Problem Set #1

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Problem 1 Classify a model from a journal (5 points).
Part (b). See References.
Part (c).

$$C(\pi_t) = \omega_t \Big[\frac{\pi_t}{1/2 - \pi_t}\Big]^{\kappa+1}, t = s, l \tag{1}$$

$$\widetilde{V}_a^{\beta}(\phi, x) = \ln(y) + \alpha_\phi \beta A + (1-\alpha_\phi)\beta\big\{Q(\phi)V_l^{\beta}(t,x) + \big[1 - Q(\phi)\big]V_l^{\beta}(\phi, x)\big\} \tag{2}$$

$$\begin{aligned}\widetilde{V}_s^{\beta}(\phi, x) =& \ln(y - z_s) + pI(s) + u\big[1 - I(s)\big] + \alpha_\phi \beta A \\ &+ (1-\alpha_\phi)\beta\big\{Q(\phi)V_l^{\beta}(t,x) + \big[1 - Q(\phi)\big]V_l^{\beta}(0, x)\big\}\end{aligned} \tag{3}$$

$$\begin{aligned}\widetilde{V}_s^{\beta}(1, x) =& \ln(y - z_s) + pI(s) + u\big[1 - I(s)\big] \\ &+ \sum_{\widehat{\phi}} R_s(\widehat{\phi})\big[1 - \gamma_s(\widehat{\phi})\big]\chi(c)\beta\big[qV_l^{\beta}(t,x) + (1-q)V_l^{\beta}(0,x)\big] \\ &+ \big\{1 - \sum_{\widehat{\phi}} R_s(\widehat{\phi})\big[1 - \gamma_s(\widehat{\phi})\big]\chi(c)\big\}\beta V_l^{\beta}(1,x)\end{aligned} \tag{4}$$

$$\begin{aligned}V_s^d =& \max_{\substack{0 \le \pi_u^d, \pi_p^d, \\ \pi_u^d + \pi_p^d < 1}} \big\{\pi_p^d \widetilde{V}_p^d(\phi, x) + \pi_u^d \widetilde{V}_u^d(\phi, x) + (1 - \pi_p^d - \pi_u^d)\widetilde{V}_a^d(\phi, x) \\ &- C(\pi_p^d) - C(\pi_u^d)\big\}, d = \iota, \beta\end{aligned} \tag{5}$$

$$\Upsilon(0, t | 1, \widehat{\phi}, c, \widehat{c}) = \big[1 - \gamma_u(\widehat{\phi})\big]\chi(c)(1-q)Q(\hat{\phi}) \tag{6}$$

$$\begin{aligned}\widetilde{V}_l^{\beta}(\phi, \widetilde{\phi}, \widetilde{c}, x) =& \ln(y - z_l) + u + l + \alpha_\phi \beta A \\ &+ (1-\alpha_\phi)(1-\epsilon)(1-\delta)(1-\alpha_{\widehat{\phi}}) \\ &\times \beta \sum_{\phi', \widehat{\phi}'} \Upsilon(\phi', \widehat{\phi}' | \phi, \widehat{\phi}, c, \widehat{c})\widetilde{V}_l^{\beta}(\phi', \widehat{\phi}', \widehat{c}, x) \\ &+ (1-\alpha_\phi)\big[1 - (1-\epsilon)(1-\delta)(1-\alpha_{\widehat{\phi}})\big] \\ &\times \beta \sum_{\phi', \widehat{\phi}'} \Upsilon(\phi', \widehat{\phi}' | \phi, \widehat{\phi}, c, \widehat{c}) V_l^{\beta}(\phi', x)\end{aligned} \tag{7}$$

$$\widetilde{V}_l^d(\phi, x) = \sum_{\widehat{\phi}, \widehat{c}} R_l(\widehat{\phi}, \widehat{c})\widetilde{V}_l^d(\phi, \widehat{\phi}, \widehat{c}, x) \tag{8}$$

$$V_l^d(\phi, x) = \max_{\pi_l^d} \big[\pi_l^d \widetilde{V}_l^d(\phi, x) + (1 - \pi_l^d)V_s^d(\phi, x) - C(\pi_l)\big] \tag{9}$$

This paper builds a choice-theoretic general equilibrium search model to analyze the HIV/AIDS epidemic in Malawi. To be specific, they divide the sexual relationships into three markets - long-term sex market, casual sex with condoms market, and casual unprotected sex market. In equilibrium, the prices as well as people's efforts to find a sex partner are determined. After that, the distribution of the health status in each period can be derived by the transition function. (1) is the cost function of people's searching efforts. (2), (3), and (4) are the value functions of the two casual sex markets and (5) describes the corresponding utility maximization problem. (6) is one of the transition functions (others are listed in appendix of the paper). (7) and (8) are the value functions of the long-term sex market and (9) describes the corresponding utility maximization problem. Note that in (2), (3), (4), and (7), the paper take $d = \beta$ as an example. If $d = \iota$, $V_l^{\beta}(\phi, x)$ on the right hand side is replaced by the expected value $\eta V_l^{\beta}(\phi, x) + (1-\eta) V_l^{\iota}(\phi, x)$.

Part (d). See Table 1 and Table 2.

Part (e). This model is dynamic, nonlinear, and deterministic.

Part (f). In the model, people are only different in terms of their time discount factors, sex as well as whether they are circumcised. However, social status of people may also affect their sex behavior. To be specific, people with higher social status may become more concerned about their sex behavior since doing casual sex could be a scandal for them if it is revealed to the public. In other words, people with higher social status may prefer long-term sex in order to avoid the risk of involving in a scandal. Moreover, social status also affect the income of people, which directly enters the value function. If social status is considered, the model would be better.

Table 1: Exogenous variables

Interpretation	Parameter
Quarterly transmission risk, unprotected sex, uncircumcised men	γ_u^m
Quarterly transmission risk, protected sex, uncircumcised men	γ_p^m
Reduction in prob. of infection if circumcised	χ
Quarterly transmission risk, unprotected sex, women	γ_u^f
Quarterly transmission risk, protected sex, women	γ_p^f
Prob. of displaying symptoms after infection	α
Quarterly death risk	δ
Prob. of death after displaying symptoms	δ_2
Prob. of divorce hazard	ϵ
Quarterly income	y
Flow utility unprotected sex	u
Flow utility protected sex	p
Flow utility long-term sex	l
Discount factors	ι, β
Value of life with AIDS	A
Prob. of switch to high discount factor	η
Search cost parameters	$\omega_s, \omega_l, \kappa$

Table 2: Endogenous variables

Interpretation	Variable
Price - protected, unprotected, long-term	z_p, z_u, z_l
Odds ratio of engaged in protected, unprotected, and long-term sex	$\pi_p^d, \pi_u^d, \pi_l^d$

Problem 2 Make your own model (5 points).
Before I build the model, I would like to first discuss the key factors that affect people's marriage decisions.

Part (d). The key factors should capture the following aspects of an individual: 1) basic features, such as age, race, education, occupation, and income (Becker 1973); 2) historical relationships, such as how many relationships he or she has been engaged in before, and whether he or she has been divorced; 3) openness towards casual sex, such as whether he or she supports casual sex, and whether he or she has tried casual sex.

Part (a). To predict whether an individual decides to get married or not, we can build a logistic regression model as follow:

$$\Pr(married_i = 1|X_i, Y_i, Z_i, u_i) = f(\beta_0 + X_i\beta_1 + Y_i\beta_2 + Z_i\beta_3 + u_i) \tag{10}$$

where $f(x) = \frac{\exp(x)}{1+\exp(x)}$, $married_i$ is a binary variable indicating whether the individual is married or not, X_i is a matrix of basic features of individual i, Y_i is a matrix of historical relationships of individual i, Z_i is a matrix of openness towards casual sex of individual i. To make use of the model, we need to use a training set to estimate the corresponding β's. After that, we could estimate the probability of marriage of each individual in our test set. For simplicity, we could assume that when the predicted probability is greater than some threshold, then the individual will get married.

Part (b). In the test set, the predicted result $married_i$ is the output of the model, so it is endogenous.

Part (c). As long as we have enough data to estimate the parameters, then the model is a complete data generating process.

Part (e). To determine which variables should be viewed as key variables, we could use existing data to estimate the parameters and see whether the parameters are statistically significant. If a factor is significant, then it may be one of the key factors. In addition, we could calculate the marginal change in probability of $married_i$ when a factor increases by one standard deviation. If the change in probability is quite large, then the factor may be one of the key factors.

Part (f). To do the preliminary test, we can divide the data into two parts. Then the parameters could be estimated with one part of data. After that, we could use the other part of data to see whether the model fits it well. If the model could successfully predict the result, it means that the factors we choose play a major role in the marriage decision of individuals, so they should be important in real life.

References

Becker, Gary S, "A theory of marriage: Part I," *Journal of Political economy*, 1973, *81* (4), 813–846.

Greenwood, Jeremy, Philipp Kircher, Cezar Santos, and Michèle Tertilt, "An equilibrium model of the African HIV/AIDS epidemic," *Econometrica*, 2019, *87* (4), 1081–1113.