - p_m^{**}$.

$$\begin{aligned} DWL &= \frac{1}{2}\left(p_b^{**} - p_m^{**}\right) \cdot \left(q^* - q^{**}\right) \\ &= \frac{1}{2} \cdot \left(\frac{28}{3} - \frac{22}{3}\right) \cdot \left(12 - \frac{32}{3}\right) \\ &= \frac{1}{2} \cdot \frac{6}{3} \cdot \frac{4}{3} \\ &= \frac{4}{3} \approx \$1.33 \end{aligned}$$



Question 2: 40 points

3. Consider the case of free-to-visit parks. Because they are free, there is no price component of the supply function, just a fixed quantity supplied for all prices. Hence, the supply function is a vertical line that intercepts the horizontal axis at the quantity supplied.

   Suppose we are given the following willingness-to-pay function for free parks:

   $$WTP(q) = 15 - \frac{3}{4}q$$

   where q is the number of people that the park can accomodate. Consider that q is very closely related to park size, so that when the town chooses q, it is in essence choosing the number of acres to dedicate to its free park.

   (**Note:** Throughout this exercise, treat q as fixed at the quantity you determine in part (c). Once you choose this q, it cannot be changed.)

   (a) [2 points] The willingness-to-pay function is equivalent to the inverse demand function. What does this mean, in your own words?

   (b) [5 points] Calculate the choke price and the quantity demanded when the price is zero. Call these $\overline{p}$ and $\overline{q}$, respectively.

   (c) [8 points] Draw the willingness-to-pay function on a graph with quantity (q) on the horizontal axis and price ($\$/q$) on the vertical axis.

   The town's mayor asks you how many people the park should accomodate. You know that the park will be free to visit and you know the willingness-to-pay function for the parks. How many visitors should the *efficient* park accomodate?

   (**Hint:** Remember that the efficient quantity maximizes *total surplus*).

   (d) The mayor proposes to enact an entrance fee of \$3. The town will reinvest the revenues into the park. These investments improve the *quality* of the existing park, but they will not make the park any bigger. Hence, they will not affect the *supply* function.

   The mayor claims that by making such improvements, the park becomes a more desireable point of interest, garnering a higher willingness-to-pay. Therefore, while the once-free park will no longer be free, there will ultimately be welfare gains from this plan. The following steps will allow you to analyze his argument.

   i. [2 ½ points] What are some examples of investments that the town could make?

   ii. [5 points] Re-draw your graph from part 1, but now include the \$3 entry price. Assuming the willingness-to-pay function doesn't change, how many visitors will there be?

   (**Hint:** You may want to turn the WTP function into a quantity demanded function as a first step).

   iii. [5 points] There will be some amount of consumer surplus that is "transferred" to the parks in terms of revenue, and some amount of consumer surplus that is lost alltogether. How much consumer surplus is transferred, and how much consumer surplus is lost? Illustrate the lost consumer surplus on your figure and calculate the total lost consumer surplus.

iv. [2 ½ points] The mayor's intuition that reinvesting in the parks will cause an increase in demand turns out to be correct. The willingness-to-pay has increased, and the new willingness-to-pay is represented by the function

$$WTP_2(q) = 30 - \frac{3}{4}q.$$

Draw this new willingness-to-pay curve on your graph.

v. [5 points] Remembering that the quantity supplied is fixed, calculate the total consumer surplus and total fee revenue under the new willingness-to-pay curve.

vi. [5 points] Is it welfare-enhancing to charge an entrance fee of \$3 and reinvest that revenue into the parks? Provide a short analysis of the policy using your insights from the previous parts of this problem. To answer this question, tell me (i) what is the net change in consumer surplus, (ii) how much fee revenue is generated?

vii. [10 bonus points] If you had to choose an optimal number of visitors for this new park to accomodate, how many would you choose? In other words, what would be q^{**}?

viii. [10 bonus points] How much more fee revenue and consumer surplus would be gained by using this new optimal accomodaiton level?

Question 3: 40 points

Solution:

(a) The inverse demand function takes a *quantity* as an input and it outputs a *price*. In particular, that price is the price at which the quantity demanded is exactly equal to the quantity of input. Hence, the inverse demand function gives us the maximum amount that the consumer is willing to pay for the good.

(b) The choke price is the price when quantity demanded is zero:

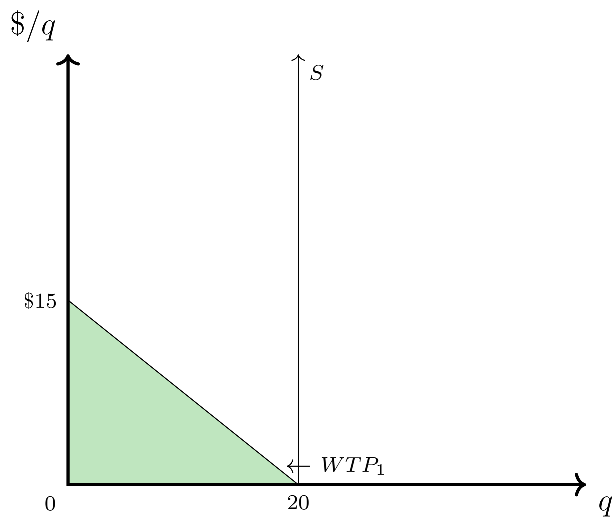
$$\overline{p} = WTP(0) = 15 - \frac{3}{4} \cdot 0 = \$15.$$

The quantity at which the willingness to pay for an additional park is found by "solving" $WTP(q) = 0$ for q:

$$\begin{aligned} WTP(\overline{q}) &= 0 \\ 15 - \frac{3}{4}\overline{q} &= 0 \\ \frac{3}{4}\overline{q} &= 15 \\ \overline{q} &= \frac{60}{3} \\ \overline{q} &= 20 \end{aligned}$$

(c) Because the parks are free to visit, the optimal quantity supplied is the quantity at which the willingness to pay for an additional park is \$0, so 20. The consumer surplus is the entire area under the willingness-to-pay function:

$$\begin{aligned} CS &= \frac{1}{2} \cdot \overline{p} \cdot 20 \\ &= \frac{1}{2} \cdot 15 \cdot 20 \\ &= \$150 \end{aligned}$$



(d) i. The number of visitors after the entrance fee is implemented is found by solving the demand function for $p = 3$. The demand functiton is $D(p) = 20 - \frac{4}{3}p$, so

$$D(3) = 20 - \frac{4}{3} \cdot 3 = 16.$$

There will be 16 visitors after the fee.

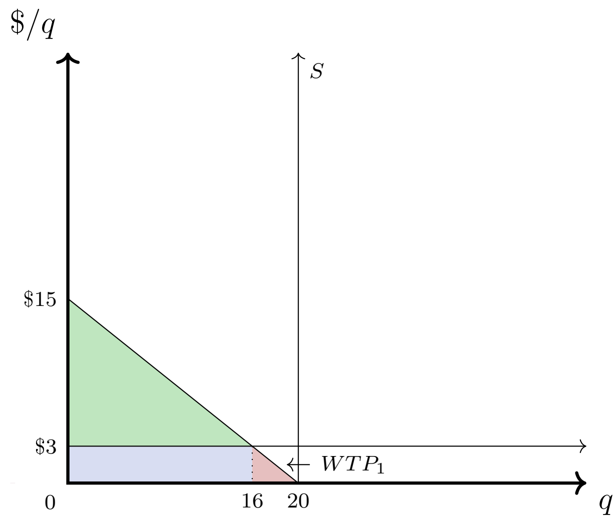
ii. To find the number of visitors, we first need to find the demand function from the willingness-to-pay function. Do this by "solving" for q as in question 1:

$$\begin{aligned} p &= 15 - \frac{3}{4} \cdot D(p) \\ \frac{3}{4}D(p) &= 15 - p \\ D(p) &= 20 - \frac{4}{3}p \end{aligned}$$

Given the \$3 entrance fee, the number of visitors is

$$q^* = D(3) = 20 - \frac{4}{3} \cdot 3 = 16.$$

This is shown in the figure below:



iii. The remaining visitors will provide revenue to the parks service in the form of their entry fees, so the transfer of consumer surplus to the parks is proportional to the new number of visitors. The fee revenue comes from consumer surplus that is "transferred" from the visitors to the park. In particular, the fee revenue is

$$FeeRev = \$3 \cdot 16 = \$48.$$

There is, however, some consumer surplus that is just purely lost. This comes from the visitors who no longer visit the park because their willingness to pay is less than the fee. This "pure" lost consumer surplus is the deadweight loss:

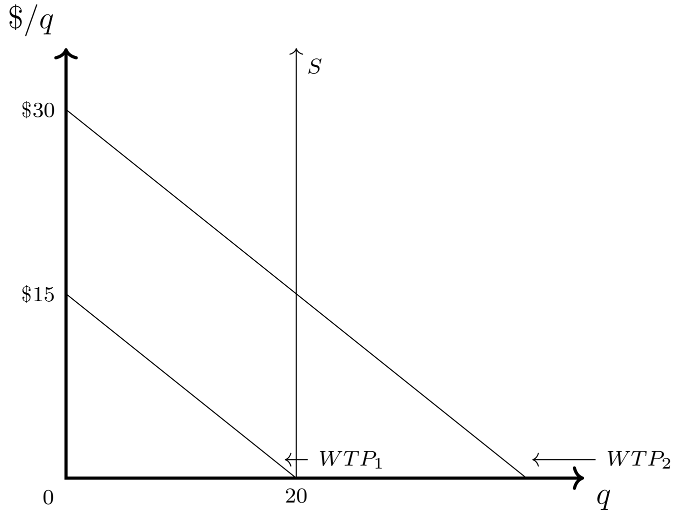
$$\begin{aligned} DWL &= \frac{1}{2} \cdot (20 - 16) \cdot \$3 \\ &= \frac{1}{2} \cdot 4 \cdot \$3 \\ &= \$6 \end{aligned}$$

The total lost consumer surplus is the sum of fee-revenue transfer plus the deadweight loss:

$$\Delta CS = \$48 + \$6 = \$54.$$

The lost consumer surplus is the area below

iv. The graph below shows the new willingness-to-pay curve:

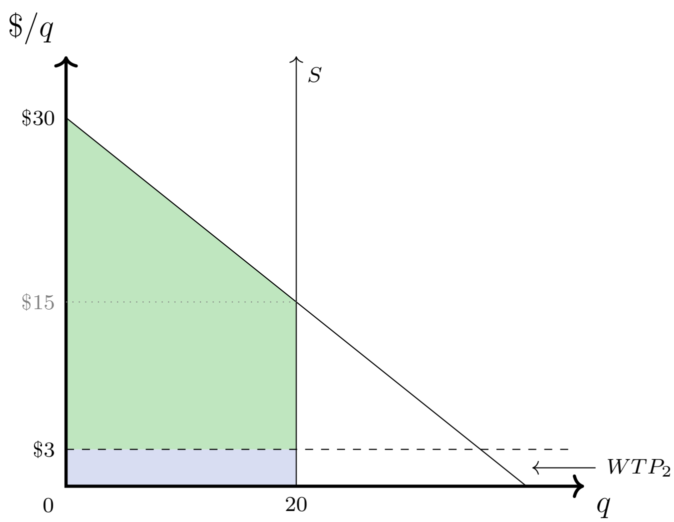


v. The fixed quantity supplied means that the park can accomodate 20 visitors. The total fee revenue is the revenue generated by 20 visitors:

$$FeeRev_2 = \$3 \cdot 20 = \$60.$$

Calculating the consumer surplus can be tricky. Note that the new willingness-to-pay curve and the supply curve intersect at a price of \$15. Hence, the consumer surplus is the sum of the square and the triangle separated at \$15.

$$\begin{aligned} CS_2 &= \frac{1}{2} \cdot (\$30 - \$15) + (\$15 - \$30) \cdot 20 \\ &= \frac{1}{2} \cdot \$15 \cdot 20 + \$12 \cdot 20 \\ &= \$150 + \$240 \\ &= \$390 \end{aligned}$$



vi. The consumer surplus with the \$3 entrance fee and the new willingness-to-pay curve is \$390, but the consumer surplus when the park was free is \$150. In addition to the new consumer surplus, the town generates \$60 of fee revenues, so the total gain is \$450. By implementing the mayor's plan, there is a gain in total surplus of \$450 and a loss of consumer surplus of \$150. Hence, the net surplus gain is \$300; this is a good policy.

vii. Given the increased willingness to pay, the optimal number of visitors for the park to accomodate is the quantity for which the willingness-to-pay is \$3. To find q^{**}, we need to get the demand function from the new willingness-to-pay function.

$$\begin{aligned} p &= 30 - \frac{3}{4} \cdot D_2(q) \\ \frac{3}{4} D_2(q) &= 30 - p \\ D_2(q) &= 40 - \frac{4}{3} p \end{aligned}$$

With this in hand, the new socially optimal quantity is easily found

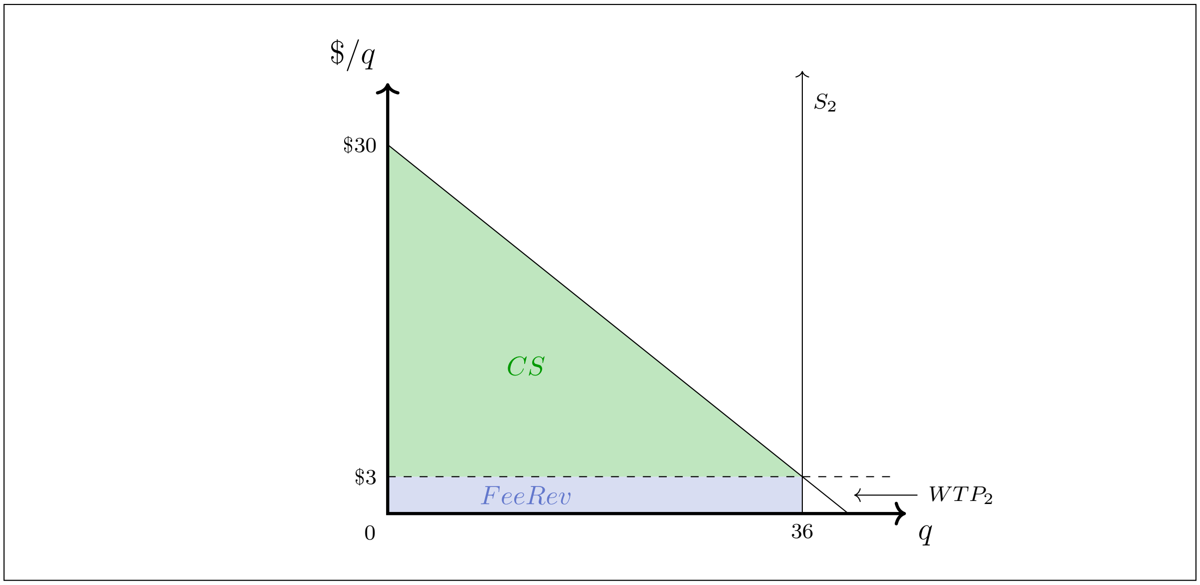
$$q^{**} = D_2(3) = 40 - \frac{4}{3} \cdot 3 = 36.$$

viii. If the town expands the park so that it can accomodate 36 visitors, the new fee revenue will be the total of the \$3 collected by them:

$$FeeRev = \$3 \cdot 36 = \$108.$$

Because the park can accomodate an additional 16 visitors, these new visitors will contribute more to the total consumer surplus. However, because they have lower willingness to pay than the earlier visitors, their marginal contribution to the consumer surplus is smaller. The new consumer surplus with the expanded park is

$$\begin{aligned} CS &= \frac{1}{2} \cdot (\$30 - \$3) \cdot 36 \\ &= \frac{1}{2} \cdot \$27 \cdot 36 \\ &= \$405 \end{aligned}$$


$\$/q$
$\$30$
S_2
CS
$\$3$
$FeeRev$
WTP_2
0
36
q