106 年台大單操輸送

Problem 1

【Solution】

(a)

(1) $\upsilon \,[=]\, m^2/s$ **(2)** $Sh \,[=]\, 1$ **(3)** $\varepsilon \,[=]\, m$

(4) $D_{AB} \,[=]\, m^2/s$ **(5)** $\tau_{yx} \,[=]\, kg/m\cdot s^2$

(b)黏滯力最主要的來源為分子間引力，若溫度升高，因為分子間距離將增加，因此造成分子間引力下降，使黏度下降。

(c) $$\underset{\substack{\text{rate of increase of}\\ \text{momentum per}\\ \text{unit volume}}}{\rho\frac{\partial \vec{v}}{\partial t}} + \underset{\substack{\text{rate of momentum}\\ \text{addition by convection}\\ \text{per unit volume}}}{\rho\vec{v}\cdot\nabla\vec{v}} = \underbrace{-\nabla P + \mu\nabla^2\vec{v}}_{\substack{\text{rate of momentum}\\ \text{addition by molecule transport}\\ \text{per unit volume}}} + \underset{\substack{\text{rate of momentum}\\ \text{addition by external force}}}{\rho\vec{g}}$$

(d) By shell balance of energy,

$$q''A\big|_x - q''A\big|_{x+dx} + \dot{q}\,Adx = 0$$

同除 $Adx \to 0$，

$$\frac{dq''}{dx} = -k\frac{d^2T}{dx^2} = \dot{q}$$

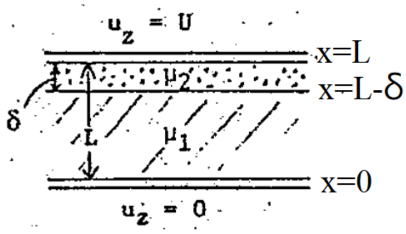
由圖之二階導數判斷：$\underbrace{\dot{q}_A = \dot{q}_B}_{=0,\ conduction\ rate=constant} < \dot{q}_C$

則 $|q_2''| = |q_3''| > |q_4''|$

((d)小題可參考：Incropera, F.; Dewitt, D.; Bergman, T.; Lavine, A. *Fundamentals of Heat and Mass Transfer*, 6th ed.; p 182, Problem 3.71.)

Problem 2

【Solution】



Assume： (1) laminar flow

(2) Incompressible Newtonian fluid

(3) no pressure gradient

(4) no gravitation force in the z direction

For **Case B**, by the equation of continuity,

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \vec{v}) = 0$$

∵ incompressible, ρ is constant

$$\nabla \bullet (\vec{v}) = \underbrace{\cancel{\frac{\partial v_x}{\partial x}}}_{v_x=0} + \underbrace{\cancel{\frac{\partial v_y}{\partial y}}}_{v_y=0} + \frac{\partial v_z}{\partial z} = 0$$

$$v_z = f(x) \text{ only}$$

By shell balance of momentum,

$$\rho v_z v_z dxdy\big|_z - \rho v_z v_z dxdy\big|_{z+dz} + \tau_{xz} dydz\big|_x - \tau_{xz} dydz\big|_{x+dx} = 0$$

同除 $dxdydz \to 0$

$$\underbrace{\cancel{\frac{-\partial(\rho v_z v_z)}{\partial z}}}_{v_z=f(z)} - \frac{\partial \tau_{xz}}{\partial x} = 0 \text{，} \tau_{xz} = -\mu \frac{dv_z}{dx} = c_1$$

$$\begin{cases} v_{z1} = \dfrac{-c_{11}}{\mu_1} x + c_{21} \\ v_{z2} = \dfrac{-c_{12}}{\mu_2} x + c_{22} \end{cases} \text{，B.C.：} \begin{cases} x = 0 \;\; v_{z1} = 0 \\ x = (L-\delta) \;\; v_{z1} = v_{z2} \\ x = (L-\delta) \;\; \tau_{xz1} = \tau_{xz2} \\ x = L \;\; v_{z2} = U \end{cases}$$

$$\begin{cases} c_{21} = 0 \\ c_{11} = c_{12} = \dfrac{-U}{\left(\dfrac{L-\delta}{\mu_1} + \dfrac{\delta}{\mu_2}\right)} \\ c_{22} = U + \dfrac{c_{11}}{\mu_2} L \end{cases}$$

$$F_B = \tau_{xz}\big|_L = -\mu_2 \frac{dv_{z2}}{dx}\bigg|_L = c_{12} = \frac{-U}{\left(\dfrac{L-\delta}{\mu_1} + \dfrac{\delta}{\mu_2}\right)}$$

For **Case A**, $\mu_1 = \mu_2$

$$F_A = \tau_{xz}\big|_L = \frac{-U}{\left(\dfrac{L-\delta}{\mu_1} + \dfrac{\delta}{\mu_2}\right)} = \frac{-U}{\left(\dfrac{L-\delta}{\mu_1} + \dfrac{\delta}{\mu_1}\right)} = \frac{-\mu_1 U}{L}$$

$$\frac{F_A}{F_B} = \frac{\left(L - \delta + \dfrac{\delta\mu_1}{\mu_2}\right)}{L}$$

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 56.)

Problem 3

【Solution】

假設反應：$C+O_2 \rightarrow CO_2$，$\delta=1-1=0$，反應前後氣體總莫耳數不變

(a) $W_{O_2}=1\ (mol\ /\ h)$, let the mass rate taken by the plant is x

$$\frac{1\times 0.21-x}{1}=0.16 \text{ ， } x=0.05\ (mole/h)$$

$$\underline{\underline{N_{O_{2,taken}}=\frac{x}{400}=1.25\times 10^{-4}\ \ (mole/h\cdot cm^2)}}$$

(b) Same us above,

$$\frac{3\times 0.21-x}{3}=0.19 \text{ ， } x=0.06\ (mole/h)$$

$$\underline{\underline{N_{O_{2,taken}}=\frac{x}{400}=1.5\times 10^{-4}\ \ (mole/h\cdot cm^2)}}$$

(c) 不同意，在此兩個氧氣提供速率下，氧氣提供速率也許皆小於代謝速率，但光從此兩個實驗無法證明這點。

Problem 4

【Solution】

(a)

$$Pr = \frac{\upsilon}{\alpha} = \frac{momentum\ diffusivity}{thermal\ diffusivity}$$

$$= \left(\frac{\delta}{\delta_t}\right)^3 = \frac{momentum\ boundary\ layer\ thickness}{thermal\ boundary\ layer\ thickness}$$

$$Re_x = \frac{\rho < v > x}{\mu} = \frac{inertial\ \ force}{viscous\ \ force}$$

$$Nu_x = \frac{h_x x}{k} = \frac{convective\ heat\ transfer}{conductive\ heat\ transfer}\ rate\ in\ radial\ direction$$

∵the temperature profile is assumed to be linear function of $\frac{y}{\delta_t}$

the temperature profile,

$$T = (T_\infty - T_s)\frac{y}{\delta_t} + T_s$$

$$\frac{d}{dx}[\int_0^{\delta_t}(T_\infty - T_s + \frac{(T_s - T_\infty)y}{\delta_t})(\frac{v_\infty y}{\delta})dy] = \alpha(\frac{T_\infty - T_s}{\delta_t})$$

$$\frac{d}{dx}[\frac{v_\infty(T_\infty - T_s)}{2\delta}\delta_t^2 + \frac{(T_s - T_\infty)\delta_t^2 v_\infty}{3\delta}] = \alpha(\frac{T_\infty - T_s}{\delta_t})$$

$$\frac{d}{dx}[v_\infty(\frac{\delta_t}{6\delta})\delta_t] = \frac{\alpha}{\delta_t}$$ ，令 $\frac{\delta_t}{\delta} = \Delta \neq f(x)$ 代入

$$v_\infty \frac{\Delta}{6}\frac{d\delta_t}{dx} = \frac{\alpha}{\delta_t}$$ ，$\delta_t = \sqrt{\frac{\alpha}{v_\infty}\left(\frac{12}{\Delta}\right)x}$

∵ $\delta = \sqrt{\frac{12\upsilon x}{v_\infty}}$ ，$\frac{\delta_t}{\delta} = \sqrt{\frac{\alpha}{\upsilon}\left(\frac{1}{\Delta}\right)} = \Delta$

$\Delta^3 = \frac{\alpha}{\upsilon} = \mathrm{Pr}^{-1}$ ，$\Delta = \frac{\delta_t}{\delta} = \mathrm{Pr}^{-\frac{1}{3}}$ ，$\delta_t = \delta\,\mathrm{Pr}^{-\frac{1}{3}} = \underline{\underline{\sqrt{\frac{12\upsilon x}{v_\infty}}\,\mathrm{Pr}^{-\frac{1}{3}}}}$

(c)

@y=0, conduction rate=convection rate

$$-k\left.\frac{dT}{dy}\right|_{y=0} = h_x(T_s - T_\infty)$$

$$-k(T_\infty - T_s) = h_x\delta_t(T_s - T_\infty)$$

$$\frac{h_x}{k} = \frac{1}{\delta_t} = \sqrt{\frac{v_\infty}{12\upsilon x}}\,\mathrm{Pr}^{\frac{1}{3}}$$

$$\frac{h_x x}{k} = Nu_x = \underline{\underline{\sqrt{\frac{1}{12}}\,\mathrm{Re}_x^{\frac{1}{2}}\,\mathrm{Pr}^{\frac{1}{3}}}}$$

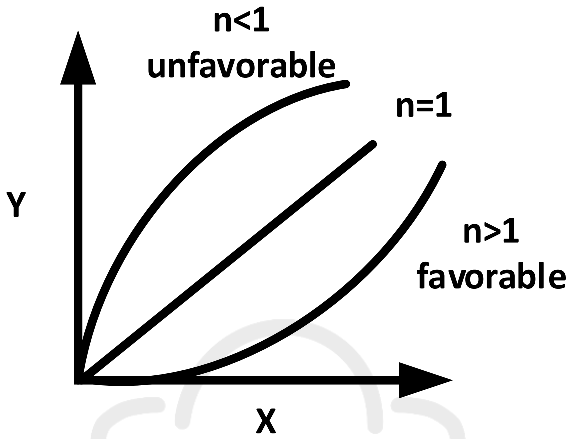
Problem 5

【Solution】

(a)

Freundlich isotherm,

$$Y = cX^n$$



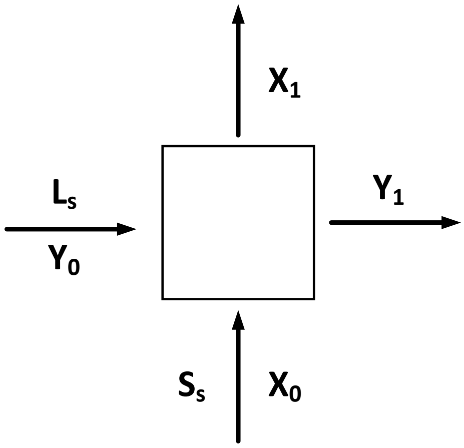
$$\begin{cases} n<1,\ unfavorable \rightarrow \text{需要高濃度}(X)\text{才有顯著的吸附效果}(Y) \\ n=1,\ linear \\ n>1,\ \ favorable \rightarrow \text{只需要低濃度}(X)\text{就有顯著的吸附效果}(Y) \end{cases}$$

根據對照，題目之 n 值為 1.5 > 1，***favorable***

※本題可在 McCabe 原文書第 7 版 p 840~841 找到類似的觀念討論，只是要注意 McCabe 所定義之 XY 定義與題目相反，因此 n 與吸附性的關係與題目相反

(本題可參考：Gavhane, K. A. *Mass Transfer* － *II*, 1st ed; p 3.6.)

(b)



$$\begin{cases} X_0 = 0 \\ L_S = 1000\ (kg) \\ Y_0 = 9.6\ (kg/kg) \\ Y_1 = 0.96\ (kg/kg) \end{cases}$$

By mass balance,

$$S_S(X_1 - X_0) = L_S(Y_0 - Y_1)$$

$$X_1 = \left(\frac{Y_1}{1.0\times10^{-4}}\right)^{\frac{1}{1.5}} = \left(\frac{0.96}{1.0\times10^{-4}}\right)^{\frac{1}{1.5}} = 451.70\ (kg/kg) \quad \text{代回}$$

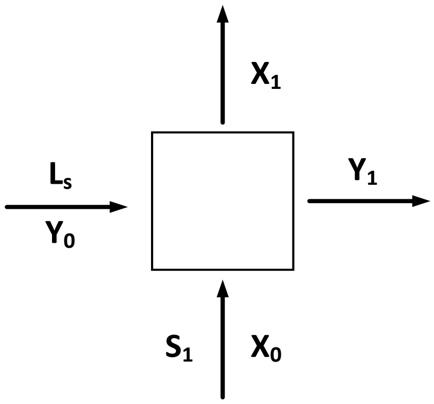
$$S_S(451.70 - 0) = 1000(9.6 - 0.96)$$

$$\underline{\underline{S_S = 19.12\ (kg)}}$$

(本題改編自：Gavhane, K. A. *Mass Transfer* － *II*, 1st ed; p 3.39~3.42, Example 3.4.)

(c)

First stage,



$$\begin{cases} X_0 = 0 \\ L_S = 1000\ (kg) \\ Y_0 = 9.6\ (kg/kg) \end{cases}$$

By mass balance,

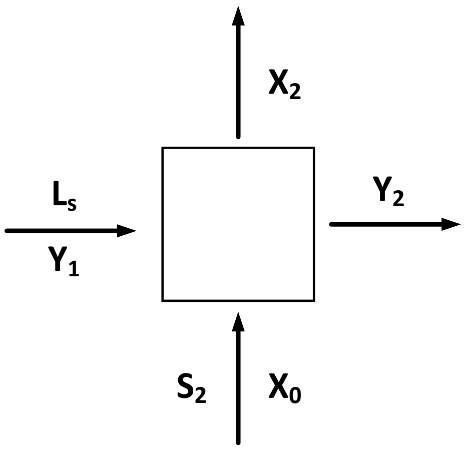
$$S_1(X_1 - X_0) = L_S(Y_0 - Y_1)$$

$$Y_1 = 1.0\times10^{-4} X_1^{1.5} \text{ 代入}$$

$$S_1(X_1 - 0) = 1000(9.6 - 1.0\times10^{-4} X_1^{1.5})$$

$$S_1 = \frac{9600}{X_1} - 0.1X_1^{0.5}$$

Second stage,



$$\begin{cases} X_0 = 0 \\ L_S = 1000\ (kg) \\ Y_2 = 0.96 \end{cases}$$

By mass balance,

$$S_2(X_2 - X_0) = L_S(Y_1 - Y_2)$$

$$Y_2 = 1.0\times10^{-4} X_2^{1.5} \text{ 代入}$$

$$S_2(X_2 - 0) = 1000(1.0\times10^{-4} X_1^{1.5} - 1.0\times10^{-4} X_2^{1.5})$$

$$S_2 = \frac{0.1}{X_2}(X_1^{1.5} - X_2^{1.5})$$

The total amount of carbon is,

$$S_1 + S_2 = \frac{9600}{X_1} - 0.1X_1^{0.5} + \frac{0.1}{X_2}(X_1^{1.5} - X_2^{1.5})$$

For minimum amount of carbon, let

$$\frac{d(S_1+S_2)}{dX_1} = 0$$

$$\frac{-9600X_2X_1^{0.5} - 0.05X_1^2X_2 + 0.15X_1^3}{X_1^{2.5}X_2} = 0 \text{ ， } X_2 = \frac{0.15X_1^3}{9600X_1^{0.5} + 0.05X_1^2}$$

又

$$X_2 = \left(\frac{Y_2}{1.0\times10^{-4}}\right)^{\frac{1}{1.5}} = \left(\frac{0.96}{1.0\times10^{-4}}\right)^{\frac{1}{1.5}} = 451.70\ (kg/kg)$$

可解出

$$X_1 = 1027.84\ (kg/kg)$$

代回：

$$(S_1+S_2)_{\min} = \frac{9600}{1027.84} - 0.1\times(1027.84)^{0.5} + \frac{0.1}{451.70}(1027.84^{1.5} - 451.70^{1.5}) = \underline{\underline{11.30\ (kg)}}$$

(本題可參考：Gavhane, K. A. *Mass Transfer* － *II*, 1st ed; p 3.38~3.39.)

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Problem 1

【Solution】

(1) a type of equipment measuring pressure.

(2) terminal

(3) steady

(4) fluid

(5) the velocity distribution is independent of the direction of the flow

(6) the transition time from unsteady-state to steady-state

(7) the type of the flow where the effect of viscous force can be neglected

(8) Bernoulli

(9) mechanics

(10) parabolic

(11) Ergun

(一) potential

(二) immiscible

(三) porosity

(四) reciprocating

(五) laminar

(六) creeping

(七) dilatant

(八) Reynolds

(九) Compressible

(十) hydraulic

Problem 2

【Solution】

A. True

B. False

For pool boiling, it is an example of natural convection

C. False

By Stefan-Boltzmann law, the radiation rate $\propto T^4$

D. True

E. True

F. False

Nusselt's film condensation theory describes the heat transfer in the case of laminar film condensation on vertical surface

G. True

H. False

$$Sh \equiv \frac{\text{mass convection rate}}{\text{mass diffusion rate}} \text{ in radial direction} = \frac{k_c L}{D_{AB}}$$

I. False

Both analogies are applicable in heat and mass transfer

J. True

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.;

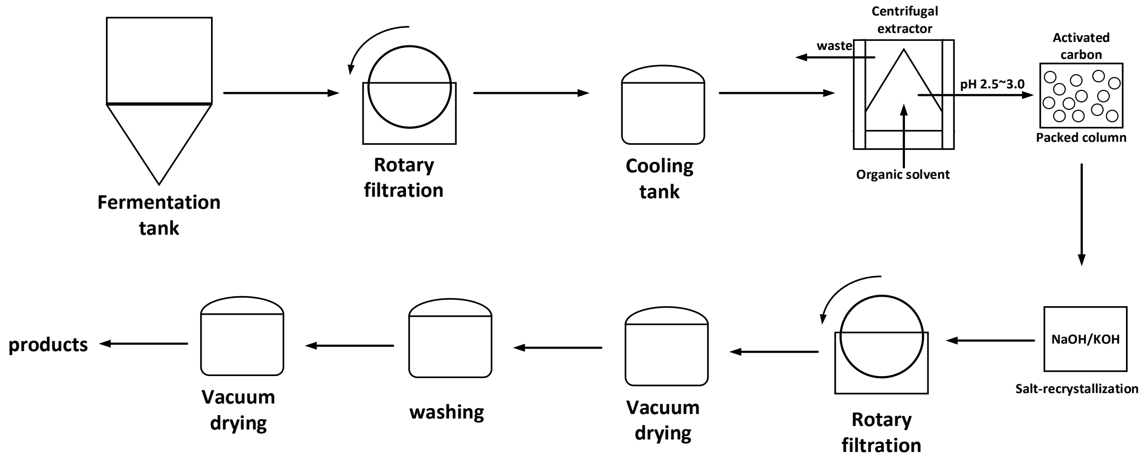
A：p 282/C：491/J：148.

Incropera, F.; Dewitt, D.; Bergman, T.; Lavine, A. *Fundamentals of Heat and Mass Transfer*, 6th ed.; D：p 690/643.

Welty, J.; Rorrer, G.; Foster, D. Fundamentals of Momentum, Heat, and Mass Transfer, 5th ed.; B：p 323/F：p 328/H：p 520/I：p.534、539.)

Problem 3

【Solution】



(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.;

p 70, Problem 2C.4.)

Problem 4

【Solution】

Forced convection relation,

$$Nu = f(\mathrm{Re}, \mathrm{Pr})$$

而

$$Nu = \frac{hL}{k}$$

$$\mathrm{Re} = \frac{\rho < v > L}{\mu}$$

$$\mathrm{Pr} = \frac{v}{\alpha} = \frac{\mu C_P}{k}$$

Total variables： $h, L, k, \rho, < v >, \mu, C_P$

Fundamental units： M, L, T, θ

共可決定 7-4=3 個無因次群，定義為 π_1, π_2, π_3

令 repeating variables： L, μ, k, ρ

For π_1

$$\pi_1 = h \times L^a \times \mu^b \times k^c \times \rho^d$$

$$\pi_1 = (M^1 T^{-3} \theta^{-1}) \times (L)^a \times (ML^{-1}T^{-1})^b \times (MT^{-3}\theta^{-1}L)^c \times (ML^{-3})^d = 1$$

$$\begin{cases} M：1+b+c+d=0 \\ L：a-b+c-3d=0 \\ T：-3-b-3c=0 \\ \theta：-1-c=0 \end{cases} \Rightarrow \begin{cases} a=1 \\ b=0 \\ c=-1 \\ d=0 \end{cases}$$

$$\pi_1 = \frac{hL}{k} = Nu$$

For π_2

$$\pi_2 = \langle v \rangle \times L^e \times \mu^f \times k^g \times \rho^h$$

$$\pi_2 = (LT^{-1}) \times (L)^e \times (ML^{-1}T^{-1})^f \times (MT^{-3}\theta^{-1}L)^g \times (ML^{-3})^h = 1$$

$$\begin{cases} M：f+g+h=0 \\ L：1+e-f+g-3h=0 \\ T：-1-f-3g=0 \\ \theta：-g=0 \end{cases} \Rightarrow \begin{cases} e=1 \\ f=-1 \\ g=0 \\ h=1 \end{cases}$$

$$\pi_2 = \frac{\langle v \rangle L\rho}{\mu} = \mathrm{Re}$$

For π_3

$$\pi_3 = C_P \times L^i \times \mu^j \times k^k \times \rho^l$$

$$\pi_3 = (L^2 T^{-2} \theta^{-1}) \times (L)^i \times (ML^{-1}T^{-1})^j \times (MT^{-3}\theta^{-1}L)^k \times (ML^{-3})^l = 1$$

$$\begin{cases} M：j+k+l=0 \\ L：2+i-j+k-3l=0 \\ T：-2-j-3k=0 \\ \theta：-1-k=0 \end{cases} \Rightarrow \begin{cases} i=0 \\ j=1 \\ k=-1 \\ l=0 \end{cases}$$

$$\pi_3 = \frac{C_P \mu}{k} = \mathrm{Pr}$$

則 3 個無因次群： $Nu, \mathrm{Re}, \mathrm{Pr}$

Problem 5

【Solution】

※注意題目給的圖有錯，內部圓柱的速度(U)應為 2m/s，非圖上所示之 0.1m/s

(a)(c)

Assume：　(1)laminar flow

(2)P=P(z)

By the equation of continuity

$$\frac{\partial \rho}{\partial t}+\nabla\bullet(\rho\vec{v})=0$$

∵ incompressible fluid, ρ is constant

$$\nabla\bullet(\vec{v})=\cancel{\underbrace{\frac{1}{r}\frac{\partial}{\partial r}(rv_r)}_{v_r=0}}+\cancel{\underbrace{\frac{1}{r}\frac{\partial}{\partial \theta}v_\theta}_{v_\theta=0}}+\cancel{\frac{\partial}{\partial z}v_z}=0$$

$$\vec{v}_z=f(r)\,\text{only}$$

By Navier-Stokes equation,

$$\rho(\underbrace{\cancel{\frac{\partial \vec{v}_z}{\partial t}}}_{s.s}+\underbrace{\cancel{\vec{v}_r}}_{\vec{v}_r=0}\frac{\partial \vec{v}_z}{\partial r}+\underbrace{\cancel{\frac{\vec{v}_\theta}{r}}}_{\vec{v}_\theta=0}\underbrace{\cancel{\frac{\partial \vec{v}_z}{\partial \theta}}}_{\vec{v}_z=f(r)}+\vec{v}_z\underbrace{\cancel{\frac{\partial \vec{v}_z}{\partial z}}}_{\vec{v}_z=f(r)})=\mu[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial \vec{v}_z}{\partial r})+\underbrace{\cancel{\frac{\partial^2 \vec{v}_z}{\partial \theta^2}}}_{\vec{v}_z=f(r)}+\underbrace{\cancel{\frac{\partial^2 \vec{v}_z}{\partial z^2}}}_{\vec{v}_z=f(r)}]-\frac{\partial p}{\partial z}+\rho\overline{g_z}$$

$$\mu\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_z}{\partial r})-\frac{\partial p}{\partial z}+\rho g_z=0$$

$$\mu\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_z}{\partial r})=\frac{\partial P}{\partial z}\begin{cases}r=r_i & v_z=U\\ r=r_o & v_z=0\end{cases}$$，積分兩次

$$v_z=\frac{r^2}{4\mu}\frac{\partial P}{\partial z}+c_1\ln r+c_2$$

代入 B.C. $$\begin{cases}c_1=\dfrac{1}{\ln(\frac{r_i}{r_0})}[U-\dfrac{1}{4\mu}\dfrac{\partial P}{\partial z}(r_1^2-r_0^2)=-4.9326-3.0829\dfrac{\partial P}{\partial z}\\ c_2=5.419+1.1368\dfrac{\partial P}{\partial z}\end{cases}$$

$$v_z=\frac{r^2}{4}\frac{\partial P}{\partial z}+(-4.9326-3.0829\frac{\partial P}{\partial z})\ln r+5.419+1.1368\frac{\partial P}{\partial z}$$

For $\frac{\partial P}{\partial z}$, $\because$ bottom is closed, then no fluid can exit the cylinder, $Q = 0$

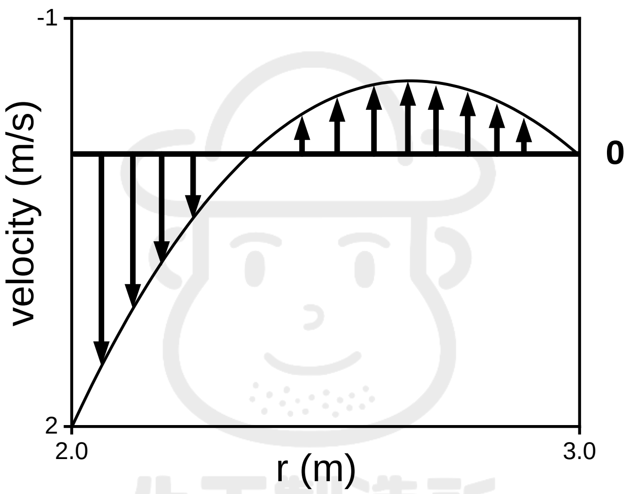
$$2\pi\int_{r_0}^{r_i} v_z r dr = 2\pi\int_{r_0}^{r_i}[\frac{r^2}{4}\frac{\partial P}{\partial z} + (-4.9326 - 3.0829\frac{\partial P}{\partial z})\ln r + 5.419 + 1.1368\frac{\partial P}{\partial z}]rdr = 0$$

$$(-0.2092\frac{\partial P}{\partial z} + 2.1657) = 0$$

$$\frac{\partial P}{\partial z} = 10.352 \text{，代回}$$

$$v_z = \underline{\underline{2.588r^2 - 36.8377\ln r + 17.1875}}$$

(b)



(※本題較簡易的想法為，因上下蓋關閉，因此在速度往下的同時，勢必會有一區往上流已平衡，直到處碰管壁(r=3)時回到 0。形狀部分則因有壓力影響呈現拋物線分布)

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 70, Problem 2C.4.)

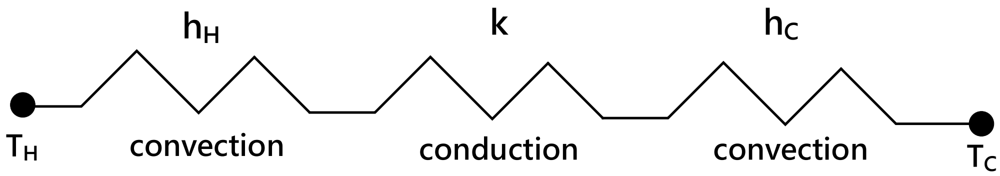
Problem 6

【Solution】

$$\because q=\frac{\Delta T}{\sum R}$$

$$\Delta T=T_H-T_C$$

For $\sum R$, by 串聯

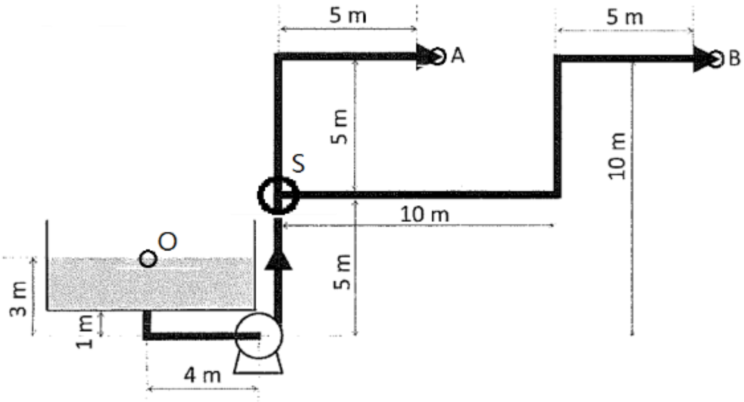


$$\sum R=\frac{1}{2\pi R_i L h_H}+\frac{\ln\frac{R_o}{R_i}}{2\pi kL}+\frac{1}{2\pi R_o L h_C}$$

$$q=\frac{\Delta T}{\sum R}=\underline{\underline{\frac{T_H-T_C}{\frac{1}{2\pi R_i L h_H}+\frac{\ln\frac{R_o}{R_i}}{2\pi kL}+\frac{1}{2\pi R_o L h_C}}}}$$

Problem 7

【Solution】



(a)

$\because Q, A$相同，$v_{suction} = v_S = 6\ (m/s)$

By mass balance,

$$\dot{m}_S = \dot{m}_A + \dot{m}_B$$

$$\frac{\pi}{4} \times D^2 \times 6 = \frac{\pi}{4} \times D^2 \times v_A + \frac{\pi}{4} \times D^2 \times v_S$$

Assume Hagen Poiseuille equation is applicable,

$$Q_A : Q_B = \frac{\Delta P_{SA}}{L_{SA}} : \frac{\Delta P_{SB}}{L_{SB}} = \frac{1atm - P_S}{L_{SA}} : \frac{1atm - P_S}{L_{SB}} = \frac{1}{L_{SA}} : \frac{1}{L_{SB}} = \frac{1}{10} : \frac{1}{20} = 2:1 = v_A : v_B$$

$$\frac{\pi}{4} \times D^2 \times 6 = \frac{\pi}{4} \times D^2 \times v_A + \frac{\pi}{4} \times D^2 \times \frac{1}{2} v_A$$

$$\begin{cases} v_A = 4\ (m/s) \\ v_B = \dfrac{1}{2} v_A = 2\ (m/s) \end{cases}$$

<check> The validity of Hagen-Poiseuille equation (laminar flow)

$$\text{Re}_S = \frac{\rho v_S D}{\mu} = \frac{1000 \times 6 \times 0.1}{0.001} = 6 \times 10^5 \quad \text{(Turbulent)}$$

$$\text{Re}_A = \frac{\rho v_A D}{\mu} = \frac{1000 \times 4 \times 0.1}{0.001} = 4 \times 10^5 \quad \text{(Turbulent)}$$

$$\text{Re}_B = \frac{\rho v_B D}{\mu} = \frac{1000 \times 2 \times 0.1}{0.001} = 2 \times 10^5 \quad \text{(Turbulent)}$$

由此可知此題假設從頭到尾皆為 turbulent flow 較為合理，且

$$\frac{k}{D}=\frac{0.002}{0.1}=0.02 \text{，turbulent 範圍內 } f=0.0125$$

By mass balance,

$$v_A=6-v_B$$

By mechanical energy balance of A, from S to A

$$\dot{m}_A[\frac{g}{g_c}(h_S-h_A)+\frac{P_S-P_A}{\rho}+\frac{1}{2g_c}(v_S^2-v_A^2)-h_f^{SA}]=0$$

$$g(h_S-h_A)+\frac{P_S-P_A}{\rho}+\frac{1}{2}(v_S^2-v_A^2)-h_f^{SA}=0-----(1)$$

同理，By mechanical energy balance of B, from S to B

$$g(h_S-h_B)+\frac{P_S-P_B}{\rho}+\frac{1}{2}(v_S^2-v_B^2)-h_f^{SB}=0-----(2)$$

$\because P_A=P_B=P_{atm}$ 、 $h_B=h_A$ ，(1)-(2)

$$\frac{1}{2}(v_B^2-v_A^2)+(h_f^{SB}-h_f^{SA})=0$$

For $h_f^{SB}-h_f^{SA}$,

$$h_f^{SB}-h_f^{SA}=\frac{4f}{2D}[L_{SB}v_B^2-L_{SA}v_A^2]=\frac{4\times 0.0125}{2\times 0.1}[20v_B^2-10v_A^2]$$

代回，並代入 mass balance,

$$\frac{1}{2}[v_B^2-(6-v_B)^2]+\frac{4\times 0.0125}{2\times 0.1}[20v_B^2-10(6-v_B)^2]=0$$

$$\boxed{\begin{cases} v_A=3.45\ (m/s) \\ v_B=2.55\ (m/s) \end{cases}}$$

(b)

By mechanical energy balance equation,

$$\dot{m}_o(\frac{g}{g_c}h_o+\underbrace{\cancel{\frac{P_o}{\rho}}}_{P_0=P_A=P_B=P_{atm}}+\underbrace{\cancel{\frac{1}{2g_c}v_o^2}}_{storage\ is\ large\ enough}+\dot{W}_s-h_f^{OS})-\dot{m}_A h_f^{SA}-\dot{m}_B h_f^{SB}$$

$$=\dot{m}_A(\frac{g}{g_c}h_A+\cancel{\frac{P_A}{\rho}}+\frac{1}{2g_c}v_A^2)+\dot{m}_B(\frac{g}{g_c}h_B+\cancel{\frac{P_B}{\rho}}+\frac{1}{2g_c}v_B^2)$$

$$6\times(gh_o+\dot{W}_s-h_f^{OS})-3.45h_f^{SA}-2.55h_f^{SB}=3.45(gh_A+\frac{1}{2}v_A^2)+2.55(gh_B+\frac{1}{2}v_B^2)$$

$$\begin{cases} h_f^{OS}=4\times0.0125\times\frac{10}{0.1}\times\frac{6^2}{2}=90 \\ h_f^{SA}=4\times0.0125\times\frac{10}{0.1}\times\frac{3.45^2}{2}=29.776 \text{ 代回} \\ h_f^{SB}=4\times0.0125\times\frac{20}{0.1}\times\frac{2.55^2}{2}=32.483 \end{cases}$$

$$6\times(9.8\times3+\dot{W}_s-90)-3.45\times29.776-2.55\times32.483$$
$$=3.45(9.8\times10+\frac{1}{2}\times3.45^2)+2.55(9.8\times10+\frac{1}{2}\times2.55^2)$$

$$\dot{W}_s=194.33\ (m^2/s^2)\text{ ， }P_P=\frac{194.33}{0.7}\times\frac{\pi}{4}\times0.1^2\times6\times1000=\underline{\underline{13082.36\ (J/s)}}$$

(c)

$$NPSH=\frac{g_c}{g}(\frac{P_0-P_v}{\rho}-h_L)+h_0$$

$$=\frac{\frac{(1-0.02)\times1.013\times10^5}{1000}-4\times0.0125\times\frac{4+1}{0.1}\times\frac{6^2}{2}}{9.8}+3=8.53>6\ (m)$$

<u>This pump is able to work without cavitation.</u>

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Problem 1

【Solution】

(a) By mass balance of the urea in the blood,

$$\text{removing rate} = 200\times1.90-195\times1.75=\underline{\underline{38.75\ (mg/min)}}$$

(b) The removed urea from the blood will enter into the dialyzing fluid, so the concentration of urea in the dialysate will be,

$$\frac{38.75}{1250}=\underline{\underline{0.031\ (mg/ml)}}$$

(c)urea concentration$=\dfrac{原有urea-去除urea}{溶液體積}$

$$\frac{5\times10^{3}(ml)\times2.9(mg/ml)-38.75(mg/min)\times60(min/hr)\times3.5hr}{5\times10^{3}(ml)}=\underline{\underline{1.2725\,(mg/ml)}}$$

Problem 2

【Solution】

Assume： (1) laminar, incompressible flow

(2) $p = p(z)$ *only*

By continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \vec{v}) = 0$$

$\because$ incompressible, $\nabla \cdot \vec{v} = 0$

$$\nabla \bullet (\vec{v}) = \underbrace{\frac{1}{r}\frac{\partial}{\partial r}(r v_r)}_{v_r = 0} + \underbrace{\frac{1}{r}\frac{\partial}{\partial \theta} v_\theta}_{v_{\theta = 0}} + \frac{\partial}{\partial z} v_z = 0$$

$$v_z = f(r) \text{ only}$$

By shell balance of momentum in the z-direction,

$$\rho v_z v_z r dr d\theta\big|_z - \rho v_z v_z r dr d\theta\big|_{z+dz} + \tau_{rz} r d\theta dz\big|_r - \tau_{rz} r d\theta dz\big|_{r+dr}$$

$$+ p r dr d\theta\big|_z - p r dr d\theta\big|_{z+dz} + \rho g r dr d\theta dz = 0$$

同除 $dr d\theta dz \to 0$,

$$\underbrace{\cancel{\frac{-\partial(r \rho v_z v_z)}{\partial z}}}_{v_z \neq f(z)} - \frac{\partial(r \tau_{rz})}{\partial r} - \frac{\partial(rp)}{\partial z} + \rho g r = 0$$

$$\frac{\partial(r\tau_{rz})}{\partial r} = -\frac{\partial(rp)}{\partial z} + \rho g r = \frac{-\partial P}{\partial z} r$$

$$\tau_{rz} = \frac{r}{2}\left(\frac{-\partial P}{\partial z}\right) + \frac{c_1}{r}$$

$\because r = \lambda R$, v_z reaches maximum, $\tau_{rz} = 0$ (*free surface*)

$$c_1 = \frac{-\lambda^2 R^2}{2}\left(\frac{-\partial P}{\partial z}\right) \text{ 代回}$$

$$\underline{\underline{\tau_{rz} = \frac{r}{2}\left(\frac{-\partial P}{\partial z}\right) - \frac{\lambda^2 R^2}{2r}\left(\frac{-\partial P}{\partial z}\right)}}$$

At $r=R$,

$$\underline{\underline{\tau_{rz}\big|_{r=R}=\frac{R}{2}\left(\frac{-\partial P}{\partial z}\right)-\frac{\lambda^2 R}{2}\left(\frac{-\partial P}{\partial z}\right)}}$$

※(1)此題不可使用下面附的 Navier-Stokes equation，因其是在 Newtonian fluid 條件下所推得

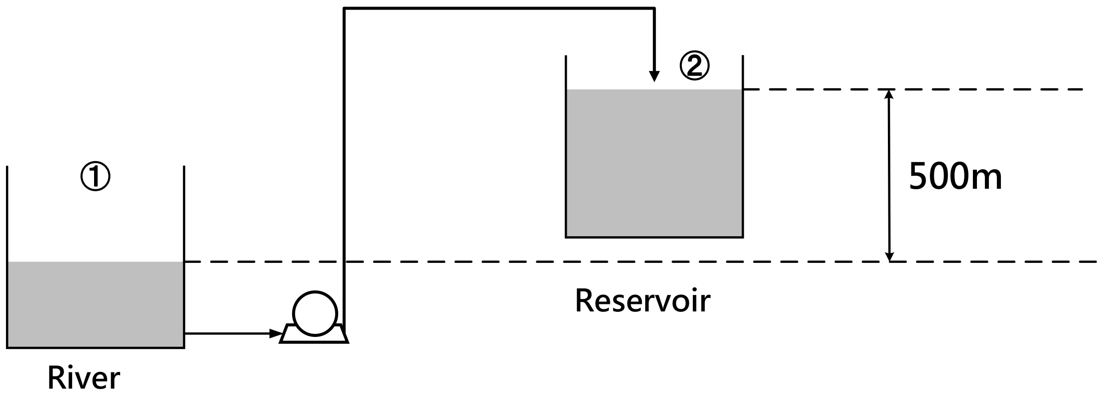
(2)本題只討論 shear stress 分布，不一定需要討論 $\frac{dv_z}{dr}$ **與距離之關係，若題目進一步要求速度分布，則一定要討論**

(本題可參考：Chen, Y.; Chen, L.; Tung, K *Semianalytical Solution for Power-Law Polymer Solution Flow in a Converging Annular Spinneret*; 2015.)

Problem 3

【Solution】

設①為 river，設②為 reservoir



For each pipe,

$$Q = 20000\ (gal/\min) \times \frac{1\ (ft^3)}{7.48\ (gal)} \times \frac{1\ (\min)}{60\ (\sec)} = 44.56\ (ft^3/s)$$

(a) By mechanical energy balance equation,

$$\dot{W}_s = \frac{1}{2g_c}(v_1^2 - v_2^2) + \frac{g}{g_c}(z_1 - z_2) + \cancel{\frac{1}{\rho}(P_1 - P_2)} - h_f$$

For v_2,

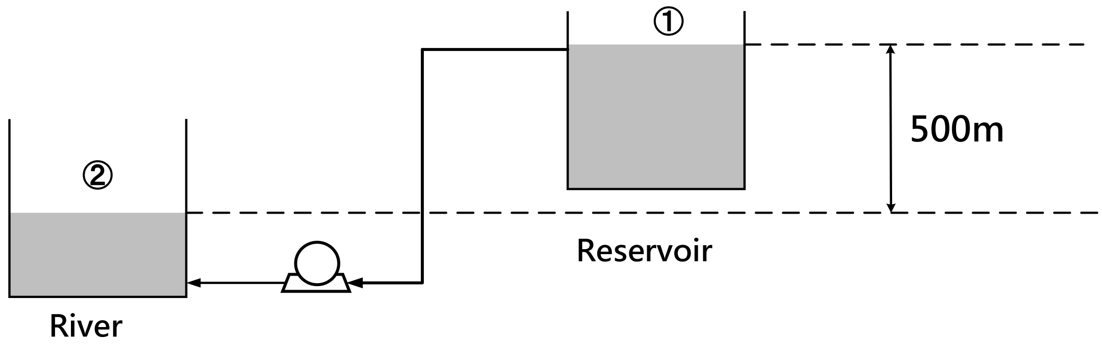
$$Q = 44.56 = \frac{\pi}{4}\left(\frac{30\ in}{12\ in/ft}\right)^2 v_2 \text{ ， } v_2 = 9.078\ (ft/s) \text{ 代回}$$

$$\dot{W}_s = \frac{1}{2 \times 32.174}(0^2 - 9.078^2) + 32.174(0 - 500) - 32.174(15)$$

$$\dot{W}_s = -16570.89\ (ft^3/s^2)$$

$$\dot{P}_P = \left|\frac{\dot{W}_s \dot{m}}{\eta} \times 2\right| = \left|\frac{\dot{W}_s \rho Q}{\eta} \times 2\right| = \left|\frac{-16570.89 \times 62.4 \times 44.56}{0.85} \times 2\right| = \underline{\underline{1.084 \times 10^8\ (lb - ft^2/s)}}$$

設①為 reservoir，設②為 river



(b) By mechanical energy balance equation,

$$\dot{W}_s = \frac{1}{2g_c}(v_1^2 - v_2^2) + \frac{g}{g_c}(z_1 - z_2) + \cancel{\frac{1}{\rho}(P_1 - P_2)} - h_f$$

$$\dot{W}_s = \frac{1}{2\times 32.174}(0^2 - 9.078^2) + 32.174(500-0) - 32.174(15)$$

$$\dot{W}_s = 15605.67\ (ft^3/s^2)$$

設 pump 效率同樣為 0.85

$$\dot{P}_t = \left|\dot{W}_s\,\dot{m}\times 2\right| = \left|\dot{W}_s\,\rho Q\times 2\right| = \left|15605.67\times 0.85\times 62.4\times 44.56\times 2\right| = \underline{\underline{7.376\times 10^7\ (lb-ft^2/s)}}$$

(c)

Overall efficiency, $\eta_o \equiv \dfrac{water\ power\ (output)}{shaft\ power\ (input)} = \dfrac{P_t}{P_P} = \dfrac{7.376\times 10^7}{1.084\times 10^8} = \underline{\underline{0.68}}$

※意即測量”水由水庫流進河中所可產生之功”與”把水重新打回水庫中”所需功的”比值”

Problem 4

【Solution】

(a) thermally fully-developed condition

$$\frac{\partial}{\partial x}\left[\frac{T_s(x)-T(x,r)}{T_s(x)-T_m(x)}\right]=0\begin{cases}T_s(x)=surface\ temperature\\ T(x,r)=temperature\\ T_m(x)=mean\ temperature\end{cases}$$

(b) Reynolds number for the internal flows

$$\mathrm{Re}\equiv\frac{inertial\ force}{viscous\ force}=\frac{\rho<v>D}{\mu}$$

(c) Reynolds analogy

$$\frac{C_f}{2}=St\begin{cases}C_f\ =\ friction\ factor\\ St=Stanton\ number\end{cases}$$

(d) Grashof number

$$Gr\equiv\frac{buoyant\ force}{viscous\ force}=\frac{\rho^2 g\beta L^3\Delta T}{\mu^2}$$

Problem 5

【Solution】

(a) By mole balance of O_2

$$N''_{O_2}(4\pi r^2)\Big|_r - N''_{O_2}(4\pi r^2)\Big|_{r+dr} = 0$$

同除 $4\pi dr \to 0$

$$\frac{-d(r^2 N''_{O_2})}{dr} = 0$$

(b) By Fick's law,

$$N''_{O_2} = -D_{AB}\frac{dC_{O_2}}{dr} + y_A(N''_{O_2} + N''_{CO_2})$$

$$\because \text{reaction, } C(s)+O_2(g)\to CO_2(g)\text{ ，} N''_{O_2} = -N''_{CO_2}$$

$$\text{則 } N''_{O_2} = -D_{AB}\frac{dC_{O_2}}{dr}$$

(c) 將(b)結果代入(a)小題結果：

$$\frac{d}{dr}\left(r^2\frac{dC_{O_2}}{dr}\right) = 0$$

$$C_{O_2} = \frac{-c_1}{r} + c_2\text{ ，} B.C.\begin{cases} r = r_0 \quad -D_{AB}\dfrac{dC_{O_2}}{dr}\Big|_{r=r_0} = -k''_1 C_{O_2}(r_0) \\ r\to\infty \quad C_{O_2} = C_{O_2}^{\infty} \end{cases}$$

B.C.1 代入：

$$-D_{AB}\frac{C_1}{r_0^2} = -k''_1 C_{O_2}(r_0)$$

$$c_1 = \frac{k''_1}{D_{AB}} r_0^2 C_{O_2}(r_0)$$

B.C.2 代入：

$$c_2 = C_{O_2}^{\infty}$$

則

$$C_{O_2} = \frac{-\frac{k_1^{"}}{D_{AB}} r_0^2 C_{O_2}(r_0)}{r} + C_{O_2}^{\infty}$$

$$C_{O_2}^{\infty} = \frac{P_{O_2}}{RT} = \frac{1\ (atm)}{8.205\times10^{-2}\times1450} = 8.405\times10^{-3}\ \ (kmol/m^3)$$

At $r = r_0$,

$$C_{O_2}(r_0) = \frac{-\frac{k_1^{"}}{D_{AB}} r_0^2 C_{O_2}(r_0)}{r_0} + C_{O_2}^{\infty}$$

$$C_{O_2}(r_0) = \frac{-0.1}{1.71\times10^{-4}}\times(1\times10^{-3})^2 C_{O_2}(r_0)\times\frac{1}{1\times10^{-3}} + 8.405\times10^{-3}$$

$$C_{O_2}(r_0) = 5.3\times10^{-3}\ \ (kmol/m^3)$$

則

$$W_A = -k_1^{"}(4\pi r_0^2)C_{O_2}(r_0) = \underline{\underline{-6.66\times10^{-9}\ \ (kmol/s)}}$$

(本題可參考：Incropera, F.; Dewitt, D.; Bergman, T.; Lavine, A. *Fundamentals of Heat and Mass Transfer*, 6th ed.; p 920, Problem 14.29.)

Problem 6

【**Solution**】

(a) By lumped analysis on the insulated vessel,

$$UA(T_h - T) = \frac{\partial}{\partial t}(\rho V_c C T) = \rho V_c C \frac{\partial T}{\partial t} \quad ,$$

Where U is the overall heat transfer coefficient

$$\frac{UA}{\rho V_c C}\int_0^t dt = \int_{T_i}^{T} \frac{dT}{T_h - T}$$

$$A = \frac{\rho V_c C}{Ut} ln \frac{T_h \text{-} T_i}{T_h \text{-} T}$$

For $U = \dfrac{1}{\dfrac{1}{h_i} + \dfrac{1}{h_0}}$

(b)

$$A = \frac{1200 \times 2.25 \times 2200}{\dfrac{1}{\dfrac{1}{10000} + \dfrac{1}{2000}} \times 60 \times 60} \ln \frac{500 - 300}{500 - 450} = 1.372 \ (m^2)$$

$$A = 1.372 = \pi(20 \times 10^{-3})L$$

$$\underline{\underline{L = 21.84\,(m)}}$$

(本題可參考：Incropera, F.; Dewitt, D.; Bergman, T.; Lavine, A. *Fundamentals of Heat and Mass Transfer*, 6th ed.; p 321, Problem 5.15.)

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Problem 1

【Solution】

(1)

(a) Rate of strain $=\dfrac{dv_x}{dy}[=]\dfrac{1}{s}$

(b) Thermal diffusivity [=] m^2/s

(c) NPSH [=] m

(d) Fanning friction factor [=] 1 (dimensionless)

(e) Reynolds stress [=] $kg/s^2\cdot m$

(2)

$$\rho\frac{\partial\vec{v}}{\partial t}+\rho\vec{v}\bullet\nabla\vec{v}=\mu\nabla^2\vec{v}-\nabla p+\rho\bar{g}$$

無因次化每一個方程式的項可得：

$$\frac{\rho U}{t_0}\frac{D\vec{v}^*}{Dt^*}=\frac{\mu U}{L^2}\nabla^{*2}\vec{v}^*-\frac{p_0}{L}\nabla^* p^*+\rho\bar{g}$$

同除 $\dfrac{\mu U}{L^2}$

$$\frac{\rho L^2}{t_0\mu}\frac{D\vec{v}^*}{Dt^*}=\nabla^{*2}\vec{v}^*-\frac{p_0 L}{\mu U}\nabla^* p^*+\frac{\rho L^2}{\mu U}\bar{g}$$

令 $\begin{cases} t_0=\dfrac{L}{U} \\ p_0=\dfrac{\mu U}{L}\end{cases}$ =>

$$\mathrm{Re}\frac{D\vec{v}^*}{Dt^*}=\nabla^{*2}\vec{v}^*-\nabla^* p^*+\frac{\rho L^2}{\mu U}\bar{g}$$

(a) For steady velocity field, the velocity profile is independent of time

Therefore,

$$\rho\frac{\partial\vec{v}}{\partial t} \text{ can be neglected}$$

(b)

$$\text{Re} \equiv \frac{inertial\ force}{viscous\ force}$$

If Re is very small, $inertial\ force << viscous\ force$

$$\frac{D\vec{v}^*}{Dt^*} \text{ can be neglected}$$

(c) For ideal flow, $inertial\ force >> viscous\ force$

$$\mu\nabla^2\vec{v} \text{ can be neglected}$$

※(a)(b)(c)三題可不須無因次化就可作答，只不過可用於強調(b)小題同時須消除兩項。

(d) Yes, the constraints of Navier-Stokes equation is constant ρ and μ, which can be satisfied by a compressible Newtonian fluid flowing incompressibly.

※此題問的是 N-S equation 能否用於描述"不可壓縮流體"，參考 Bird 第 2 版第 84 頁的敘述，只需要此系統密度 ρ 與 μ 維持恆定即可，而 compressible fluid 雖然物理本質會使得其密度隨著時間與受力狀況等條件改變，但在特定情況下一樣有可能使其維持恆定(稱為 incompressible flow)，類似於物體受力"不一定"產生"速度變化"，因為"合力可以是 0"。

(e) Yes, the origin of the Navier-Stokes equation is merely the conservation of momentum of the fluid with constant ρ and μ, and both laminar and turbulent flow is applicable.

※在 BSL 第二版 CH5 的章節介紹中(p.152)，有關於 equation of change 適用於 turbulent flow 的敘述如下：*Since the sizes of the turbulent eddies are several orders of magnitude larger than the mean free path of the molecules of the fluid, <u>the equations of change are applicable.</u>*

因紊流並不像層流一般均勻，能大範圍分析整塊流體；而是只能分割成較小塊的渦流區作分析，然而，因其尺度依舊遠大於分子尺度，還是可視為一塊一塊連續體，還是能用 equation of change 進行分析；若此時 ρ 與 μ 皆恆定，則 N-S equation 依舊可用。

(3)

The rate of change of fish concentration observed by a person fixed on a boat traveling on the lake is the "substantial derivative", which is,

$$\frac{DC}{Dt}=\frac{\partial C}{\partial t}+\vec{v}\cdot\nabla C$$

將 $C=5t+3x+2y+z$ 代入

$$\begin{aligned}\frac{DC}{Dt}&=\frac{\partial C}{\partial t}+\vec{v}\cdot\nabla C\\&=5+(5i+2j+0k)\cdot(3i+2j+k)=\underline{\underline{24\ (\#\ of\ fish/m^3\cdot\mathrm{min})}}\end{aligned}$$

※此題出自於 BSL 第二版 p.83 對於偏微分、全微分、以及質點微分的觀念，介紹如下：

(1) Observer is on a bridge recording *the rate of change of the fish concentration at a fixed location, the result is partial derivative.*

(2) Observer is on a motor boat and speed around the river, sometimes going upstream, sometimes downstream, and sometimes across the current. *At any instant, the time rate of change of the observed fish concentration is total time derivative.*

(3) Observer is on a canoe(獨木舟), just floating along with the current, observing the fish concentration. *In this case the velocity of the observer is the same as the stream. At any moment, the time rate of change is reported as substantial derivative (meaning the time rate of change is reported as one moves with the "substance").*

由此可知，若觀察者固定不動，只觀察同一片區域魚群的變化，就是偏微分；若是觀察者本身具有速度，就算固定觀看相對於觀察者同樣的相對位置的區域(比如說固定看離觀察者 10 公尺處的魚群)，此時觀察者在具有速度的情況下，實際上觀察的位置也有改變，因此在記錄數據時，也需要一併將自身速度的影響考慮進來；若速度是自由的(比方馬達船)，則用全微分；若速度與河流速度相同(比方自由漂流)，則用質點微分。再回來看此題題目，雖然題目使用了偏微分的符號，

然而其題意應解讀為”在船上的人所觀察到之濃度隨時間之變化”，即”time rate of change”，因此應自動理解為全微分或質點微分。而此題解法是將題目所給之速度條件視為與河流相同，因此採用質點微分解題。

(4)

(a)

Distribution coefficient：The ratio of mole fractions of a species in one liquid phase to another.

$$K_D = \frac{x_i^{(1)}}{x_i^{(2)}}$$

Selectivity：The ratio of the distribution coefficient of one solute to another.

$$\beta_{ij} = \frac{K_{Di}}{K_{Dj}}$$

(本題可參考：Seader, J.; Henley, E.; Roper, D. Separation Process Principles, 3rd ed; p 39.)

(b)

The **volatility** in the distillation is the equivalent of distribution coefficient in the extraction. It is defined as

$$volatility = \frac{mole\ fraction\ of\ key\ component\ in\ \boldsymbol{gas\ phase}}{mole\ fraction\ of\ key\ component\ in\ \boldsymbol{liquid\ phase}} = \frac{y_A}{x_A}$$

On the other hand, the **relative volatility** is the equivalent of selectivity, which is defined as

$$\alpha_{AB} = \frac{volatility\ of\ A}{volatility\ of\ B} = \frac{y_A / x_A}{y_B / x_B}$$

(本題可參考：Seader, J.; Henley, E.; Roper, D. Separation Process Principles, 3rd ed; p 39.)

(5)

Point X is the **loading point**, where gas begins to hinder the downward flow of liquid, and liquid begins to load the bed, replacing gas and causing a sharp pressure drop increase.

Point Y is the **flooding point,** where the gas drag force is sufficient to entrain the entire liquid.

(本題可參考：Seader, J.; Henley, E.; Roper, D. Separation Process Principles, 3rd ed; p 237.)

Problem 2

【Solution】

(1)

For a binary system with a constant average velocity in the z direction, the molar flux in the z direction relative to the molar-average velocity may also be expressed by

$$J_{A,z} = c_A(v_{A,z} - V_z) = -D_{AB}\frac{dc_A}{dz}$$

$$c_A(v_A - \bar{v}) = -D_{AB}\frac{dc_A}{dz}$$

$$c_A v_A = -D_{AB}\frac{dc_A}{dz} + c_A\bar{v}$$

∵

$$\bar{v} = \frac{1}{c}(c_A v_A + c_B v_B)$$

$$c_A\bar{v} = y_A(c_A v_A + c_B v_B) = y_A(N_A + N_B) \text{ 代回}$$

$$c_A v_A = \underline{\underline{N_A = -D_{AB}\frac{dc_A}{dz} + y_A(N_A + N_B)}}$$

(2)

Assumption：Isothermal + Isobaric $\begin{cases} total\ concentration\ C\ is\ constant ------(1) \\ the\ molar\ flux\ of\ A = the\ molar\ flux\ of\ B-(2) \end{cases}$

For (1)

$$C_A + C_B = C$$

微分

$$\frac{dC_A}{dx} + \frac{dC_B}{dx} = 0$$

$$\frac{dC_A}{dx} = -\frac{dC_B}{dx} ------(3)$$

For (2)

$$N_A = -N_B$$

$$-D_{AB}\frac{dC_A}{dx} = -(-D_{BA}\frac{dC_B}{dx}) -----(4)$$

將(3)代入(4)

$$-D_{AB}\frac{dC_A}{dx} = -(D_{BA}\frac{dC_A}{dx})$$

$$\because \frac{dC_A}{dx} \neq 0，D_{AB} = D_{BA} \text{得證}$$

(3)

假設，A 沿著 z 軸方向擴散，根據 Fick’s law

$$N_A = -D_{AB}\frac{dC_A}{dz} + y_A(N_A + \cancel{N_B})$$

$$N_A = \frac{-D_{AB}C}{1-y_A}\frac{dy_A}{dz} > 0$$

$$\because \frac{-D_{AB}C}{1-y_A} < 0 \text{ ，} \frac{dy_A}{dz} < 0$$

即表示 y_A 隨傳遞方向(z 方向)遞減

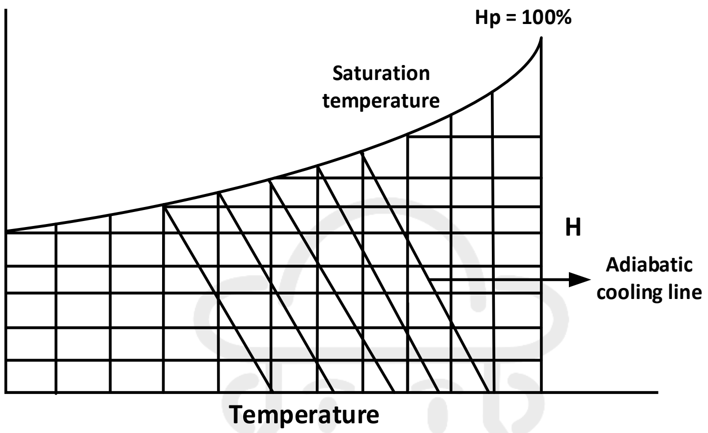
Problem 3

【**Solution**】

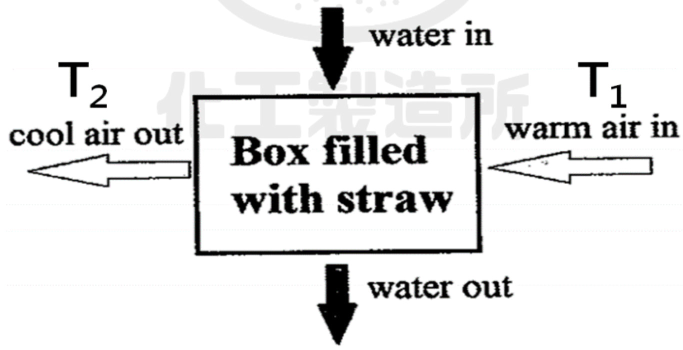
(1)

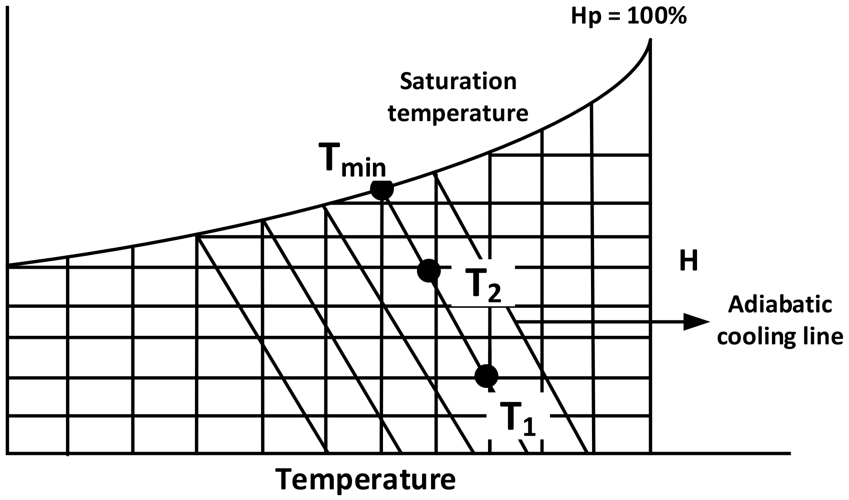
此裝置原理在於將溫暖空氣(warm air)與通入裝置並被水氣加濕後釋放到大氣中。而空氣裡的水氣會吸收人體釋放的熱而氣化使體感溫度降低，並因此達到冷氣調節的功能。

(2)



操作方法





假設進口空氣溫度、濕度狀況為 T_1；出口空氣溫度、濕度為 T_2

若此程序為絕熱加濕程序，則出口溫度 T_2 將在絕熱飽和線軌跡上

而若濕度達成 100%，則到達 T_{min}，則有最大的降溫溫差(mix temperature drop)

(3)

與印度相比之下，因台灣平均濕度較高，即 T_1 位置較靠近 T_2，所以效果將會比較差，較不適合使用。

Problem 4

【Solution】

(1)

By overall heat balance, $\cancel{in} - \cancel{out} + gen = \cancel{accu}$ (steady-state)

$$-h(T_1 - T_a)\cdot 4\pi R_C^2 + S_n \cdot \frac{4}{3}\pi R_F^3 = 0$$

$$T_1 = T_a + \frac{S_n R_F^3}{3R_C^2 h}$$

(2)

By heat balance in the aluminum cladding region, $in - out + \cancel{gen} = \cancel{accu}$

$$-k(4\pi r^2)\frac{dT_C}{dr}\bigg|_r - [-k(4\pi r^2)\frac{dT_C}{dr}]\bigg|_{r+dr} = 0$$

同除 $4\pi k dr \to 0$

$$\frac{d}{dr}(r^2\frac{dT}{dr}) = 0 \ , \begin{cases} r = R_F & -k_F\dfrac{dT_F}{dr} = -k_C\dfrac{dT_C}{dr} \\ r = R_F & S_n(\dfrac{4}{3}\pi R_F^3) = -k_C(4\pi R_F^2)\dfrac{dT_C}{dr} \\ r = R_C & T_C = T_1 \\ r = R_C & -k_C(4\pi R_C^2)\dfrac{dT_C}{dr} = h(4\pi R_C^2)[T_C(R_C) - T_a] \\ r = R_F & T_C = T_0 \end{cases}$$

將微分式積分兩次

$$\frac{dT_C}{dr} = \frac{c_1}{r^2} \text{------}(1)$$

$$T_C = -\frac{c_1}{r} + c_2 \text{------}(2)$$

取 B.C.2、B.C.4 以解出 $T_0 - T_1$

將(1)代入 B.C.2

$$c_1 = \frac{-S_n R_F^3}{3k_C}$$

將(1)、(2)代入 B.C.4

$$c_2 = T_a + c_1(\frac{1}{R_C} - \frac{k_C}{hR_C^2}) = T_a - \frac{S_n R_F^3}{3k_C}(\frac{1}{R_C} - \frac{k_C}{hR_C^2})$$

$$T_C = \frac{S_n R_F^3}{3k_C}[\frac{1}{r} - \frac{1}{R_C} + \frac{k_C}{hR_C^2}] - T_a$$

$$T_0 - T_1 = T_C(R_F) - T_C(R_C) = \underline{\underline{\frac{S_n R_F^3}{3k_C}[\frac{1}{R_F} - \frac{1}{R_C}]}}$$

※此小題因為必須先寫出可用於解出 T_c 之邊界條件，因此最後計算 $T_0 - T_1$ 時順便使用所列之邊界條件做後續的計算。然而，也有較快速的解法：

By energy balance on the cladding,

heat generated by the fissionable material into the cladding = heat conducted out by the cladding

$$\frac{4\pi}{3} R_F^3 \cdot S_n = \frac{4\pi k_C \cdot (T_0 - T_1)}{(\frac{1}{R_F} - \frac{1}{R_C})}$$

(即 B.C.2 變化形態)

整理並消去重複的項：

$$T_0 - T_1 = \underline{\underline{\frac{S_n R_F^3}{3k_C}[\frac{1}{R_F} - \frac{1}{R_C}]}}$$

(3)

By heat balance in the fissionable material region, $in - out + gen = \cancel{accu}$

$$-k_F(4\pi r^2)\frac{dT_F}{dr}\bigg|_r - [-k_F(4\pi r^2)\frac{dT_F}{dr}]\bigg|_{r+dr} + S_n(4\pi r^2 dr) = 0$$

同除 $4\pi k dr \to 0$

$$\frac{dT}{dr} = \frac{-S_n r}{3k_F} + \frac{c_3}{r^2}$$

$$T = \frac{-S_n r^2}{6k_F} - \frac{c_3}{r} + c_4 \quad \begin{cases} r = 0 & \dfrac{dT}{dr} = 0 \\ r = R_F & T = T_0 \end{cases}$$

$$\begin{cases} c_3 = 0 \\ c_4 = T_0 + \dfrac{S_n R_F^2}{6k_F} \end{cases}$$

$$T = \frac{S_n R_F^2}{6k_F}(1 - \frac{r^2}{R_F^2}) + T_0$$

$T_{\max} = T \ (r = 0)$

$$T(r=0) = T_0 + \underline{\underline{\frac{S_n R_F^2}{6k_F}}}$$

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 296-298.)

110 年台大單操輸送

Problem 1

【Solution】

1. **False**

For a steady, fully developed, laminar flow in a circular tube,

$$\frac{\bar{v}_z}{\bar{\bar{v}}_{z,max}} \approx 1-(\frac{r}{R})^{\frac{1}{2}} \ ， \ \frac{<\bar{v}_z>}{\bar{\bar{v}}_{z,max}} \approx \frac{1}{2}$$

$$P_o - P_L = (\frac{8\mu L}{\pi\rho R^4})w \propto w \quad (\mathrm{Re}<2100)$$

For **turbulent flow**,

$$\frac{\bar{v}_z}{\bar{\bar{v}}_{z,max}} \approx (1-\frac{r}{R})^{\frac{1}{7}} \ ， \ \frac{<\bar{v}_z>}{\bar{\bar{v}}_{z,max}} \approx \frac{4}{5}$$

$$P_o - P_L \approx 0.0198(\frac{2}{\pi})^{\frac{7}{4}}(\frac{\mu^{\frac{1}{4}}L}{\rho R^{\frac{19}{4}}})w^{\frac{7}{4}} \propto w^{\frac{7}{4}} \quad (10^4<\mathrm{Re}<10^5)$$

The pressure drop is not directly proportional to the mass flow rate.

(本題可參考：Bird, R.; Stewart, W.; *Lightfoot, E. Transport Phenomena*, 2nd ed.; p 154~155.)

<補充>

在 McCabe 原文書內也有提到壓降與平均速度的關係：

$$\begin{cases} laminar\ flow\ (\mathrm{Re}<2100):\ \frac{\Delta P}{L} \propto \bar{v} \\ turbulent\ flow\ (2500<\mathrm{Re}<10^6):\ \frac{\Delta P}{L} \propto (\bar{v})^{1.8} \\ very\ turbulent\ flow\ (\mathrm{Re}>10^6):\ \frac{\Delta P}{L} \propto (\bar{v})^{2} \end{cases}$$

可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*,

7th ed.; p 116.

2. **True**

The power number N_P, which is defined as,

$$N_P \equiv \frac{P}{n^3 D_a^5 \rho}\begin{cases} P = the\ power\ requirement\ (resistance\ to\ drag) \\ D_a = impeller\ diameter \\ n = angular\ velocity \\ \rho = density\ of\ the\ fluid \end{cases}$$

is ***analogous to a friction factor or a drag coefficient***. It is ***proportional to the ratio of the drag force acting on a unit area of the impeller and the inertial stress.***

(本題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 259.

Power number 之推導可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 257.)

3. **False**

With shear-thinning or pseudoplastic liquids, a turbine may give a local region or high shear rate around the impeller, but ***near the wall the shear rate is much lower and the apparent viscosity may be much higher.*** Therefore, ***the velocity will be much less*** than that with a Newtonian fluid.

(本題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 257.)

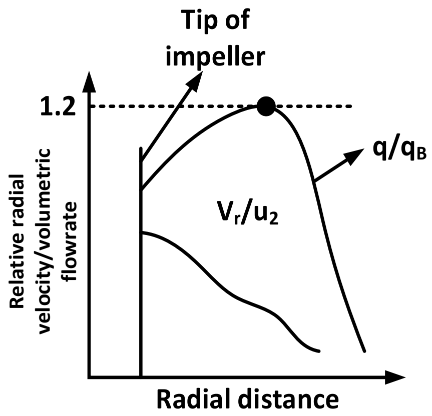
4. **True**

If the particles are very small, brownian movement appears. This is a random motion imparted to the particle by collisions between the particle and the molecules of the surrounding fluid. ***The random movement of the particle tends to suppress the effect of the force of gravity, so settileing does not occur. Application of centrifugal force reduces the relative effect of brownian movement.***

(本題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 171.)

5. **True**

參考下圖：



可以發現隨著距離 tip 越遠，r 方向的流體速度應該會逐漸下降，符合第一段敘述。

而液體的實際流量則會達到一峰值(1.2 倍 q_B)以後開始下降，也符合第二段敘述

(本題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 255, FIGURE 9.11.)

6. **True**

In agitation process, Froude number can be defined as,

$$Fr \equiv \frac{n^2 D_a}{g} = \frac{inertial\ stress}{gravitational\ force}\ per\ unit\ area\ acting\ on\ the\ fluid$$

(本題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 259.)

7. **True**

(本題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 268.)

8. **False**

在非球體的沉降過程中，因其幾何形狀對流體的方向關係會一直變動而損耗能量，有效的拖曳力(drag)會比流體流經固定方位還來的**高**

(本題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 170.)

9. **True**

The stagnation temperature of a high-speed fluid is defined as the temperature the fluid would attain were ***it brought to rest adiabatically without the development of shaft work.***

By total energy balance,

$$m[\Delta H + \Delta(\frac{u^2}{2}) + \cancel{\Delta z}] = \underset{adiabatic}{\cancel{Q}} - \underset{no\ shaft\ work}{\cancel{W}}$$

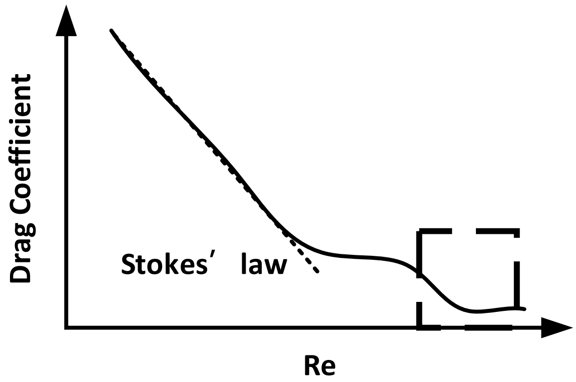
$$H_2 - H_1 + \underbrace{\cancel{\frac{u_2^2}{2} - \frac{u_1^2}{2}}}_{stagnation} = 0$$

$\underline{\underline{H_2 = H_1}}$，即進與出口之間，stagnation enthalpy 為定值

代表**進與出口之間 stagnation temperature 維持定值**

(本題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 137.)

10. <u>**True**</u>



如圖，可以觀察到 drag coefficient，在 Re 極大處($\approx 2\times10^5$)有一個小急降

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 187.

原文書將 friction coefficient 與 drag coefficient 一起討論，不過該頁指的是 drag coefficient)

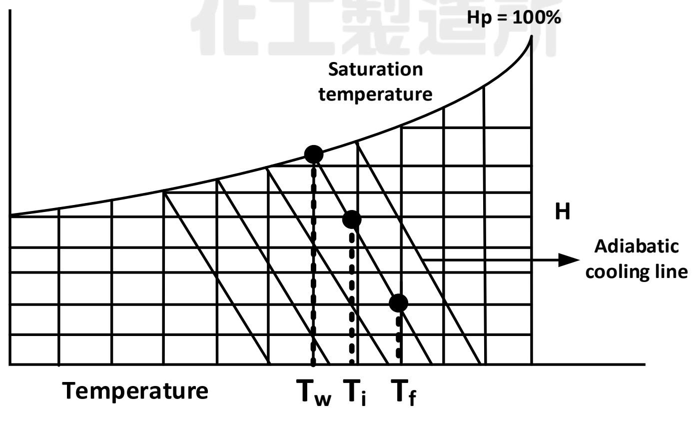
<u>Problem 2</u>

【Solution】

1. <u>**Ans：(D)**</u>

假設此過程未吸/放熱(單純利用化學物質吸收水分，且未起反應)

此過程會在絕熱飽和曲線上移動，如圖：



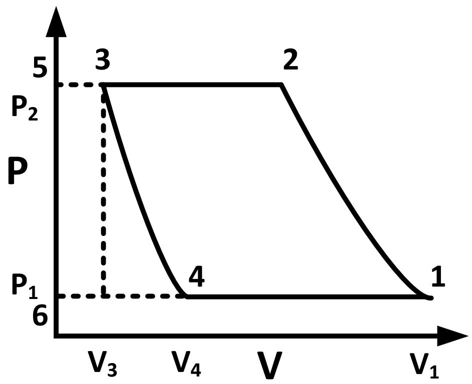
(A) wet bulb temprature (T_w)不變

(B)(D) Dry bulb temperature 增加

(C) dew point temperature 應不變，且相對於原始溫度也是降低

2. <u>Ans：(B)</u>

一個典型的 compressor 其工作路徑如下：



$$\begin{cases} 1\rightarrow 2: \textit{compression of the gas} \\ 2\rightarrow 3: \textit{constant}-\textit{pressure expulsion} \\ 3\rightarrow 4: \textit{expansion of the residual gas} \\ 4\rightarrow 1: \textit{constant}-\textit{pressure introduction of fresh gas} \end{cases}$$

The work done on the gas can be calaulated as,

$$W = W_{1256} - W_{4356}$$

Assume the compression and the expansion processes are ientropic,

$$W_{1256} = -\int_{V_1}^{V_2} PdV + P_2V_2 - P_1V_1 = -\int_{V_1}^{V_2} PdV + \int_{P_1V_1}^{P_2V_2} d(PV) = \int_{P_1}^{P_2} VdP = \int_{P_1}^{P_2} (\frac{\alpha}{P})^{\gamma} dP$$

$$= \frac{\alpha^{\frac{1}{\gamma}}}{-\frac{1}{r}+1}[(P_2)^{\frac{-1}{\gamma}+1} - (P_1)^{\frac{-1}{\gamma}+1}] = \underset{V_1}{(\frac{\alpha}{P_1})^{\frac{1}{\gamma}}} P_1 \frac{\gamma}{\gamma-1}[(\frac{P_2}{P_1})^{\frac{\gamma-1}{\gamma}} - 1] = P_1V_1\frac{\gamma}{\gamma-1}[(\frac{P_2}{P_1})^{\frac{\gamma-1}{\gamma}} - 1]$$

同理

$$W_{4356} = P_4V_4\frac{\gamma}{\gamma-1}[(\frac{P_3}{P_4})^{\frac{\gamma-1}{\gamma}} - 1] = P_1V_4\frac{\gamma}{\gamma-1}[(\frac{P_2}{P_1})^{\frac{\gamma-1}{\gamma}} - 1]$$

The work done per cycle is then,

$$W = P_1V_1\frac{\gamma}{\gamma-1}[(\frac{P_2}{P_1})^{\frac{\gamma-1}{\gamma}} - 1] - P_1V_4\frac{\gamma}{\gamma-1}[(\frac{P_2}{P_1})^{\frac{\gamma-1}{\gamma}} - 1]$$

For isentropic conditions,

$$V_4 = V_3 (\frac{P_2}{P_1})^{\frac{1}{\gamma}}$$

$$V_1 - V_4 = (V_1 - V_3) + V_3 - V_3 (\frac{P_2}{P_1})^{\frac{1}{\gamma}} = \underbrace{(V_1 - V_3)}_{V_s}[1 + \frac{V_3}{\underbrace{V_1 - V_3}_{c}} - \frac{V_3}{V_1 - V_3}(\frac{P_2}{P_1})^{\frac{1}{\gamma}}]$$

$$V_1 - V_4 = V_s[1 + c - c(\frac{P_2}{P_1})^{\frac{1}{\gamma}}] \begin{cases} V_s = swept\ volume \\ c = clearance \\ \dfrac{P_2}{P_1} = \dfrac{outlet\ pressure}{inlet\ pressure} = compression\ ratio \end{cases}$$

Define, theoretical volumetric efficiency,

$$volumetric\ efficiency \equiv \frac{volume\ of\ fluid\ discharged}{volume\ swept\ by\ the\ piston\ or\ plunger} = \underline{\underline{[1 + c - c(\frac{P_2}{P_1})^{\frac{1}{\gamma}}]}}$$

由上式可知，當 ***<u>compression ratio 變高，volumetric efficiency 變低</u>***

(本題可參考：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J.

***Chemical Engineering*, 6th ed.; Volume 1, p 351~353.)**

3. <u>Ans：(D)</u>

The assumptions of McCabe-Thiele method include,

(1) constant molar heat of vaporization

(2) no heat losses (adiabatic)

(3) no heat of mixing

(4) constant molar vapor flow and constant molar reflux flow in any section of the column

(本題可參考：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 566.)

4. <u>Ans：(A)</u>

Consider unit mass of particles containing n_i particles of characteristic dimension d_i, constituting a mass fraction of x_i, then:

$$x_i = n_i k \rho_s d_i^3 (k \text{ is a constant})$$

$$\sum x_i = 1 = \rho_s k \sum (n_i d_i^3)$$

$$n_i = \frac{1}{\rho_s k} \frac{x_i}{d_i^3}$$

The *volume-surface mean diameter*(*surface mean temperature*) can be calculated as,

$$d_s = \frac{\sum (n_i d_i) S_i}{\sum (n_i S_i)} = \frac{\sum (n_i d_i^3)}{\sum (n_i d_i^2)}$$

Replace n_i with $\frac{1}{\rho_s k} \frac{x_i}{d_i^3}$,

$$d_s = \frac{\sum x_i}{\sum \frac{x_i}{d_i}} = \frac{1}{\sum \frac{x_i}{d_i}}$$

(本題可參考：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 11~13.)

5. **Ans：(B)**

If the only purpose of the stirrer is to suspend solids, a ***propeller or other axial-flow impeller such as pitched-blade turbine would be chosen.***

Helical-ribbon and anchor impellers are used for highlt viscous liquids.

(本題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 273/249.)

6. **Ans：(B)**

As the fluid viscosity is reduced, the friction on the flow decreases. Therefore, the pressure drop of the flow ***decreases***.

7. **Ans：(C)**

The fluid pressure on a plate is independent of the geometry of the container.

So the force F from the fluid pressure on the plate X can be simply calculated as,

$$F = PA = (\rho_L g H)A = \underline{\underline{\rho_L A H g}}$$

8. **Ans：(C)**

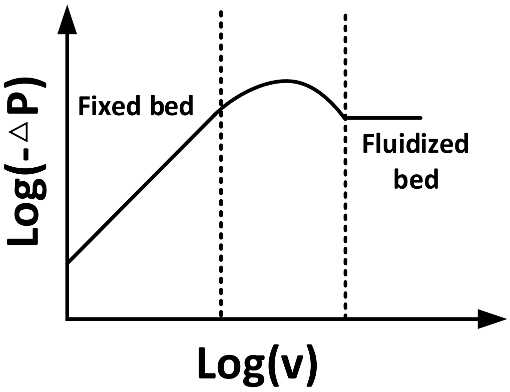
The root-mean-square components are equal for all directions at a given point. In this situation the turbulence 9s said to be ***isotropic***.

$$\overline{(u')^2} = \overline{(v')^2} = \overline{(w')^2}$$

Nearly isotropic turbulence exists when there is no velocity gradient, as at the centerline of a pipe or beyond the outer edge of a boundary layer and downstream of a grid placed in the flow.

(本題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 58~59.)

9. <u>Ans：**(B)**</u>



如上圖，可以發現隨著流體速度上升，壓降越來越大，直到固定床流體化後壓力呈現定值，而圖中轉折是固定床流體化過程中不穩定導致。

因此流體化床之壓降(水平線)運作之壓降相比固定床(上升階段)來說還要來的大

(本題可參考：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 294.)

10. <u>Ans：**(D)**</u>

<u>Problem 3</u>

【Solution】

If the suction pressure is only slightly greater than the vapore pressure of the working fluid, some liquid may flash to vapor inside the pump, which is called cavitation.
The cavitation effect will reduce the pump capacity and causes severe corrosion.

The reason why cavitation occurs tipycally only in the centrifugal pumps not displacement pumps is mainly because of the ways they operate. For centrifugal pumps, the bubbles would be unstable and broken under the impact of impeller, and thus causing the bubbles to collapse, and do damaging on the pump.

The positive displacement pump operates with a moving piston or plunger with fixed chamber (reciprocating pumps) or a rotary chamber (rotary pumps). Both types of pumps offer a stable environment for bubbles, so the bubbles will be stabilized. The main influence of the bubbles on the displacement pump is the reduction of the flow rate which can be transferred by the pump in one cycle.

事實上任何 pump 都會有 cavitation 的風險，只要吸進來的液體壓力變夠小就有機會形成泡泡或汽化造成空蝕現象，在正位移式 pump 的運作中，是利用活塞形成負壓吸收液體後往前加壓將液體推出(可參考網路上影片)，因此在形成負壓的過程中依舊有可能產生空蝕現象，然而其形成負壓時間與程度相較只往同一方向高速運轉並形成負壓吸水再離心推出之離心泵短且穩定(畢竟接下來還要往回推)，所以一般較離心泵為穩定，cavitation 的影響較不顯著。

(本題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 204/206~209.)

Problem 4

【Solution】

By the equation of continuity,

$$\frac{\partial v_x}{\partial x}+\frac{\partial v_y}{\partial y}=0 \quad \text{(incompressible)}$$

$v_x=\dfrac{2}{x}$代入

$$\frac{\partial v_y}{\partial y}=-\frac{\partial v_x}{\partial x}=-\frac{d(\frac{A}{x})}{dx}=\frac{2}{x^2}$$

$$\underline{\underline{v_y=\int\frac{2}{x^2}dy=\frac{2y}{x^2}}}$$

(本題可參考：Pritchard, P. J.; Mitchell, J. W. *Fox and McDonald's Introduction to Fluid Mechanics*; 8th ed; p 256, Problem 5.6.)

Problem 5

【Solution】

(a)

For the bed density,

$$\rho_{bed} = \frac{W_{bed}}{V_{bed}} = \frac{250000 \times 7800 \times \frac{\pi}{6}(1\times10^{-2})^3}{\frac{\pi}{4}\times 0.3^2 \times 3} = 4814.8(kg/m^3)$$

$$\varepsilon = 1 - \frac{\rho_{bed}}{\rho_p} = 1 - \frac{4814.8}{7800} = \underline{\underline{0.3827}}$$

(b)

By Ergun equation,

$$\frac{\Delta p}{L} = \frac{150\bar{v}\mu}{\Phi_s^2 D_p^2}\frac{(1-\varepsilon)^2}{\varepsilon^3} + \frac{1.75\rho\bar{v}^2}{\Phi_s D_p}\frac{1-\varepsilon}{\varepsilon^3}$$

∵ The quartz particles are spherical,

$$\Phi_s = 1$$

將各項數據代入：

$$\frac{\Delta p}{3} = \frac{150\times0.1\times1.2\times10^{-3}}{(1\times10^{-2})^2}\frac{(1-0.3827)^2}{0.3827^3} + \frac{1.75\times1.193\times10^3\times(0.1)^2}{1\times10^{-2}}\left(\frac{1-0.3827}{0.3827^3}\right)$$

$$\underline{\underline{\Delta p = 72639.69\ (N/m^2)}}$$

(c)

在 U 型管流動的酒經過流體化床時，會產生壓降，其壓降可由 U 型管右側高出 H 的酒來做補充：

$$\Delta p = \rho g H$$

$$72639.69 = 1.193\times10^3 \times 9.8H$$

$$\underline{\underline{H = 6.213(m)}}$$

Problem 6

【Solution】

(1)

Assumptions, (1) Incompressible and Newtonian fluid

(2) Steady state

(3) Fully-developed in z direction, $v_r = v_\theta = 0$

(4) θ-symmetric

By the equation of continuity,

$$\underbrace{\cancel{\frac{\partial \rho}{\partial t}}}_{incompressible} + \underbrace{\cancel{\frac{1}{r}\frac{\partial(\rho r v_r)}{\partial r}}}_{v_r=0} + \underbrace{\cancel{\frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta}}}_{v_\theta=0} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

$$\frac{\partial v_z}{\partial z} = 0 \text{ , } v_z = f(\cancel{t}, r, \cancel{\theta}, \cancel{z})$$

By the equation of change,

$\boxed{r-direction:}$

$$\rho(\cancel{\frac{\partial v_r}{\partial t}} + \cancel{v_r\frac{\partial v_r}{\partial r}} + \cancel{\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}} - \cancel{\frac{v_\theta^2}{r}} + \cancel{v_z\frac{\partial v_r}{\partial z}}) = -\frac{\partial p}{\partial r} + [\cancel{\frac{1}{r}\frac{\partial(r\tau_{rr})}{\partial r}} + \cancel{\frac{1}{r}\frac{\partial \tau_{\theta r}}{\partial \theta}} - \cancel{\frac{\tau_{\theta\theta}}{r}} + \cancel{\frac{\partial \tau_{zr}}{\partial z}}] + \cancel{\rho g_r}$$

$$\boxed{-\frac{\partial p}{\partial r} = 0}$$

$\boxed{\theta-direction:}$

$$\rho(\cancel{\frac{\partial v_\theta}{\partial t}} + \cancel{v_r\frac{\partial v_\theta}{\partial r}} + \cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}} + \cancel{\frac{v_r v_\theta}{r}} + \cancel{v_z\frac{\partial v_\theta}{\partial z}}) = -\frac{1}{r}\frac{\partial p}{\partial \theta} + [\cancel{\frac{1}{r^2}\frac{\partial(r^2\tau_{r\theta})}{\partial r}} + \cancel{\frac{1}{r}\frac{\partial \tau_{\theta\theta}}{\partial \theta}} - \cancel{\frac{\tau_{\theta r} - \tau_{r\theta}}{r}} + \cancel{\frac{\partial \tau_{z\theta}}{\partial z}}] + \cancel{\rho g_\theta}$$

$$\boxed{-\frac{1}{r}\frac{\partial p}{\partial \theta} = 0}$$

$\boxed{z-direction:}$

$$\rho(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_r}\frac{\partial v_z}{\partial r} + \cancel{\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}} + v_z\cancel{\frac{\partial v_z}{\partial z}}) = -\frac{\partial p}{\partial z} + [\frac{1}{r}\frac{\partial(r\tau_{rz})}{\partial r} + \cancel{\frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta}} + \cancel{\frac{\partial \tau_{zz}}{\partial z}}] + \cancel{\rho g_z}$$

$$\boxed{-\frac{\partial p}{\partial z} + \frac{1}{r}\frac{\partial(r\tau_{rz})}{\partial r} = 0}$$

(2)

The boundary conditions should be,

$$\begin{cases} r = R-\delta,\ \tau_{rz}^{c} = \tau_{rz}^{p} \rightarrow -\mu_c \dfrac{dv_z}{dr} = -\mu_p \dfrac{dv_z}{dr} \\ r = R-\delta,\ v_z^{c} = v_z^{p} \\ r = R,\ v_z^{p} = 0 \\ r = 0,\ \dfrac{dv_z^{c}}{dr} = 0 \end{cases}$$

(按照題意，在 vessel 內中心區域紅血球較多，且 vessel 內有一層厚度為δ的 plasma layer，因此 vessel 半徑為 R，中心區域為$R-\delta$)

Problem 7

【Solution】

(a)

Assumptions: (1) conduction only occurs on the r-direction

(2) no generation

(3) steady-state

(4) constant properties

By shell balance of energy in the control volume, ($in - out + \cancel{gen} = \cancel{accu}$)

$$q_r^{"}(\underbrace{\frac{4\pi r^2}{2}}_{hemisphere})\bigg|_r - q_r^{"}(\frac{4\pi r^2}{2})\bigg|_{r+dr} = 0$$

同除 $2\pi dr \to 0$

$$\frac{-d(r^2 q_r^{"})}{dr} = 0$$

By Fourier's law,

$$q_r^{"} = -k\frac{dT}{dr} \text{ 代入，} \boxed{k\frac{d}{dr}(r^2\frac{dT}{dr}) = 0}$$

(本題因 substrate 幾何形狀未知，且 transistor 之傳導為球座標 r 方向，因此利用 transistro 當作 control volume 來求 substrate 溫度分布)

(b)

The boundary conditions are,

$$\begin{cases} r = r_0 \to T = T_0 \\ r = \infty \to T = T_\infty \end{cases}$$

(c)

將(a)小題之結果積分

$$\frac{dT}{dr}=\frac{c_1}{r^2}\text{ ， }T=\frac{-c_1}{r}+c_2$$

代入 boundary conditions,

$$\begin{cases} c_1 = r_0(T_\infty - T_0) \\ c_2 = T_\infty \end{cases}$$

$$\underline{\underline{T=\frac{(T_0-T_\infty)r_0}{r}+T_\infty}}$$

(d)

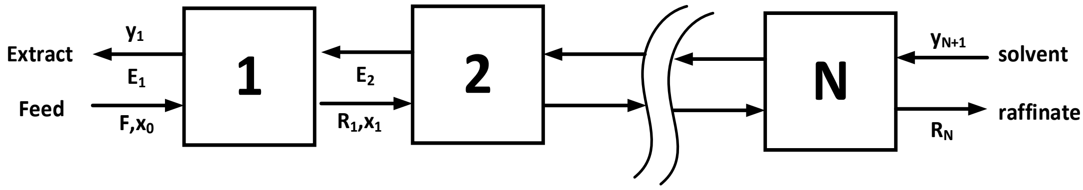
For the heat rate,

$$q_r = q_r^{"} = -k(2\pi r^2)\frac{dT}{dr} = -k(2\pi r^2)\frac{c_1}{r^2} = \underline{\underline{2k\pi r_0(T_0-T_\infty)}}$$

(本題改編自：Incropera, F.; Lavine, A.; Bergman, T. *Fundamentals of Heat and Mass Transfer*, 6th ed. ; p 181, Problem 3.18.)

Problem 8

【**Solution**】



∵ The pure solvent is equal in mass to the feed, let $F = S = 1$

(1)找出 M 點

$$\begin{cases} x_{w,M} = \dfrac{Fx_{w,0} + Sy_{w,N+1}}{F+S} = \dfrac{1\times 0.6 + 1\times 0}{1+1} = 0.3 \\ x_{a,M} = \dfrac{Fx_{a,0} + Sy_{a,N+1}}{F+S} = \dfrac{1\times 0.4 + 1\times 0}{1+1} = 0.2 \\ x_{M,M} = \dfrac{Fx_{M,0} + Sy_{M,N+1}}{F+S} = \dfrac{1\times 0 + 1\times 1}{1+1} = 0.5 \end{cases}$$

$$M = (x_w, x_a, x_M) = (0.3, 0.2, 0.5)$$

(2)找出 R_N, E_1

首先假設 H_2O 幾乎與 MIK 不互溶，因此在 E_1 內 acetone 與 MIK 之組成比例可近似為：

$$\frac{x_a}{x_M} = \frac{1\times 0.4\times 0.99}{1} = 0.396$$

亦即 $x_a \approx \dfrac{0.396}{1+0.396} \approx 0.28$、$x_M \approx \dfrac{1}{1+0.396} \approx 0.72$

在 **<u>extract line 上</u>**同時滿足 $x_a \approx 0.28$ 與 $x_M \approx 0.72$ 的附近找到點 E_1，其組成為：

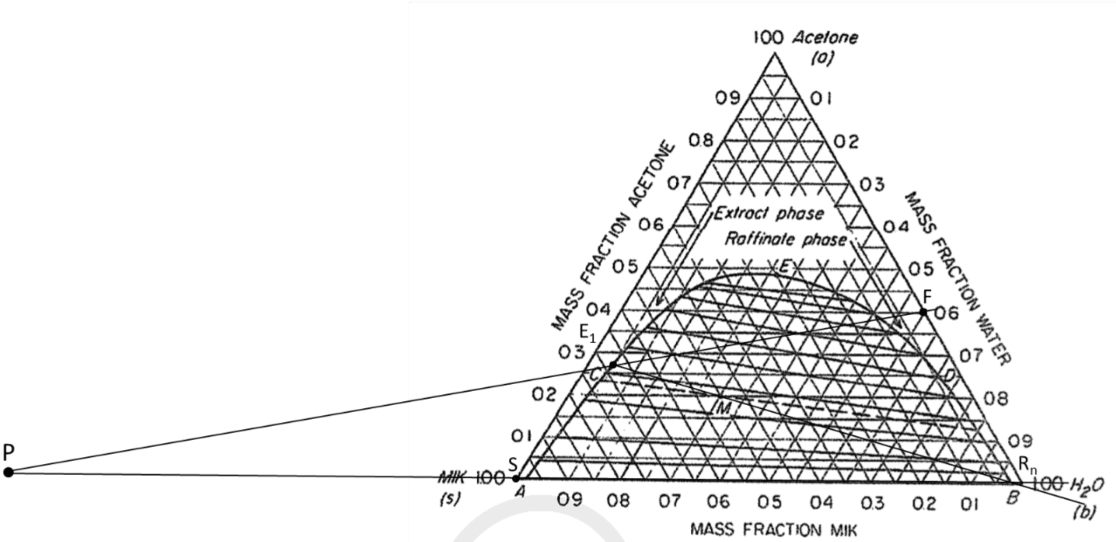
$$E_1 = \begin{cases} x_a = 0.27 \\ x_M = 0.68 \\ x_{H_2O} = 1 - 0.27 - 0.68 = 0.05 \end{cases}$$

<u>亦即先前水與 MIK 不互溶的假設合理</u>

接著連接 $\overline{E_1M}$ 與 raffinate line 交於 R_N

(3)找出 P 點

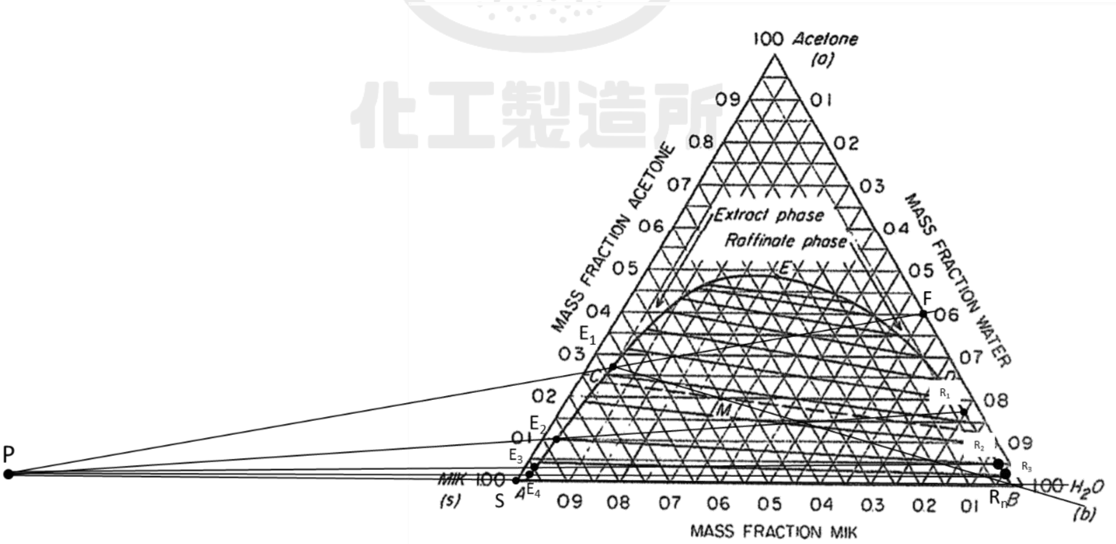
連接 $\overline{FE_1}$ 與 $\overline{SR_N}$ ，交於 P 點，做圖如下：



(4)做理論版數：

做過 E_1 之 tie line，交 raffinate line 於 R_1

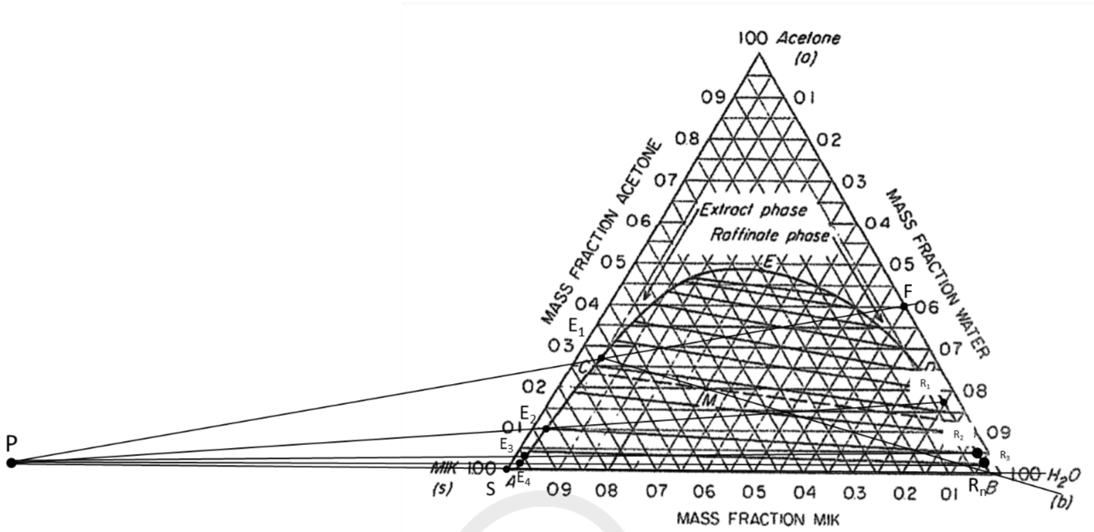
再連接 $\overline{R_1P}$ ，交 extract line 於 E_2 ，以此類推



做圖結果如上，大概需要**(a) 4 個 ideal stages**

(b)

Extract 出口(E_1)之組成大約為：



$$\boxed{\begin{cases} water = 0.05 \\ acetone = 0.275 \\ MIBK = 0.675 \end{cases}}$$

而除去 MIK 後，

$$\boxed{\begin{cases} acetone = \dfrac{0.275}{0.275+0.05} = 0.846 \\ water = 1-0.846 = 0.154 \end{cases}}$$

(此題因電腦作圖緣故有些許誤差產生，若最終答案有誤差請參考作法即可)

(本題改編自：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*,

7th ed.; p 784~786, Example 23.3.

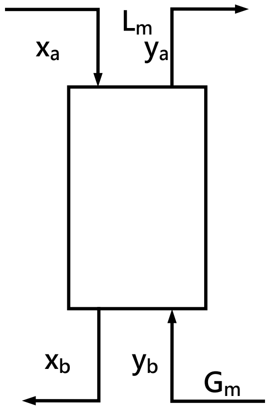
原文書之作法為利用 Ternary phase diagram 轉為 McCabe-Thiele diagram 作圖的方式

Number of ideal stages 為 <u>3.4</u>

唯其討論也很繁瑣，因此有興趣的同學可以自行翻閱，在此不另詳述)

Problem 9

【**Solution**】



$$\begin{cases} x_a = X_a = 0 \\ y_b \approx Y_b = 315\times10^{-6} \\ y_a \approx Y_a = 31\times10^{-6} \end{cases}$$

假設液相 Caustic soda (即 $NaOH$)的濃度很高，因此在CO_2被大量吸收的情況下，液相CO_2蒸氣壓很小，即$\boxed{y_a^* \approx y_b^* = 0}$

$$G_m = \frac{0.34}{29(air\ most)} = 0.0117\ (kmol/m^2s)$$

The mean molecuar weight of the liquid phase is,

$$\frac{100+900}{\frac{100}{40}+\frac{900}{18}} = 19,05$$

$$L_m^{'} = \frac{3.94}{19.05} = 0.207\ (kmol/m^2s)$$

The total height of a column can be calculated as,

$$Z = H_{OG} N_{OG}$$

$$N_{OG} = \frac{(y_b - y_a)}{\dfrac{(y_b - y_b^*) - (y_a - y_a^*)}{\ln \dfrac{y_b - y_b^*}{y_a - y_a^*}}} = \frac{(315 - 31) \times 10^{-6}}{\dfrac{(315 \times 10^{-6} - 0) - (31 \times 10^{-6} - 0)}{\ln \dfrac{315 \times 10^{-6} - 0}{31 \times 10^{-6}}}} = 2.318$$

$$H_{OG} = \frac{G_m^{'}}{K_G a P} = \frac{0.0117}{K_G a \times \underbrace{1.013 \times 10^5}_{atmospheric\ pressure}}$$

$$3 = \frac{0.0117}{K_G a \times \underbrace{1.013 \times 10^5}_{atmospheric\ pressure}} \times 2.318$$

$$\underline{\underline{K_G a = 8.945 \times 10^{-8}\ [kmol / m^3 s (N / m^2)]}}$$

(本題改編自：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 1169, Problem 12.1.)

Problem 10

【Solution】

(1) air at 294K and 40 % relative humidity

→air at $294\times\frac{9}{5}-460=69.2\ (^{\circ}\mathrm{F})$, 40 % relative humidity (A 點)

其 humidity (A 點水平向左對)為 0.006 (lb/lb)

(2) The air is heated to 366K in a reheater befre entering the dryer

→air at $366\times\frac{9}{5}-460=198.8\ (^{\circ}\mathrm{F})$ ，為 A 點水平對至溫度為$198.8\ (^{\circ}\mathrm{F})$ 的地方(B)

(3) The air is dried to 60% relative humidity adiabatically

→B 點沿絕熱飽和曲線往左上方移動至 60% relative humidity 處，為 C 點

C 點 humidity 為 0.028 (lb/lb)

Dry bulb temperature 為$104\ (^{\circ}\mathrm{F})$

整個過程，單位質量空氣可帶走之水量為

$$0.028-0.006=0.022\ (lb\ water\ /\ lb\ dryair)$$

(4)計算總共需蒸發之水量：

濕進料：

$$42\%\ water=\frac{42}{100-42}=0.724\ (lb\ water\ /\ lb\ dry\ solids)$$

出料：

$$4\%water=\frac{4}{100-4}=0.0417\ (lb\ water\ /\ lb\ dry\ solids)$$

進出料所需蒸發之水分為：$(0.724-0.0417)=0.682\ (lb\ water\ /\ lb\ dry\ solids)$

而出料含有的 dry solids 量為：

$$0.126(\frac{kg}{s})\times 2.20(\frac{lb}{kg})\times\frac{100-4}{100}=0.2667\ (lb/s)$$

總共所需的空氣質量流率：

$$\frac{0.2667\times 0.682}{0.022}=8.272\ (lb/s)$$

而 $69.2(^{\circ}\mathrm{F})$ 時，specific volume 為 $12.9\ (ft^3/lb)$

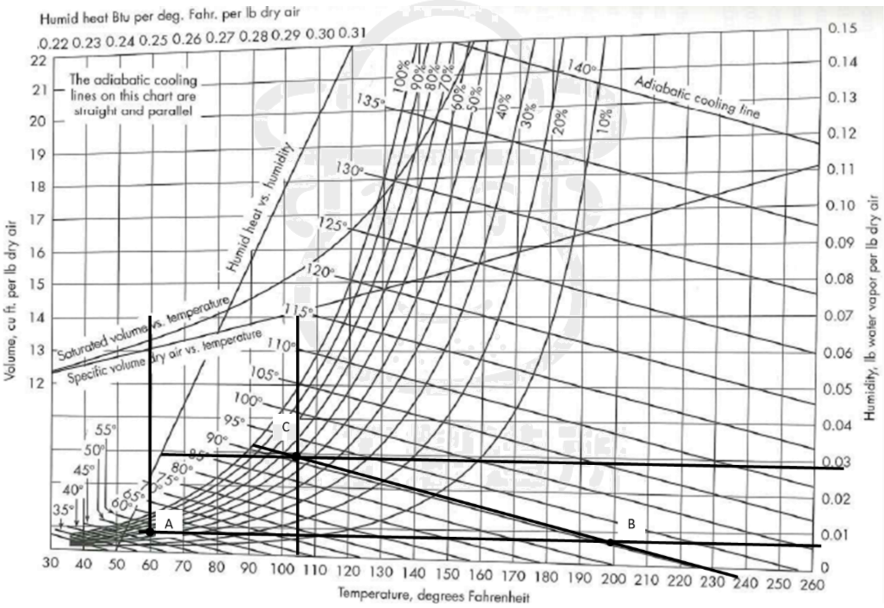
總計空氣體積流率為：$8.272\times 12.9=\underline{\underline{106.71\ (ft^3/s)}}$

乾燥空氣之 humid heat 為：$0.242\ (Btu/^{\circ}\mathrm{F}\cdot lb)$

Preheater 所需之加熱功率為：

$$0.242(198.8-69.2)\times 8.272=\underline{\underline{259.45\ (Btu/s)}}$$

作圖軌跡如下：



Second case,

(1) The water enters and leaves at 340K, which is

$$340\times\frac{9}{5}-460=152\ (^\circ\mathrm{F})$$

The inlet humidity is, 0.006 (lb/lb)

The outlet humidity (60% relative humidity) is, 0.131 (lb/lb)

整個過程，單位質量空氣可以帶走之水量為：

$$0.131-0.006=0.125\ (lb/lb)$$

總共所需的空氣質量流率：

$$\frac{0.2667\times0.682}{0.125}=1.456\ (lb/s)$$

總計空氣體積流率為：$1.456\times12.9=\underline{\underline{18.78\ (ft^3/s)}}$

Preheater 所需之加熱功率為：

$$0.242(152-69.2)\times1.456=\underline{\underline{29.17\ (Btu/s)}}$$

$$\begin{cases}(a)\ 106.71\ (ft^3/s)\\(b)\ 259.45\ (Btu/s)\\(c)\ 18.78\ (ft^3/s)\\(d)\ 29.17\ (Btu/s)\end{cases}$$

(本題改編自：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 1181, Problem 16.7.)

Problem 11

【**Solution**】

先將題目所給之 x_A, y_A 數據轉為 $y_A - x_A$ 圖

$$\begin{cases} A = benzene,\ 分子量 = 12\times 6 + 6 = 78 \\ B = toluene,\ 分子量 = 12\times 7 + 8 = 92 \end{cases}$$

計算進料、出料組成：

$$\begin{cases} x_F = \dfrac{\dfrac{40}{78}}{\dfrac{40}{78}+\dfrac{60}{92}} = 0.44 \\ x_D = \dfrac{\dfrac{0.97}{78}}{\dfrac{0.97}{78}+\dfrac{0.03}{92}} = 0.974 \\ x_W = \dfrac{\dfrac{0.02}{78}}{\dfrac{0.02}{78}+\dfrac{0.98}{92}} = 0.023 \end{cases}$$

將 x_F 、 x_D 、 x_W 標注於對角線上

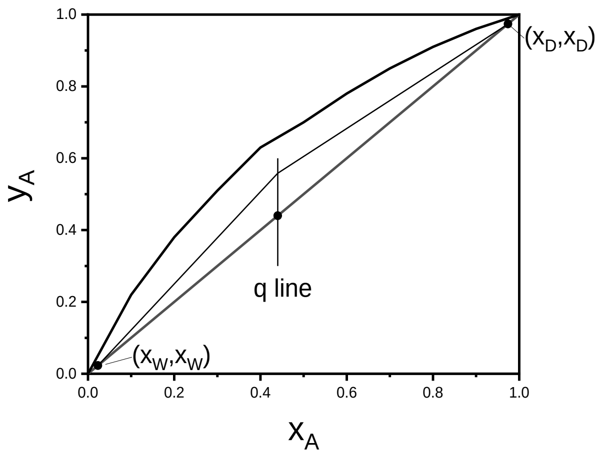
∵ The feed is liquid at boiling-point, $q = 1$, q-line 為通過 x_F 之**垂直線**

而上段操作線：

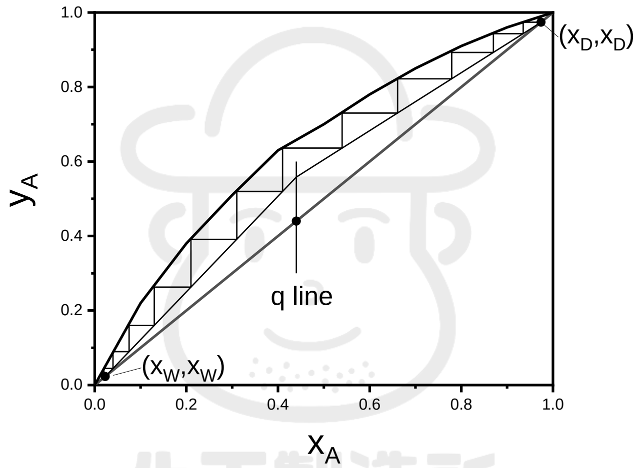
$$斜率 = \frac{R}{R+1} = \frac{3.5}{3.5+1} = 0.778$$

$$\boxed{y = 0.778x + 0.216}$$

下段操作線：為通過(q-line 與上段操作線交點)與 x_W 的直線，可作圖如下：



而其作圖計算板數的結果如下：



如圖，大概需要 **<u>12 個理論板</u>**

若是效率為 0.6，則需要

$$\frac{10}{0.6} = 20\text{個理論板}$$

計算塔頂與塔頂流量：

By mass balance,

$$F = D + W$$

$$4 = D + W$$

By mass balance of the key component,

$$Fx_F = Dx_D + Wx_W$$

$$4 \times 0.4 = 0.97D + 0.02W$$

$$\begin{cases} D = 1.6 (kg/s) \\ W = 2.4 (kg/s) \end{cases}$$

$$\boxed{\begin{cases} (a)\ D = 1.6\ (kg/s) \\ (b)\ W = 2.4\ (kg/s) \\ (c)\ 10\ plates \\ (d)\ 16.67\ plates \end{cases}}$$

(本題改編自：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 1165, Problem 11.10.)

111 年台大單操輸送

Problem 1

【Solution】

(a) Biot number, $Bi \equiv \dfrac{hL}{k_s}$

(b) Reynolds number, $Re \equiv \dfrac{\rho VL}{\mu}$

(c) Fourier number, $Fo \equiv \dfrac{\alpha t}{L^2}$

(d) Schmidt number, $Sc \equiv \dfrac{\upsilon}{D_{AB}}$

(e) Prandtl number, $Pr \equiv \dfrac{\nu}{\alpha}$

(本題可參考：Incropera, F.; Dewitt, D.; Bergman, T.; Lavine, A. Fundamentals of Heat and Mass Transfer, 6th ed.; p 376~377, Table 6.2.)

Problem 2

【Solution】

$\because$ The system is at steady state,

$$q_{conduction} = q_{radiation}$$

And the system is in series, thus

$$q_{A\to B} = \frac{T_A - T_{middle}}{R_A} = \frac{T_{middle} - T_B}{R_B} = \frac{T_A - T_B}{R_A + R_B}$$

For radiation process at A region, the total thermal resistance is,

$$R_{A,rad} + R_{B,k} = \frac{1}{\varepsilon\sigma A(1200 + T_{middle})(1200^2 + T_{middle}^2)} + \frac{L}{kA}$$

For radiation process at B region, the total thermal resistance is,

$$R_{A,k} + R_{B,rad} = \frac{L}{kA} + \frac{1}{\varepsilon\sigma A(T_{middle} + 300)(T_{middle}^2 + 300^2)}$$

$\because (1200 + T_{middle})(1200^2 + T_{middle}^2) > (T_{middle} + 300)(T_{middle}^2 + 300^2)$ ， $R_{A,rad} < R_{B,rad}$

<u>Therefore, the region A should be the radiation process.</u>

Problem 3

【Solution】

In alloys, there are more purities in the lattices, thus creating more obstacles for energy to be transferred by electrons, lowering the mean free path of the of the electron carrying thermal energy, which will decrease the thermal conductivities of alloys.

(※關於微觀的熱傳導機制，例如電子傳遞、聲子傳遞，與不同金屬、陶瓷結構的關係，可參考 Incropera, F.; Dewitt, D.; Bergman, T.; Lavine, A. *Fundamentals of Heat and Mass Transfer*, 6th ed.; p 61~64.)

<u>Problem 4</u>

【Solution】

We can make the following assumptions：

(1) 假設質傳在空間中每個方向以相同速率進行，因此簡化為一維(z)方向的 unsteady state 質傳問題。

(2) 假設杯子的幾何尺寸遠大於 dye，因此對 dye 的質傳來說可近似於無限長質傳問題，因此其 unsteady 的濃度分布為 effor function 函數：

$$\frac{C_A}{C_{A0}} = f(erf\,\frac{z}{\sqrt{4D_{AB}t}})$$

因題目所問為達到 uniform 所需時間，亦即在空間中某點達到最終稀釋倍數的時間(例如杯子體積若為 dye 的 100 倍，則 uniform 應意指空間中每一點都達到 $\frac{C_A}{C_{A0}} = \frac{1}{100}$ 的時間)。假設此點距離杯子長度為 L，則因

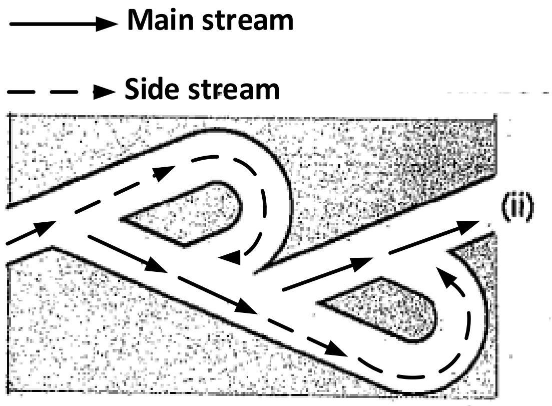
$$\frac{C_{A,uniform}}{C_{A0}} = constant = f(erf\,\frac{L}{\sqrt{4D_{AB}t}})$$

<u>代表會有相同的 t，亦即兩種情況達到 unifrom 所需要的時間相同</u>。

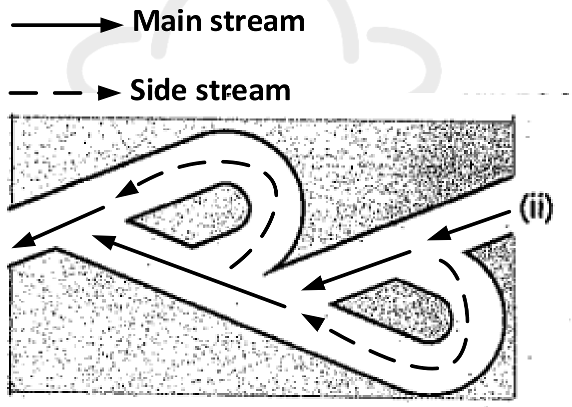
Problem 5

【Solution】

(a)



(b)



(c)

Check valve (逆止閥)是一個在管路中能夠防止液體逆流的閥件，而由(a)小題所繪製之流通情況，我們可以預期在支流流經轉彎處時，會造成一定的動量損耗以及方向改變為原本流向相反的情形，因此能夠有效的阻止流體由(i)流相(ii)；而(b)小題則不論是主流或者支流，皆具有同一個流動方向，並不會受到太大的阻礙，因此 Tesla valve 可視為能控制水流只由(ii)流向(i)的逆止閥。

Problem 6

【Solution】

Assume the heat conductivity, convection coefficient, and emissivity related to this problem are the same in both cases, then,

(1) The total heat transfer rate on the left at every moment is,

$$q_{left} = kA_k(T_{room} - T_{water}) + hA_c(T_{room} - T_{water}) + \sigma\varepsilon A(T_{room}^4 - T_{water}^4)$$

(2) The total heat transfer rate on the right at every moment is,

$$q_{right} = kA_k(T_{water} - T_{room}) + hA_c(T_{water} - T_{room}) + \sigma\varepsilon A(T_{water}^4 - T_{room}^4)$$

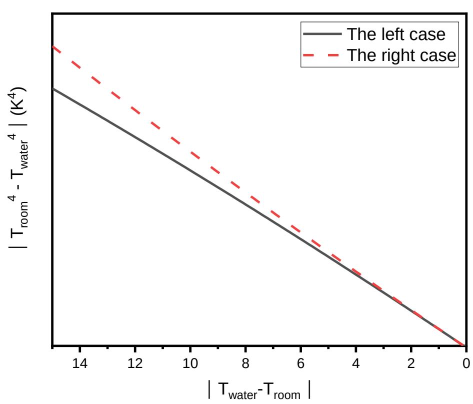
If $|T_{water} - T_{room}|$ is the same, the heat transfer rate by conduction and convecntion will be the same.

However, if we consider the contribution of radiation, we may find a different result.

If we plot $\left|T_{room}^4 - T_{water}^4\right|$ against $|T_{water} - T_{room}|$ for both cases and under the strictions,

$$\begin{cases} 273.15+10 \le T_{water} \le 273.15+25 \text{ for the left case} \\ 273.15+25 \le T_{water} \le 273.15+40 \text{ for the right case} \end{cases}$$

we can find when the value of $|T_{water} - T_{room}|$ is the same, the value of $\left|T_{room}^4 - T_{water}^4\right|$ is larger on the right case. Therefore, we can draw a conclusion that, **at every moment, the value of the heat transfer by radiation is greater on the right case than that on the left case, so the right case will reach room temperature earlier.**



Problem 7

【Solution】

Check the value of Biot number of the system,

$$Bi = \frac{h\frac{V}{A_s}}{k} = \frac{200\times\frac{\frac{4}{3}\pi\times(1\times10^{-3})^3}{4\pi\times(1\times10^{-3})^2}}{15} = 4.4\times10^{-3} << 0.1$$

The lumped-capacitance method is valid.

By energy balance,

$$-hA_s(T-T_\infty) = \rho VC\frac{dT}{dt}$$

Let $\theta = T - T_\infty$, we obtain,

$$-hA_s\theta = \rho VC\frac{d\theta}{dt}$$

$$\frac{\rho VC}{hA_s}\int_{\theta_i}^{\theta}\frac{d\theta}{\theta} = -\int_0^t dt \text{ ， } \frac{\rho VC}{hA_s}\ln\frac{\theta_i}{\theta} = t$$

$$\frac{\theta_i}{\theta} = \frac{T_i - T_\infty}{T - T_\infty} = exp[\frac{hA_s}{\rho VC}t]$$

$$\frac{95-30}{T-30} = exp[\frac{200\times4\pi\times(1\times10^{-3})^2}{7500\times\frac{4}{3}\pi\times(1\times10^{-3})^3\times500}\times1]$$

$$\underline{\underline{T \approx 85℃}}$$

Problem 8

【Solution】

(a)

By mass balance on the oxygen bubble,

$$\underline{\underline{-N_{O_2}\cdot A = C_{O_2}\cdot\frac{d}{dt}(\frac{4}{3}\pi r^3)}}$$

(b)

∵ The amount of water is in excess, we can assume the concentration of oxygen is zero in the bulk solution, so

$$N_{O_2} = k_m(C_{O_2,sat}-0)$$

代回 mass balance,

$$-k_m(C_{O_{2,sat}}-0)\cdot 4\pi r^2 = C_{O_2}\cdot\frac{d}{dt}(\frac{4}{3}\pi r^3) = 4\pi r^2\cdot C_{O_2}\cdot\frac{dr}{dt}$$

$$-k_m\frac{C_{O_{2,sat}}}{C_{O_2}} = \frac{dr}{dt}\ ,\ \int_0^t -k_m\frac{C_{O_{2,sat}}}{C_{O_2}}dt = \int_{r_1}^{r_2}dr$$

$$-k_m\frac{C_{O_{2,sat}}}{C_{O_2}}t = (r_2-r_1)$$

∵

$$C_{O_{2,sat}} = 40\ (\frac{mg}{L}) = \frac{40}{32}\times 10^{-3} = 1.25\times 10^{-3}\ (mol/L)$$

$$C_{O_2} = \frac{P}{RT} = \frac{1}{0.082\times(27+273.15)} = 0.040\ (mol/L)$$

代回：

$$-k_m\frac{1.25\times 10^{-3}}{0.040}\times 10\times 60 = (0.5-1)\times 10^{-3}$$

$$\underline{\underline{k_m = 2.98\times 10^{-5}\ (m/s)}}$$

(本題改編自：E. L. Cussler. *Diffusion: Mass Transfer in Fliud Systems*, 3rd ed.; p 242, Example 8.1-4)

Problem 9

【Solution】

(a) Firstly, by the question statement, we obtain,

$$x_F = 0.3,\ \ x_D = 0.95,\ \text{and}\ \ x_W = 0.05$$

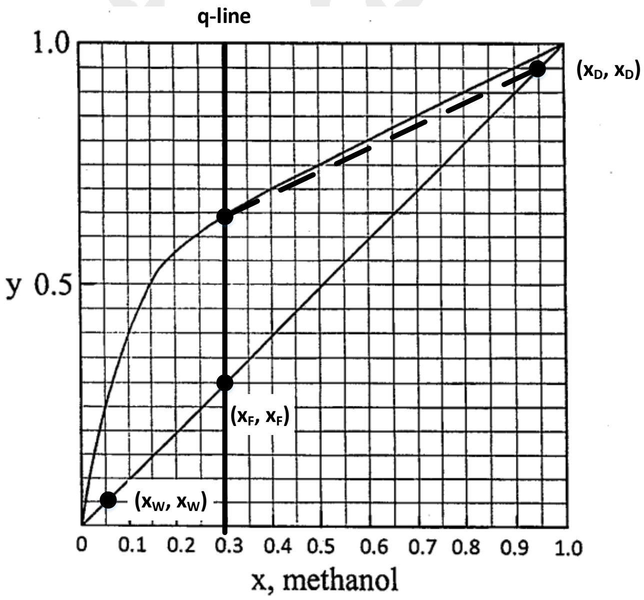
Under minimum reflux ratio, the q-line, equilibrium line, enriching line, and stripping line are all intersecting on the same point, so

(1) q-line：

$\because$ the feed is a saturated liquid, q line is a vertical line passing through (x_F, x_F).

(2) Enriching line：

The enriching line will pass through the intersection of the q-line and equilibrium line, and so as the point (x_D, x_D)



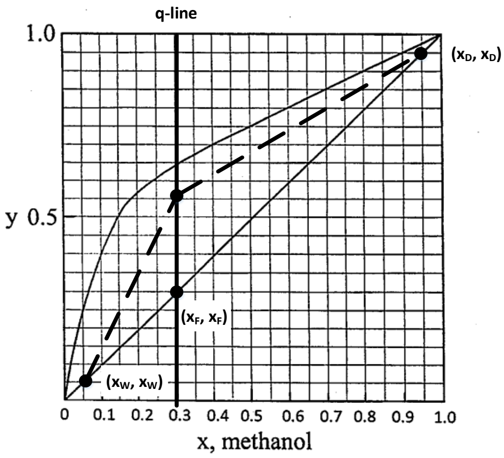
By the above figure, the enriching line (dashed) passes through (0.95, 0.95) and (0.3, 0.64). Therefore, the minimum reflux ratio can be calculated as,

$$slope = \frac{R_m}{R_m + 1} = \frac{0.95 - 0.64}{0.95 - 0.3}\ ,\ \underline{\underline{R_m = 0.91}}$$

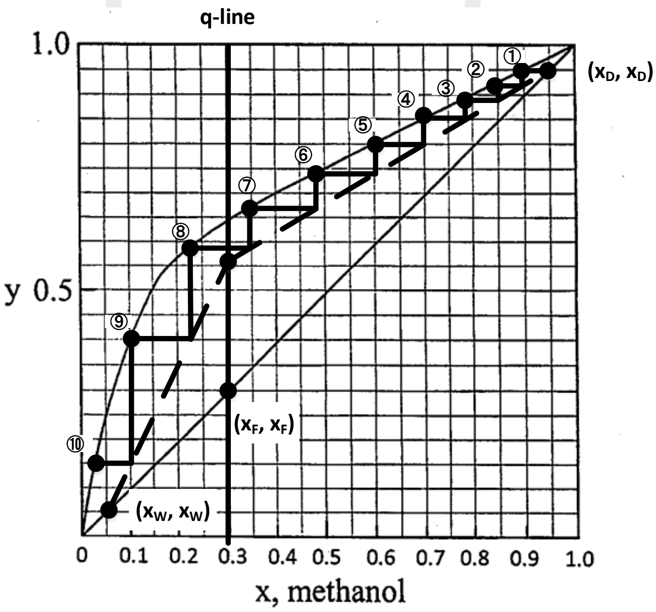
(b) If the reflux ratio is 2, then for the enriching line,

$$slope = \frac{R}{R+1} = \frac{2}{2+1} = \frac{2}{3}$$

which passes through (0.95, 0.95), so we can draw the figure as,



Therefore, by McCabe-Thiele graphical method,



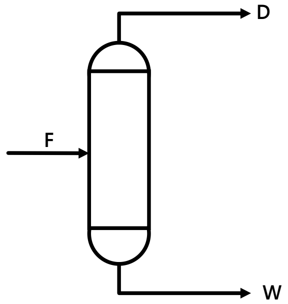
(1) <u>The optimum feed plate is at the 8th plate</u>

(2) <u>The number of theoretical stages is 10</u>

(3) <u>The composition at the 5th plate is (0.6, 0.8)</u>

(c)

By overall mass balance,



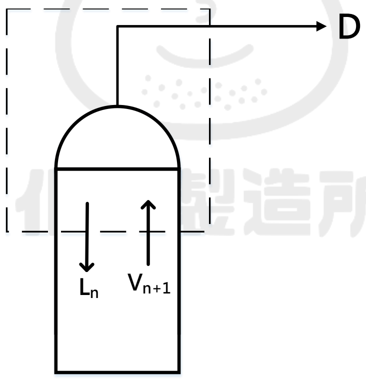
$$F = D + W \text{ ，} 1000 = D + W$$

By methanol mass balance,

$$Fx_F = Dx_D + Wx_W \text{ ，} 1000 \times 0.3 = 0.95D + 0.05W$$

$$\begin{cases} D = 277.8 \ (mol/hr) \\ W = 722.2 \ (mol/hr) \end{cases}$$

At enriching section,



By overall mass balance,

$$\underbrace{V_{n+1} = V_n}_{\text{恆莫耳溢流}} = L_n + D = 2D + D = 3D = 3 \times 277.8 = 833.4 \ (mol/hr)$$

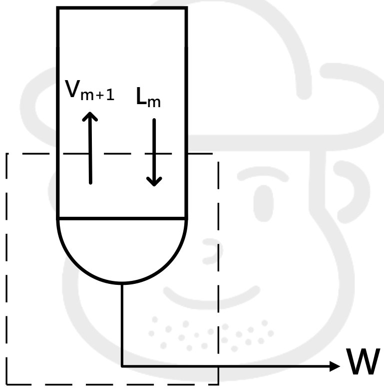
$$L_n = 2D = 555.6 \ (mol/hr)$$

At feed plate, by the definition of q-line,

$$\begin{cases} L_m = L_n + qF = L_n + F = 555.6 + 1000 = \underline{\underline{1555.6\ (mol/hr)}} \\ V_m = V_n - (1-q)F = V_n = \underline{\underline{833.4\ (mol/hr)}} \end{cases}$$

which L_m and V_m are the liquid molar flux and the vapor molar flux in the stripping section.

※在恆莫耳溢流(McCabe Thiele method 的假設之一)的情況下，整個 enriching section 的氣體與液體莫耳流量相同；整個 stripping section 也是相同情形，因此一旦確定 feed plate 區域流往或流自 stripping section 的流量，即可視為 stripping section 的流量，而若是考慮 stripping section 的質量平衡：



$$V_{m+1} = V_m = L_m - W$$

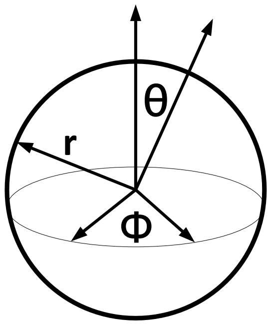
$$W = L_m - V_m = 1555.6 - 833.4 = \underline{\underline{722.2\ (mol/hr)}}$$

與上方整個塔的莫爾平衡結果相同。

(本題可參考：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed., Volume 2; p 564~566, Example 11.7 .)

Problem 10

【**Solution**】



(a)

By simplifing the ϕ-component of the equation of continuity, we get,

$$\rho(\underbrace{\cancel{\frac{\partial v_\phi}{\partial t}}}_{v_\phi \neq v_\phi(t)} + \underbrace{\cancel{v_r \frac{\partial v_\phi}{\partial r}}}_{v_r=0} + \underbrace{\cancel{\frac{v_\theta}{r}\frac{\partial v_\phi}{\partial \theta}}}_{v_\theta=0} + \underbrace{\cancel{\frac{v_\phi}{r\sin\theta}\frac{\partial v_\phi}{\partial \phi}}}_{v_\phi \neq v_\phi(\phi)} + \underbrace{\cancel{\frac{v_\phi v_r + v_\theta v_\phi \cot\theta}{r}}}_{v_\theta=v_r=0}) = -\underbrace{\cancel{\frac{1}{r\sin\theta}\frac{\partial p}{\partial \phi}}}_{p \neq p(\phi)}$$

$$+\mu[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial v_\phi}{\partial r}) + \frac{1}{r^2}\frac{\partial}{\partial \theta}(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\phi \sin\theta)) + \underbrace{\cancel{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 v_\phi}{\partial \phi^2}}}_{v_\phi \neq v_\phi(\phi)} + \underbrace{\cancel{\frac{2}{r^2\sin\theta}\frac{\partial v_r}{\partial \phi}}}_{v_r=0} + \underbrace{\cancel{\frac{2\cot\theta}{r^2\sin\theta}\frac{\partial v_\theta}{\partial \phi}}}_{v_\theta=0}] + \underbrace{\cancel{\rho g_\phi}}_{g_\phi=0}$$

整理可得：

$$\mu[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial v_\phi}{\partial r}) + \frac{1}{r^2}\frac{\partial}{\partial \theta}(\frac{1}{\sin\theta}\frac{\partial}{\partial \theta}(v_\phi \sin\theta))] = 0$$

(b)

$$\begin{cases} r = \kappa R,\ v_\phi = \Omega\kappa R\sin\theta \\ r = R,\ v_\phi = 0 \\ \theta = 0,\ v_\phi = 0 \end{cases}$$

(c)

No, if we let $r = R$, we may not absolutely get $v_\phi = 0$. The alternative solution form guessed is,

$$\boxed{v_\phi = f(r)\sin\theta}$$

※事實上，本題不需要將猜測的方程式代入 PDE 就要能夠由邊界條件看出解有誤，比較符合一般猜測解的形式的精神。一般來說在解複雜的問題時，僅有的線索只有邊界條件，單從邊界條件就應該要能判斷解的合理性，而非實際代進去做了複雜的運算才發現可能不適用。

(d)

To obtain the the troque needed to hold the outer shell stationary, we may need $\tau_{r\phi}$,

which is,

$$\tau_{r\phi} = -\mu[\cancel{\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi}} + r\frac{\partial}{\partial r}(\frac{v_\phi}{r})] = -\mu r\frac{\partial}{\partial r}(\frac{v_\phi}{r})$$

The torque needed is,

$$\boxed{\Gamma = \int_0^{2\pi}\int_0^{\pi}(\tau_{r\phi})_{r=R} R\sin\theta R^2 \sin\theta d\theta d\phi}$$

(本題改編自：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.;p 95, Example 3.6-5)

※本小題意在計算維持外球殼穩定靜止所需要的力矩。按照題意，內球沿 ϕ 方向旋轉，其會沿著 r 方向，對外球殼傳送動量，讓外球殼也傾向沿著 ϕ 方向轉。因此，為了使外球殼靜止不動，勢必需要施一個等大的力矩，來使得外球殼穩定，此即為本題所計算之 $\tau_{r\phi}$。而因此球殼系統本就沒有沿著 θ 方向運動，並且其運動系統也沿 θ 方向對稱，本就沒有，也不需要考慮 θ 方向。事實上，若以本題所猜的解代入 $\tau_{\theta\phi} = 0$

$$\tau_{\theta\phi} = -\mu[\frac{\sin\theta}{r}\frac{\partial}{\partial\theta}(\frac{v_\phi}{\sin\theta}) + \frac{1}{r\sin\theta}\frac{\partial v_\theta}{\partial\phi}] = 0$$

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Problem 1

【Solution】

True：(a)(d)(e)/ False：(b)(c)

Hagen Poiseuille equation is obtained under the following assumptions,

(1) Laminar flow

(2) Steady flow

(3) Incompressible flow

(4) Newtonian fluid

(5) Neglected entrance effect

(6) No-slip at the wall

(7) The fluid behaves as a continuum

※事實上可以利用(b)與(c)推導出圓管的速度分布與其相關性質，然而 Hagen-Poiseuille equation 特指：

$$Q = \frac{\pi R^4 \Delta P}{8\mu L}$$

條件較為嚴格

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 52.)

Problem 2

【Solution】

True：(a)(b)(c)(e)/ False：(d)

(a) The equation of continuity is merely obtained from mass balance, so applicable.

(b) The equation of motion is merely the result of momentum balance, so applicable.

(c) Navier-Stokes is derived based on the equation of motion and the assumptions of (1) constant ρ and (2) constant μ

(d) (e) By Navier-Stokes equation,

$$\rho\frac{Dv}{Dt} = -\nabla P + \rho g + \mu\nabla^2 v$$

If Reynolds number << 1, viscous term >> inertial term

$$\cancel{\rho\frac{Dv}{Dt}} = -\nabla P + \rho g + \mu\nabla^2 v$$

$$-\nabla P + \rho g + \mu\nabla^2 v = 0$$

(Stokes flow equation)

If Reynolds number >> 1, viscous term << inertial term

$$\rho\frac{Dv}{Dt} = -\nabla P + \rho g + \cancel{\mu\nabla^2 v}$$

$$\rho\frac{Dv}{Dt} + \nabla P - \rho g = 0$$

(Euler equation)

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 77~85.)

<u>Problem 3</u>

【Solution】

<u>True：(a)(b)(c)(e)/ False：(d)</u>

(a) True

(b) Stripping：溶質由液體傳遞至氣體$\leftrightarrow$Absorption：溶質由氣體傳遞至液體。

(c) True. 比如活性碳可除濕乾燥。

(d) False. Counterflow 可以提供較大的質傳驅動力，有較高的質傳速率與效率。

(e) True.

<u>Problem 4</u>

【Solution】

<u>True：(a)(b)(c)(e)/ False：(d)</u>

(a) True.

$$Sc = \frac{momentum\ diffusivity}{mass\ diffusivity}\ ,\ \Pr = \frac{momentum\ diffusivity}{thermal\ diffusivity}$$

They are analogous to each other

※而熱傳中的 Nusselt number 則與質傳之 Sherwood number 相似：

$$Nu = \frac{convective\ heat\ transfer}{thermal\ diffusion}\ ,\ Sh = \frac{convective\ mass\ transfer}{mass\ diffusion}$$

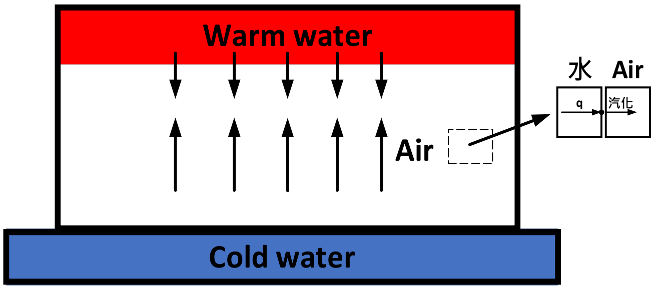
(b) True. They all have the dimension as, L^2/T

(c) True. For gaseous substances below 10 atm, their dependence of the related parameters on pressure is negligible by the prediction of collision theory.

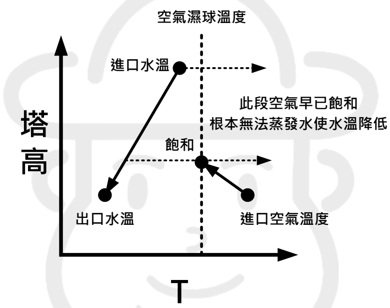
(本小題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 25/275. 推導比較偏向物化，可參考 Atkins, P.; De Paula, J.; Keeler, J. *Atkins' Physical Chemistry*, 9th ed.; p 757~758.)

(d) False. The wet-bulb temperature of the air should be below the temperature of water it makes contact; otherwise, the cooling efficiency of the tower may be lowered。

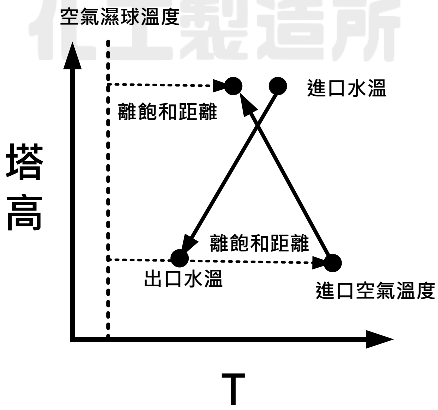
※Cooling tower 是利用乾燥空氣與水做接觸(通常為 countercurrent)，使水因水蒸氣壓梯度(水為飽和，空氣小於飽和)產生質傳至空氣中，而表面水吸收其餘水分子熱量產生汽化，使得液態水溫度降低的裝置，主要由質傳引起的蒸發降低溫度，而非靠空氣冷產生溫度梯度做冷卻。



若氣體 wet-bulb temperature 大於與其接觸之水的溫度，則空氣越容易達到飽和，無法使水有效的蒸發帶走熱量(可參考以下示意圖)：



而若是空氣濕球溫度小於水溫，則可以確保一定能夠蒸發水使水降溫：



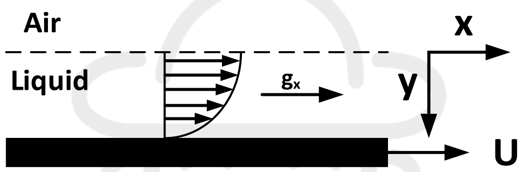
(e) True. In the nucleate boiling region, the heat transfer is facilitated by the bubbles formed on the heating surface. So it has a large heat transfer coefficient than other boiling regions. While in film boiling region, the heating surface is covered with a vapor blanket, so the interface of heating surface and the liquid is interfered, thus lowering the effective heat transfer coefficient.

(本小題可參考：Incropera, F.; Lavine, A.; Bergman, T. *Fundamentals of Heat and Mass Transfer*, 6th ed, p 624~626.)

Problem 5

【Solution】Yes：(a)(b)(d)(e)/No：(c)

Consider the flow system,



By the equation of motion, we have (no pressure drop)

$$-\frac{\partial \tau_{yx}}{\partial y}+\rho g_x=0 \text{ ， } \tau_{yx}=\rho g_x y+c_1$$

$\because y=0,\ \tau_{yx}=0$ ， $c_1=0$

$$\tau_{yx}=-\mu\frac{\partial v_x}{\partial y}=\rho g_x y \text{ ， } v_x=\frac{-\rho g_x}{2\mu}y^2+c_2$$

$\because y=B,\ v_x=U$ ， $c_2=U+\frac{\rho g_x}{2\mu}B^2$

$$v_x=\underbrace{\frac{-\rho g_x}{2\mu}y^2}_{parabolic}+\underbrace{U}_{linear,depends\ on\ y=B}+\frac{\rho g_x}{2\mu}B^2$$

(a) $g_x>0,\ U=0$ (b) $g_x>0,\ U>0$

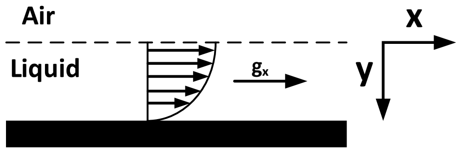
(c) $g_x=0, U=0$, 但此時應無速度，錯

(d) $g_x < 0,\ U > 0$ (e) $g_x = 0,\ U > 0$

※本題也可以利用 superposition 的觀念做答，將此系統拆分成兩大系統：

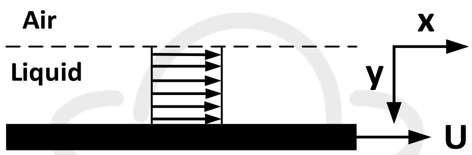
(1) 有重力影響但外板不移動

(注意上方接觸空氣，因此非對稱，只有半個拋物線)



(2) 無重力影響，但外板移動

(注意上方接觸空氣，且無其他影響，導致整體系統呈現 plug flow)

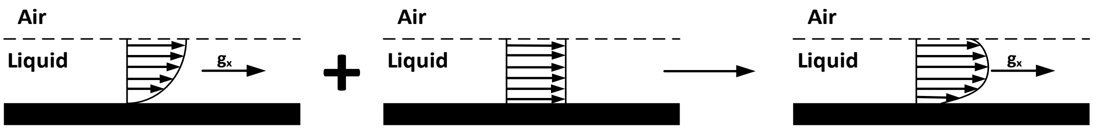


並且最終將兩種系統加起來即可：

(a)

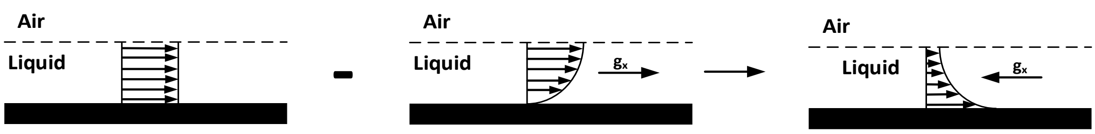
(0 代表下板不移動)

(b)



(c) 下板速度為 0，且為線性，無重力影響，因此流體應不會動

(d)



(“-“代表方向相反)

(e) plug flow 代表只有下板動，無重力影響。

Problem 6

【Solution】

(a)

Let the dimensionless velocity, time, and length is,

$$v_\theta = \frac{v_\theta}{v_0} \text{ , } \tilde{t} = \frac{t}{v_0 / l_0} \text{ , } \tilde{r} = \frac{r}{l_0}$$

The PDE becomes,

$$\underline{\underline{\frac{\rho v_0^2}{l_0}\frac{\partial v_\theta}{\partial \tilde{t}} = \frac{\mu v_0}{l_0^2}\frac{\partial}{\partial \tilde{r}}[\frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}(\tilde{r} v_\theta)]}}$$

(b)

左右同除 $\frac{\mu v_0}{l_0^2}$ ：

$$\frac{\rho v_0 l_0}{\mu}\frac{\partial v_\theta}{\partial \tilde{t}} = \frac{\partial}{\partial \tilde{r}}[\frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}(\tilde{r} v_\theta)]$$

Define,

$$\mathrm{Re} = \frac{\rho v_0 l_0}{\mu} = \frac{\textit{inertial force}}{\textit{viscous force}}$$

(c)

For the similarity analysis, the Reynolds number of both systems should be the same,

$$\mathrm{Re}_A = \mathrm{Re}_B$$

$\because v_0 = \Omega r$ ， $l_o = r$

$$\frac{\rho(\Omega R)R}{\mu} = \frac{\rho(3\Omega_B R)(3R)}{\mu} \text{ , } \underline{\underline{\Omega_B = \frac{1}{9}\Omega}}$$

Problem 7

【Solution】

Assume,

(1) Fully developed flow, $v_x = v_y = 0$

(2) $v_z \neq v_z(y)$

(3) Steady flow

(4) Constant ρ and μ

By the equation of continuity,

$$\cancel{\frac{\partial \rho}{\partial t}} + \cancel{\frac{\partial(\rho v_x)}{\partial x}} + \cancel{\frac{\partial(\rho v_y)}{\partial y}} + \frac{\partial(\rho v_z)}{\partial z} = 0$$

$$\frac{\partial v_z}{\partial z} = 0 \text{ ， } v_z = v_z(x) \text{ only}$$

By the equation of motion,

$$\rho(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_x}\frac{\partial v_z}{\partial x} + \cancel{v_y}\frac{\partial v_z}{\partial y} + v_z\cancel{\frac{\partial v_z}{\partial z}}) = -\cancel{\frac{\partial p}{\partial z}} - [\frac{\partial \tau_{xz}}{\partial x} + \cancel{\frac{\partial \tau_{yz}}{\partial y}} + \cancel{\frac{\partial \tau_{zz}}{\partial y}}] + \rho g \sin\beta$$

$$\frac{\partial \tau_{xz}}{\partial x} = \rho g \sin\beta \text{ ， } \tau_{xz} = (\rho g \sin\beta)x + c_1$$

$\because x = 0,\ \tau_{xz} = 0$ (free surface)， $\tau_{xz} = (\rho g \sin\beta)x$

To make the Bingham fluid to flow, the shear stress should be $\geq \tau_0$

$$\tau_{xz} = (\rho g \sin\beta)x \geq \tau_0$$

$x = H$ 代入：

$$(\rho g \sin\beta)H \geq \tau_0$$

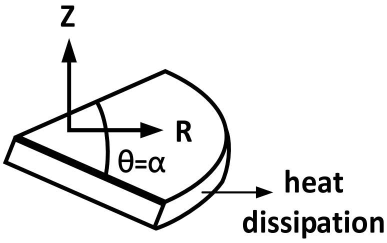
$$\sin\beta \geq \frac{\tau_0}{H\rho g} \text{ ， } \underline{\underline{\beta \geq \sin^{-1}(\frac{\tau_0}{H\rho g})}}$$

※本題需求黏滯力分布與角度之關係，因此只需要用到 equation of motion in terms of shear stress，不需要使用 Navier-Stokes equation；且本題為非牛頓流體，本來就不能用。

<u>Problem 8</u>

【Solution】

(a)



(這裡的 wedge 指長的像 pizza 一樣的形狀，為具有厚度的扇形，若是完整的圓則變成圓柱)

The Biot number is defined as,

$$Bi = \frac{h(\frac{V}{A_s})}{k} = \frac{\textit{resistance to conduction within the solid}}{\textit{resistance to convection across the film boundary}}$$

The temperature distribution of the wedge may depend on both the z and r direction, to simply the problem into the one associated only with the r direction, we should check if it's independent of z direction,

$$V = solid's\ volume = \pi R^2 \times \frac{\alpha}{2\pi} \times H = \frac{\alpha H R^2}{2}$$

$$A_s = \pi R^2 \times \frac{\alpha}{2\pi} = \frac{\alpha R^2}{2}$$

$$Bi = \frac{h(\frac{\frac{\alpha H R^2}{2}}{\frac{\alpha H^2}{2}})}{k} = \frac{hH}{k}$$

Because the thickness of the wedge is thin (thin cylindrical wedge from the question statement),

$Bi = \frac{hH}{k}$ may be very small,

we can simplify the system into an one-dimensional one (r direction)

(b)

Assume $T \neq T(\theta)$ due to symmetry and insulation, by the overall equation of energy in cylindrical coordinate (including convection in z direction in equation),

$$-k(\alpha r H)\left.\frac{dT}{dr}\right|_r - [-k(\alpha r H)\left.\frac{dT}{dr}\right|_{r+dr}] - 2\alpha r dr h(T-T_a) = 0$$

同除 $(k\alpha H)dr \to 0$,

$$\frac{d}{dr}(r\frac{dT}{dr}) - \frac{2hr}{kH}(T-T_a) = 0$$

展開並整理：

$$\frac{dT}{dr} + r\frac{d^2T}{dr^2} - (\frac{2h}{kH})r(T-T_a) = 0$$

同乘 r

$$r^2\frac{d^2T}{dr^2} + r\frac{dT}{dr} - (\frac{2h}{kH})r^2(T-T_a) = 0$$

令 $T - T_a = \theta$ ，對照題目所給方程式，可得：

$$r^2\frac{d^2\theta}{dr^2} + r\frac{d\theta}{dr} - (\frac{2h}{kH})r^2\theta = 0 \text{，} \lambda^2 = \frac{2h}{kH}$$

$$\theta = c_1 I_0(\lambda r) + c_2 K_0(\lambda r)$$

$\because r = 0,\ \theta = finite$,

$$\theta(r=0) = c_1 I_0(0) + c_2 K_0(0)$$

$K_0(0) = \infty$ ， $c_2 = 0$

$\because r = R,\ T = T_h$ 、 $\theta = T_h - T_a$

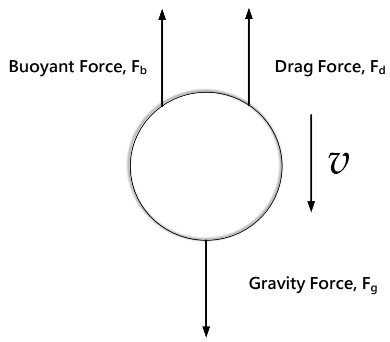
$$T_h - T_a = c_1 I_0(\lambda R) \text{，} c_1 = \frac{T_h - T_a}{I_0(\lambda R)}$$

$$T(r) = (T_h - T_a)\frac{I_0(\lambda r)}{I_0(\lambda R)} = \underline{\underline{(T_h - T_a)\frac{I_0(\sqrt{\frac{2h}{kH}}r)}{I_0(\sqrt{\frac{2h}{kH}}R)}}}$$

Problem 9

【Solution】

(a)



By force balance on the particle,

$$F = ma = m\frac{dv}{dt} = F_g - F_b - F_d$$

As the particle reaches terminal velocity,

$$m\frac{d\cancel{v}}{\cancel{dt}} = \frac{1}{6}\pi D^3 \rho_d g - \frac{1}{6}\pi D^3 \rho g - F_d = 0$$

For F_d, by the definition of friction factor,

$$F_d = \underbrace{(\pi R^2)}_{projecting\ area}(\frac{1}{2}\rho v^2) f$$

$$\frac{(\rho_d - \rho)\pi D^3 g}{6} - \frac{\pi \rho_d v^2 D^2}{8} f = 0 \text{ , } \frac{(\rho_d - \rho)Dg}{6} - \frac{\rho v^2}{8} f = 0$$

For ρ_d

$$\rho_d = \frac{2}{\frac{\pi}{6}\times 4^3} \times 1000 = 59.68\ (kg/m^3)$$

For the friction factor, try $f = 0.44$, (the diameter and density of the ball may be large enough)

$$\frac{(\rho_d - \rho)Dg}{6} - \frac{\rho v^2}{8} \times 0.44 = 0$$

$$\frac{(59.68 - 1.2)\times 4\times 10^{-2} \times 9.8}{6} - \frac{1.2v^2}{8} \times 0.44 = 0$$

$$v = 7.61\ (m/s)$$

【Check for Reynolds number】

$$\mathrm{Re} = \frac{\rho v D}{\mu} = \frac{1.2 \times 7.61 \times 4 \times 10^{-2}}{2 \times 10^{-5}} = 18620$$

<u>The value of 0.44 can be used as the friction factor in this case</u>

(a)

In this case, the force balance equation is still applicable,

$$\frac{(\rho_d - \rho)Dg}{6} - \frac{\rho v^2}{8} f = 0$$

For the friction factor, try $f = (\sqrt{24/\mathrm{Re}} + 0.54)^2$,

(the diameter and density of the ball may be small enough)

$$\frac{(\rho_d - \rho)Dg}{6} - \frac{\rho v^2}{8} \times (\sqrt{\frac{24\mu}{\rho v D}} + 0.54)^2 = 0$$

$$\frac{(1100 - 1000) \times 0.07 \times 10^{-3} \times 9.8}{6} - \frac{1000 v^2}{8} \times (\sqrt{\frac{24 \times 0.002}{1000 v \times 0.07 \times 10^{-3}}} + 0.54)^2 = 0$$

$$v = 1.31 \times 10^{-4} \ (m/s)$$

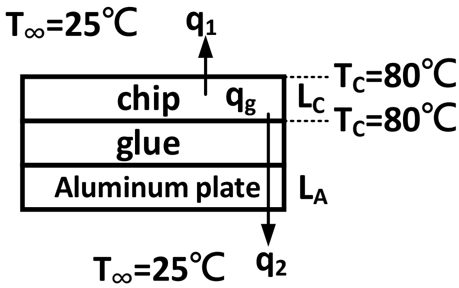
【Check for Reynolds number】

$$\mathrm{Re} = \frac{\rho v D}{\mu} = \frac{1000 \times 1.31 \times 10^{-4} \times 0.07 \times 10^{-3}}{0.002} = 4.59 \times 10^{-3}$$

<u>Which is applicable for this case.</u>

Problem 10

【Solution】



By energy balance on the chip,

$$q_g - q_1 - q_2 = 0$$

For $q_g = q_e A$ （q_e單位為W/m^2）

For q_1, because the system is in steady state,

$$q_1 = q_{con}$$

$$q_1 = hA(T_{chip} - T_\infty)$$

For q_2, because the system is in steady state,

$$q_2 = q_{glue} = q_{plate} = q_{con}$$

$$q_2 = \frac{(T_{chip} - T_\infty)}{R_{glue} + \dfrac{L_{plate}}{k_{plate}A} + \dfrac{1}{hA}}$$

代入 energy balance，且同除 A

$$q_e \cancel{A} - h\cancel{A}(T_{chip} - T_\infty) - \frac{(T_{chip} - T_\infty)}{R_{glue}A + \dfrac{L_{plate}}{k_{plate}\cancel{A}} + \dfrac{1}{h\cancel{A}}} = 0$$

$$1\times10^4 - 100(80-25) - \frac{(80-25)}{R\times(2\times10^{-2})^2 + \dfrac{2\times10^{-3}}{240} + \dfrac{1}{100}} = 0$$

$$\underline{\underline{R = 5.53\ (K/W)}}$$

(b)

Because the thickness of the aluminum plate contributes the thermal resistance of the system, which is,

$$R_{plate} = \frac{L_{plate}}{k_{plate} A}$$

where L_{plate} is the thickness of the aluminum plate.

<u>the statement would be true.</u>

<u>Problem 11</u>

【Solution】

(a)

By overall mole balance,

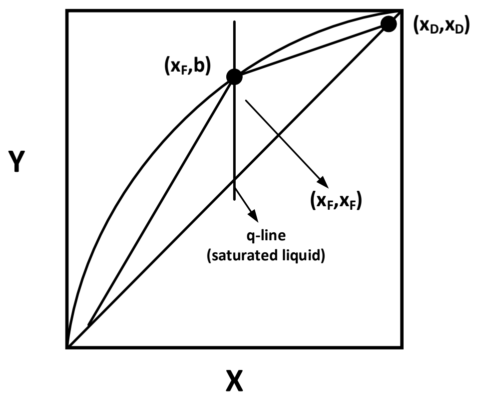
$$F = D + B \text{ ， } D + B = 100$$

By the mole balance of benzene,

$$Fx_F = Bx_B + Dx_D \text{ ， } 100 \times 0.50 = 0.90D + 0.1B$$

$$\boxed{\begin{cases} D = 50 \ (kmol/h) \\ B = 50 \ (kmol/h) \end{cases}}$$

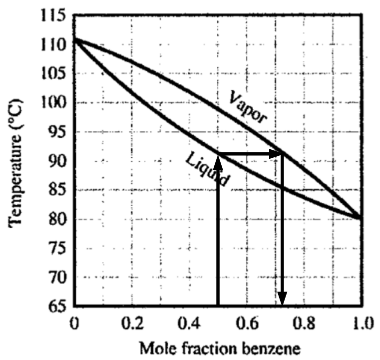
(b)



Under the minimum reflux ratio, the q-line, operation line and equilibrium line would all intersect, and the slope of operation line would be,

$$\frac{x_D - b}{x_D - x_F} = \frac{R_m}{R_m + 1}$$

By the phase diagram attached, we can read the point (x_F ,b),



For b, the vapor concentration of benzene, can be obtained as 0.71

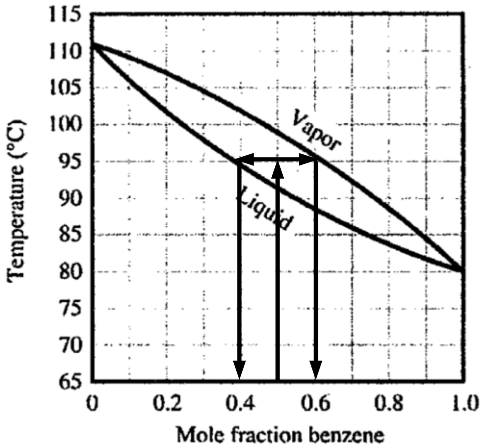
$$\frac{0.90 - 0.71}{0.90 - 0.50} = \frac{R_m}{R_m + 1} \text{ , } \underline{\underline{R_m = 0.90}}$$

(c)

If the feed is 50% saturated liquid, the composition of liquid and vapor can be calculated by the lever-arm rule,

$$\frac{L}{V}=\left|\frac{z-y}{z-x}\right|=\frac{0.5}{0.5}=1$$

By the phase diagram attached,



The compositions of liquid and vapor are,

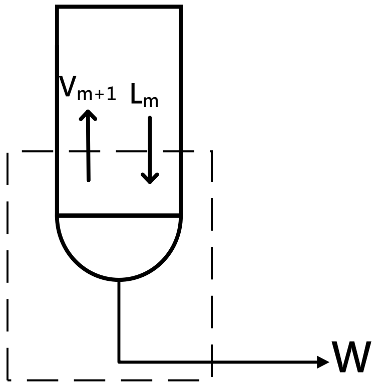
$$\begin{cases} vapor\begin{cases} Toluene=0.4 \\ Benzene=0.6 \end{cases} \\ liquid\begin{cases} Toluene=0.6 \\ Benzene=0.4 \end{cases} \end{cases}$$

The relative volatility (Benzene to Toluene) is,

$$\alpha=\frac{0.6\times 0.6}{0.4\times 0.4}=\underline{\underline{2.25}}$$

(d)

<u>For stripping section,</u>



By overall balance,

$$L_m = V_{m+1} + W$$

By key component balance,

$$L_m x_m - V_{m+1} y_{m+1} = W x_w$$

$$\text{斜率：} \frac{L_m}{V_{m+1}} = \frac{L_m}{L_m - W} > 1$$

(本小題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 672.)

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Problem 1

【Solution】

(a)

Molecular transport 意旨在微觀(microscope)的角度下，單個分子所具有的動量、能量等經由微觀的運動傳遞至周圍的傳遞方式；而 convective transport 則專指在巨觀(macroscope)下，流體經由巨觀流動(bulk motion)所產生的傳遞方式。在方程式的建立上也有不一樣的描述：

Molecular transport：係數×梯度 (例：$-\mu\dfrac{dv_x}{dy}$)

Convective transport：ρv(傳遞量) (例：$\rho v \cdot v$)

(Molecular transport refers to the transfer of momentum or energy via the microscopic motion of individual molecules to adjacent ones, while convective transport involves the bulk movement of fluids.)

(b)

在兩相的質量傳遞上，利用 overall mass transfer coefficient 的好處在於不需要處理介面上的濃度，利用 bulk 以及平衡的濃度，在搭配 overall mass transfer coefficient 下就可以直接推算質傳速率，例如：

$$K_y(y_{Ab}-y_{Ae})=k_y(y_{Ab}-y_{A0})\begin{cases} K_y = \textit{overall mass transfer coefficient based on gas phase} \\ y_{Ab} = \textit{bulk composition in gas phase} \\ y_{Ae} = \textit{gas phase composition in equilibrium with liquid at } x_{Ab} \\ y_{A0} = \textit{composition at the interphase} \end{cases}$$

(c)

For Reynolds analogy, when $\Pr \approx 1$ or $Sc \approx 1$, and no form drag, we have the following relation,

$$\frac{C_f}{2} = St = St_m$$

where

$$\begin{cases} C_f = drag\ coefficient \\ St = \dfrac{Nu}{\mathrm{Re}\,\mathrm{Pr}} \\ St_m = \dfrac{Sh}{\mathrm{Re}\,Sc} \end{cases}$$

By this analogy we can relate the key parameters of the velocity, thermal, and concentration boundary layers.

(d)

Fourier's first law of heat conduction：

$$q = -k\nabla T$$

The counterpart in momentum transport is：

$$\tau = -\mu\nabla v$$

(e)

Fourier's second law of heat conduction：

$$\frac{\partial T}{\partial t} = \alpha\nabla^2 T$$

The counterpart in mass transport is：

$$\frac{\partial C}{\partial t} = -D\nabla^2 C$$

Problem 2

【Solution】

(a)

題目給定假設：

(1) one dimensional (x-direction)

(2) steady-state

(3) incompressible

(4) fully-developed

(5) laminar

額外假設：

(6) symmetry in z-direction ($v_x \neq f(z)$)

By the equation of continuity,

$$\frac{\partial \rho}{\partial t} + \nabla \bullet (\rho \vec{u}) = 0$$

$\because$ incompressible, ρ is constant

$$\nabla \bullet (\vec{v}) = \frac{\partial u_x}{\partial x} + \underbrace{\cancel{\frac{\partial u_y}{\partial y}}}_{u_y=0} + \underbrace{\cancel{\frac{\partial u_z}{\partial z}}}_{u_z=0} = 0$$

$$u_x = f(y) \text{ only}$$

By shell balance of momentum,

$$\rho u_x u_x dydz\big|_x - \rho u_x u_x dydz\big|_{x+dx} + \tau_{yx} dxdz\big|_y - \tau_{yx} dxdz\big|_{y+dy} + pdydz\big|_x - pdydz\big|_{x+dx} = 0$$

同除 $dxdydz \to 0$

$$\underbrace{\cancel{\frac{-\partial(\rho u_x u_x)}{\partial x}}}_{u_x = f(y)} - \frac{\partial \tau_{yx}}{\partial y} - \frac{dp}{dx} = 0 \text{ ，}$$

$\tau_{xz} = -\mu \dfrac{du_x}{dy}$ ，且令 modified pressure $P = p + \rho g z$ 代入：

$$\mu \frac{d^2 u_x}{dy^2} = (\frac{dP}{dx}) = constant$$

之所以最後可以訂為 constant 原因在於左式對 y 微分(視為 y 函數)，而壓力項對 x 微分(視為 x 函數)，因此在此情況下，微分後應為常數(同時可視為 x 與 y 的函數)。事實上 v_x 為 y 函數沒錯，modified pressure 若同時討論其他方向，也會得到只是 x 有關的函數的結論：

$$\text{y-component：} -\frac{dP}{dy} = 0 \ \text{(非 y 函數)}$$

$$\text{z-component：} -\frac{dP}{dz} = 0 \ \text{(非 z 函數)}$$

若在此使用一般的壓力，非 modified pressure，則在 z 方向的討論上就會出現 p 與 z 有關的結論，因此就不會有(a)小題的關係，也就是(a)小題之 P 改用大寫的原因。

(b)

根據(a)小題結論：

$$\mu \frac{d^2u_x}{dy^2} = (\frac{dP}{dx})$$

$$u_x = \frac{1}{2\mu}(\frac{dP}{dx})y^2 + c_1 y + c_2 \begin{cases} y = \frac{a}{2},\ u_x = 0 \\ y = 0,\ \frac{du_x}{dy} = 0\ (symmetry) \end{cases},\ \begin{cases} c_1 = 0 \\ c_2 = -\frac{1}{2\mu}(\frac{dP}{dx})(\frac{a}{2})^2 \end{cases}$$

$$u_x = \frac{1}{2\mu}(\frac{dP}{dx})y^2[1-(\frac{a/2}{y})^2]$$

The average velocity can be calculated as,

$$u_m = \frac{1}{2\mu}(\frac{dP}{dx})\frac{\int_0^W \int_{\frac{-a}{2}}^{\frac{a}{2}} y^2[1-(\frac{a/2}{y})^2]dydz}{\int_0^W \int_{\frac{-a}{2}}^{\frac{a}{2}} dydz} = \frac{1}{2\mu}(\frac{dP}{dx})(-\frac{a^2}{6}) = \underline{\underline{-\frac{a^2}{12\mu}(\frac{dP}{dx}) = \frac{a^2}{12\mu}(\frac{\Delta P}{L})}}$$

$$u_x = \frac{1}{2\mu}(\frac{dP}{dx})[y^2-(\frac{a}{2})^2] = -\frac{a^2}{12\mu}(\frac{dP}{dx})\times 6[(\frac{1}{2})^2-(\frac{y}{a})^2] = u_m[\frac{3}{2}-6(\frac{y}{a})^2]$$

$$u_x = u_m[\frac{3}{2}-6(\frac{y}{a})^2] = \frac{3}{2}u_m[1-4(\frac{y}{a})^2] = \underline{\underline{\frac{3}{2}u_m[1-(\frac{y}{a/2})^2]}}$$

(c)(d)

Define,

$$D_h \equiv \frac{4A_c}{P} = \frac{4aW}{2W} = 2a$$

Define Moody (or Darcy) friction factor,

$$f = 4\times \underbrace{\frac{-\mu \left.\dfrac{du_x}{dy}\right|_{y=\frac{a}{2}}}{\rho u_m^2/2}}_{Fanning\ friction\ factor} = \frac{48u_m\mu}{\rho u_m^2 a} = \frac{48}{\dfrac{\rho u_m a}{\mu}} = \frac{96}{\dfrac{\rho u_m (2a)}{\mu}} = \frac{96}{\dfrac{\rho u_m D_h}{\mu}} = \frac{96}{\mathrm{Re}_{Dh}}$$

The coefficient *C* is <u>96</u> for the parallel-plate channel here.

(本小題改編自：Incropera, F.; Lavine, A.; Bergman, T. *Fundamentals of Heat and Mass Transfer*, 6th ed, p 534, Problem 8.5.)

<u>Problem 3</u>

【Solution】

(a)

Because the reaction occurs heterogenously,

$$N_A^{"} = R_A^{"} = -k^{"}C_A[=]\frac{mole}{m^2\cdot s}$$

$$\underline{\underline{k^{"}[=]\frac{mole}{m^2\cdot s}\times\frac{m^3}{mole}[=]\frac{m}{s}}}$$

(b)

Assume：

(1) The process is in steady state

(2) One-dimensional transfer

By the equation of continuity,

$$-\vec{\nabla}\cdot\vec{N}_A^{"}+\underbrace{\cancel{R_A^{"'}}}_{no\ homogeneous\ reaction}=\underbrace{\cancel{\frac{\partial C_A}{\partial t}}}_{steady\ state}$$

$$-\vec{\nabla}\cdot\vec{N}_A^{"}=0$$

By Fick's law, we have,

$$\vec{N}_A^{"}=\vec{J}_A^{*}+\underbrace{\cancel{x_A(\vec{N}_A^{"}+\vec{N}_B^{"})}}_{stationary\ medium}=-D_{AB}\frac{dC_A}{dx}=-D_{AB}C\frac{dx_A}{dx}$$

代回：

$$\boxed{\frac{d^2x_A}{dx^2}=0}$$

(c)

In this case, the appropriate boundary conditions are,

$$\begin{cases}x=0,\ N_A^{"}=R_A^{"}=-CD_{AB}\dfrac{dx_A}{dx}=-k^{"}C_A(0)\\ x=L,\ x_A=x_{AL}\end{cases}$$

By the result in subquestion (b), we have,

$$x_A=c_1x+c_2$$

Insert into B.C. (1),

$$D_{AB}c_1=k^{"}c_2\ ，\ c_2=\frac{D_{AB}}{k^{"}}c_1$$

Insert in to B.C. (2),

$$x_{AL}=c_1L+c_2\ ，\ x_{AL}=c_1L+\frac{D_{AB}}{k^{"}}c_1=c_1(L+\frac{D_{AB}}{k^{"}})$$

$$c_1 = \frac{x_{AL}}{(L + \frac{D_{AB}}{k^"})}\text{，}c_2 = \frac{D_{AB}x_{AL}}{k^"(L + \frac{D_{AB}}{k^"})}$$

代回：

$$x_A = \frac{x_{AL}}{(L + \frac{D_{AB}}{k^"})}x + \frac{D_{AB}x_{AL}}{k^"(L + \frac{D_{AB}}{k^"})}$$

$$\frac{x_A}{x_{AL}} = \frac{x}{(L + \frac{D_{AB}}{k^"})} + \frac{D_{AB}}{k^"(L + \frac{D_{AB}}{k^"})} = \frac{(x + \frac{D_{AB}}{k^"})}{(L + \frac{D_{AB}}{k^"})}$$

$$\frac{x_A}{x_{AL}} = \frac{(x + \frac{D_{AB}}{k^"})}{(L + \frac{D_{AB}}{k^"})} = \frac{(\frac{k^"}{D_{AB}}x + 1)}{(\frac{k^"L}{D_{AB}} + 1)} = \underline{\underline{\frac{(1 + \frac{k^"}{D_{AB}}x)}{(1 + \frac{k^"}{D_{AB}}L)}}}$$

(d)

$$N^"_{A,x}(0) = -CD_{AB}\left.\frac{dx_A}{dx}\right|_{x_A=0} = \frac{-CD_{AB}x_{AL}(\frac{k^"}{D_{AB}})}{(1 + \frac{k^"}{D_{AB}}L)} = \underline{\underline{-\frac{k^"Cx_{AL}}{(1 + \frac{k^"}{D_{AB}}L)}}}$$

(e)

If the process is <u>**"reaction limited"**</u>,

$$\underline{\underline{\frac{x_A(0)}{x_{AL}} = \lim_{k^" \to 0}\frac{(1 + \frac{k^"}{D_{AB}} \times 0)}{(1 + \frac{k^"}{D_{AB}}L)} \approx 1}}$$

也就是在觸媒表面上的 A 濃度幾乎不改變，因為反應速率很慢。

$$\underline{\underline{N^"_{A,x}(0) = -\lim_{k^" \to 0}\frac{k^"Cx_{AL}}{(1 + \frac{k^"}{D_{AB}}L)} = 0}}$$

也就是在傳送至觸媒表面上的 A flux 非常小，因為幾乎沒有消耗。

(f)

If the process is “diffusion limited”,

$$\underline{\underline{\frac{x_A(0)}{x_{AL}} = \lim_{k''\to\infty} \frac{(1+\dfrac{k''}{D_{AB}}\times 0)}{(1+\dfrac{k''}{D_{AB}}L)} = \frac{1}{\infty} \approx 0}}$$

也就是在觸媒表面上的 A 因為反應過快，幾乎一碰到觸媒就全部反應。

$$\underline{\underline{N''_{A,x}(0) = -\lim_{k''\to\infty} \frac{k''Cx_{AL}}{(1+\dfrac{k''}{D_{AB}}L)} \approx \frac{k''Cx_{AL}}{\dfrac{k''L}{D_{AB}}} = \frac{D_{AB}}{L}Cx_{AL}}}$$

也就是在此狀況下，質傳速率幾乎為此系統可達到的最大值，因最高濃度為 x_{AL}，且觸媒表面

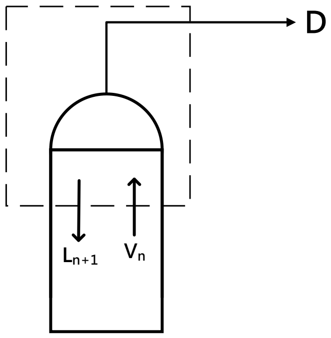
濃度為 0，根據 Fick’s law 可寫為：

$$N''_{A,x}(0) = \frac{D_{AB}}{L}C(x_{AL}-0) = \frac{D_{AB}}{L}Cx_{AL}$$

Problem 4

【Solution】

Construct the operation line, top column first,



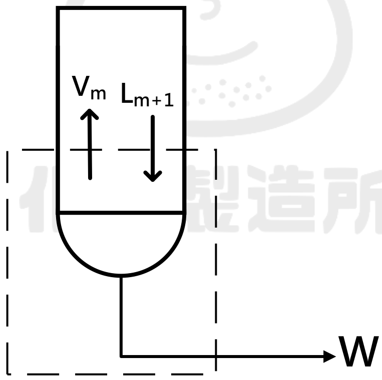
By mole balance for the light component,

$$y_n V_n = L_{n+1} x_{n+1} + D x_d \text{ , } y_n = \frac{L_{n+1}}{V_n} x_{n+1} + \frac{D}{V_n} x_d$$

By the assumption of constant molar flow rate, let $L_{n+1} = L_n$

$$\boxed{y_n = \frac{L_n}{V_n} x_{n+1} + \frac{D}{V_n} x_d}$$

Again, for the bottom column,



By mole balance for the light component,

$$y_m V_m + W x_w = L_{m+1} x_{m+1} \text{ , } y_n = \frac{L_{m+1}}{V_m} x_{m+1} + \frac{W}{V_m} x_w$$

By the assumption of constant molar flow rate, let $L_{m+1} = L_m$

$$\boxed{y_m = \frac{L_m}{V_m} x_{m+1} - \frac{W}{V_m} x_m}$$

To begin with, we calculate the value of V_n and L_n, by overall mole balance,

$$D+W=100$$

By overall mole balance for key component,

$$Dx_d+Wx_w=100\times 0.4$$

$$0.9\times D+0.1W=40$$

$$D=37.5\ kmol\ ，\ W=62.5\ kmol$$

The reflux ratio is 3, therefore,

$$L_n=RD=3D=112.5\ kmol$$

$$V_n=L_n+D=112.5+37.5=150\ kmol$$

Therefore, the top operating line is,

$$\boxed{y_n=\frac{112.5}{150}x_{n+1}+\frac{37.5}{150}\times 0.9=0.75x_{n+1}+0.225}$$

For bottom operating line, since the feed enters as saturated liquid at boiling point,

$$L_m=L_n+F=112.5+100=212.5\ kmol$$

$$V_m=L_m-W=212.5-67.5=150\ kmol$$

$$\boxed{y_m=\frac{212.5}{150}x_{m+1}-\frac{67.5}{150}\times 0.1=1.415x_{m+1}-0.042}$$

Since the top product leaves the column after condensed, $y_t=x_d=0.90$ in such case, and $x_t=0.79$ by the figure attached. (此時對照題目附圖，為第 1 板)

第 2 板：

$$y_{t-1}=0.75x_t+0.225=0.75\times 0.79+0.225=0.818$$

$$x_{t-1}=0.644\ \text{ from the figure}$$

第 3 板：

$$y_{t-2}=0.75x_{t-1}+0.225=0.75\times0.644+0.225=0.708$$

$$x_{t-2}=0.492 \quad \text{from the figure}$$

第 4 板：

$$y_{t-3}=0.75x_{t-2}+0.225=0.75\times0.492+0.225=0.594$$

$$x_{t-3}=0.382 \quad \text{from the figure}$$

因 feed 的 composition 為 0.4，此時計算出之 x_{t-3} 很接近 feed 的組成，因此後續計算改使用下塔操作線：

第 5 板：

$$y_{t-4}=1.415x_{t-3}-0.042=1.415\times0.382-0.042=0.498$$

$$x_{t-4}=0.298 \quad \text{from the figure}$$

第 6 板：

$$y_{t-5}=1.415x_{t-4}-0.042=1.415\times0.298-0.042=0.379$$

$$x_{t-5}=0.208 \quad \text{from the figure}$$

第 7 板：

$$y_{t-6}=1.415x_{t-5}-0.042=1.415\times0.208-0.042=0.252$$

$$x_{t-6}=0.120 \quad \text{from the figure}$$

第 8 板：

$$y_{t-7}=1.415x_{t-6}-0.042=1.415\times0.120-0.042=0.127$$

$$x_{t-7}=0.048 \quad \text{from the figure}$$

此時已經低於題目所要求之塔底分率應為 0.10，故總板數應為 8 板，而若算上 reboiler，**<u>則總理論板數應為 8-1 = 7 板，且 feed 應於第 4 或 5 板進入。</u>**

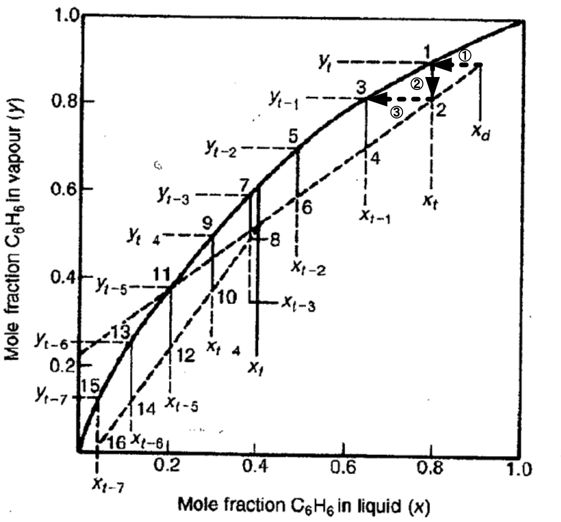
本題所規定使用之 Lewis-Sorel method 其實可視為 McCabe-Thiele method 的公式版本，應兩者是基於相同的假設與公式做理論板數的推求，若考慮 Lewis-Sorel 的上板操作線(即題目附圖之 2,4,6,8 所連成的線)，其公式為：

$$y_n = \frac{L_n}{V_n} x_{n+1} + \frac{D}{V_n} x_d$$

此時令 $x_{n+1} = x_d$ 代入：

$$y_n = \frac{L_n}{V_n} x_d + \frac{D}{V_n} x_d = \frac{L_n + D}{V_n} x_d = x_d$$

也就是上段操作線通過 (x_d, x_d)，因此跟原文書或是補習班推導 McCabe-Thiele 方法之操作線是一模一樣的過程。而下段操作線也可以用一樣的方式推出。另外，假設在這題我們使用 McCabe-Thiele 方法作圖，可以得到這樣的過程：



可以發現①：$y_t = x_d$ 對過去

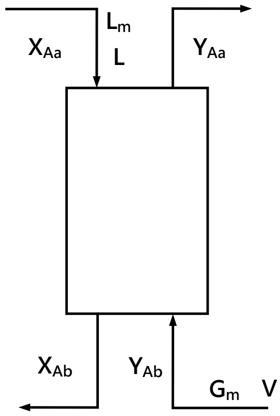
②：訂好 y_t 後，往 x_t 對過去，也就是 McCabe-Thiele 的往下畫

③：訂好 x_t 後，計算出 y_{t-1}，也就是②往下畫後就能夠訂得出來的 y 值，再往左邊對的話，就可以訂出相對應的 x_{t-1}。另外，根據附圖也可知道大概在第 4 板的位置為進料板，總共需要 8 個平衡單元、8-1=7 個理論板。

(本題可參考：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 563~566, Example 11.7。與 McCabe-Thiele 的比較可參考同本書之 p 566~568.)

Problem 5

【Solution】



$$G_m = \text{gas molar flow rate on a solute-free basis.}$$

$$L_m = \text{liquid flow rate on a solute-free basis.}$$

By the question statement, we have

$$X_{Aa} = 0.06 \text{ 、 } X_{Ab} = 0.001 \text{ 、 } Y_{Ab} = 0$$

For **minimum steam consumptions**, the operating line will intersect with equilibrium line, therefore,

$$Y_{Aa} = 3X_{Aa} = 3 \times 0.06 = 0.18$$

By mole balance for A

$$L_m(X_{Aa} - X_{Ab}) = G_m(Y_{Aa} - Y_{Ab})$$

$$(\frac{G_m}{L_m})_{\min} = \frac{X_{Aa} - X_{Ab}}{Y_{Aa} - Y_{Ab}} = \frac{0.06 - 0.001}{0.18 - 0} = 0.328$$

For **actual steam consumptions**, Y_{Aa} is no longer on the equilibrium line, therefore by Kremser equation (applicable when the equilibrium line is straight),

$$N = \frac{\ln[(Y_{Aa} - Y_{Aa}^*)/(Y_{Ab} - Y_{Ab}^*)]}{\ln[(Y_{Ab}^* - Y_{Aa}^*)/(Y_{Ab} - Y_{Aa})]} \text{ (N 為理論板數)}$$

$$Y_{Aa}^* = 3X_{Aa} = 3\times 0.06 = 0.18 \text{ ，} Y_{Ab}^* = 3X_{Ab} = 0.003$$

代回：

$$30\times 0.3 = \frac{\ln[(Y_{Aa}-0.18)/(0-0.003)]}{\ln[(0.003-0.18)/(0-Y_{Aa})]} \text{ ，} Y_{Aa} = 0.129$$

Therefore, the actual steam consumptions can be calculated as,

$$(\frac{G_m}{L_m}) = \frac{X_{Aa}-X_{Ab}}{Y_{Aa}-Y_{Ab}} = \frac{0.06-0.001}{0.129-0} = 0.457$$

The ratio of the specific and minimum steam consumptions is thereby,

$$\frac{(G_m/L_m)_{act}}{(G_m/L_m)_{\min}} = \frac{0.457}{0.328} = \underline{\underline{1.39}}$$

若

$$\frac{(G_m/L_m)_{act}}{(G_m/L_m)_{\min}} = 2 \text{ ，} (\frac{G_m}{L_m})_{act} = (\frac{G_m}{L_m})_{\min}\times 2 = 0.328\times 2 = 0.656$$

$$(\frac{G_m}{L_m}) = \frac{X_{Aa}-X_{Ab}}{Y_{Aa}-Y_{Ab}} = \frac{0.06-0.001}{Y_{Aa}-0} = 0.656 \text{ ，} Y_{Aa} = 0.09$$

代回 Kremser equation：

$$N = \frac{\ln[(0.09-0.18)/(0-0.003)]}{\ln[(0.003-0.18)/(0-0.09)]} = 5.03$$

The actual number of plates is,

$$\underline{\underline{N_{act} = \frac{5.03}{0.3} = 16.76 \approx 17 \text{ } plates}}$$

事實上，本題之 Kremser equation 有很多不同形式，都可以算出一樣的答案，以下做不同的示範：

【法②】By using stripping/absorption factor

$$\frac{X_{Aa}-X_{Ab}}{X_{Aa}-\frac{Y_{Ab}}{m}}=\frac{(1/A)^{N+1}-(1/A)}{(1/A)^{N+1}-1}$$

where $A=\frac{L_m}{mG_m}$ and m is the slope of the equilibrium line

$$\frac{0.06-0.001}{0.06-\frac{0}{3}}=\frac{(3G_m/L_m)^{30\times0.3+1}-(3G_m/L_m)}{(3G_m/L_m)^{30\times0.3+1}-1}$$

$$\frac{G_m}{L_m}=0.457$$

If

$$\frac{(G_m/L_m)_{act}}{(G_m/L_m)_{\min}}=2\text{，}(\frac{G_m}{L_m})_{act}=(\frac{G_m}{L_m})_{\min}\times2=0.328\times2=0.656$$

代回：

$$\frac{0.06-0.001}{0.06-\frac{0}{3}}=\frac{(3\times0.656)^{N+1}-(30.656)}{(3\times0.656)^{N+1}-1}$$

$$N=5.024\text{，}\underline{\underline{N_{act}=\frac{5.024}{0.3}=16.74\approx17\ plates}}$$

一般解法所使用之方程式來自於 McCabe 原文書的整理，可參考 McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 653~658.

補充之法②來自於台大單操所使用之原文書，可參考 Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 704~706. 兩個公式大同小異，不過是整理出的形式不同。

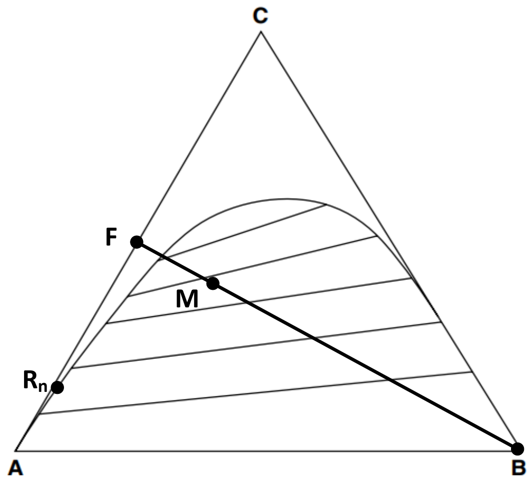
(本題改編自：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 708~709, Example 12.6.)

Problem 6

【**Solution**】

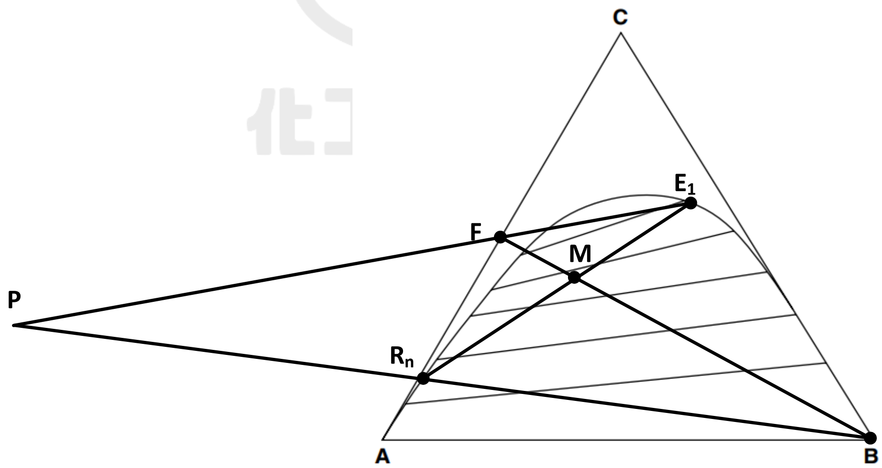
首先，訂出 feed 與 Solvent 的點，並且根據 feed 與 solvent 的進料比率訂出 M 點，

而最終 raffinate 之點為 R_n (在平衡線上)：

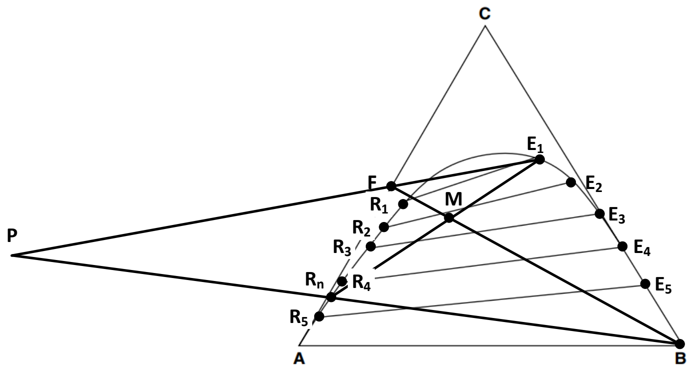


其中 $\dfrac{FM}{MB}=\dfrac{25}{100}$ (lever rule)

連線 MR_n ，交另一端平衡線於 E_1 ，且 E_1F 與 BR_n 交於 P 點：



從 E_1 出發，沿平衡線至左邊可得 R_1，再利用 PR_1 連線交至平衡線右邊可得 E_2，依此類推：由 E_n 出發沿平衡線至左邊可得 R_n ，再由 PR_n 連線至右邊可得 E_{n+1} 。



(基本上認真畫過，可以發現 R 跟 E 點就差不多都在題目本來就畫好的平衡線端點上)

根據繪圖結果，可知需要 **<u>5 個 ideal stages</u>**

第一個 extract (E_1)的組成為 $\boxed{\begin{cases} A = 0.09 \\ B = 0.33 \\ C = 0.58 \end{cases}}$

(數據直接參考原文書，實際作圖結果因人而異)

(本題改編自：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 736~737, Example 13.2.)

Problem 7

【Solution】

Due to the absence of information regarding the values of the activity coefficients for the system, we make the assumption that the system behaves ideally. Consequently, according to Raoult's law, we can express the following:

$$y_i P_i = x_i P_i^{\circ}$$

By the Antoine equation,

$$\log_{10} P_b^{\circ} = 6.90565 - \frac{1211.033}{(338 - 273.15) + 220.79}$$

$$\log_{10} P_t^{\circ} = 6.95334 - \frac{1343.943}{(338 - 273.15) + 219.377} ,$$

$$P_b^{\circ} = 463.37\ (mmHg) = 61.78\ (kN / m^2)$$

$$P_t^{\circ} = 167.85\ (mmHg) = 22.38\ (kN / m^2)$$

代入，計算總壓：

$$P_{total} = x_b P_b^{\circ} + x_t P_t^{\circ} = 0.5(61.78 + 22.38) = 42.1\ (kN / m^2)$$

$$y_b = \frac{P_b}{P_{total}} = \frac{0.5 \times 62.78}{42.1} = \underline{\underline{0.746}}$$

$$y_t = \frac{P_t}{P_{total}} = \frac{0.5 \times 22.38}{42.1} = \underline{\underline{0.254}}$$

∵

$$42.1 < 101.3$$

The liquid will not vaporize unless the total pressured is decreased from

$101.3\ kN / m^2$ to $42.1\ kN / m^2$.

(本題改編自：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 549~550, Example 11.3.)

Problem 8

【Solution】

By energy balance between the water and gas sides,

$$q = \dot{m}_c\, c_{pc}(T_{co} - T_{ci}) = \dot{m}_h\, c_{ph}(T_{hi} - T_{ho})$$

$$180 \times 4.19 \times (90 - 15) = 900 \times 1.05 \times (150 - T_{ho}) \text{ ， } \underline{\underline{T_{ho} = 90.1°\text{C}}}$$

To calculate the overall heat transfer coefficient based on the outside tube diameter, consider the possible thermal resistance,

$$\frac{1}{U(\pi d_0 L)} = \underbrace{\frac{1}{h_h(\pi d_0 L)}}_{\text{outside convection}} + \underbrace{\frac{\ln(d_o / d_i)}{2k_w(\pi L)}}_{\text{tube conduction}} + \underbrace{\frac{1}{h_c(\pi d_i L)}}_{\text{inside convection}} + \underbrace{\frac{f_{fouling}}{(\pi d_i L)}}_{\text{fouling}}$$

$$\frac{1}{U} = \frac{1}{h_h} + \frac{d_0 \ln(d_o / d_i)}{2k_w} + \frac{d_0}{h_c d_i} + \frac{d_0 f_{fouling}}{d_i}$$

$$\frac{1}{U} = \frac{1}{120} + \frac{32 \times 10^{-3} \ln(\frac{32}{25})}{2 \times 381} + \frac{32}{1200 \times 25} + \frac{32 \times 0.002}{25} \text{ ， } \underline{\underline{U = 83.54\ (W / m^2 K)}}$$

To calculate the true mean temperature difference, ΔT_m

$$q = \dot{m}_c\, c_{pc}(T_{co} - T_{ci}) = UA\Delta T_m$$

$$\frac{180 \times 1000}{3600} \times 4.19 \times (90 - 15) = 83.54 \times 5 \times \Delta T_m \text{ ， } \underline{\underline{\Delta T_m = 37.6°\text{C}}}$$

※此題所需求之 true mean temperature difference 與常用之 log mean temperature difference 不同，不過皆是用來計算一個熱交換器的熱交換量。一般來說，log mean temperature 所適用之情況(不須做而外修正)為 counter current 的配置，也就是考慮沿著熱交換器流向，所能達到的最大平均溫度差。而 true mean temperature difference 則是在一個配置下，實際所能達到平均溫度差。在一般熱交換器的介紹中，有一個 correction factor, *F*, 即：

$$q = UAF(LMTD) \quad (F \le 1)$$

此 *F* 就是用於代表 true mean temperature difference 與 log mean temperature difference 的偏離程度，若 *F* 越接近 1，代表該配置越接近 counter current 的配置，具有越高的熱交換潛力。

(本題改編自：Shah, R.; Sekulic, D. *Fundamentals of Heat Exchanger Design*, p 112~114, Example 3.1.

關於 true mean temperature difference 與 log mean temperature difference 的差異與關聯，可參考同本書之 p 187~189。)

Problem 9

【Solution】

If there is no longitudinal wall heat conduction (沿著流向的壁熱傳)，$\lambda_x = 0$

根據 $C^* = 1$、$(\eta_o hA)_h / (\eta_o hA)_c = 1$、$\lambda_x = 0$、$NTU = 6$，查表可得：

TABLE 4.1 Reduction in Crossflow Exchanger Effectiveness ($\Delta\varepsilon/\varepsilon$) Due to Longitudinal Wall Heat Conduction for $C^* = 1$.

$\frac{\lambda_x}{\lambda_y}$	$\frac{(\eta_o hA)_x}{(\eta_o hA)_y}$	NTU	ε for $\lambda_x = 0$	$\Delta\varepsilon/\varepsilon$ for $\lambda_x =$: 0.005	0.010	0.015	0.020	0.025	0.030	0.040	0.060	0.080	0.100	0.200	0.400
0.5	0.5	1.00	0.4764	0.0032	0.0062	0.0089	0.0115	0.0139	0.0162	0.0204	0.0276	0.0336	0.0387	0.0558	0.0720
		2.00	0.6147	0.0053	0.0103	0.0150	0.0194	0.0236	0.0276	0.0350	0.0481	0.0592	0.0689	0.1026	0.1367
		4.00	0.7231	0.0080	0.0156	0.0227	0.0294	0.0357	0.0418	0.0530	0.0726	0.0892	0.1036	0.1535	0.2036
		6.00	0.7729	0.0099	0.0192	0.0279	0.0360	0.0437	0.0510	0.0644	0.0877	0.1072	0.1239	0.1810	0.2372
		8.00	0.8031	0.0114	0.0220	0.0319	0.0411	0.0497	0.0578	0.0728	0.0984	0.1197	0.1377	0.1988	0.2580
		10.00	0.8238	0.0127	0.0243	0.0351	0.0451	0.0545	0.0633	0.0793	0.1066	0.1290	0.1480	0.2115	0.2724
		50.00	0.9229	0.0246	0.0446	0.0616	0.0764	0.0897	0.1017	0.1229	0.1569	0.1838	0.2057	0.2765	0.3427
		100.00	0.9476	0.0311	0.0543	0.0732	0.0893	0.1034	0.1160	0.1379	0.1729	0.2001	0.2223	0.2942	0.3666
	1.0	1.00	0.4764	0.0029	0.0055	0.0079	0.0102	0.0123	0.0143	0.0180	0.0244	0.0296	0.0341	0.0490	0.0634
		2.00	0.6147	0.0055	0.0097	0.0141	0.0182	0.0221	0.0258	0.0327	0.0449	0.0553	0.0643	0.0959	0.1286
		4.00	0.7231	0.0078	0.0151	0.0220	0.0284	0.0346	0.0404	0.0512	0.0702	0.0863	0.1002	0.1491	0.1991
		$\boxed{6.00}$	$\boxed{0.7729}$	0.0097	0.0188	0.0273	0.0353	0.0428	0.0499	0.0630	0.0857	0.1049	0.1213	0.1779	0.2344
		8.00	0.8031	0.0113	0.0217	0.0314	0.0404	0.0489	0.0569	0.0716	0.0968	0.1178	0.1356	0.1965	0.2560
		10.00	0.8238	0.0125	0.0240	0.0347	0.0446	0.0538	0.0624	0.0782	0.1052	0.1274	0.1462	0.2096	0.2708
		50.00	0.9229	0.0245	0.0445	0.0614	0.0763	0.0895	0.1015	0.1226	0.1566	0.1834	0.2053	0.2758	0.3405
		100.00	0.9476	0.0310	0.0543	0.0731	0.0892	0.1033	0.1159	0.1378	0.1727	0.1999	0.2221	0.2933	0.3619
	2.0	1.00	0.4764	0.0027	0.0051	0.0074	0.0095	0.0116	0.0135	0.0170	0.0232	0.0285	0.0330	0.0489	0.0652
		2.00	0.6147	0.0048	0.0092	0.0134	0.0173	0.0211	0.0247	0.0313	0.0432	0.0533	0.0621	0.0938	0.1274
		4.00	0.7231	0.0076	0.0147	0.0213	0.0277	0.0336	0.0393	0.0499	0.0685	0.0844	0.0982	0.1468	0.1971
		6.00	0.7729	0.0095	0.0184	0.0268	0.0346	0.0420	0.0490	0.0619	0.0844	0.1033	0.1196	0.1760	0.2328
		8.00	0.8031	0.0111	0.0214	0.0309	0.0398	0.0482	0.0561	0.0706	0.0956	0.1164	0.1342	0.1949	0.2548
		10.00	0.8238	0.0124	0.0238	0.0343	0.0440	0.0532	0.0617	0.0774	0.1041	0.1262	0.1450	0.2083	0.2698
		50.00	0.9229	0.0245	0.0444	0.0613	0.0761	0.0893	0.1013	0.1223	0.1563	0.1831	0.2050	0.2754	0.3401
		100.00	0.9476	0.0310	0.0542	0.0731	0.0891	0.1032	0.1158	0.1377	0.1725	0.1997	0.2219	0.2928	0.3600

可得 effectiveness, $\varepsilon = 0.7729$

$$\varepsilon_{hot} = \frac{\Delta T_{hot}}{\Delta T_{\max}} = \frac{T_{hi} - T_{ho}}{T_{hi} - T_{ci}} = \frac{360 - T_{ho}}{360 - 25} = 0.7729 \text{，} \underline{\underline{T_{ho} = 101.1°C}}$$

$$\varepsilon_{cold} = \frac{\Delta T_{cold}}{\Delta T_{\max}} = \frac{T_{co} - T_{ci}}{T_{hi} - T_{ci}} = \frac{T_{co} - 25}{360 - 25} = 0.7729 \text{，} \underline{\underline{T_{co} = 283.9°C}}$$

Take longitudinal conduction into account, we have,

$$\lambda_c = \lambda_h = 0.04 \text{，} \frac{\lambda_c}{\lambda_h} = 1$$

By the table,

1.0	0.5	1.00	0.4764	0.0020	0.0039	0.0057	0.0074	0.0090	0.0106	0.0136	0.0190	0.0237	0.0280	0.0436	0.0609
		2.00	0.6147	0.0034	0.0067	0.0098	0.0128	0.0157	0.0185	0.0238	0.0336	0.0423	0.0501	0.0803	0.1154
		4.00	0.7231	0.0053	0.0103	0.0152	0.0198	0.0243	0.0287	0.0369	0.0520	0.0653	0.0773	0.1229	0.1753
		6.00	0.7729	0.0066	0.0129	0.0189	0.0246	0.0302	0.0355	0.0456	0.0637	0.0797	0.0940	0.1472	0.2070
		8.00	0.8031	0.0076	0.0148	0.0217	0.0283	0.0346	0.0406	0.0520	0.0723	0.0900	0.1057	0.1634	0.2270
		10.00	0.8238	0.0085	0.0165	0.0241	0.0313	0.0382	0.0448	0.0571	0.0790	0.0979	0.1145	0.1751	0.2410
		50.00	0.9229	0.0170	0.0316	0.0445	0.0562	0.0667	0.0765	0.0940	0.1233	0.1474	0.1677	0.2378	0.3092
		100.00	0.9476	0.0218	0.0395	0.0543	0.0673	0.0789	0.0894	0.1080	0.1385	0.1632	0.1840	0.2546	0.3273
	1.0	1.00	0.4764	0.0020	0.0038	0.0055	0.0072	0.0088	0.0103	0.0132	0.0183	0.0228	0.0268	0.0412	0.0567
		2.00	0.6147	0.0034	0.0066	0.0097	0.0127	0.0156	0.0183	0.0236	0.0331	0.0417	0.0493	0.0786	0.1125
		4.00	0.7231	0.0053	0.0103	0.0152	0.0198	0.0243	0.0286	0.0368	0.0517	0.0650	0.0769	0.1220	0.1742
		6.00	0.7729	0.0066	0.0129	0.0189	0.0246	0.0301	0.0354	0.0455	0.0636	0.0795	0.0936	0.1466	0.2064
		8.00	0.8031	0.0076	0.0149	0.0217	0.0283	0.0346	0.0406	0.0519	0.0722	0.0898	0.1054	0.1630	0.2266
		10.00	0.8238	0.0085	0.0165	0.0241	0.0313	0.0382	0.0448	0.0571	0.0789	0.0978	0.1143	0.1749	0.2407
		50.00	0.9229	0.0170	0.0316	0.0445	0.0562	0.0667	0.0765	0.0940	0.1233	0.1473	0.1677	0.2377	0.3090
		100.00	0.9476	0.0218	0.0395	0.0543	0.0673	0.0789	0.0894	0.1080	0.1385	0.1632	0.1840	0.2546	0.3270
	2.0	1.00	0.4764	0.0020	0.0039	0.0057	0.0074	0.0090	0.0106	0.0136	0.0190	0.0237	0.0280	0.0436	0.0609
		2.00	0.6147	0.0034	0.0067	0.0098	0.0128	0.0157	0.0185	0.0238	0.0336	0.0423	0.0501	0.0803	0.1154
		4.00	0.7231	0.0053	0.0103	0.0152	0.0198	0.0243	0.0287	0.0369	0.0520	0.0653	0.0773	0.1229	0.1753
		6.00	0.7729	0.0066	0.0129	0.0189	0.0246	0.0302	0.0355	0.0456	0.0637	0.0797	0.0940	0.1472	0.2070
		8.00	0.8031	0.0076	0.0148	0.0217	0.0283	0.0346	0.0406	0.0520	0.0723	0.0900	0.1057	0.1634	0.2270
		10.00	0.8238	0.0085	0.0165	0.0241	0.0313	0.0382	0.0448	0.0571	0.0790	0.0979	0.1145	0.1751	0.2410
		50.00	0.9229	0.0170	0.0316	0.0445	0.0562	0.0667	0.0765	0.0940	0.1233	0.1474	0.1677	0.2378	0.3092
		100.00	0.9476	0.0218	0.0395	0.0543	0.0673	0.0789	0.0894	0.1080	0.1385	0.1632	0.1840	0.2546	0.3273

$$\left(\frac{\Delta\varepsilon}{\varepsilon}\right)_{\lambda_x=0.04}=0.0455=\frac{\varepsilon_{\lambda=0}-\varepsilon_{\lambda=0.04}}{\varepsilon_{\lambda=0}}=\frac{0.7729-\varepsilon_{\lambda=0.04}}{0.7729}$$

$$\varepsilon_{\lambda=0.04}=0.7377$$

$$\varepsilon_{hot}=\frac{\Delta T_{hot}}{\Delta T_{\max}}=\frac{T_{hi}-T_{ho}}{T_{hi}-T_{ci}}=\frac{360-T_{ho}}{360-25}=0.7377\text{，}\underline{\underline{T_{ho}=112.9°C}}$$

$$\varepsilon_{cold}=\frac{\Delta T_{cold}}{\Delta T_{\max}}=\frac{T_{co}-T_{ci}}{T_{hi}-T_{ci}}=\frac{T_{co}-25}{360-25}=0.7377\text{，}\underline{\underline{T_{co}=272.1°C}}$$

$$\boxed{\begin{cases} without\ longitudinal\ conduction \rightarrow T_{ho}=101.1°C,\ T_{co}=283.9°C \\ with\ longitudinal\ conduction \rightarrow T_{ho}=112.9°C,\ T_{co}=272.1°C \end{cases}}$$

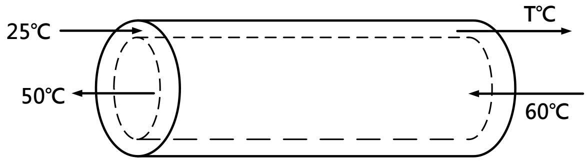
※此題所需求之 effectiveness 可由查表與公式算出，然而表格並未附在題目上，公式也過於繁瑣不太建議背，不過一題也才 4 分，所以考試當下不寫應該無所謂，只需要自己練習時能夠理解他背後的概念即可。

(本題改編自：Shah, R.; Sekulic, D. *Fundamentals of Heat Exchanger Design*, p 243~244, Example 4.1. 表格出自同本書之 Table 4.1。)

<u>Problem 10</u>

【Solution】

(a) Countercurrent configuration



By energy balance, we can determine the outlet temperature of the cooling water, T,

$$Q = \dot{m}_h \cancel{C_{ph}} (60-50) = \dot{m}_c \cancel{C_{pc}} (T-25)$$

$$0.34 \times (60-50) = 0.3 \times (T-25) \text{ ， } T = 36.3℃$$

To calculate the necessary tube length, we need LMTD,

$$LMTD = \frac{(50-25)-(60-36.3)}{\ln(\frac{50-25}{60-36.3})} = 24.33℃$$

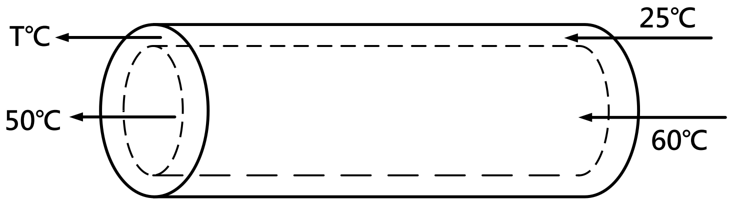
By energy balance, we have

$$Q = \dot{m}_h C_{ph} (60-50) = UA(LMTD) = U(\pi DL)(LMTD)$$

$$0.34 \times 1000 \times 4.18 \times (60-50) = 1600 \times (\pi \times 0.025L) \times 24.33$$

$$\underline{\underline{L = 4.65\ (m)}}$$

(b) Co-current configuration



By energy balance, we can determine the outlet temperature of the cooling water, T,

$$Q = \dot{m}_h \cancel{C_{ph}}(60-50) = \dot{m}_c \cancel{C_{pc}}(T-25)$$

$$0.34 \times (60-50) = 0.3 \times (T-25) \text{ ， } T = 36.3℃$$

To calculate the necessary tube length, we need LMTD,

$$LMTD = \frac{(50-36.3)-(60-25)}{\ln(\frac{50-36.3}{60-25})} = 22.68℃$$

By energy balance, we have

$$Q = \dot{m}_h C_{ph}(60-50) = UA(LMTD) = U(\pi DL)(LMTD)$$

$$0.34 \times 1000 \times 4.18 \times (60-50) = 1600 \times (\pi \times 0.025L) \times 22.68$$

$$\underline{\underline{L = 4.98\ (m)}}$$

(本題改編自：Cao, E. *Heat Transfer in Process Engineering*, p 93~94, Example 5-2.)

Problem 11

【Solution】

Consider energy balance of the thermometer,

$$q_{con,thm\to air}+q_{rad,thm\to wall}=0$$

$$h(T_{bulb}-T_{air})+\sigma\varepsilon(T_{bulb}^4-T_{wall}^4)=0$$

$$8.3(20-T_{air})+5.669\times10^{-8}\times0.9[(20+273.15)^4-(5+273.15)^4]=0$$

$$\underline{\underline{T_{air}=28.6°\mathrm{C}}}$$

(本題可參考：Mills, A.F.; Coimbra, C.F.M. *Basic Heat and Mass Transfer*, 3rd ed., p 46~47, Problem 1.35.)

114 年台大單操輸送

<u>Problem 1</u>

【Solution】

(a)

By the Newtonain's law of viscosity,

$$\tau_{rz}=-\mu\frac{dv_z}{dr}=\frac{(P_0-P_L)R}{2L}[(\frac{r}{R})-\frac{1-\kappa^2}{2\ln\frac{1}{\kappa}}(\frac{R}{r})]$$

$$v_z=-\frac{(P_0-P_L)R}{4\mu L}[(\frac{r^2}{R})-\frac{1-\kappa^2}{\ln\frac{1}{\kappa}}R\ln(r)]+C$$

$$\begin{cases} r=R,\ v_z=0 \\ r=\kappa R,\ v_z=0 \end{cases}$$

然而由題目所給的τ，$r=\kappa R$應已代入過解τ

$\because r=R,\ v_z=0$

$$0=-\frac{(P_0-P_L)R^2}{4\mu L}[1-\frac{1-\kappa^2}{\ln\frac{1}{\kappa}}\ln(R)]+C$$

$$C=\frac{(P_0-P_L)R^2}{4\mu L}[1-\frac{1-\kappa^2}{\ln\frac{1}{\kappa}}\ln(R)]$$

代回：

$$\underline{\underline{v_z=\frac{(P_0-P_L)R^2}{4\mu L}[1-(\frac{r^2}{R^2})-\frac{1-\kappa^2}{\ln\frac{1}{\kappa}}\ln(\frac{R}{r})]}}$$

(b)

$\because v_z = v_{z,\max}$ when $r = r_{\max}$

$$\left.\frac{dv_z}{dr}\right|_{r=r_{\max}} = \left.\frac{\tau_{rz}}{-\mu}\right|_{r=r_{\max}} = -\frac{(P_0-P_L)R}{2\mu L}[(\frac{r_{\max}}{R})-\frac{1-\kappa^2}{2\ln\frac{1}{\kappa}}(\frac{R}{r_{\max}})]=0$$

$$[(\frac{r_{\max}}{R})-\frac{1-\kappa^2}{2\ln\frac{1}{\kappa}}(\frac{R}{r_{\max}})]=0 \text{，} \frac{r_{\max}^2}{R}-AR=0$$

$$(\text{令 } A=\frac{1-\kappa^2}{2\ln\frac{1}{\kappa}})$$

$$r_{\max} = \pm\sqrt{A} = \pm\sqrt{\frac{1-\kappa^2}{2\ln\frac{1}{\kappa}}} = \underline{\underline{\sqrt{\frac{1-\kappa^2}{2\ln\frac{1}{\kappa}}}}}$$

(※座標原點定位在套管中心，因此不會有負的半徑，取正)

(c)

For the average velocity, we have,

$$<v_z>=\frac{\int_0^{2\pi}\int_{\kappa R}^{R} v_z r dr d\theta}{\int_0^{2\pi}\int_{\kappa R}^{R} r dr d\theta}=\frac{(P_0-P_L)R^2}{4\mu L}\frac{\int_{\kappa R}^{R}[r-(\frac{r^3}{R^2})-\frac{1-\kappa^2}{\ln\frac{1}{\kappa}}r\ln(\frac{R}{r})]dr}{\frac{1}{2}R^2(1-\kappa^2)}$$

$$=\frac{(P_0-P_L)R^3}{4\mu L}\frac{\int_{\kappa R}^{R}[\frac{r}{R}-(\frac{r^3}{R^3})+\frac{1-\kappa^2}{\ln\frac{1}{\kappa}}(\frac{r}{R})\ln(\frac{r}{R})]dr}{\frac{1}{2}R^2(1-\kappa^2)}$$

For $\int_{\kappa R}^{R}[\frac{r}{R}-(\frac{r^3}{R^3})+\frac{1-\kappa^2}{\ln\frac{1}{\kappa}}(\frac{r}{R})\ln(\frac{r}{R})]dr$,

$$\int_{\kappa R}^{R}[\frac{r}{R}-(\frac{r^3}{R^3})+\frac{1-\kappa^2}{\ln\frac{1}{\kappa}}(\frac{r}{R})\ln(\frac{r}{R})]dr=\frac{1}{2}R(1-\kappa^2)-\frac{1}{4}[R(1-\kappa^4)]+\frac{R}{4}\frac{1-\kappa^2}{\ln\frac{1}{\kappa}}[-2\kappa^2\ln\kappa-(1-\kappa^2)]$$

$$<v_z>=\frac{(P_0-P_L)R^3}{4\mu L}\frac{\frac{1}{2}R\cancel{(1-\kappa^2)}-\frac{1}{4}[R\cancel{(1-\kappa^2)}(1+\kappa^2)]+\frac{R}{4}\frac{\cancel{1-\kappa^2}}{\ln\frac{1}{\kappa}}[-2\kappa^2\ln\kappa-(1-\kappa^2)]}{\frac{1}{2}R^2\cancel{(1-\kappa^2)}}$$

$$<v_z>=\frac{(P_0-P_L)R^3}{4\mu L}\frac{2\cancel{(1-\kappa^2)}-[\cancel{(1-\kappa^2)}(1+\kappa^2)]+\frac{\cancel{1-\kappa^2}}{\ln\frac{1}{\kappa}}[-2\kappa^2\ln\kappa-(1-\kappa^2)]}{2R\cancel{(1-\kappa^2)}}$$

$$<v_z>=\frac{(P_0-P_L)R^2}{8\mu L}[2-(1+\kappa^2)+2\kappa^2-\frac{(1-\kappa^2)}{\ln\frac{1}{\kappa}}]=\frac{(P_0-P_L)R^2}{8\mu L}[(\kappa^2+1)-\frac{(1-\kappa^2)}{\ln\frac{1}{\kappa}}]$$

$$=\frac{(P_0-P_L)R^2}{8\mu L}[\frac{(1-\kappa^4)}{(1-\kappa^2)}-\frac{(1-\kappa^2)}{\ln\frac{1}{\kappa}}]$$

(※考試當下可能不用整理到這麼簡便，這邊是用來輔助同學對照原文書結果)

For the mass flow rate,

$$w=\rho R^2(1-\kappa^2)<v_z>=\frac{\rho(P_0-P_L)R^4}{8\mu L}[(1-\kappa^4)-\frac{(1-\kappa^2)^2}{\ln\frac{1}{\kappa}}]$$

(d)

For the force exerted, it will be contributed by both the ***inner*** and ***outer surface***

$$F_z=(2\pi RL)\tau_{rz}\big|_R-(2\pi\kappa RL)\tau_{rz}\big|_{\kappa R}$$

$$=(2\pi RL)\frac{(P_0-P_L)R}{2L}[1-\frac{1-\kappa^2}{2\ln\frac{1}{\kappa}}]-(2\pi\kappa RL)\frac{(P_0-P_L)R}{2L}[(\kappa)-\frac{1-\kappa^2}{2\ln\frac{1}{\kappa}}(\frac{1}{\kappa})]$$

$$F_z=\pi R^2(P_0-P_L)(1-\kappa^2)$$

(※ $r=\kappa R$ 部分前面有負號來源於在 κR 位置的流體，其對牆壁的施力與座標方向反向，因此 shear stress 前面有負號。此外，根據此結果，我們可以看出針對此套管的流體，其施加於牆壁的力，即為套管截面積×壓差；壓力在流體運動方向往上施力，此力同樣也會因為流體與牆壁接觸而作用於牆壁上)

(e)

For the volumetric flow rate,

$$Q = \frac{w}{\rho} = \frac{(P_0 - P_L)R^4}{8\mu L}[(1-\kappa^4) - \frac{(1-\kappa^2)^2}{\ln\frac{1}{\kappa}}]$$

where $\kappa = \frac{1.26}{2.8} = 0.45$

$$Q = \frac{3.7\times10^4 \times (2.8\times10^{-2})^4}{8\times0.05655\times8.2}[(1-0.45^4) - \frac{(1-0.45^2)^2}{\ln(\frac{1}{0.45})}] = \underline{\underline{9.9\times10^{-4}\ \ (m^3/s)}}$$

(本小題為原文書章節範例：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 53~56.)

Problem 2

【**Solution**】

(a)

Assume：

(1) The flow is steady and fully developed in θ-direction

(2) $v_r = v_z = 0$.

(3) incompressible Newtonian fluid.

By the equation of continuity in cylindrical coordinate,

$$\cancel{\frac{\partial \rho}{\partial t}} + \cancel{\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)} + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta) + \cancel{\frac{\partial}{\partial z}(\rho v_z)} = 0$$

$$\frac{\partial v_\theta}{\partial \theta} = 0，v_\theta = f(\cancel{t}, r, \cancel{\theta}, \cancel{z}) = f(r)\ \text{only}$$

Simplify the Navier Stokes equations:

【$r-$**component**】:

$$\rho(\cancel{\frac{\partial v_r}{\partial t}} + \cancel{v_r\frac{\partial v_r}{\partial r}} + \cancel{\frac{v_\theta}{r}\frac{\partial v_r}{\partial \theta}} + \cancel{v_z\frac{\partial v_r}{\partial z}} - \frac{v_\theta^2}{r}) = -\frac{\partial p}{\partial r} + [\cancel{\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}(r v_r))} + \cancel{\frac{1}{r^2}\frac{\partial^2 v_r}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_r}{\partial z^2}} - \cancel{\frac{2}{r^2}\frac{\partial v_\theta}{\partial \theta}}] + \cancel{\rho g_r}$$

【$\theta-$**component**】:

$$\rho(\cancel{\frac{\partial v_\theta}{\partial t}} + \cancel{v_r\frac{\partial v_\theta}{\partial r}} + \cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}} + \cancel{v_z\frac{\partial v_\theta}{\partial z}} + \cancel{\frac{v_\theta v_r}{r}})$$

$$= \cancel{\frac{-1}{r}\frac{\partial p}{\partial \theta}} + \mu[\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}(r v_\theta)) + \cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_\theta}{\partial z^2}} + \cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}}] + \cancel{\rho g_\theta}$$

【$z-$**component**】:

$$\rho(\cancel{\frac{\partial v_z}{\partial t}} + \cancel{v_r\frac{\partial v_z}{\partial r}} + \cancel{\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}} + \cancel{v_z\frac{\partial v_z}{\partial z}}) = -\frac{\partial p}{\partial z} + [\cancel{\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial v_z}{\partial r})} + \cancel{\frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2}} + \cancel{\frac{\partial^2 v_z}{\partial z^2}}] - \rho g_z$$

(※重力向下，為-z 方向)

【$r-$component】：

$$-\rho(\frac{v_\theta^2}{r})=-\frac{\partial p}{\partial r}$$

【$\theta-$component】：

$$\mu[\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta))]=0$$

【$z-$component】：

$$\frac{\partial p}{\partial z}=-\rho g_z$$

(b)

$$\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta))=0\text{，}v_\theta=\frac{c_1}{2}r+\frac{c_2}{r}\begin{cases}r=0,v_\theta=0\\r=R,v_\theta=\Omega R\end{cases}$$

$$c_1=2\Omega\text{，}c_2=0$$

$$\underline{\underline{v_\theta=\Omega r}}$$

(c)

$$\frac{\partial p}{\partial r}=\frac{\rho v_\theta^2}{r}=\rho\Omega^2 r$$

$$\frac{\partial p}{\partial z}=-\rho g_z$$

Integrate the z-direction,

$$p=-\rho g_z z+f(r)$$

代回 r 方向，

$$f^{'}(r)=\rho\Omega^2 r$$

Integrate with r,

$$f=\frac{\rho\Omega^2 r^2}{2}+C$$

And the pressure distribution becomes,

$$p(r,z)=-\rho g_z z+\frac{\rho\Omega^2 r^2}{2}+C$$

Because the liquid is open to the atmosphere at the top, when $r=0$, $z=z_0$ and $p=p_{atm}$

$$p(0,z_0)=-\rho g_z z_0+C=p_{atm}$$

$$C=p_{atm}+\rho g_z z_0$$

代回：

$$\boxed{p(r,z)-p_{atm}=-\rho g_z(z-z_0)+\frac{\rho\Omega^2r^2}{2}}$$

(d)

At the interface, the pressure is p_{atm}. Therefore, for every z that represents the interface height ($z_{interface}$),

$$p_{atm}-p_{atm}=-\rho g_z(z_{interface}-z_0)+\frac{\rho\Omega^2r^2}{2}=0$$

$$\boxed{(z_{interface}-z_0)=(\frac{\Omega^2}{2g_z})r^2}$$

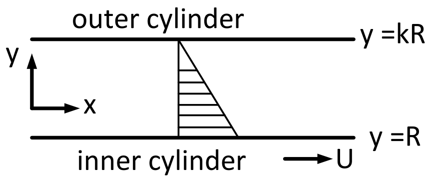
(本小題為原文書章節例題：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 93~95, Example 3.6-4。在(d)小題中，原文書並未額外定義 $z_{interface}$。然而(c)小題所推出的壓力通式的 z 為任何位置的 z 皆可用；(d)小題是限定在液氣介面的液體區域才可用，其高度會有一定限制，與(c)小題的不同，因此這裡多定義一個新的位置參數來做辨別。)

Problem 3

【Solution】

(a)

If the slit between the two cylinders is small $(\kappa-1) << 1$, the curvature effects can be negelcted:



By the question statement, the velocity profile can be taken to be linear with respect to the radius:

$$v_x = c_1 y + c_2 \begin{cases} y = R,\ v_x = U \\ y = \kappa R,\ v_x = 0 \end{cases}, \quad \begin{cases} c_1 = -\dfrac{U}{R(\kappa-1)} \\ c_2 = \dfrac{U\kappa}{\kappa-1} \end{cases}$$

$$v_x = \frac{U\kappa}{\kappa-1}(1-\frac{y}{\kappa R})$$

Therefore, the viscous heat dissipation ($\mu\Phi_v$) is,

$$\mu\Phi_v = \mu(\frac{\partial v_x}{\partial y})^2 = \underline{\underline{\mu[\frac{U}{R(\kappa-1)}]^2}}$$

(b)

Assume:

(1) steady state heat transfer

(2) Newtonian fluid with constant properties

(3) only one-dimension heat transfer (y direction)

(4) neglected curvature effects

By the equation of energy,

$$\cancel{\rho C_P(\frac{\partial T}{\partial t} + v_x\frac{\partial T}{\partial x} + v_y\frac{\partial T}{\partial y} + v_z\frac{\partial T}{\partial z})} = k(\cancel{\frac{\partial^2 T}{\partial x^2}} + \frac{\partial^2 T}{\partial y^2} + \cancel{\frac{\partial^2 T}{\partial z^2}}) + \mu\Phi_v$$

$$k(\frac{\partial^2 T}{\partial y^2})+\mu\Phi_v = k(\frac{\partial^2 T}{\partial y^2})+\mu[\frac{U}{R(\kappa-1)}]^2 = 0$$

$$\boxed{T = -\frac{\mu}{2k}[\frac{U}{R(\kappa-1)}]^2 y^2 + c_1 y + c_2 \begin{cases} y = R,\ \dfrac{dT}{dy} = 0 \\ y = \kappa R,\ -k\dfrac{dT}{dy} = h(T-T_a) \end{cases}}$$

(內圓柱的熱傳可忽略，因此寫作微分 =0)

(c)

$$T = -\frac{\mu}{2k}[\frac{U}{R(\kappa-1)}]^2 y^2 + c_1 y + c_2 \begin{cases} y = R,\ \dfrac{dT}{dy} = 0 \\ y = \kappa R,\ -k\dfrac{dT}{dy} = h(T-T_a) \end{cases}$$

將 B.C.1 代入：

$$\frac{dT}{dy} = -\frac{\mu}{k}[\frac{U}{R(\kappa-1)}]^2 R + c_1 = 0\ ，\ \boxed{c_1 = \frac{\mu}{k}[\frac{U}{R(\kappa-1)}]^2 R}$$

將 B.C.2 代入：

$$\mu[\frac{U}{R(\kappa-1)}]^2(\kappa R) - \mu[\frac{U}{R(\kappa-1)}]^2 R = h[T(\kappa R) - T_a]$$

$$\boxed{T(\kappa R) = \frac{\mu U^2}{hR(\kappa-1)} + T_a}$$

因此：

$$T(\kappa R) = -\frac{\mu}{2k}[\frac{U}{R(\kappa-1)}]^2(\kappa R)^2 + c_1\kappa R + c_2 = -\frac{\mu}{2k}[\frac{U}{R(\kappa-1)}]^2(\kappa R)^2 + \frac{\mu}{k}[\frac{U}{R(\kappa-1)}]^2 \kappa R^2 + c_2$$

$$c_2 = T(\kappa R) + \frac{\mu}{2k}[\frac{U}{(\kappa-1)}]^2(\kappa^2 - 2\kappa) = \frac{\mu U^2}{hR(\kappa-1)} + T_a + \frac{\mu}{2k}[\frac{U}{(\kappa-1)}]^2(\kappa^2-2\kappa)$$

$$T = -\frac{\mu}{2k}[\frac{U}{R(\kappa-1)}]^2 y^2 + \frac{\mu}{k}[\frac{U}{R(\kappa-1)}]^2 Ry + \frac{\mu U^2}{hR(\kappa-1)} + T_a + \frac{\mu}{2k}[\frac{U}{(\kappa-1)}]^2(\kappa^2-2\kappa)$$

$$\boxed{T = \frac{\mu}{2k}[\frac{U}{(\kappa-1)}]^2[\frac{2y}{R} - (\frac{y}{R})^2] + \frac{\mu}{2k}[\frac{U}{(\kappa-1)}]^2(\kappa^2-2\kappa) + \frac{\mu U^2}{hR(\kappa-1)} + T_a}$$

Problem 4

【Solution】

(a) Assume the diffusion is steady and only in r-direction, by mole balance of A

$$N_A''(4\pi r^2)\Big|_r - N_A''(4\pi r^2)\Big|_{r+dr} = 0$$

同除 $4\pi dr \to 0$

$$\frac{-d(r^2 N_A'')}{dr} = 0$$

$$N_A'' = -D_{AB}\frac{dC_A}{dr} + \underbrace{\cancel{x_A(N_A''+N_B'')}}_{slightly\ soluble} = -D_{AB}C\frac{dx_A}{dr}$$

$$\text{則 } N_A'' = -D_{AB}C\frac{dx_A}{dr}$$

代入莫耳平衡式：

$$\frac{d}{dr}(r^2\frac{dx_A}{dr}) = 0$$

$$x_A = \frac{-c_1}{r} + c_2 \text{ ， } B.C.\begin{cases} r = R & x_A = x_{A0} \\ r \to \infty & x_A = x_{A\infty} \end{cases} \text{ ， } \begin{cases} c_1 = R(x_{A\infty} - x_{A0}) \\ c_2 = x_{A\infty} \end{cases}$$

$$\boxed{x_A = \frac{R(x_{A0} - x_{A\infty})}{r} + x_{A0}}$$

(b) At the interphase,

$$-D_{AB}\cancel{C}\frac{dx_A}{dr}\Big|_{r=R} = k\cancel{C}(x_{A0} - x_{A\infty}) = D_{AB}\frac{(x_{A0} - x_{A\infty})}{R}$$

$$\frac{kR}{D_{AB}} = 1$$

Define the Sherwood number as,

$$\boxed{Sh = \frac{convective\ mass\ transfer\ rate}{diffusive\ mass\ transfer\ rate} = \frac{kD}{D_{AB}} = \frac{k(2R)}{D_{AB}} = 2}$$

(c)

【法 1】

By the definition of molar flux,

$$N_A\big|_{r=R} = C_A v_A\big|_{r=R} = \underbrace{C_A(v_A - v_r^*)\big|_{r=R}}_{diffusion} + \underbrace{C_A v_r^*\big|_{r=R}}_{advection=bulk\ motion}$$

$$C_A v_r^*\big|_{r=R} = N_A\big|_{r=R} - C_A(v_A - v_r^*)\big|_{r=R}$$

By Fick's law,

$$N_A'' = -D_{AB}C\frac{dx_A}{dr} + x_A(N_A'' + \underbrace{\cancel{N_B''}}_{stagnant})$$

$$N_A''\big|_{r=R} = \frac{-D_{AB}C}{(1-x_A)}\frac{dx_A}{dr}\bigg|_{r=R}$$

Also,

$$C_A(v_A - v_r^*)\big|_{r=R} = -D_{AB}C\frac{dx_A}{dr}\bigg|_{r=R}$$

$$C_A v_r^*\big|_{r=R} = N_A\big|_{r=R} - C_A(v_A - v_r^*)\big|_{r=R} = \frac{-D_{AB}C}{(1-x_A)}\frac{dx_A}{dr}\bigg|_{r=R} + D_{AB}C\frac{dx_A}{dr}\bigg|_{r=R} = D_{AB}C(1-\frac{1}{1-x_A})\frac{dx_A}{dr}\bigg|_{r=R}$$

$$\boxed{v_r^*\big|_{r=R} = D_{AB}\frac{(1-\frac{1}{1-x_A})}{x_A}\frac{dx_A}{dr}\bigg|_{r=R} = \underline{\underline{(-\frac{D_{AB}}{1-x_A})\frac{dx_A}{dr}\bigg|_{r=R}}}}$$

【法 2】

The average velocity induced by the mass transfer process is,

$$\boxed{v^*\big|_{r=R} = \frac{C_A v_A + \cancel{C_B v_B}}{C}\bigg|_{r=R} = \frac{C_A v_A}{C}\bigg|_{r=R} = \frac{N_A}{C}\bigg|_{r=R} = \underline{\underline{\frac{-D_{AB}}{(1-x_A)}\frac{dx_A}{dr}\bigg|_{r=R}}}}$$

Problem 5

【**Solution**】

(a) To consider the axial direction of mass transfer, by the equation of continuity,

$$\frac{\partial C_i}{\partial t}+\cancel{v_r\frac{\partial C_i}{\partial r}}+\cancel{\frac{v_\theta}{r}\frac{\partial C_i}{\partial \theta}}+v_z\frac{\partial C_i}{\partial z}=E_D[\frac{1}{r}\frac{\partial}{\partial r}(\cancel{r\frac{\partial C_i}{\partial r}})+\cancel{\frac{1}{r^2}\frac{\partial^2 C_i}{\partial \theta^2}}+\frac{\partial^2 C_i}{\partial z^2}]\cancel{+r_A}$$

$$\boxed{\frac{\partial C_i}{\partial t}=E_D\frac{\partial^2 C_i}{\partial z^2}-v_z\frac{\partial C_i}{\partial z}}$$

(b) Consider the facts that solute may be retained in the porous particles, the concentration of $\overline{C}_{pore}$ may also changes with time and location. Therefore, for a given z, the total solute concentration at the convrol volume can be described by:

$$C_z=(1-\phi)C_i+\phi\varepsilon\overline{C}_{pore}$$

Assume the particle distribution is uniform in z-direction, $\phi\neq\phi(z)$

$$\underbrace{(1-\phi)\frac{\partial C_i}{\partial t}+\phi\varepsilon\frac{\partial \overline{C}_{i,pore}}{\partial t}}_{control\ \ volume\text{中總溶質濃度的累積}}=\underbrace{E_D\frac{\partial^2 C_i}{\partial z^2}}_{control\ \ volume\text{中藉由流體}diffusion\text{跑出去的濃度}}-\underbrace{v_z\frac{\partial C_i}{\partial z}}_{control\ \ volume\text{中藉由流體}convection\text{跑出去的濃度}}$$

(c)

Assume the diffusion within one particle is:

(1) spherical coordinate (i.e. the particle is spherical)

(2) one-dimensional in r-direction (θ and ϕ symmetric)

(3) no convection (by the question statement)

(4) constant properties

(5) no chemical reaction

By the general equation of continuity in spherical coordinates,

$$(\frac{\partial C_{pore}}{\partial t}+\cancel{v\cdot\nabla C_{pore}})=D_P\nabla^2 C_{pore}+\cancel{r_{pore}}=D_P\nabla^2 C_{pore}$$

$$\boxed{\frac{\partial C_{pore}}{\partial t}=D_P[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial C_{pore}}{\partial r})]}$$

(d)

The average concentration can be calculated by,

$$\overline{C}_{pore} = \frac{\int_0^{\pi}\int_0^{2\pi}\int_0^{R} C_{pore} r^2 \sin\theta drd\theta d\phi}{\int_0^{\pi}\int_0^{2\pi}\int_0^{R} r^2 \sin\theta drd\theta d\phi} = \underline{\underline{\frac{3}{R^3}\int_0^{R} C_{pore} r^2 dr}}$$

(※因為 $C_{pore}(r,z,t)$ 對 r 積分了，所以運算後的 $\overline{C}_{pore}$ 並沒有 r 的影響)

(e)

因為流體內的溶質會質傳給顆粒進入孔洞，因此對於顆粒來說：

$$\phi\varepsilon\frac{\partial C_{pore}}{\partial t} = \phi k_f a_s (C_i - \overline{C}_{pore}) \text{ ， } \varepsilon\frac{\partial C_{pore}}{\partial t} = k_f a_s (C_i - \overline{C}_{pore})$$

where a_s is the area of particle per unit particle volume

將其代回(b)小題式子，即：

$$(1-\phi)\frac{\partial C_i}{\partial t} + \phi\varepsilon\frac{\partial \overline{C}_{i,pore}}{\partial t} = E_D\frac{\partial^2 C_i}{\partial z^2} - v_z\frac{\partial C_i}{\partial z}$$

得：

$$\boxed{(1-\phi)\frac{\partial C_i}{\partial t} = E_D\frac{\partial^2 C_i}{\partial z^2} - v_z\frac{\partial C_i}{\partial z} - k_f a_s (C_i - \overline{C}_{pore})}$$