106 年中央單操輸送

<u>Problem 1</u>

【Solution】

For steel pipe and other rough pipes,

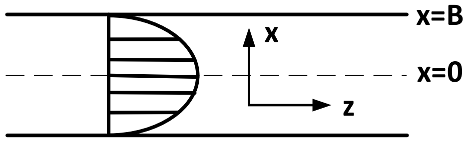
$$f = \begin{cases} f(\mathrm{Re}) \ \textit{for laminar flow} \\ f\left(\dfrac{\varepsilon}{D}\right) \ \textit{for turbulent flow} \ (\mathrm{Re} > 10^6) \end{cases}$$

ε is the relative roughness factor

(本題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 107/113~116.)

Problem 2

【Solution】



By the equation of motion,

$$\cancel{\rho\frac{D\vec{v}}{Dt}} = -\nabla P - [\nabla\cdot\overset{=}{\tau}] + \cancel{\rho\vec{g}}$$

For flow in z-direction,

$$\nabla\cdot\overset{=}{\tau} + \nabla P = \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial P}{\partial z} = 0$$

$$\tau_{xz} = (\frac{-\partial P}{\partial z})x + c_1$$

For $0 \le x \le B$,

$$\tau_{xz} = m(\frac{-dv_z}{dx})^n \text{ 代入}$$

$$m(\frac{-dv_z}{dx})^n = (\frac{-\partial P}{\partial z})x + c_1 \text{ ，} (\frac{-dv_z}{dx})^n = \frac{1}{m}(\frac{-\partial P}{\partial z})x + \alpha_1$$

$$v_z = \frac{-n}{n+1}[\frac{1}{m}(\frac{-\partial P}{\partial z})x]^{\frac{n+1}{n}} + \beta_1 x + \beta_2 \begin{cases} x=0, \ \dfrac{dv_z}{dx} = 0 \\ x=B, \ v_z = 0 \end{cases}$$

$$\begin{cases} \beta_1 = 0 \\ \beta_2 = \dfrac{n}{n+1}[\dfrac{1}{m}(\dfrac{-\partial P}{\partial z})]^{\frac{1}{n}} B^{\frac{n+1}{n}} \end{cases}$$

$$\underline{\underline{v_z = \frac{n}{n+1}[\frac{1}{m}(\frac{-\partial P}{\partial z})]^{\frac{1}{n}} B^{\frac{n+1}{n}}[1-(\frac{x}{B})^{\frac{n+1}{n}}]}}$$

For $Q(-B \le x \le B) = 2Q(0 \le x \le B)$

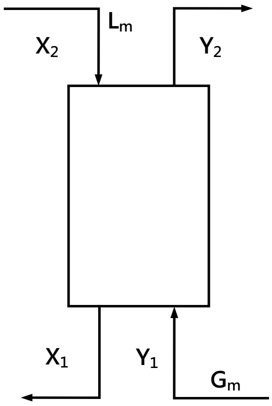
$$Q = 2\int_0^W \int_0^B v_z dxdy = \underline{\underline{\frac{2WB^2 n}{2n+1}[(\frac{-\partial P}{\partial z})B]^{\frac{1}{n}}}}$$

$$\text{where, } (\frac{-\partial P}{\partial z}) = \frac{P_0 - P_L}{L}$$

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 243, Example 8.3-2.)

<u>Problem 3</u>

【Solution】



$$X_2 = 0 \text{ , } Y_1 = 0.04 \text{ , } X_1 = 0.015$$

For Y_2, which is 3% of Y_1, we have $Y_2 = 0.04 \times 0.03 = 0.0012$

(a) To calculate the height of the absorber,

$$Z = \underbrace{\frac{G_m'}{K_G'' a}}_{H_{OG}} \underbrace{\frac{Y_1 - Y_2}{(Y - Y_e)_{lm}}}_{N_{OG}}$$

For $(Y - Y_e)_{lm}$, we have,

$$(Y - Y_e)_{lm} = \frac{(Y_1 - Y_{1e}) - (Y_2 - Y_{2e})}{\ln(\frac{Y_1 - Y_{1e}}{Y_2 - Y_{2e}})} = \frac{(0.04 - 2.5 \times 0.015) - (0.0012 - 0)}{\ln(\frac{0.04 - 2.5 \times 0.015}{0.0012 - 0})} = 1.77 \times 10^{-3}$$

代回：

$$Z = \frac{0.017}{0.03} \cdot \frac{(0.04 - 0.0012)}{1.77 \times 10^{-3}} = 12.41\ (m)$$

(b)

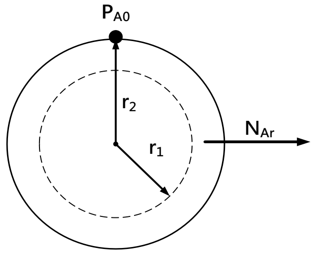
The number of transfer units is the log-mean term for the calculation of height, which is,

$$N_{OG} = \frac{Y_1 - Y_2}{(Y - Y_e)_{lm}} = \frac{(0.04 - 0.0012)}{1.77 \times 10^{-3}} = \underline{\underline{21.91}}$$

(本題改編自：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 1170, Problem 12.4.)

Problem 4

【Solution】



Assume,

$$\begin{cases} naphthalene = A \\ air = B \end{cases}$$

By pseudo-steady-state assumption on the naphthalene particle,

$$-(4\pi r^2)N_{Ar}\Big|_r = \frac{d(\frac{4}{3}\pi r^3\rho)}{Mdt} = \frac{4\pi r^2\rho}{M}\frac{dr}{dt}$$

$$N_{Ar}\Big|_r = \frac{-\rho}{M}\frac{dr}{dt}$$

For N_{Ar} , by Fick's law,

$$N_{Ar} = -\mathscr{D}_{AB}\frac{dC_{Ar}}{dr} + \underbrace{\cancel{y_A}}_{very\ dilute}(N_{Ar}+N_{Br}) = \frac{-\mathscr{D}_{AB}P}{RT}\frac{dy_{Ar}}{dr}$$

$$N_{Ar} = \frac{w_{Ar}}{4\pi r^2} = \frac{-\mathscr{D}_{AB}P}{RT}\frac{dy_{Ar}}{dr}, \ \frac{w_{Ar}}{4\pi}\int_r^{\infty}\frac{dr}{r^2} = \frac{-\mathscr{D}_{AB}P}{RT}\int_{y_{A0}}^{0} dy_{Ar}$$

$$w_{Ar} = 4\pi r\frac{\mathscr{D}_{AB}P}{RT}(y_{A0}-0)$$

$$N_{Ar} = \frac{w_{Ar}}{4\pi r^2} = \frac{\mathscr{D}_{AB}P_{A0}}{RTr} \text{ 代回}$$

$$\frac{\mathscr{D}_{AB}P_{A0}}{RTr} = \frac{-\rho}{M}\frac{dr}{dt} \text{ , } dt = \frac{-\rho RT}{\mathscr{D}_{AB}P_{A0}M}rdr \text{ , } \int_0^t dt = \frac{-\rho RT}{\mathscr{D}_{AB}P_{A0}M}\int_{r_0}^{r_1} rdr$$

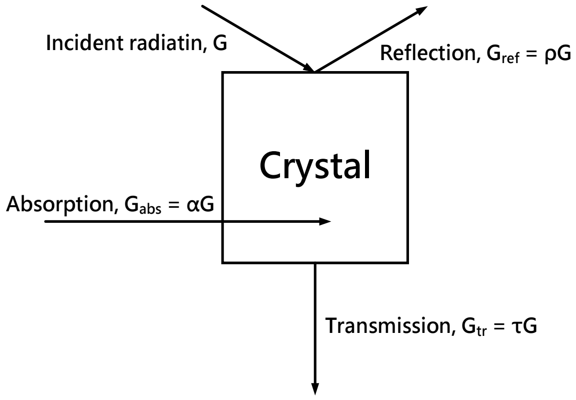
$$t = \frac{\rho RT}{2\mathscr{D}_{AB}P_{A0}M}(r_0^2 - r_1^2)$$

$$= \frac{1.145\times10^3\times8.314\times(273+60)}{2\times8.19\times10^{-6}\times245\times128\times10^{-3}}[(\frac{3}{2}\times10^{-2})^2-(\frac{1.5}{2}\times10^{-2})^2]$$

$$= \underline{\underline{1041390.4\ (s)}}$$

Problem 5

【Solution】



(a)

By energy balance on the crystal,

$$\underbrace{\alpha G}_{energy\ absorbed} - \underbrace{E(emissive\ power)}_{energy\ release} = 0$$

Assume the crystal is opaque,

$$E - \alpha G = \varepsilon\sigma(T_c^4 - T_\infty^4) - \alpha G = 0$$

$$0.8 \times 5.67 \times 10^{-8}(T_c^4 - 300^4) - 0.08 \times 1 \times 10^{13} \times 10^6 \times 10 \times 1.6 \times 10^{-16} = 0$$

$$\underline{\underline{T_c = 436.4\ (K)}}$$

(b)

By energy balance of the crystal,

$$(G - G_{ref} - E)A = \frac{\partial(\rho V C_P T_c)}{\partial t}$$

$$[\alpha G - \varepsilon\sigma(T_c^4 - 300^4)] = \rho \underbrace{\frac{V}{A}}_{L} C_P \frac{\partial T_c}{\partial t}$$

$$\int_0^t dt = \int_{T_0}^{T_1} \frac{\rho L C_P}{[\alpha G - \varepsilon\sigma(T_c^4 - 300^4)]} dT_c$$

$$t = \int_{300}^{273+40} \frac{1.2 \times 10^{-3} \times 0.2 \times 6}{0.08 \times 1 \times 10^{13} \times 10 \times 1.6 \times 10^{-16} - 0.8 \times 5.67 \times 10^{-8} \times 10^{-6}(T_c^4 - 300^4)} dT_c = \underline{\underline{15.02\ (s)}}$$

(c)

Assume this case is similar with the case of fluid flowing above a plate,

$$\delta(x)=\frac{5x}{\sqrt{\mathrm{Re}_x}}=\frac{5x}{\sqrt{\frac{v_\infty x}{\nu}}}=\frac{5x}{\sqrt{\frac{1.5x}{1.9\times10^{-6}}}}=5.627\times10^{-3}\sqrt{x}$$

$\because (\frac{\delta}{\delta_T})^3=\mathrm{Pr}$,

$$[\frac{5.627\times10^{-3}\sqrt{x}}{\delta_T(x)}]=0.69^{\frac{1}{3}}$$

$$\underline{\underline{\delta_T(x)=6.368\times10^{-3}\sqrt{x}}}$$

$$\boxed{\mathrm{Pr}=\frac{momentum\ boundary\ layer\ thickness}{thermal\ boundary\ layer\ thickness}}$$

(d)

For Biot number,

$$Bi=\frac{hL_e}{k_s},$$

$$\text{Where }\begin{cases}L_e=charateristic\ length\\ k_s=the\ conductivity\ of\ \ the\ crystal\end{cases}$$

但此題並未給定 crystal 的 conductivity 值，應該沒辦法往下算

若題目給定 crystal 之 thermal conductivity，且檢查結果 $Bi<0.1$，則 crystal 經由液態氮向外之熱傳可用 lumped-capacity analysis 做計算，即

$$\underbrace{(G-G_{ref}-E)A}_{radiation}-\underbrace{hA(T-T_{100K})}_{convection}=\frac{\partial(\rho VC_PT_c)}{\partial t}$$

For h , we use average heat transfer coefficient over the length of the crystal, which is

$$\bar{h}_L = \frac{\int_0^L \bar{h}_x dx}{L} = \frac{\int_0^L \frac{k}{\delta_t} dx}{L} = \frac{9.8\times10^{-3}}{0.2\times10^{-3}} \times \int_0^L \frac{1}{6.368\times10^{-3}\sqrt{x}}$$

$$= \frac{9.8\times10^{-3}}{0.2\times10^{-3}} \times \frac{2}{6.638\times10^{-3}} \cdot \sqrt{0.2\times10^{-3}} = 208.78\ (W/m^2\cdot K)$$

代回：

$$[\alpha G - \varepsilon\sigma(T_c^4 - 100^4)] - h(T - T_{100K}) = \rho \underset{L}{\frac{V}{A}} C_P \frac{\partial T_c}{\partial t}$$

$$\int_0^t dt = \int_{100}^{T_1} \frac{\rho L C_P}{[\alpha G - \varepsilon\sigma(T_c^4 - 100^4) - h(T - T_{100K})]} dT_c$$

$$t = \int_{100}^{273.15+30} \frac{1.2\times10^{-3}\times0.2\times6}{0.08\times1\times10^{13}\times10\times1.6\times10^{-16} - 0.8\times5.67\times10^{-8}\times10^{-6}(T_c^4 - 100^4) - 208.78\times10^{-6}(T_c - 100)} dT_c$$

$$< 0$$

<u>The sample will never reach 30℃ during the test.</u>

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Problem 1

【Solution】

(a)

The continuum assumption is valid when the molecule mean free path is much less than the length scale of the tube.

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 52.)

(b)

$$\text{Scaling factors：}\begin{cases} v = velocity\ of\ \ the\ fluid \\ \rho = density\ of\ \ the\ fluid \\ \mu = viscosity\ of\ \ the\ fluid \\ D = diameter\ of\ \ the\ tube \end{cases}$$

$$Re = \frac{\rho v D}{\mu} = \frac{inertial\ force}{viscous\ force}$$

(c)

$$\begin{cases} I.C.\ ：t = 0,\ v = 0 \\ B.C.\ ：r = \dfrac{D}{2},\ v = 0 \\ B.C.\ ：r \to \infty,\ v = v_\infty \end{cases}\text{，令}\begin{cases} t^* = \dfrac{t}{t_0} \\ v^* = \dfrac{v}{v_\infty} \\ r^* = \dfrac{r}{D} \end{cases}$$

$$\boxed{\begin{cases} t^* = 0,\ v^* = 0 \\ r = \dfrac{1}{2},\ v^* = 0 \\ r \to \infty,\ v^* = 1 \end{cases}}$$

(d)

By the equation of change of momentum,

$$\rho\frac{Dv}{Dt}=\mu\nabla^2 v-\nabla P+\underbrace{\cancel{\rho g}}_{neglected}\text{，令}\begin{cases}P^*=\dfrac{P}{P_B}\\ \dfrac{D}{Dt}=\dfrac{D}{Dt^*}\cdot\dfrac{1}{t_0}\\ \nabla=\dfrac{1}{D}\nabla^*\\ \nabla^2=\dfrac{1}{D^2}\nabla^{*2}\end{cases}$$

$$\rho\frac{v_\infty}{t_0}\frac{Dv^*}{Dt^*}=\frac{v_\infty\mu}{D^2}\nabla^{*2}v^*-\frac{P_B}{D}\nabla^*P^*\text{，}\frac{Dv^*}{Dt^*}=\frac{t_0\mu}{\rho D^2}\nabla^{*2}v^*-\frac{P_Bt_0}{\rho v_\infty D}\nabla^*P^*$$

令 $t_0=\dfrac{D}{v_\infty}$

$$\frac{Dv^*}{Dt^*}=\frac{\mu}{\rho v_\infty D}\nabla^{*2}v-\frac{P_B}{\rho v_\infty^2}\nabla^*P^*=\frac{1}{Re}\nabla^{*2}v^*-\frac{P_B}{\rho v_\infty^2}\nabla^*P^*$$

(e)

$$\begin{cases}velocity: Re=\dfrac{\rho v_\infty D}{\mu}\\ pressure:\dfrac{P_B}{\rho v_\infty^2}\end{cases}\rightarrow v_\infty\propto\frac{1}{D}\ \textit{in the same Re value}$$

(f)

For real system,

$$\mathrm{Re}_{real}=\frac{v_\infty D}{v}=\frac{40\times10^{-2}\times1\times10^{-3}}{3\times10^{-6}}=133.33$$

For mock system,

$$\mathrm{Re}_{mock}=\frac{v_\infty D}{v}=\frac{v_\infty\times10\times10^{-2}}{1\times10^{-6}}=133.33$$

$$\underline{\underline{v_{\infty,mock}=1.33\times10^{-3}\ (m/s)}}$$

Problem 2

【Solution】

(a)

Wiedemann-Franz-Lorenz equation,

$$\frac{k}{k_e T}=L=constant\begin{cases}k=thermal\ conductivity\\k_e=electrical\ conductivity\\L=Lorenz\ number\approx 22\sim 29\times 10^{-9}(volt^2/K^2)\end{cases}$$

for pure metal at 0℃ and little changes with temperature above 0℃.

$$\begin{cases}non-metal:water>engine\ \text{oil}\\metal:k\propto k_e, k_{e,copper}>k_{e,iron}\rightarrow k_{copper}>k_{iron}\end{cases}$$

Thermal conductivity,

$$\boxed{copper>iron>water>engine\ oil}$$

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 280~281.)

(b)

(i)

Correct, $k = 1.9891\times10^{-4}\dfrac{\sqrt{\dfrac{T}{M}}}{\sigma^2\Omega_k}$, independent of pressure.

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 275.)

(ii)

Correct, $k = 1.9891\times10^{-4}\dfrac{\sqrt{\dfrac{T}{M}}}{\sigma^2\Omega_k} \propto \sqrt{T}$

(c)

$$Pr = \frac{\nu}{\alpha} = \frac{momentum\ diffusivity}{thermal\ diffusivity} = Prandtl\ number$$

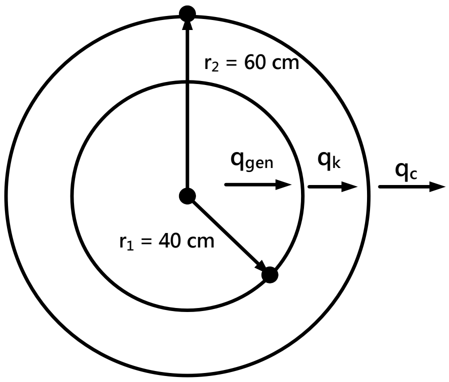
(d)

$$Bi = \frac{h(\frac{V}{A})}{k} = \frac{internal\ thermal\ resistance}{external\ thermal\ resistance}$$

$$Fo = \frac{\alpha t}{(\frac{V}{A})^2} = \frac{heat\ diffusion\ rate}{heat\ storage\ rate}$$

Problem 3

【Solution】



(a)

At the tube surface along y-direction, T is constant = 25℃

$$\boxed{\frac{dT}{dy}=0}$$

(b)

By the equation of continuity of energy in r-direction,

$$\rho C_p(\cancel{\frac{\partial T}{\partial t}}+\cancel{\vec{v}\cdot\nabla T})=k\nabla^2 T+\cancel{\mu\Phi_v}$$

$$k[\frac{1}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})]=0 \quad \text{(only r-direction conduction)}$$

積分兩次

$$T=c_1\ln r+c_2\begin{cases} r=r_1, T=T_2 \\ r=r_2, h(T_1-T_{air})=-k\left.\dfrac{dT}{dr}\right|_{r=r_2}\end{cases}$$

$$\begin{cases} T_2=c_1\ln r_1+c_2 \\ h(T_1-T_{air})=-k\dfrac{c_1}{r_2}\end{cases}\rightarrow\begin{cases} c_1=\dfrac{h(T_{air}-T_1)r_2}{k} \\ c_2=T_2+\dfrac{h(T_1-T_{air})r_2}{k}\ln r_1\end{cases}$$

$$T=\frac{h(T_{air}-T_1)r_2}{k}\ln\frac{r}{r_1}+T_2$$

$r = r_2$ 代入：

$$T = \frac{h(T_{air} - T_1)r_2}{k}\ln\frac{r_2}{r_1} + T_2 = T_1$$

$$\frac{100(298 - T_1)\times 60\times 10^{-2}}{50}\ln\frac{60}{40} + (200 + 273) = T_1$$

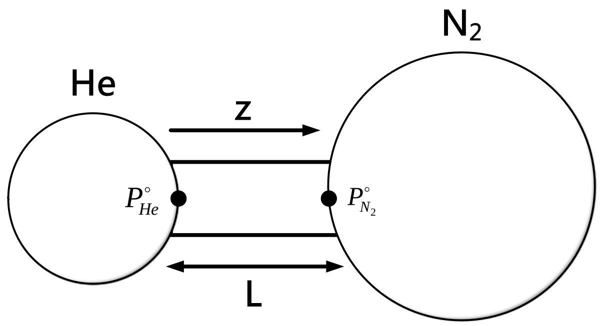
$$\underline{\underline{T_1 = 415.72\ (K) = 142.72\ (℃)}}$$

(c)

$$q^{"} = q_c^{"} = h(T_1 - T_{air}) = 100(142.72 - 25) = \underline{\underline{11772.16\ (W/m^2)}}$$

Problem 4

【Solution】



(a)

By Fick's law,

$$N_{He} = -\mathscr{D}_{He-N_2}\frac{dC_{He}}{dz} + \underbrace{\cancel{y_{He}}}_{very\ dilute}(N_{He} + N_{N_2})$$

$$N_{He} = -\mathscr{D}_{He-N_2}\frac{dC_{He}}{dz} = \frac{-\mathscr{D}_{He-N_2}}{RT}\frac{dP_{He}}{dz}$$

$$\int_0^L N_{He}dz = \int_{P_{He}^{\circ}}^{P_{He,L}} \frac{-\mathscr{D}_{He-N_2}}{RT}dP_{He}$$

$$N_{He} = \frac{-\mathscr{D}_{He-N_2}}{RTL}(P_{He,L} - P_{He}^{\circ}) = \frac{-0.894\times10^{-4}}{8.314\times340\times8\times10^{-2}}(0.012-0.055)\times1.013\times10^{5}$$

$$=1.72\times10^{-3}\ (mole/m^2\cdot s) = \underline{\underline{1.72\times10^{-6}\ (kgmole/m^2\cdot s)}}$$

(本題也可以直接假設 equimolar-counter-diffusion 來解題)

(b)

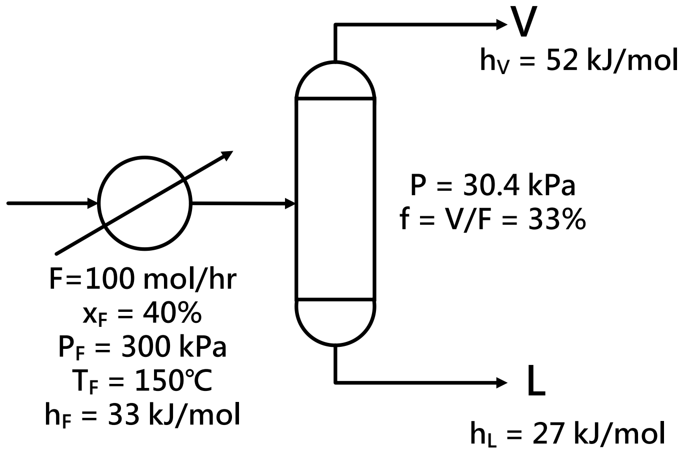
∵ The pressure of the system is constant and there are only two components,

the system can be considered as an equimolar-counter-diffusion system.

$$\underline{\underline{N_{N_2} = -N_{He} = -1.72\times10^{-6}\ (kgmole/m^2\cdot s)}}$$

Problem 5

【Solution】

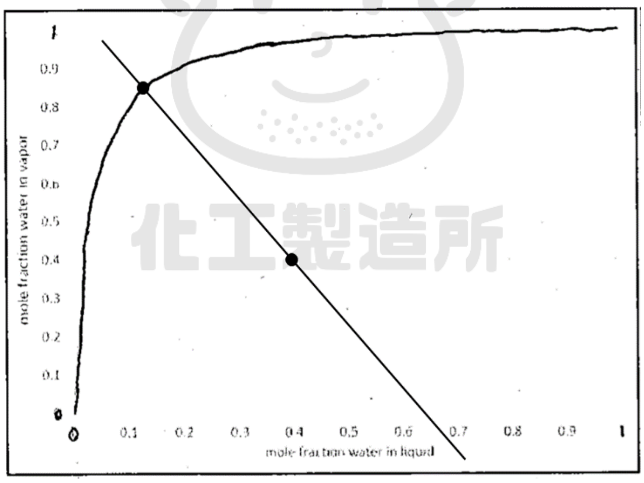


(a)

For operating line,

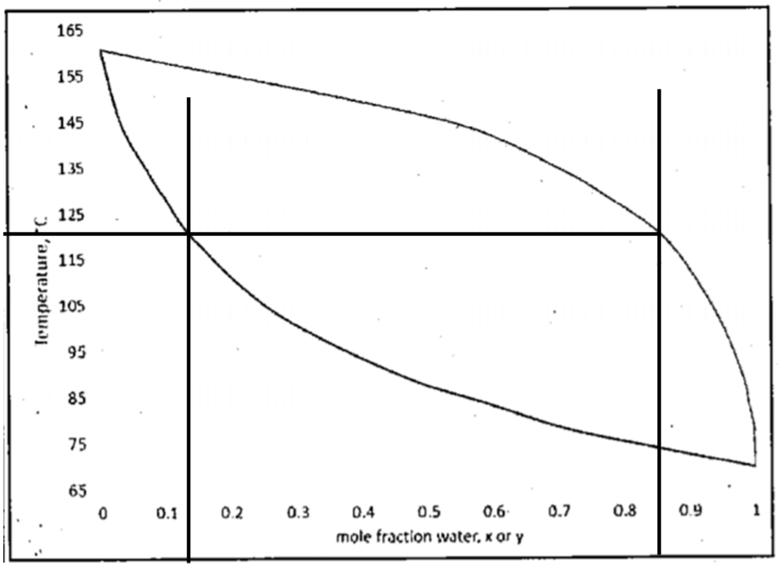
$$y_A = \frac{f-1}{f} x_A + \frac{x_F}{f} = \frac{0.33-1}{0.33} x_A + \frac{0.4}{0.33} = -2x_A + 1.21$$

在 $y_A - x_A$ 圖上畫過 $(x_F, x_F) = (0.4, 0.4)$ 、斜率為 2 之直線



可知 operating line 與 equilibrium line 交於 $(x_A, y_A) = (0.12, 0.85)$

再對至 $x_A - y_A - T$ 圖可得溫度為 <u>**123℃**</u>



(b)

By energy balance,

$$Fh_F + Q = Lh_L + Vh_v$$

By mole balance,

$$\begin{cases} F = L + V \\ Fx_F = Lx_A + Vy_A \end{cases}$$

$$\begin{cases} 100 = L + V \\ 100 \times 0.4 = 0.12L + 0.85V \end{cases}, \begin{cases} L = 61.64 \\ V = 38.36 \end{cases}$$

代回 Energy balance,

$$100 \times 33 \times 10^3 + Q = 61.64 \times 27 \times 10^3 + 38.36 \times 52 \times 10^3$$

$$\underline{\underline{Q = 359000\ (kJ)}}$$

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Problem 1

【**Solution**】

(a)

$$Re = \frac{inertial\ force}{viscous\ force}$$

(b)

$$f = \frac{drag\ force}{inertial\ force}$$

(c)

$$Re_h = \frac{4R_h < v > \rho}{\mu}, \text{ where } R_h = \frac{cross - sectional\ area}{wetted\ area}$$

$$f = (\frac{R_h}{L})(\frac{P_0 - P_L}{\frac{1}{2}\rho < v >^2})$$

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 183.)

(d)

當 Reynolds number 變大，inertial force >> viscous force

則 $f \propto \frac{1}{inertial\ force}$ 就變小，但不代表 shear stress 變小，只是相對於 inertial force 來說影響程度變小

(e)

By Hagen-Poiseuille equation, (假設在圓管內流動)

$$Q = \frac{\pi(P_0 - P_L)R^4\rho}{8\mu \mathrm{L}} = \rho\pi R^2 < v >$$

$$(P_0 - P_L) = \frac{8\mu L\rho\pi R^2 < v >}{\pi R^4 \rho} = \frac{8\mu L < v >}{R^2}$$

$$f = \frac{D}{4L} \times \frac{8\mu L < v >}{(\frac{D}{2})^2} \times \frac{2}{\rho < v >^2} = \frac{16\mu}{\rho < v > D} = \frac{16}{\mathrm{Re}}$$

(f)

The thickness of the viscous sublayer will become less with increasing Reynolds number, or even less than the roughness of the pipe. Therefore the effect of roughness is dominate.

(g)

The dependence of f on L/D arises form the development of the time-averaged velocity distribution from its flat entry shape toward more rounded profiles.

For laminar flow with small enough Re, the shape of the velocity distribution is "fully-developed". In the transportation of fluids, the entrance length is usually a small fraction of the total.

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 181.)

(h)

The same reason as **(f)**, if the roughness of the wall increases, it will hinder the transportation of the flow.

(i)

The value of D will contribute the resistance of the transportation of the flow.

(j)

If a rough pipe is smoothed, the friction factor is reduced. When further smoothing brings about no further reduction in ther frction factor for a given Reynolds number, the tube is said to be “hydraulically smooth”.

(本題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering,* 7th ed.; p 114.)

Problem 2

【Solution】

(a) Incorrect

Stream function 不可用於 compressible flow，這樣才有 exact equation

$$\nabla \cdot v = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0 \text{可以使用}$$

(b) Incorrect

Velocity potential 可用於 3D 流場

(c) Incorrect

$$\begin{cases} v_x = \dfrac{\partial \phi}{\partial x} = 5x^2 - 5y^2 = \dfrac{\partial \psi}{\partial y} \\ v_y = \dfrac{\partial \phi}{\partial y} = -10xy = -\dfrac{\partial \psi}{\partial x} \end{cases} ，\underline{\underline{\psi = 5x^2 y - \frac{5}{3} y^3}}$$

(d) Correct

$$\vec{v} = \nabla \phi = u_\infty L\left(\frac{2x - 3y^2}{L^3}\overrightarrow{e_x} - \frac{6xy}{L^3}\overrightarrow{e_y}\right)$$

For irrotational flow, $\nabla \times \vec{v} = 0$

<check>:

$$\nabla \times \vec{v} = 0\overrightarrow{e_x} + 0\overrightarrow{e_y} + (-6y + 6y)\overrightarrow{e_z} = 0$$

(e) Correct

Problem 3

【Solution】

(a)

$$Bi = \frac{h(\frac{V}{A})}{k} = \frac{internal\ resistance}{external\ resistance} \text{ to heat transfer}$$

$$Nu = \frac{hL}{k_f} = \frac{conductive\ resistance}{convective\ resistance} \text{ of the fluid}$$

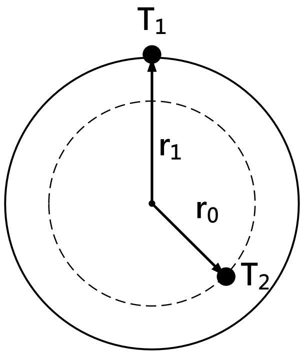
Biot number has the conductivity of the main object while Nu uses that of the external fluid.

(b)

$$Nu = f(Re, Gr, Pr)\begin{cases} Re = \dfrac{inertial\ \ force}{viscous\ \ force} \\ Gr = \dfrac{buoyant\ \ force}{viscous\ \ force} \\ Pr = \dfrac{momentum\ diffusivity}{thermal\ diffusivity} \end{cases}$$

Problem 4

【Solution】



Let $k = a + bT \begin{cases} a = \dfrac{k_1 - k_0}{T_1 - T_0} \\ b = k_0 - aT_0 = \dfrac{k_0 T_1 - k_1 T_0}{T_1 - T_0} \end{cases}$

Assume：(1)Steady-State

(2)only r-direction heat conduction

By Fourier's law,

$$q_r = -k(2\pi rL)\frac{dT}{dr} = -(a+bT)(2\pi rL)\frac{dT}{dr}$$

$$\int_{r_0}^{r_1} \frac{q_r}{r} dr = -2\pi L \int_{T_0}^{T_1} (a+bT) dT$$

$$q_r \ln\frac{r_1}{r_0} = 2\pi L[a(T_0 - T_1) + \frac{b}{2}(T_0^2 - T_1^2)]$$

$$q_r = \frac{2\pi L}{\ln\frac{r_1}{r_0}}[a(T_0 - T_1) + \frac{b}{2}(T_0^2 - T_1^2)]$$

$$= \frac{2\pi L}{\ln\frac{r_1}{r_0}}[(k_0 - k_1) + \frac{(T_0 + T_1)}{2}(k_1 T_0 - k_0 T_1)]$$

(※本題解法可與 104 年中央化材所 Problem 4 互相對照)

Problem 5

【Solution】

(a)

By Fick's first law,

$$N_{Ar} = \frac{w_A}{4\pi r^2} = -\mathscr{D}_{AB} c \frac{dy_A}{dr} + \underbrace{\cancel{y_A}}_{very\ dilute} (N_A + N_B)$$

$$\int_a^\infty \frac{w_A}{4\pi r^2} dr = -\mathscr{D}_{AB} c \int_{X_{A1}}^0 dy_A$$

$$\frac{w_A}{4\pi}(\frac{1}{a}) = \mathscr{D}_{AB} c X_{A1} \text{ , } w_A = 4\pi \mathscr{D}_{AB} c X_{A1} a$$

$$\underline{\underline{N_{Ar} = \frac{w_A}{4\pi r^2} = \frac{\mathscr{D}_{AB} c X_{A1} a}{r^2}}}$$

(b)

$$\underline{\underline{w_A = 4\pi \mathscr{D}_{AB} c X_{A1} a}}$$

(c)

For $a(t)$, by pseudo-steady-state assumption

$$w_A = \frac{-d}{dt}(\frac{4}{3}\frac{\rho}{M_A}\pi a^3) = -4\pi a^2 \frac{\rho}{M_A}\frac{da}{dt}$$

$$\mathscr{D}_{AB} c X_{A1} = -a \frac{\rho}{M_A}\frac{da}{dt} \text{ , } \int_0^t \mathscr{D}_{AB} c X_{A1} dt = -\int_{a_o}^a \frac{\rho}{M_A} a da = \frac{1}{2}\frac{\rho}{M_A}(a_0^2 - a^2)$$

$$\underline{\underline{a(t) = \sqrt{a_0^2 - \frac{2M_A \mathscr{D}_{AB} c X_{A1} t}{\rho}}}}$$

$$w_A \propto a$$

If a of the particles is small, the diffusion rate will be small and it will stabilize the particles.

(d)

$$N_{Ar}(a_0)=\frac{\mathcal{D}_{AB}cX_{A1}a_0}{a_0^2}=\frac{\mathcal{D}_{AB}cX_{A1}}{a_0}$$

(e)

$$t\sim\frac{a_0^2}{\mathcal{D}_{AB}}$$

(f)

$$t_0=\frac{\rho a_0^2}{2\mathcal{D}_{AB}cX_{A1}M_A}$$

(g)

$$\frac{t}{t_0}\sim\frac{\dfrac{a_0^2}{\mathcal{D}_{AB}}}{\dfrac{\rho a_0^2}{\mathcal{D}_{AB}cX_{A1}M_A}}\sim\frac{cX_{A1}M_A}{\rho}$$

(在這裡 2 不必要寫，因為不影響數值的 order 太多)

(h)

$$\frac{t}{t_0}<<1\text{，}\frac{cX_{A1}M_A}{\rho}<<1$$

(time for change in size is large)

(下接本小題說明)

※(e)~(h)小題的過程，可以粗糙的由 Fick's second law 說明，亦即：

$$\frac{\partial C_A}{\partial t} = D_{AB}\frac{\partial^2 C_A}{\partial z^2}$$

(以直角坐標來想)

其中，若要保留時間項，代表此時我們考慮的時間尺度會造成明顯的系統變化(比如濃度、顆粒尺寸等)，此時時間為重要的參數，即為 unsteady 的狀態。亦可由 t_0 (f 小題)做估算，也就是以完全溶解的時間尺度看，因為完全溶解代表顆粒消失，此為明確的尺寸變化參數。而單純以某個幾何尺寸下擴散(比如從 A 擴散到 B)的時間，則由右邊項作無因次化可看出($t \sim D_{AB}/L^2$)，即(e)小題結論。因此若是尺寸改變的時間(t_0)遠大於 t (舉粗糙的例子來說，假設顆粒尺寸改變 1 nm 所需要的時間是 2 小時，然而溶質傳出去的時間只需要 1 秒。)，代表我們可以將範圍限縮到某個瞬間(1 秒)去看質傳的情況，時間項則由最後積分的邊界條件或是其餘方程式納入，也就是平常做 pseudo-steady state 的做法。也就是(a)小題在算質傳通量時，不考慮時間，先算出某個瞬間的質傳通量。然而完整的溶解過程尺寸一定會變化，所以在(c)小題採用質量平衡，將時間項(亦即 $a(t)$)納入，最後呈現出完整的半徑-時間關係式，此時能採用此種做法的標準就是：

$$\boxed{\frac{t}{t_0} << 1}$$

然而若是顆粒太小顆，或是完全蒸發或昇華太快(像常見的樟腦昇華問題)，代表尺寸劇烈變化的時間非常短，這時候我們就很難找到一個瞬間可以使用穩態假設，此時時間項就必須要完整考慮。

109 年中央單操輸送

<u>Problem 1</u>

【Solution】

(a)

$$\tau = \tau_{yx} = \mu \frac{\partial u_x}{\partial y} = \mu(3-3y^2)$$

$$= \rho\upsilon(3-3y^2) = (10^{-3}\times10^{-6})\times(7\times10^{-7}\times10^{4})(3-3y^2)$$

$$= \underline{\underline{7\times10^{-6}(3-3y^2)}}\ (kg/cm\cdot s^2)$$

(b)

y = 0.8mm 代入 τ_{yx}

$$\tau_{yx}\Big|_{y=0.8} = 7\times10^{-6}(3-3\times0.8^2) = \underline{\underline{7.56\times10^{-6}\ \ (kg/cm\cdot s^2)}}$$

(c)

$$\tau_{xx} = -\mu[2\frac{\partial u_x}{\partial x}] = 0$$

There is no momentum flux in the x-direction.

Problem 2

【**Solution**】

(a)

For Stokes' law to be applicable,

$$f=\frac{24}{Re}=\frac{24\mu}{\rho v_{\infty}D_P}=\frac{24\mu}{\rho\sqrt{\frac{4(\rho_P-\rho)gD_P}{3\rho f}}D_P}(Re<1)$$

$$f^{\frac{1}{2}}=\frac{24\mu}{\rho\sqrt{\frac{4(\rho_P-\rho)gD_P}{3\rho}}D_P}>24^{\frac{1}{2}}$$

$$\begin{cases}\mu=1.8\times10^{-5}(kg/m\cdot s)\\ \rho=1.18(kg/m^3) \qquad \text{代入}\\ \rho_P=4000(kg/m^3)\end{cases}$$

$$\frac{24\times1.8\times10^{-5}}{1.18\sqrt{\frac{4(4000-1.18)\times9.8D_P}{3\times1.18}}D_P}>24^{0.5}$$

$$\underline{\underline{D_P<5.0\times10^{-5}\ (m)=50\mu m}}\text{得證}$$

(b)

For $D_P=10\mu m$, Stokes' law is applicable,

$$v_t=\sqrt{\frac{4(\rho_P-\rho)gD_P}{3f\rho}}\text{，}f=\frac{24}{\mathrm{Re}}=\frac{24\mu}{\rho v_t D_P}$$

$$v_t=\frac{gD_P^2(\rho_P-\rho)}{18\mu}=\frac{9.8\times(10\times10^{-6})^2(4000-1.18)}{18\times1.8\times10^{-5}}=\underline{\underline{0.012\ (m/s)}}$$

For $D_P=140.250\ (\mu m)$, Stokes' law isn't applicable,

$$v_t=\sqrt{\frac{4(\rho_P-\rho)gD_P}{3f\rho}}\text{，}\mathrm{Re}=\frac{\rho v_t D_P}{\mu}$$

$$\mathrm{Re}=\frac{\rho D_P}{\mu}\sqrt{\frac{4(\rho_P-\rho)gD_P}{3f\rho}}$$

$D_P = 140\times10^{-6}\ (m)$ 代入

$$\mathrm{Re} = \frac{1.18\times140\times10^{-6}}{1.80\times10^{-5}}\sqrt{\frac{4(4000-1.18)\times9.8\times140\times10^{-6}}{3\times1.18f}} = \frac{22.85}{\sqrt{f}}$$

令：

$$\begin{cases} f=1 \rightarrow \mathrm{Re}=22.85 \\ f=100 \rightarrow \mathrm{Re}=2.285 \end{cases}\text{兩點畫一直線}$$

$$\begin{cases} f=5.22 \\ \mathrm{Re}=10 \end{cases}\text{時會與題目圖形曲線得到交點}$$

$$10 = \frac{1.18v_t\times140\times10^{-6}}{1.8\times10^{-5}}\ \text{，}\ \underline{\underline{v_t = 1.089\ (m/s)}}$$

$D_P = 250\times10^{-6}(m)$ 代入

$$\mathrm{Re} = \frac{1.18\times250\times10^{-6}}{1.80\times10^{-5}}\sqrt{\frac{4(4000-1.18)\times9.8\times250\times10^{-6}}{3\times1.18f}} = \frac{54.528}{\sqrt{f}}$$

令：

$$\begin{cases} f=1 \rightarrow \mathrm{Re}=54.528 \\ f=100 \rightarrow \mathrm{Re}=5.428 \end{cases}\text{兩點畫一直線}$$

$$\begin{cases} f=1.858 \\ \mathrm{Re}=40 \end{cases}\text{時會與題目圖形曲線得到交點}$$

$$40 = \frac{1.18v_t\times250\times10^{-6}}{1.8\times10^{-5}}\ \text{，}\ \underline{\underline{v_t = 2.44\ (m/s)}}$$

之所以可以利用 Re 與 f 做圖畫出直線即是因為：

$$\mathrm{Re}=\frac{Dv_t\rho}{\mu}\text{ ， }f=\frac{4}{3}\frac{aD}{v_t^2}\left(\frac{\rho_p-\rho}{\rho}\right)$$

$$\frac{\mathrm{Re}}{f}=\frac{3}{4}\frac{v_t^3\rho^2}{\mu a(\rho_p-\rho)}\text{ ， }\mathrm{Re}=\frac{v_t^3\rho^2}{\mu a(\rho_p-\rho)}f=Cf$$

只不過這題我們並不知道 C，因為不知道速度 v_s

因此才改用 $\mathrm{Re}\sqrt{f}=$ 定值的方式求出符合 Re 與 f 關係式的兩點

再畫出直線就可反推 C 得到斜率，也就是 v_s

而這題也可以順利求得正確的 Reynolds number

本題另種題型可以參考： Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 188, Example 6.3.1.

Problem 3

【Solution】

(a)

$$q_{c,x_0} = q_{k,x_0\to x_1} = q_{k,x_1\to x_2} = q_{k,x_2\to x_3} = q_{c,x_3}$$

$$\frac{T_a - T_0}{\dfrac{1}{h_0}} = \frac{T_0 - T_1}{\dfrac{x_1 - x_0}{k^{01}}} = \frac{T_1 - T_2}{\dfrac{x_2 - x_1}{k^{12}}} = \frac{T_2 - T_3}{\dfrac{x_3 - x_2}{k^{23}}} = \frac{T_3 - T_h}{\dfrac{1}{h_3}}$$

$$\frac{T_a - T_h}{\dfrac{1}{h_0} + \dfrac{x_1 - x_0}{k^{01}} + \dfrac{x_2 - x_1}{k^{12}} + \dfrac{x_3 - x_2}{k^{23}} + \dfrac{1}{h_3}} = U(T_a - T_h)$$

$$\underline{\underline{U = \left(\frac{1}{h_0} + \frac{x_1 - x_0}{k^{01}} + \frac{x_2 - x_1}{k^{12}} + \frac{x_3 - x_2}{k^{23}} + \frac{1}{h_3}\right)^{-1}}}$$

(b)

At steady state,

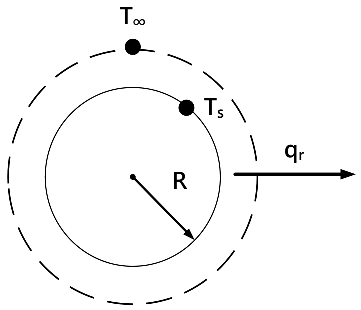
$$q_{01} = q_{12} = q_{23}$$

$$-k^{01}\left.\frac{dT}{dx}\right|_{01} = -k^{12}\left.\frac{dT}{dx}\right|_{12} = -k^{23}\left.\frac{dT}{dx}\right|_{23}$$

$$\because \left.\frac{dT}{dx}\right|_{01} < \left.\frac{dT}{dx}\right|_{23} < \left.\frac{dT}{dx}\right|_{12} \text{，} \underline{\underline{k_{01} > k_{23} > k_{12}}}$$

Problem 4

【Solution】



(a)

$$\text{Assume：}\begin{cases} steady\text{-}state \\ no\ heat\ generation \\ only\ r-direction\ conduction \end{cases}$$

By equation of change of energy in spherical cooridinate,

$$\rho \hat{C}_P\left(\cancel{\frac{\partial T}{\partial t}} + \cancel{\vec{v}\cdot\nabla T}\right) = k\nabla^2 T + \cancel{\mu\phi_v}$$

$$k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \cancel{\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right)} + \cancel{\frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}}\right] = 0$$

積分 2 次：

$$T = \frac{-c_1}{r} + c_2 \text{ ，} \begin{cases} r = R, T = T_s \\ r = \infty, T = T_\infty \end{cases} \text{，} \begin{cases} c_1 = R(T_\infty - T_s) \\ c_2 = T_\infty \end{cases}$$

$$\underline{\underline{T = \frac{R}{r}(T_s - T_\infty) + T_\infty}}$$

(b)

$@r = R$， $q_k = q_c$

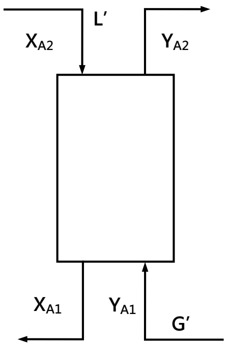
$$-k\left.\frac{dT}{dr}\right|_{r=R} = h(T_s - T_\infty)$$

$$\frac{R}{R^2}k(T_s - T_\infty) = h \text{ ，} \frac{hR}{k} = 1 \text{ ，} \underline{\underline{Nu = \frac{hD}{k} = \frac{2hR}{k} = 2}}$$

Problem 5

【Solution】

(a)



(b)

假設空氣量>>acetone，因此分子量$_{G'}$ $\approx$ 分子量$_{air}$ $= 28.8$

$$\begin{cases} G' = \dfrac{1\times10^3}{28.8} = 34.72\ (mole/m^2\cdot s) \\ L' = \dfrac{1.6\times10^3}{18} = 88.89\ (mole/m^2\cdot s) \end{cases} , \begin{cases} y_{A1} = 0.015 \\ y_{A2} = 0.00015 \\ x_{A2} = 0 \end{cases}$$

By mole balance of acetone,

$$34.72(0.015-0.00015) = 88.89(x_{A1}-0)$$

$$x_{A1} = 5.8\times10^{-3}$$

$$\begin{cases} x_{A1}^* = \dfrac{y_{A1}}{1.75} = \dfrac{0.015}{1.75} = 8.57\times10^{-3} \\ x_{A2}^* = \dfrac{y_{A2}}{1.75} = \dfrac{0.00015}{1.75} = 8.57\times10^{-5} \end{cases}$$

$$N_{OL} = \frac{(x_{A1}-x_{A2})}{\left[\dfrac{(x_{A1}^*-x_{A1})-(x_{A2}^*-x_{A2})}{\ln\dfrac{x_{A1}^*-x_{A1}}{x_{A2}^*-x_{A2}}}\right]} = \frac{5.8\times10^{-3}-0}{\left[\dfrac{(8.57\times10^{-3}-5.8\times10^{-3})-(8.57\times10^{-5}-0)}{\ln\dfrac{8.57\times10^{-3}-5.8\times10^{-3}}{8.57\times10^{-5}-0}}\right]} = \underline{\underline{7.509}}$$

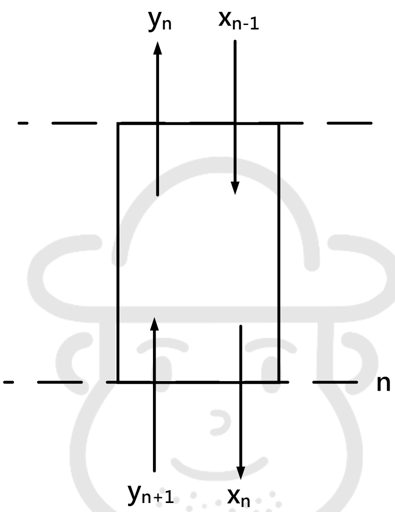
(c)

$$\begin{cases} y_{A1}^* = 1.75x_1 = 0.010 \\ y_{A2}^* = 1.75x_2 = 0 \end{cases}$$

$$N_{OG} = \frac{y_{A1} - y_{A2}}{\left[\dfrac{(y_{A1} - y_{A1}^*) - (y_{A2} - y_{A2}^*)}{\ln \dfrac{y_{A1} - y_{A1}^*}{y_{A2} - y_{A2}^*}}\right]} = \frac{0.015 - 0.00015}{\left[\dfrac{(0.015 - 0.010) - (0.00015 - 0)}{\ln \dfrac{0.015 - 0.010}{0.00015 - 0}}\right]} = \underline{\underline{10.64}}$$

Problem 6

【Solution】



For total reflux, $y_{n+1} = x_n$

$$\frac{y_A}{y_B} = \alpha_{AB} \frac{x_A}{x_B}$$

$$\frac{y_n}{1 - y_n} = \alpha_{AB} \frac{x_{n+1}}{1 - x_{n+1}} \quad , \quad \frac{x_n}{1 - x_n} = \alpha_{AB} \frac{x_{n+1}}{1 - x_{n+1}}$$

$$\frac{x_D}{1 - x_D} = \alpha_{AB} \frac{x_1}{1 - x_1} = (\alpha_{AB})^2 (\frac{x_2}{1 - x_2}) = (\alpha_{AB})^3 (\frac{x_3}{1 - x_3}) = (\alpha_{AB})^n (\frac{x_n}{1 - x_n}) = (\alpha_{AB})^{n+1} (\frac{x_w}{1 - x_w})$$

$$n = \frac{\ln[\dfrac{x_D}{1 - x_D} \cdot \dfrac{1 - x_w}{x_w}]}{\ln \alpha_{AB}} - 1 = \frac{\ln[\dfrac{0.9}{1 - 0.9} \cdot \dfrac{1 - 0.1}{0.1}]}{\ln 2.4} - 1 = \underline{\underline{4.02\ (plates)}}$$

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Problem 1

【Solution】

(a)

$$Reynolds\ number,\ Re \equiv \frac{inertial\ force}{viscous\ force}$$

(b)

Molecular momentum flux,

$$\tau = -\sum_{n=1}^{Z} m_n (v_x|_{y-} - v_x|_{y+}) \begin{cases} v_x = x - directional\ average\ velocity \\ m_n = the\ mass\ of\ the\ molecule \end{cases}$$

(本題可參考：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer*, 5th ed.; p 84.)

(c)

The Prandtl mixing length plays roughly the same role in turbulent flow as the mean free path in kinetic theory,

$$\tau_{yx}^{(t)} = -\rho l^2 \left|\frac{d\overline{v_x}}{dy}\right| \frac{d\overline{v_x}}{dy} \begin{cases} wall\ turbulence \to l = \kappa_1 y \\ free\ turbulence \to l = \kappa_2 b \end{cases}$$

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 163.)

(本題可參考：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer*, 5th ed.; p158~159.)

(d)

For a two dimensional, incompressible and irrotational flow, we can introduce a function so that,

$$\vec{v} = -\nabla \phi$$

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 126~127.)

(e)

For a turbulent flow, the turbulent momentum flux tensor,

$$\overline{\tau^{(t)}} = \rho\overline{v'v'}$$

which is also called ***Reynolds stress***

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 158.)

(f)

The term form drag can be interpreted as the force exerted by the fluid on the solid, acting normal to the surface.

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 60.)

(g)

The fanning friction factor,

$$f = \frac{1}{4}\left(\frac{D}{L}\right)\left(\frac{P_0 - P_L}{\frac{1}{2}\rho < v >^2}\right)$$

can be calculated from the experimental data and used as a prediction of friction drag

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 178~179.)

(h)

By introducing the eddy diffusivity, ε_M

we can calculate the Reynolds stress as in the case of laminar flow,

$$(\tau_{yx})_{turb} = \rho\varepsilon_M \frac{d\bar{v}_x}{dy}$$

Eddy diffusivity is the analogy of the kinetic viscosity in laminar flow, which is a property of the flow and not of the fluid.

(本題可參考：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer*, 5th ed.; p158.)

(i)

The divergence of velocity is,

$$\nabla \cdot \vec{v}$$

Which will be zero for an incompressible fluid when considering its continuity equation.

(j)

For a flow with very low speeds or large viscosity, its Reynolds number

$$Re = \frac{inertial\ \ force}{viscous\ \ force} << 1$$

This type of flow is called creeping flow.

Problem 2

【**Solution**】

By Navier-Stokes equation,

$$\rho\frac{Du}{Dt}=\mu\nabla^2 u-\nabla p+\rho g$$

將題目所列之 dimensionless relations 代入

$$\frac{Du}{Dt}=\frac{u_\infty}{\frac{L}{u_\infty}}\frac{Du^*}{Dt^*}=\frac{u_\infty^2}{L}\frac{Du^*}{Dt^*}$$

$$\mu\nabla^2 u=\frac{\mu u_\infty}{L^2}\nabla^{*2}u^*$$

$$\nabla p=\frac{\rho u_\infty^2}{L}\nabla^* p^*$$

=>

$$\rho\frac{u_\infty^2}{L}\frac{Du^*}{Dt^*}=\frac{\mu u_\infty}{L^2}\nabla^{*2}u^*-\frac{\rho u_\infty^2}{L}\nabla^* p^*+\rho g$$

同除 $\frac{\rho u_\infty^2}{L}$

$$\frac{Du^*}{Dt^*}=\frac{\mu}{\rho u_\infty L}\nabla^{*2}u^*-\nabla^* p^*+\frac{gL}{u_\infty^2}$$

$$\boxed{\frac{Du^*}{Dt^*}=\frac{1}{Re}\nabla^{*2}u^*-\nabla^* p^*+\frac{gL}{u_\infty^2}}$$

Problem 3

【Solution】

(a)

$$\varepsilon = 1 - \frac{\rho_b}{\rho_p} = 1 - \frac{962}{1600} = 0.398$$

(b)

$$D_P = \frac{6}{a_v} = \frac{6}{S_p / V_p}$$

$$S_p = 2(\frac{\pi D^2}{4}) + \pi D h = \frac{3}{2}\pi D^2 \ \ (total\ surface\ area)$$

$$V_p = \frac{\pi}{4} D^2 h = \frac{\pi}{4} D^3$$

$$D_p = \frac{6}{\frac{3}{2}\pi D^2 \times \frac{4}{\pi D^3}} = D = \underline{\underline{0.02\ (m)}}$$

(c)

$$a = a_v(1-\varepsilon) = \frac{6}{D}(1-\varepsilon) = \frac{6}{0.02}(1-0.398) = \underline{\underline{180.6\ (m^{-1})}}$$

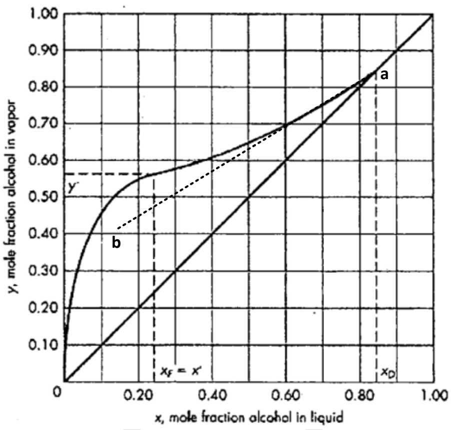
(本題可參考：Geankoplis, C. *Transport Processes and Separation Process Principles*, 4th ed.; p 126, Example 3.1-3.)

<u>Problem 4</u>

【Solution】

<u>Minimum reflux ratio</u>

Operation line that is tangent to the equilibrium line corresponds to the line ***ab*** in the following figure,



<u>Minimum operating ratio</u>

Operation line corresponds to the diagonal line shown on the figure.

(※對於 operation line 會超過 equilibrium line 之 minimum reflux ratio line 可參考：

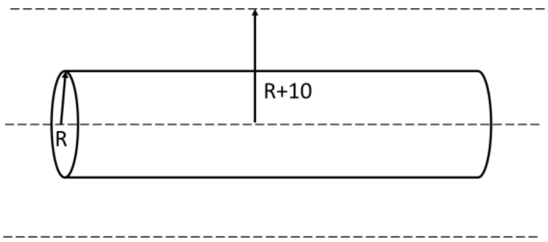
McCabe, W.; Smith, J.; Harriott, P. ***Unit Operations of Chemical Engineering*****, 7th ed.;**

p 688~691.)

Problem 5

【Solution】

(a)



To describe the system, we introduce cylindrical coordinates.

Assumptions,

(1) Pipe is long enough so that the diffusion is only in r-direction

(2) No reaction

(3) Steady-state

(b)

By the general equation of continuity,

$$\underbrace{\cancel{\frac{\partial C_A}{\partial t}}}_{s.s} + \nabla \cdot N_A = \underbrace{\cancel{R_A}}_{no\ reaction}$$

$$\boxed{\frac{d}{dr}(rN_A) = 0}$$

(c)

By Fick's flux equation,

$$N_A = -cD_{AB}\frac{dy_A}{dr} + y_A(N_A + \cancel{N_B})$$

$$\boxed{N_A = \frac{-cD_{AB}}{1-y_A}\frac{dy_A}{dr}}$$

(d)

N_A 代入(b)之 general differential equation,

$$\frac{d}{dr}(r\frac{-cD_{AB}}{1-y_A}\frac{dy_A}{dr})=0$$

$$\boxed{\frac{d}{dr}(\frac{r}{1-y_A}\frac{dy_A}{dr})=0}$$

(本題可參考：Welty, J.; Rorrer, G.; Foster, D. *Fundamentals of Momentum, Heat, and Mass Transfer*, 5th ed.; p 448, Problem 25.6.)

Problem 6

【Solution】

(a)

By entropy balance of the steam passing through the turbine,

$$\underbrace{\cancel{\frac{dS}{dt}}}_{s.s}=\sum_i \dot{M}_k\hat{S}_k+\underbrace{\cancel{\sum\frac{\dot{Q}}{T}}}_{adiabatic}+\cancel{\dot{S}_{gen}}$$

$$\boxed{\hat{S}_{in}=\hat{S}_{out}}$$

By energy balance,

$$\dot{M}\,\Delta H=\dot{W}_s=-F\cdot v$$

(在穩定航行下船所需要的前進功率)

$$20\times(H_{out}-H_{in})=-3.86\times10^{6}\times37\times\frac{1000}{3600}$$

$$\boxed{H_{out}-H_{in}=-1983.61}$$

此時若可找出一 T、P 狀態下之 steam 滿足 entropy 以及 energy balance 即可繼續作答，唯獨題目少給了 1 bar、100℃時 steam 之熱力學參數，無法繼續往下算

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Problem 1

【Solution】

(a) The net positive suction head (NPSH) is by which the value of the pressure in the pump inlet exceeding the vapor pressure.

The pump must word under enough NPSH because some of the liquid in the pump may flash into vapor inside the pump, causing cavitation and damaging the pump.

(b)

(1) Venturi meter : The device measures the pressure drop when the fluid passes through a throat, where the velocity of the fluid increases. By Bernoulli equation, the pressure drop and the velocity difference can be related together.

(2) Orifice meter：The basic principle of the designation of orifice meter is the same as the venture meter.

(3) Rotameter：The fluid flows with constant pressure drop, but the area where the fluid passes through is varied. Therefore, the float inside the rotameter will be suspended and can indicate the flow rate under that condition.

((b)小題題可參考：McCabe, W.; Smith, J.; Harriott, P. *Unit Operations of Chemical Engineering*, 7th ed.; p 214~225.)

<u>Problem 2</u>

【Solution】

By the equation of continuity,

$$\underbrace{\frac{\partial \rho}{\partial t}}_{incompressible}+\frac{1}{r}\frac{\partial}{\partial r}\underbrace{\cancel{(\rho r v_r)}}_{v_r=0}+\frac{1}{r}\frac{\partial}{\partial \theta}\underbrace{\cancel{(\rho v_\theta)}}_{v_\theta=0}+\frac{\partial}{\partial z}(\rho v_z)=0$$

$$v_z=v_z(r,\cancel{\theta},\cancel{z},\cancel{t})$$

By momentum balance in the falling film region,

$$\rho v_z v_z r dr d\theta\big|_z-\rho v_z v_z r dr d\theta\big|_{z+dz}+\tau_{rz} r d\theta dz\big|_r-\tau_{rz} r d\theta dz\big|_{r+dr}+\rho g r dr d\theta dz=0$$

同除 $dr d\theta dz \to 0$,

$$\underbrace{\cancel{\frac{-\partial(r\rho v_z v_z)}{\partial z}}}_{v_z\neq f(z)}-\frac{\partial(r\tau_{rz})}{\partial r}+\rho g r=0$$

$$\frac{\partial(r\tau_{rz})}{\partial r}=+\rho g r\text{，}\tau_{rz}=\frac{1}{2}\rho g r+\frac{c_1}{r}$$

$\because \tau_{rz}=-\mu(\frac{dv_z}{dr})$

$$-\mu(\frac{dv_z}{dr})=\frac{1}{2}\rho g r+\frac{c_1}{r}$$

$$v_z=-\frac{\rho g}{4\mu}r^2+C_1\ln r+C_2\begin{cases}r=R, v_z=0\\ r=aR, \dfrac{dv_z}{dr}=0\end{cases}$$

$$\begin{cases}C_1=\dfrac{\rho g}{2\mu}a^2R^2\\ C_2=\dfrac{\rho g}{4\mu}R^2-\dfrac{\rho g}{2\mu}a^2R^2\ln R\end{cases}$$

代回：

$$v_z=-\frac{\rho g}{4\mu}r^2+C_1\ln r+C_2=\underline{\underline{\frac{\rho g R^2}{4\mu}[1-(\frac{r}{R})^2+2a^2\ln\frac{r}{R}]}}$$

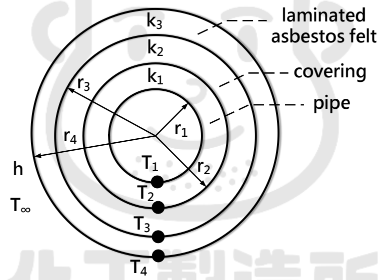
For the mass flow rate,

$$
\begin{aligned}
w &= \int_0^{2\pi}\int_R^{aR} \rho v_z r dr d\theta = \frac{\pi\rho^2 gR^2}{2\mu}\int_R^{aR} r - \frac{r^3}{R^2} + 2a^2 r \ln\frac{r}{R} dr \\
&= \frac{\pi\rho^2 gR^2}{2\mu}\left[\frac{1}{2}(r^2)\Big|_R^{aR} - \frac{1}{4R^2}(r^4)\Big|_R^{aR} + 2a^2\left(\frac{r^2}{2}\ln\frac{r}{R} - \frac{r^2}{4}\right)\Big|_R^{aR}\right] \\
&= \frac{\pi\rho^2 gR^2}{2\mu}\left[\frac{1}{2}R^2(a^2-1) - \frac{1}{4}R^2(a^4-1) + 2a^2R^2\left(\frac{a^2}{2}\ln a - \frac{a^2}{4} + \frac{1}{4}\right)\right. \\
&= \frac{\pi\rho^2 gR^4}{8\mu}\left[2(a^2-1) - (a^4-1) + 4a^2\ln a - 2a^4 + 2a^2\right] \\
&= \underline{\underline{\frac{\pi\rho^2 gR^4}{8\mu}\left(-1 + 4a^2 - 3a^4 + 4a^4\ln a\right)}}
\end{aligned}
$$

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 64, Problem 2B.6.)

<u>Problem 3</u>

【Solution】



(a)

Assume the system is in steady state, by Ohm's law

$$
\frac{q}{L} = \frac{\Delta T}{L\sum R}
\begin{cases}
\Delta T = T_1 - T_4 = 900 - 122 = 778\ (^\circ F) \\
L\sum R = \dfrac{\ln\dfrac{r_2}{r_1}}{2\pi k_1} + \dfrac{\ln\dfrac{r_3}{r_2}}{2\pi k_2} + \dfrac{\ln\dfrac{r_4}{r_3}}{2\pi k_3}
\end{cases}
$$

$$
L\sum R = \frac{\ln\dfrac{2.37}{2.07}}{2\pi\times 23.5} + \frac{\ln\dfrac{\frac{2.37}{2}+1.25}{\frac{2.37}{2}}}{2\pi\times 0.058} + \frac{\ln\dfrac{\frac{2.37}{2}+1.25+2.5}{\frac{2.37}{2}+1.25}}{2\pi\times 0.042} = 4.654
$$

$$
\frac{q}{L} = \frac{\Delta T}{L\sum R} = \frac{778}{4.654} = \underline{\underline{167.16\ (Btu/ft\cdot hr)}}
$$

(b)

By Ohm's law

$$\frac{q}{L}=\frac{\Delta T}{L\sum R}\begin{cases}\Delta T=T_1-T_3=900-T_3\\ L\sum R=\dfrac{\ln\frac{r_2}{r_1}}{2\pi k_1}+\dfrac{\ln\frac{r_3}{r_2}}{2\pi k_2}\end{cases}$$

$$L\sum R=\frac{\ln\frac{2.37}{2.07}}{2\pi\times 23.5}+\frac{\ln\left(\frac{\frac{2.37}{2}+1.25}{\frac{2.37}{2}}\right)}{2\pi\times 0.058}=1.977$$

$$\frac{q}{L}=\frac{\Delta T}{L\sum R}=\frac{900-T_3}{1.977}=167.16 \text{，} \underline{\underline{T_3=569.48\ (^\circ F)}}$$

(c)

$$\frac{q}{L}=\frac{hA(T_4-T_\infty)}{L}$$

$$\frac{h\times 2\pi\underbrace{\left(\frac{\frac{2.37}{2}+1.25+2.5}{12}\right)}_{inch\ to\ ft}L\times(122-86)}{L}=167.16 \text{，} \underline{\underline{h=1.797\ (Btu/hr\cdot ft^2\cdot{}^\circ F)}}$$

Problem 4

【Solution】

(b)

$$\frac{1}{K_G}=\frac{m}{k_L}+\frac{1}{k_G}=\frac{10}{5\times 10^{-4}}+\frac{1}{0.01}\text{ ， }\underline{\underline{K_G=4.975\times 10^{-5}\ (kmol/m^2\cdot s\cdot atm)}}$$

$$\frac{1}{K_L}=\frac{1}{k_L}+\frac{1}{mk_G}=\frac{1}{5\times 10^{-4}}+\frac{1}{10\times 0.01}\text{ ， }\underline{\underline{K_L=4.975\times 10^{-4}\ (kmol/m^2\cdot s\cdot (kmol/m^3))}}$$

(a)

$$N'=K_L(x_L-x_L^*)=K_L(x_L-\frac{y_G}{m})=4.975\times 10^{-4}(1\times 10^{-3}\times 10^{-3}-\frac{0}{10})=\underline{\underline{4.975\times 10^{-10}\ (kmol/m^2\cdot s)}}$$

(c) **<u>The tower is a stripping tower</u>**

(d)

By **<u>Two-film theory</u>**

(1) The interface is at equilibrium.

(2) The two phases are in direct contact with each other.

(3) A concentration gradient will exist in each phase.

(本題可參考：Geankoplis, C. *Transport Processes and Separation Process Principles*, 4th ed.; p 636~637.)

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Problem 1

【Solution】

(a) (1) Constant molar heat of vaporization.

(2) No heat losses.

(3) No heat of mixing.

(4) Constant molar flow rate everywhere.

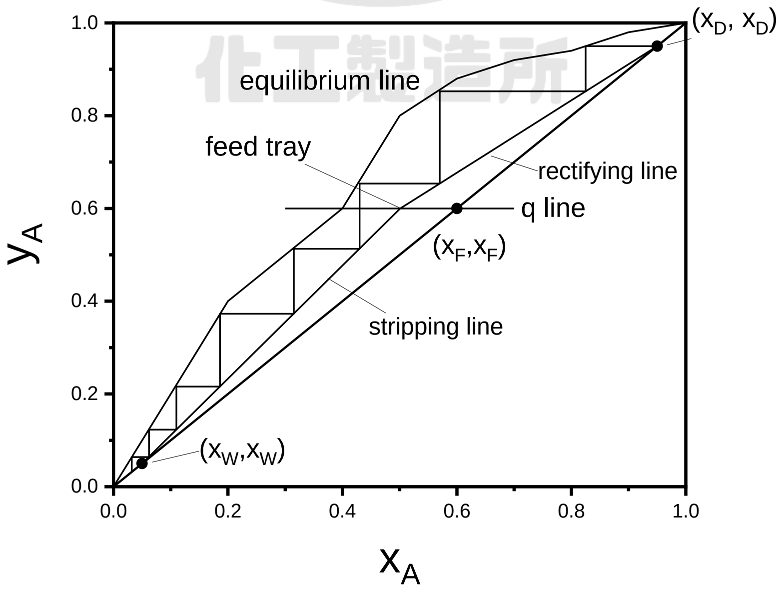
(5) Negligible pressure drop.

(6) The plates are ideally behaved.

If the top and bottom products are nearly pure in extreme compositions, and, if the two components have similar boiling points, the method could be used.

(本題可參考：Coulson, J.; Richardson, J.; Backhurst, J.; Harker, J. *Chemical Engineering*, 5th ed.; Volume 2, p 566.)

(b)



(此圖是借用其他題目的數據繪製而成，平衡線形狀可以簡單繪製曲線即可，且 q-line 部分不一定為水平線，主要強調通過 feed point，抑或可自行假設進料狀況。)

Problem 2

【Solution】

Assume the system is in steady state, constant P and T,

By shell balance of A,

$$N_{Ar}(4\pi r^2)\Big|_r - N_{Ar}(4\pi r^2)\Big|_{r+dr} = 0$$

同除 $4\pi dr \to 0$

$$\frac{d}{dr}(r^2 N_{Ar}) = 0$$

$$r^2 N_{Ar} = constant = r_1^2 N_{Ar1}$$

By Fick's law,

$$N_{Ar} = -D_{AB}C\frac{dx_A}{dr} + x_A(N_{Ar} + \underbrace{\cancel{N_{Br}}}_{stagnant})$$

$$N_{Ar} = \frac{-D_{AB}C}{(1-x_A)}\frac{dx_A}{dr} \text{ 代回}$$

$$r^2 N_{Ar} = r^2\frac{-D_{AB}C}{(1-x_A)}\frac{dx_A}{dr} = r_1^2 N_{Ar1}$$

$$r_1^2 N_{Ar1}\int_{r_1}^{r_2}\frac{1}{r^2}dr = -D_{AB}C\int_{x_{A1}}^{x_{A2}}\frac{1}{(1-x_A)}dx_A$$

$$r_1^2 N_{Ar1}(\frac{1}{r_1}-\frac{1}{r_2}) = D_{AB}C\ln(\frac{1-x_{A2}}{1-x_{A1}})$$

$$r_1^2 N_{Ar1}(\frac{r_2-r_1}{r_1 r_2}) = D_{AB}C\ln(\frac{1-x_{A2}}{1-x_{A1}})$$

$$r_1^2 N_{Ar1} = D_{AB}C(\frac{r_1 r_2}{r_2-r_1})\ln(\frac{1-x_{A2}}{1-x_{A1}})$$

$$\underline{\underline{N_{Ar1} = \frac{D_{AB}C}{r_2-r_1}(\frac{r_2}{r_1})\ln(\frac{1-x_{A2}}{1-x_{A1}})}}$$

(本題改編自：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 572, 18.B.7.)

Problem 3

【Solution】

(a)

(1)

The Navier-Stokes equation is based simplified from the equation of motion based on the assumptions of Newtonian fluid with constant ρ and μ .

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 84.)

(2)

With small Reynolds number, the viscous effect is dominant, the Navier-Stokes equation can be simplified to,

$$-\nabla P + \mu\nabla^2 \underline{v} + \rho \underline{g} = 0$$

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 85.)

(b)

(1) True

The gravity effect can be included in the modified pressure term.

(2) False

Only applicable to Newtonian fluid. For non-Newtonian fluids, the Hagen Poiseuille equation should be modified to a more general from.

(3) True

The equation can be used when the flow is laminar.

(本題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 52.)

(c)

(1) Yes

The equation of continuity is obtained from merely mass balance, regardless of the fluid type.

(2) Yes

The equation of motion is based on the momentum balance, regardless of the fluis type.

(3) No

The Stokes equation is based on the Navier-Stokes equation with small Reynolds number, but the flow in this question is compressible, which can not be described by Navier Stokes equation.

(本小題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 85.)

Problem 4

【Solution】

(a)

By the equation of motion in z-direction, assume steady state,

$$\rho\left(\cancel{\frac{\partial v_z}{\partial t}}+\cancel{v_r}\frac{\partial v_z}{\partial r}+\frac{v_\theta}{\cancel{r}}\cancel{\frac{\partial v_z}{\partial \theta}}+v_z\cancel{\frac{\partial v_z}{\partial z}}\right)=\cancel{\frac{-\partial p}{\partial z}}-\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})+\frac{1}{\cancel{r}}\cancel{\frac{\partial \tau_{\theta z}}{\partial \theta}}+\cancel{\frac{\partial \tau_{zz}}{\partial z}}\right]+\rho g_z$$

$$-\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})\right]+\rho g_z=0$$

$$\left[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})\right]=-\rho g\sin(30^\circ)=-\frac{\rho g}{2}$$

(重力與 z 方向相反，有負號)

$$\tau_{rz}=-\frac{\rho g}{4}r+\frac{c_1}{r}$$

$\because r=0,\ \tau_{rz}$ is finite, $c_1=0$

$$\tau_{rz}=-\frac{\rho g}{4}r$$

At r = R,

$$\left|\tau_{rz}\right|=\frac{\rho gR}{4}>\tau_0$$

<u>**The fluid is able to flow down.**</u>

(b)

By the equation of motion in z-direction, assume steady state,

$$\rho(\cancel{\frac{\partial v_z}{\partial t}}+\cancel{v_r}\frac{\partial v_z}{\partial r}+\cancel{\frac{v_\theta}{r}\frac{\partial v_z}{\partial \theta}}+v_z\cancel{\frac{\partial v_z}{\partial z}})=\frac{-\partial p}{\partial z}-[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})+\cancel{\frac{1}{r}\frac{\partial \tau_{\theta z}}{\partial \theta}}+\cancel{\frac{\partial \tau_{zz}}{\partial z}}]+\rho g_z$$

$$[\frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rz})]=\frac{-\partial p}{\partial z}+\rho g_z=\frac{P_0-P_L}{L}-\frac{\rho g}{2}$$

$$\tau_{rz}=(\frac{P_0-P_L}{2L}-\frac{\rho g}{4})r+\frac{c_2}{r}$$

$\because r=0,\ \tau_{rz}$ is finite, $c_2=0$

$$\tau_{rz}=(\frac{P_0-P_L}{2L}-\frac{\rho g}{4})r$$

Because the fluid should moves upward,

$$\tau_{rz}\big|_R=(\frac{P_0-P_L}{2L}-\frac{\rho g}{4})R>\tau_0$$

$$\frac{P_0-P_L}{2L}R>\frac{\rho g}{8}R+\frac{\rho g}{4}R$$

$$\frac{P_0-P_L}{2L}>\frac{3\rho g}{8}\text{ ，}\underline{\underline{P_0-P_L>\frac{3\rho gL}{4}}}$$

<u>Problem 5</u>

【Solution】

(1)

In this forced convection process, the two dimensionless numbers should be identical,

$$\mathrm{Re}=\textit{Reynolds number}$$

$$\mathrm{Pr}=\textit{Prandtl number}$$

(2)

$$\mathrm{Re}=\frac{\textit{inertial force}}{\textit{viscous force}}$$

$$\mathrm{Pr}=\frac{\textit{momentum diffusivity}}{\textit{thermal diffusivity}}$$

(3)

Conceptually, the heat transfer in this case involves conduction from the chip and the convection outward by the fluid flow. With the same heat transfer condition, the flow condition should be the same for both system accounting for the heat convection. With the same Prandtl number, the situation of thermal boundary layer within which the conduction occurs may be the same.

(4)

$$Nu = f(\mathrm{Re}, \mathrm{Pr})$$

When the two dimensionless group are the same, the Nusselt number, which is a function of the two parameters, should give the same functional form between the two systems. By the definition of Nusselt number,

$$Nu = \frac{hL}{k_f}$$

the Nusselt number can give the value of heat transfer coefficient for a given system.

(5)

At the edge of thermal boundary layer, we have,

$$hA(T_s - T_\infty) = \int_0^W \int_0^L -kA \left.\frac{\partial (T - T_s)}{\partial y}\right|_{y=0} dxdz = \underbrace{\int_0^W \int_0^L \rho C_P v_x (T - T_\infty) dydz}_{convection}$$

$$h = \int_0^W \int_0^L \rho C_P v_x [T^* - 1] dydz$$

$$\because T^* = f(\frac{y}{\Delta\delta}) \quad (\Delta = \frac{\delta_T}{\delta})$$

$$h = \int_0^W \int_0^L \rho C_P v_x [f(\frac{y}{\Delta\delta}) - 1] dydz = f(\mathrm{Re}, \mathrm{Pr})$$

$$\frac{hL}{k} = Nu = f(\mathrm{Re}, \mathrm{Pr})$$

If the two system have the same Re and Pr, the function from of Nusselt number should be the same because they should give the same value of Nusselt number.

(本小題可參考：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 388~390.)

(6)

※題目應改為<u>"in what velocity should you operate the air flow......"</u>，因模擬系統是在空氣下進行。

Because the Reynolds number should be the same between the model and the real system,

$$\frac{\rho_m v_m L_m}{\mu_m} = \frac{\rho_r v_r L_r}{\mu_r}$$

For model system,

$$\rho_m(300K) = 1.1614\ (kg/m^3)\ ,\ L_m = 10\ cm\ ,\ \mu_m = 184.6\times10^{-7}\ (Pa)$$

For real system,

$$\rho_r(290K) = \frac{1}{1.001}\times10^3\ (kg/m^3)\ ,\ L_r = 1\ (mm)\ ,\ \mu_r = 1080\times10^{-6}\ (Pa)$$

$$\frac{1.1614\times v_m\times10\times10}{184.6\times10^{-7}} = \frac{\frac{1000}{1.001}\times0.8\times1}{1080\times10^{-6}}$$

$$\underline{\underline{v_m = 0.117\ (m/s)}}$$

(7)

For the Prandtl number,

$$\Pr_{water} = \frac{(\frac{\mu}{\rho})}{(\frac{k}{\rho c_p})} = \frac{\mu c_p}{k} = \frac{1080\times10^{-6}\times4.184\times10^3}{598\times10^{-3}} = 7.56$$

$$\Pr_{air} = \frac{\upsilon}{\alpha} = \frac{15.89\times10^{-6}}{22.5\times10^{-6}} = 0.706$$

<u>Because the Prandtl number of the two systems are not the same, the choice may not be right.</u>

(8)

At steady state, assume the air in 27℃ is a suitable choice,

$$h(T-T_{\infty})=q^{"}$$

$$h(55-27)=200\text{ ，}\ h=7.143\ (W/m^2\cdot K)$$

The Nusselt number of the two system should be the same,

$$Nu_m=Nu_r$$

$$\frac{h_m L_m}{k_m}=\frac{h_r L_r}{k_r}$$

$$\frac{7.143\times 10\times 10}{26.3\times 10^{-3}}=\frac{h_r\times 1}{598\times 10^{-3}}$$

$$\underline{\underline{h_r=16241.2\ (W/m^2K)}}$$

(9)

As the air flow passes through the surface of the sink, the temperature should be higher because it's heated by the surface of the sink with higher temperature, under the steady state situation,

$$h(T-T_{\infty})=q^{"}$$

As $T_{\infty}\uparrow$, $T-T_{\infty}\downarrow$

$h\uparrow$ along the path.

(10)

(以下假設 heat sink 為立方體)

【Check the Biot number of the model system】

The Biot number is defined as,

$$Bi = \frac{h(\frac{V}{A_s})}{k} = \frac{\text{resistance to conduction within the solid}}{\text{resistance to convection across the film boundary}}$$

For k, the value of which can be taken as 46 in the temperature range,

$$Bi = \frac{h(\frac{L}{2})}{k_a} = \frac{7.143\times 10\times 10^{-2}}{46\times 2} = 7.76\times 10^{-3} < 0.1$$

(加熱面積為雙面，所以要除以 2)

(Assume the air in 27℃ is a suitable choice)

The lumped-parameter analysis is applicable in this case, by overall energy balance,

$$\rho V c_p \frac{\partial T}{\partial t} = hA(T - T_\infty)$$

$$\frac{\rho V c_p}{2hA}\int_{T_i}^{T_f} \frac{dT}{(T - T_\infty)} = \int_0^t dt$$

$$t = \frac{\rho L c_p}{2h}\ln\left(\frac{T_i - T_\infty}{T_f - T_\infty}\right)$$

$$t = \frac{3970\times 10\times 10^{-2}\times 765}{2\times 5.263}\ln\left(\frac{65-27}{40-27}\right) = \underline{\underline{22803.6\ (s)}}$$

(c_p 在操作溫度範圍內假設與 300K 下數值差不多。)

(11)

$$q = \rho c_p V(T_f - T_i) = 3970\times 765\times (10\times 10^{-2})^3 \times (65-40) = \underline{\underline{75926\ (J)}}$$

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Problem 1

【Solution】

(1)

$$\underline{\text{此題題目有誤，應將方程式改為}}\ \frac{\partial \rho}{\partial t}+\nabla\bullet(\rho v)=0$$

(a) False

(b) False

The equation of continuity is derived from mass balance, regardless of the steadiness.

(c) False

(2)

(a) True

(b) False

(c) False

Problem 2

【Solution】

(1)

Let the dimensionless velocity, time, and length is,

$$v_\theta=\frac{v_\theta}{v_0}\ ,\ \tilde{t}=\frac{t}{v_0/l_0}\ ,\ \tilde{r}=\frac{r}{l_0}$$

The PDE becomes,

$$\frac{\rho v_0^2}{l_0}\frac{\partial v_\theta}{\partial \tilde{t}}=\frac{\mu v_0}{l_0^2}\frac{\partial}{\partial \tilde{r}}[\frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}(\tilde{r}v_\theta)]$$

左右同除$\dfrac{\mu v_0}{l_0^2}$：

$$\frac{\rho v_0 l_0}{\mu}\frac{\partial v_\theta}{\partial \tilde{t}}=\frac{\partial}{\partial \tilde{r}}[\frac{1}{\tilde{r}}\frac{\partial}{\partial \tilde{r}}(\tilde{r}v_\theta)]$$

Define,

$$\mathrm{Re}=\frac{\rho\Omega R^2}{\mu}=\frac{inertial\ force}{viscous\ force}$$

$$(v_0=\Omega R\text{ ， }l_0=R)$$

(c)

For the similarity analysis, the Reynolds number of both systems should be the same,

$$\mathrm{Re}_R=\mathrm{Re}_{2R}$$

$\because v_0=\Omega r$ ， $l_o=r$

$$\frac{\rho(2\Omega R)R}{\mu}=\frac{\rho(2\Omega_{2R}R)(2R)}{\mu}\text{ ， }\underline{\underline{\Omega_{2R}=\frac{\Omega}{2}}}$$

<u>Problem 3</u>

【Solution】

(a)

To determine the height of the interface,

Assume：

(1) The flow is steady and fully developed in θ-direction

(2) $v_r=v_z=0$.

(3) Incompressible Newtonian fluid.

By the equation of continuity,

$$\cancel{\frac{\partial\rho}{\partial t}}+\cancel{\frac{1}{r}\frac{\partial}{\partial r}(\rho r v_r)}+\frac{1}{r}\frac{\partial}{\partial\theta}(\rho v_\theta)+\cancel{\frac{\partial}{\partial z}(\rho v_z)}=0$$

$$\frac{\partial v_\theta}{\partial\theta}=0,\ \ v_\theta=f(\cancel{t},r,\cancel{\theta},\cancel{z})=f(r)\text{only}$$

By the Navier Stokes equation in θ direction,

$$\rho\left(\cancel{\frac{\partial v_\theta}{\partial t}}+\cancel{v_r\frac{\partial v_\theta}{\partial r}}+\cancel{\frac{v_\theta}{r}\frac{\partial v_\theta}{\partial \theta}}+\cancel{v_z\frac{\partial v_\theta}{\partial z}}+\cancel{\frac{v_\theta v_r}{r}}\right)$$

$$=\cancel{\frac{-1}{r}\frac{\partial p}{\partial \theta}}+\mu\left[\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)+\cancel{\frac{1}{r^2}\frac{\partial^2 v_\theta}{\partial \theta^2}}+\cancel{\frac{\partial^2 v_\theta}{\partial z^2}}+\cancel{\frac{2}{r^2}\frac{\partial v_r}{\partial \theta}}\right]+\cancel{\rho g_\theta}$$

$$\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(rv_\theta)\right)=0\text{ ， }v_\theta=\frac{c_1}{2}r+\frac{c_2}{r}\begin{cases}r=0, v_\theta=0\\ r=R, v_\theta=\Omega R\end{cases}$$

$$c_1=2\omega\text{ ， }c_2=0$$

$$v_\theta=\Omega r$$

(b)

To determine the height of the interface, by Navier-Stokes equation with r and z components,

$$\frac{-\partial p}{\partial r}=\frac{-\rho v_\theta^2}{r}$$

$$\frac{-\partial p}{\partial z}=-\rho(-g_z)$$

Integrate the z-direction,

$$p=-\rho g_z z+f(r)$$

代回 r 方向，

$$\frac{\partial p}{\partial r}=f'(r)=\frac{\rho v_\theta^2}{r}=\frac{\rho[\Omega r]^2}{r}=\rho\Omega^2 r$$

Integrate with r,

$$f=\frac{\rho\Omega^2 r^2}{2}+C$$

And the pressure distribution becomes,

$$p(r,z)=-\rho g_z z+\frac{\rho\Omega^2 r^2}{2}+C$$

Because the liquid is open to the atmosphere at the top,

$$p(r,h(r)) = p(R,h(R)) = P_0, \quad p(r,h(r)) = p(R,h(R))$$

$$-\rho g_z h(r) + \frac{\rho\Omega^2 r^2}{2} + C = -\rho g_z h(R) + \frac{\rho\Omega^2 R^2}{2} + C$$

$$-\rho g_z (h - h(R)) = \frac{\rho\Omega^2 R^2}{2}[1-(\frac{r}{R})^2]$$

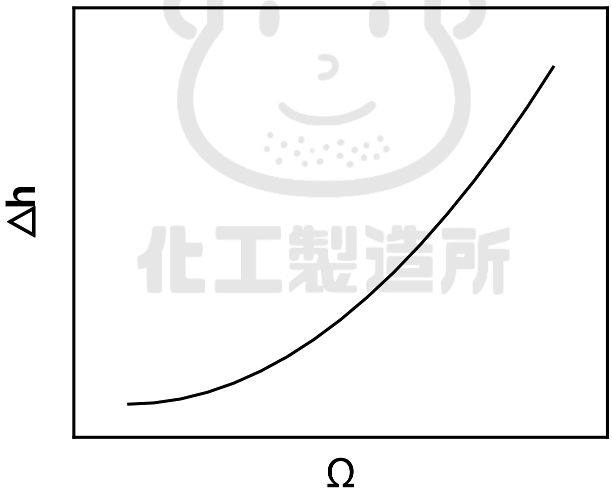
$$h(r) = \frac{\Omega^2 R^2}{2g_z}[(\frac{r}{R})^2 - 1] + h(R)$$

$\Delta h = h(R) - h(0)$

$$\underline{\underline{\Delta h = h(R) - h(0) = \frac{\Omega^2 R^2}{2g_z}}}$$

(c)

$\Delta h \propto \Omega^2$，為拋物線：



(本題改編自：Bird, R.; Stewart, W.; Lightfoot, E. *Transport Phenomena*, 2nd ed.; p 110, Problem 3B.15.)

Problem 4

【**Solution**】

Assumptions：

(1) Steady state heat transfer, convective heat transfer = latent heat of the water in the plumage.

(2) The vaporization process of the liquid water occurs at 32°C (liquid water film temperature).

(3) Constant properties.

By the steady state heat transfer assumption:

$$h(T_\infty - T_s)A_s t = \lambda(\Delta m) \text{ ， } t = \frac{\lambda(\Delta m)}{h(T_\infty - T_s)A_s}$$

For the heat transfer coefficient h,

$$\overline{Nu_L} = 0.034\,\mathrm{Re}_L^{4/5}\,\mathrm{Pr}^{1/3}$$

$$\frac{hL}{k_f} = 0.034(\frac{\rho VL}{\mu})^{0.8}(\mathrm{Pr})^{1/3}$$

For k_f, by Table A.4,

$$\frac{(273.15+37)-300}{350-300} = \frac{k_f - 26.3\times10^{-3}}{30.0\times10^{-3} - 26.3\times10^{-3}} \text{ ， } k_f = 27.1\times10^{-3}\ (W/m\cdot K)$$

For ρ, by Table A.4,

$$\frac{(273.15+37)-300}{350-300} = \frac{\rho - 1.1614}{0.9950 - 1.1614} \text{ ， } \rho = 1.128\ (kg/m^3)$$

For μ, by Table A.4,

$$\frac{(273.15+37)-300}{350-300} = \frac{\mu - 184.6\times10^{-7}}{208.2\times10^{-7} - 184.6\times10^{-7}} \text{ ， } \mu = 1.89\times10^{-5}\ (N\cdot s/m^2)$$

For Pr, by Table A.4,

$$\frac{(273.15+37)-300}{350-300} = \frac{\mathrm{Pr} - 0.707}{0.700 - 0.707} \text{ ， } \mathrm{Pr} = 0.705$$

代回：

$$\frac{hL}{k_f} = 0.034(\frac{\rho VL}{\mu})^{0.8}(\Pr)^{1/3}$$

$$h = 0.034(\frac{\rho V}{\mu})^{0.8}(\Pr)^{1/3} k_f L^{-0.2}$$

$$= 0.034 \times (\frac{1.128 \times 30}{1.89 \times 10^{-5}})^{0.8} \times (0.705)^{1/3} \times 27.1 \times 10^{-3} \times (0.04)^{-0.1}$$

$$= 113.63\ (W/m^2 \cdot K)$$

For the latent heat of water, by Table A.6, we have $\lambda = 2426\ (kJ/kg)$

$$t = \frac{\lambda(\Delta m)}{h(T_\infty - T_s)A_s} = \frac{2426 \times 10^3 \times (0.05 \times 0.5)}{113.63(37-32) \times 0.04} = 2668\ (s)$$

The maximum allowable distance is,

$$\underline{\underline{L = 2668 \times 30 = 80064\ (m)}}$$

(本題改編自：Incropera, F.; Lavine, A.; Bergman, T. *Fundamentals of Heat and Mass Transfer*, 6th ed, p 396, Problem 6.58. 原文書給定之參考解答是利用質傳的方式作切入，計算出所需之時間與距離，同學可嘗試與熱傳方式做比較。)

Problem 5

【**Solution**】

Assume:

(1) Steady-state, one dimensional diffusion

(2) Constant properties

(3) Uniform temperature and pressure

(4) The air is stagnant

By shell balance of A,

$$N_{Az}\cdot A\big|_{z}-N_{Az}\cdot A\big|_{z+dz}=0$$

同除 $dz\to 0$

$$\frac{-dN_{Az}}{dz}=0\text{ ， }N_{Az}\neq f(z)$$

For N_{Az}, by Fick's law,

$$N_{Az}=-cD_{AB}\frac{dx_A}{dz}+x_A(N_{Az}+\cancel{N_{Bz}})$$

$$N_{Az}=-\frac{cD_{AB}}{1-x_A}\frac{dx_A}{dz}$$

代回：

$$\frac{d}{dz}\left(\frac{1}{1-x_A}\frac{dx_A}{dz}\right)=0$$

積分兩次：

$$-\ln(1-x_A)=c_1z+c_2\begin{cases}z=0,\ x_A=x_{A,sat}\\ z=L,\ x_A=0\end{cases}$$

$$\begin{cases}c_1=\dfrac{\ln(1-x_{A,sat})}{L}\\ c_2=-\ln(1-x_{A,sat})\end{cases}$$

For the evaporation rate,

$$N_{Az} = -\frac{cD_{AB}}{1-x_A}\frac{dx_A}{dz} = -\frac{cD_{AB}\ln(1-x_{A,sat})}{L}$$

For $x_{A,sat}$, the saturated pressure is, (by Table A.6)

$$p_{sat} = 0.1053\ (bars)$$

$$x_{A,sat} = \frac{p_{sat}}{P_{total}} = \frac{0.1053}{0.25\times 1.013} = 0.410$$

代入：

$$\frac{0.257\times 10^3}{18\times 3600} = -\frac{cD_{AB}\ln(1-x_{A,sat})}{L} = -\frac{PD_{AB}\ln(1-x_{A,sat})}{RTL} = -\frac{0.25\times 1.013\times 10^5 D_{AB}\ln(1-0.410)}{8.314\times 320\times 150\times 10^{-3}}$$

$$\underline{\underline{D_{AB} = 1.18\times 10^{-4}\ (m^2/s)}}$$

(本題改編自：Incropera, F.; Lavine, A.; Bergman, T. *Fundamentals of Heat and Mass Transfer*, 6th ed, p 918, Problem 14.13.)

Problem 6

【Solution】

(a)

The heat equation is the results of **energy balance equation/relationship of the system.**

$$\underbrace{\frac{1}{r}\frac{\partial}{\partial r}(kr\frac{\partial T}{\partial r}) + \frac{1}{r^2}\frac{\partial}{\partial \phi}(k\frac{\partial T}{\partial \phi}) + \frac{\partial}{\partial z}(k\frac{\partial T}{\partial z})}_{heat\ transfer\ by\ conduction} + \underset{heat\ generation}{q} = \underbrace{\rho c_p \frac{\partial T}{\partial t}}_{increase\ in\ energy}$$

(b)

Constant ρ and c_p

(c)

The heat transfer from the top and bottom ends of the cup is negligible, therefore we may neglect the impact of z direction,

$$\frac{1}{r}\frac{\partial}{\partial r}(kr\frac{\partial T}{\partial r})+\frac{1}{r^2}\frac{\partial}{\partial \phi}(k\frac{\partial T}{\partial \phi})+q=\rho c_p\frac{\partial T}{\partial t}$$

(d)

Assumptions：

(1) No extra heat generation sources needed be considered,

$$q=0$$

(2) Axial symmetry for the cup,

$$T\neq T(\phi)$$

(3) Within the short period of time, the temporal variation of the temperature is negligible,

$$\cancel{\rho c_p\frac{\partial T}{\partial t}}$$

(4) Constant properties

$$\frac{k}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})=0$$

(e)

$$\frac{k}{r}\frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})=0\ ，\ \frac{\partial}{\partial r}(r\frac{\partial T}{\partial r})=0$$

$$T=c_1\ln r+c_2\begin{cases}r=0,\ T\ \text{is finite}\\ r=R,\ T=T_1\end{cases}\ ，\ \begin{cases}c_1=0\\ c_2=T_1\end{cases}$$

$$\underline{\underline{T=T_1}}$$

(f)

$$\underline{\underline{T=T_1}}$$

(g)

按題意，假設：

(1) 按題意要找一個最安全的 PLA 容器厚度，因造成 PLA 升溫的來源主要為容器內的咖啡，因此若咖啡維持在最熱的情況下，其經由 PLA 輻射出去在杯壁造成的溫度 $<35^\circ\mathrm{C}$，就代表其他較低溫度也能滿足此假設。

$$\frac{d^2T}{dx^2}=0$$

$$T=c_1x+c_2\begin{cases}x=0,\ T=T_1\\ x=L,-k\dfrac{dT}{dx}=\sigma\varepsilon(T_0^4-T_\infty^4)\end{cases}$$

(第二個邊界條件按題意也需要對流項，然而沒給對流係數)

代入 B.C.1：

$$c_2=T_1$$

代入 B.C.2：

$$-kc_1=\sigma\varepsilon[(T_0^4-T_\infty^4)]，\ c_1=-\frac{\sigma\varepsilon}{k}(T_0^4-T_\infty^4)$$

代回：

$$T=-\frac{\sigma\varepsilon}{k}(T_0^4-T_\infty^4)x+T_1$$

因顧客要求標準杯壁溫度必須小於 $35^\circ\mathrm{C}$，因此：

$$T(L)=-\frac{\sigma\varepsilon}{k}(T_0^4-T_\infty^4)L+T_1\le(273.15+35)$$

$$L\ge\frac{k[T_1-(273.15+35)]}{\sigma\varepsilon(T_0^4-T_\infty^4)}$$

$$(35+273.15)=-\frac{5.67\times10^{-8}\times0.8}{0.193}[(90+273.15)^4-(25+273.15)^4]L+(90+273.15)$$

$$\underline{\underline{L\ge0.20\ (m)}}$$