Name:

Group members: Last updated: 10/01/2025

ARE 336 – Problem set 2 – Externalities and public/common goods

Due **October 3, 2025**

This problem set has 4 questions, for a total of 26 points (with the possibility to earn up to 2 bonus points).

1. Many outdoor recreation activities take place on private land with the owner's approval. Organizations like the Carolina Climbers Coalition (CCC) work with landowners to keep these avenues of communication open to benefit us all. But what happens when a landowner doesn't want climbers on her land? Sometimes, like the Maibauer bouldering area, the CCC will raise funds and purchase this land.

   This sounds a lot like Coasian bargaining! This raises the question: How did the CCC know that this was a welfare-enhancing purchase? In this question, we will go through a stylized model of the CCC's decision process.

   Suppose that the owner of the Maibauer area has built a house on the land that she rents for passive income. This rental revenue reliably earns hear \$6,000 each year, but she needs to spend \$1,000 in upkeep. From the owner's perspective, as long as the property is earning a profit, she'll keep it; she has no familial ties to the land and no value for the land outside of its profitability. Assuming a discount rate of 5%, the net present value of the the land is

$$NPV = \sum_{t=0}^{\infty} \frac{1}{(1.05)^t} \cdot \$5,000 = \$105,000.$$

   This means that she is *just as happy* keeping the land as she is getting a \$105,000 cash payment right now.

   Now consider the CCC. They want to make an offer to the land owner, but they need to raise revenue and they only want to do this if they know that it will provide a net benefit to climbers. For simplicity, assume there are only two types of relevant climbers: locals and out-of-towners. Naturally, the locals stand to benefit much more than the out-of-towners if the CCC buys the land, but the out-of-towners will still benefit. Suppose there are $N_L = 2,000$ local climbers and $N_O = 4,000$ out-of-town climbers.

   (a) [1 point] Normally we talk about "marginal" benefits, but this time we are just talking about benefits. Why is this?

   > **Solution:** When we talk about marginal benefits, we are focused on the benefit of adding one more unit. Because this purchase is an "all or nothing" deal, we are not focused on the additional benefit of one more unit.

   (b) [2 points] Suppose that everybody knows exactly how good the bouldering is on the Maibauer property. Under this assumption, locals are willing to pay \$50 and out-of-towners are willing to pay \$10 to secure the land. If everybody made a donation to the CCC of an amount equal to their willingness to pay for access to the Maibauer boulders, how much revenue could the CCC collect? Is this enough to fund the purchase of the Maibauer land?

Solution: The total donation revenue is

$$\begin{aligned} TR &= N_L \cdot \$50 + N_O \cdot \$10 \\ &= 2,000 \cdot \$50 + 4,000 \cdot \$10 \\ &= \$100,000 + \$40,000 \\ &= \$140,000 \end{aligned}$$

This is enough to purchase the land from the current owner.

(c) Now suppose that the CCC isn't allowed to see the land before purchase and the quality of the bouldering (either "great" or "bad") is not known. Formally, suppose that the boulders are "great" with a 50% probability; hence, they are "bad" with a 50% probability. Suppose that, if the boulders are great, the locals value access to them at \$50 and the out-of-towners value the access at \$10. However, if the boulders are bad, the locals value having access to the boulders at \$25 and the out-of-towners value the access at \$1.

i. [1 point] Complete the following payoff matrix:

Climbers

Boulder quality	Locals	Out of towners
Good		
Bad		

(**Hint:** The payoff matrix represents the payoffs in the different states of the world. There is no expected values or probability in this.)

Solution: The benefits of accessing the boulders varies by both the "type" of climber and by the quality of the boulders:

Climbers

Boulder quality	Locals	Out of towners
Good	\$50	\$10
Bad	\$25	\$1

ii. [1 point] What is the expected benefit to each type of climber of purchasing the property? (**Hint:** A random variable X which can take on the value x_1 with probability p and can take on the value

x_2 with probability $1-p$ has an expected value $\mathbb{E}[X] = p \cdot x_1 + (1-p) \cdot x_2$.

Solution: The expected value for locals and out of towners is not the same. The locals receive a benefit of \$50 if the boulders turn out to be good, but only \$25 if the boulders turn out to be bad. Thus, their expected benefit is

$$\mathbb{E}[B_L] = 0.5 \cdot \$50 + 0.5 \cdot \$25 = \$37.5$$

The out of towners receive a benefit of \$10 if the boulders turn out to be good, but only \$1 if the boulders turn out to be bad. Thus, their expected benefit is

$$\mathbb{E}[B_O] = 0.5 \cdot \$10 + 0.5 \cdot \$1 = \$5.5$$

iii. [1 point] Assume that each climber makes a contribution to CCC equal to his expected benefit. Fill out the following payoff matrix:

Boulder quality \ Climbers	Locals	Out of towners
Good		
Bad		

What do you notice about the payoffs?
(**Hint:** Fill out the "net benefits" in the cells, assuming that the true quality of the boulders is labeled in the rows.)

Solution: The net benefit depends on the observed state of the world, but the donation of each climber depends only on his type. We can calculate the values of this cell by subtracting the expected benefit (i.e., the climber's donation) from the cells we put in the above payoff matrix.

Boulder quality \ Climbers	Locals	Out of towners
Good	\$12.5	\$4.5
Bad	-\$12.5	-\$4.5

Notice that the negative payoff locals receive when the boulders are bad are the exact same magnitude as the positive payoff they receive when the boulders are good. This is an artifact of boulder quality being a two-outcome random variable with equal probabilities; if there were more than two types of quality and/or $p \neq 0.5$, this "payoff equivalence" would not hold. (very interesting!)

iv. [1 point] Suppose that the CCC has to collect its funding before it can make a move to purchase the land. Thus everybody makes their contribution based on their *expected benefit* of having access to the land. How much revenue will the CCC raise? Is this enough to purchase the Maibauer property?

Solution: No the CCC will not raise enough revenue to buy the property:

$$TR = N_L \cdot \mathbb{E}[B_L] + N_O \cdot \mathbb{E}[B_O] = 2,000 \cdot \$37.5 + 4,000 \cdot \$5.5 = \$97,000.$$

Question 1: 7 points

2. Mimi-Radio is a public radio station broadcasting to the Triangle. It is freely broadcast on public FM airwaves. Assume that there are two types of consumers: Big Fans (BF) and Moderate Fans (MF). Their marginal benefit curves are as follows:

$$MB_{BF} = 10 - \frac{1}{2}q$$
$$MB_{MF} = 10 - \frac{3}{2}q$$

where q is the hours of programming listened to. The marginal cost of providing programming is a constant \$10 per hour.

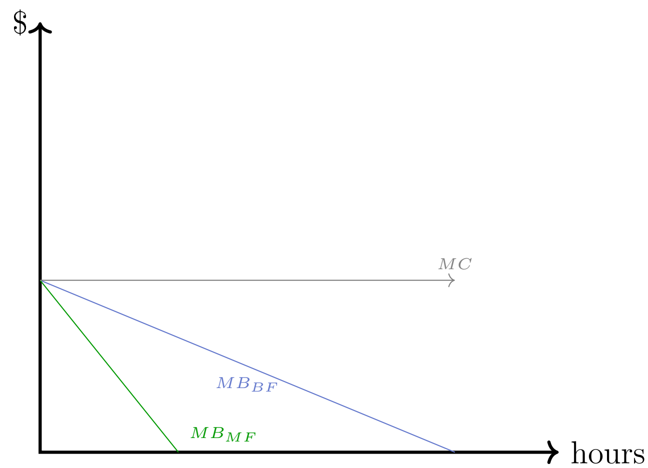
For simplicity, assume that the relative size of the two groups are normalized to be 1.

(a) [1 point] Is Mimi-Radio a public good? Explain.

> **Solution:** Yes, Mimi-Radio is a public good. It is for sure non-rivalrous because one person's consumption does not affect another's. It is non-excludable because it is broadcast on airwaves which are available to everybody.

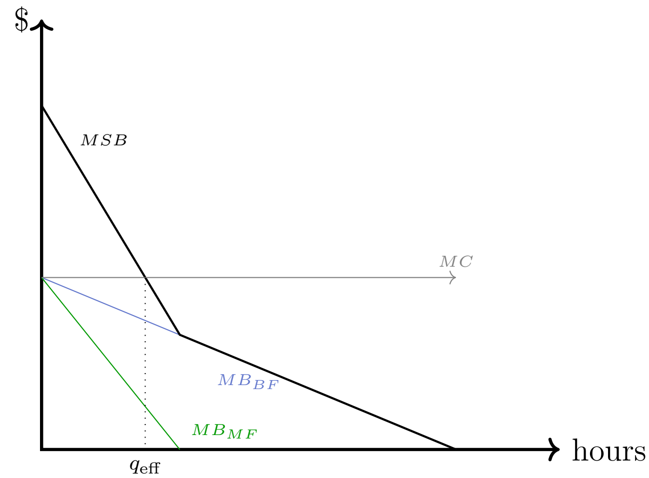
(b) [2 points] Draw the marginal benefit curves of both types of consumers and the marginal cost of radio provision on a graph with "hours" on the horizontal axis. Do a *vertical summation* of the two marginal benefit curves to create the social marginal benefit curve.

Assume that the two types of consumers have to supply all of their own Mimi-Radio (i.e., they have to pay the marginal cost of each q). How many hours of programming would the Big Fans, acting in their own self-interest, provide (if they had to purchase hours of Mimi-Radio)? What about the Moderate Fans?

> **Solution:**
>
> 
>
>
> Both types of fan would provide 0 hours of programming because their choke price is \$10.

(c) [2 points] What is the *efficient quantity* of radio programming?

Solution:



Because there is just 1 person of each type, the social marginal benefit is the sum:

$$SMB = MB_{BF} + MB_{MF} = 10 - \frac{1}{2}q + 10 - \frac{3}{2}q = 20 - 2q.$$

The efficient quantity equates marginal cost with the social marginal benefits. That is,

$$\begin{aligned} MC &= SMB(q_{\text{eff.}}) \\ 10 &= 20 - 2q_{\text{eff.}} \\ 2q_{\text{eff.}} &= 10 \\ q_{\text{eff.}} &= 5 \end{aligned}$$

(d) [1 point] Without intervention, why might we expect the provision of radio to be less than socially optimal?

Solution: Because nobody will produce anything! More generally, we can expect there to be suboptimal provision for public goods because of free riding.

Question 2: 6 points

3. Mountaineer Village (MV) is an apartment complex in Boone, North Carolina with 3,000 single-occupant units (each fully rented). MV is thinking of installing a swimming pool. If it does, it will add an additional charge to each unit's rent. Building and operating the pool will cost MV $5,000 per day.

   The occupants of the units have different valuations of the pool. These valuations represent the daily rent increase they are willing to accept for the pool. The breakdown of their willingness-to-pay is as follows:

No. individuals	WTP per day
1,000	$3
1,000	$2
1,000	$1

   Suppose also that the intended pool is large enough so that whatever number of occupants come on any day will not affect what people are willing to pay for the pool (i.e., there is no congestion externality). Also assume that only MV occupants will be allowed in the pool (i.e., no outsiders can come).

   (a) [1 point] Is this a situation of rivalry or non-rivalry?
   (**Hint:** Focus only on this model, not any prior expectations you may have about congestion.)

   > **Solution:** This is a situation of non-rivalry because the only impact on the households' utility is the existence of the pool. As far as we know, one person's consumption of the pool has no impact on anybody else's.

   (b) [1 point] Is this a situation of excludability or non-excludability?

   > **Solution:** This is a situation of excludability. The apartment complex can charge an entry fee or it can prevent people from entering the pool.

   (c) [1 point] Would building the pool be an efficient use of resources?
   (**Hint:** Remember that "efficiency" relies on the benefits and costs of something, not the actual funding scheme.)

   > **Solution:** Yes. The pool costs $5,000 to build, but it generates a total of $6,000 in consumer surplus. It has positive net benefits.

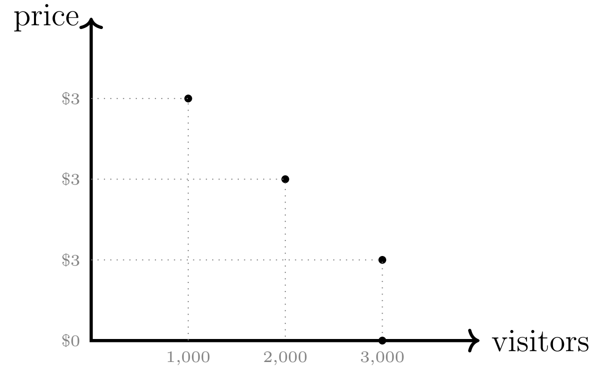
   (d) Consider four possible prices for admission to the pool: $3, $2, $1, $0.
   (**Hint:** These are prices to enter the pool – the occupants pay this everytime they visit the pool.)

      i. [1 point] What number of units will pay at each of these prices? Plot these points on a graph with "visitors" on the horizontal axis and "price" on the vertical axis.
      (**Hint:** These are individual pairs of price and quantity, do not connect them as a demand function.)

Solution: The visitation is broken down in the following table:

Price ($)	visitors
3	1,000
2	2,000
1	3,000
0	3,000

and is shown graphically:



ii. [1 point] Will any of these prices cover the cost of providing the pool?

Solution: The total revenue brought in is the price times the number of visitors at that price. Thus,

$$TR(p = 3) = \$3 \cdot 1,000 = \$3,000$$
$$TR(p = 2) = \$2 \cdot 2,000 = \$4,000$$
$$TR(p = 1) = \$1 \cdot 3,000 = \$3,000$$
$$TR(p = 0) = \$0 \cdot 3,000 = \$0$$

None of the prices will afford the construction of the pool.

iii. [1 point] Which of these prices will lead to the efficient number of visitors?
Recall that a **price** is efficient only if it is *not inefficient*, and a price is inefficient if there exists another price that can improve welfare without making anybody worse off. There can be more than one efficient price.
(**Hint:** Recall that the efficient quantity is *for sure* the quantity where price equals marginal cost.)

Solution: The efficient quantity occurs when the price equals the marginal cost for sure. The marginal cost is 0, so the efficient quantity is 3,000 visitors. Therefore, both $1 and $0 lead to the efficient number of visitors.

(e) [1 point] Is there any pricing scheme for admission to this pool that would both cover the pool's cost *and* achieve the efficient number of visitors?
(**Hint:** There are two possible answers here. Either one is sufficient as long as your answer is well-reasoned.)

Solution: If everybody needs to be charged the same price, no. The maximum possible revenue that the pool can earn with a uniform price is $4,000.

However, if the pool knows everybody's willingness to pay, then it can charge each visitor their exact willingness to pay. This is called "perfect price discrimination" and it allows the firm to "capture" all of the visitors' consumer surplus. By practicing perfect price discrimination, the pool can earn $6,000 which will cover the construction cost of the pool.

(f) [2 points] Suppose that this pool has a capacity of only 2,000 individuals per day. If more than 2,000 individuals come, the willingness to pay of each individual falls to $0.50 per day. Now what is the efficient pricing scheme for the pool?

Solution: The efficient pricing scheme maximizes social surplus.

Because the willingness to pay drops to $0.50 when the number of visitors surpasses 2,000, we should think about what the maximum possible consumer surplus is when more than 2,000 visitors come. There are only 3,000 residents in the apartment complex so the maximum possible surplus created when over 2,000 visitors come is $0.50 \cdot 3,000 = \$1,500$. We know from the questions above that we can get better consumer surplus when the price is $2.

We know that charging a price at or below $2 generates more consumer surplus than charging a price above $2. We also know that if we charge a price less than or equal to $1, there will be 3,000 visitors so the price leading to the efficient number of visitors must be greater than $1. There is no change in the number of visitors for prices between $2 and just over $1, so the efficient price here is any price p such that $1 < p \leq 2$.

(g) [1 bonus point] Suppose that MV can add an additional charge to its residents, but it has to charge everyone the same amount. Would a uniform rent increase of $1.67 and making the pool open to all renters be an efficient outcome?

Solution: We know this is not an efficient because only 2,000 visitors will come and the efficient quantity is 3,000.

(h) [1 bonus point] Suppose again that MV can add an additional charge to its residents, but it has to charge each unit the same amount. Would all renters be better off under a uniform rent increase of $1.67, relative to the scenario where there is no pool at all?

Solution: The renters with a willingness to pay below $1.67 will be worse off. Because there are 1,000 renters with a willingness to pay below $1.67, all renters would not be better off under this scenarios.

Question 3: 9 points

4. Assume that the constant marginal cost of providing a public pool is \$2. The enjoyment of pool users falls with more people at the pool such that the utility of individual i of visiting the pool is

$$u_i = 11 - Q$$

where Q is the number of people in the pool.

(a) [2 points] How many people, acting in their own self-interest, will enter the pool? What is the total social net benefit at this number?
(**Hint:** This is the "open access" number.)

Solution: When everybody is acting in their own self-interest, they will ignore the congestion externality that their presence creates. Individual i will enter the pool as long as $u_i(Q) \geq 0$. Thus, 11 people will enter the pool. The twelfth individual will not.

The total social benefits are the benefits of everybody at the pool minus the costs of providing the pool for that many people. The total benefits are $u_i(11) \cdot 11 = 0$ and the total costs are $\$2 \cdot 11 = \$22$.

Hence, the social net benefits are \$-22.

(b) [2 points] What would be the socially optimal number of people in the pool? What is the social net benefit at this number?
(**Hint:** This is the "efficient" number.)

Solution: The socially optimal number of visitors considers the private marginal benefits and the congestion externality. Thus the social marginal benefit of the Q^{th} individual is $u_i(Q) = 11 - Q - Q = 11 - 2Q$. The socially optimal number of visitors equates the marginal cost with the social marginal benefit.

$$\begin{aligned} 11 - 2Q_{\text{eff.}} &= 2 \\ 2Q_{\text{eff.}} &= 9 \\ Q_{\text{eff.}} &= 4.5 \end{aligned}$$

The social net benefits are the social benefits minus the cost:

$$\begin{aligned} SNB(Q_{\text{eff.}}) &= u_i(Q_{\text{eff.}}) \cdot Q_{\text{eff.}} - 2 \cdot Q_{\text{eff.}} \\ &= (11 - Q_{\text{eff.}}) \cdot Q_{\text{eff.}} - 2 \cdot Q_{\text{eff.}} \\ &= 6.5 \cdot 4.5 - 9 \\ &= \$20.25 \end{aligned}$$

Question 4: 4 points