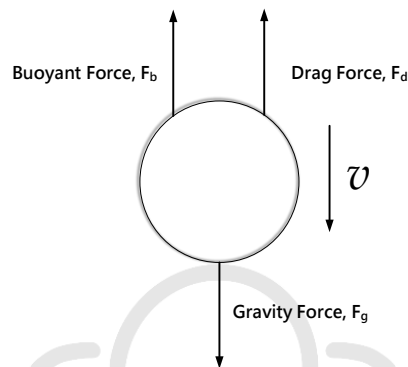# 106 年成大單操輸送

Problem 1
【Solution】

(a)

By force balance on the solid sphere,

By Stokes' law,

$$F_d = 3\pi\mu Dv$$

Because the drag force will increase as the velocity increases, the velocity will continue to increase until the force balance satisfies, which can be represented as,

$$\frac{dv}{dt} = 0 \quad , \quad v = \text{constant} = v_t$$

(b)

將 $\begin{cases} F_g = \dfrac{4}{3}\pi R^3 \rho_s g \\ F_b = \dfrac{4}{3}\pi R^3 \rho_f g \end{cases}$ 代入 Force balance equation,

$$\frac{4}{3}\pi R^3 \rho_s g = \frac{4}{3}\pi R^3 \rho_f g + 6\pi\mu R v_t$$

$$\mu = \frac{2R^2(\rho_s - \rho_f)g}{9v_t} \propto \frac{1}{v_t}$$

即在不同流體下使同樣的球體達到其相對應的終端速度，就可推算流體之黏度

Problem 2

【Solution】

(a)

$$\rho v_z v_z (2\pi r dr)\big|_z - \rho v_z v_z (2\pi r dr)\big|_{z+dz} + \tau_{rz}(2\pi r dz)\big|_r - \tau_{rz}(2\pi r dz)\big|_{r+dr}$$

$$+ p(2\pi r dr)\big|_z - p(2\pi r dr)\big|_{z+dz} + (2\pi r dr dz)\rho g_z = \frac{\partial}{\partial t}(\rho v_z \cdot 2\pi r dr dz)$$

同除 $2\pi dr dz \to 0$

$$\boxed{-\frac{\partial}{\partial z}(\rho r v_z v_z) - \frac{\partial}{\partial r}(r\tau_{rz}) - r\frac{\partial p}{\partial z} + r\rho g_z = \frac{\partial}{\partial t}(\rho r v_z)}$$

(b)

Assumptions are,

(1) Steady state

(2) Fully-developed in z-direction, $v_r = v_\theta = 0$

(3) Incompressible, Newtonian flow

By equation of continuity,

$$\cancel{\frac{\partial \rho}{\partial t}} + \frac{1}{r}\frac{\partial}{\partial r}\cancel{(\rho r v_r)} + \frac{1}{r}\frac{\partial}{\partial \theta}\cancel{(\rho v_\theta)} + \frac{\partial}{\partial z}(\rho v_z) = 0$$

$$\frac{\partial}{\partial z}(v_z) = 0 \quad, \quad v_z \neq f(z)$$

For the general differential equation,

$$-\cancel{\frac{\partial}{\partial z}(\rho r v_z v_z)} - \frac{\partial}{\partial r}(r\tau_{rz}) - r\frac{\partial p}{\partial z} + \cancel{r\rho g_z} = \cancel{\frac{\partial}{\partial t}(\rho r v_z)}$$

$$\frac{\partial}{\partial r}(r\tau_{rz}) = -r\frac{\partial p}{\partial z} \quad, \quad \tau_{rz} = -\mu\frac{dv_z}{dr}$$

$$\frac{d}{dr}\left(r\mu\frac{dv_z}{dr}\right) = r\left(\frac{\partial p}{\partial z}\right)\begin{cases} r = \dfrac{d}{2}, \ v_z = V \\[2mm] r = \dfrac{D}{2}, \ v_z = 0 \end{cases}$$

$$v_z = \frac{r^2}{4\mu}\left(\frac{\partial P}{\partial z}\right) + c_1 \ln r + c_2 \begin{cases} c_1 = \dfrac{V - \dfrac{1}{16\mu}\left(\dfrac{\partial p}{\partial z}\right)(d^2 - D^2)}{\ln \dfrac{d}{D}} \\[3em] c_2 = -\dfrac{D^2}{16\mu}\left(\dfrac{\partial p}{\partial z}\right) - \dfrac{V - \dfrac{1}{16\mu}\left(\dfrac{\partial p}{\partial z}\right)(d^2 - D^2)}{\ln \dfrac{d}{D}} \ln \dfrac{D}{2} \end{cases}$$

$$\boxed{v_z = -\frac{D^2}{16\mu}\left(\frac{\partial p}{\partial z}\right)\left[1 - \left(\frac{2r}{D}\right)^2\right] + \left[\frac{V - \dfrac{1}{16\mu}\left(\dfrac{\partial p}{\partial z}\right)(d^2 - D^2)}{\ln \dfrac{d}{D}}\right]\cdot \ln \frac{2r}{D}}$$

$$\left(\text{where, } \frac{\partial p}{\partial z} = \frac{P_{atm} - P_1}{L}\right)$$

(c)

$$F_z = \int_0^L \int_0^{2\pi} (\tau_{rz}\cdot r)\Big|_{r=\frac{D}{2}}\, d\theta\, dz$$

$$= -\left[\frac{D^2}{8}\left(\frac{\partial p}{\partial z}\right) + \frac{\mu V - \dfrac{1}{16}\left(\dfrac{\partial p}{\partial z}\right)(d^2 - D^2)}{\ln \dfrac{d}{D}}\right]\int_0^L \int_0^{2\pi} d\theta\, dz$$

$$= -2\pi L\left[\frac{D^2}{8}\left(\frac{\partial p}{\partial z}\right) + \frac{\mu V - \dfrac{1}{16}\left(\dfrac{\partial p}{\partial z}\right)(d^2 - D^2)}{\ln \dfrac{d}{D}}\right]$$

(本題可參考：**Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 65.**)

Problem 3

【**Solution**】

(a)

If there is an object with uniform temperature distribution, is can be analyzed with so-called lumped analysis by considering that the temperature is only a function of time, not position.

(b)

For us to use the lumped-analysis, the Biot number of the object should be less than 0.1
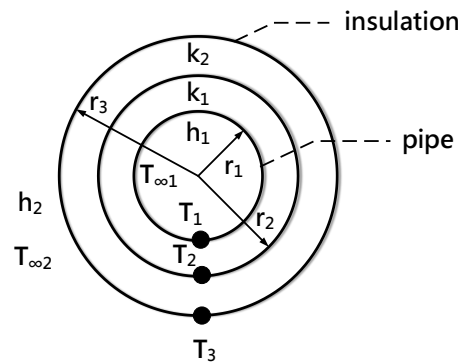
(c)

$$Bi = \frac{h_f (V/A)}{k_s} = \frac{h_f (\frac{D}{6})}{k_s} = \frac{300 \times \frac{6 \times 10^{-3}}{6}}{2} = 0.15$$

$\because Bi = 0.15 > 0.1$，***The lumped analysis is not applicable***

Problem 4

【Solution】

(a)

By Ohm's law

$$\frac{q}{L} = \frac{\Delta T}{L \sum R}$$

$$\begin{cases} \Delta T = T_{\infty 1} - T_{\infty 2} = 300 - 15 = 285 \ (K) \\ L \sum R = \dfrac{1}{2\pi r_1 h_1} + \dfrac{\ln \dfrac{r_2}{r_1}}{2\pi k_1} + \dfrac{\ln \dfrac{r_3}{r_2}}{2\pi k_2} + \dfrac{1}{2\pi h_2 r_3} \end{cases}$$

$$L \sum R = \frac{1}{2\pi \times 2.5 \times 10^{-2} \times 50} + \frac{\ln \dfrac{3}{2.5}}{2\pi \times 100} + \frac{\ln \dfrac{6}{3}}{2\pi \times 0.1} + \frac{1}{2\pi \times 6 \times 10^{-2} \times 20} = 1.3634$$

$$\frac{q}{L} = \frac{\Delta T}{L \sum R} = \frac{285}{1.3634} = \underline{\underline{209.03 \ (J / s \cdot m)}}$$

(b)

$\because$ *in series*

$$\frac{q}{L} = \frac{T_1 - T_2}{\underbrace{\dfrac{\ln \dfrac{r_2}{r_1}}{2\pi k_1}}_{pipe\ shell}} = \frac{T_2 - T_3}{\underbrace{\dfrac{\ln \dfrac{r_3}{r_2}}{2\pi k_2}}_{insulation}} = \frac{T_1 - T_2}{\dfrac{\ln \dfrac{3}{2.5}}{2\pi \times 100}} = \frac{T_2 - T_3}{\dfrac{\ln \dfrac{6}{3}}{2\pi \times 0.1}} = 209.03$$

$$Temperature\ drops\ between \begin{cases} pipe\ shell = T_1 - T_2 = 0.0606 \ (K) \\ insulation = T_2 - T_3 = 230.60 \ (K) \end{cases}$$

【**Solution**】

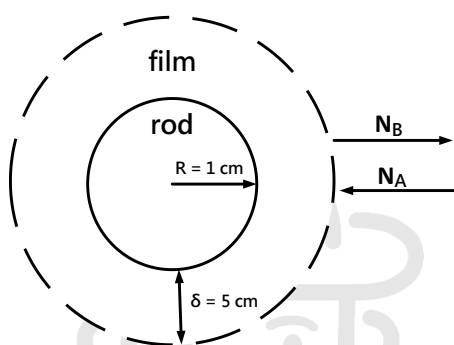**(a)**

Assumptions,

(1) Steady state along r-direction

(2) The total concentration is constant everywhere

設 $\begin{cases} A = O_2 \\ B = CO_2 \end{cases}$

By mole balance of A,

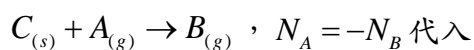$$N_{Ar} \cdot 2\pi rL\big|_r - N_{Ar} \cdot 2\pi rL\big|_{r+dr} + 0 = 0$$

同除 $2\pi Ldr \to 0$

$$\frac{d}{dr}(rN_{Ar}) = 0$$

By Fick's first law,

$$N_{Ar} = -D_{AB}C\frac{dy_A}{dr} + y_A(N_A + N_B)$$

∵ *reaction*

$$C_{(s)} + A_{(g)} \to B_{(g)} \ , \ N_A = -N_B \text{ 代入}$$

$$N_{Ar} = -D_{AB}C\frac{dy_A}{dr} \text{ 代回 differential equation}$$

$$\frac{d}{dr}\left(-rD_{AB}C\frac{dy_A}{dr}\right) = 0 \ , \ \boxed{\frac{d}{dr}\left(r\frac{dy_A}{dr}\right) = 0 \begin{cases} r = R, \ y_A = y_{As} \\ r = R+\delta, \ y_A = y_{A0} \end{cases}}$$

(b)

對(a)小題之 differential equation 積分兩次

$$y_A = c_1 \ln r + c_2 \begin{cases} c_1 = \dfrac{y_{A0} - y_{As}}{\ln(1+\dfrac{\delta}{R})} \\[4mm] c_2 = y_{A0} - \dfrac{y_{A0} - y_{As}}{\ln(1+\dfrac{\delta}{R})} \ln(R+\delta) \end{cases}$$

$$y_A = y_{A0} - \frac{y_{A0} - y_{As}}{\ln(1+\dfrac{\delta}{R})} \cdot \ln(\frac{R+\delta}{r})$$

At surface r=R, production rate of CO_2 = −diffusion rate of O_2, that is

$$N_{CO_2}\Big|_{r=R} = -N_{O_2}\Big|_{r=R}$$

$$N_{O_2} = \frac{W_{Ar}}{2\pi r L} = -D_{AB} C \frac{dy_A}{dr}$$

$$\frac{W_A}{2\pi L}\int_R^{R+\delta} \frac{dr}{r} = -D_{AB} C \int_{y_{As}}^{y_{A0}} dy_A$$

$$W_A = \frac{-2\pi L D_{AB} C}{\ln(1+\dfrac{\delta}{R})} \cdot (y_{A0} - y_{As}) \begin{cases} C = \dfrac{P}{RT} = \dfrac{3\times 1013 \times 10^2}{8.314 \times 1000} = 36.553 \ (mol/m^3) \\[3mm] R = 0.01m \\ L = 0.3m \\ \delta = 0.005m \\ y_{As} = 0 \ (diffusion\ limited) \\ D_{AB} = 1.0\times 10^{-5} \ (m^2/s) \\ y_{A0} = 0.21 \end{cases}$$

$$W_B = -W_A = \underline{\underline{3.57 \times 10^{-4} \ (mol/s)}}$$

(c)

At surface $r = R$, surface reaction rate of O_2 = diffusion rate of O_2

$$2\pi rL \cdot r_A^{"}\Big|_{r=R} = W_A\Big|_{r=R}$$

$$2\pi rL \cdot r_A^{"}\Big|_{r=R} = \frac{-2\pi LD_{AB}C}{\ln(1+\frac{\delta}{R})} \cdot (y_{A0} - y_{As}), \text{ where } r_A^{"} = -k^{"}Cy_{As}$$

$$Rk^{"}Cy_{As} = \frac{D_{AB}C}{\ln(1+\frac{\delta}{R})} \cdot (y_{A0} - y_{As}), \text{ where } k^{"} = 1.5 \times 10^{-2}(1/s)$$

$$y_{As} = 0.02965$$

故 reaction rate

$$W_A = -2\pi RLk^{"}Cy_{As}$$

$$= -2\pi(1 \times 10^{-2})(0.3)(1.5 \times 10^{-2})(36.553)(0.02965)$$

$$= -3.06 \times 10^{-4} \ (mol/s)$$

(本題可參考：**Welty, J.; Rorrer, G.; Foster, D.** *Fundamentals of Momentum, Heat, and Mass Transfer*, 5th ed.; p 494, Problem 26.27.)

Problem 6

【Solution】

(a)

By mole balance of the key component,

$$1 \cdot x_{Fi} = f \cdot y_i + (1-f)x_{Bi}$$

By equilibrium relation,

$$y_i = K_i x_{Bi}$$

Combine,

$$1 \cdot x_{Fi} = f \cdot K_i x_{Bi} + (1-f)x_{Bi}$$

$$x_{Fi} = x_{Bi}[f(K_i-1)+1]$$

$$x_{Bi} = \frac{x_{Fi}}{f(K_i-1)+1}$$

(b)

將 f 與 K 值代入 C_6, C_7, C_8 檢查

$$\begin{cases} x_{C_6} = \dfrac{0.4}{0.7(2.23-1)+1} = 0.215 \\[2mm] x_{C_7} = \dfrac{0.27}{0.7(1.04-1)+1} = 0.263 \\[2mm] x_{C_8} = \dfrac{0.33}{0.7(0.46-1)+1} = 0.531 \end{cases}$$

檢查

$$x_{C_6} + x_{C_7} + x_{C_8} \approx 1 \text{，此數據合理}$$

For composition of vapor product,

$$\begin{cases} y_{C_6} = 2.23 \times 0.215 = 0.479 \\ y_{C_7} = 1.04 \times 0.263 = 0.274 \\ y_{C_8} = 0.46 \times 0.531 = 0.244 \end{cases}$$

(c)

設參考溫度為 105°C = 378K

(此種假設就不必計算出口之液態熱量，因為出口液體溫度也為 105°C)

By energy balance, $in - out + \cancel{gen} = \cancel{accu}$

$$1 \cdot \sum x_{F_i} C_{pi}(T - 378) = f \cdot \sum_i y_i \Delta H_{vi}$$

$$[(0.4 \times 62) + (0.27 \times 70) + (0.33 \times 78)](T - 378)$$

$$= 0.7[(0.479 \times 6370) + (0.274 \times 7510) + (0.244 \times 8560)]$$
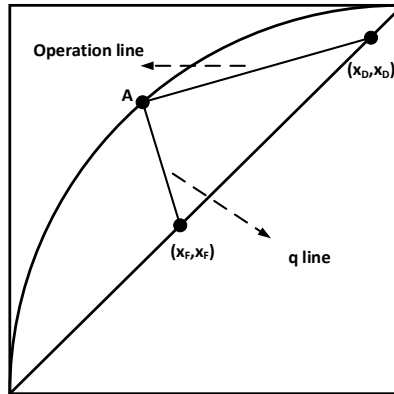
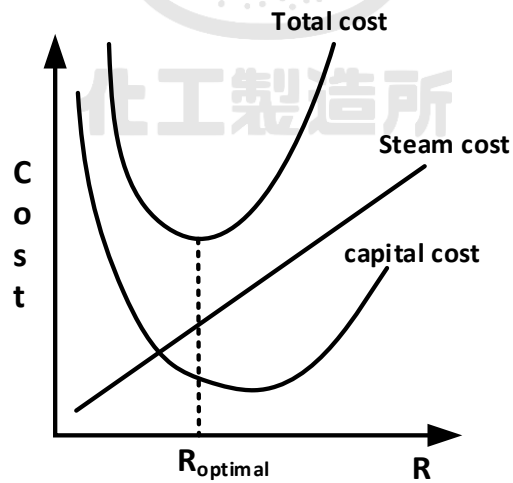$$\underline{\underline{T = 450.56 \ (K)}}$$

Problem 7

【Solution】

(a)

先點出塔頂產物點 (x_D, x_D)，將操作線與 q-line(可從進料狀況知道斜率，且通過 (x_F, x_F))及平衡線之交點(A)連線，即可得到上段操作線，而操作線斜率為

$\dfrac{R}{R+1}$，此時之 $R = R_{min}$。

(b)

(本題可參考：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*,

5th ed. Volume 2; p 571.)

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Problem 1
【Solution】

(a)

At the surface, $r = R$ and at $\theta = 45°$

$$\vec{v} = (R^3 w / R^2)\sin(45°)\vec{e_\phi} = \frac{\sqrt{2}}{2}wR\vec{e_\phi}$$

(b)

$$\begin{cases} v_\theta = 0 \\ v_\phi = (\dfrac{R^3 w}{r^2})\sin\theta \ 代入 \ \tau_{ij} \\ v_r = 0 \end{cases}$$

$$\tau_{rr} = \tau_{\theta\theta} = \tau_{\phi\phi} = \tau_{r\theta} = \tau_{\theta r} = \tau_{\phi\theta} = \tau_{\theta\phi} = 0$$

$$\tau_{r\phi} = \tau_{\phi r} = \mu[\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi} + r\frac{\partial}{\partial r}(\frac{v_\phi}{r})]$$

$$= R^3 w\mu\sin\theta(-3\frac{1}{r^3})$$

$$= -3w\mu\sin\theta(\frac{R}{r})^3$$

(c)

According to (b)

$$-\tau_{r\phi}\Big|_{\theta=45°} = \frac{3\sqrt{2}}{2}w\mu(\frac{R}{r})^3$$

The above relation shows that the $\phi-direction$ momentum is transferred in the r-direction.

(d)

$$vorticity = \nabla \times \vec{v}\Big|_{r=R} =$$

$$\left(\left(\frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(v_\phi\sin\theta) - \frac{\partial v_\phi}{\partial\phi}\right]\right)\vec{e}_r + \left(\frac{1}{r\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{1}{r}\frac{\partial}{\partial r}(rv_\phi)\right)\vec{e}_\theta + \frac{1}{r}\left(\frac{\partial}{\partial r}(rv_\theta) - \frac{\partial v_r}{\partial\theta}\right)\vec{e}_\phi\right)\Bigg|_{r=R}$$

$$= \frac{1}{r\sin\theta}\left[(R^3 w / r^2)\cdot 2\sin\theta\cos\theta]\vec{e}_r + [(R^3 w / r^3)\sin\theta]\vec{e}_\theta + 0\right|_{r=R}$$

$$= 2w\left(\frac{R}{r}\right)^3\cos\theta\vec{e}_r + w\left(\frac{R}{r}\right)^3\sin\theta\vec{e}_\theta\Bigg|_{r=R}$$

$$= \underline{\underline{2w\cos\theta\vec{e}_r + w\sin\theta\vec{e}_\theta}}$$

Problem 2

【Solution】

(a)

By Navier-Stokes equation in θ-direction,

$$\rho(\cancel{\frac{\partial v_\theta}{\partial t}} + \cancel{v_r \frac{\partial v_\theta}{\partial r}} + \frac{v_\theta}{r}\cancel{\frac{\partial v_\theta}{\partial \theta}} + \cancel{\frac{v_\theta v_r}{r}} + \cancel{v_z \frac{\partial v_\theta}{\partial z}})$$

$$= \cancel{\frac{-1}{r}\frac{\partial p}{\partial \theta}} + \cancel{\rho g_\theta} + \mu[\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)) + \cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}} + \cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}} + \cancel{\frac{\partial^2 v_\theta}{\partial z^2}}]$$

$$\mu[\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta))] = 0$$

積分兩次

$$v_\theta = \frac{c_1}{r} + c_2 r$$

(b)

$$\begin{cases} r = R_i, \ v_\theta = wR_i \\ r = R_o, \ v_\theta = 0 \end{cases} \quad \text{代入並解出 } c_1 \text{ 與 } c_2$$

$$\begin{cases} c_1 = \dfrac{w}{(\dfrac{1}{R_i^2} - \dfrac{1}{R_o^2})} = \dfrac{(\dfrac{4000}{60} \times 2\pi)}{[\dfrac{1}{(1.7\times 10^{-2})^2} - \dfrac{1}{(1.8\times 10^{-2})^2}]} = 1.121 \\[4mm] c_2 = \dfrac{w}{(1 - \dfrac{R_o^2}{R_i^2})} = \dfrac{(\dfrac{4000}{60} \times 2\pi)}{[1 - (\dfrac{1.8\times 10^{-2}}{1.7\times 10^{-2}})^2]} = -3458.74 \end{cases}$$

$$v_\theta = \frac{1.121}{r} - 3458.74r$$

(c)

$$|T_z| = \left|\int_0^L \int_0^{2\pi} (\tau_{r\theta} \cdot r) r\,d\theta dz\right|, \quad |\tau_{r\theta}| = \left|-\mu[r\frac{\partial}{\partial r}(\frac{v_\theta}{r}) + \cancel{\frac{1}{r}\frac{\partial v_r}{\partial \theta}}]\right| = \left|\frac{-2.24\mu}{r^2}\right|$$

$$|T_z| = 2.24\mu\int_0^L \int_0^{2\pi} d\theta dz = 4.48\mu\pi L$$

$$0.05 = 4.48\mu\pi \times 36\times 10^{-3}, \quad \mu = 0.098 \ (kg/m\cdot s)$$

Problem 3

【Solution】

(a)

$$Nu = \frac{h_f L}{k_f} = \frac{convection\ heat\ transfer}{conduction\ heat\ transfer}\ in\ the\ radial\ direction$$

(b)

A lumped-analysis system is system with uniform temperature distribution, is can be analyzed by considering that the temperature is only a function of time, not position. Usually, if the Biot number of the system <0.1, we can use lumped-parameter analysis.

(c)

After the nucleate boiling regime, a vapor film forms around the wire which provides a considerable resistance to heat transfer； thus, the heat flux decreases
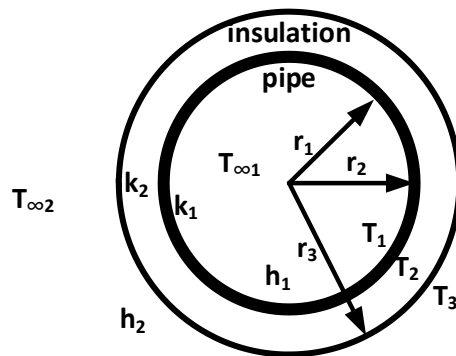
(d)

Heat exchanger effectiveness is defined as,

$$\varepsilon = \frac{actual\ heat\ transfer\ rate}{heat\ transfer\ rate\ with\ infinite\ heat\ transfer\ area}$$

(e)

The radiant heat flux from a blackbody is proportional to the fourth power of the absolute temperature at its surface

$$\frac{q}{A} = \sigma T^4$$

Problem 4
【Solution】

(a)

For conduction rate, by Fourier's first law,

$$q = -kA\frac{dT}{dr} \ \ (for\ cylindrical\ coordinate)$$

$$q = -k(2\pi rL)\frac{dT}{dr} \ , \ \ \frac{q}{2\pi L}\int_{r_i}^{r_o}\frac{dr}{r} = -k\int_{T_i}^{T_o} dT$$

$$\frac{q}{2\pi L}\ln\frac{r_o}{r_i} = k(T_i - T_o)$$

$$q = \frac{T_i - T_o}{\dfrac{\ln(\dfrac{r_o}{r_i})}{2\pi kL}}$$

For convection rate, by Newton's law of cooling,

$$q = hA(T_i - T_o) = \frac{T_i - T_o}{\dfrac{1}{2\pi rLh}}$$

For heat transfer in series, the heat transfer rate is the same everywhere in the system whether by conduction or convection, that is

$$q = \frac{T_{\infty,1} - T_{\infty,2}}{\sum R} = \frac{T_{\infty,1} - T_1}{\dfrac{1}{2\pi r_1 L h_1}} = \frac{T_1 - T_2}{\dfrac{\ln \dfrac{r_2}{r_1}}{2\pi k_1 L}} = \frac{T_2 - T_3}{\dfrac{\ln \dfrac{r_3}{r_2}}{2\pi k_2 L}} = \frac{T_3 - T_{\infty,2}}{\dfrac{1}{2\pi r_3 L h_2}}$$

$$又 \ T_{\infty,1} - T_{\infty,2} = (T_{\infty,1} - T_1) + (T_1 - T_2) + (T_2 - T_3) + (T_3 - T_{\infty,2})$$

$$\boxed{\sum R = \frac{1}{2\pi r_1 L h_1} + \frac{\ln \dfrac{r_2}{r_1}}{2\pi k_1 L} + \frac{\ln \dfrac{r_3}{r_2}}{2\pi k_2 L} + \frac{1}{2\pi r_3 L h_2}}$$

(b)

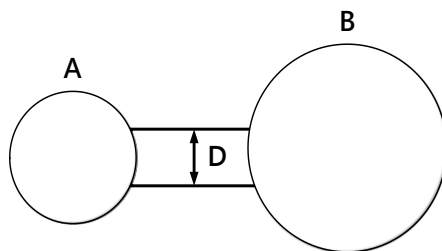For critical radius of insulation, let

$$\frac{\partial}{\partial r_3}\left(\sum R\right) = 0$$

$$\frac{1}{2\pi k_2 L r_3} + \frac{1}{2\pi L h_2}\frac{-1}{r_3^2} = 0$$

$$\underline{\underline{r_3 = \frac{k_2}{h_2}}} = critical \ radius \ of \ insulation$$

Problem 5

【Solution】

設 $A = N_2$, $B = O_2$

(a)

Assumptions：

(1) Steady-state

(2) Total concentration and total pressure is constant

(3) No reaction between A and B

(4) There is only z-direction diffusion needed to be considered

(5) ∵The difference of the partial pressure of A and B is the same, and they are in

opposite direction => A and B are in equimolar counter diffusion.

By mole balance of A

$$(N_{Az} \cdot \pi r^2)\big|_z - (N_{Az} \cdot \pi r^2)\big|_{z+dz} + 0 = 0$$

同除 $\pi r^2 dz \to 0$

$$\frac{d}{dz}(N_{Az}) = 0$$

By Fick's first law,

$$N_{Az} = -D_{AB}C\frac{dy_A}{dz} + y_A \underbrace{(N_A + N_B)}_{equimolar\ counter\ diffusion} = -D_{AB}C\frac{dy_A}{dz}$$

$\because N_{Az} \neq f(z)$，兩端可直接積分：

$$N_{Az} \int_0^L dz = -D_{AB} C \int_{y_{A0}}^{y_A} dy_A$$

$$N_{Az} = \frac{D_{AB} C}{L}(y_{A0} - y_A) = \frac{D_{AB} P}{LRT}(y_{A0} - y_A) = \underline{\underline{\frac{D_{AB}}{LRT}(P_{A0} - P_A)}}$$

(b)

$$W_{Az} = \frac{\pi}{4} D^2 \cdot N_{Az} = \frac{\pi}{4} D^2 \cdot \frac{D_{AB} P}{LRT}(y_{A0} - y_A)$$

$$= \frac{\pi}{4}(0.1)^2 \cdot \frac{2.0 \times 10^{-5} \times 1.013 \times 10^{-5}}{8.314 \times 298 \times 2}(0.8 - 0.15)$$

$$= \underline{\underline{2.09 \times 10^{-6} \ (mol/s)}}$$

(c)

$$N_{Az} = \frac{D_{AB} P}{RTL}(y_{A0} - y_A) = \frac{D_{AB} C}{L}(y_{A0} - y_A) = \frac{D_{AB}}{L}(C_{A0} - C_A)$$

$$= k(C_{A0} - C_A)$$

$$k = \frac{D_{AB}}{L} = \frac{2.0 \times 10^{-5}}{2} = \underline{\underline{1.0 \times 10^{-5} \ (m/s)}}$$

Problem 6

【Solution】

(a)

By energy balance,

$$H(T) - H(T_s) + \underset{adiabatic}{0} = 0 \text{ , } H(T) = H(T_s)$$

Set the reference temperature as T_s

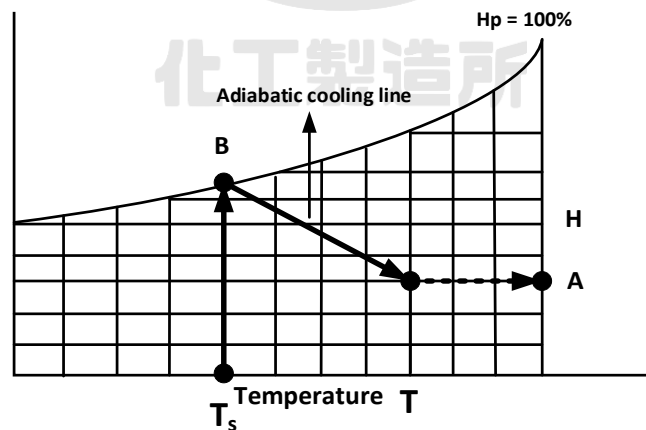$$C_s(T - T_s) + \mathcal{H}\lambda_s = C_s(T_s - T_s) + \mathcal{H}_s\lambda_s$$

$$\boxed{\frac{C_s}{\lambda_s} = \frac{\mathcal{H}_s - \mathcal{H}}{T - T_s}}$$

(b)

If $Pr \approx Sc$ throughout the whole process, (usually in air-water system), adiabatic

saturation line will be the same of wet bulb temperature line. Therefore,
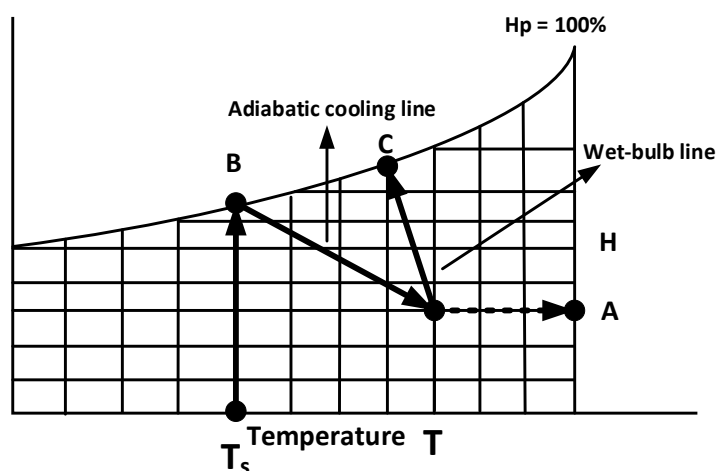
$$T_s \approx T_{wbt}$$

And the humidity of inlet gas can be found by the following procedures,

(1) 於橫軸上定好T_s後，往上對至$H_p = 100\%$的曲線，交點在 B

(2) 從 B 點出發，延 adiabatic cooling line 往下走至溫度=T

(3) 於該點向右讀出 Humidity 為 A。

(2)

若 $\Pr \neq Sc$，則 adiabatic saturation line 與 wet bulb temperature line 不重合，則

For wet-bulb temerature,

(1) 於橫軸上定好 T_s 後，往上對至 $H_p = 100\%$ 的曲線，交點在 B

(2) 從 B 點出發，延 adiabatic cooling line 往下走至溫度=T

(3) 延 wet-bulb line (psychrometric line)對至 humidity 為 100%處，即為 wet bulb temperature。(點 C)

※定義來說，**adiabatic saturation temperature** 為經過熱力學平衡並吸濕後，所能達到的最低溫度，此為一平衡狀態；而 **wet bulb temperature** 則為 **steady state**，即大量氣體通過濕球時，濕球上液體氣化，並且達到與氣體同樣溫度所需吸收之熱與氣體所放出之潛熱所得到的溫度，有定義上的差別。一般來說，**wet bulb temperature** 可以用以下關聯計算：

$$\frac{H - H_{wb}}{T - T_{wb}} = -\frac{h_y}{M_B k_y \lambda_w} = \frac{-c_p}{\lambda}(\frac{Sc}{\Pr})^m$$

若(1) $\Pr \approx Sc$，則

$$\frac{H - H_{wb}}{T - T_{wb}} = \frac{-c_p}{\lambda}$$

若(2)系統為 **air-water system**，則：

$$\frac{H - H_{wb}}{T - T_{wb}} = \frac{-c_s}{\lambda}$$

(與 adiabatic cooling line 相同)

結合(1)與(2)兩個條件可推得:

$$\frac{h_y}{M_B k_y} \approx -c_s$$

即為 Lewis relation,滿足時代表 wet-bulb line 與 adiabatic cooling line 重合

另外,關於 wet bulb temperature 的找法,可參考 McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 627~628.。即與 adiabatic saturation temperature 相同的做法,是 wet bulb temperature 與 100 percent line 的交點,且除了 air-water system 以外,幾乎所有 air-organic solvents 的系統之 wet bulb line 斜率都比 adaibatic saturation line 高。
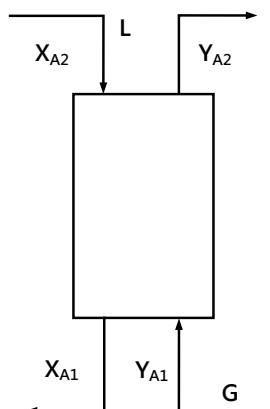
(Adiabatic saturation line 尋找絕熱飽和溫度過程可參考:McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 622~623.

Wet bulb temperature 與 adiabatic saturation temperature 的定義與差別可參考同本書

p 619/624~625.)

【**Solution**】

由題意：

$$
\begin{cases}
y_{A1} = 0.5 \rightarrow Y_{A1} = \dfrac{0.5}{1-0.5} = 1 \\[2mm]
x_{A2} = 0.01 \rightarrow X_{A2} = \dfrac{0.01}{1-0.01} = 0.01 \\[2mm]
y_{A2} = \dfrac{0.5 \times 0.04}{0.5 \times 0.04 + 0.5} = 0.0384 \rightarrow Y_{A2} = \dfrac{0.0384}{1-0.0384} = 0.04
\end{cases}
$$

For minimum flow rate of the entering oil, the operating line should intersect with the equilibrium line

$$
y_{A1} = 2x_{A1} = 0.5 \text{，} \quad x_{A1} = 0.25 \rightarrow X_{A1} = \dfrac{0.25}{1-0.25} = 0.333
$$

By the operating line,

$$
\frac{Y_{A1} - Y_{A2}}{X_{A1} - X_{A2}} = \frac{L^{'}}{G^{'}}
$$

$$
\frac{1-0.04}{0.333-0.01} = \frac{L_a \times 0.99}{100 \times 0.5} \text{，} \quad \underline{\underline{L_a = 150 \ (mol/s)}}
$$

At the minimum flow rate, ***the number of ideal plates is infinity***

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Problem 1

【Solution】

(a)

$$Re \equiv \frac{inertial\ force}{viscous\ force}$$

(b)

$$\begin{cases} I : viscous\ force \\ IV: inertial\ force \end{cases}$$

(c)

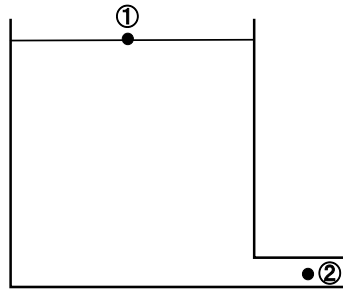$$C_D = \frac{viscous\ force}{inertial\ force} \propto \frac{1}{Re}$$

(本題可參考：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer*, 5th ed.; p 138-141.)

Problem 2

【Solution】

(a)

By mechanical energy balance between 1 and 2,

$$\cancel{W_s} = \frac{1}{2g_c}(\cancel{v_1^2} - v_2^2) + \frac{g}{g_c}\underbrace{(z_1 - z_2)}_{H} + \frac{1}{\rho}\cancel{(P_1 - P_2)}_{P_1 = P_2 = P_0} - \cancel{h_f}$$

By mass balance,

$$\rho A_1 v_1 = \rho A_2 v_2 \quad , \quad \because A_1 \gg A_2 \quad , \quad v_1 \ll v_2$$

$$\frac{v_2^2}{2g_c} = \frac{g}{g_c}H \quad , \quad v_2 = <v> = \underline{\underline{\sqrt{2gH}}}$$

(b)

By pseudo-steady state assumption,

$$-\rho A_h <v> = \frac{d(\rho A_c H)}{dt} \begin{cases} A_h 為出口端截面積 \\ A_c 為上端開口面積 \end{cases}$$

$$A_h \sqrt{2gH} = A_c \frac{dH}{dt}$$

$$-\int_0^t \frac{A_h}{A_c}\sqrt{2g}\,dt = \int_{H_1}^{H_2} \frac{1}{\sqrt{H}}\,dH$$

$$\boxed{t = \frac{2(\sqrt{H_1} - \sqrt{H_2})A_c}{A_h\sqrt{2g}}}$$

(c)

By mass balance

$$\rho a \cdot Ca\sqrt{2gH} = \rho(\pi r^2)u \quad , \quad \boxed{H = \frac{\pi^2 r^4 u^2}{2C^2 a^4 g}}$$
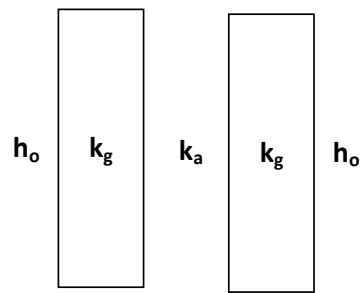
(本題改編自：Nakayama Y. *Introduction to Fluid Mechanics*, 2nd ed; p 66~68.)

另原文書在此題假設速率為 $C\sqrt{2gH}$ ，最終答案會有所不同

$$\boxed{H = (\frac{\pi v}{Ca\sqrt{2g}})^2 r^4}$$

【Solution】

$$q = \frac{\Delta T}{\sum R} \text{ , } \Delta T = 300 - 250 = 50 \ (K)$$

$$\sum R = \frac{1}{h_i A} + \frac{L_g}{k_g A} + \frac{L_a}{k_a A} + \frac{L_g}{k_g A} + \frac{1}{h_o A}$$

$$= \frac{1}{25 \times 2 \times 4} + \frac{0.5 \times 10^{-2}}{0.78 \times (2 \times 4)} \times 2 + \frac{1 \times 10^{-2}}{0.026 \times (2 \times 4)} + \frac{1}{20 \times (2 \times 4)}$$

$$= 0.0609$$

$$q = \frac{50}{0.0609} = \underline{\underline{820.62 \ (J \ / \ s)}}$$

Problem 4

【Solution】

(a)

$$B.C. \begin{cases} x = 0, \ T = T_s \\ x = \delta, \ T = T_0 \\ x = \delta, \ \dfrac{\partial T}{\partial x} = 0 \end{cases}$$

(b)

將三個 B.C.代入 $T(x)$

$$\begin{cases} T_s = A + B \cdot 0 + C \cdot 0^2 = A \\ T_0 = T_s + B\delta + C\delta^2 \\ 0 = B + 2C\delta \end{cases} \rightarrow \begin{cases} A = T_s \\ B = \dfrac{-2(T_s - T_0)}{\delta} \\ C = \dfrac{T_s - T_0}{\delta^2} \end{cases}$$

$$\boxed{T(x) = T_s - \frac{2(T_s - T_0)}{\delta} x + \frac{(T_s - T_0)}{\delta^2} x^2}$$

(c)

By energy balance

$$q''\big|_x - q''\big|_{x+dx} = \frac{\partial}{\partial t}(\rho C_p T dx)$$

同除 $dx \to 0$

$$\frac{-dq''}{dx} = \frac{\partial}{\partial t}(\rho C_p T) \quad , \quad -\int_{q''}^{0} dq'' = \rho C_p \frac{\partial}{\partial t} \int_0^L T dx$$

$$q'' = \rho C_p \frac{\partial}{\partial t} \int_0^L T dx$$

$$= \rho C_p \frac{\partial}{\partial t} [\int_0^\delta T dx + \int_\delta^L T_0 dx]$$

$$= \rho C_p \frac{\partial}{\partial t} [\int_0^\delta T dx + T_0(L - \delta)]$$

$$= \underline{\underline{\int_0^\delta \rho C_p \frac{\partial T}{\partial t} dx - T_0 \rho C_p \frac{\partial \delta}{\partial t}}}$$

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(d)

$$q''\Big|_{x=0} = \frac{q}{A}\Big|_{x=0} = -k\frac{dT}{dx}\Big|_{x=0} = \int_0^\delta \rho C_p \frac{\partial T}{\partial t}dx - T_0\rho C_p \frac{\partial \delta}{\partial t}$$

將 $T(x) = T_s - \frac{2(T_s - T_0)}{\delta}x + \frac{(T_s - T_0)}{\delta^2}x^2$ 代入

$$k \cdot \frac{2(T_s - T_0)}{\delta} = \frac{1}{3}\int_0^\delta \rho C_p[T_s - \frac{2(T_s - T_0)}{\delta}x + \frac{(T_s - T_0)}{\delta^2}x^2]dx - T_0\rho C_p \frac{\partial \delta}{\delta t}$$

$$k \cdot \frac{2(T_s - T_0)}{\delta} = \frac{1}{3}\frac{d\delta}{dt}(T_s - T_0)\rho C_p$$

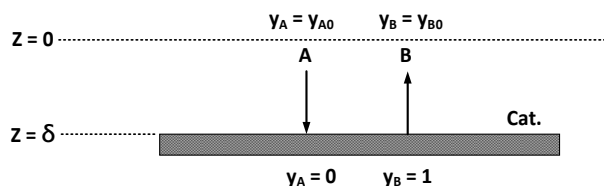$$6\frac{k}{\rho C_p} = \delta\frac{d\delta}{dt} = 6\alpha \text{ , } 6\alpha dt = \delta d\delta$$

$$6\alpha t = \frac{1}{2}\delta^2 \text{ , } \underline{\underline{\delta = \sqrt{12\alpha t}}}$$

(本題可參考：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass*

transfer, **5th ed.; p 266~268.)**

Problem 5
【Solution】

Assume：instantaneous reaction upon reaching the catalytic surface

(a)

By shell balance of B

$$N_B A_c \big|_z - N_B A_c \big|_{z+dz} - k_1 y_B A_c dz = 0$$

同除 $A_c dz \rightarrow 0$

$$\frac{-dN_B}{dz} - k_1 y_B = 0 \; ----(1) \begin{cases} z = 0, \; y_B = y_{B0} \\ z = \delta, \; y_B = 1 \end{cases}$$

By Fick's law

$$N_B = -D_{AB} C \frac{dy_B}{dz} + y_B (N_A + N_B)$$

∵ *from the heterogeneous reaction at interface*

$$A \rightarrow B \; , \; N_A = -N_B \; 代入上式$$

$$N_B = -D_{AB} C \frac{dy_B}{dz} \; 代入 (1)$$

$$D_{AB} C \frac{d^2 y_B}{dz^2} - k_1 y_B = 0$$

$$\frac{d^2 y_B}{dz^2} - \frac{k_1}{D_{AB} C} y_B = 0 \; , \; 令 \; \frac{k_1}{D_{AB} C} = m^2$$

$$\frac{d^2 y_B}{dz^2} - m^2 y_B = 0 \quad , \quad y_B = c_1 \cosh mz + c_2 \sinh mz \begin{cases} c_1 = y_{B0} \\ c_2 = \dfrac{1 - y_{B0} \cosh m\delta}{\sinh m\delta} \end{cases}$$

$$\boxed{y_B = y_{B0} \cosh mz + (1 - y_{B0} \cosh m\delta) \frac{\sinh mz}{\sinh m\delta}} \quad , \quad m = \sqrt{\frac{k_1}{D_{AB} C}}$$

(b)

$$\because N_A = -N_B = D_{AB}C \frac{dy_B}{dz}\bigg|_{z=0}$$

$$D_{AB}C \frac{dy_B}{dz}\bigg|_{z=0} = \frac{D_{AB}Cm(1-y_{B0}\cosh m\delta)}{\sinh m\delta} \quad , \quad m = \sqrt{\frac{k_1}{D_{AB}C}}$$

(c)

By shell balance of A

$$N_A A_c\big|_z - N_A A_c\big|_{z+dz} + (k_1 y_B - k_1' y_A)A_c dz = 0$$

同除 $A_c dz \rightarrow 0$

$$\frac{-dN_A}{dz} + (k_1 y_B - k_1' y_A) = 0 \text{----}(2) \begin{cases} z=0, \ y_A = y_{A0} \\ z=\delta, \ y_A = 0 \end{cases}$$

By Fick's law

$$N_A = -D_{AB}C \frac{dy_A}{dz} + y_A(N_A + N_B)$$

$\because$ *reaction*

$$A \rightleftharpoons B \ , \ N_A = -N_B \text{ 代入上式}$$

$$N_A = -D_{AB}C \frac{dy_A}{dz} \ , \ \text{且} \ y_B = 1 - y_A \text{ 代入}(2)$$

$$D_{AB}C \frac{d^2 y_A}{dz^2} + k_1 - (k_1 + k_1')y_A = 0$$

$$\frac{d^2 y_A}{dz^2} + \frac{k_1}{D_{AB}C} - \frac{(k_1 + k_1')}{D_{AB}C}y_A = 0 \begin{cases} \dfrac{k_1}{D_{AB}C} = m \\ \dfrac{k_1 + k_1'}{D_{AB}C} = n^2 \end{cases}$$

$$\frac{d^2 y_A}{dz^2} - n^2 y_A = -m$$

$$\boxed{y_A = c_1 \cosh nz + c_2 \sinh nz + \frac{m}{n^2} \begin{cases} c_1 = y_{A0} - \dfrac{m}{n^2} \\ c_2 = \dfrac{-\dfrac{m}{n^2} - \left(y_{A0} - \dfrac{m}{n^2}\right)\cosh n\delta}{\sinh n\delta} \end{cases}} \begin{cases} \dfrac{k_1}{D_{AB}C} = m \\ \dfrac{k_1 + k_1'}{D_{AB}C} = n^2 \end{cases}$$

(本題改編自：**Welty, J.; Rorrer, G.; Foster, D.** *Fundamentals of Momentum, Heat, and Mass Transfer*, **5th ed.; p 495, Problem 26.29/26.30.**)

Problem 6

【Solution】

(a)

Inlet air：$T_w = 75°F$, $T_{yb} = 90°F$ => $\begin{cases} \lambda_1 = 1075 \ (Btu/lb)(參考原文書代定值) \\ C_{s1} = 0.245 \ (Btu/°F \cdot lb \ dry \ air) \\ H_1 = 0.018 \ (lb \cdot water/air) \end{cases}$

Outlet air：$T_w = 90°F$, $T_{yb} = 100°F$ => $\begin{cases} \lambda_2 = 1075 \ (Btu/lb)(參考原文書代定值) \\ C_{s2} = 0.251 \ (Btu/°F \cdot lb \ dry \ air) \\ H_2 = 0.028 \ (lb \cdot water/air) \end{cases}$

Inlet water：$T_{xa} = 100°F$, $\lambda_3 = 1075 \ (Btu/lb)(參考原文書代定值)$

By energy balance

$$空氣吸熱 = 水放熱$$

$$G_y\{[C_{s2}(T_{yb2} - T_{ref}) + H_2\lambda_2] - [C_{s1}(T_{ya1} - T_{ref}) + H_1\lambda_1]\} = G_x C_L(T_{xa} - T_{xb})$$

$$C_L = the \ specific \ heat \ of \ water, \ assumed \ to \ be \ 1$$

令 $T_{ref} = 32°F$ 代入

$$1000[0.251(100-32) + 0.028 \times 1075 - 0.245(90-32) - 0.018 \times 1075] = 1100(100-T)$$

$$\underline{\underline{T = 87.63°F}}$$

(b)

水可被降溫至最低 75°F(與空氣入口溫相同，就不會再繼續被降溫)

此時之 water flow rate 為
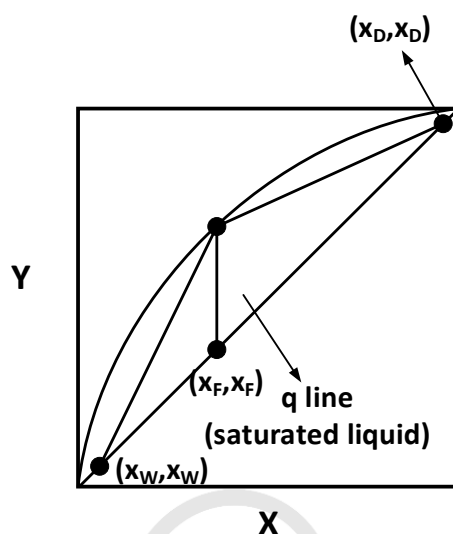
$$1000[0.251(100-32) + 0.028 \times 1075 - 0.245(90-32) - 0.018 \times 1075] = G_x(100-75)$$

$$G_x = \underline{\underline{544.32 \ (lb/h \cdot ft^2)}}$$

(本題改編自：**McCabe, W.; Smith, J.; Harriott, P.** *Unit Operations of Chemical Engineering*, **7th ed.; p 637~638, Example 19.3.)**

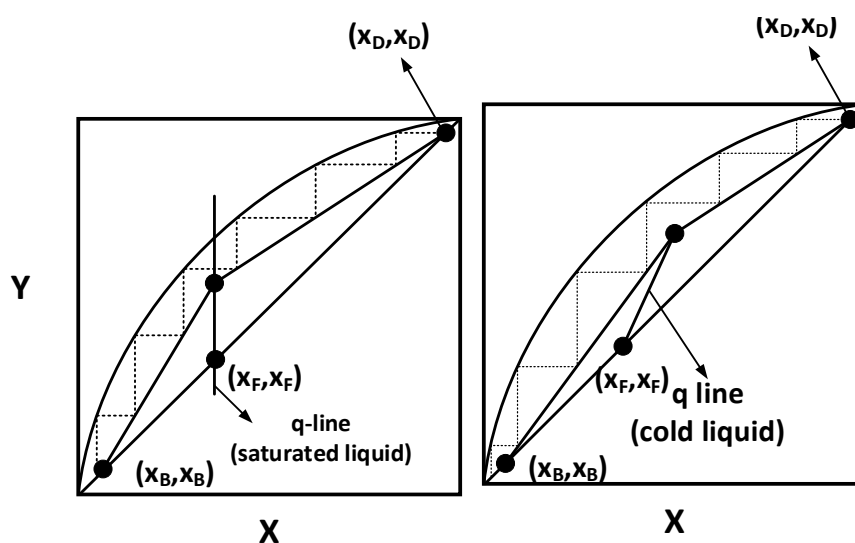【**Solution**】

(a)

(1) 首先定出 saturated liquid 所代表之 q-line，即通過(x_F, x_F)之垂直線

(2) 將 q-line 與平衡線之交點往(x_D, x_D)連線，即可求出上段操作線

(3) 由上段操作線之斜率＝$\dfrac{R}{R+1}$，即可求出此時之 reflux ratio，為 minimum reflux ratio.

(b)

(g) 將 saturated liquid feed 改為 sub-cooled liquid feed，**板數變少**(雖然最後都是

回答大約 5+1 板，但很明顯可以看出 sub-cooled liquid 最後一段超出 x_w 許

多。)

原因為：

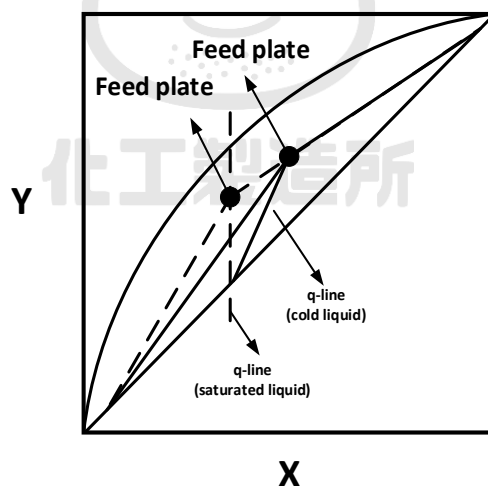① Feed line 向右傾斜，因而導致 rectifying section 板數變少。

② 而主要原因為塔內整體液體量變多，導致 stripping section 之操作線斜率變低，

從而導致板數也變少。

③進料熱焓相對降低，造成溫度降低，導致分離效果變好，因而只需要較少理論

板做分離。

(有關 q line 斜率對理論板數的影響，可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit*

*Operations of Chemical Engineering***, 7th ed.; p 683~686, Example 21.2-(b).)**

(ii)

如圖，由原本之 saturated liquid feed 改為 sub-cooled liquid feed，則 **feed plate 之**

位置將上升

(3)

If water is used as the cooling medium in the condenser and the condensate is not subcooled, then

$$\dot{m}_w = \frac{V\lambda}{\Delta T \cdot c_{pw}} \begin{cases} V = vapor\ rate\ at\ the\ top\ plate \\ \lambda = molar\ latent\ heat\ of\ the\ mixture \\ \Delta T = temperature\ difference\ of\ water \\ c_{pw} = heat\ capacity\ of\ water \end{cases}$$

冷卻水量將**不變**

(4)

If saturated steam is used as the heating medium, then

$$\dot{m}_s = \frac{\bar{V}\lambda}{\lambda_s} \begin{cases} \bar{V} = vapor\ rate\ from\ reboiler \\ \lambda = molar\ latent\ heat\ of\ the\ mixture \\ \lambda_s = latent\ heat\ of\ steam \end{cases}$$

$\because \bar{V} = V - (1-q)F$ ， $q\uparrow$ ， $\bar{V}\uparrow$

再沸器所需要之蒸氣量將**上升**

(有關進料狀態在固定其他狀況下，對此二者之影響，可參考：**McCabe, W.; Smith, J.; Harriott, P.** *Unit Operations of Chemical Engineering*, **7th ed.; p 683~686, Example 21.2-(c)(d).**)

(5)

蒸餾塔之能量提供來源為：(1)再沸器蒸氣↑ ＋(2) preheater 能量損耗↓

綜合起來**總能量損耗量改變不多**

(6)

The reasons for preheating the feed, in most cases, are to ***keep the vapor flow rate about the same in both sections*** of the column and to ***make use of the energy in a hot-liquid stream such as the bottom product***.

(此小題可參考：**McCabe, W.; Smith, J.; Harriott, P.** *Unit Operations of Chemical Engineering*, **7th ed.; p 683~686, Example 21.2-(d)之最後討論.**)

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Problem 1

【Solution】

(a)

$$C_D = \frac{F_D / A_P}{\rho v_\infty^2 / 2} = \frac{drag\ force}{inertial\ force}$$

(b) For sphere, $F_D = 3\pi\mu R v_\infty$

$$A_P = \frac{\pi D_P^2}{4}\ (投影面積)$$

$$C_D = \frac{6\pi\mu R v_\infty \times 4}{\pi D_P^2} \times \frac{2}{\rho v_\infty^2} = \frac{24\mu}{\rho v_\infty D_P} = \frac{24}{Re_D}$$

(本題可參考：Geankoplis, C. *Transport Processes and Separation Process Principles*, 4th ed.;

p 116.)

Problem 2

【**Solution**】

(a)

∵ *incompressible*

$$\nabla \cdot \vec{v} = 0 = \frac{1}{r}\frac{\partial(rv_r)}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta} = \frac{1}{r}\frac{\partial v_r}{\partial r} + \frac{\partial v_r}{\partial r} + \frac{1}{r}\frac{\partial v_\theta}{\partial \theta}$$

而根據 Stream Function 定義，可寫出一函數 ψ 使得

$$\begin{cases} v_r = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \\ v_\theta = -\frac{\partial \psi}{\partial r} \end{cases}$$

Define *vorticity*,

$$\vec{\omega} = 2\omega_z = \underbrace{[\nabla \times \vec{v}]_z}_{irrotational = 0} = [\frac{\partial v_\theta}{\partial r} + \frac{v_\theta}{r} - \frac{\partial v_r}{\partial \theta}] = -[\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r^2}\frac{\partial \psi}{\partial \theta} + \frac{1}{r}\frac{\partial^2 \psi}{\partial \theta^2}] = -\nabla^2 \psi$$

$$\boxed{\nabla^2 \psi = 0}$$

(本題可參考：**Welty, J.; Rorrer, G.; Foster, D.** *Fundamentals of Momentum, Heat, and Mass Transfer*, **5th ed.; p 114~116.**)

(b)

If we consider a path in xy plane, then the stream function is

$$d\Psi(x,y) = \frac{\partial \Psi}{\partial x}dx + \frac{\partial \Psi}{\partial y}dy = -v_y dx + v_x dy$$

If $\Psi = constant$, then,
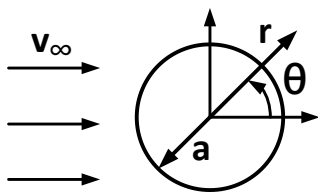
$$-v_y dx + v_x dy = 0 \quad , \quad \frac{dy}{dx}\bigg|_{\Psi = constant} = \frac{v_y}{v_x}$$

We can see that the slope of $\Psi = constant$ is the same as that of streamline.

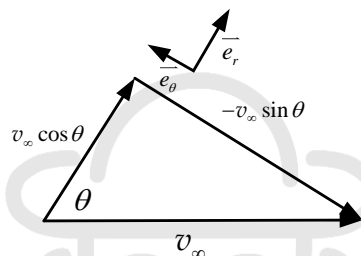Therefore, the function $\Psi(x,y)$ can be used to represent the streamline.

(本題可參考：**Welty, J.; Rorrer, G.; Foster, D.** *Fundamentals of Momentum, Heat, and Mass Transfer*, **5th ed.; p 114~116.**)

(c)

$$(1) \ v_r\big|_{r=a} = 0 \to \frac{\partial \psi}{\partial \theta}\bigg|_{r=a} = 0 \ , \ (2) \ v_\theta\big|_{\theta=0} = 0 \to \frac{\partial \psi}{\partial r}\bigg|_{\theta=0} = 0$$

$$(3) \ v\big|_{r\to\infty} = finite \quad (4) \ v\big|_{r\to\infty} = v_\infty$$

$$\begin{cases} v_r = v_\infty \cos\theta = \frac{1}{r}\frac{\partial \psi}{\partial \theta} \to \psi = v_\infty r \sin\theta \\ v_\theta = -v_\infty \sin\theta = \frac{-\partial \psi}{\partial r} \to \psi = v_\infty r \sin\theta \end{cases}$$

令 $\psi = f(r)\sin\theta$，代入 Stream funtion

$$r^2 \frac{d^2 f}{dr^2} + r\frac{df}{dr} - f = 0$$

令 $f = cr^m$ 代入

$$f = c_1 r + \frac{c_2}{r} \begin{cases} r = a, f = 0 \\ r \to \infty, f = v_\infty r \end{cases} \ , \ \begin{cases} c_1 = v_\infty \\ c_2 = -v_\infty a^2 \end{cases}$$

$$f = v_\infty r[1 - \frac{a^2}{r^2}] \begin{cases} \psi = v_\infty r(1 - \frac{a^2}{r^2})\sin\theta \\ v_r = v_\infty \cos\theta(1 - \frac{a^2}{r^2}) \\ v_\theta = -v_\infty(1 + \frac{a^2}{r^2})\sin\theta \end{cases}$$

(本題可參考：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer*, **5th ed.; p 116~117.)**

Problem 3
【**Solution**】

(a)

$$\text{Economy} : \frac{V}{S} = \frac{vapor\ production\ rate}{heating\ steam\ consumption\ rate}$$

(b)

Stefan-Boltzmann law

$$\frac{q}{A} = \sigma T^4, \text{ where } \sigma \text{ is the Stefan-Boltzmann constant}$$

(c)

$$\text{Number of transfer unit (NTU)} = \frac{UA}{C_{\min}}$$

$$\begin{cases} U = overall\ heat\ transfer\ coefiicient \\ A = heat\ transfer\ area \\ C_{min} = smaller\ heat\ capacity\ coefficient\ between\ hot\ and\ cold\ fluids \end{cases}$$

((a)小題可參考：本題可參考：**Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J.** *Chemical Engineering*, **5th ed.; Volume 2, p 782.**)

((b)~(c)小題可參考：**Welty, J.; Rorrer, G.; Foster, D.** *Fundamentals of Momentum, Heat, and Mass Transfer*, **5th ed.; p 209/347~349.**)

(d)
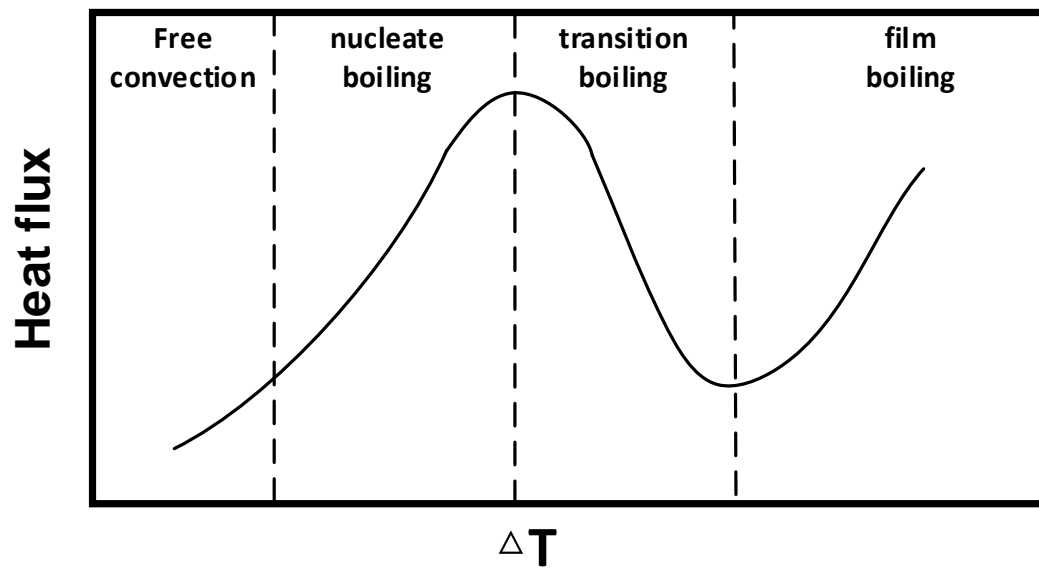
After the nucleate boiling, a vapor film forms around the wire, which provides a considerable resistance to heat transfer, then the heat flux would decrease. It's called the transitional region of boiling.
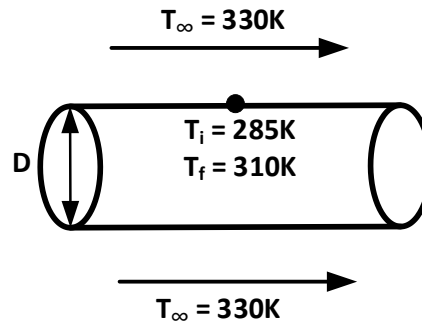
(本題可參考：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer*, 5th ed.; p 323-325.)

Problem 4

【Solution】

【check】

$$Bi = \frac{h(\frac{V}{A})}{k_s} = \frac{hD}{4k} = \frac{121.64 \times 0.5 \times 10^{-2}}{4 \times 500} << 0.1$$

The lumped-parameter analysis is applicable

By lumped-parameter analysis: $\cancel{in} - \cancel{out} + \cancel{gen} = accu$

$$-hA(T - 330) = \rho V C_P \frac{\partial T}{\partial t}$$

$$h\pi DL(330 - T) = \rho \times \frac{\pi}{4} D^2 \times L C_P \frac{\partial T}{\partial t}$$

$$\int_0^t h\,dt = \frac{1}{4} \rho D C_P \int_{T_i}^{T_f} \frac{dT}{330 - T}$$

$$h = \frac{\rho D C_P}{4t} \ln \frac{330 - T_i}{330 - T_f} = \frac{8000 \times 0.5 \times 10^{-2} \times 300}{4 \times 20} \ln \frac{330 - 285}{330 - 310}$$

$$h = \underline{\underline{121.64 \ (W/m^2 \cdot K)}}$$

Problem 5

【Solution】

(a)

Assumptions:

(1) The process is in steady-state.

(2) The effective diffusion coefficient, D_{Ae} is constant everywhere in the system.

(3) The reactant is very dilute.

(4) There are only r and z-direction diffusion.

(b)

By the equation of continuity of species A,

$$\rho(\underbrace{\frac{\partial C_A}{\partial t}}_{s.s} + \cancel{v_r}\frac{\partial C_A}{\partial r} + \cancel{\frac{v_\theta}{r}}\frac{\partial C_A}{\partial \theta} + \cancel{v_z}\frac{\partial C_A}{\partial z}) = -[\frac{1}{r}\frac{\partial}{\partial r}(rN_{Ar}) + \frac{1}{r}\cancel{\frac{\partial N_{A\theta}}{\partial \theta}} + \frac{\partial N_{Az}}{\partial z}] + R_A$$

$$N_{Ar} = -D_{Ae}\frac{\partial C_A}{\partial r} + \underbrace{\cancel{y_A}}_{dilute}(N_A + N_B + N_C + N_D) = -D_{Ae}\frac{\partial C_A}{\partial r}$$

且 $R_A = -k_1 C_A$ ，代入得

$$\boxed{D_{Ae}[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial C_A}{\partial r}) + \frac{\partial^2 C_A}{\partial z^2}] - k_1 C_A = 0}$$

(c)

$$r: \begin{cases} B.C.: \ r = 0, \ C_A \ is \ finite \\ B.C.: \ r = R, \ N_{Ar} = -D_{Ae}\frac{\partial C_A}{\partial r}\bigg|_{r=R} = k_s C_A \end{cases}$$
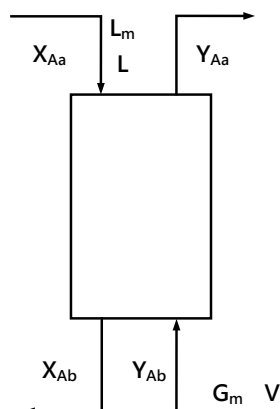
$$z: \begin{cases} B.C.: \ z = 0, C_A = C_{A\infty} \\ B.C.: \ z = L, N_{Az} = -D_{Ae}\frac{\partial C_A}{\partial z}\bigg|_{z=L} = 0 \ (impermeable) \end{cases}$$

(d)

$$\boxed{k_1 \ [=] \ \frac{1}{s}, \ k_s \ [=] \ \frac{cm}{s}}$$

Problem 6
【Solution】

G_m = gas molar flow rate on a solute-free basis.

L_m = liquid flow rate on a solute-free basis.

$\because$ the concentration of ammonia (denoted as A) is dilute, let

$$x_{Ai} \approx X_{Ai} = \frac{x_{Ai}}{1 - x_{Ai}} \quad , \quad y_{Ai} \approx Y_{Ai} = \frac{y_{Ai}}{1 - y_{Ai}}$$

(a)

$y_{Ab} = 0 \quad , \quad x_{Aa} = 0.03$

$$L_m = L(1 - x_{Aa}) = 100 \times (1 - 0.03) = 97 \ (mole / time)$$

$$G_m = V(1 - y_{Ab}) = V$$

For x_{Ab}, by mole balance of A in the liquid phase,

$$L_m x_{Aa}(1 - 0.90) = L_m x_{Ab} \quad , \quad x_{Ab} = 0.1 \times 0.03 = 0.003$$

To calculate the minimum flow rate, we assume y_{Aa} is in equilibrium with x_{Aa}

$$y_{Aa} = 0.8 x_{Aa} = 0.8 \times 0.03 = 0.024$$

By total mole balance of A,

$$G_m(y_{Aa} - y_{Ab}) = L_m(x_{Aa} - x_{Ab})$$

$$G_m = \frac{L_m(x_{Aa} - x_{Ab})}{(y_{Aa} - y_{Ab})} = \frac{97 \times (0.03 - 0.003)}{(0.024 - 0)} = 109.125 \ (mole / time) = V$$

(1) That is, the the vapor flow rate is **109.125 mole** per 100 mole of liquid feed.

(2) **The number is ideal stages needed is infinity**

(b)

If the air input is 1.5 times of the liquid input, then by total mole balance of A,

$$G_m(y_{Aa} - y_{Ab}) = L_m(x_{Aa} - x_{Ab})$$

$$y_{Aa} = \frac{L_m}{G_m}(x_{Aa} - x_{Ab}) + y_{Ab} = \frac{L \times 0.97}{V}(0.03 - 0.003) + 0 = \frac{0.97}{1.5}(0.03 - 0.003) + 0 = 0.01746$$

Operating line：$y = \dfrac{0.97}{1.5}(x - 0.003) + 0 = 0.647x - 0.002$

【法① -用 y 表示之 Kremser equation】

∵ The slope of operating line is lower than equilibrium line,

$$N = \frac{\ln[(y_{Aa} - y_{Aa}^*) / (y_{Ab} - y_{Ab}^*)]}{\ln[(y_{Ab}^* - y_{Aa}^*)/(y_{Ab} - y_{Aa})]}$$

$y_{Ab} = 0$，$y_{Aa} = 0.01746$，$y_{Ab}^* = 0.8x_{Ab} = 0.8 \times 0.003 = 0.0024$，

$y_{Aa}^* = 0.8x_{Aa} = 0.8 \times 0.03 = 0.024$ 代回：

$$N = \frac{\ln[(0.01746 - 0.024) / (0 - 0.0024)]}{\ln[(0.0024 - 0.024) / (0 - 0.01746)]} = 4.711，大約需 5 個 ideal stages$$

【法② -用 x 表示之 Kremser equation】

The use of y is the conventional variable in gas adsorption, and can be used for stripping also. However, in practice, equations in x are more common, which is

$$N = \frac{\ln[(x_{Aa} - x^*_{Aa})/(x_{Ab} - x^*_{Ab})]}{\ln[(x_{Aa} - x_{Ab})/(x^*_{Aa} - x^*_{Ab})]}$$

$x_{Ab} = 0.003$ ， $x_{Aa} = 0.03$ ， $x^*_{Ab} = \dfrac{y_{Ab}}{0.8} = \dfrac{0}{0.8} = 0$ ， $x^*_{Aa} = \dfrac{y_{Aa}}{0.8} = \dfrac{0.01746}{0.8} = 0.021825$

代回：

$$N = \frac{\ln[(0.03 - 0.021825)/(0.003 - 0)]}{\ln[(0.03 - 0.003)/(0.021825 - 0)]} = 4.711 \text{，} \underline{\textbf{大約需 5 個 ideal stages}}$$

※根據 **McCabe, W.; Smith, J.; Harriott, P.** *Unit Operations of Chemical Engineering*, **7th ed.; p 653~657.**的敘述，若是兩相於理想單元中達到平衡，才各自前往下一單元，且操作線與平衡線皆為直線的情況下，即可使用 **Kremser equation**。若是 **absorption(液體吸收氣體)**，則通常使用 **y** 來表示，且(1)操作線斜率>平衡線，此時吸收因子>1，較能有效吸收溶質，此時則使用題目所給之方程式；若是(2)操作線斜率<平衡線，此時吸收因子<1，則需各自將分子分母的 ln 項做倒數，各別 ln 出來的結果才會都是正值，然而最終結果與直接用題目給的形式算出來的結果沒有差別。若是 **stripping(氣體吸收液體)**，則通常使用 **x** 來表示，且(1)平衡線>操作線斜率，則氣提因子>1，較能有效氣提出溶質，此時使用法②之形式；若是(2)平衡線<操作線斜率，則一樣倒數做答，但結果沒有差別。另外，由法①與法②做比較，採用 x 與 y 形式都能得到相同結果，不過是習慣用法的差別而已。

(c)

【法① -利用 y 表示之 Kremser equation 做計算】

Murphree efficiency, η_M, is the change in vapor composition from one plate to the next divided by the change would have occurred if the vapor leaving is in equilibrium with the liquid phase.

$$\eta_M = \frac{y_n - y_{n+1}}{y_n^* - y_{n+1}}$$

$$\begin{cases} y_n = \textit{actual concentration of vapor leaving plate n} \\ y_{n+1} = \textit{actual concentration of vapor entering plate n} \\ y_n^* = \textit{concentration of vapor in equilibrium with liquid leaving downpipe from plate n} \end{cases}$$

Let,

y_n = the equilibrium concentration of vapor modified by Murphree efficiency, y^{*M}

y_n^* = the concentration of vapor by theoretical equilibrium line, y^*

$$\eta_M = \frac{y^{*M} - y}{y^* - y} \quad , \quad y^{*M} = \eta_M(y^* - y) + y$$

$$y_{Ab} = 0 \quad , \quad y_{Aa} = 0.01746 \quad , \quad y_{Ab}^* = 0.0024 \quad , \quad y_{Aa}^* = 0.024$$

$$\begin{cases} y_{Ab}^{*M} = \eta_M(y_{Ab}^* - y_{Ab}) + y_{Ab} = 0.7 \times (0.0024 - 0) + 0 = 1.68 \times 10^{-3} \\ y_{Aa}^{*M} = \eta_M(y_{Aa}^* - y_{Aa}) + y_{Aa} = 0.7 \times (0.024 - 0.01746) + 0.01746 = 0.022038 \end{cases}$$

By Kremser equation (modified by Murphree efficiency),

$$N = \frac{\ln[(y_{Aa} - y_{Aa}^{*M})/(y_{Ab} - y_{Ab}^{*M})]}{\ln[(y_{Ab}^{*M} - y_{Aa}^{*M})/(y_{Ab} - y_{Aa})]}$$

$$N = \frac{\ln[(0.01746 - 0.022038)/(0 - 1.68 \times 10^{-3})]}{\ln[(1.68 \times 10^{-3} - 0.022038)/(0 - 0.01746)]} = 6.528$$

需 7 個 ideal stages

【法② -利用 x 表示之 Kremser equation 做計算】

此時須將 Murphree efficiency 改用 x 的形式，即

$$\eta_M = \frac{x_n - x_{n-1}}{x_n - x_n^*}$$

(可參考 Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 631.)

Let,

x_{n-1} = the equilibrium concentration of liquid modified by Murphree efficiency, x^{*M}

x_n^* = the concentration of vapor by theoretical equilibrium line, x^*

$$\eta_M = \frac{x - x^{*M}}{x - x^*} \ , \ x^{*M} = x - \eta_M(x - x^*)$$

$$x_{Ab} = 0.003 \ , \ x_{Aa} = 0.03 \ , \ x_{Ab}^* = 0 \ , \ x_{Aa}^* = 0.021825$$

$$\begin{cases} x_{Ab}^{*M} = x_{Ab} - \eta_M(x_{Ab} - x_{Ab}^*) = 0.003 - 0.7 \times (0.003 - 0) = 9 \times 10^{-4} \\ x_{Aa}^{*M} = x_{Aa} - \eta_M(x_{Aa} - x_{Aa}^*) = 0.03 - 0.7 \times (0.03 - 0.021825) = 0.0242775 \end{cases}$$

By Kremser equation (modified by Murphree efficiency),

$$N = \frac{\ln[(x_{Aa} - x_{Aa}^{*M})/(x_{Ab} - x_{Ab}^{*M})]}{\ln[(x_{Aa} - x_{Ab})/(x_{Aa}^{*M} - x_{Ab}^{*M})]}$$

$$N = \frac{\ln[(0.03 - 0.0242775)/(0.003 - 9 \times 10^{-4})]}{\ln[(0.03 - 0.003)/(0.0242775 - 9 \times 10^{-4})]} = 6.958$$

<u>需 7 個 ideal stages</u>

【法③ - 將 Murphree 效率轉成總效率】

For the special case when the equilibrium and operating lines are straight, the following equation can be applied:

$$\eta_o = \frac{\ln[1+\eta_M(\frac{mV}{L}-1)]}{\ln(\frac{mV}{L})} \begin{cases} \eta_o = overall\ efficiency \\ \eta_M = Murphree\ efficiency \\ V = the\ gas\ flow\ rate \\ L = the\ liquid\ flow\ rate \\ m = the\ slope\ of\ the\ EL \end{cases}$$

$$= \frac{\ln[1+0.7\times(0.8\times1.5-1)]}{\ln(0.8\times1.5)} = 0.719$$

$$\eta_o = \frac{number\ of\ ideal\ plates}{number\ of\ actual\ plates}$$

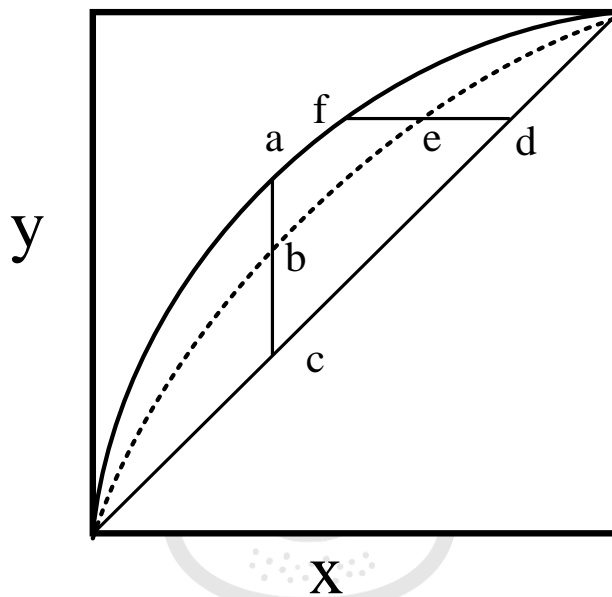$$\frac{number\ of\ ideal\ plates}{\eta_o} = \frac{4.711}{0.719} = 6.556$$

需 7 個 ideal stages

【補充 – Murphree efficiency】

一般來說，在非理想的情況下，離開某單元的液體與氣體並不會達到平衡線所表示的完美平

衡，而有些許落差，此落差即是由 efficiency 所表示，又可分為 overall, Murphree, 以及 local

三種。以 Murphree 來說，其與正常平衡線的關係如下(實線為正常平衡線，虛線為考慮效率所

繪製)：

其中，若是以 vapor 定義之 Murphree 效率為：$\overline{bc} / \overline{ac}$；若是以 liquid 定義之 Murphree 效率

則為 $\overline{ed} / \overline{fd}$。由圖可見，以氣相定義之 Murphree 效率不一定等於液體，因此(c)小題法①與法

②計算結果才會有相對應的差異。

(本題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th

ed.; p 712-717/p 656.)

110 年成大單操輸送

Problem 1
【Solution】

(a)

By the equation of continuity,

$$\underset{\substack{incompressible}}{\cancel{\frac{\partial \rho}{\partial t}}} + \frac{1}{r}\frac{\partial}{\partial r}\underset{v_r=0}{(r\rho \cancel{v_r})} + \frac{1}{r}\frac{\partial(\rho v_\theta)}{\partial \theta} + \underset{v_z=0}{\cancel{\frac{\partial(\rho v_z)}{\partial z}}} = 0$$

$$\frac{\partial v_\theta}{\partial \theta} = 0 \text{ , } v_\theta = v_\theta(r, \cancel{\theta}, z, \cancel{t})$$

$$\underline{\underline{v_\theta = v_\theta(r, z)}}$$

The reason for we have to consider the r and z direction effect of v_θ is that the upper disk is rotating but stationary for lower disk, thus we have to consider the change of v_θ in z direction. Moreover, in this system, the rotating speed is related to the radius, which could be $v_\theta \propto \Omega r$. Therefore, these components should be considered.

(b)

$\theta - direction$

$$\rho(\cancel{\frac{\partial v_\theta}{\partial t}} + \cancel{v_r \frac{\partial v_\theta}{\partial r}} + \frac{v_\theta}{r}\cancel{\frac{\partial v_\theta}{\partial \theta}} + \cancel{\frac{v_r v_\theta}{r}} + \cancel{v_z \frac{\partial v_\theta}{\partial z}})$$

$$= \frac{-1}{r}\frac{\partial p}{\partial \theta} + \cancel{\rho g_\theta} + \mu[\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)) + \frac{1}{r^2}\cancel{\frac{\partial^2 v_\theta}{\partial \theta^2}} + \frac{2}{r^2}\cancel{\frac{\partial v_r}{\partial \theta}} + \frac{\partial^2 v_\theta}{\partial z^2}]$$

$$\frac{1}{r}\frac{\partial p}{\partial \theta} = \mu[\frac{\partial}{\partial r}(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)) + \frac{\partial^2 v_\theta}{\partial z^2}] = f(r,z)$$

$\because p(\theta) = p(\theta + 2\pi)$

$$p(\theta) = rf \cdot \theta + c = p(\theta + 2\pi) = rf \cdot (\theta + 2\pi) + c \text{ , } f = 0$$

(可直接用對稱所以不與 θ 方向有關做省略)

$$\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\frac{\partial^2 v_\theta}{\partial z^2}\right]=f(r,z)=0 \quad \begin{cases} r=0, v_\theta=0 \\ z=0, v_\theta=0 \\ z=H, v_\theta=\Omega r \end{cases}$$

(c)

$$\because z=H, v_\theta=\Omega r \propto r$$

$$v_\theta=(r,H)=cg(H)r^n \propto r$$

$$\underline{\underline{n=1}}$$

$$\because \frac{d^2 g}{dz^2}=0, \quad g(z)=\alpha z+\beta \text{ 代回}$$

$$v_\theta(r,z)=C(\alpha z+\beta)r=(Dz+E)r$$

$$\begin{cases} z=0, v_\theta=0 \\ z=H, v_\theta=\Omega r \end{cases} \rightarrow \begin{cases} E=0 \\ D=\dfrac{\Omega}{H} \end{cases}$$

$$\boxed{v_\theta=\Omega\frac{z}{H}r}$$

(d)

$$T_z=\iint(-\tau_{z\theta}\cdot r)dA=\int_0^R\int_0^R\mu\frac{dv_\theta}{dz}\cdot r\cdot dr d\theta$$

$$=2\pi\mu\int_0^R\frac{\Omega}{H}r^3 dr=\underline{\underline{\frac{\pi\mu\Omega R^4}{2H}}}$$

(e)

$$\nabla\times\vec{v}=\left(\frac{1}{\cancel{r}}\frac{\cancel{\partial v_z}}{\partial\theta}-\frac{\partial v_\theta}{\partial z}\right)\vec{e_r}+\left(\frac{\cancel{\partial v_r}}{\cancel{\partial z}}-\frac{\cancel{\partial v_z}}{\cancel{\partial r}}\right)\vec{e_\theta}+\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)-\frac{1}{\cancel{r}}\frac{\cancel{\partial v_r}}{\partial\theta}\right)\vec{e_z}$$

$$=\frac{-\partial v_\theta}{\partial z}\vec{e_r}+\left[\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right]\vec{e_z}=\frac{-\Omega r}{H}\vec{e_r}+\frac{2\Omega z}{H}\vec{e_z}\neq 0$$

The flow is rotational

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 106~107,

Problem 3B.5)

Problem 2
【Solution】

The fin effectiveness is defined as,

$$\varepsilon_f = \frac{fin\ heat\ tranfer\ rate}{heat\ transfer\ rate\ without\ the\ fin} = \sqrt{\frac{kP}{hA}}$$

(For infinite-long fin)

$$\begin{cases} (1) the\ shape\ of\ the\ fin\ (P,\ A_c) \\ (2) the\ material\ of\ the\ fin\ (k,h) \end{cases}$$

(本題可參考：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer*, 5th ed.; p 233~238.)


Problem 3
【Solution】

When the lumped parameter analysis is applicable, we can assume the thermal properties of the object are uniform, so is the temperature distribution and we can focus on the effect of external convection.

For the lumped parameter analysis to be applicable, the Biot number (Bi) of the system should be less than 0.1.

(本題可參考：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer*, 5th ed.; p 254.)

Problem 4
【Solution】
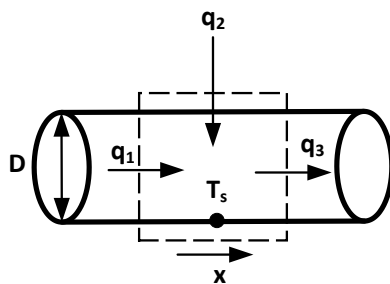
By energy balance of the control volume,

$$q_1 - q_3 + q_2 = 0$$

$$\left.\frac{\rho\pi D^2}{4}vC_pT\right|_x - \left.\frac{\rho\pi D^2}{4}vC_pT\right|_{x+dx} + h\pi D dx(T_s - T) = 0$$

同除 $dx \to 0$

$$\frac{-\rho\pi D^2}{4}vC_p\frac{dT}{dx} + h\pi D(T_s - T) = 0$$

$$\frac{dT}{(T_s - T)} = \frac{4h}{\rho D v C_p}dx \quad , \quad \int_{T_0}^{T_L}\frac{dT}{(T_s - T)} = \int_0^L\frac{4h}{\rho D v C_p}dx$$

$$\ln\frac{T_s - T_0}{T_s - T_L} = \frac{h}{\rho v C_p}\frac{4L}{D}$$

(2)

By Reynold analogy,

$$St = \frac{f}{2} = 0.003 = \frac{h}{\rho C_p v}$$

$$\ln\frac{100 - 20}{100 - T_L} = 0.003 \times \frac{4 \times 250}{2} \quad , \quad T_L = 82.15 \ (℃)$$

(本題改編自：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer,* 5th ed.; p 291~293, Example 1.)

Problem 5

【Solution】

(a)

Assume：

(1) Steady state

(2) One-dimensional diffusion (z-direction)

(3) $\begin{cases} vapor = A \\ air = B \end{cases} \rightarrow B\ is\ stagnant$

(4) No homogeneous reaction

(b)

By mole balance of A in the control volume,

$$N_{Az} \cdot dxdy \big|_z - N_{Az} \cdot dxdy \big|_{z+dz} = 0$$

同除 $dxdydz \rightarrow 0$

$$\frac{dN_{Az}}{dz} = 0 \quad \text{(general equation)}$$

By Fick's flux equation,

$$N_{Az} = -cD_{AB} \frac{dy_A}{dz} + y_A(N_{Az} + \cancel{N_{Bz}})$$

$$N_{Az} = \frac{-cD_{AB}}{1-y_A} \frac{dy_A}{dz}$$

Combine,

$$\frac{d}{dz}\left[\frac{cD_{AB}}{1-y_A}\frac{dy_A}{dz}\right] = 0$$

$$\underline{\underline{\frac{d}{dz}\left[\frac{cD_{AB}}{1-y_A}\frac{dy_A}{dz}\right] = 0}}$$

(c)

Boundary conditions,

$$\begin{cases} z = 0, \ y_A = y_A^0 \\ z = L, \ y_A = 0 \end{cases}$$

$$\ln(\frac{1}{1-y_A}) = c_1 z + c_2 \begin{cases} c_1 = \dfrac{\ln(1-y_A^0)}{L} \\ c_2 = \ln(\dfrac{1}{1-y_A^0}) \end{cases}$$

$$\frac{z}{L}\ln(\frac{1}{1-y_A^0}) = \ln(\frac{1-y_A}{1-y_A^0})$$

$$\frac{1-y_A}{1-y_A^0} = (\frac{1}{1-y_A^0})^{\frac{z}{L}}$$

$$N_{Az} = \frac{-cD_{AB}}{1-y_A}\frac{dy_A}{dz}\bigg|_{z=0} = \frac{cD_{AB}}{1-y_A^0}\cdot(1-y_A^0)\cdot\frac{1}{L}\cdot\ln(\frac{1}{1-y_A^0})$$

$$= \frac{cD_{AB}}{L}\ln(\frac{1}{1-y_A^0}) = \frac{cD_{AB}}{L}\ln\frac{1}{y_B^0} = \frac{cD_{AB}}{L}\frac{(C-C_B^0)}{\dfrac{(C-C_B^0)}{\ln\dfrac{C}{C_B^0}}} = \underline{\underline{\frac{cD_{AB}(C-C_B^0)}{L\ln C_{B,lm}}}}$$

(d)
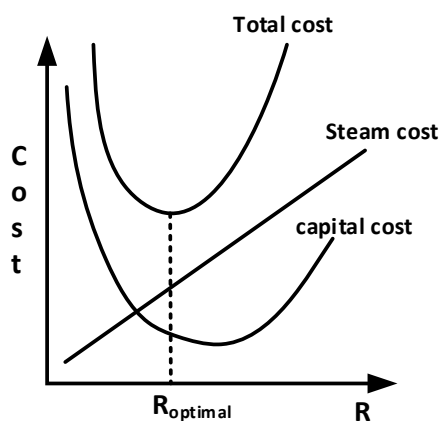
$$\underline{\underline{N_{Az} = \frac{D_{AB}P}{RTL}\cdot\frac{(P-P_B^0)}{\ln P_{B,lm}}}} \quad , \quad \ln P_{B,lm} = \frac{(P-P_B^0)}{\ln\dfrac{P}{P_B^0}}$$

(e)

$$N_{Az} = \frac{3\times10^{-5}\times1.5\times1.013\times10^5}{8.314\times323.15\times1.2}\cdot\ln\frac{1.5}{1.5(1-0.2)} = \underline{\underline{3.155\times10^{-4} \ (mol/m^2\cdot s)}}$$

Problem 6
【Solution】
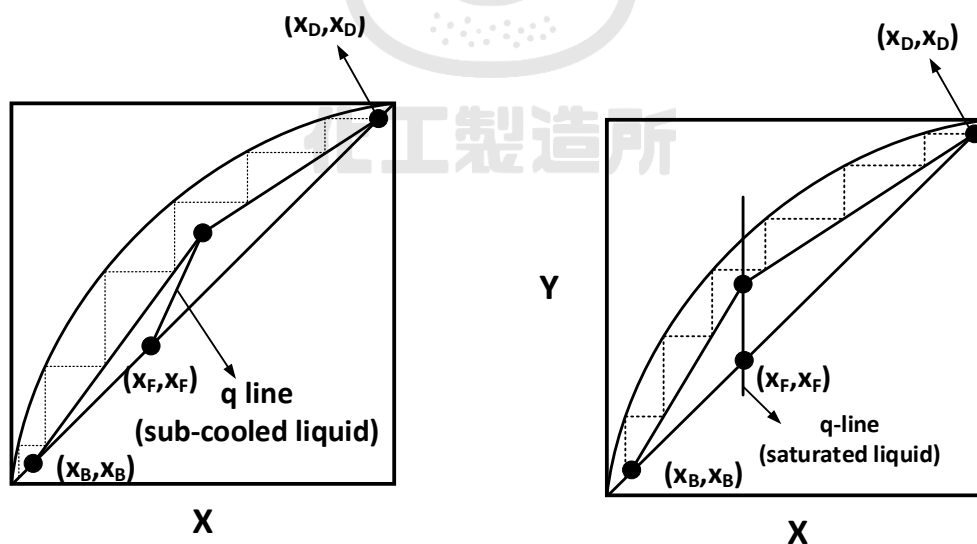
(a)

如圖，若：

$$\begin{cases} R < R_{economic} \rightarrow R\uparrow \\ R > R_{economic} \rightarrow R\downarrow \end{cases}$$

(b)

(1) 將 sub-cooled liquid feed 改為 saturated liquid feed，**板數變多**(雖然最後都是

回答大約 5+1 板，但很明顯可以看出 cold liquid 最後一段超出 X_w 許多。)

原因為：

① Feed line 向左傾斜，因而導致 rectifying section 板數變多。

② 而主要原因為塔內整體液體量變少，導致 stripping section 之斜率變大，從而導致板數也變多。

③進料熱焓相對提升，造成溫度升高，導致分離效果變差，因而需要較多理論板做分離。

(有關 q line 斜率對理論板數的影響，可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 683~686, Example 21.2-(b).)

(2)

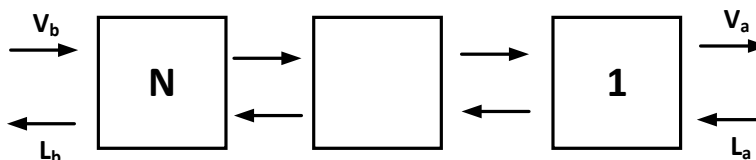蒸餾塔之能量提供來源為：(1)再沸器蒸氣↓＋(2) preheater 能量損耗↑

綜合起來**總能量損耗量改變不多**

(此小題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 683~686, Example 21.2 之最後討論.)

Problem 7

【Solution】

(a)

Let A = solid, B = oil, C = benzene, assume the solid is not extracted by the solvent,

At the exit (L_b)

$$\frac{B}{B+C} = 0.1 \quad , \quad \frac{B+C}{A} = 0.3$$

$$B = 0.1 \cdot (B+C) = 0.1 \cdot 0.3A = 0.03 \times 1000 = 30 \ (kg)$$

The mass of removed oil is,

$$500 - 30 = \underline{\underline{470}} \ (g)$$

(b)

If we have minimum solvent flow rate, then, the V_a is in equilibrium with L_a

The benzene concentration in solution, $y_{a,B}$ in L_a is,

$$y_{a,B} = \frac{100}{100 + 500} = x_{a,B} = 0.167$$

$$V_a = \underset{oil\ removed}{470} \times \frac{1}{1 - 0.167} = 564 \ (kg)$$

By mass balance of benzene in the system,

$$L_{b,B} = 1000 \times 0.3 \times (1 - 0.1) = 270 \ (kg)$$

$$L_{a,B} = 100 \ (kg)$$

$$V_{b,B} + L_{a,B} = V_{a,B} + L_{b,B} \quad , \quad V_{b,B} = 564 \times 0.167 + 270 - 100 = \underline{\underline{264}} \ (kg)$$

In this case, ***the number of ideal stages is infinity***

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Problem 1

【Solution】

By the definition of NPSH,

$$NPSH = \frac{P_0 - P_v}{\rho \frac{g}{g_c}} + h_{pump} - h_{loss} \quad , \quad h_{pump} = L_{pipe}$$

For $h_{loss} = h_{loss,pipe} + h_{loss,elbow}$

$$h_{loss} = \frac{4f}{2Dg}(L_{pipe}v^2) \ [=]m$$

$$v = \frac{300}{60 \times 60 \times \frac{\pi}{4}(6 \times 2.54 \times 10^{-2})^2} = 4.568 \ (m/s)$$

$$(1 \ inch = 2.54 \ cm)$$

$$h_{loss} = \frac{4 \times 0.012}{2 \times 6 \times 2.54 \times 10^{-3} \times 9.8} \times 4.568^2 \cdot L_{pipe} + 2 \times 0.1$$

代回：

$$NPSH = \frac{P_0 - P_v}{\rho \frac{g}{g_c}} + h_{pump} - h_{loss} \geq 15$$
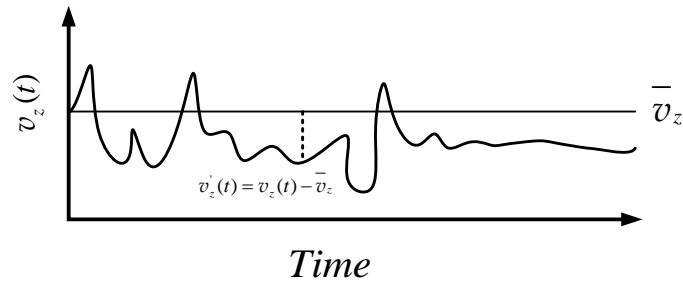
$$\frac{101300 - 3200}{9.8 \times 1000} + L_{pipe} - (\frac{\frac{4 \times 0.012}{2 \times 6 \times 2.54 \times 10^{-2}} \times 4.568^2 \cdot L_{pipe}}{9.8} + 2 \times 0.1) \geq 15$$

$$\underline{\underline{L_{pipe} \geq 7.81 \ (m)}}$$

Problem 2
【Solution】

Consider, a turbulent flow as follows,

$$\overline{v}_z = time-smoothed\ velocity = \frac{1}{t_0}\int_{t-\frac{1}{2}t_0}^{t+\frac{1}{2}t_0} v_z(s)ds$$

即是說，因紊流的每個質點較不穩定，且會隨著時間到處浮動，因此，解析上

必須採用"時間平均"的概念定義所要分析的質點，即為 time-smoothed velocity

(出自 Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 156~157.)

By Prandtl's 1/7 power law,

$$\frac{\overline{v}_z}{v_{z,\max}} = (\frac{y}{R})^{1/7} = (\frac{R-r}{R})^{1/7} = (1-\frac{r}{R})^{1/7}$$

(y = 0 為管壁)

(y 表示之表達式出自 Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass*

***Transfer*, 5th ed.; p 163.**

r 表示之表達式於 Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 154.

有提及，兩者相同。)

To derive the relationship between the average velocity and maximum velocity within

a tube,

$$Q = 2\pi\int_0^R \overline{v}_z \cdot rdr$$

$\because r = R - y$ ， $dr = d(-y)$ 代入

$$Q = 2\pi \overline{v}_{z,\max} \int_R^0 (\frac{y}{R})^{1/7} \cdot (R-y) d(-y) = 2\pi \overline{v}_{z,\max} R^2 \int_0^R (\frac{y}{R})^{1/7} \cdot (1-\frac{y}{R}) d(\frac{y}{R})$$

$$= 2\pi \overline{v}_{z,\max} R^2 \int_0^R [(\frac{y}{R})^{1/7} - (\frac{y}{R})^{8/7}] d(\frac{y}{R})$$

$$= 2\pi \overline{v}_{z,\max} R^2 \cdot (\frac{7}{8} - \frac{7}{15}) = \frac{49}{60} \pi \overline{v}_{z,\max} R^2$$

For average velocity,

$$Q = \pi R^2 \overline{v}_{avg} = \frac{49}{60} \pi \overline{v}_{z,\max} R^2$$

$$\overline{v}_{avg} = \frac{49}{60} \overline{v}_{z,\max} \approx 0.817 \overline{v}_{z,\max}$$

(1) (本題可參考：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*,

5th ed.; Volume 1, p 84.)

※**Prandtl's 1/7 power-law** 來自於流體於紊流狀態($\mathrm{Re} < 10^7$)下，流經一平板時，靠近板邊所產

生之 shear stress，其經驗表達式為：

$$\tau_0 = 0.0225 \rho \overline{v}_{x,\max}^2 (\frac{\upsilon}{\overline{v}_{x,\max} \delta})^{1/4}$$

Assume a simple power relation is applicable,

$$(\frac{\overline{v}_x}{\overline{v}_{x,\max}}) = (\frac{y}{\delta})^f$$

$$\tau_0 = 0.0225 \rho \overline{v}_{x,\max}^2 (\frac{\upsilon}{\delta})^{1/4} = 0.0225 \rho^{0.75} \mu^{0.25} \delta^{-0.25} \overline{v}_{x,\max}^{1.75}$$

$$= 0.0225 \rho^{0.75} \mu^{0.25} \delta^{-0.25} \overline{v}_x^{1.75} (\frac{\delta}{y})^{1.75f}$$

$$= 0.0225 \rho^{0.75} \mu^{0.25} \delta^{1.75f - 0.25} \overline{v}_x^{1.75} y^{-1.75f}$$

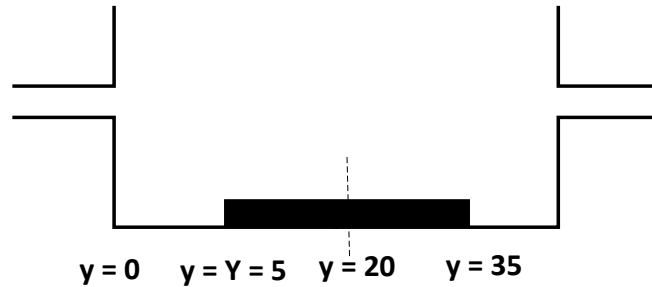**Let the index of $\delta = 0$ (the shear stress is defined only by the velocity at y from the surface)**

$1.75f - 0.25 = 0$ ∨ ， $f = \frac{1}{7}$

Problem 3

【Solution】

(1) Schmidt number is defined as,

$$Sc \equiv \frac{\nu}{D_{AB}} = \frac{6.0 \times 10^{-3}}{2.0 \times 10^{-6}} = \underline{\underline{3.0 \times 10^3}}$$

y = 0   y = Y = 5   y = 20   y = 35

Firstly, calculate the Reynolds number, (approximated by the local Re_x at the trailing edge (y = 35),

$$\mathrm{Re}_x = \frac{\upsilon_\infty x}{\upsilon} = \frac{35 \times \upsilon_\infty}{6 \times 10^{-3}}$$

For υ_∞

$$\upsilon_\infty = \frac{Q}{A} = \frac{20}{(2 - 0.02) \times \underset{\text{容器寬}}{10}} = 1.01 \ (cm/s)$$

Therefore,

$$\mathrm{Re}_x = \frac{35 \times \upsilon_\infty}{6 \times 10^{-3}} = \frac{35 \times 1.01}{6 \times 10^{-3}} = 5891.7$$

(laminar)

For the configuration of the system (hydrodynamic boundary layer develops first) in laminar flow, the local Sherwood number is,

$$Sh_y = 0.332 \mathrm{Re}_y^{1/2} [\frac{Sc}{1 - (\frac{Y}{y})^{3/4}}]^{1/3} = \frac{ky}{D_{AB}}$$

The total mass transfer rate over the overall region is,

$$\overline{k_c} = \frac{0.332(\frac{\upsilon_\infty}{\nu})^{1/2} Sc^{1/3} D_{AB} \int_Y^y \frac{dy}{y^{1/2}[1 - (\frac{Y}{y})^{3/4}]^{1/3}} dy}{L}$$

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$$Sh_L = \frac{\overline{k_c}L}{D_{AB}} = 0.332(\frac{\upsilon_\infty}{\nu})^{1/2} Sc^{1/3} \int_Y^y \frac{dy}{y^{1/2}[1-(\frac{Y}{y})^{3/4}]^{1/3}} dy$$

For the integral,

$$\int_Y^y \frac{1}{y^{1/2}[1-(\frac{Y}{y})^{3/4}]^{1/3}} dy = \int_5^{35} \frac{1}{y^{1/2}[1-(\frac{5}{y})^{3/4}]^{1/3}} dy \approx 9.91$$

(計算機下限直接輸入 5 會導致分母有 0，積分發散，改由 5.001 代替。)

$$Sh_L = \frac{\overline{k_c}L}{D_{AB}} = 0.332(\frac{1.01}{6.0\times10^{-3}})^{1/2}(3.0\times10^3)^{1/3} \times 9.91 = \underline{\underline{615.5}}$$

(b)

$$\overline{k_c} = \frac{Sh_L \cdot D_{AB}}{L} = \frac{615.5 \times 2.0\times10^{-6}}{30} = \underline{\underline{4.1\times10^{-5}}} \ (cm/s)$$

The average flux is,

$$\overline{N_A} = \overline{k_c}(C_{A,s} - C_{A,\infty}) = 4.1\times10^{-5}(0.05-0) = \underline{\underline{2.05\times10^{-5}}} \ (g/cm^2 \cdot s)$$

(c)

By the definition of local Sherwood number,

$$Sh_y = 0.332 \, \mathrm{Re}_y^{1/2}[\frac{Sc}{1-(\frac{Y}{y})^{3/4}}]^{1/3} = \frac{k_y y}{D_{AB}}$$

$$k_y = \underline{\underline{\frac{0.332 \, \mathrm{Re}_y^{1/2}[\frac{Sc}{1-(\frac{Y}{y})^{3/4}}]^{1/3} D_{AB}}{y}}}$$

(d)

By mass flux balance of the control volume,

$$\frac{\partial(\rho l)}{\partial t} = -k_c C_{A,s}$$

$$\rho\frac{dl}{dt} = \frac{0.332\,\mathrm{Re}_y^{1/2}[\dfrac{Sc}{1-(\dfrac{Y}{y})^{3/4}}]^{1/3} D_{AB}}{y} C_{A,s}$$

兩邊對 t 積分，且 $t=0,\ l=l_0$ 代入：

$$\rho\frac{dl}{dt} = \frac{0.332\,\mathrm{Re}_y^{1/2}[\dfrac{Sc}{1-(\dfrac{Y}{y})^{3/4}}]^{1/3} D_{AB}}{y} C_{A,s}$$

$$l = \frac{0.332\,\mathrm{Re}_y^{1/2}[\dfrac{Sc}{1-(\dfrac{Y}{y})^{3/4}}]^{1/3} D_{AB}}{\rho y} C_{A,s}t + l_0$$

(e)

For the hydrodynamic boundary layer thickness,

$$\delta = \frac{5y}{\sqrt{\mathrm{Re}_y}} = \frac{5\times(5+15)}{\sqrt{\dfrac{v_\infty y}{\nu}}} = \frac{5\times 20}{\sqrt{\dfrac{20}{\dfrac{(2-0.02)\times 10}{6.0\times 10^{-3}}\times 20}}} = \underline{\underline{1.72\ (cm)}}$$

By the definition of Schmidt number, for this system,

$$[\frac{Sc}{1-(\dfrac{Y}{y})^{3/4}}]^{1/3} = \frac{\delta}{\delta_c}$$

$$[\frac{3.0\times 10^3}{1-(\dfrac{5}{5+15})^{\frac{3}{4}}}]^{\frac{1}{3}} = \frac{1.72}{\delta_c} \quad,\quad \underline{\underline{\delta_c = 0.101\ (cm)}}$$

(本題可參考：**Welty, J.; Rorrer, G.; Foster, D.** *Fundamentals of Momentum, Heat, and Mass Transfer*, **5th ed.; p 572~574, Example 1.**)

Problem 4

【Solution】

(a)

By overall mole balance,

$$F = D + B \text{，} D + B = 1000$$

By the mole balance of benzene,

$$Fx_F = Bx_B + Dx_D \text{，} 1000 \times 0.30 = 0.98D + 0.1B$$

$$\begin{cases} D = 227.3 \ (kmol / h) \\ B = 772.7 \ (kmol / h) \end{cases}$$

(b)

The reflux ratio is, by definition,

$$R_D = \frac{L}{D}, \text{ where } L \text{ is the liquid flow rate in the rectifying section}$$

By mole balance in the rectifying section,

$$V = L + D = R_D D + D = (R_D + 1)D$$

$$R_D = \frac{V}{D} - 1 = \frac{753}{227.3} - 1 = \underline{\underline{2.31}}$$

(c)

By the definition of q-line,

$$\begin{cases} \overline{L} = L + qF \\ \overline{V} = V - (1-q)F \end{cases}$$

$$q = \frac{heat \ needed \ to \ vaporize \ 1 \ mole \ of \ feed}{molar \ latent \ heat \ of \ the \ feed} = \frac{38.6(97-40) + 7885}{7885} = 1.28$$

$$\begin{cases} \overline{L} = L + qF = R_D D + qF = 2.31 \times 227.3 + 1.28 \times 1000 = \underline{\underline{1805.7}} \ (kmol / h) \\ \overline{V} = V - (1-q)F = 753 - (1-1.28) \times 1000 = \underline{\underline{1033}} \ (kmol / h) \end{cases}$$

(d)

$$q_r = \overline{V}\lambda = 1033 \times (0.9 \times 8050 + 0.1 \times 7500) = \underline{\underline{8.26 \times 10^6}} \ (kcal / h)$$

(e)

題目應意指在相同其他條件下，只將 feed 由 subcooled liquid 改為 saturated liquid 的變化：

$$q_c = V\lambda = 753 \times (0.98 \times 7500 + 0.02 \times 8050) = \underline{\underline{5.65 \times 10^6 \ (kcal/h)}}$$

(f)

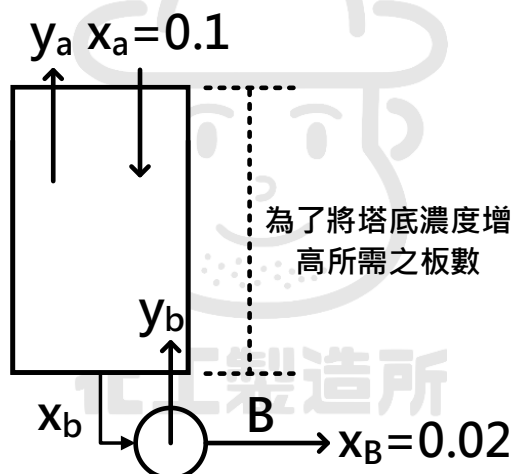題目應意指在相同其他條件下，只將 feed 由 subcooled liquid 改為 saturated vapor 的變化：

因以質量平衡觀點來說，進料 thermal condition 並不影響最終出料組成/流量，亦即 D 維持相同。

此外，在 V 同樣維持 753 kg/hr 的情況下，其 <u>reflux ratio 應維持相同</u>。

(g)

題目所給之平衡關係應為液相 benzene(x)與氣相 benzene(y)之平衡關係，特指 benzene 即因在題意下濃度較低，才有線性平衡關係可用，以下圖說明：

因須將塔底 toluene 濃度由 0.9 增為 0.98，因此 benzene 濃度須從 0.9 降低為 0.02，因此需多增加板數，且 benezene 濃度較低，因此平衡關係應由 benzen 出發。

For y_b, the liquid and vapor phase is in equilibrium in the reboiler, then

$$y_b = 2x_B = 2 \times 0.02 = 0.04$$

(不可直接將 0.02 做為 y_b，因再沸器內液氣平衡，因此進入塔內之氣體分率必須按照平衡關係算出)

For x_b, by mole balance of benzene around the reboiler,
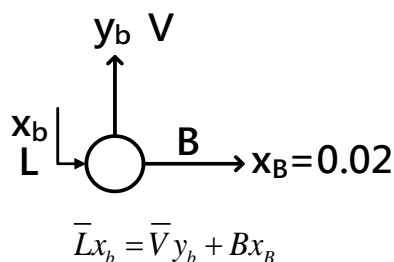
$$\overline{L}x_b = \overline{V}y_b + Bx_B$$

Because we can use the result of problem (c),

$$\begin{cases} \overline{L} = 1805.7 \ (kmol/h) \\ \overline{V} = 1033 \ (kmol/h) \end{cases}$$

For B, by overall mole balance around the reboiler,

$$B = \overline{L} - \overline{V} = 1805.7 - 1033 = 772.7 \ (kmol/h)$$

$$x_b = \frac{\overline{V}y_b + Bx_B}{\overline{L}} = \frac{1033 \times 0.04 + 772.7 \times 0.02}{1805.7} = 0.027$$

$$y_b^* = 0.027 \times 2 = 0.054$$

For y_a, by mole balance of benzene throught the section,

$$\overline{L}x_a = \overline{V}y_a + Bx_B$$

$$y_a = \frac{\overline{L}x_a - Bx_B}{\overline{V}} = \frac{1805.7 \times 0.1 - 772.7 \times 0.02}{1033} = 0.16$$

For y_a^*, $y_a^* = 2x_a = 0.2$

By Kremser equation,

$$N = \frac{\ln[(y_a - y_a^*)/(y_b - y_b^*)]}{\ln[(y_b^* - y_a^*)/(y_b - y_a)]} = \frac{\ln[(0.16 - 0.2)/(0.04 - 0.054)]}{\ln[(0.054 - 0.2)/(0.04 - 0.16)]} = \underline{\underline{5.35}}$$

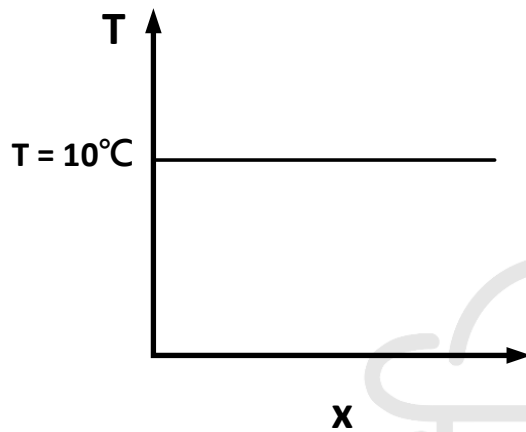To increase the concentration of toluene from 0.9 to 0.98, we need approximately 6 ideal plates.

(本題所附之 **Kremser eqaution** 為操作線斜率小於平衡線所用之形式，已經倒數過，且與最終結

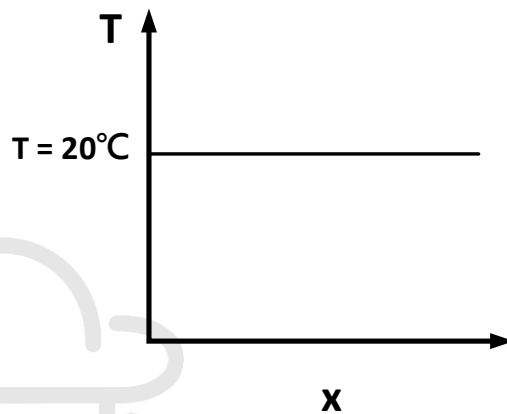果也無關，詳情可參考 109 年成大單操第 6 題之最後討論。本小題改編自成大期中考題。)

Problem 5
【Solution】
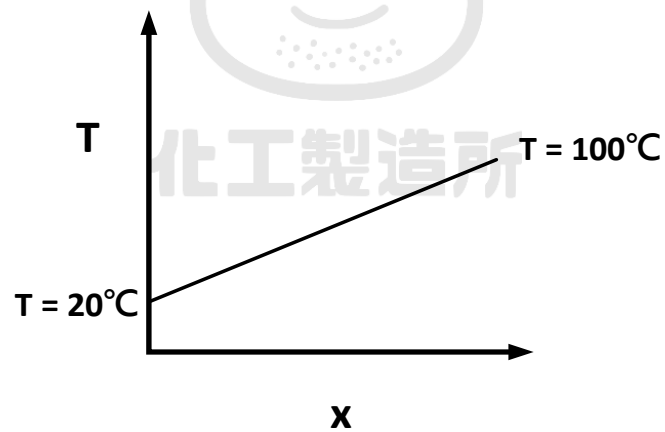
$t \to \infty$, the system reaches steady state.

(a)   **(b)**   **(c)**

(c)

Problem 6

【Solution】

(a)

The log mean temperature difference is,

$$LMTD = \frac{(120-20)-(120-60)}{\ln(\frac{120-20}{120-60})} = \underline{\underline{78.3 \ (K)}}$$

(b)

By energy balance,

$$q = \dot{m}_w C_{p,w}(T_{w,o} - T_{w,i}) = UA(LMTD)$$

$$U = \frac{\dot{m}_w C_{p,w}(T_{w,o} - T_{w,i})}{A(LMTD)} = \frac{0.5 \times 4178(60-20)}{\pi \times 100 \times 10^{-3} \times 10 \times 78.3} = \underline{\underline{339.7 \ (W/m^2 \cdot K)}}$$
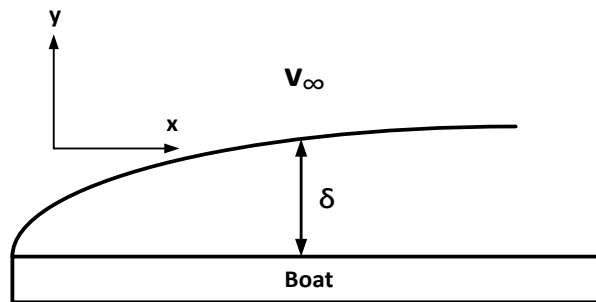
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Problem 1
【Solution】

(a)

本題情境可視為船不動，液體流動的邊界層問題：

For the boundary conditions,

$$\begin{cases} y = 0, \ v_x = 0 \\ y = \delta, \ v_x = v_\infty \\ y = \delta, \ \dfrac{dv_x}{dy} = 0 \end{cases}$$

代入：

$$\begin{cases} \alpha = 0 \\ \beta\delta + \gamma\delta^2 = v_\infty \\ \beta + 2\gamma\delta = 0 \end{cases} , \quad \begin{cases} \alpha = 0 \\ \beta = \dfrac{2v_\infty}{\delta} \\ \gamma = -\dfrac{v_\infty}{\delta^2} \end{cases}$$

The velocity profile is,

$$v_x = 2v_\infty(\frac{y}{\delta}) - v_\infty(\frac{y}{\delta})^2$$

※若為 3 次方近似，即：

$$v_x = a + by + cy^2 + dy^3$$

則需要多一個邊界條件，為：

$$y = 0, \ \frac{d^2 v_x}{dx^2} = 0$$

因根據 momentum balance,

$$v_x \frac{\partial v_x}{\partial y} + v_y \frac{\partial v_y}{\partial y} = v_\infty \frac{\partial v_\infty}{\partial x} + \mu \frac{\partial^2 v_x}{\partial y^2}$$

$\because y = 0, \ v_x = v_y = 0$ ，且 $v_\infty \neq f(x)$

$$\cancel{v_x \frac{\partial v_x}{\partial y}} + \cancel{v_y \frac{\partial v_y}{\partial y}} = \cancel{v_\infty \frac{\partial v_\infty}{\partial x}} + \mu \frac{\partial^2 v_x}{\partial y^2}$$

$$\boxed{\frac{\partial^2 v_x}{\partial y^2} = 0}$$

其中 $v_\infty \dfrac{\partial v_\infty}{\partial x}$ 項由壓力項演變而來，可參考：

Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, **2nd ed.; p 135.**

(b)

By the von Karman approximation,

$$\frac{\tau_0}{\rho} = (\cancel{\frac{dv_\infty}{dx}}) \int_0^\delta (v_\infty - v_x) dy + \frac{d}{dx} \int_0^\delta v_x (v_\infty - v_x) dy$$

where,

$$\tau_0 = \mu \frac{\partial v_x}{\partial y}\bigg|_{y=0} = \frac{2\mu v_\infty}{\delta}$$

$$\frac{\mu}{\rho}(\frac{2v_\infty}{\delta}) = \frac{d}{dx} \int_0^\delta [2v_\infty(\frac{y}{\delta}) - v_\infty(\frac{y}{\delta})^2][v_\infty - 2v_\infty(\frac{y}{\delta}) + v_\infty(\frac{y}{\delta})^2] dy$$

$$\frac{\mu}{\rho}(\frac{2v_\infty}{\delta}) = \frac{d}{dx}\{v_\infty^2 \int_0^\delta [2(\frac{y}{\delta}) - 5(\frac{y}{\delta})^2 + 4(\frac{y}{\delta})^3 - (\frac{y}{\delta})^4] dy\}$$

$$\frac{\mu}{\rho}(\frac{2v_\infty}{\delta}) = \frac{d}{dx}[v_\infty^2(1-\frac{5}{3}+1-\frac{1}{5})\delta] = \frac{d}{dx}(\frac{2}{15}v_\infty^2\delta) = \frac{2}{15}v_\infty^2\frac{d\delta}{dx}$$

$$\frac{15\mu}{\rho v_\infty} = \delta\frac{d\delta}{dx} \quad, \quad \frac{15\mu x}{\rho v_\infty} = \frac{1}{2}\delta^2$$

$$\underline{\underline{\delta = \sqrt{\frac{30\mu x}{\rho v_\infty}}}}$$

(b)

$$F = \int_0^W\int_0^L \tau_0 dxdz = \int_0^W\int_0^L \mu\frac{\partial v_x}{\partial y}\bigg|_{y=0} dxdz = \int_0^W\int_0^L \frac{2\mu v_\infty}{\delta} dxdz$$

$$F = \int_0^W\int_0^L \frac{2\mu v_\infty}{\delta} dxdz = \int_0^W\int_0^L \frac{2\mu v_\infty}{\sqrt{\frac{30\mu x}{\rho v_\infty}}} dxdz = \sqrt{\frac{2}{15}\rho\mu v_\infty^3}\int_0^W\int_0^L \frac{1}{\sqrt{x}} dxdz$$

$$F = \sqrt{\frac{2}{15}\rho\mu v_\infty^3}\int_0^W\int_0^L \frac{1}{\sqrt{x}} dxdz = W\sqrt{\frac{8}{15}L\rho\mu v_\infty^3}$$

$$F = W\sqrt{\frac{8}{15}L\rho\mu v_\infty^3} = 1.5\times\sqrt{\frac{8}{15}\times3\times1000\times1.30\times10^{-3}\times15^3} = \underline{\underline{125.68\ (N)}}$$

Problem 2

【Solution】

(a)

Assume the fluid is incompressible, for pitot tube flowmeter,

$$v_{r=0} = \sqrt{\frac{2g_c(p_s - p_o)}{\rho}} = \sqrt{\frac{2 \times 1 \times 2100}{1260}} = \underline{\underline{1.825}} \ (m/s)$$

(關於皮托管的原理與公式,可參考:**McCabe, W.; Smith, J.; Harriott, P.** *Unit Operations of Chemical Engineering*, **7th ed.; p 237~238.**)

(b)

To determine the relationship between the average velocity and the maximum velocity ($r = 0$), calculate the Reynold number with maximum velocity first.

$$\text{Re}_{max} = \frac{\rho v_{max} D}{\mu} = \frac{1260 \times 1.825 \times 40 \times 10^{-3}}{50 \times 10^{-3}} = 1840$$

The result shows that the overall flow in the tube is laminar, where the results is valid.

$$\frac{v_{avg}}{v_{max}} = \frac{1}{2} \ , \ v_{avg} = \frac{1}{2} v_{max} = \frac{1}{2} \times 1.825 = \underline{\underline{0.913}} \ (m/s)$$

$$\text{Re} = \frac{\rho v_{avg} D}{\mu} = \frac{1260 \times 0.913 \times 40 \times 10^{-3}}{50 \times 10^{-3}} = \underline{\underline{920}}$$

For mass flow rate,

$$w_A = \rho(\frac{\pi}{4}D^2)v_{avg} = 1260 \times \frac{\pi}{4} \times (40 \times 10^{-3})^2 \times 0.913 = \underline{\underline{1.445}} \ (kg/s)$$

(c)

For orifice meter,

$$v_{r=0} = \frac{C_o}{\sqrt{1-\beta^4}} \sqrt{\frac{2(p_a - p_b)}{\rho}}$$

$$\beta = \frac{D_{orifice}}{D_{pipe}} = \frac{24}{40} = 0.6$$

$$1.825 = \frac{C_o}{\sqrt{1-0.6^4}} \sqrt{\frac{2 \times 450}{1260}}$$

$$\underline{\underline{C_o = 2.015}}$$

The definition of the discharge coefficient is,

$$C_o = \frac{actual\ flow\ rate\ with\ firction\ loss}{ideal\ flow\ rate}$$

The result of $C_o > 1$ indicates that the flow rate with fricloss is larger than that of the ideal one, which is possible. <u>The orifice meter is borken.</u>

(關於 orifice meter 的原理與公式，可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 227~229. Discharge coefficient 概念可參考同本書於 Venturi meter 之介紹，在 226 頁。)

Problem 3
【Solution】

To determine which is thicker, we need to calculate the Prandtl number of the fluid

first, whose definition is,

$$\Pr = \frac{\upsilon}{\alpha} = \frac{momentum\ diffusivity}{therma\ diffusivity}$$

$$\begin{cases} \Pr > 1, & \delta > \delta_t \\ \Pr \approx 1, & \delta \approx \delta_t \\ \Pr < 1, & \delta < \delta_t \end{cases}$$

Problem 4
【Solution】

(a) If the lumped-parameter analysis is applicable,

$$Bi = \frac{h(\dfrac{V}{A})}{k} < 0.1$$

The Biot number in this case is,

$$Bi = \frac{400 \times [\dfrac{\dfrac{1}{6}\pi(2\times10^{-3})^3}{\pi(2\times10^{-3})^2}]}{100} = \frac{400 \times \dfrac{1}{6} \times 2\times10^{-3}}{100} = 0.001333$$

$\because Bi <<< 0.1$

The lumped-parameter analysis is applicable.

(b) By the lumped-parameter analysis,

$$\frac{\pi}{6}\rho C D^3 \frac{\partial T}{\partial t} = h(\pi D^2)(T_\infty - T)$$

$$\int_{T_0}^{T_f} \frac{dT}{(T_\infty - T)} = \int_0^t (\frac{6h}{D\rho C})dt$$

$$\ln(\frac{T_\infty - T_0}{T_\infty - T_f}) = \frac{6h}{D\rho C}t$$

(c)

$T = 123^\circ\text{C}$ 及各項數據代入：

$$\ln(\frac{125-25}{125-123}) = \frac{6 \times 400}{2 \times 10^{-3} \times 10000 \times 300} t$$

$$\underline{\underline{t = 9.78 \ (s)}}$$

(d)

若 $k = 1 \ W/m \cdot K$，則：

$$Bi = \frac{400 \times [\frac{\frac{1}{6}\pi(2 \times 10^{-3})^3}{\pi(2 \times 10^{-3})^2}]}{1} = \frac{400 \times \frac{1}{6} \times 2 \times 10^{-3}}{1} = 0.1333 > 0.1$$

The internal resistance can't be neglected, and the governing equation is,

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T$$

$$\text{where} \quad T = T(t, r, \theta, \phi)$$

If we only consider the r-direction temperature profile under the assumption of symmetry,

$$\underline{\underline{\frac{\partial T}{\partial t} = \alpha[\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 \frac{\partial T}{\partial r})]}}$$

The boundary conditions and initial condition is,

$$\begin{cases} t = 0, \ T = T_0 \\ r = 0, \ \frac{\partial T}{\partial r} = 0 \\ r = R, \ -k\frac{\partial T}{\partial r} = h(T - T_\infty) \end{cases}$$

(e)

By energy balance of the thermocouple,

$$q_{rad} - q_{conv} = 0$$

$$\sigma A(T_{wall}^4 - T_{tc}^4) - hA(T_{tc} - T_\infty) = 0$$

$$5.676 \times 10^{-8}[(600 + 273.15)^4 - T_{ct}^4] - 400[T_{ct} - (125 + 273.15)] = 0$$

$$\underline{\underline{T_{ct} = 473.4 \ (K) = 200.3 \ (^\circ C)}}$$

Problem 5

【Solution】

(a)

In the present process, we know,

$$y_A = 0.100 \text{ , } x_A = 0.006$$

Compared with the equilibrium data,

$$y_{Ae} = 0.10 \text{ , } x_{Ae} = 0.00250$$

$$x_A > x_{Ae}$$

Methylene will be stripped from the liquid phase to gas phase.

(b)

By Henry's law,

$$p_A = Hc_A = HCx_A$$

將任一平衡數據代入，這裡選 $y_A = 0.10$，$x_A = 0.00250$

$$1.80 \times 0.10 = H \times \frac{992.3 \times 10^3}{18} \times 0.00250$$

$$\underline{\underline{H = 1.306 \times 10^{-3} \ (atm/(mol/m^3))}}$$

(c)

$$\frac{1}{K_x} = \frac{1}{k_x} + \frac{1}{H'k_y}$$

For H', which is the Henry's law constant in terms of mole fraction,

$$H' = \frac{y_A}{x_A} = \frac{0.10}{0.00250} = 40$$

代入：

$$\frac{1}{K_x} = \frac{1}{0.150} + \frac{1}{40 \times 0.008}$$

$$K_x = 0.102 \ (gmol/m^2 \cdot s)$$

$$K_L = \frac{K_x}{C} = \frac{0.102}{\dfrac{992.3 \times 10^3}{18}} = \underline{\underline{1.85 \times 10^{-6} \ (m/s)}}$$

(d)

$$N_A = K_L(C_{A\infty} - C_{AL}^*) = K_L C(x_A - x_{Ae})$$

$$N_A = 1.852 \times 10^{-6} \times \frac{992.3 \times 10^3}{18}(0.006 - 0.0025) = \underline{\underline{3.57 \times 10^{-4} \ (mol/m^2 \cdot s)}}$$

(e)

$$N_A = k_x(x_A - x_i)$$

$$3.57 \times 10^{-4} = 0.15(0.006 - x_{Ai}) \quad , \quad \underline{\underline{x_{Ai} = 3.62 \times 10^{-3}}}$$

$$N_A = k_y(y_{Ai} - y_A)$$

$$3.57 \times 10^{-4} = 0.008(y_{Ai} - 0.1) \quad , \quad \underline{\underline{y_{Ai} = 0.144}}$$

(本題改編自：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer,* 5th ed.; p 565, Problem 29.12.)

Problem 6

【**Solution**】

(i) With a high vapor velocity, the entrainment will becomes excessive, causing the a large drop in plate efficeincy for a small increase in vapor rate. At that limit, **entrainment flooding point** is reached. When the pressure drop on a plate is too high and the liquid in the downcomer backs up to the plate above, the flow from that plate is inhibited, and the liquid level is increased as well, which is called **downcomer flooding**.

(簡單來說，就是往上的氣體流量太大，造成太強的壓降阻礙液體流出造成堆積)

The solutions to such a problem will be:

(1) Decrease the vapor flow rate by decreasing the reboiler duty.

(2) Increase the liquid/vapor reflux ratio to increase liquid flow rate.

(3) Increase the plate spacing delays the onset of downcomer flooding.

(4) The flooding velocity is much lower at high pressure than that of atmospheric pressure, so operating under lower pressure will help.

(ii) At low vapor velocities, the pressure drop is not great enough to prevent liquid from flowing down through some of the holes. This condition is called **weeping** and is **more likely** to occur if there is a slight gradient in liquid head aross the plate. Under such condition, the contact between the vapor and liquid will reduce.

(氣體流量太低，導致無法有效阻擋液體從洞往下流，**最常發生**在 plate 傾斜導致有水平方向液壓差的情況，液體將會直接從與板高度差最大的地方往下流，且氣體會走液壓差最小的路徑，導致無法均勻的接觸以至於效率變低，如下圖。)

The solutions to such a problem will be:

(1) Increase the vapor flow rate by increasing the reboiler duty.

(2) Decrease the liquid/vapor reflux ratio to decrease liquid flow rate.

(3) Level the plates, and use unform hole sizes.

(4) Use smaller holes, and lower the fraction open area, but may increase pressure drop and reduce the maximum flow rate.
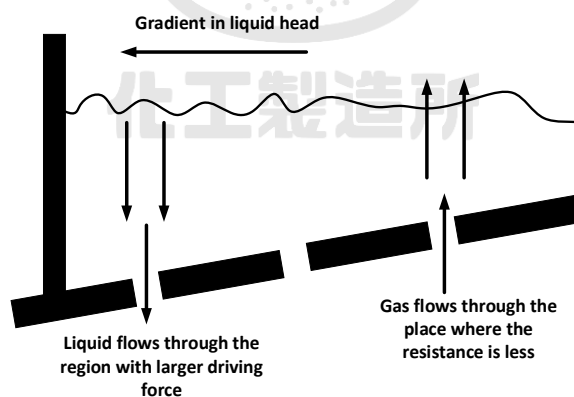
(本題可參考：**McCabe, W.; Smith, J.; Harriott, P.** *Unit Operations of Chemical Engineering*, **7th ed.; p 707~709.**)

(本題於 103 年成大單操輸送 Problem 7 出過。)

Problem 7

【Solution】

(a)

With the minimum water flow rate, the final composition must be on the equilibrium line.

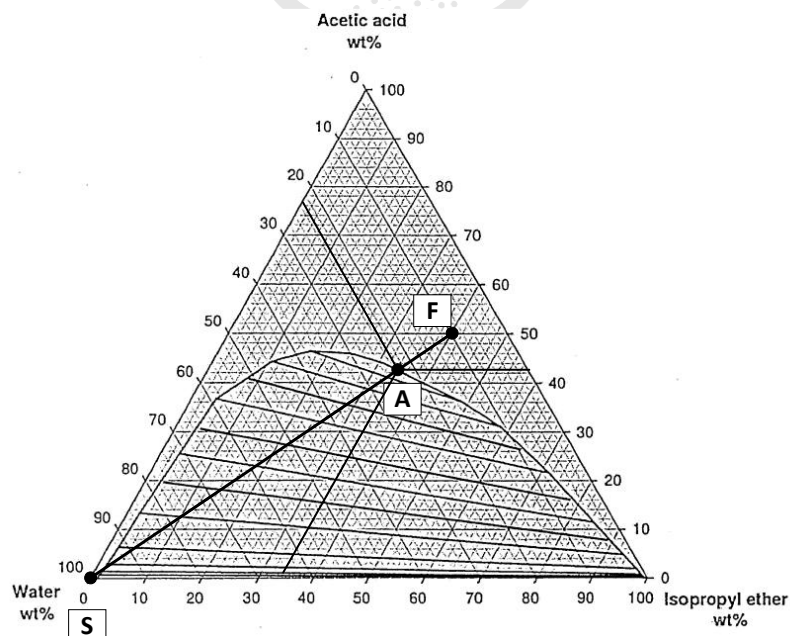$$F = \begin{cases} acetic\ acid = 0.5 \\ isopropyl\ ether = 0.4 \\ water = 0.1 \end{cases}, \quad S = pure\ water$$

$$A = final\ composition \begin{cases} water = 0.235 \\ acetic\ acid = 0.425 \\ isopropyl\ ether = 0.34 \end{cases}$$

Assume the pure water required is $x\ g\ /\ 100\ g\ mixture$,

by water balance,
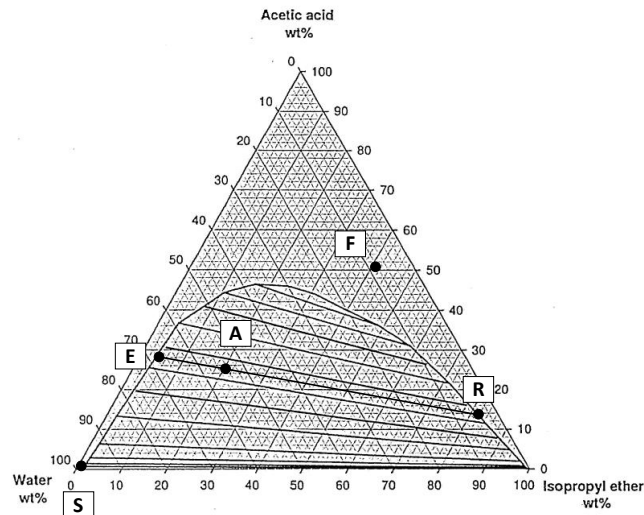
$$100 \times 0.1 + x = (100 + x) \times 0.235$$

$$\underline{x = 17.64\ (g)}$$

(b)

$\because$ *flow rate* : *feed* = *water*

$$\overline{FA} = \overline{SA} \text{ on the phase diagram}$$

並沿 A 做 tie line,

$$E = extract \begin{cases} water = 0.72 \\ acetic\ acid = 0.24 \\ isopropyl\ ether = 0.04 \end{cases}$$

$$R = raffinate \begin{cases} water = 0.14 \\ acetic\ acid = 0.04 \\ isopropyl\ ether = 0.82 \end{cases}$$

(c)

$$\frac{\overline{EA}}{\overline{RA}} \approx \frac{2}{7}$$

$$E : R = 7 : 2$$

$$E_{acetic\ acid} = (100 + 100) \times \frac{7}{7+2} \times 0.24 = \underline{\underline{37.33\ (g)}}$$

(本小題改編自成大期中考題。)

113 年成大單操輸送

Problem 1

【**Solution**】

(a)

By the definition of the velocity potential function,

$$v_x = \frac{\partial \phi}{\partial x} = u_\infty L(\frac{3x^2}{L^3} - \frac{3y^2}{L^3}) \quad 、 \quad v_y = \frac{\partial \phi}{\partial y} = -u_\infty L(\frac{6xy}{L^3})$$

By the definition of the stream function,

$$v_x = \frac{\partial \Psi}{\partial y} = u_\infty L(\frac{3x^2}{L^3} - \frac{3y^2}{L^3}) \quad , \quad \Psi = u_\infty L(\frac{3x^2 y}{L^3} - \frac{y^3}{L^3}) + f(x) + C$$

$$v_y = -\frac{\partial \Psi}{\partial x} = -u_\infty L(\frac{6xy}{L^3}) \quad , \quad \Psi = u_\infty L(\frac{3x^2 y}{L^3}) + g(y) + C$$

比較：

$$\Psi = u_\infty L(\frac{3x^2 y}{L^3} - \frac{y^3}{L^3}) \quad (令 C = 0)$$

(b)

$$\nabla \cdot v = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = u_\infty L(\frac{6x}{L^3}) - u_\infty L(\frac{6x}{L^3}) = 0$$

The continuity equation is satisfied.

※以數學來說，若有 Stream function 的先決條件即為 $\nabla \cdot v = 0$；而 volocity potential function

存在，則代表應有 $\nabla \times v = 0$ 的關係。

(本題可參考：**Welty, J.; Rorrer, G.; Foster, D.** *Fundamentals of Momentum, Heat, and Mass*

Transfer, **5th ed.; p 114~115/117~118.**)

(c)

$$\nabla \times v = \frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} = -u_\infty L(\frac{6y}{L^3}) + u_\infty L(\frac{6y}{L^3}) = 0$$

The flow is irrotational.

※可參考上題觀念，應可預期因為 velocity potential 存在，代表此 flow 應為 irrotational。

(d)

$\because \nabla \cdot v = 0$, The flow is irrotational. However, the flow need not be steady

incompressible flow because when $\nabla \cdot v = 0$, the continuity becomes,

$$\frac{\partial \rho}{\partial t} + v \cdot \nabla \rho + \rho \cancel{\nabla \cdot v} = 0$$

which doesnt't ensure a steady incompressible flow.

Problem 2
【Solution】

(a)

Laminar flow: A flow in smooth layers.

Newtonian: A fluid having constant μ.

Steady: A flow that doesn't vary with time.

Continuous: When at any time, the discharge at every section of the stream is the same.

Incompressible: A flow in which its density remains constant throughout its course.

Fully-developed: A flow where its velocity profile doesn't change along the axis of the flow.

(b)

Consider the equation of continuity,

$$\frac{\partial \cancel{\rho}}{\cancel{\partial t}} + \frac{\partial}{\cancel{\partial x}}(\cancel{\rho v_x}) + \frac{\partial}{\partial y}(\rho v_y) + \frac{\partial}{\cancel{\partial z}}(\cancel{\rho v_z}) = 0$$

$$\frac{\partial}{\partial y}(\rho v_y) = 0 \quad , \quad v_y \neq f(y)$$

By the equation of motion in y direction,

$$\rho(\cancel{\frac{\partial v_y}{\partial t}} + \cancel{v_x}\frac{\partial v_y}{\partial x} + v_y\cancel{\frac{\partial v_y}{\partial y}} + v_z\cancel{\frac{\partial v_y}{\partial z}}) = -\cancel{\frac{\partial p}{\partial y}} + \mu[\frac{\partial^2 v_y}{\partial x^2} + \cancel{\frac{\partial^2 v_y}{\partial y^2}} + \cancel{\frac{\partial^2 v_y}{\partial z^2}}] - \rho g$$

$$\mu\frac{\partial^2 v_y}{\partial x^2} - \rho g = 0 \begin{cases} x = 0, \ v_y = u_w \\ x = \delta, \ \dfrac{\partial v_y}{\partial x} = 0 \end{cases}$$

$$v_y = \frac{\rho g}{2\mu}x^2 + c_1 x + c_2 \begin{cases} c_1 = \dfrac{-\rho g}{\mu}\delta \\ c_2 = u_w \end{cases}$$

$$v_y = \frac{\rho g}{2\mu}x^2 + \frac{-\rho g}{\mu}\delta x + u_w$$

For the average velocity,

$$v_{avg} = \frac{\int_0^\delta (\frac{\rho g}{2\mu}x^2 + \frac{-\rho g}{\mu}\delta x + u_w)dx}{\int_0^\delta dx} = \frac{\frac{\rho g}{6\mu}\delta^3 - \frac{\rho g}{2\mu}\delta^3 + u_w\delta}{\delta} = \underline{\underline{-\frac{\rho g}{3\mu}\delta^2 + u_w}}$$

For the shear stress at the belt-fluid interface,

$$\tau\big|_{x=0} = -\mu\frac{\partial v_y}{\partial x}\bigg|_{x=0} = \underline{\underline{\rho g \delta}}$$

(本題可參考：**Munson, B.** *Fundamentals of Fluid Mechanics*, 6th ed.; p 312~313, Example 6.9.)

Problem 3
【Solution】

(a)

By Wien's Displacement Law,

$$\lambda_{peak}T = 2.898 \times 10^{-3} \ (m \cdot K)$$

$$\lambda_{peak} = \frac{2.898 \times 10^{-3}}{1227 + 273} = 1.932 \times 10^{-6} \ (m) = \underline{\underline{1932 \ (nm)}}$$

(b)

The total emissive power can be calculated by the Stefan-Boltzmann law,

$$E_b = \sigma T^4 = 5.676 \times 10^{-8} \times (1227 + 273)^4 = \underline{\underline{2.87 \times 10^5 \ (W/m^2)}}$$

(c)

First, we calculate the total radiated power emitted by the black ball

$$I_b = \sigma T^4 A_{ball}$$

The radiation intensity decreases with distance as the power disperses over a spherical surface. Thus, the radiation intensity per unit area at the detector's location is given by:

$$I = \frac{I_b}{4\pi d^2}$$

Consequently, the total radiant power received by the detector is:

$$I_D = \frac{I_b}{4\pi d^2} \times A_D = \frac{\sigma T^4 A_{ball}}{4\pi d^2} A_D = \frac{5.676 \times 10^{-8} \times (1227 + 273)^4 (0.02)^2}{2^2} \times 0.4 \times 10^{-4} = \underline{\underline{2.87 \times 10^{-4} (W)}}$$

(d)

The Biot number in this case is,

$$Bi = \frac{h(\frac{V}{A})}{k} = \frac{600 \times [\dfrac{\frac{4}{3}\pi(1 \times 10^{-2})^3}{4\pi(1 \times 10^{-2})^2}]}{100} = 0.02 < 0.1$$

<u>The lumped-parameter analysis is applicable.</u>

By the lumped-parameter analysis, (neglect effect of radiation)

$$\frac{4}{3}\pi\rho C r^3 \frac{\partial T}{\partial t} = h(4\pi r^2)(T_\infty - T)$$

$$\int_{T_0}^{T_f} \frac{dT}{(T_\infty - T)} = \int_0^t (\frac{3h}{r\rho C})dt$$

$$\ln(\frac{T_\infty - T_0}{T_\infty - T_f}) = \frac{3h}{r\rho C}t$$

$$\ln[\frac{300 - (1227 + 273)}{300 - T_f}] = \frac{3 \times 600}{1 \times 10^{-2} \times 10000 \times 200000} \times 90 \quad, \quad \underline{\underline{T_f = 1490\ (K) = 1217\ (°C)}}$$

(本題(a)~(c)小題可參考：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer,* **5th ed.; p 394, Problem 23.4.**)

Problem 4

【Solution】

(a)

Assume:

(1) one-directional mass transfer in z axis.

(2) water is insoluble in the film, $N_B = 0$

(3) no reaction occurred

By shell balance of ethanol (A),

$$N_A A_c \big|_z - N_A A_c \big|_{z+dz} = 0$$

同除 $A_c dz \to 0$

$$\frac{-dN_A}{dz} = 0$$

By Fick's law

$$N_A = -D_{AB} C \frac{dx_A}{dz} + x_A(N_A + \cancel{N_B}) \quad , \quad N_A = -\frac{D_{AB} C}{1 - x_A} \frac{dx_A}{dz}$$

代回：

$$\frac{d}{dz}\left[\frac{1}{1 - x_A} \frac{dx_A}{dz}\right] = 0 \quad \begin{cases} z = z_1, \ x_A = x_{A1} \\ z = z_2, \ x_A = x_{A2} \end{cases}$$

(b)

$$x_{A1} = \frac{\dfrac{5.4}{46}}{\dfrac{5.4}{46} + \dfrac{100 - 5.4}{18}} = \underline{\underline{0.0218}}$$

$$x_{A2} = \frac{\dfrac{14.6}{46}}{\dfrac{14.6}{46} + \dfrac{100 - 14.6}{18}} = \underline{\underline{0.0627}}$$

(c)

$$\frac{d}{dz}\left[\frac{1}{1-x_A}\frac{dx_A}{dz}\right]=0 \quad \begin{cases} z=z_1,\ x_A=x_{A1} \\ z=z_2,\ x_A=x_{A2} \end{cases}$$

積分 2 次：

$$\ln(1-x_A)=C_1 z+C_2 \quad,\quad C_1=\frac{\ln\left(\dfrac{1-x_{A1}}{1-x_{A2}}\right)}{z_1-z_2}$$

For the steady-state flux of ethanol,

$$N_A=-\frac{D_{AB}C}{1-x_A}\frac{dx_A}{dz}=\frac{D_{AB}C\ln\left(\dfrac{1-x_{A1}}{1-x_{A2}}\right)}{z_1-z_2}$$

For C, we use the average concentration between 1 and 2,

$$C_1=\frac{982.7\times10^3}{0.0218\times46+(1-0.0218)\times18}=52803.8\ mole/m^3$$

$$C_2=\frac{968.7\times10^3}{0.0627\times46+(1-0.0627)\times18}=49034.2\ mole/m^3$$

$$C=\frac{C_1+C_2}{2}=50919.0\ mole/m^3$$

$$N_A=\frac{D_{AB}C\ln\left(\dfrac{1-x_{A1}}{1-x_{A2}}\right)}{z_1-z_2}=\frac{7.40\times10^{-10}\times50919\times\ln\left(\dfrac{1-0.0218}{1-0.0627}\right)}{3\times10^{-3}}=\underline{\underline{5.36\times10^{-4}}}\ (mole/m^2\cdot s)$$

For water, it's is insoluble in the liquid film,

$$\underline{\underline{N_B=0}}$$

(d)

When the temperature rises, so does the diffusivity of ethanol, leading to an anticipated

increase in the steady-state flux of ethanol.

(e)

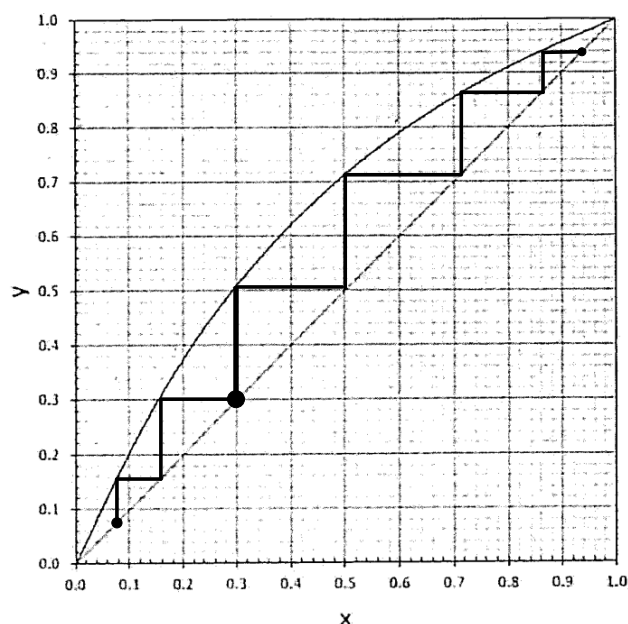$$N_A = kC(x_{A2} - x_{A1}) \text{ , } kC(x_{A2} - x_{A1}) = 5.36 \times 10^{-4}$$

$$k = \frac{5.36 \times 10^{-4}}{(0.0627 - 0.0218) \times 50919} = \underline{\underline{2.57 \times 10^{-7} \ (m/s)}}$$

Problem 5
【Solution】

(a)

假設題意為 top plate 可達到最高的 ethanol 濃度為多少；bottom plate 可達到之最低 ethanol 濃度

為多少，則假設無限回流比，則此時可用固定數量的板，達到最高與最低的塔頂/底濃度。

讀圖，可知最高塔頂之 benzene 濃度為 $x_D = 0.94$、最低可達成之塔底濃度為 0.08

(電腦作圖，可能有些許誤差，請以自己實際作圖為主)。

$$\begin{cases} x_{D,\max} = 0.94 \\ x_{W,\min} = 0.08 \end{cases}$$

※如果將題意理解為此時可達到之塔底/塔頂濃度為多少，就會有非常多種可能，例如：

這兩種都可以達成題目所要求之上塔 4 板、下塔 2 板的要求。若以這概念去想，需要試一陣子，而接續之子題目也依據你所畫出的結果去討論即可，在這裡我採用上面的想法去看這題。

(b)

此時回流比為無限大(∞)

(c)

By overall mole balance,

$$F = W + D$$

By mole balance of benzene,

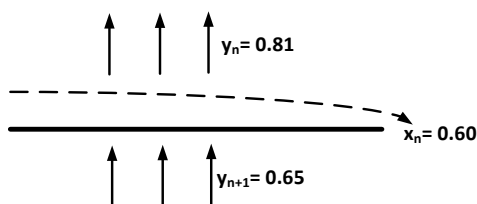$$Fx_F = Wx_W + Dx_D$$

$$0.3F = 0.08W + 0.94D$$

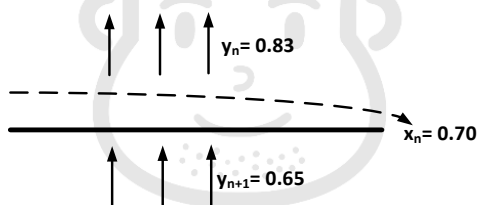$$\boxed{\begin{cases} W = 0.745F \\ D = 0.255F \end{cases}}$$

Problem 6

【Solution】

(a)

By the definition of Murphree effeciency,

$$\eta_M = \frac{y_n - y_{n+1}}{y_n^* - y_{n+1}} = \begin{cases} y_n = \textit{actual concentration of vapor leaving plate } n \\ y_{n+1} = \textit{actual concentration of vapor entering plate } n \\ y_n^* = \textit{concentration of vapor in equiibrium with liquid leaving plate } n \end{cases}$$

$$\eta_M = \frac{y_n - y_{n+1}}{y_n^* - y_{n+1}} = \frac{0.81 - 0.65}{0.6(2 - 0.6) - 0.65} = \underline{\underline{0.842}}$$

(b)

By the definition of local effeciency,

$$\eta' = \frac{y_n' - y_{n+1}'}{y_{en}^* - y_{n+1}'} = \begin{cases} y_n' = \textit{actual concentration of vapor leaving plate } n \\ y_{n+1}' = \textit{actual concentration of vapor entering plate } n \\ y_{en}^* = \textit{concentration of vapor in equiibrium with liquid at same location} \end{cases}$$
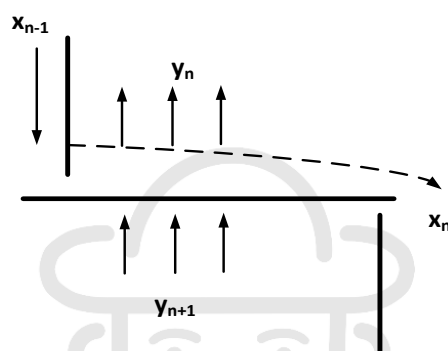
$$\eta' = \frac{y_n' - y_{n+1}'}{y_{en}^* - y_{n+1}'} = \frac{0.83 - 0.65}{0.7(2 - 0.7) - 0.65} = \underline{\underline{0.692}}$$

(c)

當 local efficiency 很大時，有可能會造成 y_n 變大，因此造成 x_n 變小，根據公式：

$$\eta_M = \frac{y_n - y_{n+1}}{y_n^* - y_{n+1}} = \frac{y_n(\uparrow) - y_{n+1}}{\underbrace{x_n(2-x_n)}_{\downarrow\downarrow} - y_{n+1}} > 1$$

※在 **109** 成大單操輸送 **Problem 6** 的題後討論中，已經討論過 **Murphree effeiciency** 的用處，其定義已在本題做描述。以氣體定義來說，從其分母可以看出來，基本上是以離開某板的液體，所對應到的平衡氣體濃度(假想)所創造的質傳驅動力，來作效率的描述,亦即若某板的交換情況為：
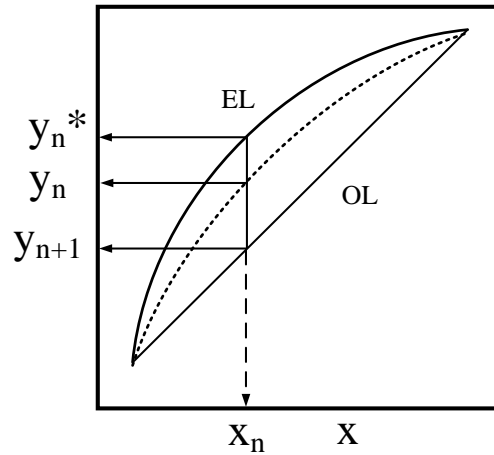

以氣相來說，下板氣體以平均濃度為 y_{n+1} 的組成上來後，在達成熱力學平衡時，理論上可以以 y_n^* 的濃度作為 y_n 離開(此 y_n^* 則是與 x_n 濃度達成熱力學平衡時的濃度)，然而因考慮到實際板表現與理論板有差別，因此會以實際上真正的 y_n 為組成濃度離開，此時就可以計算 **Murphree** 效率為：
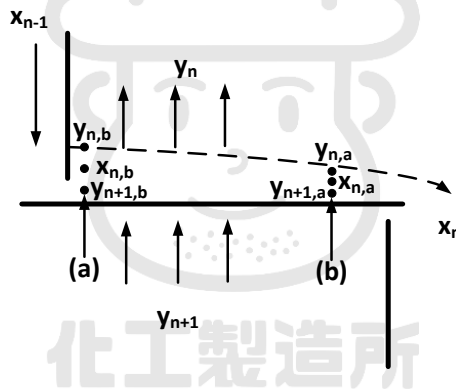
$$\eta_M = \frac{y_n - y_{n+1}}{y_n^* - y_{n+1}}$$

如上所述，Murphree 效率主要以整個板的平均濃度作為計算的參數，然而一個板上的每一個

點，其相對應的液體/氣體濃度可能都會有所差異。因此，若我們想明確知道板上某個點的情況

(當然實際很難測定)，我們也相對應的，定義了某一點的效率的算法，這就是 local efficiency，

若我們在板上的兩點(a)與(b)計算該兩點的 local efficiency，則可示意為：

其 local efficiency 分別為：

$$(a) = \eta_a' = \frac{y_{n,a} - y_{n+1,a}}{y_{en,a}^* - y_{n+1,a}} \quad , \quad y_{en,a}^* \text{ 由 } x_{n,a} \text{ 計算平衡得出}$$

$$(b) = \eta_b' = \frac{y_{n,b} - y_{n+1,b}}{y_{en,b}^* - y_{n+1,b}} \quad , \quad y_{en,b}^* \text{ 由 } x_{n,b} \text{ 計算平衡得出}$$

因此，不論分子或分母，local efficiency 皆因不同點，而有不同參數；反之，Murphree efficiency

基本上分子就是以整個板平均為主，此外分母則是以"離開該板的液體/氣體"作為定義，是為兩

者不同的地方。一般來說，在尺寸較大、質傳效率差、以及混合較不均勻的塔中，液體在沿著水

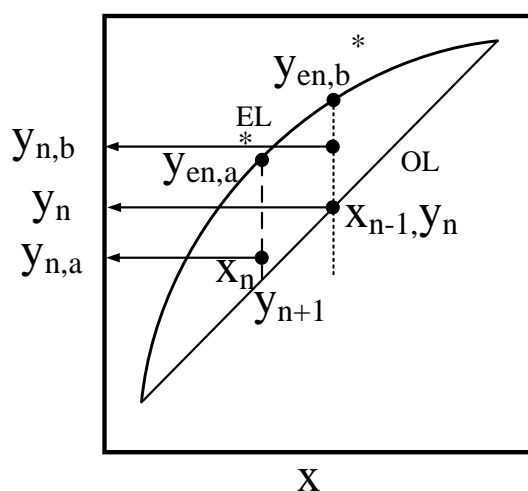平方向，會有明顯的濃度落差，亦即進口附近液體濃度靠近 x_{n-1}、出口位置靠近 x_n。因此，在

進口附近，其接觸的氣體會受到較濃的液體影響，而有比平均更高的濃度；反之，在出口附近，

因其通過的氣體，接觸的液體濃度相對較低，因此其通過後氣體濃度，比平均更低，即：
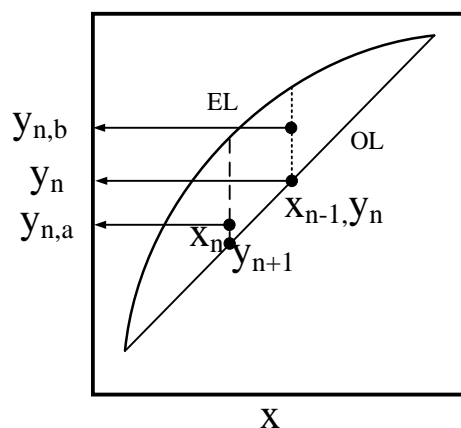
其中，y_n 為整個板的平均濃度，$y_{n,b}$ 則為該板接近液體進口端的氣體濃度($y_{n,b} > y_n$)，而 $y_{n,a}$

為接近該板出口端的氣體濃度($y_{n,a} < y_n$)。然而，不論在哪個點，根據定義，其效率分母皆為該

點最大可能的濃度差，假設下板進來的氣體均勻，在每個點都以 y_{n+1} 的濃度進來，則(a)與(b)的
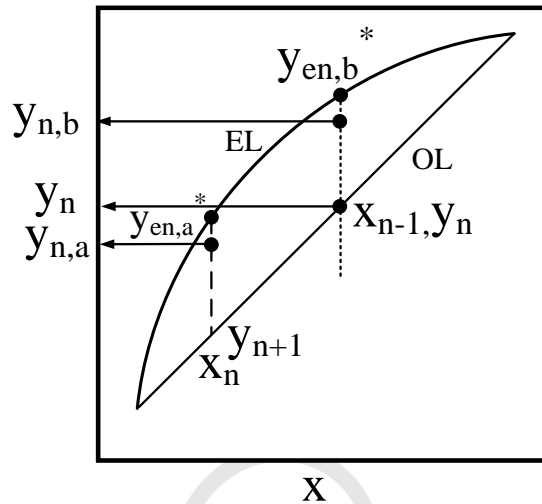
local efficiency 為：

$$\eta_a^{'} = \frac{y_{n,a} - y_{n+1}}{y_{en,a}^{*} - y_{n+1}} < 1$$

$$\eta_b^{'} = \frac{y_{n,b} - y_{n+1}}{y_{en,b}^{*} - y_{n+1}} < 1$$

因此，我們不難發現，$\boxed{\text{local efficiency 必定}\ <1}$

然而，<u>**Murphree Efficiency 就沒有此限制**</u>，我們假設同樣在進出口端的液體有濃度差，且因為其氣液混合比較均勻，以及質傳效率比較高，因此 local efficiency 皆比較高，則：

因此時混合較好的關係，我們可以預期有較多液體中的溶質質傳到氣體之中，因此其 x_n 相比前面的例子來說，應該會左移一些($x_n\downarrow$，也就是解答中的 x_n 下降；以及 y_n 平均起來應該會上升，亦即解答中的分子上升)。若我們此時要計算 **Murphree efficiency**，則根據定義為：

$$\eta_M = \frac{y_n - y_{n+1}}{y_n^* - y_{n+1}} = \frac{y_n - y_{n+1}}{y_{en,a}^* - y_{n+1}}$$

其中，因為 **Murphree efficiency** 的分母固定為定義出口液體(x_n)所平衡的氣體濃度，因此以圖上的對應來說大約為 $x_{en,a}^*$，又以圖上的情況來說，我們不難看出 y_n 有機會$> y_{en,a}^*$，因此，得出了一個重要的結論：$\boxed{\text{Murphree efficiency 可能}\ >1}$。一般此種情況可藉由增加 local efficiency 的方式達成，常見的操作手段為增加塔的直徑。
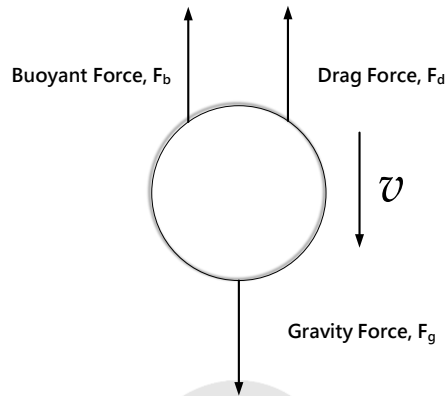
此外，因為兩種效率分母定義不同的關係，$\boxed{\text{Murphree > local}}$，因 Murphree 定義在出口液體濃度，通常為一個板濃度最小的液體區，因此會得到效率值較大的結果。

以上討論摘錄至成大化工系所提供的單操講義 **ppt**(應可直接搜尋觀看)，以及 **McCabe, W.; Smith, J.; Harriott, P.** *Unit Operations of Chemical Engineering*, **7th ed.; p712~714**。請務必親自查看。

114 年成大單操輸送

Problem 1

【Solution】

By force balance on the particle,

$$F = ma = m\frac{dv}{dt} = F_g - F_b - F_d$$

As the particle reaches terminal velocity,

$$m\frac{d\cancel{v}}{\cancel{dt}} = mg - \frac{1}{6}\pi D_s^3 \rho_L g - (\frac{1}{2}\rho_L v^2)(\underbrace{\frac{\pi}{4}D_s^2}_{投影面積})C_D = 0$$

【假定此時為 Stokes drag region】

$$(\rho_s - \rho_L)(\frac{1}{6}\pi D_s^3)g - (\frac{1}{2}\rho_L v^2)(\frac{\pi}{4}D_s^2)\frac{24}{\dfrac{\rho_L v D_s}{\mu}} = 0$$

$$(4500-1260)[\frac{1}{6}\pi(41\times10^{-3})^3]\times9.8 - (\frac{1}{2}\times1260\times v^2)[\frac{\pi}{4}(41\times10^{-3})^2]\frac{24}{\dfrac{1260\times v\times41\times10^{-3}}{1.46}} = 0$$

$$v = 2.03\ (m/s)$$

【檢查 Reynolds number】

$$Re = \frac{1260\times2.03\times41\times10^{-3}}{1.46} = 71.83，不適用假設$$

【以 Re = 70，對應 $C_D = 1.5$】

代回

$$(\rho_s - \rho_L)(\frac{1}{6}\pi D_s^3)g - (\frac{1}{2}\rho_L v^2)(\frac{\pi}{4}D_s^2)C_D = 0$$

$$(4500 - 1260)[\frac{1}{6}\pi(41 \times 10^{-3})^3] \times 9.8 - (\frac{1}{2} \times 1260 \times v^2)[\frac{\pi}{4}(41 \times 10^{-3})^2] \times 1.5 = 0$$

$$v = 0.958 \ (m/s)$$

【檢查 Reynolds number】

$$Re = \frac{1260 \times 0.958 \times 41 \times 10^{-3}}{1.46} = 33.89 \ ，\text{非 Re} = 70$$

【以 Re = 30，對應 $C_D = 2$】

代回

$$(\rho_s - \rho_L)(\frac{1}{6}\pi D_s^3)g - (\frac{1}{2}\rho_L v^2)(\frac{\pi}{4}D_s^2)C_D = 0$$

$$(4500 - 1260)[\frac{1}{6}\pi(41 \times 10^{-3})^3] \times 9.8 - (\frac{1}{2} \times 1260 \times v^2)[\frac{\pi}{4}(41 \times 10^{-3})^2] \times 2 = 0$$

$$v = 0.83 \ (m/s)$$

【檢查 Reynolds number】

$$Re = \frac{1260 \times 0.83 \times 41 \times 10^{-3}}{1.46} = 29.36 \approx 30$$

$$\underline{\underline{v = 0.83 \ (m/s)}}$$

Problem 2

【Solution】

The kinetic correction factor for Mechanical energy balance equation is,

$$\alpha = \frac{\dot{E}_k}{m} = \frac{\int_A v^3 dA}{v_{avg}^3 A} = \frac{2\pi \int_0^R v^3 r dr}{v_{avg}^3 (\pi R^2)} = \frac{2\int_0^R [v_{max}(1-\frac{r}{R})^{\frac{1}{6}}]^3 r dr}{(\frac{72}{91}v_{max})^3 R^2} = \frac{2\int_0^1 [v_{max}(1-\varepsilon)^{\frac{1}{6}}]^3 \varepsilon d\varepsilon}{(\frac{72}{91}v_{max})^3} = \underline{\underline{1.077}}$$

※一般機械能平衡方程式的動能項皆以平均速度作為代表代入計算，然而因為實際流體速度具

有一定分布，因此若是單純以平均速度作為參數代入，在具有明顯速度分布的流場中將會產生一

定誤差，因此需要在動能項前面加入一個修正係數，來修正為真正的流體動能，例如 laminar flow

的修正常數為 2；turbulent flow 的修正係數接近為 1。此可參考 McCabe, W.; Smith, J.; Harriott,

P. *Unit Operations of Chemical Engineering*, **7th ed.; p 89**。此外，關於此題所提及之 Prandtl's

power law，在 111 年成大單操輸送 Problem 2 有出現過，可參考該題相關概念。

Problem 3

【Solution】

(a)

The bulk average temperature of the fluid A is,

$$T_{avg} = \frac{\int_0^{2\pi} \int_0^{R_o} u \cdot T r dr d\theta}{\int_0^{2\pi} \int_0^{R_o} u \cdot r dr d\theta} = \frac{2\int_0^{R_o} T r dr}{R_o^2} = \frac{2\int_0^{R_o} \{T_s + m[1-(\frac{r}{R_o})^4]\} r dr}{R_o^2}$$

$$\underline{\underline{T_{avg} = T_s + \frac{2m}{3}}}$$

(b)

Near the inside wall,

$$q_{conduction} = q_{convection}$$

$$-k \frac{dT}{dr}\Big|_{r=R_o} = h\underbrace{(T\big|_{r=R_o} - T_{avg})}_{\text{高溫->低溫（設 m < 0）}} \text{ , } k \times \frac{4m}{R_o} = h[(T_s + \frac{2m}{3}) - T_s] \text{ , } \underline{\underline{h = \frac{6k}{R_o}}}$$

(c)

By the definition of Nusselt number,

$$Nu = \frac{hD}{k} = \frac{h(2R_o)}{k} = \underline{\underline{12}}$$

(d)

Assume the inlet temperature is uniform, by energy balance from $z=0$ to $z=L$,

$$\underbrace{u(\pi R_o^2)C_p T_0}_{in} - \underbrace{u(\pi R_o^2)C_p T_{avg}(z)}_{out} + q_{gen} = 0$$

【**Case (1)** – constant wall temperature T_s】

$$q_{gen} = h(2\pi R_o dz)(T_s - T_{avg})$$

$$u(\pi R_o^2)C_p T_{avg}(z)\Big|_z - u(\pi R_o^2)C_p T_{avg}(z)\Big|_{z+dz} + h(2\pi R_o dz)(T_s - T_{avg}) = 0$$

同除 $dz \to 0$

$$-u(\pi R_o^2)C_p \frac{dT_{avg}}{dz} + h(2\pi R_o)(T_s - T_{avg}) = 0$$

$$\frac{d(T_{avg})}{(T_s - T_{avg})} = \frac{2h}{uR_o C_p} dz$$

$$\ln(\frac{T_s - T_L}{T_s - T_0}) = -\frac{2h}{uR_o C_p} L$$

$$\underline{\underline{T_L = T_s - (T_s - T_0)\exp(-\frac{2h}{uR_o C_p}L)}}$$

【**Case (2)** – constant heat flux q_0 】

$$q_{gen} = q_0(2\pi R_o dz)$$

$$u(\pi R_o^2)C_p T_{avg}(z)\big|_z - u(\pi R_o^2)C_p T_{avg}(z)\big|_{z+dz} + q_0(2\pi R_o dz) = 0$$
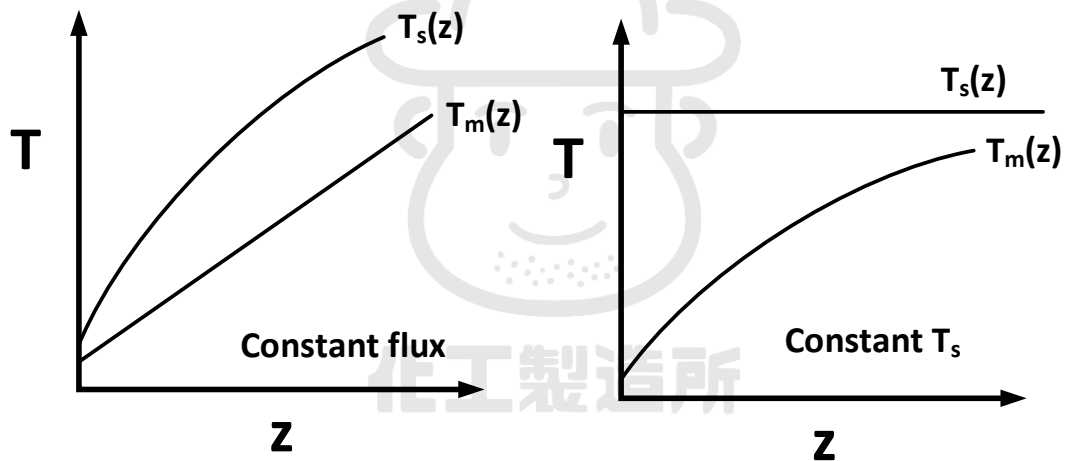
同除 $dz \to 0$

$$-u(\pi R_o^2)C_p \frac{dT_{avg}}{dz} + q_0(2\pi R_o) = 0$$

$$d(T_{avg}) = \frac{2q_0}{uR_o C_p} dz$$

$$\underline{\underline{T_L = T_0 + \frac{2q_0}{uR_o C_p} L}}$$

(e)

(**本題可參考：Incropera, F.; Lavine, A.; Bergman, T.** *Fundamentals of Heat and Mass Transfer*,

6th ed., p 483~493)

Problem 4

【Solution】

(a)

In region *I*, we can feasibly assume,

(1) The system is in steady state

(2) The diffusion process is 1-D in r-direction (θ symmetric)

(3) Constant properties

(4) The surrounding fluid flows so rapidly that it instantly carries species B and C away from the surface, making their concentrations negligible in the bulk stream.

(5) The catalyst (component D) is stagnant.

(6) Uniform geometry

(b)

For N_A, by Fick's law,

$$N_A = -D_A \frac{dC_A}{dr} + y_A (N_A + N_B + N_C + \cancel{N_D})$$

(其中，以該種情況來說，$N_A \neq -N_B \neq -N_C$)

$$\boxed{N_A = -D_A \frac{dC_A}{dr} + y_A (N_A + N_B + N_C)}$$

If the process occurs in a dilute system, by Fick's law,

$$\boxed{N_A = -D_A \frac{dC_A}{dr} + \cancel{y_A} (N_A + N_B + N_C) = -D_A \frac{dC_A}{dr}}$$

(c)

By mass balance of A in region *I*,

$$N_A(2\pi r^2)\big|_r - N_A(2\pi r^2)\big|_{r+dr} + r_A(2\pi r^2)dr = 0$$

同除 $2\pi dr \to 0$ ：

$$-\frac{d(N_A r^2)}{dr} + r_A r^2 = 0$$

In the control volume, A is reacted into B following 1st order mechanism,

$$r_A = -k_1 C_A$$

$$\boxed{-\frac{d(N_A r^2)}{dr} - k_1 C_A r^2 = 0}$$

(d)

For species A,

$$\begin{cases} r = R_1, C_A = 0 \ (diffusion\ limited) \\ r = R_2, C_A = C_{AO} \end{cases}$$
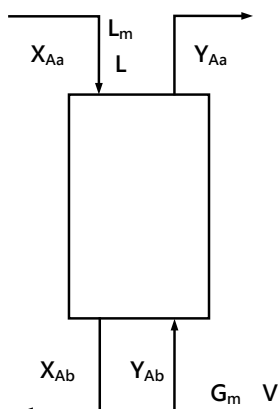
For species B,

$$\begin{cases} r = R_1, \dfrac{dC_B}{dr} = 0 \ (impermeable) \\ r = R_2, C_B = 0 \ (negligible,\ from\ assumption\ of\ question\ (a)) \end{cases}$$

(本題改編自：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer*, 7th ed., Problem 25.10。)

Problem 5

【Solution】

G_m = gas molar flow rate on a solute-free basis.

L_m = liquid flow rate on a solute-free basis.

∵ the concentration of ammonia (denoted as A) is dilute, let

$$x_{Ai} \approx X_{Ai} = \frac{x_{Ai}}{1-x_{Ai}} \quad , \quad y_{Ai} \approx Y_{Ai} = \frac{y_{Ai}}{1-y_{Ai}} \quad , \quad L \approx L_m \quad , \quad G \approx G_m$$

(a)

$$x_{Aa} = 0 \quad , \quad y_{Ab} = 0.01$$

For y_{Aa}, by mole balance of A in the gas phase,

$$y_{Aa} = 0.1 \times (1 - 0.97) = 0.0003$$

Initially, to calculate the minimum flow rate, we assume y_{Aa} is in equilibrium with x_{Aa},

assume the equilibrium line slope is m,

$$x_{Ab} = \frac{1}{m} y_{Ab} = \frac{1}{m} \times 0.01 = \frac{0.01}{m}$$

By total mole balance of A,

$$(\frac{L}{G})_{min} = \frac{(y_{Aa} - y_{Ab})}{(x_{Aa} - x_{Ab})} = \frac{0.0003 - 0.01}{0 - \frac{0.01}{m}} = \frac{0.0097m}{0.01} = 0.97m$$

Therefore, for the liquid rate at 1.25 times the minimum,

$$1.25(\frac{L}{G})_{\min} = 1.2125m$$

The operating line becomes,

$$\frac{L}{G}(x_{Aa} - x_{Ab}) = (y_{Aa} - y_{Ab})$$

$$1.2125m(0 - x) = (y - 0.01)$$

$$\underline{\underline{y = -1.2125mx + 0.01}}$$

(b)

The number of transfer unit is defined as,

$$N_y = \frac{y_b - y_a}{\dfrac{(y_b - y_b^*) - (y_a - y_a^*)}{\ln(\dfrac{y_b - y_b^*}{y_a - y_a^*})}}$$

$$y_{Ab} = 0.01 \text{ , } y_{Aa} = 0.0003$$

$$y_{Aa}^* = mx_{Aa} = m \times 0 = 0$$

For x_{Ab}, by operating line,

$$y_{Ab}^* = mx_{Ab}$$

$$0.0003 = -1.2125mx_{Ab} + 0.01 \text{ , } x_{Ab} = \frac{0.008}{m}$$

$$y_{Ab}^* = mx_{Ab} = 0.008$$

代回：

$$N_y = \frac{0.01 - 0.0003}{\dfrac{(0.01 - 0.008) - (0.0003 - 0)}{\ln(\dfrac{0.01 - 0.008}{0.0003 - 0})}} = \underline{\underline{10.82}}$$

Problem 6

【Solution】

(a)

For constant rate zone, $R = constant$

$$R = -\frac{m_s}{A}\frac{dX}{dt}$$

移項積分：

$$\int_0^{t_T} dt = -\frac{m_s}{AR}\int_{X_1}^{X_2} dX \quad , \quad \boxed{t_T = \frac{m_s(X_1 - X_2)}{AR}}$$

(b)

The total drying process involves two periods,

$$t_T = \underset{X_1 \to X_c}{t_c} + \underset{X_c \to X_2}{t_f}$$

For constant rate zone,

$$t_c = \frac{m_s(X_1 - X_c)}{AR}$$

For falling rate zone,

$$R = -\frac{m_s}{A}\frac{dX}{dt} = aX$$

移項積分：

$$\int_0^{t_f} dt = -\frac{m_s}{aA}\int_{X_c}^{X_2} \frac{dX}{X}$$

$$t_f = \frac{m_s}{aA}\ln(\frac{X_c}{X_2})$$

The total drying time is,

$$t_T = t_c + t_f = \frac{m_s(X_1 - X_c)}{AR} + \frac{m_s}{aA}\ln(\frac{X_c}{X_2})$$

(c)

The total drying time is,

$$t_T = t_c + t_f = \frac{m_s(X_1 - X_c)}{AR} + \frac{m_s}{aA}\ln(\frac{X_c}{X_2})$$

$$8 = \frac{m_s(0.35 - 0.20)}{AR} + \frac{m_s}{aA}\ln(\frac{0.20}{0.10}) = \frac{m_s}{A}[\frac{(0.35 - 0.20)}{R} + \frac{1}{a}\ln(\frac{0.20}{0.10})]$$

At the critical content, the drying rate is continuious,

$$R = a \times 0.2$$

Therefore,

$$8 = \frac{m_s}{A}[\frac{(0.35 - 0.20)}{a \times 0.2} + \frac{1}{a}\ln(\frac{0.20}{0.10})] = \frac{m_s}{aA}[\frac{(0.35 - 0.20)}{0.2} + \ln(\frac{0.20}{0.10})]$$

$$8 = \frac{m_s}{aA}[\frac{(0.35 - 0.20)}{0.2} + \ln(\frac{0.20}{0.10})]$$

$$\frac{m_s}{aA} = 5.543$$

For the drying process from 40% to 5%,

$$t_T = \frac{m_s}{aA}[\frac{(0.40 - 0.20)}{0.2} + \ln(\frac{0.20}{0.05})] = 5.543 \times [\frac{(0.40 - 0.20)}{0.2} + \ln(\frac{0.20}{0.05})] = \underline{\underline{13.23\ (hr)}}$$