106 年台科大工程數學與輸送現象

Problem 1

【**Solution**】

(a)

$$\Pr \equiv \frac{momentum\ duffusivity}{thermal\ diffusivity} = \frac{\upsilon}{\alpha} = \frac{\mu C_p}{k}$$

(b)

$$Nu \equiv \frac{conductive\ thermal\ resistance}{convective\ thermal\ resistance} = \frac{hL}{k}$$

(c)

$$St = \frac{h}{\rho v_\infty C_p}$$

(d)

$$\mathrm{Re} \equiv \frac{inertial\ force}{viscous\ force} = \frac{\rho v_\infty L}{\mu}$$

(e)

$$\underline{\underline{St = \frac{Nu}{\mathrm{Re}\,\Pr}}}$$

Problem 2

【**Solution**】

By shell balance of energy outside the sphere, assuming k is constant,

$$q_r(4\pi r^2)\Big|_r - q_r(4\pi r^2)\Big|_{r+dr} = 0$$

同除 $4\pi dr \to 0$

$$\frac{d(r^2 q_r)}{dr} = 0 \text{ ， } \frac{d}{dr}(r^2 k\frac{dT}{dr}) = 0\begin{cases} r = R, T = T_0 \\ r \to \infty, T = T_\infty \end{cases}$$

$$T = \frac{-c_1}{r} + c_2\begin{cases} c_1 = (T_\infty - T_0)R \\ c_2 = T_\infty \end{cases}$$

$$T = (T_0 - T_\infty)\frac{R}{r} + T_\infty$$

$@r = R,\ conduction = convection$

$$q_r = -k\frac{dT}{dr}\Big|_{r=R} = \underline{\underline{\frac{k}{R}(T_0 - T_\infty)}}$$

Problem 3

【**Solution**】

For constant rate drying,

$$t = \frac{(X_1 - X_2)}{AR}$$

$$X_1 = \frac{100\times 30}{100(1-0.3)} = 0.428\ (\frac{kg\ water}{kg\ dry\ solid})$$

For w_2

$$\frac{m_w}{(100)(1-0.3)+m_w} = 0.20 \text{ ， } m_w = 17.5$$

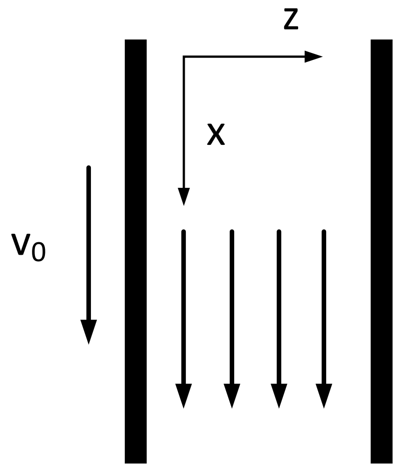
$$X_2 = \frac{m_w}{100(1-0.3)} = 0.25$$

$$t_C = \frac{(0.428-0.25)\times 70}{0.03\times 70\times 0.001} = \underline{\underline{5952.38\ (s)}}$$

(本題可參考：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 922, Example 16.4.)

Problem 4

【**Solution**】



(a)

Assumptions：

(1) incompressible/Newtonian

(2) steady state

(3) fully developed in x-direction

(4) $v_x \neq f(y)$

By equation of continuity,

$$\underbrace{\cancel{\frac{\partial \rho}{\partial t}}}_{incompressible} + \frac{\partial(\rho v_x)}{\partial x} + \underbrace{\cancel{\frac{\partial(\rho v_y)}{\partial y}}}_{v_y=0} + \underbrace{\cancel{\frac{\partial(\rho v_z)}{\partial z}}}_{v_z=0} = 0$$

$$\frac{\partial(\rho v_x)}{\partial x} = 0 \text{ ，} v_x \neq f(x)$$

By the momentum balance in the control volume,

$$(\rho v_x v_x)dydz\big|_x-(\rho v_x v_x)dydz\big|_{x+dx}+\tau_{zx}dxdy\big|_z-\tau_{zx}dxdy\big|_{z+dz}+pdydz\big|_x-pdydz\big|_{x+dx}+\rho gdxdydz=0$$

同除 $dxdydz\to 0$

$$\cancel{\underbrace{\frac{-\partial(\rho v_x v_x)}{\partial x}}_{v_x\neq f(x)}}-\frac{\partial\tau_{zx}}{\partial z}-\frac{\partial p}{\partial x}+\rho g=0$$

$\because$ *Newtonian,* $\tau_{zx}=-\mu\dfrac{dv_x}{dz}$

$$\mu\frac{d^2v_x}{dz^2}-\frac{\partial p}{\partial x}+\rho g=0$$

$\because \dfrac{\partial p}{\partial x}\propto v_x$ with proportionality = -1

$$\underline{\underline{\mu\frac{d^2v_x}{dz^2}+v_x+\rho g=0}}$$

(b)

$$\begin{cases} z=0, v_x=v_0 \\ z=B, v_x=0 \end{cases}$$

(c)

令 $v_x+\rho g=f$

$$\mu\frac{d^2f}{dz^2}+f=0$$

$$f=c_1\cos\frac{z}{\sqrt{\mu}}+c_2\sin\frac{z}{\sqrt{\mu}}\begin{cases} z=0,\ f=v_0+\rho g \\ z=B,\ f=\rho g \end{cases}$$

$$\begin{cases} c_1=v_0+\rho g \\ c_2=\dfrac{\rho g-(v_0+\rho g)\cos\dfrac{B}{\sqrt{\mu}}}{\sin\dfrac{B}{\sqrt{\mu}}} \end{cases}$$

$$v_x=f-\rho g=\underline{\underline{c_1\cos\frac{z}{\sqrt{\mu}}+c_2\sin\frac{z}{\sqrt{\mu}}-\rho g}}$$

(d)

$$< v_x >= \frac{\int_0^B \int_0^W v_x dydz}{\int_0^B \int_0^W dydz} = \frac{W[\sqrt{\mu}(c_1 \sin\frac{B}{\sqrt{\mu}} - c_2 \cos\frac{B}{\sqrt{\mu}}) - \rho g]}{BW}$$

$$= \frac{\sqrt{\mu}}{B}(v_0 + \rho g)\sin\frac{B}{\sqrt{\mu}} - \frac{\sqrt{\mu}}{B}[\frac{\rho g - (v_0 + \rho g)\cos\frac{B}{\sqrt{\mu}}}{\sin\frac{B}{\sqrt{\mu}}}]\cos\frac{B}{\sqrt{\mu}} - \frac{\rho g}{B}$$

(e)

$@\, z = 0$

$$\tau_{zx} = -\mu \frac{dv_x}{dz}\bigg|_{z=0} = \frac{c_1}{\sqrt{\mu}} = \frac{\rho g - (v_0 + \rho g)\cos\frac{B}{\sqrt{\mu}}}{\sqrt{\mu}\sin\frac{B}{\sqrt{\mu}}}$$

$@\, z = B$

$$\tau_{zx} = -\mu \frac{dv_x}{dz}\bigg|_{z=B} = \frac{-c_1}{\sqrt{\mu}}\sin\frac{B}{\sqrt{\mu}} + \frac{c_2}{\sqrt{\mu}}\cos\frac{B}{\sqrt{\mu}} = \frac{-(v_0 + \rho g)}{\sqrt{\mu}}\sin\frac{B}{\sqrt{\mu}} + [\frac{\rho g - (v_0 + \rho g)\cos\frac{B}{\sqrt{\mu}}}{\sqrt{\mu}\sin\frac{B}{\sqrt{\mu}}}]\cos\frac{B}{\sqrt{\mu}}$$

Problem 5

【Solution】

$$D[=]L \qquad d[=]L \qquad \sigma[=]\frac{N}{m^2}[=]\frac{M}{T^2}$$

$$\rho[=]\frac{kg}{m^3}[=]\frac{M}{L^3} \quad \mu[=]\frac{kg}{m\cdot s}[=]\frac{M}{LT} \quad g[=]\frac{m}{s^2}[=]\frac{L}{T^2}$$

Fundamental units： M,L,T

Total variable： D,d,ρ,σ,μ,g

共可決定$6-3=3$個無因次群，定義為π_1,π_2,π_3

令 repeating unit 為 d,ρ,g

For π_1

$$\pi_1 = Dd^a\rho^b g^c$$

$$L\cdot L^a\cdot(ML^{-3})^b\cdot(LT^{-2})^c=1$$

$$\begin{cases} L:1+a-3b+c=0 \\ T:-2c=0 \\ M:b=0 \end{cases} => \begin{cases} a=-1 \\ b=0 \\ c=0 \end{cases}$$

$$\underline{\underline{\pi_1=\frac{D}{d}}}$$

For π_2

$$\pi_2 = \sigma d^d\rho^e g^f$$

$$(MT^{-2})\cdot L^d\cdot(ML^{-3})^e\cdot(LT^{-2})^f=1$$

$$\begin{cases} L:d-3e+f=0 \\ T:-2-2f=0 \\ M:1+e=0 \end{cases} => \begin{cases} d=-2 \\ e=-1 \\ f=-1 \end{cases}$$

$$\underline{\underline{\pi_2=\frac{\sigma}{d^2\rho g}}}$$

For π_3

$$\pi_3 = md^g \rho^h g^i$$

$$(ML^{-1}T^{-1}) \cdot L^g \cdot (ML^{-3})^h \cdot (LT^{-2})^i = 1$$

$$\begin{cases} L: -1 + g - 3h + i = 0 \\ T: -1 - 2i = 0 \\ M: 1 + h = 0 \end{cases} \Rightarrow \begin{cases} g = \dfrac{-3}{2} \\ h = -1 \\ i = -\dfrac{1}{2} \end{cases}$$

$$\underline{\underline{\pi_3 = \frac{\mu}{d^{\frac{3}{2}} \rho g^{\frac{1}{2}}}}}$$

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Problem 1

【**Solution**】

將原式同乘 y^{-4}

$$xy^{-4}\frac{dy}{dx}+y^{-3}=3x^2$$

令 $u=y^{-3}$

$$\frac{du}{dx}=-3y^{-4}\frac{dy}{dx}$$

$$y^{-4}\frac{dy}{dx}=-\frac{1}{3}\frac{du}{dx}$$

$$\frac{-1}{3}x\frac{du}{dx}+u=3x^2$$

同乘 $\frac{-3}{x^4}$，

$$\frac{1}{x^3}\frac{du}{dx}-\frac{3}{x^4}u=\frac{-9}{x^2}$$

$$\frac{d}{dx}\left(\frac{u}{x^3}\right)=\frac{-9}{x^2}$$

$$\frac{u}{x^3}=\frac{9}{x}+C_1$$

$$u=9x^2+C_1x^3$$

$$y^{-3}=9x^2+C_1x^3$$

代入 I.C.

$$C_1=-1$$

$$\underline{\underline{y^{-3}=9x^2-x^3}}$$

Problem 2

【**Solution**】

$$f(t)=\begin{cases}0,\ 0<t<2\\ t,\ t\ge 2\end{cases}=t\cdot H(t-2)\text{, H(t)為 Heaviside function}$$

將原式做拉式轉換，令 $Y(s)=L\left[y(t)\right]$

$$s^2Y(s)-sy(0)-y'(0)+9Y(s)=-\frac{d}{ds}\left(\frac{e^{-2s}}{s}\right)=e^{-2s}\left(\frac{2}{s}+\frac{1}{s^2}\right)$$

$$Y(s)=e^{-2s}\left(\frac{2}{s(s^2+9)}+\frac{1}{s^2(s^2+9)}\right)+\frac{1}{s^2+9}$$

$$Y(s)=e^{-2s}\left(\frac{2}{9}\frac{1}{s}-\frac{2}{9}\frac{s}{s^2+9}+\frac{1}{9}\frac{1}{s^2}-\frac{1}{9}\frac{1}{s^2+9}\right)+\frac{1}{s^2+9}$$

$$y(t)=\left(\frac{2}{9}-\frac{2}{9}\cos\left(3(t-2)\right)+\frac{1}{9}(t-2)-\frac{1}{27}\sin\left(3(t-2)\right)\right)H(t-2)+\frac{1}{3}\sin(3t)$$

$$\underline{\underline{y(t)=\left(\frac{1}{9}t-\frac{2}{9}\cos\left(3(t-2)\right)-\frac{1}{27}\sin\left(3(t-2)\right)\right)H(t-2)+\frac{1}{3}\sin(3t),t>0}}$$

Problem 3

【**Solution**】

令 u(x,y,z)=X(x)Y(y)Z(z)代入 $\frac{\partial^2u}{\partial x^2}+\frac{\partial^2u}{\partial y^2}+\frac{\partial^2u}{\partial z^2}=0$

$$X''YZ+XY''Z+XYZ''=0$$

同乘 $\frac{1}{XYZ}$，

$$\frac{X''}{X}=-\frac{Y''Z+YZ''}{YZ}=-\lambda$$

$$X''+\lambda X=0$$

$$Y''Z+YZ''-\lambda YZ=0$$

$$\frac{Y''}{Y}=-\frac{Z''-\lambda Z}{Z}=-\alpha$$

$$Y''+\alpha Y=0,\ Z''-(\lambda+\alpha)Z=0$$

B.C. $X(0)=X(1)=Y(0)=Y(2)=Z(0)=0$

1. $\lambda=0$ 代入 $X''+\lambda X=0$

$$X(x)=C_1x+C_2$$

代入 B.C.

$$C_1=C_2=0$$

2. $\lambda=-k^2, k>0$ 代入 $X''+\lambda X=0$

$$X(x)=C_1\cosh(kx)+C_2\sinh(kx)$$

代入 B.C.

$$C_1=C_2=0$$

3. $\lambda=k^2, k>0$ 代入 $X''+\lambda X=0$

$$X(x)=C_1\cos(kx)+C_2\sin(kx)$$

代入 B.C.

$$C_1=0$$

令 $C_2\neq 0$

$$\sin(k)=0\Rightarrow k=m\pi\Rightarrow\lambda=m^2\pi^2,\forall m\in\mathbb{N}$$

同理，討論 $Y''+\alpha Y=0$ 的情況

$$\alpha=\frac{n^2\pi^2}{4},\forall n\in\mathbb{N}$$

代入 $Z''-(\lambda+\alpha)Z=0$

$$Z''-\left(m^2+\frac{n^2}{4}\right)\pi^2 Z=0$$

$$Z=C_1\cosh\left(\pi\sqrt{m^2+\frac{n^2}{4}}z\right)+C_2\sinh\left(\pi\sqrt{m^2+\frac{n^2}{4}}z\right)$$

代入 B.C.

$$C_1 = 0$$

$$Z = C_2 \sinh\left(\pi\sqrt{m^2+\frac{n^2}{4}}z\right)$$

$$u(x,y,z) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}\sinh\left(\pi\sqrt{m^2+\frac{n^2}{4}}z\right)\sin(m\pi x)\sin\left(\frac{n\pi y}{2}\right)$$

代入 B.C.

$$u(x,y,4) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} C_{mn}\sinh\left(4\pi\sqrt{m^2+\frac{n^2}{4}}\right)\sin(m\pi x)\sin\left(\frac{n\pi y}{2}\right) = xy$$

$$C_{mn}\sinh\left(4\pi\sqrt{m^2+\frac{n^2}{4}}\right) = \left(2\int_0^1 x\sin(m\pi x)\,dx\right)\left(\int_0^2 y\sin\left(\frac{n\pi y}{2}\right)dy\right)$$

$$C_{mn} = \frac{8(-1)^{m+n+2}}{mn\pi^2\sinh\left(4\pi\sqrt{m^2+\frac{n^2}{4}}\right)}$$

$$u(x,y,z) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{8(-1)^{m+n+2}}{mn\pi^2}\right)\frac{\sinh\left(\pi\sqrt{m^2+\frac{n^2}{4}}z\right)}{\sinh\left(4\pi\sqrt{m^2+\frac{n^2}{4}}\right)}\sin(m\pi x)\sin\left(\frac{n\pi y}{2}\right)$$

Problem 4

【Solution】

$$(2i)^{\frac{1}{3}}=\left(2e^{i\left(\frac{\pi}{2}+2n\pi\right)}\right)^{\frac{1}{3}}=2^{\frac{1}{3}}e^{i\left(\frac{\pi}{6}+\frac{2n\pi}{3}\right)}$$

$$=2^{\frac{1}{3}}\left(\cos\left(\frac{\pi}{6}+\frac{2n\pi}{3}\right)+i\sin\left(\frac{\pi}{6}+\frac{2n\pi}{3}\right)\right),n=0,1,2$$

1. $n=0$

$$(2i)^{\frac{1}{3}}=2^{\frac{1}{3}}\left(\cos\left(\frac{\pi}{6}\right)+i\sin\left(\frac{\pi}{6}\right)\right)=\underline{\underline{2^{\frac{1}{3}}\left(\frac{\sqrt{3}}{2}+\frac{1}{2}i\right)}}$$

2. $n=1$

$$(2i)^{\frac{1}{3}}=2^{\frac{1}{3}}\left(\cos\left(\frac{5\pi}{6}\right)+i\sin\left(\frac{5\pi}{6}\right)\right)=\underline{\underline{2^{\frac{1}{3}}\left(\frac{-\sqrt{3}}{2}+\frac{1}{2}i\right)}}$$

3. $n=2$

$$(2i)^{\frac{1}{3}}=2^{\frac{1}{3}}\left(\cos\left(\frac{3\pi}{2}\right)+i\sin\left(\frac{3\pi}{2}\right)\right)=\underline{\underline{-2^{\frac{1}{3}}i}}$$

Problem 5

【**Solution**】

$$\tau = -\mu\frac{dv_x}{dr} = -\mu(\frac{-2v_{\max}r}{R^2})$$

For the shear stress on the wall,

$$\tau\Big|_{r=R} = -\mu(\frac{-2v_{\max}r}{R^2})\Bigg|_{r=R} = 0.01\times(\frac{2\times1.2}{\frac{0.25}{2}}) = \underline{\underline{0.192\ (g/cm\cdot s^2)}}$$

Problem 6

【**Solution**】

$$\frac{D_{KA-235}}{D_{KA-238}} = \frac{97.0\bar{r}\sqrt{\frac{T}{M_{235}}}}{97.0\bar{r}\sqrt{\frac{T}{M_{238}}}} = \sqrt{\frac{M_{238}}{M_{235}}} = \sqrt{\frac{352.038}{349.028}} = \underline{\underline{1.0043}}$$

Problem 7

【**Solution**】

(1)

By the definition of Reynolds number,

$$\mathrm{Re} = \frac{\rho v D}{\mu} \propto D\ \text{(管子前後總截面積不變，}v\text{不變)}$$

$\because D\downarrow$

$$\mathrm{Re}\downarrow\text{，}\underline{\textbf{turbulent} \rightarrow \textbf{laminar}}$$

(2)

$$f \propto \frac{1}{\mathrm{Re}}\text{，}\mathrm{Re}\downarrow, f\uparrow$$

$$\Delta P\uparrow$$

Problem 8

【Solution】

(A)

$$\Delta T_{lm} = \frac{(98.5-46.5)-(49.0-15.5)}{\ln(\dfrac{98.5-46.5}{49.0-15.5})} = \underline{\underline{42.07℃}}$$

For ΔT_m,

$$\begin{cases} Y = \dfrac{46.5-15.5}{98.5-15.5} = 0.373 \\ Z = \dfrac{98.5-49.0}{46.5-15.5} = 1.60 \end{cases} ，F_T = 0.82$$

$$\Delta T_m = 42.07 \times 0.82 = \underline{\underline{34.5℃}}$$

(B)

By energy balance,

$$\dot{m}_c C_{pc} \Delta T_c = UA\Delta T_m$$

$$12.6 \times (46.5-15.5) \times 4180 = 3500 A \times 34.5$$

$$\underline{\underline{A = 13.52\ (m^2)}}$$

Problem 9

【Solution】

By overall mole balance,

$$F = L + V$$

$$100 = 58 + L \text{，} L = 42$$

By mole balance of A,

$$Fx_F = Vy_A + Lx_A$$

$$100 \times 0.55 = 58y_A + 42x_A$$

$$y_A = -0.724x_A + 0.948$$

$$\because \alpha_{AB} = \frac{\frac{y_A}{x_A}}{(\frac{1-y_A}{1-x_A})} = 2.86$$

$$\frac{y_A}{x_A} = \frac{1-y_A}{1-x_A} \times 2.86 \text{，代入}$$

$$\frac{-0.724x_A + 0.948}{x_A} = \frac{1-(-0.724x_A + 0.948)}{(1-x_A)} \times 2.86 \rightarrow \begin{cases} x_A = 0.401 \\ y_A = 0.657 \end{cases}$$

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Problem 1

【**Solution**】

$$xy\frac{dy}{dx}=4x^2+y^2$$

同乘 $\frac{1}{x^2}$，

$$\frac{y}{x}\frac{dy}{dx}=4+\left(\frac{y}{x}\right)^2$$

令 $u=\frac{y}{x}$

$$\frac{dy}{dx}=u+x\frac{du}{dx}$$

$$u\left(u+x\frac{du}{dx}\right)=4+u^2\Rightarrow xu\frac{du}{dx}=4\Rightarrow udu=\frac{4}{x}dx$$

$$\frac{1}{2}u^2=\ln\left(x^4\right)+C_1$$

$$\frac{1}{2}\left(\frac{y}{x}\right)^2=\ln\left(x^4\right)+C_1$$

代入 I.C.可得 $C_1=\frac{1}{2}$

$$\frac{1}{2}\left(\frac{y}{x}\right)^2=\ln\left(x^4\right)+\frac{1}{2}\Rightarrow y^2=x^2\left(2\ln\left(x^4\right)+1\right)$$

$$\underline{\underline{y=\sqrt{x^2\left(2\ln\left(x^4\right)+1\right)}}}$$

(因為有給 I.C.，故負不合)

Problem 2

【**Solution**】

將原式同乘 x，可得$x^3\dfrac{dy}{dx}+3x^2y=1\Rightarrow\dfrac{d}{dx}\left(x^3y\right)=1\Rightarrow x^3y=C_1$

代入 I.C.可得$C_1=2$

$$\underline{\underline{x^3y=2}}$$

Problem 3

【**Solution**】

$$L\left\{t^2\left[H(t-2)-H(t-3)\right]\right\}=\frac{d^2}{ds^2}\left(L\left\{H(t-2)-H(t-3)\right\}\right)$$

$$=\frac{d^2}{ds^2}\left(\frac{e^{-2s}-e^{-3s}}{s}\right)=\frac{d}{ds}\left(\frac{-2e^{-2s}+3e^{-3s}}{s}-\frac{e^{-2s}-e^{-3s}}{s^2}\right)$$

$$=\frac{4e^{-2s}-9e^{-3s}}{s}-\frac{-4e^{-2s}+6e^{-3s}}{s^2}+\frac{2e^{-2s}-2e^{-3s}}{s^3}$$

Problem 4

【**Solution**】

1. 求y_h

令$D=\dfrac{d}{dx}$代入$y''-2y'+2y=0$

$$\left(D^2-2D+2\right)y=0\text{，令 }D^2-2D+2=0$$

$$D=1\pm i\Rightarrow y_h=e^x\left(C_1\cos(x)+C_2\sin(x)\right)$$

2. 求y_p，由 Heaviside 反微分運算，可知

$$y_p=\frac{1}{D^2-2D+2}\left(e^x\sin(x)\right)=e^x\frac{1}{D^2+1}\left(\sin(x)\right)$$

$$=\frac{-xe^x\cos(x)}{2}=\frac{-xe^x\cos(x)}{2}$$

3. $y=e^x\left(C_1\cos(x)+C_2\sin(x)\right)-\dfrac{xe^x\cos(x)}{2}$

Problem 5

【**Solution**】

因為兩直線之方向向量皆會與平面法向量垂直

所以平面法向量可由兩直線之方向向量做外積求得。

$$\vec{L}_1 = 7\hat{i} - 6\hat{j} + 6\hat{k}, \vec{L}_1 = 6\hat{i} - 5\hat{j} - 3\hat{k} \text{ 分別為 } L_1\text{、}L_2 \text{ 之方向向量}$$

$$\vec{N} = \vec{L}_1 \times \vec{L}_2 \text{ 為平面之法向量}$$

$$\vec{N} = 48\hat{i} + 57\hat{j} + \hat{k}$$

$$\text{平面方程式 } E: 48x + 57y + z = k$$

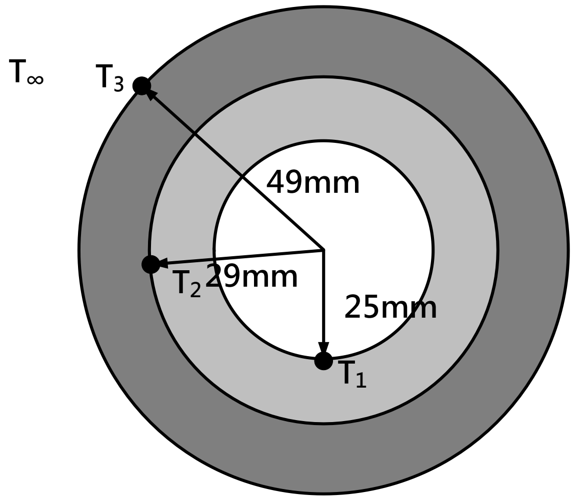
因為平面必過L_1、L_2經過的點，故平面會過(-1,4,-6)

$$k = 174$$

$$\text{平面方程式 } E: 48x + 57y + z = 174$$

<u>Problem 6</u>

【Solution】



$$\begin{cases} T_\infty = 30^\circ\mathrm{C} \\ T_1 = 85^\circ\mathrm{C} \\ T_2 = ? \\ T_3 = 50^\circ\mathrm{C} \end{cases}$$

(a)

By Fourier's law,

$$q_k = -k(2\pi rL)\frac{dT}{dr} \neq f(r)$$

$$\int_{r_1}^{r_2} \frac{q_k}{r} dr = -2\pi kL \int_{T_1}^{T_2} dT$$

$$q_k = \frac{2\pi kL(T_1 - T_2)}{\ln\frac{r_2}{r_1}} = \frac{\Delta T}{R}$$

$$R = \frac{\ln(\frac{r_1}{r_2})}{2\pi kL} = \frac{\Delta r}{\frac{k(2\pi\Delta rL)}{\ln\frac{r_2}{r_1}}} = \underline{\underline{\frac{\Delta r}{kA_{lm}}}}$$

(b)

By Ohm’s law,

$$q=\frac{T_1-T_3}{\sum R}=\frac{T_1-T_3}{\frac{1}{h_1(2\pi r_1 L)}+\frac{\ln\frac{r_2}{r_1}}{2\pi k_s L}+\frac{\ln\frac{r_3}{r_2}}{2\pi k_i L}}$$

For h_i

$\because$ @85℃，$\upsilon=0.344\times10^{-6}(m^2/s)$，$\Pr=2.08$，$k=0.673$

$$\mathrm{Re}=\frac{vD}{\upsilon}=\frac{5\times50\times10^{-3}}{0.344\times10^{-6}}=7.267\times10^{5}$$

$$Nu=\frac{hD}{k}=0.027\,\mathrm{Re}^{0.8}\,\Pr^{\frac{1}{3}}\left(\frac{\mu_b}{\mu_w}\right)$$

(The viscosity of hot water is not varied with temperature)

$$\frac{h\times50\times10^{-3}}{0.673}=0.027\times(7.267\times10^{5})^{0.8}\times2.03^{\frac{1}{3}}$$

$$h=22419.2\ (W/m^2\cdot k)$$

代回

$$q=\frac{85-50}{\frac{1}{22491.2\times(2\pi\times25\times10^{-3})}+\frac{\ln\frac{25+4}{25}}{2\pi\times45\times10}+\frac{\ln\frac{25+4+20}{25+4}}{2\pi\times0.18\times10}}=\underline{\underline{753.35\ (W)}}$$

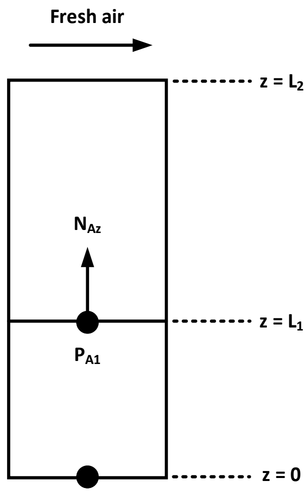
(c)

$$q=\frac{T_2-T_3}{\frac{\ln\frac{49}{29}}{2\pi\times0.18\times10}}=753.35=\frac{T_2-50}{\frac{\ln\frac{49}{29}}{2\pi\times0.18\times10}}$$

$$\underline{\underline{T_2=84.94\ (℃)}}$$

<u>Problem 7</u>

【Solution】



Assume： (1) one-directional diffusion

(2) steady state

(3) $\begin{cases} A = H_2O \\ B = air \end{cases}$ and B is stagnant

By Fick's law,

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz} + y_A(N_{Az} + \underbrace{\cancel{N_{Bz}}}_{stagnant})$$

$$\int_{L_1}^{L_2} N_{Az} dz = -D_{AB}c\int_{y_{A1}}^{y_{A2}} \frac{dy_A}{1-y_A}$$

$$N_{Az} = \frac{D_{AB}c}{L_2 - L_1}\ln\frac{1-y_{A2}}{1-y_{A1}} = \frac{D_{AB}P}{RT(L_2 - L_1)}\ln\frac{P-P_{A2}}{P-P_{A1}}$$

For P_{A2},

@ 20°C and dew point, by the humidity chart,

$$H = 0.015\frac{kg\ H_2O}{kg\ air}$$

$$P_{A2} = \frac{\frac{0.015}{18}}{\frac{1}{28.8} + \frac{0.015}{18}} \times 1.013 \times 10^5 = 2374.22\ (Pa)$$

將數據代回：

$$N_{Az}=\frac{2.5\times10^{-6}\times1.013\times10^{5}}{8.314\times303.15\times(3.0-1.5)}\ln\frac{1.013\times10^{5}-2374.22}{1.013\times10^{5}-4245}=7.67\times10^{-7}\,(mol/m^{2}\cdot s)$$

$$w_{Az}=N_{Az}A=7.67\times10^{-7}\times\frac{\pi}{4}\times1^{2}=\underline{\underline{6.027\times10^{-7}\ \ (mol/s)}}$$

After 20 days,

by quasi-steady-state mole balance,

$$-N_{Az}A_{z}=\frac{d}{dt}\left(\frac{\rho AL_{1}}{M}\right)=\frac{\rho A}{M}\frac{d}{dt}(L_{1}-L_{2})$$

$$\frac{-D_{AB}P}{RT(L_{2}-L_{1})}\ln\frac{P-P_{A2}}{P-P_{A1}}=\frac{\rho A}{M}\frac{d}{dt}(L_{1}-L_{2})$$

$$\frac{D_{AB}P}{RT}\ln\frac{P-P_{A2}}{P-P_{A1}}\int_{0}^{t}dt=\frac{\rho A}{M}\int_{\Delta z_{1}}^{\Delta z_{2}}(L_{2}-L_{1})d(L_{2}-L_{1})$$

$$\frac{D_{AB}P}{RT}\ln\frac{P-P_{A2}}{P-P_{A1}}t=\frac{\rho A}{2M}(\Delta z_{2}^{2}-\Delta z_{1}^{2})$$

$$\frac{2.5\times10^{-6}\times1.013\times10^{5}}{8.314\times303.15}\ln\frac{1.013\times10^{5}-2374.22}{1.013\times10^{5}-4245}\times20\times86400=\frac{1.0\times10^{3}\times\frac{\pi}{4}\times1^{2}}{2\times18\times10^{-3}}(\Delta z_{2}^{2}-2.5^{2})$$

$$\Delta z_{2}^{2}=2.50003\ (m)$$

Liquid level：$3-2.50003=\underline{\underline{0.49997\ (m)}}$

(本題可參考：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer,* 5th ed.; p 448, Problem 26.3.)

Problem 8

【Solution】

By continuity equation,

$$\underbrace{\frac{\partial \rho}{\partial t}}_{incompressible}+\frac{1}{r}\frac{\partial}{\partial r}\underbrace{\cancel{(\rho r v_r)}}_{v_r=0}+\frac{1}{r}\frac{\partial}{\partial \theta}(\rho v_\theta)+\frac{\partial}{\partial z}\underbrace{\cancel{(\rho v_z)}}_{v_z=0}=0$$

$$\frac{\partial v_\theta}{\partial \theta}=0 \rightarrow v_\theta = f(\cancel{t}, r, \cancel{\theta}, \cancel{z})$$

By N-S equation in cylinder coordinate,

r-direction,

$$-\rho\frac{v_\theta^2}{r}=\frac{-\partial p}{\partial r}\ ,\ \ p=p(r)$$

z-direction,

$$\frac{-\partial p}{\partial z}=0\ ,\quad p\neq p(z)$$

θ-direction,

$$\rho\left(\cancel{\frac{\partial v_\theta}{\partial t}}+\cancel{v_r\frac{\partial v_r}{\partial r}}+\cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}}+\cancel{\frac{v_r v_\theta}{r}}+\cancel{v_z\frac{\partial v_\theta}{\partial z}}\right)=$$

$$\frac{-1}{r}\frac{\partial p}{\partial \theta}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{r}(r v_\theta)\right)+\cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial r^2}}+\cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}}+\cancel{\frac{\partial^2 v_\theta}{\partial z^2}}\right]+\cancel{\rho g_\theta}$$

$$\mu\left[\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{r}(r v_\theta))\right]=\frac{1}{r}\frac{\partial p}{\partial \theta}=f(r)$$

$$p=rf(r)\theta+c$$

$\because p(\theta)=p(\theta+2\pi)$

$$rf(r)\theta+c=rf(r)(\theta+2\pi)+c$$

$$f(r)=0$$

$$\boxed{\mu\left[\frac{\partial}{\partial r}\frac{1}{r}\frac{\partial}{r}(r v_\theta))\right]=0\begin{cases} r=R_i,\ v_\theta=R_i\Omega \\ r=R_o,\ v_\theta=0\end{cases}}$$

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Problem 1

【Solution】

令 $x=e^t, D=\dfrac{d}{dt}$ 代入原式

$$\left(D(D-1)-D+1\right)y(t)=t$$

1. 求 y_h

令 $D(D-1)-D+1=(D-1)^2=0$

$$D=1,1\Rightarrow y_h=C_1e^t+C_2te^t$$

$$y_h=C_1x+C_2x\ln(x), x>0$$

2. 求 y_p，由未定係數法，令 $y_p=At+b$ 代入原式比較係數

$$A=1, b=2\Rightarrow y_p=t+2$$
$$y_p=\ln(x)+2, x>0$$

$$\underline{\underline{y=C_1x+C_2x\ln(x)+\ln(x)+2, x>0}}$$

Problem 2

【Solution】

令 Tank1 與 Tank2 的濃度分別為 x 與 y，糖含量分別為 m_1 與 m_2

$$\begin{cases}\dfrac{dm_1}{dt}=\dfrac{d(200x)}{dt}=-6x+3y\\ \dfrac{dm_2}{dt}=\dfrac{d(100y)}{dt}=4x-4y+5\delta(t-3)\end{cases}, x(0)=y(0)=\frac{1}{20}$$

$$\begin{cases}\dfrac{dx}{dt}=-\dfrac{3}{100}x+\dfrac{3}{200}y\\ \dfrac{dy}{dt}=\dfrac{1}{25}x-\dfrac{1}{25}y+\dfrac{1}{20}\delta(t-3)\end{cases}$$

將方程組做拉式轉換，令 $X(s)=L\left[x(t)\right], Y(s)=L\left[y(t)\right]$

$$\begin{cases} sX(s)-x(0)=-\dfrac{3}{100}X(s)+\dfrac{3}{200}Y(s) \\ sY(s)-y(0)=\dfrac{1}{25}X(s)-\dfrac{1}{25}Y(s)+\dfrac{1}{20}e^{-3s} \end{cases}$$

$$\begin{cases} \left(s+\dfrac{3}{100}\right)X(s)-\dfrac{3}{200}Y(s)=\dfrac{1}{20} \\ -\dfrac{1}{25}X(s)+\left(s+\dfrac{1}{25}\right)Y(s)=\dfrac{1}{20}\left(e^{-3s}+1\right) \end{cases}$$

利用克拉瑪定理

$$X(s)=\frac{\frac{1}{20}s+\frac{11}{4000}}{s^2+\frac{7}{100}s+\frac{3}{5000}}+e^{-3s}\left(\frac{\frac{3}{4000}}{s^2+\frac{7}{100}s+\frac{3}{5000}}\right)$$

$$x(t)=\frac{e^{\frac{-7}{200}t}}{100}\left(5\cosh\left(\frac{t}{40}\right)+4\sinh\left(\frac{t}{40}\right)\right)+\frac{3e^{\frac{-7}{200}(t-3)}}{100}\sinh\left(\frac{t-3}{40}\right)H(t-3)$$

$$Y(s)=\frac{\frac{1}{20}s+\frac{7}{2000}}{s^2+\frac{7}{100}s+\frac{3}{5000}}+e^{-3s}\left(\frac{\frac{1}{20}s+\frac{3}{2000}}{s^2+\frac{7}{100}s+\frac{3}{5000}}\right)$$

$$y(t)=\frac{e^{\frac{-7}{200}t}}{100}\left(5\cosh\left(\frac{t}{40}\right)+7\sinh\left(\frac{t}{40}\right)\right)+\frac{e^{\frac{-7}{200}(t-3)}}{100}\left(5\cosh\left(\frac{t-3}{40}\right)-\sinh\left(\frac{t-3}{40}\right)\right)H(t-3)$$

$$\underline{\underline{m_1=2e^{\frac{-7}{200}t}\left(5\cosh\left(\frac{t}{40}\right)+4\sinh\left(\frac{t}{40}\right)\right)+6e^{\frac{-7}{200}(t-3)}\sinh\left(\frac{t-3}{40}\right)H(t-3)}}$$

$$\underline{\underline{m_2=e^{\frac{-7}{200}t}\left(5\cosh\left(\frac{t}{40}\right)+7\sinh\left(\frac{t}{40}\right)\right)+e^{\frac{-7}{200}(t-3)}\left(5\cosh\left(\frac{t-3}{40}\right)-\sinh\left(\frac{t-3}{40}\right)\right)H(t-3)}}$$

Problem 3

【Solution】

$$\text{flux}=\iint_s \vec{F}\cdot\hat{n}dA=\int_0^{2\pi}\int_0^{\cos^{-1}\left(\frac{1}{3}\right)}\vec{r}\cdot\frac{\vec{r}}{r}r^2\sin\theta d\theta d\phi=\underline{\underline{36\pi}}$$

Problem 4

【**Solution**】

$$\nabla^2 T=\frac{\partial^2 T}{\partial r^2}+\frac{1}{r}\frac{\partial T}{\partial r}+\frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2}=0,\ T(r,0)=T(r,\pi)=0$$

由特徵函數展開法可令$T(r,\theta)=\sum_{n=1}^{\infty}C_n(r)\sin(n\theta)$代入$r^2\frac{\partial^2 T}{\partial r^2}+r\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial \theta^2}=0$

$$\sum_{n=1}^{\infty}\left(r^2C_n''(r)+rC_n'(r)-n^2C_n(r)\right)\sin(n\theta)=0$$

$$r^2C_n''(r)+rC_n'(r)-n^2C_n(r)=0$$

$$C_n(r)=a_nr^{-n}+b_nr^n$$

$$T(r,\theta)=\sum_{n=1}^{\infty}\left(a_nr^{-n}+b_nr^n\right)\sin(n\theta)$$

$$\because T(0,\theta)\text{ is bounded}\quad \therefore a_n=0$$

$$\because T(C,\theta)=\sum_{n=1}^{\infty}b_nC^n\sin(n\theta)=T_0$$

$$\therefore b_n=\frac{2}{\pi C^n}\int_0^{\pi}T_0\sin(n\theta)d\theta=\frac{2T_0}{n\pi C^n}\left(1-(-1)^n\right)$$

$$\underline{\underline{T(r,\theta)=\sum_{n=1}^{\infty}\left(\frac{2T_0}{n\pi}\left(1-(-1)^n\right)\right)\left(\frac{r}{C}\right)^n\sin(n\theta)}}$$

Problem 5

【**Solution**】

(a) temperature gradient

(b) conduction/convection/radiation

(c) $Nu \equiv \dfrac{hL}{k} = \dfrac{conductive\ thermal\ resistance}{convective\ thermal\ resistance}$

(d) $Nu = f(\mathrm{Re}, \mathrm{Pr})$

Problem 6

【**Solution**】

∵ Up to the break through point, the unused bed length is 4cm,

$$LUB = 4\ cm \text{ ，} L_b = 14 - 4 = 10\ (cm)$$

If the required t_b is increased to 10hr,

$$\frac{t_b^{'}}{t_b} = \frac{10}{3.8} = \frac{L_b^{'}}{L_b} = \frac{L_b^{'}}{10}$$

$$L_b^{'} = 26.3\ (cm)$$

$$L^{'} = 26.3 + LUB = 26.3 + 4 = \underline{\underline{30.3\ (cm)}}$$

Problem 7

【**Solution**】

(a) (1) uniform surface

(2) monolayered-adsorption

(3) no interaction between adsorbed sites

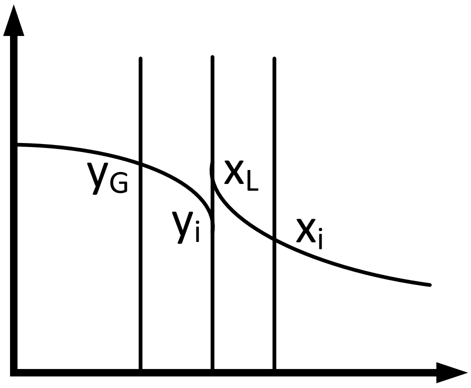
(b)

$$q \equiv \frac{heat\ needed\ to\ vaporize\ 1\ mole\ of\ feed}{molar\ latent\ heat\ of\ the\ feed}$$

(c)

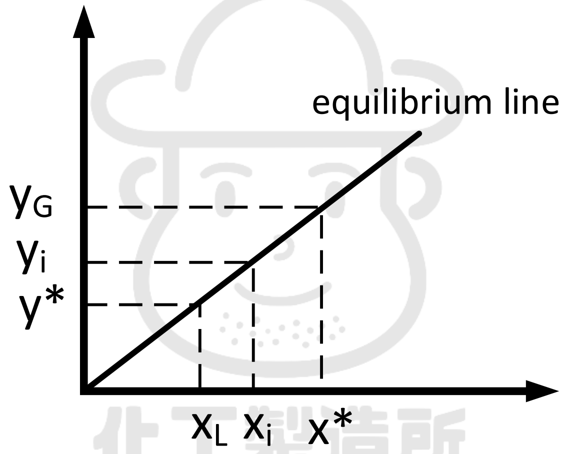
∵ NH_3 在水中溶解度極大，因此液相所受之阻力較小，**<u>質傳由氣相控制</u>**

※根據雙膜理論，其質傳過程可如下圖表示：



$$N_A = k_y(y_F - y_i) = k_x(x_i - x_L)$$

若繪製成平衡關係圖，則：

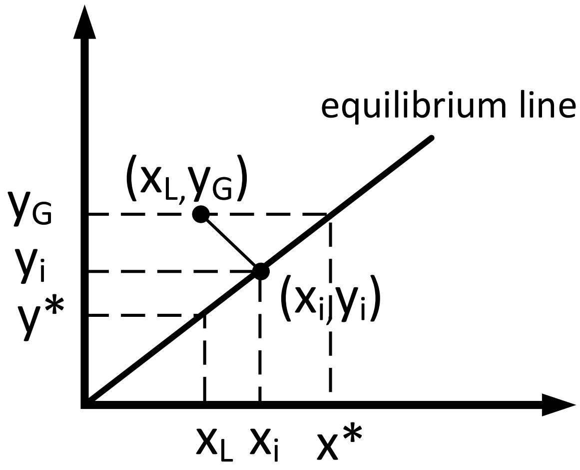


(平衡線沒有一定要直線，只是比較好想像整個過程)

其中，較好測定的濃度為氣相與液相的 bulk 濃度(y_G **與** x_L **)，因此可以將上式經過一些搬移：**

$$k_y(y_G - y_i) = k_x(x_i - x_L)\text{，}-\frac{k_x}{k_y} = \frac{y_G - y_i}{x_i - x_L}$$

亦即 (x_L, y_G) **與** (x_i, y_i) **會在同一條直線上，斜率為** $-k_x / k_y$



(一般假設介面阻力很小，所以會達成平衡，亦即 x_i, y_i 會與平衡線相交)

由此，我們就能夠藉由 x_L 與 y_G 以及已知的 N_A，反推 x_i 與 y_i，就能夠得知分別的 k_x 與 k_y 為多少。然而，這個過程並不這麼好算，尤其是萬一氣相濃度差或液相自己的濃度差太小，會有很大的誤差，因此也有另外一派做法，也就是大家所熟悉的 overall mass transfer coefficient 的算法，亦即利用 y_G 與 y^* (與液相 bulk 平衡的假想氣相濃度)或利用 x_L 與 x^* (與氣相 bulk 平衡的假想液相濃度)所定義的質傳係數。

$$N_A = K_y(y_G - y^*) = K_x(x_L - x^*)$$

而連結兩個系統的關係，則由線性關係進行(注意，這裡我說的是線性關係，不是完完全全的 Henry's law，原文書是以曲線平衡線做內插的方式進行。)

$$k_y(y_G - y_i) = k_x(x_i - x_L)$$

$$y_G - y^* = (y_G - y_i) + (y_i - y^*)$$

根據圖形，我們知道

$$\frac{y_i - y^*}{x_i - x_L} = m \text{，} y_i - y^* = m(x_i - x_L)$$

代回：

$$y_G - y^* = (y_G - y_i) + m(x_i - x_L)$$

因此：

$$N_A = K_y(y_G - y^*) = k_y(y_G - y_i) = k_x(x_i - x_L)\begin{cases}(y_G - y^*) = \dfrac{N_A}{K_y} \\ (y_G - y_i) = \dfrac{N_A}{k_y} \\ (x_i - x_L) = \dfrac{N_A}{k_x}\end{cases}$$

代回：

$$\frac{N_A}{K_y} = \frac{N_A}{k_y} + \frac{mN_A}{k_x} \text{，} \boxed{\frac{1}{K_y} = \frac{1}{k_y} + \frac{m}{k_x}}$$

同樣的，若考慮液相，也有類似關係，即：

$$\boxed{\frac{1}{K_x} = \frac{1}{mk_y} + \frac{1}{k_x}}$$

因此，根據溶質在液氣相的關係，有以下兩種情況：

(1) 溶質在<u>液相溶解度極高</u>(代表 m 很小)：

此時

$$\frac{1}{K_y} = \frac{1}{k_y} + \cancel{\frac{m}{k_x}} \approx \frac{1}{k_y} \text{ 或，} \frac{1}{K_x} = \frac{1}{mk_y} + \cancel{\frac{1}{k_x}} \approx \frac{1}{mk_y}$$

，亦即<u>整體質傳由氣相控制</u>

(2) 溶質在<u>液相溶解度極低</u>(代表 m 很大)：

此時

$$\frac{1}{K_y} = \cancel{\frac{1}{k_y}} + \frac{m}{k_x} \approx \frac{m}{k_x} \text{ 或，} \frac{1}{K_x} = \cancel{\frac{1}{mk_y}} + \frac{1}{k_x} \approx \frac{1}{k_x}$$

，亦即<u>整體質傳由液相控制</u>

(本題相關討論全部可參考：**Geankoplis, C.** ***Transport Processes and Separation Process Principles***, **4th ed.; p 594~601.)**

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Problem 1

【Solution】

$$y'+\frac{2}{x+1}y=3,\text{同乘}(x+1)^2$$

$$(x+1)^2y'+2(x+1)y=3(x+1)^2$$

$$\frac{d}{dx}\left((x+1)^2y\right)=3(x+1)^2$$

$$(x+1)^2y=(x+1)^3+C$$

代入 I.C.可得 $C=2$

$$(x+1)^2y=(x+1)^3+2$$

$$y=(x+1)+\frac{2}{(x+1)^2}$$

Problem 2

【Solution】

令 $Y(s)=L\left[y(t)\right],F(s)=L\left[f(t)\right]$ ，做 Laplace Transform

$$s^2Y(s)-sy(0)-y'(0)-2\left(sY(s)-y(0)\right)-8Y(s)=F(s)$$

$$\left(s^2-2s-8\right)Y(s)=F(s)+s-2\Rightarrow(s-4)(s+2)Y(s)=F(s)+s-2$$

$$Y(s)=\frac{F(s)}{(s-4)(s+2)}+\frac{s-2}{(s-4)(s+2)}$$

$$Y(s)=\frac{1}{6}F(s)\left(\frac{1}{s-4}-\frac{1}{s+2}\right)+\frac{1}{3}\left(\frac{1}{s-4}+\frac{2}{s+2}\right)$$

$$y(t)=L^{-1}\left[Y(s)\right]=\frac{1}{6}f(t)*\left(e^{4t}-e^{-2t}\right)+\frac{1}{3}\left(e^{4t}-2e^{-2t}\right),t\geq0$$

$$\underline{\underline{y(t)=\frac{1}{6}\int_0^t f(t-\tau)\left(e^{4\tau}-e^{-2\tau}\right)d\tau+\frac{1}{3}\left(e^{4t}-2e^{-2t}\right),t\geq0}}$$

Problem 3

【**Solution**】

$$\int_c \vec{F}\cdot d\vec{R} = \int_c 2xdx + ydy - zdz$$

令 $C:\begin{cases} x = 2\cos\theta \\ y = 2\sin\theta, \quad \theta\in[0,\frac{\pi}{2}] \\ z = 0 \end{cases}$，則 $dx = -2\sin\theta d\theta,\ dy = 2\cos\theta d\theta,\ dz = 0$

$$\int_c 2xdx + ydy - zdz = \int_0^{\frac{\pi}{2}}\left((4\cos\theta)(2\sin\theta)+(2\sin\theta)(2\cos\theta)\right)d\theta$$

$$= \int_0^{\frac{\pi}{2}} 12\sin\theta\cos\theta d\theta = \underline{\underline{6}}$$

Problem 4

【**Solution**】

利用特徵函數展開法(the expansion of eigenfunction)，

令 $u(x,t) = \sum_{n=1}^{\infty} C_n(t)\cos\left(\frac{n\pi x}{L}\right)$代入$\frac{\partial u}{\partial t} - 4\frac{\partial^2 u}{\partial x^2} = 0$

$$\sum_{n=1}^{\infty}\left(C_n{}'(t) + \left(\frac{2n\pi}{L}\right)^2 C_n(t)\right)\cos\left(\frac{n\pi x}{L}\right) = 0$$

令$C_n{}'(t) + \left(\frac{2n\pi}{L}\right)^2 C_n(t) = 0 \Rightarrow C_n(t) = a_n e^{-(\frac{2n\pi}{L})^2 t}$

$$\underline{\underline{u(x,t) = \sum_{n=1}^{\infty} a_n e^{-(\frac{2n\pi}{L})^2 t}\cos\left(\frac{n\pi x}{L}\right)}}$$

I.C.代入

$$u(x,0) = \sum_{n=1}^{\infty} a_n\cos\left(\frac{n\pi x}{L}\right) = f(x) \Rightarrow \underline{\underline{a_n = \frac{2}{L}\int_0^L f(x)\cos\left(\frac{n\pi x}{L}\right)dx}}$$

Problem 5

【Solution】

$$P_A(\lambda) = \det(A - \lambda I) = \begin{vmatrix} 1-\lambda & -2 \\ 2 & 0-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - \lambda + 4 = 0 \Rightarrow \underline{\underline{\lambda = \frac{1 \pm \sqrt{15}i}{2}}}$$

Problem 6

【Solution】

(a)

Assume only $\begin{cases} v = v_x(t, y) \\ T = T(t, y) \\ w = w_A(t, y),\ \textit{no reaction} \end{cases}$

Momentum,

$$\frac{\partial v_x}{\partial t} = \frac{-1}{\rho}\frac{\partial p}{\partial x} + \upsilon\left[\frac{\partial^2 v_x}{\partial y^2}\right] + g_x$$

Heat,

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho C_p}\left(\frac{\partial v_x}{\partial y}\right)^2$$

Mass,

$$\frac{\partial w_A}{\partial t} = D_{AB}\frac{\partial^2 w_A}{\partial y^2}$$

$$\boxed{\upsilon \ [=] \ \alpha \ [=] \ D_{AB} \ [=] \ m^2 / s}$$

(b)

題目似乎有誤，應改為：

$$\nabla = \frac{\partial}{\partial r}\overrightarrow{\delta_r} + \frac{1}{r}\frac{\partial}{\partial \theta}\overrightarrow{\delta_\theta} + \frac{\partial}{\partial z}\overrightarrow{\delta_z}$$

令 $\vec{v} = v_r(r,\theta)\overrightarrow{\delta_r} + v_\theta(r,\theta)\overrightarrow{\delta_\theta}$

$\because$ *incompressible flow*, $\nabla \cdot \vec{v} = 0$

$$\begin{aligned}
\nabla \cdot \vec{v} &= (\frac{\partial}{\partial r}\overrightarrow{\delta_r} + \frac{1}{r}\frac{\partial}{\partial \theta}\overrightarrow{\delta_\theta}) \cdot (v_r\overrightarrow{\delta_r} + v_\theta\overrightarrow{\delta_\theta}) \\
&= \frac{\partial v_r}{\partial r}(\overrightarrow{\delta_r} \cdot \overrightarrow{\delta_r}) + \frac{v_r}{r}\underbrace{\frac{\partial \overrightarrow{\delta_r}}{\partial \theta}}_{\overrightarrow{\delta_\theta}} \cdot \overrightarrow{\delta_\theta} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}(\overrightarrow{\delta_\theta} \cdot \overrightarrow{\delta_\theta}) + \frac{v_\theta}{r}\underbrace{\frac{\partial \overrightarrow{\delta_\theta}}{\partial \theta}}_{-\overrightarrow{\delta_r}} \cdot \overrightarrow{\delta_\theta} \\
&= \frac{\partial v_r}{\partial r} + \frac{v_r}{r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} = 0
\end{aligned}$$

整理：

$$\frac{1}{r}\frac{\partial (rv_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} = 0 \text{ ，} \frac{\partial (rv_r)}{\partial r} + \frac{\partial v_\theta}{\partial \theta} = 0$$

因此假設：

$$rv_r = f\frac{\partial \Psi}{\partial \theta} \text{ ，} v_r = \frac{f}{r}\frac{\partial \Psi}{\partial \theta} \text{ ；} v_\theta = g\frac{\partial \Psi}{\partial r}$$

(f 與 g 為函數)

代入：

$$(\frac{\partial f}{\partial r}\frac{\partial \Psi}{\partial \theta} + \frac{\partial g}{\partial \theta}\frac{\partial \Psi}{\partial r}) + (f\frac{\partial^2 \Psi}{\partial r \partial \theta} + g\frac{\partial^2 \Psi}{\partial \theta \partial r}) = 0$$

$$f\frac{\partial^2 \Psi}{\partial r \partial \theta} + g\frac{\partial^2 \Psi}{\partial \theta \partial r} = 0 \text{ ，} f = -g$$

令 $f = -g = -1$ 代入滿足

則可知若

$$\begin{cases} v_r = \dfrac{-1}{r}\dfrac{\partial \psi}{\partial \theta} \\ v_\theta = \dfrac{\partial \psi}{\partial r} \end{cases}$$
，可滿足 continuity equation

ψ 即為 stream function

<u>Problem 7</u>

【Solution】

(a)

$$H \equiv \frac{water\ vapor\ (kg)}{dry\ air\ (kg)}$$

$$H_p = \frac{actual\ humidity}{saturation\ humidity} \times 100$$

$$H_R \equiv \frac{actual\ partial\ pressure\ of\ water}{saturation\ pressure\ of\ water\ vapor} \times 100$$

(b)

$$t = \frac{L_s}{AR_C}\Delta X = \frac{L_s(X_1 - X_2)}{AR_C}$$

For R_C,

$$R_C = \frac{h}{\lambda_w}(T - T_w) \times 3600$$

For h,

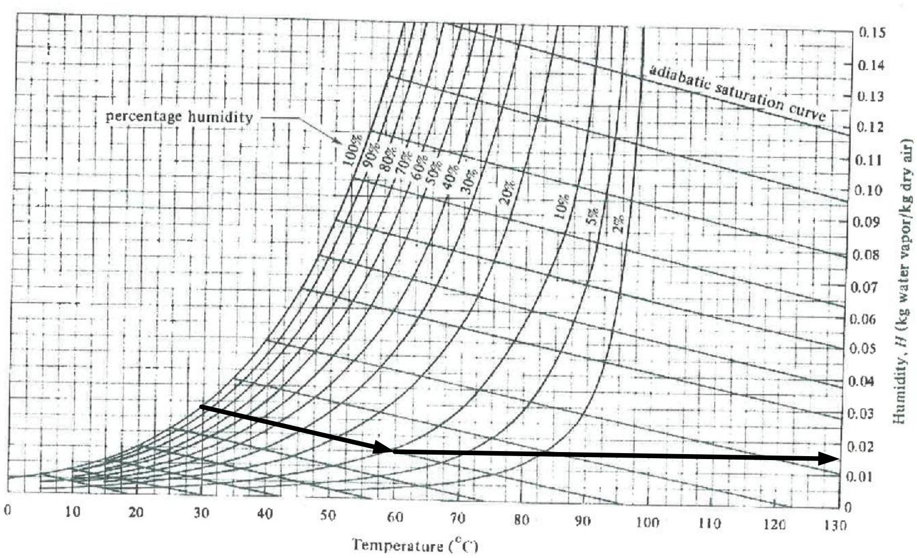
$$h = 0.0204 \times G^{0.8}$$

$$G = v_{air}\rho_{air}$$

where V_H is the humid volume, which is calculated as,

$$V_H = \frac{22.4}{273}T \times (\frac{1}{28.8} + \frac{1}{18}H) = \frac{22.4}{273} \times (273.15 + 60) \times (\frac{1}{28.8} + \frac{1}{18} \times 0.015) = \underline{\underline{0.972\ (m^3 / kg\ dry\ air)}}$$

where H is the humidity for air with a dry bulb temperature of 60°C and wet bulb temperature of 29.4°C, illustrated as the figure below:



Therefore, the density of the air is calculated as,

$$\rho = \frac{w}{V_H} = \frac{1+0.015}{0.972} = 1.044\ (kg/m^3)$$

The heat transfer coefficient is then,

$$h = 0.0204 \times G^{0.8} = 0.0204 \times (3.05 \times 1.044 \times 3600)^{0.8} = 36.07\ (W/m^2K)$$

For $\lambda_w = \lambda_w\ (29.4℃)$

$$\lambda_w(27℃) = 2550.18 - 113.25 = 2437.55\ (kJ/kg)$$

$$\lambda_w(30℃) = 2556.3 - 123.79 = 2430.51\ (kJ/kg)$$

By 內插法

$$\frac{29.4-30}{27-30} = \frac{\lambda_w - 2430.51}{2437.55 - 2430.51}\ ，\ \lambda_w = 2431.918\ (kJ/kg)$$

代回：

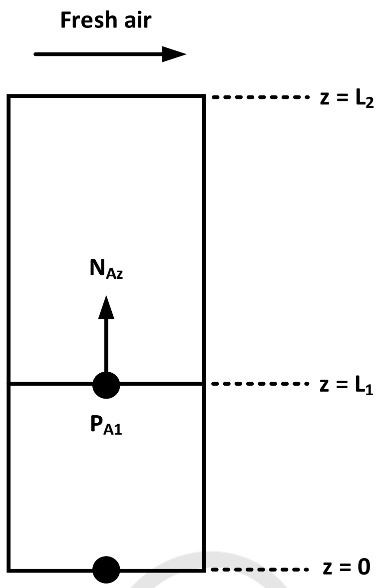
$$R_C = \frac{36.07}{2431.918 \times 10^3}(60-29.4) \times 3600 = 1.634\ (\frac{kg \cdot H_2O}{h \cdot m^2})$$

再代回：

$$t = \frac{11.34 \times (0.35-0.22)}{0.61^2 \times 1.634} = \underline{\underline{2.42\ (hr)}}$$

Problem 7

【Solution】



Assume： (1) one-directional diffusion

(2) steady state

(3) $\begin{cases} A = A \\ B = air \end{cases}$ and B is stagnant

By Fick's law,

$$N_{Az} = -cD_{AB}\frac{dy_A}{dz} + y_A(N_{Az} + \underbrace{\cancel{N_{Bz}}}_{stagnant})$$

$$\int_{L_1}^{L_2} N_{Az}dz = -D_{AB}c\int_{y_{A1}}^{y_{A2}}\frac{dy_A}{1-y_A}$$

$$N_{Az} = \frac{D_{AB}c}{L_2 - L_1}\ln\frac{1-y_{A2}}{1-y_{A1}} = \frac{D_{AB}P}{RT(L_2 - L_1)}\ln\frac{P-P_{A2}}{P-P_{A1}}$$

For P_{A2},

@ 0°C and dew point, by humidity the chart,

$$H = 0.005 \frac{kg\ H_2O}{kg\ air}$$

$$P_{A2} = \frac{\frac{0.005}{18}}{\frac{1}{28.8} + \frac{0.005}{18}} \times 755 = 5.992\ (mmHg)$$

by quasi-steady-state mole balance,

$$-N_{Az} A_z = \frac{d}{dt}\left(\frac{\rho A L_1}{M}\right) = \frac{\rho A}{M} \frac{d}{dt}(L_1 - L_2)$$

$$\frac{-D_{AB} P}{RT(L_2 - L_1)} \ln \frac{P - P_{A2}}{P - P_{A1}} = \frac{\rho A}{M} \frac{d}{dt}(L_1 - L_2)$$

$$\frac{D_{AB} P}{RT} \ln \frac{P - P_{A2}}{P - P_{A1}} \int_0^t dt = \frac{\rho A}{M} \int_{\Delta z_1}^{\Delta z_2} (L_2 - L_1) d(L_2 - L_1)$$

$$\frac{D_{AB} P}{RT} \ln \frac{P - P_{A2}}{P - P_{A1}} t = \frac{\rho A}{2M} (\Delta z_2^2 - \Delta z_1^2)$$

For Δz_2

$$\Delta z_2 = \Delta z_1 + \frac{\Delta V}{\Delta A} = 17.1 + \frac{0.0208}{0.82} = 17.134\ (cm)$$

$$\frac{D_{AB} \times \frac{755}{760} \times 1}{82.05 \times 273.15} \ln \frac{755 - 5.99}{755 - 33} \times 10 = \frac{1.59 \times 0.82}{2 \times 154} (17.134^2 - 17.1^2)$$

$$\underline{\underline{D_{AB} = 303.987\ (cm^2 / h)}}$$

(本題與 108 年台科大單操輸送 Problem 7 相似)

(本題可參考：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer,* 5th ed.; p 448, Problem 26.3.)

111 年台科大工程數學與輸送現象

Problem 1

【**Solution**】

$$(e^y - x^2)y' + \cos(2x) = 2xy$$

$$e^y dy - x\left(xdy + 2ydx\right) + \cos(2x)dx = 0$$

$$e^y dy - d\left(x^2 y\right) + \cos(2x)dx = 0$$

$$\int e^y dy - \int d\left(x^2 y\right) + \int \cos(2x)dx = 0$$

$$e^y - x^2 y + \frac{1}{2}\sin(2x) = C$$

代入 I.C.可得 $C = e^2$

$$\underline{\underline{e^y - x^2 y + \frac{1}{2}\sin(2x) = e^2}}$$

Problem 2

【**Solution**】

令 $f(t) = \begin{cases} 0, & t < 1 \\ t, & t \ge 1 \end{cases} = t \times H(t-1)$ ，H(t)為 Heaviside function

此題用拉式轉換，所以 t 應該要大於 0，做 Laplace Transform

$$s^2 Y(s) - sy(0) - y'(0) + 4Y(s) = e^{-s}\left(\frac{1}{s} + \frac{1}{s^2}\right)$$

$$\left(s^2 + 4\right)Y(s) = e^{-s}\left(\frac{1}{s} + \frac{1}{s^2}\right)$$

$$Y(s) = \frac{1}{4}e^{-s}\left(\frac{1}{s} + \frac{1}{s^2} - \frac{s+1}{s^2+4}\right)$$

$$\underline{\underline{y(t) = L^{-1}[Y(s)] = \frac{1}{4}\left(t - \frac{1}{2}\sin(2(t-1)) - \cos\left(2(t-1)\right)\right)H(t-1), t \ge 1}}$$

Problem 3

【Solution】

not passing through the origin(continuous in C)

By Green's theorem,

$$\oint_c \frac{-2y}{x^2+y^2}dx+\frac{2x}{x^2+y^2}dy=\iint_D\left[\frac{\partial}{\partial x}\left(\frac{2x}{x^2+y^2}\right)-\frac{\partial}{\partial y}\left(\frac{-2y}{x^2+y^2}\right)\right]dxdy=\iint_D(0)\,dxdy=\underline{\underline{o}}$$

Problem 4

【Solution】

$$\because f(x)=H(x)e^{-4x}$$

$$\therefore F\left[f(x)\right]=F\left[H(x)e^{-4x}\right]=\underline{\underline{\frac{1}{4+iw}}}$$

Problem 5

【Solution】

利用特徵函數展開法(the expansion of eigenfunction)，

令 $y(x,t)=\sum_{n=1}^{\infty}C_n(t)\sin\left(\frac{n\pi x}{L}\right)$代入$\frac{\partial^2 y}{\partial t^2}-4\frac{\partial^2 y}{\partial x^2}=0$

$$\sum_{n=1}^{\infty}\left(C_n''(t)+\left(\frac{2n\pi}{L}\right)^2C_n(t)\right)\sin\left(\frac{n\pi x}{L}\right)=0$$

$$C_n''(t)+\left(\frac{2n\pi}{L}\right)^2C_n(t)=0\Rightarrow C_n(t)=\alpha_n\cos\left(\frac{2n\pi}{L}t\right)+\beta_n\sin\left(\frac{2n\pi}{L}t\right)$$

$$y(x,t)=\sum_{n=1}^{\infty}\left(\alpha_n\cos\left(\frac{2n\pi}{L}t\right)+\beta_n\sin\left(\frac{2n\pi}{L}t\right)\right)\sin\left(\frac{n\pi x}{L}\right)$$

$$y_t(x,t)=\sum_{n=1}^{\infty}\left(\frac{-2n\pi}{L}\alpha_n\sin\left(\frac{2n\pi}{L}t\right)+\frac{2n\pi}{L}\beta_n\cos\left(\frac{2n\pi}{L}t\right)\right)\sin\left(\frac{n\pi x}{L}\right)$$

I.C.代入

$$y_t(x,0)=\sum_{n=1}^{\infty}\frac{2n\pi}{L}\beta_n\sin\left(\frac{n\pi x}{L}\right)=0\Rightarrow\beta_n=0$$

$$y(x,t)=\sum_{n=1}^{\infty}\alpha_n\cos\left(\frac{2n\pi}{L}t\right)\sin\left(\frac{n\pi x}{L}\right)$$

I.C.代入

$$y(x,0)=\sum_{n=1}^{\infty}\alpha_n \sin\left(\frac{n\pi x}{L}\right)=f(x)\Rightarrow \alpha_n=\frac{2}{L}\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx$$

$$y(x,t)=\frac{2}{L}\sum_{n=1}^{\infty}\left(\int_0^L f(x)\sin\left(\frac{n\pi x}{L}\right)dx\right)\cos\left(\frac{2n\pi}{L}t\right)\sin\left(\frac{n\pi x}{L}\right)$$

Problem 6

【Solution】

(a)

By the equation of continuity,

$$\underbrace{\frac{\partial\rho}{\partial t}}_{incompressible}+\frac{1}{r}\frac{\partial}{\partial r}\underbrace{\cancel{(\rho r v_r)}}_{v_r=0}+\frac{1}{r}\frac{\partial}{\partial\theta}\underbrace{\cancel{(\rho v_\theta)}}_{v_\theta=0}+\frac{\partial}{\partial z}(\rho v_z)=0$$

$$v_z=v_z(r,\cancel{\theta},\cancel{z},\cancel{t})$$

By momentum balance,

$$\rho v_z v_z r dr d\theta\big|_z-\rho v_z v_z r dr d\theta\big|_{z+dz}+\tau_{rz} r d\theta dz\big|_r-\tau_{rz} r d\theta dz\big|_{r+dr}+p r dr d\theta\big|_{dz}-p r dr d\theta\big|_{z+dz}=0$$

同除 $drd\theta dz\to 0$,

$$\underbrace{\cancel{\frac{-\partial(r\rho v_z v_z)}{\partial z}}}_{v_z\neq f(z)}\frac{\partial(r\tau_{rz})}{\partial r}-r\frac{dp}{dz}=0$$

$$\frac{\partial(r\tau_{rz})}{\partial r}=r\underbrace{\frac{d(p)}{dz}}_{constant\ A}=rA$$

$$\because \tau_{rz}=-\mu\left(\frac{dv_z}{dr}\right)$$

$$-\mu\frac{d}{dr}\left[r\left(\frac{dv_z}{dr}\right)\right]=rA$$

(b)

由(a)小題推導之 governing equation 積分可得：

$$v_z=\frac{-r^2}{4\mu}A+C_1\ln r+C_2\begin{cases}r=\kappa R, v_z=0\\ r=R, v_z=0\end{cases}\begin{cases}C_1=\dfrac{R^2(\kappa^2-1)A}{4\mu\ln\kappa}\\ C_2=\dfrac{-R^2(\kappa^2-1)A}{4\mu\ln\kappa}\ln R+\dfrac{R^2A}{4\mu}\end{cases}$$

得：$$\underline{\underline{v_z=\frac{R^2A}{4\mu}(1-\frac{r^2}{R^2})+\frac{R^2(\kappa^2-1)A}{4\mu\ln\kappa}\ln\frac{r}{R}}}$$

Problem 7

【Solution】

(a) By mole balance of fragrance (A),

$$N_A^{"}(4\pi r^2)\Big|_r-N_A^{"}(4\pi r^2)\Big|_{r+dr}=0$$

同除 $4\pi dr\to 0$

$$\frac{-d(r^2N_A^{"})}{dr}=0$$

$$N_A^{"}=-D_{AB}\frac{P}{RT}\frac{dy_A}{dr}+y_A(N_A^{"}+\underbrace{\cancel{N_B^{"}}}_{stagnant})$$

$$則\ N_A^{"}=-\frac{D_{AB}P}{RT}\cdot\frac{1}{1-y_A}\cdot\frac{dy_A}{dr}$$

代入：

$$\frac{d}{dr}[r^2\frac{dy_A}{(1-y_A)dr}]=0\text{ ，}\frac{1}{1-y_A}\cdot\frac{dy_A}{dr}=\frac{C_1}{r^2}$$

$$\ln(1-y_A)=\frac{C_1}{r}+c_2\begin{cases}r=r_1, y_A=y_{A0}\\ r\to\infty, y_A=0\end{cases}$$

則 $C_1 = r_1 \ln(1-y_{A0})$ 代回：

$$W_A\Big|_{r=r_1} = (4\pi r_1^2)\cdot N_A''\Big|_{r=r_1} = (4\pi r_1^2)\cdot -\frac{D_{AB}P}{RT}\cdot\frac{C_1}{r_1^2} = -\frac{4\pi D_{AB}Pr_1}{RT}\ln(1-y_{A0}) = -\frac{4\pi D_{AB}Pr_1}{RT}\ln(\frac{P-P_{A0}}{P})$$

$$W_A\Big|_{r=r_1} = -\frac{4\pi\times 6.92\times10^{-6}\times1.013\times10^{5}\times1.0\times10^{-3}}{8.314\times318}\ln(\frac{1-\frac{0.555}{760}}{1}) = \underline{\underline{2.43\times10^{-9}\ \ (mole/s)}}$$

Problem 8

【Solution】

(a)

Assume the temperature distribution is linear in y direction, which is,

$$T(y)=(T_v-T_w)\frac{y}{\delta}+T_w$$

By energy balance on the film, (per unit length)

$$q_{conduction}=q_{condensation}$$

$$-k_f\cdot dx\cdot\left.\frac{dT}{dy}\right|_{y=0}=-h_v dm$$

For dm, by momentum balance in the film, (per unit length)

$$\tau_{yx}dx\big|_{dy}-\tau_{yx}dx\big|_{y+dy}+\rho_v g dxdy=0$$

同除 $dxdy\to 0$

$$\frac{-d\tau_{yx}}{dy}+\rho_v g=0$$

$\tau_{yx}=-\mu\dfrac{dv_x}{dy}$ 代入：

$$v_x=\frac{-\rho g}{2\mu}y^2+C_1 y+C_2\begin{cases}y=0,v_x=0\\ y=\delta,\dfrac{dv_x}{dy}=0\end{cases}$$

$$\begin{cases}C_1=\dfrac{\rho g}{\mu}\delta\\ C_2=0\end{cases}，\ v_x=\frac{\rho g}{\mu}(\delta y-\frac{y^2}{2})$$

$$m=\int_0^{\delta}\rho_v v_x dy=\frac{\rho_v^2 g\delta^3}{3\mu_v}\quad\text{(per unit length)}$$

代回 energy balance 結果：

$$-k_f\cdot dx\cdot(\frac{T_v-T_w}{\delta})=-h_v d(\frac{\rho_v^2 g\delta^3}{3\mu_v})=\frac{h_v\rho_v^2 g\delta^2}{\mu_v}d\delta$$

$$k_f(T_v-T_w)\int_0^x dx=\frac{h_v\rho_v^2 g}{\mu_v}\int_0^{\delta}\delta^3 d\delta\ ，\ \underline{\underline{\delta=[\frac{4\mu_v k_f(T_v-T_w)}{h_v\rho_v^2 g}]^{1/4}}}$$

(b)

At P = 68.9 kPa, the saturated temperature is,

$$\frac{T_v - 85}{90-85} = \frac{68.9-57.83}{70.14-57.83} \text{，} T_v = 89.5℃ = 362.64\ (K)$$

For the viscosity, we use the average value between T_v and T_w

$$T = T_w \text{，} \mu_w = 0.3310$$

$$T = T_v \text{，} \mu_v = 0.3183$$

$$\mu_{avg} = \frac{0.3310+0.3183}{2} = 0.3247\ (cp)$$

For the specific gravity, we use the average value between T_v and T_w

$$T = T_w \text{，} \rho_w = 967.8\ (kg/m^3)$$

$$T = T_v \text{，} \rho_v = 985.6\ (kg/m^3)$$

(Specific volume 倒數)

$$\rho_{avg} = 976.7\ (kg/m^3)$$

For the thermal conductivity, we use the average value between T_v and T_w

$$T = T_w \text{，} k_w = 0.673\ (W/m\cdot K)$$

$$T = T_v \text{，} k_v = 0.676\ (W/m\cdot K)$$

$$k_{avg} = 0.674\ (W/m\cdot K)$$

For the latent heat, 由 $T = 89.5℃$ 查表內插可知

$$h_v = 2651.1 - 353.8 = 2297.3\ (kJ/kg)$$

代回：

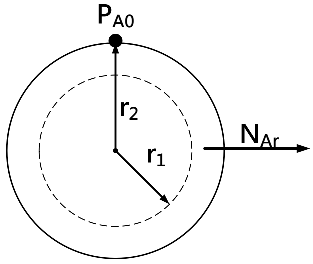
$$\delta = [\frac{4\mu_v k_f (T_v - T_w)}{h_v \rho_v^2 g}]^{1/4} = [\frac{4\times 0.3247\times 10^{-3}\times 0.674\times(89.5-86.11)}{2297.3\times 10^3\times 976.7^2\times 9.8}]^{1/4} = \underline{\underline{1.08\times 10^{-4}\ (m)}}$$

112 年台科大工程數學與輸送現象

<u>Problem 1</u>

【Solution】

(a)



Assume,

$$\begin{cases} naphthalene = A \\ air = B \end{cases}$$

(1) The system is in pseudo-steady state.

(2) Constant temperature and pressure.

(3) $P_{A1} << 1\, atm$, the naphthalene is very dilute.

(4) The partial pressure of naphthalene is negligible in the bulk.

By mole balance on the naphthalene sphere

$$r_{evaporate} = (4\pi r^2) N_{Ar}\Big|_{r=R(t)}$$

For N_{Ar}, by Fick's law,

$$N_{Ar} = -\mathscr{D}_{AB}\frac{dC_{Ar}}{dr} + \underbrace{\cancel{y_A}}_{very\ dilute}(N_{Ar} + N_{Br}) = \frac{-\mathscr{D}_{AB}P}{RT}\frac{dy_{Ar}}{dr}$$

$$N_{Ar} = \frac{w_{Ar}}{4\pi r^2} = \frac{-\mathscr{D}_{AB}P}{RT}\frac{dy_{Ar}}{dr}\ ,\ \frac{w_{Ar}}{4\pi}\int_r^{\infty}\frac{dr}{r^2} = \frac{-\mathscr{D}_{AB}P}{RT}\int_{y_{A1}}^{0} dy_{Ar}$$

$$w_{Ar} = N_{Ar}(4\pi r^2) = 4\pi r\frac{\mathscr{D}_{AB}P}{RT}(y_{A1} - 0) = 4\pi r\frac{\mathscr{D}_{AB}P_{A1}}{RT}$$ 代回：

$$r_{evaporate} = 4\pi[R(t)]\frac{\mathscr{D}_{AB}P_{A1}}{RT}\ (kmol/s) = \underline{\underline{4\pi[R(t)]\frac{\mathscr{D}_{AB}MP_{A1}}{RT}\ (kg/s)}}$$

(b)

By the general equation of continuity,

$$\cancel{\frac{\partial C_A}{\partial t}} + \nabla \cdot (N_A) = \cancel{R_A}$$

$$\nabla \cdot (N_A) = 0$$

$N_{Ar} = \dfrac{-\mathscr{D}_{AB} P}{RT} \dfrac{dy_{Ar}}{dr}$ 代入：

$$\frac{1}{r^2} \frac{\partial}{\partial r}\left(r^2 \frac{-\mathscr{D}_{AB} P}{RT} \frac{dy_{Ar}}{dr}\right) = 0 \text{ ， } \frac{\partial}{\partial r}\left(r^2 \frac{dP_{Ar}}{dr}\right) = 0$$

$$P_{Ar} = \frac{c_1}{r} + c_2 \begin{cases} r = R(t), P_{Ar} = P_{A1} \\ r = \infty, P_{Ar} = 0 \end{cases} \text{ ， } \underline{\underline{P_{Ar} = \frac{R(t)}{r} P_{A1}}}$$

(c)

By pseudo-steady state assumption on the naphthalene sphere,

$$-(4\pi r^2) N_{Ar}\Big|_{r=R(t)} = \frac{d\left(\frac{4}{3}\pi [R(t)]^3 \rho\right)}{dt} = 4\pi [R(t)]^2 \rho \frac{dR(t)}{dt}$$

$$4\pi [R(t)] \frac{\mathscr{D}_{AB} M P_{A1}}{RT} = -4\pi [R(t)]^2 \rho \frac{dR(t)}{dt}$$

$$\int_0^{\tau} dt = \frac{-RT\rho}{\mathscr{D}_{AB} M P_{A1}} \int_{r_1}^{0} [R(t)] dR(t)$$

$$t = \frac{RT\rho}{2\mathscr{D}_{AB} M P_{A1}} r_1^2 = \frac{8.314 \times (273.15 + 27) \times 1.14 \times 10^3}{2 \times 7 \times 10^{-6} \times 128 \times \frac{0.6}{760} \times 10^5} \times (2 \times 10^{-3})^2 = \underline{\underline{80.43\ (s)}}$$

<u>Problem 2</u>

【Solution】

(a) Correct

Backward-feed evaporator 是新鮮進料與加熱用蒸氣進料方向相反的一種蒸發設計，在蒸氣剛進入系統時，溫度較高，此時若濃產物黏度較高，就能夠有效降低其黏度，並且增加熱交換效率。

(本題可參考：Geankoplis, C. *Transport Processes and Separation Process Principles*, 4th ed.; p 533.)

(b) Correct

(本題可參考：Geankoplis, C. *Transport Processes and Separation Process Principles*, 4th ed.; p 565~566.)

(c) Correct

The bound moisture in the solid exerts a vapor pressure less than that of liquid water at the same temperature.

(本題可參考：Geankoplis, C. *Transport Processes and Separation Process Principles*, 4th ed.; p 575.)

(d) Incorrect

$$q = \frac{heat\ needed\ to\ vaporize\ 1\ mol\ of\ feed\ at\ entering\ conditions}{molar\ latent\ heat\ of\ vaporization\ of\ feed}$$

(本題可參考：Geankoplis, C. *Transport Processes and Separation Process Principles*, 4th ed.; p 710.)

(e) Incorrect

Reflux can increase the purity of the overhead product, but may increase the cost of cooling water.

(f) Correct

(本題可參考：Geankoplis, C. *Transport Processes and Separation Process Principles*, 4th ed.; p 765.)

<u>Problem 3</u>

【Solution】

(a)

$$\underline{\underline{\tau_{zr} = -\mu \frac{dv_r}{dz}}}$$

(b)

<u>Momentum Flux</u>

(c)

In the same way, write the continuity equation in spherical coordinates, let $v_\phi = 0$

$$\nabla \bullet v = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2 v_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v_\theta \sin\theta) = 0$$

$$\frac{\sin\theta}{r}\frac{\partial}{\partial r}(r^2 v_r) + \frac{\partial}{\partial\theta}(v_\theta \sin\theta) = 0$$

因此假設：

$$r^2 v_r = f\frac{\partial\Psi}{\partial\theta}\text{，}\boxed{v_r = \frac{f}{r^2}\frac{\partial\Psi}{\partial\theta}}\text{；}v_\theta \sin\theta = g\frac{\partial\Psi}{\partial r}\text{，}\boxed{v_\theta = \frac{g}{\sin\theta}\frac{\partial\Psi}{\partial r}}$$

(f 與 g 為函數)

代入：

$$\frac{\sin\theta}{r}\left(f\frac{\partial^2\Psi}{\partial r\partial\theta} + \frac{\partial f}{\partial r}\frac{\partial\Psi}{\partial\theta}\right) + \left(g\frac{\partial^2\Psi}{\partial\theta\partial r} + \frac{\partial g}{\partial\theta}\frac{\partial\Psi}{\partial r}\right) = 0$$

$$\left(\frac{\sin\theta}{r}\frac{\partial f}{\partial r}\frac{\partial\Psi}{\partial\theta} + \frac{\partial g}{\partial\theta}\frac{\partial\Psi}{\partial r}\right) + \left(\frac{\sin\theta}{r}f\frac{\partial^2\Psi}{\partial r\partial\theta} + g\frac{\partial^2\Psi}{\partial\theta\partial r}\right) = 0$$

$$f\frac{\sin\theta}{r}\frac{\partial^2\Psi}{\partial r\partial\theta} + g\frac{\partial^2\Psi}{\partial\theta\partial r} = 0\text{，}f\frac{\sin\theta}{r} = -g$$

令 $f=-\dfrac{1}{\sin\theta}$ ，$g=\dfrac{1}{r}$ 代入也滿足上式左半部

(※ $\dfrac{\partial\Psi}{\partial\theta}$ 為 θ 函數且 $\dfrac{\partial\Psi}{\partial r}$ 為 r 函數，因此兩者若相加 $=0$，其前面的係數直接令為

0 會比較簡單，即 $\dfrac{\partial f}{\partial r}=0$ ，$f=f(\theta)$ ；$\dfrac{\partial g}{\partial\theta}=0$ ，$g=g(r)$)

因此，若：

$$\begin{cases} v_r=\dfrac{-1}{r^2\sin\theta}\dfrac{\partial\Psi}{\partial\theta} \\ v_\theta=\dfrac{1}{r\sin\theta}\dfrac{\partial\Psi}{\partial r} \end{cases}$$ ，可滿足 continuity equation

ψ 即為 stream function

(d)

To determine the height of the interface,

Assume：

(1) The flow is steady and fully developed in θ-direction

(2) $v_r=v_z=0$.

(3) Incompressible Newtonian fluid.

By the equation of continuity,

$$\cancel{\frac{\partial\rho}{\partial t}}+\cancel{\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)}+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta)+\cancel{\frac{\partial}{\partial z}(\rho v_z)}=0$$

$$\frac{\partial v_\theta}{\partial\theta}=0\text{ ，}v_\theta=f(\cancel{t},r,\cancel{\theta},\cancel{z})=f(r)\,\text{only}$$

By the Navier Stokes equation in θ direction,

$$\rho(\cancel{\frac{\partial v_\theta}{\partial t}}+\cancel{v_r\frac{\partial v_\theta}{\partial r}}+\cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}}+\cancel{v_z\frac{\partial v_\theta}{\partial z}}+\cancel{\frac{v_\theta v_r}{r}})$$

$$=\cancel{\frac{-1}{r}\frac{\partial p}{\partial \theta}}+\mu[\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta))+\cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}}+\cancel{\frac{\partial^2 v_\theta}{\partial z^2}}+\cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}}]+\cancel{\rho g_\theta}$$

$$\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta))=0\ ，\ v_\theta=\frac{c_1}{2}r+\frac{c_2}{r}\begin{cases}r=0,v_\theta=0\\ r=R,v_\theta=\omega R\end{cases}$$

$$c_1=2\omega\ ，\ c_2=0$$

$$v_\theta=\omega r$$

To determine the height of the interface, by Navier-Stokes equation with r and z components,

$$\frac{-\partial p}{\partial r}=\frac{-\rho v_\theta^2}{r}$$

$$\frac{-\partial p}{\partial z}=-\rho(-g_z)$$

Integrate the z-direction,

$$p=-\rho g_z z+f(r)$$

代回 r 方向，

$$\frac{\partial p}{\partial r}=f'(r)=\frac{\rho v_\theta^2}{r}=\frac{\rho[\omega r]^2}{r}=\rho\omega^2 r$$

Integrate with r,

$$f=\frac{\rho\omega^2r^2}{2}+C$$

And the pressure distribution becomes,

$$p(r,z)=-\rho g_z z+\frac{\rho\omega^2r^2}{2}+C$$

Because the liquid is open to the atmosphere at the top,

$$p(r,h(r))=p(R,h(R))=P_0\text{，}\quad p(r,h(r))=p(R,h(R))$$

$$-\rho g_z h(r)+\frac{\rho\omega^2r^2}{2}+C=-\rho g_z h(R)+\frac{\rho\omega^2R^2}{2}+C$$

$$-\rho g_z(h-h(R))=\frac{\rho\omega^2R^2}{2}[1-(\frac{r}{R})^2]$$

$$h(r)=\frac{\omega^2R^2}{2g_z}[(\frac{r}{R})^2-1]+h(R)$$

∵ The total volume is V_0

$$V_0 = \int_0^R 2\pi r[h(r)]dr = \frac{\pi\omega^2 R^2}{g_z}\int_0^R r[(\frac{r}{R})^2 - 1] + h(R)dr$$

$$V_0 = \frac{\pi\omega^2 R^2}{g_z}(-\frac{1}{4R^2}) + R \cdot h(R)$$

$$h(R) = \frac{V_0 + \frac{\pi\omega^2}{4g_z}}{R}$$

代回：

$$\underline{\underline{h(r) = \frac{\omega^2 R^2}{2g_z}[(\frac{r}{R})^2 - 1] + \frac{V_0 + \frac{\pi\omega^2}{4g_z}}{R}}}$$

(本題改編自：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 110, Problem 3B.15.)

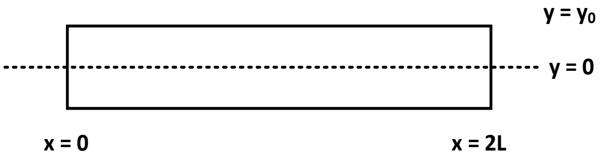
<u>Problem 4</u>

【Solution】

(a)

The positive sign is used to ensure the heat flux is in the same direction with the temperature gradient.

(b)



By the general energy equation, assume steady state

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$$

$$\begin{cases} x=0,\ T=T_b \\ x=2L,\ T=T_a \\ y=0,\ \dfrac{\partial T}{\partial y}=0\ (symmetry) \\ y=y_0,\ -k\dfrac{\partial T}{\partial y}=h(T-T_\infty) \end{cases}$$

Let,

$$\theta=\frac{T-T_b}{T_\infty-T_b}$$

$$\frac{\partial^2 \theta}{\partial x^2}+\frac{\partial^2 \theta}{\partial y^2}=0 \begin{cases} x=0,\ \theta=0 \\ x=2L,\ \theta=\theta_a \\ y=0,\ \dfrac{\partial \theta}{\partial y}=0 \\ y=y_0,\ -k\dfrac{\partial \theta}{\partial y}=h\theta \end{cases}$$

By separation of variable, let

$$\theta(x,y)=X(x)Y(y)$$

$$\frac{1}{X}\frac{d^2X}{dx^2}=\frac{-1}{Y}\frac{d^2Y}{dy^2}=\lambda^2$$

For $Y(y)$,

$$Y(y) = A\cos(\lambda y) + B\sin(\lambda y)\begin{cases} y = 0,\ \dfrac{dY}{dy} = 0 & ---(1) \\ y = y_0,\ \dfrac{dY}{dy} = \dfrac{hY}{-k} & ---(2) \end{cases}$$

By B.C. (1)

$$-A\lambda\sin(0) + B\lambda\cos(0) = 0 \text{，} \boxed{B = 0}$$

By B.C. (2)

$$-A\lambda\sin(\lambda y_0) = \frac{-h}{k}[A\cos(\lambda y_0)]$$

$$\lambda = \frac{h\cos(\lambda y_0)}{k\sin(\lambda y_0)}$$

Let $\lambda = \beta_n$

$$\beta_n = \frac{h\cos(\beta_n y_0)}{k\sin(\beta_n y_0)}$$

$$Y(y) = A\cos(\beta_n y)$$

For $X(x)$,

$$X(x) = C\sinh(\beta_n x) + D\cosh(\beta_n x)\begin{cases} x = 0,\ X = 0 & ---(3) \\ y = 2L,\ X = \theta_a & ---(4) \end{cases}$$

By B.C. (3)

$$C\sinh(0) + D\lambda\cosh(0) = 0 \text{，} \boxed{D = 0}$$

$$\theta(x,y) = C\sinh(\beta_n x)\cdot A\cos(\beta_n y) = \alpha_n \sinh(\beta_n x)\cos(\beta_n y)$$

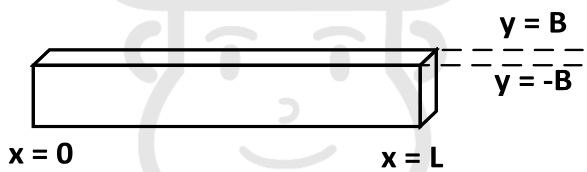
By B.C. (4)

$$\theta_a = \alpha_n \sinh(2L\beta_n)\cos(\beta_n y)$$

$$\boxed{\alpha_n = \frac{\int_0^{y_0} \theta_a \cos(\beta_n y)dy}{\int_0^{y_0} \cos^2(\beta_n y)dy}}$$

$$\boxed{\theta(x,y) = \sum_{n=1}^{\infty} \alpha_n \sinh(\beta_n x)\cos(\beta_n y)} \text{，} \boxed{\alpha_n = \frac{\int_0^{y_0} \theta_a \cos(\beta_n y)dy}{\int_0^{y_0} \cos^2(\beta_n y)dy}}$$

※本題不需也不能將對流項如同一般 fin 題型一樣，納入微分方程式裡。因在所有熱傳問題中，對流項都應該擺在邊界條件，除非取的 control volume 能夠納入對流的影響，如大部分的 fin 解溫度分布問題。然而，在一般 fin 問題的簡化上，是假設 fin 溫度分布只與 x 方向有關(可參考 BSL 第二版 p.308 上方表格，有所有 fin 問題簡化的條件)，因此假設同樣以二維方程式，且對流項置於邊界條件時：



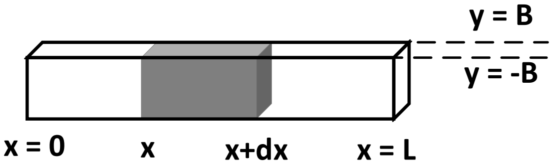
$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \begin{cases} y = -B, \quad \underbrace{-k\frac{\partial T}{\partial y} = -h(T - T_\infty)}_{fin\text{溫度高，向}-y\text{方向傳熱，因此}h\text{ 前面有負號}} \\ y = B, \ -k\frac{\partial T}{\partial y} = h(T - T_\infty) \end{cases}$$

$$\frac{\partial^2}{\partial x^2}\underbrace{[\int_{-B}^{B} T dy]}_{T \neq T(y)} + \int_{-B}^{B} \frac{\partial^2 T}{\partial y^2} dy = 0 \text{，} \frac{\partial^2}{\partial x^2}(2BT) + \left.\frac{\partial T}{\partial y}\right|_{-B}^{B} = 0$$

$$(2B)\frac{\partial^2 T}{\partial x^2} - \frac{2h(T - T_\infty)}{k} = 0$$

$$\boxed{\frac{\partial^2 T}{\partial x^2} - \frac{h(T - T_\infty)}{kB} = 0}$$

此形式剛好與直接使用 control volume 做熱量平衡結果相同，如：



By energy balance,

$$(2WB)q_x\big|_x-(2WB)q_x\big|_{x+dx}-2h(Wdx)(T-T_\infty)=0$$

同除 $2BWdx\to 0$

$$\frac{-dq_x}{dx}-\frac{h}{B}(T-T_\infty)=0$$

$\because q_x=-k\dfrac{\partial T}{\partial x}$

$$k\frac{d^2T}{dx^2}-\frac{h}{B}(T-T_\infty)=0$$

$$\boxed{\frac{d^2T}{dx^2}-\frac{h}{kB}(T-T_\infty)=0}$$

然而，此方法必須建立在 T = T(x)，與 y 方向無關的前提下，才可做使用。

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Problem 1

【**Solution**】

1. 求 y_h

令 $y=e^{mx}$ 代入 $y''-4y'+4y=0$

$$(m^2-4m+4)e^{mx}=0 \text{，令 } m^2-4m+4=(m-2)^2=0 \text{，} m=2,2$$

$$y_h=C_1e^{2x}+C_2xe^{2x}$$

2. 求 y_p，由 Heaviside 反微分運算，可知

$$y_p=\frac{1}{(D-2)^2}\left(4e^{2x}-9e^{-x}\right)=4e^{2x}\frac{1}{D^2}(1)-e^{-x}=4e^{2x}\left(\frac{1}{2}x^2\right)-e^{-x}$$

$$y_p=2x^2e^{2x}-e^{-x}$$

$$\underline{\underline{y=C_1e^{2x}+C_2xe^{2x}+2x^2e^{2x}-e^{-x}}}$$

Problem 2

【**Solution**】

令 $f(t)=\begin{cases}0, & 0\le t<4\\ 12, & t\ge 4\end{cases}=12H(t-4)$，$H(t)$ 為 Heaviside function

做 Laplace Transform

$$s^2Y(s)-sy(0)-y'(0)-2\left(sY(s)-y(0)\right)-3Y(s)=12e^{-4s}\frac{1}{s}$$

$$\left(s^2-2s-3\right)Y(s)=12e^{-4s}\frac{1}{s}+s-2$$

$$Y(s)=12e^{-4s}\frac{1}{s\left(s^2-2s-3\right)}+\frac{s-2}{\left(s^2-2s-3\right)}=12e^{-4s}\frac{1}{s(s-3)(s+1)}+\frac{s-2}{(s-3)(s+1)}$$

$$Y(s)=e^{-4s}\left(\frac{-4}{s}+\frac{1}{s-3}+\frac{3}{s+1}\right)+\frac{1}{4}\left(\frac{1}{s-3}+\frac{3}{s+1}\right)$$

$$\underline{\underline{y(t)=\left(-4+e^{3(t-4)}+3e^{-(t-4)}\right)H(t-4)+\frac{1}{4}\left(e^{3t}+3e^{-t}\right),t\ge 0}}$$

Problem 3

【Solution】

(a) $\nabla\cdot\vec{F}=\left(\frac{\partial}{\partial x}\hat{i}+\frac{\partial}{\partial y}\hat{j}+\frac{\partial}{\partial z}\hat{k}\right)\cdot\left(2xy\hat{i}+xe^{y}\hat{j}+2z\hat{k}\right)=\underline{\underline{2y+xe^{y}+2}}$

(b) $\nabla\times\vec{F}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 2xy & xe^{y} & 2z\end{vmatrix}=\underline{\underline{\left(e^{y}-2x\right)\hat{k}}}$

(c)因為旋轉場不會發散，所以$\nabla\cdot\left(\nabla\times\vec{F}\right)$為 $\underline{0}$

Problem 4

【Solution】

利用分離變數法，令$u(x,t)=X(x)T(t)$代入 B.C.

先處理邊界條件: $\begin{cases}u(0,t)=X(0)T(t)=0\\ u(2,t)=X(2)T(t)=0\end{cases}\Rightarrow X(0)=X(2)=0$

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}\Rightarrow\frac{\frac{\partial u}{\partial t}}{u}=\frac{\frac{\partial^2 u}{\partial x^2}}{u}\Rightarrow\frac{XT'}{XT}=\frac{X''T}{XT}\Rightarrow\frac{T'}{T}=\frac{X''}{X}=-\lambda$$

$$\therefore\begin{cases}X''+\lambda X=0\\ T'+\lambda T=0\end{cases}$$，$X''+\lambda X=0$ 為特徵方程式

1. 令$\lambda=0$

$$X''(x)=0\Rightarrow X(x)=C_1+C_2x$$

$$\text{B.C.代入}\Rightarrow C_1=C_2=0$$

2. 令$\lambda=-k^2,k>0$

$$X''(x)-k^2X(x)=0\Rightarrow X(x)=C_1\cosh(kx)+C_2\sinh(kx)$$

B.C.代入 $\Longrightarrow C_1=C_2=0$

3. 令 $\lambda = k^2, k > 0$

$$X''(x) + k^2 X(x) = 0 \Rightarrow X(x) = C_1\cos(kx) + C_2\sin(kx)$$

$$\text{B.C.代入} \Longrightarrow C_1 = 0,\ \text{令}\ C_2 \neq 0 \Rightarrow k = \frac{n\pi}{2}, n = 1,2,3,\ldots$$

$$\therefore \lambda_n = \left(\frac{n\pi}{2}\right)^2 \Rightarrow X_n(x) = \sin(\frac{n\pi x}{2}), n = 1,2,3,\ldots$$

代入 $T' + \lambda T = 0 \Rightarrow T' + \left(\frac{n\pi c}{2}\right)^2 T = 0$

$$T_n(t) = a_n e^{-\left(\frac{n\pi c}{2}\right)^2 t}$$

$$u_n(x,t) = X_n(x)T_n(t) = a_n e^{-\left(\frac{n\pi c}{2}\right)^2 t}\sin(\frac{n\pi x}{2})$$

$$u(x,t) = \sum_{n=1}^{\infty} a_n e^{-\left(\frac{n\pi c}{2}\right)^2 t}\sin(\frac{n\pi x}{2})$$

I.C.代入

$$u(x,0) = \sum_{n=1}^{\infty} a_n \sin(\frac{n\pi x}{2}) = \begin{cases} x, & 0<x<1 \\ 0, & 1<x<2 \end{cases}$$

$$a_n = \frac{2}{2}\left(\int_0^1 x\sin(\frac{n\pi x}{2})dx + \int_1^2 0\cdot\sin(\frac{n\pi x}{2})dx\right)$$

$$= \int_0^1 x\sin(\frac{n\pi x}{2})dx = \frac{-2}{n\pi}\cos(\frac{n\pi}{2}) + \frac{4}{n^2\pi^2}\sin(\frac{n\pi}{2})$$

$$\underline{\underline{u(x,t) = \sum_{n=1}^{\infty} a_n e^{-\left(\frac{n\pi c}{2}\right)^2 t}\sin(\frac{n\pi x}{2})}}$$

Problem 5

【Solution】

Known: $T_{av}=20°C,\ v=4\text{ m/s},\ D_o=6\text{ cm},\ D_i=5\text{ cm},\ \text{thickness } t=0.6\text{ cm}$

$$T_{steam}=110°C,\ L_{pipe}=10\text{ m},\ k_s=50\text{ W/m}\cdot\text{K},\ k_{ins}=0.2\text{ W/m}\cdot\text{K}$$

By interpolation approach, we can obtain the water properties at $20\circ C$

$$\rho\approx 997.4\text{ kg/m}^3,\ C_p\approx 4.185\text{ kJ/kg}\cdot\text{K}$$

$$\mu=\mu_b\ \text{(bulk average)}\approx 1.024\times10^{-3}\text{ pa}\cdot\text{s}$$

$$k\approx 0.5973\text{ W/m}\cdot\text{K},\ \ \text{Pr}\approx 7.21$$

To calculate μ_w (fluid viscosity at the wall), we need to obtain the water viscosity at

$110\circ C$.

Also, by interpolation approach,

$$\mu_w=0.2655\times10^{-3}\text{ pa}\cdot\text{s}$$

$$\text{Re}=\frac{997.4\times4\times0.05}{1.024\times10^{-3}}\approx 194804.7$$

Therefore, we should use the equation

$$\text{Nu}=\frac{hD_i}{k}=0.027\,\text{Re}^{0.8}\,\text{Pr}^{1/3}\left(\frac{\mu_b}{\mu_w}\right)^{0.14}$$

$$\frac{h(0.05)}{0.5973}=0.027(194804.7)^{0.8}(7.21)^{1/3}\left(\frac{1.024}{0.2655}\right)^{0.14}$$

$$h=12832.7\text{ W/m}^2\cdot\text{K}$$

By Ohm's Law,

$$\sum R_{thermal} = \frac{1}{h\left(\pi D_i L_{pipe}\right)} + \frac{\ln\left(\frac{D_o}{D_i}\right)}{2\pi k_s L_{pipe}} + \frac{\ln\left(\frac{D_o + 2t}{D_o}\right)}{2\pi k_{ins} L_{pipe}} = 1.462\times10^{-2}$$

$$q = \frac{\Delta T}{\sum R_{thermal}} = \frac{110-20}{1.462\times10^{-2}} = 6155.95\ \mathrm{J/s}$$

(本題公式講解可參考：Geankoplis, C. *Transport Processes and Separation Process Principles*, 3th ed.; p 238~240，公式代法可參考此範圍中例題.)

Problem 6

【Solution】

(a) False

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 53-55.)

(b) True

(本題可參考：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer,* 5th ed.; p 111.)

(c) True

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 14.)

(d) True

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 516.)

(e) True

From a fundamental view point of view, the assumption that the concentration gradients are the driving forces as given by Fick's laws is not correct. Instead, the gradient of the chemical potential is the real driving force.

(本題可參考：Mehrer, H. *Diffusion in Solids: Fundamentals, Methods, Materials, Diffusion-Controlled Processes*, p 170.)

(f) False

Schmidt number is the ratio of the momentum and mass diffusivities, while the ratio of the convective mass transfer to the diffusive mass transfer is Sherwood number.

(本題可參考：Incropera, F.; Dewitt, D.; Bergman, T.; Lavine, A. *Fundamentals of Heat and Mass Transfer,* 6th ed.; p 377.)

(g) False

It should be the percentage relative humidity, not the percentage humidity.

(本題可參考：Geankoplis, C. *Transport Processes and Separation Process Principles*, 5th ed.; p 1039.)

(h) False

The basic temperature profile of a distillation column is hotter at the bottom and cooler at the top.

(本題可參考網站：https://neutrium.net/unit-operations/distillation-fundamentals/)

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Problem 1

【**Solution**】

令 $D=\dfrac{d}{dx}$ 代入 O.D.E.

$$\left(D^2-4\right)y=-7e^{2x}+x$$

1. 求 y_h

$$D^2-4=0$$

$$D=2,-2$$

$$y_h=C_1\cosh(2x)+C_2\sinh(2x)$$

2. 求 y_p，由 Heaviside 反微分運算，可知

$$y_p=\frac{1}{D^2-4}\left(-7e^{2x}\right)+\frac{1}{D^2-4}x$$

$$y_p=\frac{-7}{(D+2)(D-2)}\left(e^{2x}\right)-\frac{1}{4}\left(1+\frac{D^2}{4}+\left(\frac{D^2}{4}\right)^2+\ldots\right)x$$

$$y_p=\frac{-7}{4}xe^{2x}-\frac{1}{4}x$$

$$y=C_1\cosh(2x)+C_2\sinh(2x)-\frac{7}{4}xe^{2x}-\frac{1}{4}x$$

代入 I.C.

$$C_1=1,C_2=\frac{5}{2}$$

$$\underline{\underline{y=\cosh(2x)+\frac{5}{2}\sinh(2x)-\frac{7}{4}xe^{2x}-\frac{1}{4}x}}$$

Problem 2

【**Solution**】

令 $Y(s)=L\left[y(t)\right]$，將原式進行拉氏轉換

$$sY(s)-y(0)-4Y(s)=\frac{1}{s}$$

$$(s-4)Y(s)=\frac{1}{s}+1$$

$$Y(s)=\frac{1}{s(s-4)}+\frac{1}{s-4}=\frac{-\frac{1}{4}}{s}+\frac{\frac{5}{4}}{s-4}$$

$$\underline{\underline{y(t)=-\frac{1}{4}+\frac{5}{4}e^{4t},t>0}}$$

Problem 3

【**Solution**】

由特徵函數展開法，可令 $y(x,t)=\sum_{n=1}^{\infty}C_n(t)\sin\left(\frac{n\pi x}{5}\right)$ 代入 P.D.E.

$$\sum_{n=1}^{\infty}\left(C_n''(t)+\frac{4n^2\pi^2}{25}C_n(t)\right)\sin\left(\frac{n\pi x}{5}\right)=0$$

$$C_n''(t)+\frac{4n^2\pi^2}{25}C_n(t)=0$$

$$C_n(t)=\alpha_n\cos\left(\frac{2n\pi t}{5}\right)+\beta_n\sin\left(\frac{2n\pi t}{5}\right)$$

$$y(x,t)=\sum_{n=1}^{\infty}\left(\alpha_n\cos\left(\frac{2n\pi t}{5}\right)+\beta_n\sin\left(\frac{2n\pi t}{5}\right)\right)\sin\left(\frac{n\pi x}{5}\right)$$

代入 I.C.

$$\alpha_n=0$$

$$\beta_n=\frac{2}{5}\int_4^5\frac{5}{2n\pi}(5-x)\sin\left(\frac{n\pi x}{5}\right)dx=\frac{25\sin\left(\frac{4n\pi}{5}\right)+5n\pi\cos\left(\frac{4n\pi}{5}\right)}{n^3\pi^3}$$

$$\underline{\underline{y(x,t)=\sum_{n=1}^{\infty}\left(\frac{25\sin\left(\frac{4n\pi}{5}\right)+5n\pi\cos\left(\frac{4n\pi}{5}\right)}{n^3\pi^3}\right)\sin\left(\frac{2n\pi t}{5}\right)\sin\left(\frac{n\pi x}{5}\right)}}$$

Problem 4

【Solution】

(a)

The driving force for heat transfer is **temperature gradient**.

(b)

The Langmuir isotherm is based on the following fundamental assumptions:

(1) Adsorption is limited to monolayer coverage, with no further adsorption occurring once all surface sites are occupied.

(2) All adsorption sites are identical, and the surface is homogeneous in terms of energy distribution.

(3) The probability of adsorption at a given site is independent of the occupancy of adjacent sites.

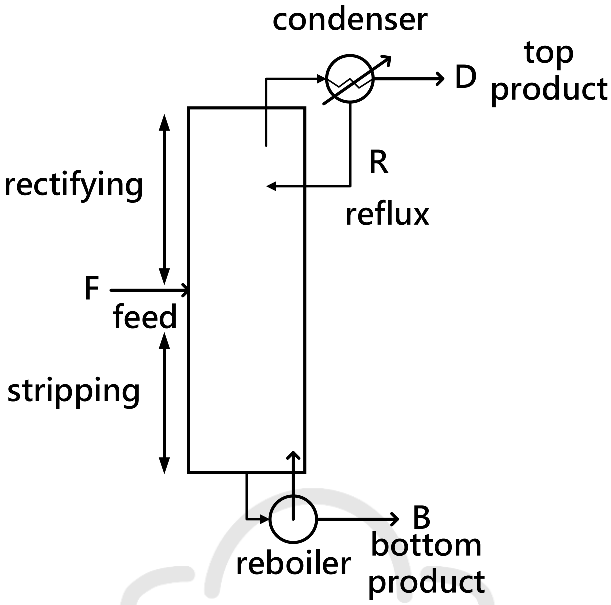
(4) No lateral interactions exist between adsorbed molecules, meaning adsorption occurs without cooperative or repulsive effects.

((b)小題可參考：Atkins, P.; De Paula, J.; Keeler, J. Atkins' *Physical Chemistry*, 9th ed.; p 889.)

Problem 5

【Solution】

(a)



(b)

$$q=\frac{heat\ needed\ to\ vaporize\ 1\ mole\ of\ feed}{molar\ latent\ heat\ of\ the\ feed}\begin{cases} q>1=cold\ feed \\ q=0=saturated\ vapor \\ q<0=superheated\ vapor \end{cases}$$

Problem 6

【Solution】

(a)

$$Nu=\frac{hL}{k_f}=\frac{convective\ heat\ transfer\ rate}{conductive\ heat\ transfer\ rate}$$

All the related parameters are related to the liquid phase.

(b)

$$Sh=\frac{\upsilon}{D_{AB}}=\frac{momentum\ diffusivity}{mass\ diffusivity}=\left(\frac{momentum\ boundary\ layer}{mass\ transfer\ boundary\ layer}\right)^n$$

(c)

By overall mass transfer coefficient in the gas phase, we have,

$$\frac{1}{K_y}=\frac{1}{k_y}+\frac{m}{k_x}$$

Due to the large solubility of SO_2 in water, we have $m<<1$

$$\frac{1}{K_y}=\frac{1}{k_y}+\cancel{\frac{m}{k_x}}\approx\frac{1}{k_y}$$

亦即<u>整體質傳由氣相控制</u>

也可使用液相總質傳係數做討論：

$$\frac{1}{K_x}=\frac{1}{mk_y}+\cancel{\frac{1}{k_x}}\approx\frac{1}{mk_y}$$

(本題與 109 台科大單操輸送 Problem 7-(c)雷同，相關討論全部可參考：Geankoplis, C. *Transport Processes and Separation Process Principles*, 4th ed.; p 594~601.)