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# Operations Research

Patrick Gustav Blaneck

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# 1 Linear Programming

## Definition: Linear Programming

*Linear Programming* is the problem of optimizing (maximizing or minimizing) a *linear objective function* subject to a set of *linear functional constraints*.

**Given:**  $A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m, c \in \mathbb{R}^n$

**Find:**  $x^* \in \mathbb{R}^n$  where  $x^* = \arg \max \{c^T x \mid Ax \leq b\}$

## Bonus: Linear Programming Solvers

Software that solves linear programs - *linear programming solvers* - also generate lots of important auxiliary information (as well as the optimum):

- sensitivity analysis
- shadow prices
- alternative optima
- ...

## Theorem: Ellipsoid Method

A LP of dimension  $n$  can be solved in  $\mathcal{O}(L^2 \cdot n^6)$  time [khachiyan1979], where  $L = \#$  bits in the input.

## Theorem: Interior Point Method

A LP of dimension  $n$  can be solved in a *numerically stable* way in  $\mathcal{O}(L^2 \cdot n^{3.5})$  time [karmarkar1984].

## Definition: Integer Linear Programs (ILP)

**Given:**  $A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m, c \in \mathbb{R}^n$

**Find:**  $x^* \in \mathbb{Z}^n$  where  $x^* = \arg \max \{c^T x \mid Ax \leq b\}$

## Example: Integer Linear Program for VERTEX COVER

### VERTEX COVER

**Given:** Graph  $G = (V, E)$

**Find:** VERTEX COVER, i.e.  $V' \subseteq V$  such that every edge has at least one endpoint in  $V'$ .

### Integer Linear Program:

For  $v \in V$ , let  $x_v \in \{0, 1\}$ .

Goal: minimize  $\sum_{v \in V} x_v$ .

Constraints: for every edge  $uv \in E$ , we require  $x_u + x_v \geq 1$ .