https://matse.paddel.xyz/spicker

Operations Research

Patrick Gustav Blaneck

Last Change: April 8, 2021

Contents

1 Linear Programming

Definition: Linear Programming

Linear Programming is the problem of optimizing (maximizing or minimizing) a *linear objective function* subject to a set of *linear functional constraints*.

Given: $A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m, c \in R^n$

Find: $x^* \in \mathbb{R}^n$ where $x^* = \arg\max\{c^T x \mid Ax \leq b\}$

Bonus: Linear Programming Solvers

Software that solves linear programs - *linear programming solvers* - also generate lots of important auxiliary information (as well as the optimum):

- sensitivity analysis
- shadow prices
- alternative optima
- ...

Theorem: Ellipsoid Method

A LP of dimension n can be solved in $\mathcal{O}(L^2 \cdot n^6)$ time [2], where $L = \#$ bits in the input.

Theorem: Interior Point Method

A LP of dimension n can be solved in a *numerically stable* way in $\mathcal{O}(L^2 \cdot n^{3.5})$ time [1].

Definition: Integer Linear Programs (ILP)

Given: $A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m, c \in R^n$

Find: $\underline{x^* \in \mathbb{Z}^n}$ where $x^* = \arg\max\{c^T x \mid Ax \leq b\}$

Example: Integer Linear Program for Vertex Cover

Vertex Cover
Given: Graph $G = (V, E)$
Find: Vertex Cover, i.e. $V' \subseteq V$ such that every edge has at least one endpoint in V'.

Integer Linear Program:

For $v \in V$, let $x_v \in \{0, 1\}$.

Goal: minimize $\sum_{v \in V} x_v$.

Constraints: for every edge $uv \in E$, we require $x_u + x_v \geq 1$.

References

[1] Narendra Karmarkar. "A new polynomial-time algorithm for linear programming". In: *Proceedings of the sixteenth annual ACM symposium on Theory of computing*. 1984, pp. 302–311.

[2] Leonid Genrikhovich Khachiyan. "A polynomial algorithm in linear programming". In: *Doklady Akademii Nauk*. Vol. 244. 5. Russian Academy of Sciences. 1979, pp. 1093–1096.