European Transport Simulator:

transport equations, variables
and Fortran implementation

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General comments

Grids:

ETS uses two types of grids for the toroidal flux coordinate, not normalized ρ and normalized x, which are defined as:

$$\rho = \sqrt{\frac{\Phi}{\pi B_0}} \quad \text{[m]};$$

Φ [Wb] is the toroidal flux and B_0 [T] is the magnetic field measured at the characteristic major radius of the device R_0 [m];

$x = \dfrac{\rho}{\rho_b}$ [-], where ρ_b is the coordinate of plasma magnetic boundary.

$$\frac{\partial}{\partial \rho} = \frac{\partial}{\partial x}\frac{\partial x}{\partial \rho} = \frac{1}{\rho_b}\frac{\partial}{\partial x}$$

All interfaces between different modules and subroutines use ρ as the primary coordinate, internally, transport equations are solved using x coordinate.

Use of the normalized coordinate x introduces an artificial non-physical convection term in all transport equations. Any change in the boundary value of $\rho_b(t)$ results in rearrangement of all plasma characteristics throughout the whole cross-section. In many circumstances, as the initial phase of discharge, L-H transition, strong additional heating, etc., the correction is not negligible.

Form of transport equations and adiabatic compression terms:

Consider the equation:

$$\left.\frac{\partial}{\partial t}\right|_{\rho}\left(\frac{\partial V}{\partial \rho}\cdot\langle f\rangle\right)+\frac{\partial}{\partial \rho}\left(\frac{\partial V}{\partial \rho}\cdot\left\langle|\nabla\rho|^2\right\rangle\cdot\Gamma_{\rho}\right)=\frac{\partial V}{\partial \rho}\cdot S\,, \tag{0.1}$$

where:

- $\Gamma_{\rho} = \left\langle f\,(\vec{u}-\vec{u}_{\rho})\nabla\rho\right\rangle \Big/ \left\langle|\nabla\rho|^2\right\rangle$ is the flux through the surface $\rho = const$. One shall note, that it is different to the diffusive flux $\Gamma_D = \left\langle f\,(\vec{u}-\vec{u}_{\Phi})\nabla\rho\right\rangle \Big/ \left\langle|\nabla\rho|^2\right\rangle$. The difference between Γ_ρ and Γ_D can be highlighted by noting that, in absence of collisions, $\Gamma_D \equiv 0$ whereas Γ_ρ can be anything. On the other hand under the condition of $\dot{B}_0 = \dfrac{dB_0}{dt} = 0$, $\Gamma_D \equiv \Gamma_\rho$.
- here $\vec{u}$ is the plasma mass velocity and $\vec{u}_\rho$ is the velocity of the surface $\rho = const$
- S is the source density.

For negligible variations of f on a flux-surface, the difference between Γ_ρ and Γ_D is proportional to

$$\left\langle\left(\vec{u}_\rho-\vec{u}_\Phi\right)\cdot\nabla\rho\right\rangle = \left.\frac{\partial\rho}{\partial t}\right|_{\Phi} = \frac{\dot{B}_0}{2B_0}\rho\,.$$

Thus the transport equation can be re-written in the form:

$$\left.\frac{\partial}{\partial t}\right|_{\rho}\left(\frac{\partial V}{\partial \rho}\langle f\rangle\right)+\frac{\partial}{\partial \rho}\left(\frac{\partial V}{\partial \rho}\left[\left\langle|\nabla\rho|^{2}\right\rangle\Gamma_{D}-\frac{\dot{B}_{0}}{2B_{0}}\rho\langle f\rangle\right]\right)=\frac{\partial V}{\partial \rho}\cdot S$$

or as

$$\left(\left.\frac{\partial}{\partial t}\right|_{\rho}-\frac{\dot{B}_{0}}{2B_{0}}\frac{\partial}{\partial \rho}\rho\right)\cdot\left(\frac{\partial V}{\partial \rho}\langle f\rangle\right)+\frac{\partial}{\partial \rho}\left(\frac{\partial V}{\partial \rho}\left\langle|\nabla\rho|^{2}\right\rangle\left(-D\frac{\partial}{\partial \rho}\langle f\rangle+\upsilon\langle f\rangle\right)\right)=\frac{\partial V}{\partial \rho}\cdot S\,, \qquad (0.2)$$

on the assumption of

$$\Gamma_{D}=-D\frac{\partial}{\partial \rho}\langle f\rangle+\upsilon\langle f\rangle.$$

This is a standard representation of the transport equation used in ETS for the fixed ρ grid

If we introduce following notations:

$$k_{\Phi}(t)=\frac{\dot{\Phi}_{b}}{2\Phi_{b}}=k_{B}(t)+k_{\rho}(t),\quad k_{B}(t)=\frac{\dot{B}_{0}}{2B_{0}}=\frac{1}{2B_{0}}\cdot\frac{dB}{dt},\quad k_{\rho}(t)=\frac{\dot{\rho}_{b}}{\rho_{b}}=\frac{1}{\rho_{b}}\cdot\frac{d\rho_{b}}{dt} \qquad (0.3)$$

The equation can be rewritten as:

$$\left.\frac{\partial}{\partial t}\right|_{\rho}\cdot\left(\frac{\partial V}{\partial \rho}\langle f\rangle\right)+\frac{\partial}{\partial \rho}\left(\frac{\partial V}{\partial \rho}\left[\left\langle|\nabla\rho|^{2}\right\rangle\left(-D\frac{\partial}{\partial \rho}\langle f\rangle+\upsilon\langle f\rangle\right)-k_{B}\rho\langle f\rangle\right]\right)=\frac{\partial V}{\partial \rho}\cdot S\,. \qquad (0.4)$$

Using (0.3) and

$$\left.\frac{\partial a}{\partial t}\right|_{\rho}=\left.\frac{\partial a}{\partial t}\right|_{x}+\frac{\partial a}{\partial x}\cdot\left.\frac{\partial x}{\partial t}\right|_{\rho},$$

equation (0.4) can be rewritten for the normalised coordinate, x:

$$\left.\frac{\partial}{\partial t}\right|_{x}\cdot\left(\frac{\partial V}{\partial \rho}\langle f\rangle\right)+\frac{\partial}{\partial \rho}\left(\frac{\partial V}{\partial \rho}\left[\left\langle|\nabla\rho|^{2}\right\rangle\left(-D\frac{\partial}{\partial \rho}\langle f\rangle+\upsilon\langle f\rangle\right)-k_{\Phi}\rho\langle f\rangle\right]\right)=\frac{\partial V}{\partial \rho}\cdot\left(S-k_{\rho}\langle f\rangle\right). \qquad (0.5)$$

It is also useful to define some transformations as:

$$\frac{3}{2}\left(\frac{\partial V}{\partial \rho}\right)^{-5/3}\cdot\left(\left.\frac{\partial}{\partial t}\right|_{\rho}-\frac{\dot{B}_{0}}{2B_{0}}\frac{\partial}{\partial \rho}\cdot\rho\right)\cdot\left[\left(\frac{\partial V}{\partial \rho}\right)^{5/3}\langle f\rangle\right]=$$

$$\frac{3}{2}\left(\frac{\partial V}{\partial \rho}\right)^{-5/3}\left.\frac{\partial}{\partial t}\right|_{x}\left[\left(\frac{\partial V}{\partial \rho}\right)^{5/3}\langle f\rangle\right]-\left(\frac{\partial V}{\partial \rho}\right)^{-1}\frac{\partial}{\partial \rho}\cdot\left[\left(\frac{\partial V}{\partial \rho}\right)\cdot\frac{3}{2}\frac{\dot{\Phi}_{b}}{2\Phi_{b}}\rho\cdot\langle f\rangle\right]+\left[\frac{3}{2}\frac{\dot{\rho}_{b}}{\rho_{b}}-\frac{\dot{\Phi}_{b}}{2\Phi_{b}}\rho\frac{\partial}{\partial \rho}\ln\frac{\partial V}{\partial \rho}\right]\langle f\rangle \qquad (0.6)$$

and:

$$\rho \frac{\partial}{\partial \rho} \ln \frac{\partial V}{\partial \rho} = 1 + \rho \frac{\partial}{\partial \rho} \ln\left(\frac{1}{\rho} \frac{\partial V}{\partial \rho} \right) = 1 + \rho \frac{\partial}{\partial \rho} \ln\left(\frac{4\pi^2 B_0}{F\left\langle \frac{1}{R^2} \right\rangle} \right) = 1 - \rho \frac{\partial}{\partial \rho} \ln\left(F\left\langle \frac{1}{R^2} \right\rangle \right) \tag{0.7}$$

Generalized form of transport equations:

All numerical solvers used within the ETS are adapted to solve the transport equation in the form:

$$\frac{a(x)\cdot Y(x,t) - b(x)\cdot Y(x,t-1)}{h} + \frac{1}{c(x)} \frac{\partial}{\partial x}\left(-d(x)\cdot \frac{\partial Y(x,t)}{\partial x} + e(x)\cdot Y(x,t) \right) = \\ = f(x) - g(x)\cdot Y(x,t) \tag{0.8}$$

where Y is the function, t is the time and x is the radial coordinate.

The boundary conditions shall be provided in the form:

$$v(x_{bnd})\cdot \left.\frac{\partial Y(x,t)}{\partial x}\right|_{bnd} + u(x_{bnd})\cdot Y(x_{bnd},t) = w(x_{bnd})$$

Therefore the following set of quantities is expected by the numerical solver:

$$x, \quad a(x), \quad b(x), \quad c(x), \quad d(x), \quad e(x), \quad f(x), \quad g(x), \quad h, \quad Y(x,t-1)$$

$$u(1:2), \quad v(1:2), \quad w(1:2),$$

where index *1* refers to the plasma axis and *2* to the plasma boundary.

The solver will return:

$$Y(x,t), \quad \frac{\partial Y(x,t)}{\partial x}$$

Current diffusion equation

Variables and units:

ψ - flux function [Wb]

$j_{\parallel}$ - parallel current density [A/m^2]

j_{tor} - toroidal current density [A/m^2]

Q_{OH} - ohmic heating power [W]

q - safety factor [-]

$E_{\parallel}$ - parallel electric field [V/m]

$\sigma_{\parallel}$ - parallel conductivity [Ohm^{-1} m^{-1}]

μ_0 - permeability of free space ;

R_0 - characteristic major radius of the device [m]

B_0 - magnetic field measured at R_0 [T]

F - diamagnetic function [m·T]

Equation:

For a constant ρ grid, the equation

is $\sigma_{//}\left(\left.\frac{\partial\Psi}{\partial t}\right|_{\rho}-\frac{\rho\dot{B}_0}{2B_0}\frac{\partial\Psi}{\partial\rho}\right)=\frac{F^2}{\mu_0 B_0\rho}\frac{\partial}{\partial\rho}\left[\frac{V'}{4\pi^2}\left\langle\left|\frac{\nabla\rho}{R}\right|^2\right\rangle\frac{1}{F}\frac{\partial\Psi}{\partial\rho}\right]-\frac{V'}{2\pi\rho}\left(j_{ni,\exp}+j_{ni,imp}\cdot\Psi\right)$

Where the diamagnetic function, $F=RB_{\varphi}$, the geometry coefficients V' and $\left\langle\left|\frac{\nabla\rho}{R}\right|^2\right\rangle$ are provided by the equilibrium solver.

For a constant normalized grid, this becomes

$$\sigma_{//}\left.\frac{\partial\Psi}{\partial t}\right|_{x}=\frac{F^2}{\mu_0 B_0\rho}\frac{\partial}{\partial\rho}\left[\frac{V'}{4\pi^2}\left\langle\left|\frac{\nabla\rho}{R}\right|^2\right\rangle\frac{1}{F}\frac{\partial\Psi}{\partial\rho}\right]-\frac{V'}{2\pi\rho}\left(j_{ni,\exp}+j_{ni,imp}\cdot\Psi\right)+\rho\sigma_{//}\cdot\frac{\dot{\Phi}_b}{2\Phi_b}\cdot\frac{\partial\Psi}{\partial\rho} \tag{1.1}$$

Equation (1.1) can be reformulated as:

$$\begin{aligned}\sigma_{//}\left.\frac{\partial\Psi}{\partial t}\right|_{x}=&\frac{F^2}{\mu_0 B_0\rho}\frac{\partial}{\partial\rho}\left[\frac{V'}{4\pi^2}\left\langle\left|\frac{\nabla\rho}{R}\right|^2\right\rangle\frac{1}{F}\frac{\partial\Psi}{\partial\rho}+\frac{\mu_0 B_0\rho^2\sigma_{//}}{F^2}\cdot\frac{\dot{\Phi}_b}{2\Phi_b}\cdot\Psi\right]-\frac{V'}{2\pi\rho}\left(j_{ni,\exp}+j_{ni,imp}\cdot\Psi\right)\\&+\sigma_{//}\left(2\rho\cdot\frac{1}{F}\cdot\frac{\partial F}{\partial\rho}-\rho\cdot\frac{1}{\sigma_{//}}\cdot\frac{\partial\sigma_{//}}{\partial\rho}-2\right)\cdot\frac{\dot{\Phi}_b}{2\Phi_b}\cdot\Psi\end{aligned} \tag{1.2}$$

non inductive current includes contributions in generic form from external sources (*computed by other modules, like ICRH, ECRH, NBI, neutrals and etc*.), in the form of explicit and implicit terms:

$$j_{ni,\exp} = \sum_{isource=1}^{nsource} m_{source} \cdot j_{isource,\exp} \tag{1.3}$$

$$j_{ni,imp} = \sum_{isource=1}^{nsource} m_{source} \cdot j_{isource,imp} \ . \tag{1.4}$$

This summation is done in the workflow by ***Source_Combiner***, where individual weights, m_{source}, can be adjusted.

Boundary Conditions:

On axis, *at* $\rho = 0$, ETS assumes:

$$\left.\frac{\partial \psi}{\partial \rho}\right|_{\rho=0} = 0 \tag{1.5}$$

At the edge, *at* $\rho = \rho_{bnd}$, following options (distinguished by boundary condition type) are available:

type=1 (value)

$$\left.\psi\right|_{\rho=\rho_{bnd}} = \psi_{bnd}, \tag{1.6}$$

where ψ_{bnd} is the value of poloidal flux at the outer boundary.

type=2 (total current inside $\rho = \rho_{bnd}$)

$$\left.\frac{\partial \psi}{\partial \rho}\right|_{\rho=\rho_{bnd}} = \frac{4\pi^2 \mu_0}{V' \left\langle \left| \frac{\nabla \rho}{R} \right|^2 \right\rangle} I_{bnd} \tag{1.7}$$

where I_{bnd} is the value of plasma current inside the boundary.

type=3 (loop voltage at $\rho = \rho_{bnd}$)

$$\left.\frac{\partial \psi}{\partial t}\right|_{\rho=\rho_{bnd}} = U_{loop,bnd} - \left.\frac{\partial \Psi}{\partial \rho}\right|_{\rho=\rho_{bnd}} \cdot \frac{\partial \rho_b}{\partial t} \tag{1.8}$$

where $U_{loop,bnd}$ is the plasma loop voltage at the boundary.

type=4 (generic)

$$v_{gen} \left.\frac{\partial \psi}{\partial \rho}\right|_{\rho=\rho_{bnd}} + u_{gen} \left.\psi\right|_{\rho=\rho_{bnd}} = w_{gen} \tag{1.9}$$

Definitions for quantities derived from current equation:

Poloidal components of the magnetic field and flux function:

$$B_{pol} = \frac{|\nabla\rho|}{2\pi R_0}\frac{\partial\psi}{\partial\rho} \tag{1.10}$$

Safety factor:

$$q = \frac{2\pi B_0 \rho}{(\partial\Psi / \partial\rho)} \tag{1.12}$$

Total current:

(toroidal) $$j_{tor} = \frac{2\pi R_0}{\mu_0 V'}\cdot\frac{\partial}{\partial\rho}\left(H\frac{\partial\psi}{\partial\rho}\right); \quad H = \frac{V'}{4\pi^2}\cdot\left\langle\left(\frac{\nabla\rho}{R}\right)^2\right\rangle \tag{1.13}$$

(parallel) $$j_{\parallel} = \frac{2\pi}{\mu_0 R_0 V'}\cdot\left(\frac{F}{B_0}\right)^2\frac{\partial}{\partial\rho}\left(\frac{R_0 B_0}{F}H\frac{\partial\psi}{\partial\rho}\right) \tag{1.14}$$

Ohmic heating & parallel electric field:

$$Q_{OH} = \sigma_{\parallel} E_{\parallel}^{\ 2} \qquad E_{\parallel} = \frac{1}{\sigma_{\parallel}}\left(j_{\parallel} - j_{ni,\exp} - j_{ni,imp}\cdot\Psi\right) \tag{1.15}$$

If equation (1.2) is not solved, *ets will extrapolate the quantities using safety factor profile as primary quantity:*

$$\Psi = \int_0^{\rho}\frac{2\pi B_0}{q}\rho\,\partial\rho\ ; \tag{1.16}$$

$$j_{tor} = \frac{2\pi R_0}{\mu_0 V'}\cdot\frac{\partial}{\partial\rho}\left(H\frac{2\pi B_0\rho}{q}\right); \tag{1.17}$$

$$j_{\parallel} = \frac{2\pi}{\mu_0 R_0 V'}\cdot\left(\frac{F}{B_0}\right)^2\frac{\partial}{\partial\rho}\left(\frac{R_0 B_0}{F}H\frac{2\pi B_0\rho}{q}\right) \tag{1.18}$$

$$Q_{OH} = \frac{1}{\sigma_{\parallel}}\left(j_{\parallel} - j_{ni,\exp} - j_{ni,imp}\cdot\Psi\right)^2 \tag{1.19}$$

Translation to generalized form for interface with numerical solver:

Rewrite the equation (1.2) using the time discretization: $\frac{\Psi - \Psi^-}{\tau}$,where Ψ^- is the value of the poloidal flux at previous time and τ is the time step.

$$\sigma_{//} \left. \frac{\Psi - \Psi^-}{\tau} \right|_x + \frac{F^2}{\mu_0 B_0 \rho \rho_b} \frac{\partial}{\partial x}\left[-\frac{V'}{4\pi^2}\left\langle \left| \frac{\nabla \rho}{R} \right|^2 \right\rangle \frac{1}{F\rho_b} \frac{\partial \Psi}{\partial x} - \frac{\mu_0 B_0 \rho^2 \sigma_{//}}{F^2} \cdot \frac{\dot{\Phi}_b}{2\Phi_b} \cdot \Psi \right] = -\frac{V'}{2\pi\rho} j_{ni,\exp}$$

$$- \left[\frac{V'}{2\pi\rho} j_{ni,imp} + \sigma_{//} \left(2 - 2\rho \cdot \frac{1}{F} \cdot \frac{\partial F}{\partial \rho} + \rho \cdot \frac{1}{\sigma_{//}} \cdot \frac{\partial \sigma_{//}}{\partial \rho} \right) \cdot \frac{\dot{\Phi}_b}{2\Phi_b} \right] \cdot \Psi \qquad (1.20)$$

Then, using the equation (0.8), coefficients for the interface with numerical solver are defined as:

$$a(x) = \sigma_{//}$$

$$b(x) = \sigma_{//}$$

$$c(x) = \frac{\mu_0 B_0 \rho \rho_b}{F^2}$$

$$d(x) = \frac{V'}{4\pi^2}\left\langle \left| \frac{\nabla \rho}{R} \right|^2 \right\rangle \frac{1}{F\rho_b} \qquad (1.21)$$

$$e(x) = -\frac{\mu_0 B_0 \rho^2 \sigma_{//}}{F^2} \cdot \frac{\dot{\Phi}_b}{2\Phi_b}$$

$$f(x) = -\frac{V'}{2\pi\rho} j_{ni,\exp}$$

$$g(x) = \frac{V'}{2\pi\rho} j_{ni,imp} + \sigma_{//} \left(2 - 2\rho \cdot \frac{1}{F} \cdot \frac{\partial F}{\partial \rho} + \rho \cdot \frac{1}{\sigma_{//}} \cdot \frac{\partial \sigma_{//}}{\partial \rho} \right) \cdot \frac{\dot{\Phi}_b}{2\Phi_b}$$

$$h = \tau$$

$$Y^{t-1}(\rho) = \Psi^-$$

boundary conditions (1.22)
at the axis:

$$v(1) = 1; \quad u(1) = 0; \quad w(1) = 0$$

At the edge, $x = 1$:

***type=1* (value)**

$$\psi\big|_{x=1} - \psi_{bnd} = 0$$

$$v(2) = 0; \quad u(2) = 1; \quad w(2) = \psi_{bnd}$$

***type=2* (total current inside $x = 1$)**

$$\left.\frac{\partial\psi}{\partial x}\right|_{x=1} = \frac{4\pi^2\mu_0}{V'\left\langle\left|\frac{\nabla\rho}{R}\right|^2\right\rangle} I_{bnd}\cdot\rho_b$$

$$v(2) = 1; \quad u(2) = 0; \quad w(2) = \frac{4\pi^2\mu_0}{V'\left\langle\left|\frac{\nabla\rho}{R}\right|^2\right\rangle} I_{bnd}\cdot\rho_b$$

type=3 (loop voltage at $x = 1$)

$$\psi\big|_{x=1} - \left(\tau U_{loop,bnd} + \psi^{-}_{bnd}\right) = 0$$

$$v(2) = 0; \quad u(2) = 1; \quad w(2) = \tau U_{loop,bnd} + \psi^{-}_{bnd}$$

type=4 (generic)

$$\frac{v_{gen}}{\rho_b}\left.\frac{\partial\Psi}{\partial x}\right|_{bnd} + u_{gen}\Psi_{bnd} = w_{gen}$$

$$v(2) = \frac{v_{gen}}{\rho_b}; \quad u(2) = u_{gen}; \quad w(2) = w_{gen}$$

Variables used inside fortran routines:

(Routine: MAIN_PLASMA, Subroutine: CURRENT)

Variable	TYPE%NAME used in ETS data flow	Internal name used in CURRENT routine	Units
τ	EVOLUTION%TAU	TAU	s
$\dot{B}_0$	GEOMETRY%BTPRIME	BTPRIME	T/s
B_0	GEOMETRY%BT	BT	T
R_0	GEOMETRY%R0	R0	m
ρ	GEOMETRY%RHO	RHO	m

x	GEOMETRY%RHO_NORM	RHO_NORM	-
V'	GEOMETRY%VPR	VPR	m^2
$\left\langle \left\lvert \frac{\nabla \rho}{R} \right\rvert^2 \right\rangle$	GEOMETRY%G3	G3	m^{-2}
F	GEOMETRY%FDIA	FDIA	T·m
H	--	H	
π	--	PI	
μ_0	--	MU0	H/m
Ψ	PROFILES%PSI	PSI	V·s
Ψ^-	EVOLUTION%PSIM	PSIM	V·s
q	PROFILES%QSF	QSF	--
j_{tor}	PROFILES%CURR_TOR	CURR_TOR	A/m^2
$j_{\parallel}$	PROFILES %CURR_PAR	CURR_PAR	A/m^2
$j_{ni,\exp}$	--	CURR_NI_EXP	A/m^2
$j_{ni,imp}$	--	CURR_NI_IMP	$A/(V\cdot s\cdot m^2)$
$j_{source,1}$	SOURCES%CURR_NI_EXP	--	A/m^2
$j_{source,2}$	SOURCES %CURR_NI_IMP	--	$A/(V\cdot s\cdot m^2)$
Q_{OH}	SOURCES%QOH	QOH	W/m^3
$\sigma_{\parallel}$	TRANSPORT%SIGMA *or* SOURCES%SIGMA *or* COLLISIONS%SIGMA	SIGMA	$(Ohm\cdot m)^{-1}$
$E_{\parallel}$	PROFILES %E_PAR	E_PAR	V/m
a	SOLVER%A	A	--
b	SOLVER%B	B	--
c	SOLVER%C	C	--
d	SOLVER%D	D	--
e	SOLVER%E	E	--
f	SOLVER%F	F	--
g	SOLVER%G	G	--
h	SOLVER%H	H	--
v(1:2)	SOLVER%V	V	--
u(1:2)	SOLVER%U	U	--

w(1:2)	SOLVER%W	W	--
Y^{t-1}	SOLVER%YM	W	--
solution	SOLVER%Y	Y	--
derivative of solution	SOLVER%DY	DY	--

Functions	**Internal name used in CURRENT routine**
$\frac{\sigma_{//}\mu_0\rho^2}{F^2}$	FUN1
$\frac{\partial}{\partial\rho}\frac{\sigma_{//}\mu_0\rho^2}{F^2}$	DFUN1
$\frac{V'}{4\pi^2}\left\langle\left\vert\frac{\nabla\rho}{R}\right\vert^2\right\rangle$	FUN2
$\frac{2\pi B_0}{q}$	FUN3
$H\frac{\partial\psi}{\partial\rho}$	FUN4
$\frac{\partial}{\partial\rho}\left(H\frac{\partial\psi}{\partial\rho}\right)$	DFUN4
$\frac{R_0B_0}{F}H\frac{\partial\psi}{\partial\rho}$	FUN5
$\frac{\partial}{\partial\rho}\left(\frac{R_0B_0}{F}H\frac{\partial\psi}{\partial\rho}\right)$	DFUN5

Ion density

Variables and units:

n_i - ion density [m^{-3}]

Γ_i - ion flux [s^{-1}]

γ_i - ion flux contributing to heat transport [A/m^2]

Γ_{Si} - integral source (approximation for the flux if equation is not solved) [s^{-1}]

$n_{i,\mathrm{int}}$ - interpretative ion density [m^{-3}], required if equation is not solved

D_i - ion diffusion coefficient [m^2/s]

V_i^{pinch} - ion pinch velocity [m/s]

$S_{i,\exp}$ - explicit part of ion source density [s^{-1} m^{-3}]

$S_{i,imp}$ - implicit part of ion source density [s^{-1}];

Equation:

$$\left(\left.\frac{\partial}{\partial t}\right|_{\rho} - \frac{\dot{B}_0}{2B_0}\cdot\frac{\partial}{\partial\rho}\rho\right)(V' n_i) + \frac{\partial}{\partial\rho}\Gamma_i = V'\left(S_{i,\exp} - S_{i,imp}\cdot n_i\right) \tag{2.1}$$

where the total flux defined as:

$$\Gamma_i = V'\left\langle\left|\nabla\rho\right|^2\right\rangle\left(-D_i\frac{\partial n_i}{\partial\rho} + n_i V_i^{pinch}\right); \tag{2.2}$$

where total transport coefficients, D_i and V_i^{pinch}, are defined as a sum of individual contributions from different transport models weighted with coefficients m_{model}:

$$D_i = \sum_{i\,model=1}^{n\,model} m_{model}\cdot D_{i,i\,model}\ ; \tag{2.3}$$

$$V_i^{pinch} = \sum_{i\,model=1}^{n\,model} m_{model}\cdot V_{i,i\,model}^{pinch}\ . \tag{2.4}$$

(*Combination of individual contributions,* $D_{i,i\,model}$ and $V_{i,i\,model}^{pinch}$*, into* D_i *and* V_i^{pinch} *is done outside of the ETS equation solver in **Transport combiner***)

Sources in equation (2.3) include contributions in generic form from NBI, recycling and puffed neutrals, ripple, ergodization, and other possible sources, in the form of explicit and implicit terms:

$$S_{i,\exp} = \sum_{isource=1}^{nsource} m_{source} \cdot S_{i,isource,1} \tag{2.5}$$

$$S_{i,imp} = \sum_{isource=1}^{nsource} m_{source} \cdot S_{i,isource,2} \tag{2.6}$$

This summation is done in the workflow by ***Source_Combiner***, where individual weights, m_{source}, can be adjusted.

Equation (2.1) can be rewritten for the normalized grid coordinate as:

$$\left.\frac{\partial}{\partial t}\right|_x (V' n_i) + \frac{\partial}{\partial \rho}\left[V'\left(\left\langle |\nabla\rho|^2\right\rangle\left(-D_i \frac{\partial n_i}{\partial \rho} + n_i V_i^{pinch}\right) - \frac{\dot{\Phi}_b}{2\Phi_b}\cdot \rho n_i\right)\right] = V'\left[S_{i,\exp} - \left(S_{i,imp} + \frac{\dot{\rho}_b}{\rho_b}\right)\cdot n_i\right]. \tag{2.7}$$

Boundary Conditions:

NOTE! Specified positive values for $\nabla n_{i,bnd}$ and L_{ni} correspond to "normal" profile with density decreasing towards the edge

On axis*, at* $\rho = 0$, ETS assumes:

$$\left.\frac{\partial n_i}{\partial \rho}\right|_{x=0} = 0 \text{ and } V_i^{pinch} = 0 \tag{2.8}$$

At the edge*, at* $\rho = \rho_{bnd}$, following options (distinguished by boundary condition type) are available:

type=1 (value)

$$\left. n_i \right|_{x=1} = n_{i,bnd} \tag{2.9}$$

type=2 (gradient)

$$\left.\frac{\partial n_i}{\partial \rho}\right|_{x=1} = -\nabla n_{i,bnd} \tag{2.10}$$

type=3 (scale length)

$$\left.\frac{1}{(\partial \ln n_i / \partial \rho)}\right|_{x=1} = -L_{ni} \tag{2.11}$$

type=4 (flux)

$$\left. V'\left\langle |\nabla\rho|^2\right\rangle\left(-D_i \frac{\partial n_i}{\partial \rho} + n_i V_i^{pinch}\right)\right|_{x=1} = \Gamma_{i,bnd} \tag{2.12}$$

type=5 (generic)

$$v_{gen}\left.\frac{\partial n_i}{\partial \rho}\right|_{x=1} + u_{gen} n_i\big|_{x=1} = w_{gen} \tag{2.13}$$

Definitions for quantities derived from ion density equation:

Ion diffusion equation provides the fulx contributing to ion heat transport, this is produced based on the input from attached transport models:

$$\gamma_i = \sum_{imodel=1}^{nmodel} c_{1,imodel} V'\left\langle|\nabla\rho|^2\right\rangle\left(-D_{i,imodel}\frac{\partial n_i}{\partial \rho} + n_i V_{i,imodel}^{pinch}\right), \tag{2.14}$$

where $c_{1,imodel}$ are the coefficients 1, 3/2 or 5/2 also provided by transport models.

If equation (2.7) is not solved (NI_BND_TYPE=0), interpretative density, $n_{i,\mathrm{int}}$***, should be specified and flux is calculated from the integral of sources:***

$$n_i = n_{i,\mathrm{int}}; \tag{2.15}$$

$$\Gamma_i = \Gamma_{Si}; \tag{2.16}$$

$$\gamma_i = \frac{3}{2}\Gamma_{Si}; \tag{2.17}$$

where τ is the time step, V'^{-} and $n_{i,\mathrm{int}}^{-}$ are taken from the previous time step, and

$$\Gamma_{Si} = \frac{\dot{\Phi}_b}{2\Phi_b}\cdot\left(\rho V' n_{i,\mathrm{int}}\right) + \rho_b\int_0^x\left(V' S_{i,\exp} + \frac{V'^{-} n_{i,\mathrm{int}}^{-}}{\tau} - n_{i,\mathrm{int}} V'\left(\frac{1}{\tau} + S_{i,imp} + \frac{\dot{\rho}_b}{\rho_b}\right)\right)\partial x; \tag{2.18}$$

Translation to generalized form for interface with numerical solver:

Rewrite the equation (2.7) using the time discretization: $\frac{\partial V' n_i}{\partial t} = \frac{V' n_i - V'^{-} n_i^{-}}{\tau}$,where V'^{-} and n_i^{-} are taken at the previous time and τ is the time step:

$$\frac{V' n_i - V'^{-} n_i^{-}}{\tau} + \frac{1}{\rho_b}\frac{\partial}{\partial x}\left[V'\left(\left\langle|\nabla\rho|^2\right\rangle\left(-\frac{D_i}{\rho_b}\frac{\partial n_i}{\partial x} + n_i V_i^{pinch}\right) - \frac{\dot{\Phi}_b}{2\Phi_b}\cdot\rho n_i\right)\right] = \\ = V' S_{i,\exp} - \left(S_{i,imp} + \frac{\dot{\rho}_b}{\rho_b}\right)\cdot V' n_i \tag{2.19}$$

Then, using the equation (0.8), coefficients for the interface with numerical solver are defined as:

$$a(x) = V'$$

$$b(x) = V'^{-}$$

$$c(x) = \rho_b$$

$$d(x) = V'\left\langle |\nabla\rho|^2 \right\rangle \frac{D_i}{\rho_b}$$

$$e(x) = V'\left\langle |\nabla\rho|^2 \right\rangle V_i^{pinch} - \frac{\dot{\Phi}_b}{2\Phi_b} \cdot \rho V' \qquad (2.20)$$

$$f(x) = V' S_{i,\exp}$$

$$g(x) = V'\left(S_{i,imp} + \frac{\dot{\rho}_b}{\rho_b} \right)$$

$$h = \tau$$

$$Y^{t-1}(\rho) = n_i^{-}$$

boundary conditions:
at the axis, $x = 0$:
the choice of the boundary condition is connected with the choice on numerical solver, two options are available:

$$\left. \frac{\partial n_i}{\partial \rho} \right|_{x=0} = 0$$

(2.21)

$$v(1) = 1; \quad u(1) = 0; \quad w(1) = 0$$

Or:

$$\left. \Gamma_i \right|_{x=0} = 0$$

$$v(1) = -\frac{\left. D_i \right|_{x=0}}{\rho_b}; \quad u(1) = \left. V_i^{pinch} \right|_{x=0}; \quad w(1) = 0$$

at the edge, $x = 1$:
<u>**type=1**</u> **(value)**

$$\left. n_i \right|_{x=1} = n_{i,bnd}$$

$$u(2) = 0; \quad v(2) = 1; \quad w(2) = n_{i,bnd}$$

type=2 (gradient)

$$\left.\frac{\partial n_i}{\partial x}\right|_{x=1} = -\nabla n_{i,bnd} \cdot \rho_b$$

$$v(2)=1; \quad u(2)=0; \quad w(2)=-\nabla n_{i,bnd} \cdot \rho_b$$

type=3 (scale lenght)

$$\left.\frac{\rho_b}{(\partial \ln n_i / \partial x)}\right|_{x=1} = -L_{ni}$$

$$v(2)=1; \quad u(2)=\rho_b / L_{ni}; \quad w(2)=0$$

type=4 (flux)

$$V'\left\langle |\nabla\rho|^2 \right\rangle \left(-\frac{D_i}{\rho_b}\frac{\partial n_i}{\partial x} + n_i V_i^{pinch} \right)\Bigg|_{x=1} = \Gamma_{i,bnd}$$

$$v(2) = -V'\left\langle |\nabla\rho|^2 \right\rangle \frac{D_i}{\rho_b}; \quad u(2) = V'\left\langle |\nabla\rho|^2 \right\rangle V_i^{pinch}; \quad w(2) = \Gamma_{i,bnd}$$

type=5 (generic)

$$\left.\frac{v_{gen}}{\rho_b}\frac{\partial n_i}{\partial x}\right|_{bnd} + u_{gen} n_{i,bnd} = w_{gen}$$

$$v(2) = \frac{v_{gen}}{\rho_b}; \quad u(2) = u_{gen}; \quad w(2) = w_{gen}$$

Variables used inside fortran routines:

(Routine: MAIN_PLASMA, Subroutine: ION_DENSITY)

Variable	TYPE%NAME used in ETS data flow	Internal name used in ION_DENSITY routine	Units
τ	EVOLUTION%TAU	TAU	s
$\dot{B}_0$	GEOMETRY%BTPRIME	BTPRIME	T/s
B_0	GEOMETRY%BT	BT	T
ρ	GEOMETRY%RHO	RHO	m
V'	GEOMETRY%VPR	VPR	m^2

V'^{-}	EVOLUTION%VPRM	VPRM	m^2
$\left\langle \lvert\nabla\rho\rvert^2 \right\rangle$	GEOMETRY%G1	G1	--
n_i	PROFILES%NI	NI	m^{-3}
n_i^-	EVOLUTION%NI	NIM	m^{-3}
Γ_i	PROFILES%FLUX_NI	FLUX	1/s
γ_i	PROFILES%FLUX_NI_CONV	FLUX_NI_CONV	1/s
Γ_{Si}	PROFILES%INT_SOURCE_NI	INT_SOURCE	1/s
$S_{i,\exp}$	-	SI_EXP	$m^{-3}s^{-1}$
$S_{i,imp}$	-	SI_IMP	1/s
$S_{i,isource1}$	SOURCES%SI_EXP	-	$m^{-3}s^{-1}$
$S_{i,isource2}$	SOURCES%SI_IMP	-	1/s
D_i	--	DIFF	m^2/s
V_i^{pinch}	--	VCONV	m/s
$D_{i,i\,model}$	TRANSPORT%DIFF_NI	DIFF_MOD	m^2/s
$V_{i,i\,model}^{pinch}$	TRANSPORT%VCONV_NI	VCONV_MOD	m/s
$c_{1,i\,model}$	TRANSPORT%C1	C1	--
a	SOLVER%A	A	--
b	SOLVER%B	B	--
c	SOLVER%C	C	--
d	SOLVER%D	D	--
e	SOLVER%E	E	--
f	SOLVER%F	F	--
g	SOLVER%G	G	--
h	SOLVER%H	H	--
v(1:2)	SOLVER%V	V	--
u(1:2)	SOLVER%U	U	--
w(1:2)	SOLVER%W	W	--
Y^{t-1}	SOLVER%YM	W	--
solution	SOLVER%Y	Y	--
derivative of solution	SOLVER%DY	DY	--

Functions	**Internal name used in ION_DENSITY routine**
$\frac{1}{\rho}\left(V' S_{i,\exp} + \frac{V'^{-}\, n_{i,\mathrm{interpretative}}^{-}}{\tau} - n_{i,\mathrm{interpretative}} V' \cdot \left(\frac{1}{\tau} + S_{i,imp}\right)\right)$	FUN1
$\int_0^{\rho}\left(V' S_{i,\exp} + \frac{V'^{-}\, n_{i,\mathrm{interpretative}}^{-}}{\tau} - n_{i,\mathrm{interpretative}} V' \cdot \left(\frac{1}{\tau} + S_{i,imp}\right)\right)\partial\rho$	INTFUN1

Electron density

Variables and units:

n_e - electron density [m^{-3}]

Γ_e - electron flux [s^{-1}]

γ_e - electron flux contributing to heat transport [A/m^2]

Γ_{Se} - integral source (approximation for the flux if equation is not solved) [s^{-1}]

$n_{e,\mathrm{int}}$ - interpretative electron density [m^{-3}], required if equation is not solved

D_e - electron diffuselectron coefficient [m^2/s]

V_e^{pinch} - electron pinch velocity [m/s]

$S_{e,\exp}$ - explicit part of electron source density [s^{-1} m^{-3}]

$S_{e,imp}$ - implicit part of electron source density [s^{-1}];

Equation:

$$\left(\frac{\partial}{\partial t}\bigg|_\rho - \frac{\dot{B}_0}{2B_0}\cdot\frac{\partial}{\partial\rho}\rho\right)(V'n_e) + \frac{\partial}{\partial\rho}\Gamma_e = V'\left(S_{e,\exp} - S_{e,imp}\cdot n_e\right) \tag{3.1}$$

where the total flux defined as:

$$\Gamma_e = V'\left\langle|\nabla\rho|^2\right\rangle\left(-D_e\frac{\partial n_e}{\partial\rho} + n_e V_e^{pinch}\right); \tag{3.2}$$

where total transport coefficients, D_e and V_e^{pinch}, are defined as a sum of individual contributions from different transport models weighted with coefficients m_{model}:

$$D_e = \sum_{imodel=1}^{nmodel} m_{model}\cdot D_{e,imodel}; \tag{3.3}$$

$$V_e^{pinch} = \sum_{imodel=1}^{nmodel} m_{model}\cdot V_{e,imodel}^{pinch}. \tag{3.4}$$

(*Combinationof individual contributions,* $D_{e,imodel}$ and $V_{e,imodel}^{pinch}$, *into* D_e *and* V_e^{pinch} *is done outside of the ETS equation solver in* ***Transport combiner***)

Sources in equation (3.3) include contributions in generic form from NBI, recycling and puffed neutrals and other possible sources, in the form of explicit and implicit terms:

$$S_{e,\exp} = \sum_{isource=1}^{nsource} m_{source} \cdot S_{e,isource,\exp}$$
(3.5)

$$S_{e,imp} = \sum_{isource=1}^{nsource} m_{source} \cdot S_{e,isource,imp}$$
(3.6)

This summation is done in the workflow by ***Source_Combiner***, where individual weights, m_{source}, can be adjusted.

Equation (3.1) can be rewritten for the normalized grid coordinate as:

$$\left.\frac{\partial}{\partial t}\right|_x (V' n_e) + \frac{\partial}{\partial \rho}\left[V'\left(\left\langle|\nabla\rho|^2\right\rangle\left(-D_e\frac{\partial n_e}{\partial \rho} + n_e V_e^{pinch}\right) - \frac{\dot{\Phi}_b}{2\Phi_b}\cdot \rho n_e\right)\right] = V'\left[S_{e,\exp} - \left(S_{e,imp} + \frac{\dot{\rho}_b}{\rho_b}\right)\cdot n_e\right]. \quad (3.7)$$

Boundary Conditions:

NOTE! Specified positive values for $\nabla n_{e,bnd}$ and L_{ne} correspond to "normal" profile with density decreasing towards the edge

On axis*, at* $\rho = 0$, ETS assumes:

$$\left.\frac{\partial n_e}{\partial \rho}\right|_{x=0} = 0 \text{ and } V_e^{pinch} = 0 \quad (3.8)$$

At the edge*, at* $\rho = \rho_{bnd}$, following options (distinguished by boundary condition type) are available:

type=1 (value)

$$\left. n_e \right|_{x=1} = n_{e,bnd} \quad (3.9)$$

type=2 (gradient)

$$\left.\frac{\partial n_e}{\partial \rho}\right|_{x=1} = -\nabla n_{e,bnd} \quad (3.10)$$

type=3 (scale lenght)

$$\left.\frac{1}{(\partial \ln n_e / \partial \rho)}\right|_{x=1} = -L_{ne} \quad (3.11)$$

type=4 (flux)

$$\left. V'\left\langle|\nabla\rho|^2\right\rangle\left(-D_e\frac{\partial n_e}{\partial \rho} + n_e V_e^{pinch}\right)\right|_{x=1} = \Gamma_{e,bnd} \quad (3.12)$$

type=5 (generic)

$$v_{gen}\left.\frac{\partial n_e}{\partial \rho}\right|_{x=1} + u_{gen} n_e = w_{gen} \tag{3.13}$$

Definitions for quantities derived from electron density equation:

If equation (3.7) is not solved , interpretative density, $n_{e,\mathrm{int}}$, should be specified and flux is calculated from the integral of sources:

$$n_e = n_{e,\mathrm{int}}; \tag{3.14}$$

$$\Gamma_e = \Gamma_{Se}; \tag{3.15}$$

$$\gamma_e = \frac{3}{2}\Gamma_{Se}; \tag{3.16}$$

where τ is the time step, V'^- and $n_{e,\mathrm{int}}^-$ are taken from the previous time step, and

$$\Gamma_{Se} = \frac{\dot{\Phi}_b}{2\Phi_b}\cdot\left(\rho V' n_{e,\mathrm{int}}\right) + \rho_b \int_0^x \left(V' S_{e,\exp} + \frac{V'^- n_{e,\mathrm{int}}^-}{\tau} - n_{e,\mathrm{int}} V' \cdot \left(\frac{1}{\tau} + S_{e,imp} + \frac{\dot{\rho}_b}{\rho_b} \right) \right) \partial x; \tag{3.17}$$

Translation to generalized form for interface with numerical solver:

Rewrite the equation (3.7) using the time discretization: $\frac{\partial V' n_e}{\partial t} = \frac{V' n_e - V'^- n_e^-}{\tau}$,where V'^- and n_e^- are taken at the previous time and τ is the time step:

$$\frac{V' n_e - V'^- n_e^-}{\tau} + \frac{1}{\rho_b}\frac{\partial}{\partial x}\left[V' \left(\left\langle \left|\nabla\rho\right|^2 \right\rangle \left(-\frac{D_e}{\rho_b}\frac{\partial n_e}{\partial x} + n_e V_e^{pinch} \right) - \frac{\dot{\Phi}_b}{2\Phi_b}\cdot \rho n_e \right) \right] = \\ = V' S_{e,\exp} - \left(S_{e,imp} + \frac{\dot{\rho}_b}{\rho_b} \right) \cdot V' n_e \tag{3.18}$$

Then, using the equation (0.8), coefficients for the interface with numerical solver are defined as:

$a(x) = V'$

$b(x) = V'^-$

$c(x) = \rho_b$

$$d(x) = V'\left\langle |\nabla\rho|^2 \right\rangle \frac{D_e}{\rho_b}$$

$$e(x) = V'\left\langle |\nabla\rho|^2 \right\rangle V_e^{pinch} - \frac{\dot{\Phi}_b}{2\Phi_b} \cdot \rho V' \tag{3.19}$$

$$f(x) = V' S_{e,\exp}$$

$$g(x) = V'\left(S_{e,imp} + \frac{\dot{\rho}_b}{\rho_b} \right)$$

$$h = \tau$$

$$Y^{t-1}(\rho) = n_e^-$$

boundary conditions:
at the axis, $x = 0$:
the choice of the boundary condition is connected with the choice on numerical solver, two options are available:

$$\left. \frac{\partial n_e}{\partial x} \right|_{x=0} = 0 \tag{3.20}$$

$$v(1) = 1; \quad u(1) = 0; \quad w(1) = 0$$

Or:

$$\left. \Gamma_e \right|_{x=0} = 0$$

$$v(1) = -\left. \frac{D_e}{\rho_b} \right|_{x=0}; \quad u(1) = \left. V_e^{pinch} \right|_{x=0}; \quad w(1) = 0$$

at the edge, $x = 1$:
<u>type=1</u> (value)

$$\left. n_e \right|_{x=1} = n_{e,bnd}$$

$$u(2) = 0; \quad v(2) = 1; \quad w(2) = n_{e,bnd}$$

<u>type=2</u> (gradient)

$$\left. \frac{\partial n_e}{\partial x} \right|_{x=1} = -\nabla n_{e,bnd} \cdot \rho_b$$

$$v(2) = 1; \quad u(2) = 0; \quad w(2) = -\nabla n_{e,bnd} \cdot \rho_b$$

<u>type=3</u> (scale lenght)

$$\left.\frac{\rho_b}{(\partial \ln n_e / \partial x)}\right|_{x=1} = -L_{ne}$$

$$v(2) = 1; \quad u(2) = \rho_b / L_{ne}; \quad w(2) = 0$$

type=4 (flux)

$$V'\left\langle |\nabla\rho|^2 \right\rangle \left. \left(-\frac{D_e}{\rho_b}\frac{\partial n_e}{\partial x} + n_e V_e^{pinch} \right)\right|_{x=1} = \Gamma_{e,bnd}$$

$$v(2) = -V'\left\langle |\nabla\rho|^2 \right\rangle \frac{D_e}{\rho_b}; \quad u(2) = V'\left\langle |\nabla\rho|^2 \right\rangle V_e^{pinch}; \quad w(2) = \Gamma_{e,bnd}$$

type=5 (generic)

$$\left.\frac{v_{gen}}{\rho_b}\frac{\partial n_e}{\partial x}\right|_{bnd} + u_{gen} n_{e,bnd} = w_{gen}$$

$$v(2) = \frac{v_{gen}}{\rho_b}; \quad u(2) = u_{gen}; \quad w(2) = w_{gen}$$

Variables used inside fortran routines:

(Routine: MAIN_PLASMA, Subroutine: ELECTRON_DENSITY)

Variable	TYPE%NAME used in ETS data flow	Internal name used in ELECTRON_DENSITY routine	Units
τ	EVOLUTELECTRON%TAU	TAU	s
$\dot{B}_0$	GEOMETRY%BTPRIME	BTPRIME	T/s
B_0	GEOMETRY%BT	BT	T
ρ	GEOMETRY%RHO	RHO	m
V'	GEOMETRY%VPR	VPR	m^2
V'^-	EVOLUTELECTRON%VPRM	VPRM	m^2
$\left\langle \|\nabla\rho\|^2 \right\rangle$	GEOMETRY%G1	G1	--
n_i	PROFILES%NI	NI	m^{-3}
n_i^-	EVOLUTELECTRON%NI	NIM	m^{-3}
Γ_i	PROFILES%FLUX_NI	FLUX	1/s

γ_i	PROFILES%FLUX_NI_CONV	FLUX_NI_CONV	1/s
Γ_{Si}	PROFILES%INT_SOURCE_NI	INT_SOURCE	1/s
$S_{i,\exp}$	-	SI_EXP	$m^{-3}s^{-1}$
$S_{i,imp}$	-	SI_IMP	1/s
$S_{i,isource1}$	SOURCES%SI_EXP	-	$m^{-3}s^{-1}$
$S_{i,isource2}$	SOURCES%SI_IMP	-	1/s
D_i	--	DIFF	m^2/s
V_i^{pinch}	--	VCONV	m/s
$D_{i,i\,model}$	TRANSPORT%DIFF_NI	DIFF_MOD	m^2/s
$V_{i,i\,model}^{pinch}$	TRANSPORT%VCONV_NI	VCONV_MOD	m/s
$c_{1,i\,model}$	TRANSPORT%C1	C1	--
a	SOLVER%A	A	--
b	SOLVER%B	B	--
c	SOLVER%C	C	--
d	SOLVER%D	D	--
e	SOLVER%E	E	--
f	SOLVER%F	F	--
g	SOLVER%G	G	--
h	SOLVER%H	H	--
v(1:2)	SOLVER%V	V	--
u(1:2)	SOLVER%U	U	--
w(1:2)	SOLVER%W	W	--
Y^{t-1}	SOLVER%YM	W	--
solution	SOLVER%Y	Y	--
derivative of solution	SOLVER%DY	DY	--

Functions	**Internal name used in ELECTRON_DENSITY routine**
$\frac{1}{\rho}\left(V' S_{i,\exp} + \frac{V'^{-} n_{i,interpretative}^{-}}{\tau} - n_{i,interpretative} V' \left(\frac{1}{\tau} + S_{i,imp}\right)\right)$	FUN1

$\int_0^{\rho}\left(V'S_{i,\exp}+\frac{V'^{-}n_{i,interpretative}^{-}}{\tau}-n_{i,interpretative}V'\cdot\left(\frac{1}{\tau}+S_{i,imp}\right)\right)\partial\rho$	INTFUN1

Quasi-neutrality condition

Variables and units:

n_i - ion density [m^{-3}]

Γ_i - ion flux [s^{-1}]

γ_i - ion flux contributing to heat transport [A/m^2]

Z_{ion} - ion charge [-]

n_e - electron density [m^{-3}]

Γ_e - electron flux [s^{-1}]

γ_e - electron flux contributing to heat transport [A/m^2]

n_{imp} - impurity density [m^{-3}]

Γ_{imp} - impurity flux [s^{-1}]

Z_{imp} - impurity charge [-]

Z_{eff} - plasma effective charge [-]

Equations:

Plasma quasi-neutrality can be described by following conditions:

$$n_e = \sum_{ion} Z_{ion}\cdot n_{ion} + \sum_{imp} Z_{imp}\cdot n_{imp} \quad , \tag{4.1}$$

$$\Gamma_e = \sum_{ion} Z_{ion}\cdot \Gamma_{ion} + \sum_{imp} Z_{imp}\cdot \Gamma_{imp} \quad , \tag{4.2}$$

where indexes *e, ion* end *imp* refer to electrons, ions and impurity ions retrospectively. Impurity contribution is considered by standalone ***Impurity module*** and provided to the main transport solver as input.

Definitions for quantities derived from quasi-neutrality conditions:

Quasi-neutrality conditions in ETS can be used to calculate any particle plasma component (electrons or ions) or several of them (several ions).

Electrons computed from quasi-neutrality:

Density and the flux are obtained from equations (4.1) and (4.2), plasma effective charge is computed as:

$$Z_{eff} = \frac{\sum_{ion} Z_{ion}^2 \cdot n_{ion} + \sum_{imp} Z_{imp}^2 \cdot n_{imp}}{n_e} \tag{4.4}$$

Electron flux contributing to heat transport is obtained as:

$$\gamma_e = \sum_{ion} Z_{ion} \cdot \gamma_{ion} \tag{4.5}$$

Ins computed from quasi-neutrality:
Density and the flux are obtained from:

$$(n \cdot Z)_{sum} = n_e - \sum_{ion} Z_{ion} \cdot n_{ion} - \sum_{imp} Z_{imp} \cdot n_{imp} \ , \tag{4.6}$$

$$(\Gamma \cdot Z)_{sum} = \Gamma_e - \sum_{ion} Z_{ion} \cdot \Gamma_{ion} - \sum_{imp} Z_{imp} \cdot \Gamma_{imp} \ , \tag{4.7}$$

Here index *ion* refers to ion density and fluxes obtained from solving the diffusion equation (2.7), index *sum* refers to the sum of ions for which the predictive equation is not solved. Individual contributions for these ions are obtained even from:

$$n_{ion} = \frac{(n \cdot Z)_{sum}}{N_{ion}} \cdot \frac{1}{Z_{ion}} \ , \tag{4.8}$$

$$\Gamma_{ion} = \frac{(\Gamma \cdot Z)_{sum}}{N_{ion}} \cdot \frac{1}{Z_{ion}} \ , \tag{4.9}$$

where N_{ion} is the number of ions to be predicted from the quasi neutrality condition.

Or from defining the weights, w_{ion}, between contributions from different ions, then individual contributions are obtained as:

$$n_{ion} = \frac{(n \cdot Z)_{sum}}{\sum_{ion} w_{ion}} \cdot w_{ion} \ , \tag{4.10}$$

$$\Gamma_{ion} = \frac{(\Gamma \cdot Z)_{sum}}{\sum_{ion} w_{ion}} \cdot w_{ion} \ . \tag{4.11}$$

Variables used inside fortran routines:

(Routine: MAIN_PLASMA, Subroutine: QUASI_NEUTRALITY)

Variable	TYPE%NAME used in ETS data flow	Internal name used in QUASI_NEUTRALITY routine	Units
ρ	GEOMETRY%RHO	RHO	m
n_e	PROFILES%NE	NE	m^{-3}
Γ_e	PROFILES%FLUX_NE	FLUX_NE	1/s
γ_e	PROFILES%FLUX_NE_CONV	FLUX_NE_CONV	1/s
Z_{eff}	PROFILES%ZEFF	ZEFF	--
Z_{ion}	PROFILES%ZION	ZION	--
Z_{ion}^2	PROFILES%ZION2	ZION2	--
n_{ion}	PROFILES%NI	NI	m^{-3}
Γ_{ion}	PROFILES%FLUX_NI	FLUX_NI	1/s
γ_{ion}	PROFILES%FLUX_NI_CONV	FLUX_NI_CONV	1/s
Z_{imp}	IMPURITY%ZIMP	ZIMP	--
Z_{imp}^2	IMPURITY%ZIMP2	ZIMP2	--
n_{imp}	IMPURITY%NZ	NZ	m^{-3}
Γ_{imp}	IMPURITY %FLUX_NZ	FLUX_NZ	1/s

Ion temperatures

Variables and units:

T_i - ion temperature [eV]

q_i - ion conductive heat flux [W]

$T_i \cdot \gamma_i$ - ion convective heat flux [W]

H_i - total ion heat flux [W]

$H_{i,\text{int}}$ - total ion heat flux calculated as the integral of sources [W], if equation is not solved

χ_i - ion heat conductivity [m^2/s]

V_{Ti}^{pinch} - ion heat pinch velocity [m/s]

$Q_{i,\exp}$ - explicit part of ion heat source density [eV·m^{-3}s^{-1}]

$Q_{i,imp}$ - implicit part of ion heat source density [m^{-3}s^{-1}];

Q_{ei} - electron-ion exchange frequency [m^{-3}s^{-1}];

Q_{zi} - impurity-ion exchange frequency [m^{-3}s^{-1}];

$Q_{\gamma i}$ - flow exchange terms [m^{-3}s^{-1}];

Equation:

$$\frac{3}{2}\left(\frac{\partial}{\partial t}-\frac{\dot{B}_0}{2B_0}\cdot\frac{\partial}{\partial\rho}\rho\right)\left(n_i T_i V'^{\frac{5}{3}}\right)+V'^{\frac{2}{3}}\frac{\partial}{\partial\rho}\left(q_i+T_i\,\gamma_i\right)=V'^{\frac{5}{3}}\left[Q_{i,\exp}-Q_{i,imp}\cdot T_i+Q_{ei}+Q_{zi}+Q_{\gamma i}\right] \tag{5.1}$$

where conductive and convective heat flux are defined as:

$$q_i = V'\left\langle\left|\nabla\rho\right|^2\right\rangle\left[n_i\left(-\chi_i\frac{\partial T_i}{\partial\rho}+T_i V_{Ti}^{pinch}\right)\right] \quad \text{and} \quad T_i\,\gamma_i\,, \tag{5.2}$$

γ_i is defined by equation (2.14)

Total heat flux:

$$H_i = q_i + T_i\,\gamma_i \tag{5.3}$$

Total transport coefficients are defined as a sum of individual contributions from different transport models weighted with coefficients m_{model}:

$$\chi_i = \sum_{i\,\text{model}=1}^{n\,\text{model}} m_{\text{model}}\cdot\chi_{i,i\,\text{model}} \tag{5.4}$$

$$V_{Ti}^{pinch} = \sum_{i\,model=1}^{n\,model} m_{model} \cdot V_{Ti,i\,model}^{pinch} \qquad (5.5)$$

(*Combination of individual contributions,* $\chi_{i,i\,model}$ *and* $V_{Ti,i\,model}^{pinch}$*, into* χ_i *and* V_{Ti}^{pinch} *is done outside of the ETS equation solver in* ***Transport combiner***)

Right hand side of equation (5.1) includes contributions in generic form from external sources (*computed by other modules, like ICRH, ECRH, NBI, neutrals and etc*.), in the form of explicit and implicit terms:

$$Q_{i,\exp} = \sum_{isource=1}^{nsource} m_{source} \cdot Q_{i,isource,\exp}$$

(5.6)

$$Q_{i,imp} = \sum_{isource=1}^{nsource} m_{source} \cdot Q_{i,isource,imp} \qquad (5.7)$$

This summation is done in the workflow by ***Source_Combiner***, where individual weights, m_{source}, can be adjusted.

Number of source terms are computed by the ETS internally, this includes exchange terms with electrons,

$$Q_{ei} = \nu_{ei}\left(T_e - T_i\right), \qquad (5.8)$$

and impurity ions,

$$Q_{zi} = \sum_z \nu_{zi}\left(T_z - T_i\right) = \sum_z \nu_{zi} T_z - T_i \cdot \sum_z \nu_{zi}\ , \qquad (5.9)$$

where ν_{ei} and $\sum_z \nu_{zi}$ are the exchange frequency.

The flow exchange terms are imported from the attached transport models:

$$Q_{\gamma i} = \sum_{i\,model=1}^{n\,model} Q_{\gamma i,i\,model}\ . \qquad (5.10)$$

Making use of equations (0.6-0.7), equation (5.1) can be rewritten for the normalized coordinate x :

$$\frac{3}{2}\frac{\partial}{\partial t}\bigg|_x n_i T_i V'^{\frac{5}{3}} + V'^{\frac{2}{3}} \frac{\partial}{\partial \rho}\left[V'\left\langle\left|\nabla\rho\right|^2\right\rangle\left(n_i\left(-\chi_i \frac{\partial T_i}{\partial \rho} + T_i V_{Ti}^{pinch}\right)\right) + T_i\,\gamma_i - V'\frac{3\dot{\Phi}_b}{4\Phi_b}\cdot\rho n_i T_i\right] =$$
$$= V'^{\frac{5}{3}}\left[Q_{i,\exp} - Q_{i,imp}\cdot T_i + Q_{ei} + Q_{zi} + Q_{\gamma i} - \left[\frac{3}{2}\frac{\dot{\rho}_b}{\rho_b} - \frac{\dot{\Phi}_b}{2\Phi_b}\left(1 - \rho\frac{\partial}{\partial\rho}\ln\left(F\cdot\left\langle\frac{1}{R^2}\right\rangle\right)\right)\right]\cdot n_i T_i\right] \qquad (5.11)$$

Boundary Conditions:

NOTE! Specified positive values for $\nabla T_{i,bnd}$ ***and*** L_{Ti} ***correspond to "normal" profile with density decreasing towards the edge***

on axis, $x = 0$:

$$\left.\frac{\partial T_i}{\partial \rho}\right|_{x=0} = 0 \text{, assuming } V_{Ti}^{pinch} = 0 \tag{5.12}$$

at the edge, $x = 1$

type=1 (value)

$$T_i\big|_{\rho=\rho_{bnd}} = T_{i,bnd}$$

type=2 (gradient)

$$\left.\frac{\partial T_i}{\partial \rho}\right|_{\rho=\rho_{bnd}} = -\nabla T_{i,bnd}$$

type=3 (scale lenght)

$$\left.\frac{1}{(\partial \ln T_i / \partial \rho)}\right|_{\rho=\rho_{bnd}} = -L_{Ti}$$

type=4 (flux)

$$(q_i + T_i \gamma_i)\big|_{\rho=\rho_{bnd}} = f_{Ti,bnd}$$

type=5 (generic)

$$v_{gen}\left.\frac{\partial T_i}{\partial \rho}\right|_{bnd} + u_{gen} T_{i,bnd} = w_{gen}$$

Definitions for quantities derived from ion temperature equation:

If equation (5.11) is not solved, interpretative temperature should be specified and various flux components are calculated from the integral of sources:

$$T_i = T_{i,\text{interpretative}}; \tag{5.13}$$

$$T_i \gamma_i = \gamma_i \cdot T_{i,\text{interpretative}} \tag{5.14}$$

$$H_i = H_{i,\text{int}} \tag{5.15}$$

$$q_i = H_{i,\text{int}} - \gamma_i \cdot T_{i,\text{interpretative}} \tag{5.16}$$

where:

$$H_{i,\text{int}} = \frac{3\dot{\Phi}_b}{4\Phi_b} \cdot \rho n_i V' T_{i,\text{interpretative}}$$
$$+\rho_b \cdot \int_0^\rho V' \left[\frac{3}{2} \frac{n_i^- T_{i,\text{interpretative}}^-}{\tau} \left(\frac{V'^-}{V'} \right)^{\frac{5}{3}} + Q_{i,\exp} + Q_{\gamma i} + \nu_{ei} T_e + \sum_z \nu_{zi} T_z \right] \partial x$$
$$-\rho_b \cdot \int_0^\rho V' \left[\frac{3}{2} \frac{n_i}{\tau} + Q_{i,imp} + \nu_{ei} + \sum_z \nu_{zi} + \left[\frac{3}{2} \frac{\dot{\rho}_b}{\rho_b} - \frac{\dot{\Phi}_b}{2\Phi_b} \left(1 - \rho \frac{\partial}{\partial \rho} \ln \left(F \cdot \left\langle \frac{1}{R^2} \right\rangle \right) \right) \right] \cdot n_i \right] \cdot T_{i,\text{interpretative}} \cdot \partial x$$

(5.17)

Translation to generalized form for interface with numerical solver:

Equation (5.11) can be rewritten using time discretization, $\frac{\partial}{\partial t} n_i T_i V'^{\frac{5}{3}} = \frac{n_i T_i V'^{\frac{5}{3}} - n_i^- T_i^- V'^{-\frac{5}{3}}}{\tau}$, as

$$\frac{3}{2} \frac{n_i T_i V'^{\frac{5}{3}} - n_i^- T_i^- V'^{-\frac{5}{3}}}{\tau} + V'^{\frac{2}{3}} \frac{\partial}{\partial \rho} \left[V' \left\langle |\nabla \rho|^2 \right\rangle \left(n_i \left(-\chi_i \frac{\partial T_i}{\partial \rho} + T_i V_{Ti}^{pinch} \right) \right) + T_i \gamma_i - V' \frac{3\dot{\Phi}_b}{4\Phi_b} \cdot \rho n_i T_i \right] =$$
$$= V'^{\frac{5}{3}} \left[Q_{i,\exp} - Q_{i,imp} \cdot T_i + Q_{ei} + Q_{zi} + Q_{\gamma i} - \left[\frac{3}{2} \frac{\dot{\rho}_b}{\rho_b} - \frac{\dot{\Phi}_b}{2\Phi_b} \left(1 - \rho \frac{\partial}{\partial \rho} \ln \left(F \cdot \left\langle \frac{1}{R^2} \right\rangle \right) \right) \right] \cdot n_i T_i \right] \quad , \quad (5.18)$$

where τ is the time step, T_i^-, n_i^- and V'^- are taken from the previous time step.

To make it compatible with the form defined by the equation (0.8), it can be:

$$\frac{\left(\frac{3}{2} n_i V' \right) \cdot T_i - \left(\frac{3}{2} n_i^- \frac{\left(V'^- \right)^{\frac{5}{3}}}{V'^{\frac{2}{3}}} \right) \cdot T_i^-}{\tau}$$
$$+ \frac{1}{\rho_b} \frac{\partial}{\partial x} \left[-V' \left\langle |\nabla \rho|^2 \right\rangle \frac{\chi_i n_i}{\rho_b} \cdot \frac{\partial T_i}{\partial x} + \left(V' \left\langle |\nabla \rho|^2 \right\rangle n_i V_{Ti}^{pinch} + \gamma_i - V' \frac{3\dot{\Phi}_b}{4\Phi_b} \cdot \rho n_i \right) \cdot T_i \right] \quad (5.19)$$
$$= V' \left[Q_{i,\exp_i} + Q_{ei} + Q_{zi} + Q_{\gamma i} - \left[Q_{i,imp} + \left[\frac{3}{2} \frac{\dot{\rho}_b}{\rho_b} - \frac{\dot{\Phi}_b}{2\Phi_b} \left(1 - \rho \frac{\partial}{\partial \rho} \ln \left(F \cdot \left\langle \frac{1}{R^2} \right\rangle \right) \right) \right] \cdot n_i \right] \cdot T_i \right]$$

Then, taking in account (5.8) and (5.9), coefficients for the interface with numerical solver can be used as:

$$a(x) = \frac{3}{2} V' n_i \quad (5.20)$$

$$b(x)=\frac{3}{2}n_i^{-}\left(\frac{V'^{-\frac{5}{3}}}{V'^{\frac{2}{3}}}\right)$$

$$c(x)=\rho_b$$

$$d(x)=V'\left\langle\left|\nabla\rho\right|^2\right\rangle\frac{n_i\chi_i}{\rho_b}$$

$$e(x)=V'\left\langle\left|\nabla\rho\right|^2\right\rangle n_i V_{Ti}^{pinch}+\gamma_i-V'\frac{3\dot{\Phi}_b}{4\Phi_b}\cdot\rho n_i$$

$$f(x)=V'\left[Q_{i,\exp}+Q_{\gamma i}+\nu_{ei}T_e+\sum_z \nu_{zi}T_z\right]$$

$$g(x)=V'\left[Q_{i,imp}+\nu_{ei}+\sum_z \nu_{zi}+\left[\frac{3}{2}\frac{\dot{\rho}_b}{\rho_b}-\frac{\dot{\Phi}_b}{2\Phi_b}\left(1-\rho\frac{\partial}{\partial\rho}\ln\left(F\cdot\left\langle\frac{1}{R^2}\right\rangle\right)\right)\right]\cdot n_i\right]$$

$$h=\tau$$

$$Y^{t-1}(\rho)=T_i^{-}$$

boundary conditions:

at the axis, $x=0$:

the choice of the boundary condition is connected with the choice on numerical solver, two options are available:

$$\left.\frac{\partial T_i}{\partial x}\right|_{x=0}=0 \qquad (5.21)$$

$$v(1)=1; \quad u(1)=0; \quad w(1)=0$$

at the edge, $x=1$:

<u>type=1</u> (value)

$$T_i\big|_{x=1}=T_{i,bnd}$$

$$v(2)=0; \quad u(2)=1; \quad w(2)=T_{i,bnd}$$

<u>type=2</u> (gradient)

$$\left.\frac{\partial T_i}{\partial x}\right|_{x=1}=-\nabla T_{i,bnd}\cdot\rho_b$$

$$v(2)=1; \quad u(2)=0; \quad w(2)=-\nabla T_{i,bnd}\cdot\rho_b$$

type=3 (scale length)

$$\left.\frac{\rho_b}{(\partial \ln T_i / \partial x)}\right|_{x=1} = -L_{Ti}$$

$$v(2)=1; \quad u(2)=\rho_b / L_{Ti}; \quad w(2)=0$$

type=4 (flux)

$$\left.(q_i + T_i \gamma_i)\right|_{x=1} = f_{Ti,bnd}$$

$$v(2) = -\frac{n_i \chi_i V'}{\rho_b}\left\langle |\nabla\rho|^2 \right\rangle; \quad u(2) = n_i V_{Ti}^{pinch} V' \left\langle |\nabla\rho|^2 \right\rangle + \gamma_i; \quad w(2) = f_{Ti,bnd}$$

type=5 (generic)

$$\left.\frac{v_{gen}}{\rho_b}\frac{\partial T_i}{\partial x}\right|_{bnd} + u_{gen} T_{i,bnd} = w_{gen}$$

$$v(2) = \frac{v_{gen}}{\rho_b}; \quad u(2) = u_{gen}; \quad w(2) = w_{gen}$$

Variables used inside fortran routines:

Variable	TYPE%NAME used in ETS data flow	Internal name used in ION_TEMPERATURE routine	Units
τ	EVOLUTION%TAU	TAU	s
$\dot{B}_0$	GEOMETRY%BTPRIME	BTPRIME	T/s
B_0	GEOMETRY%BT	BT	T
ρ	GEOMETRY%RHO	RHO	m
V'	GEOMETRY%VPR	VPR	m^2
V'^{-}	EVOLUTION%VPRM	VPRM	m^2
$\frac{\partial V'}{\partial \rho}$	--	DVPR	m
$\left\langle \|\nabla\rho\|^2 \right\rangle$	GEOMETRY%G1	G1	--

T_i	PROFILES%TI	TI	eV
T_i^-	EVOLUTION%TI	TIM	eV
n_i	PROFILES%NI	NI	m^{-3}
n_i^-	EVOLUTION%NI	NIM	m^{-3}
γ_i	PROFILES%FLUX_NI_CONV	FLUX_NI	1/s
H_i	PROFILES%FLUX_TI	FLUX_TI	W
$T_i\gamma_i$	PROFILES%FLUX_TI_CONV	FLUX_TI_CONV	W
q_i	PROFILES%FLUX_TI_COND	FLUX_TI_COND	W
$H_{i,\mathrm{int}}$	PROFILES%INT_SOURCE_TI	INT_SOURCE	W
$Q_{i,\exp}$	--	QI_EXP	$eV\cdot m^{-3}s^{-1}$
$Q_{i,imp}$	--	QI_IMP	$m^{-3}s^{-1}$
$Q_{i,isource,1}$	SOURCES%QI_EXP	--	$eV\cdot m^{-3}s^{-1}$
$Q_{i,isource,2}$	SOURCES%QI_IMP	--	$m^{-3}s^{-1}$
ν_{ei}	COLLISIONS%VEI	VEI	$m^{-3}s^{-1}$
$\nu_{ei}T_e$	COLLISIONS %QEI	QEI	$eV\cdot m^{-3}s^{-1}$
$\sum_z \nu_{zi}$	COLLISIONS %VZI	VZI	$m^{-3}s^{-1}$
$\sum_z \nu_{zi}T_z$	COLLISIONS %QZI	QZI	$eV\cdot m^{-3}s^{-1}$
$Q_{\gamma i}$	--	QGI	$eV\cdot m^{-3}s^{-1}$
$Q_{\gamma i,i\,model}$	TRANSPORT%QGI	--	$eV\cdot m^{-3}s^{-1}$
χ_i	--	DIFF	m^2/s
V_{Ti}^{pinch}	--	VCONV	m/s
$\chi_{i,i\,model}$	TRANSPORT%DIFF_TI	--	m^2/s
$V_{Ti,i\,model}^{pinch}$	TRANSPORT%VCONV_TI	--	m/s
a	SOLVER%A	A	--
b	SOLVER%B	B	--
c	SOLVER%C	C	--
d	SOLVER%D	D	--
e	SOLVER%E	E	--
f	SOLVER%F	F	--

g	SOLVER%G	G	--
h	SOLVER%H	H	--
v(1:2)	SOLVER%V	V	--
u(1:2)	SOLVER%U	U	--
w(1:2)	SOLVER%W	W	--
Y^{t-1}	SOLVER%YM	W	--
solution	SOLVER%Y	Y	--
derivative of solution	SOLVER%DY	DY	--

Functions	**Internal name used in ION_TEMPERATURE routine**
$\frac{V}{\rho}'\left(\frac{3}{2}\frac{n_i^{-}T_{i,\mathrm{interpretative}}^{-}}{\tau}\left(\frac{V'^{-}}{V'}\right)^{\frac{5}{3}}+Q_{i,\exp}+Q_{\gamma i}+\nu_{ei}T_e+\sum_z \nu_{zi}T_z - \left[\frac{3}{2}\frac{n_i}{\tau}+Q_{i,imp}+\nu_{ei}+\sum_z \nu_{zi}-\rho n_i\cdot\frac{\dot{B}_0}{2B_0}\cdot\frac{\partial V'}{\partial\rho}\right]\cdot T_{i,\mathrm{interpretative}}\right)$	FUN1
$\int_0^{\rho} V'\left[\frac{3}{2}\frac{n_i^{-}T_{i,\mathrm{interpretative}}^{-}}{\tau}\left(\frac{V'^{-}}{V'}\right)^{\frac{5}{3}}+Q_{i,\exp}+Q_{\gamma i}+\nu_{ei}T_e+\sum_z \nu_{zi}T_z\right]\partial\rho - \int_0^{\rho} V'\left[\frac{3}{2}\frac{n_i}{\tau}+Q_{i,imp}+\nu_{ei}+\sum_z \nu_{zi}-\rho n_i\cdot\frac{\dot{B}_0}{2B_0}\cdot\frac{\partial V'}{\partial\rho}\right]\cdot T_{i,\mathrm{interpretative}}\cdot\partial\rho$	INTFUN1

Electron energy transport equation

Variables and units:

T_e - electron temperature [eV]

q_e - electron conductive heat flux [W]

$T_e \cdot \gamma_e$ - electron convective heat flux [W]

H_e - total electron heat flux [W]

$H_{e,\mathrm{int}}$ - total electron heat flux calculated as the integral of sources [W], if equation is not solved

χ_e - electron heat conductivity [m^2/s]

V_{Te}^{pinch} - electron heat pinch velocity [m/s]

$Q_{e,\exp}$ - explicit part of electron heat source density [eV·m^{-3}s^{-1}]

$Q_{e,imp}$ - implicit part of electron heat source density [m^{-3}s^{-1}];

Q_{ie} - ion- electron exchange frequency [m^{-3}s^{-1}];

$Q_{\gamma i}$ - flow exchange terms [m^{-3}s^{-1}];

Equation:

$$\frac{3}{2}\left(\frac{\partial}{\partial t}-\frac{\dot{B}_0}{2B_0}\cdot\frac{\partial}{\partial\rho}\rho\right)\left(n_eT_eV'^{\frac{5}{3}}\right)+V'^{\frac{2}{3}}\frac{\partial}{\partial\rho}\left(q_e+T_e\,\gamma_e\right)=V'^{\frac{5}{3}}\left[Q_{e,\exp}-Q_{e,imp}\cdot T_e+Q_{ie}-Q_{\gamma i}\right] \tag{6.1}$$

where conductive and convective heat flux are defined as:

$$q_e=V'\left\langle\left|\nabla\rho\right|^2\right\rangle\left[n_e\left(-\chi_e\frac{\partial T_e}{\partial\rho}+T_eV_{Te}^{pinch}\right)\right] \quad \text{and} \quad T_e\,\gamma_e\,, \tag{6.2}$$

γ_e is defined by equation (3.14)

Total heat flux:

$$H_e=q_e+T_e\,\gamma_e \tag{6.3}$$

Total transport coefficients are defined as a sum of individual contributions from different transport models weighted with coefficients m_{model}:

$$\chi_e=\sum_{i\,\mathrm{model}=1}^{n\,\mathrm{model}}m_{\mathrm{model}}\cdot\chi_{e,i\,\mathrm{model}} \tag{6.4}$$

$$V_{Te}^{pinch}=\sum_{i\,\mathrm{model}=1}^{n\,\mathrm{model}}m_{\mathrm{model}}\cdot V_{Te,i\,\mathrm{model}}^{pinch} \tag{6.5}$$

(*Combination of individual contributions,* $\chi_{e,i\,model}$ *and* $V_{Te,i\,model}^{pinch}$*, into* χ_e *and* V_{Te}^{pinch} *is done outside of the ETS equation solver in* ***Transport combiner***)

Right hand side of equation (6.1) includes contributions in generic form from external sources (*computed by other modules, like ICRH, ECRH, NBI, neutrals and etc*.), in the form of explicit and implicit terms:

$$Q_{e,\exp} = \sum_{isource=1}^{nsource} m_{source} \cdot Q_{e,isource,\exp} \tag{6.6}$$

$$Q_{e,imp} = \sum_{isource=1}^{nsource} m_{source} \cdot Q_{e,isource,imp} \tag{6.7}$$

This summation is done in the workflow by ***Source_Combiner***, where individual weights, m_{source}, can be adjusted.

Number of source terms are computed by the ETS internally, this includes exchange terms with electrons,

$$Q_{ie} = \sum_i \nu_{ei}\left(T_i - T_e\right), \tag{6.8}$$

where ν_{ei} is the exchange frequency.

The flow exchange terms are imported from the attached transport models:

$$Q_{\gamma i} = \sum_{i\,model=1}^{n\,model} Q_{\gamma i,i\,model}\ . \tag{6.9}$$

Making use of equations (0.6-0.7), equation (5.1) can be rewritten for the normalized coordinate x:

$$\begin{aligned}\frac{3}{2}\frac{\partial}{\partial t}\bigg|_x n_e T_e V'^{\frac{5}{3}} + V'^{\frac{2}{3}}\frac{\partial}{\partial\rho}\left[V'\left\langle\left|\nabla\rho\right|^2\right\rangle\left(n_e\left(-\chi_e\frac{\partial T_i}{\partial\rho} + T_e V_{Te}^{pinch}\right)\right) + T_e\gamma_e - V'\frac{3\dot{\Phi}_b}{4\Phi_b}\cdot\rho n_e T_e\right] = \\ = V'^{\frac{5}{3}}\left[Q_{e,\exp} - Q_{e,imp}\cdot T_e + Q_{ie} - Q_{\gamma i} - \left[\frac{3}{2}\frac{\dot{\rho}_b}{\rho_b} - \frac{\dot{\Phi}_b}{2\Phi_b}\left(1-\rho\frac{\partial}{\partial\rho}\ln\left(F\cdot\left\langle\frac{1}{R^2}\right\rangle\right)\right)\right]\cdot n_e T_e\right]\end{aligned} \tag{6.10}$$

Boundary Conditions:

NOTE! Specified positive values for $\nabla T_{e,bnd}$ ***and*** L_{Te} ***correspond to "normal" profile with density decreasing towards the edge***

on axis, $x = 0$:

$$\frac{\partial T_e}{\partial\rho}\bigg|_{x=0} = 0\text{, assuming } V_{Te}^{pinch} = 0 \tag{6.11}$$

at the edge, $x = 1$

type=1 (value)

$$T_e\big|_{\rho=\rho_{bnd}} = T_{e,bnd}$$

type=2 (gradient)

$$\left.\frac{\partial T_e}{\partial \rho}\right|_{\rho=\rho_{bnd}} = -\nabla T_{e,bnd}$$

type=3 (scale lenght)

$$\left.\frac{1}{(\partial \ln T_e / \partial \rho)}\right|_{\rho=\rho_{bnd}} = -L_{Te}$$

type=4 (flux)

$$\left(q_e + T_e \gamma_e\right)\big|_{\rho=\rho_{bnd}} = H_{Te,bnd}$$

type=5 (generic)

$$v_{gen} \left.\frac{\partial T_e}{\partial \rho}\right|_{bnd} + u_{gen} T_{e,bnd} = w_{gen}$$

Definitions for quantities derived from electron temperature equation:

If equation (6.10) is not solved, interpretative temperature should be specified and various flux components are calculated from the integral of sources:

$$T_e = T_{e,\text{interpretative}}; \tag{6.12}$$

$$T_e \gamma_e = \gamma_e \cdot T_{e,\text{interpretative}} \tag{6.13}$$

$$H_e = H_{e,\text{int}} \tag{6.14}$$

$$q_e = H_{e,\text{int}} - \gamma_e \cdot T_{e,\text{interpretative}} \tag{6.15}$$

where:

$$\begin{aligned}
H_{e,\text{int}} = {} & \frac{3\dot{\Phi}_b}{4\Phi_b} \cdot \rho n_e V' T_{e,\text{interpretative}} \\
& + \rho_b \cdot \int_0^\rho V' \left[\frac{3}{2} \frac{n_e^{-} T_{e,\text{interpretative}}^{-}}{\tau} \left(\frac{V'^{-}}{V'} \right)^{\frac{5}{3}} + Q_{e,\exp} - Q_{\gamma i} + \sum_i \nu_{ei} T_i \right] \partial x \\
& - \rho_b \cdot \int_0^\rho V' \left[\frac{3}{2} \frac{n_e}{\tau} + Q_{e,imp} + \sum_i \nu_{ei} + \left[\frac{3}{2} \frac{\dot{\rho}_b}{\rho_b} - \frac{\dot{\Phi}_b}{2\Phi_b} \left(1 - \rho \frac{\partial}{\partial \rho} \ln \left(F \cdot \left\langle \frac{1}{R^2} \right\rangle \right) \right) \right] \cdot n_e \right] \cdot T_{e,\text{interpretative}} \cdot \partial x
\end{aligned} \tag{6.16}$$

Translation to generalized form for interface with numerical solver:

Equation (5.11) can be rewritten using time discretization, $\frac{\partial}{\partial t} n_i T_i V'^{\frac{5}{3}} = \frac{n_i T_i V'^{\frac{5}{3}} - n_i^- T_i^- V'^{-\frac{5}{3}}}{\tau}$, as

$$\frac{3}{2}\frac{n_i T_i V'^{\frac{5}{3}} - n_i^- T_i^- V'^{-\frac{5}{3}}}{\tau} + V'^{\frac{2}{3}} \frac{\partial}{\partial \rho}\left[V' \left\langle |\nabla\rho|^2 \right\rangle \left(n_i \left(-\chi_i \frac{\partial T_i}{\partial \rho} + T_i V_{Ti}^{pinch} \right)\right) + T_i \gamma_i - V' \frac{3\dot{\Phi}_b}{4\Phi_b} \cdot \rho n_i T_i \right] =$$
$$= V'^{\frac{5}{3}} \left[Q_{i,\exp} - Q_{i,imp} \cdot T_i + Q_{ei} + Q_{zi} + Q_{\gamma i} - \left[\frac{3}{2}\frac{\dot{\rho}_b}{\rho_b} - \frac{\dot{\Phi}_b}{2\Phi_b}\left(1 - \rho \frac{\partial}{\partial \rho} \ln\left(F \cdot \left\langle \frac{1}{R^2} \right\rangle \right)\right)\right] \cdot n_i T_i \right] , \quad (6.17)$$

where τ is the time step, T_e^-, n_e^- and V'^- are taken from the previous time step.

To make it compatible with the form defined by the equation (0.8), it can be:

$$\frac{\left(\frac{3}{2} n_e V'\right) \cdot T_e - \left(\frac{3}{2} n_e^- \frac{V'^{-\frac{5}{3}}}{V'^{\frac{2}{3}}} \right) \cdot T_e^-}{\tau}$$
$$+ \frac{1}{\rho_b}\frac{\partial}{\partial x}\left[-V' \left\langle |\nabla\rho|^2 \right\rangle \frac{\chi_e n_e}{\rho_b} \cdot \frac{\partial T_e}{\partial x} + \left(V' \left\langle |\nabla\rho|^2 \right\rangle n_e V_{Te}^{pinch} + \gamma_e - V' \frac{3\dot{\Phi}_b}{4\Phi_b} \cdot \rho n_e \right) \cdot T_e \right] \quad (6.18)$$
$$= V' \left[Q_{e,\exp_i} + Q_{ie} - Q_{\gamma i} - \left[Q_{e,imp} + \left[\frac{3}{2}\frac{\dot{\rho}_b}{\rho_b} - \frac{\dot{\Phi}_b}{2\Phi_b}\left(1 - \rho \frac{\partial}{\partial \rho} \ln\left(F \cdot \left\langle \frac{1}{R^2} \right\rangle \right)\right)\right] \cdot n_e \right] \cdot T_e \right]$$

Then, taking in account (6.8) and (6.9), coefficients for the interface with numerical solver can be used as:

$$a(x) = \frac{3}{2} V' n_e \quad (6.19)$$

$$b(x) = \frac{3}{2} n_e^- \left(\frac{V'^{-\frac{5}{3}}}{V'^{\frac{2}{3}}} \right)$$

$$c(x) = \rho_b$$

$$d(x) = V' \left\langle |\nabla\rho|^2 \right\rangle \frac{n_e \chi_e}{\rho_b}$$

$$e(x) = V' \left\langle |\nabla\rho|^2 \right\rangle n_e V_{Te}^{pinch} + \gamma_e - V' \frac{3\dot{\Phi}_b}{4\Phi_b} \cdot \rho n_e$$

$$f(x) = V'\left[Q_{e,\exp} - Q_{\gamma i} + \sum_i \nu_{ei} T_i\right]$$

$$g(x) = V'\left[Q_{e,imp} + \sum_i \nu_{ei} + \left[\frac{3}{2}\frac{\dot{\rho}_b}{\rho_b} - \frac{\dot{\Phi}_b}{2\Phi_b}\left(1 - \rho\frac{\partial}{\partial\rho}\ln\left(F \cdot \left\langle \frac{1}{R^2} \right\rangle\right)\right)\right] \cdot n_e\right]$$

$$h = \tau$$

$$Y^{t-1}(\rho) = T_e^-$$

boundary conditions:

at the axis, $x = 0$:

the choice of the boundary condition is connected with the choice on numerical solver, two options are available:

$$\left.\frac{\partial T_e}{\partial x}\right|_{x=0} = 0 \qquad (6.20)$$

$$v(1) = 1; \quad u(1) = 0; \quad w(1) = 0$$

at the edge, $x = 1$:

type=1 (value)

$$T_e\big|_{x=1} = T_{e,bnd}$$

$$v(2) = 0; \quad u(2) = 1; \quad w(2) = T_{e,bnd}$$

type=2 (gradient)

$$\left.\frac{\partial T_e}{\partial x}\right|_{x=1} = -\nabla T_{e,bnd} \cdot \rho_b$$

$$v(2) = 1; \quad u(2) = 0; \quad w(2) = -\nabla T_{e,bnd} \cdot \rho_b$$

type=3 (scale length)

$$\left.\frac{\rho_b}{(\partial \ln T_e / \partial x)}\right|_{x=1} = -L_{Te}$$

$$v(2) = 1; \quad u(2) = \rho_b / L_{Te}; \quad w(2) = 0$$

type=4 (flux)

$$\left(q_e + T_e \gamma_i\right)\Big|_{x=1} = H_{Te,bnd}$$

$$v(2) = -\frac{n_e \chi_e V'}{\rho_b}\left\langle\left|\nabla\rho\right|^2\right\rangle; \quad u(2) = n_e V_{Te}^{pinch} V'\left\langle\left|\nabla\rho\right|^2\right\rangle + \gamma_e; \quad w(2) = H_{Te,bnd}$$

<u>type=5</u> (generic)

$$\frac{v_{gen}}{\rho_b}\frac{\partial T_e}{\partial x}\bigg|_{bnd} + u_{gen} T_{e,bnd} = w_{gen}$$

$$v(2) = \frac{v_{gen}}{\rho_b}; \quad u(2) = u_{gen}; \quad w(2) = w_{gen}$$

Variables used inside fortran routines:

Variable	TYPE%NAME used in ETS data flow	Internal name used in ELECTRON_TEMPERATURE routine	Units
τ	EVOLUTION%TAU	TAU	s
$\dot{B}_0$	GEOMETRY%BTPRIME	BTPRIME	T/s
B_0	GEOMETRY%BT	BT	T
ρ	GEOMETRY%RHO	RHO	m
V'	GEOMETRY%VPR	VPR	m^2
V'^-	EVOLUTION%VPRM	VPRM	m^2
$\frac{\partial V'}{\partial \rho}$	--	DVPR	m
$\left\langle\left\|\nabla\rho\right\|^2\right\rangle$	GEOMETRY%G1	G1	--
T_e	PROFILES%TE	TE	eV
T_e^-	EVOLUTION%TE	TEM	eV
n_e	PROFILES%NE	NE	m^{-3}
n_e^-	EVOLUTION%NE	NEM	m^{-3}
γ_e	PROFILES%FLUX_NE_CONV	FLUX_NE	1/s
H_e	PROFILES%FLUX_TE	FLUX_TE	W

$T_e \gamma_e$	PROFILES%FLUX_TE_CONV	FLUX_TE_CONV	W
q_e	PROFILES%FLUX_TE_COND	FLUX_TE_COND	W
$H_{e,\text{int}}$	PROFILES%INT_SOURCE_TE	INT_SOURCE	W
$Q_{e,\exp}$	--	QE_EXP	$eV \cdot m^{-3}s^{-1}$
$Q_{e,imp}$	--	QE_IMP	$m^{-3}s^{-1}$
$Q_{e,isource,1}$	SOURCES%QE_EXP	--	$eV \cdot m^{-3}s^{-1}$
$Q_{e,isource,2}$	SOURCES%QE_IMP	--	$m^{-3}s^{-1}$
$\sum_i \nu_{ei}$	COLLISIONS%VIE	VIE	$m^{-3}s^{-1}$
$\sum_i \nu_{ei} T_i$	COLLISIONS %QIE	QIE	$eV \cdot m^{-3}s^{-1}$
$Q_{\gamma i}$	--	QGI	$eV \cdot m^{-3}s^{-1}$
$Q_{\gamma i,i\,\mathrm{mod}el}$	TRANSPORT%QGI	--	$eV \cdot m^{-3}s^{-1}$
χ_e	--	DIFF	m^2/s
V_{Te}^{pinch}	--	VCONV	m/s
$\chi_{e,i\,\mathrm{mod}el}$	TRANSPORT%DIFF_TE	--	m^2/s
$V_{Te,i\,\mathrm{mod}el}^{pinch}$	TRANSPORT%VCONV_TE	--	m/s
a	SOLVER%A	A	--
b	SOLVER%B	B	--
c	SOLVER%C	C	--
d	SOLVER%D	D	--
e	SOLVER%E	E	--
f	SOLVER%F	F	--
g	SOLVER%G	G	--
h	SOLVER%H	H	--
v(1:2)	SOLVER%V	V	--
u(1:2)	SOLVER%U	U	--
w(1:2)	SOLVER%W	W	--
Y^{t-1}	SOLVER%YM	W	--
solution	SOLVER%Y	Y	--
derivative of solution	SOLVER%DY	DY	--
		Internal name used in	

Functions	**ELECTRON_TEMPERATURE routine**
$\frac{V}{\rho}'\left(\frac{3}{2}\frac{n_e^{-}T_{e,\mathrm{interpretative}}^{-}}{\tau}\left(\frac{V'^{-}}{V'}\right)^{\frac{5}{3}}+Q_{e,\exp}+\sum_i \nu_{ei}T_i-Q_{\gamma i} -\left[-\frac{3}{2}\frac{n_e}{\tau}Q_{e,imp}+\sum_i \nu_{ei}-\rho n_e\cdot\frac{\dot{B}_0}{2B_0}\frac{1}{V'}\cdot\frac{\partial V'}{\partial \rho}\right]\cdot T_{e,\mathrm{interpretative}}\right)$	FUN1
$\int_0^{\rho} V'\left(\frac{3}{2}\frac{n_e^{-}T_{e,\mathrm{interpretative}}^{-}}{\tau}\left(\frac{V'^{-}}{V'}\right)^{\frac{5}{3}}+Q_{e,\exp}+\sum_i \nu_{ei}T_i-Q_{\gamma i}\right)\partial\rho - \int_0^{\rho} V'\left(-\frac{3}{2}\frac{n_e}{\tau}Q_{e,imp}+\sum_i \nu_{ei}-\rho n_e\cdot\frac{\dot{B}_0}{2B_0}\frac{1}{V'}\cdot\frac{\partial V'}{\partial \rho}\right)\cdot T_{e,\mathrm{interpretative}}\partial\rho$	INTFUN1

Rotation transport equation

Variables and units:

$u_{i,\varphi}$ - ion toroidal rotation velocity [m/s]

$\omega_{i,\varphi}$ - ion angular toroidal velocity [1/s]

M_i - ion toroidal momentum [kg*m/s]

Φ_i - ion toroidal momentum flux [kg*m2/s2]

$\Phi_{i,conv}$ - convective component of ion toroidal momentum flux [kg*m2/s2]

$\Phi_{i,cond}$ - conductive component of ion toroidal momentum flux [kg*m2/s2]

M_{tot} - total ion toroidal momentum [kg*m/s]

Φ_{tot} - total ion toroidal momentum flux [kg*m2/s2]

Equation:

$$\left(\left.\frac{\partial}{\partial t}\right|_{\rho} - \frac{\dot{B}_0}{2B_0}\cdot\frac{\partial}{\partial\rho}\rho\right)\left(V'\langle R\rangle m_i n_i u_{i,\varphi}\right) + \frac{\partial}{\partial\rho}\Phi_i = V'\left(U_{i,\varphi,\exp} - U_{i,\varphi,imp}\cdot u_{i,\varphi} + U_{zi,\varphi}\right) \tag{7.1}$$

where total flux is defined as:

$$\Phi_i = V'\left\langle\left|\nabla\rho\right|^2\right\rangle m_i n_i \langle R\rangle\cdot\left(-\chi_{u\varphi,i}\frac{\partial u_{i,\varphi}}{\partial\rho} + u_{i,\varphi}V_{u\varphi,i}^{pinch}\right) + m_i\langle R\rangle u_{i,\varphi}\Gamma_i \tag{7.2}$$

with convective and conductive components:

$$\Phi_{i,conv} = m_i\langle R\rangle u_{i,\varphi}\Gamma_i \tag{7.3}$$

$$\Phi_{i,cond} = V'\left\langle\left|\nabla\rho\right|^2\right\rangle m_i n_i \langle R\rangle\cdot\left(-\chi_{u\varphi,i}\frac{\partial u_{i,\varphi}}{\partial\rho} + u_{i,\varphi}V_{u\varphi,i}^{pinch}\right) \tag{7.4}$$

Total transport coefficients are defined as a sum of individual contributions from different transport models weighted with coefficients m_{model}:

$$\chi_{u\varphi,i} = \sum_{i\,model=1}^{n\,model} m_{model}\,\chi_{u\varphi,i\,model} \tag{7.5}$$

$$V_{u\varphi}^{pinch} = \sum_{i\,model=1}^{n\,model} m_{model}\,V_{u\varphi,i\,model}^{pinch} \tag{7.6}$$

(*Combination of individual contributions,* $\chi_{u\varphi,i\,\mathrm{model}}$ *and* $V_{u\varphi,i\,\mathrm{model}}^{pinch}$*, into* $\chi_{u\varphi}$ *and* $V_{u\varphi}^{pinch}$ *is done outside of the ETS equation solver in* ***Transport combiner***)

Right hand side of equation (7.1) includes contributions in generic form from external sources in the form of explicit and implicit terms:

$$U_{i,\varphi,\exp} = \sum_{isource=1}^{nsource} m_{source} U_{i,\varphi,isource,\exp} \tag{7.7}$$

$$U_{i,\varphi,imp} = \sum_{isource=1}^{nsource} m_{source} U_{i,\varphi,isource,imp} \tag{7.8}$$

This summation is done in the workflow by ***Source_Combiner***, where individual weights, m_{source}, can be adjusted.

momentum exchange with other ion components:

$$U_{zi,\varphi} = \langle R \rangle \sum_z \frac{m_{zi} n_z}{\tau_{zi}} \left(u_{z,\varphi} - u_{i,\varphi} \right) \tag{7.9}$$

various collisions quantities ($\langle R \rangle \sum_z \frac{m_{zi} n_z}{\tau_{zi}}$ and $\langle R \rangle \sum_z \frac{m_{zi} n_z u_{z,\varphi}}{\tau_{zi}}$) should be provided by stand alone COLLISIONS module

Equation (7.1) can be rewritten in the form:

$$\begin{aligned}
&\left.\frac{\partial}{\partial t}\right|_x \left(V' \langle R \rangle m_i n_i u_{i,\varphi} \right) \\
&\quad + \frac{\partial}{\partial \rho} \left(V' \left\langle \left| \nabla \rho \right|^2 \right\rangle m_i n_i \langle R \rangle \cdot \left(-\chi_{u\varphi,i} \frac{\partial u_{i,\varphi}}{\partial \rho} + u_{i,\varphi} V_{u\varphi,i}^{pinch} \right) + m_i \langle R \rangle u_{i,\varphi} \Gamma_i - \frac{\dot{\Phi}_b}{2\Phi_b} V' m_i n_i \langle R \rangle u_{i,\varphi} \rho \right) \\
&\quad = V' \left(U_{i,\varphi,\exp} + U_{zi,\varphi} - \left(U_{i,\varphi,imp} + \frac{\dot{\rho}_b}{\rho_b} m_i n_i \langle R \rangle \right) \cdot u_{i,\varphi} \right)
\end{aligned} \tag{7.10}$$

Boundary Conditions:

NOTE! Specified positive values for $\nabla u_{\varphi,bnd}$ ***and*** $L_{u\varphi}$ ***correspond to "normal" profile with density decreasing towards the edge***

on axis, $\rho = 0$:

$$\left.\frac{\partial u_{i,\varphi}}{\partial \rho}\right|_{\rho=0} = 0 \tag{7.11}$$

at the edge, $\rho = \rho_{bnd}$:

type=1 (value)

$$\left. u_{i,\varphi}\right|_{\rho=\rho_{bnd}} = u_{\varphi,bnd}$$

type=2 (gradient)

$$\left.\frac{\partial u_{i,\varphi}}{\partial \rho}\right|_{\rho=\rho_{bnd}} = -\nabla u_{\varphi,bnd}$$

type=3 (scale lenght)

$$\left.\frac{1}{\left(\partial \ln u_{i,\varphi} / \partial \rho\right)}\right|_{\rho=\rho_{bnd}} = -L_{u\varphi}$$

type=4 (flux)

$$\left.\Phi\right|_{\rho=\rho_{bnd}} = f_{u\varphi,bnd}$$

type=5 (generic)

$$v_{gen}\left.\frac{\partial u_{i,\varphi,bnd}}{\partial \rho}\right|_{bnd} + u_{gen} u_{i,\varphi,bnd} = w_{gen}$$

Definitions for quantities derived from rotation equation:

total toroidal momentum:

$$M_i = m_i n_i \langle R \rangle u_{i,\phi} \tag{7.12}$$

angular velocity for plasma component *i*:

$$\omega_{i,\varphi} = \frac{u_{i,\varphi}}{\langle \mathrm{R} \rangle} \tag{7.13}$$

total momentum and flux are defined as:

$$M_{tot} = \sum_i M_i \; ; \quad \Phi_{tot} = \sum_i \Phi_i \tag{7.14}$$

If equation (7.10) is not solved, interpretative toroidal velocity should be specified, momentum and flux are assumed:

$$u_{i,\varphi} = u_{i,\varphi,\text{interpretative}} \; ; \tag{7.15}$$

$$\omega_{i,\varphi} = \frac{u_{i,\varphi,\text{interpretative}}}{\langle R \rangle} \; ; \tag{7.16}$$

$$M_i = m_i n_i \langle R \rangle u_{i,\varphi,\text{interpretative}} \; ; \tag{7.17}$$

$$\Phi_i = \Phi_{i,\text{int}} \tag{7.18}$$

$$\Phi_{i,conv} = m_i \langle R \rangle \Gamma_i u_{i,\varphi,\text{interpretative}} \tag{7.19}$$

$$\Phi_{i,cond} = \Phi_{i,\text{int}} - m_i \langle R \rangle \Gamma_i u_{i,\varphi,\text{interpretative}} \tag{7.20}$$

$$M_{tot} = \sum_i M_i \; ; \quad \Phi_{tot} = \sum_i \Phi_i \tag{7.21}$$

where:

$$\begin{aligned} \Phi_{i,\text{int}} &= \frac{\dot{\Phi}_b}{2\Phi_b} V' m_i n_i \langle R \rangle \rho_b x u_{i,\varphi,\text{interpretative}} + \\ &+ \rho_b \int_0^x V' \left(U_{i,\varphi,\exp} + \langle R \rangle \sum_z \frac{m_{zi} n_z u_{z,\varphi}}{\tau_{zi}} + \frac{\langle R \rangle^- m_i n_i^- u_{i,\varphi,\text{interpretative}}^-}{\tau} \left(\frac{V'^-}{V'} \right) \right) \partial x \\ &- \rho_b \int_0^x V' \left(U_{i,\varphi,imp} - \langle R \rangle \frac{m_i n_i}{\tau} - \langle R \rangle \sum_z \frac{m_{zi} n_z}{\tau_{zi}} - \frac{\dot{\rho}_b}{\rho_b} m_i n_i \langle R \rangle \right) u_{i,\varphi,\text{interpretative}} \partial x \end{aligned} \tag{7.22}$$

Translation to generalized form for interface with numerical solver:

Equation (6.10) can be rewritten using time discretization,

$$\left. \frac{\partial}{\partial t} \right|_x \left(V' \langle R \rangle m_i n_i u_{i,\varphi} \right) = \frac{V' \langle R \rangle m_i n_i u_{i,\varphi} - V'^- \langle R \rangle^- m_i n_i^- u_{i,\varphi}^-}{\tau} \, ,$$

where V'^-, n_i^-, $\langle R \rangle^-$ and $u_{i,\varphi}^-$ are taken at the previous time step as:

$$
\begin{aligned}
& m_i \frac{V' n_i \langle R\rangle u_{i,\varphi} - V'^{-} n_i^{-} \langle R\rangle^{-} u_{i,\varphi}^{-}}{\tau} \\
& \quad + \frac{\partial}{\partial \rho}\left(V'\left\langle |\nabla\rho|^2 \right\rangle m_i n_i \langle R\rangle \cdot \left(-\chi_{u\varphi,i} \frac{\partial u_{i,\varphi}}{\partial \rho} + u_{i,\varphi} V_{u\varphi,i}^{pinch} \right) + m_i \langle R\rangle u_{i,\varphi} \Gamma_i - \frac{\dot{\Phi}_b}{2\Phi_b} V' m_i n_i \langle R\rangle u_{i,\varphi} \rho \right) \;, \\
& \quad = V'\left(U_{i,\varphi,\exp} + \langle R\rangle \sum_z \frac{m_{zi} n_z}{\tau_{zi}} \left(u_{z,\varphi} - u_{i,\varphi} \right) - \left(U_{i,\varphi,imp} + \frac{\dot{\rho}_b}{\rho_b} m_i n_i \langle R\rangle \right) \cdot u_{i,\varphi} \right)
\end{aligned} \tag{7.23}
$$

or as:

$$
\begin{aligned}
& m_i \frac{V' n_i \langle R\rangle u_{i,\varphi} - V'^{-} n_i^{-} \langle R\rangle^{-} u_{i,\varphi}^{-}}{\tau} \\
& \quad + \frac{1}{\rho_b} \frac{\partial}{\partial x}\left(- \frac{V'\left\langle |\nabla\rho|^2 \right\rangle m_i n_i \langle R\rangle \chi_{u\varphi,i}}{\rho_b} \frac{\partial u_{i,\varphi}}{\partial x} + \left(V'\left\langle |\nabla\rho|^2 \right\rangle n_i V_{u\varphi,i}^{pinch} + \Gamma_i - \frac{\dot{\Phi}_b}{2\Phi_b} V' n_i \rho \right) \cdot \langle R\rangle m_i \cdot u_{i,\varphi} \right) \\
& \quad = V'\left(U_{i,\varphi,\exp} + \langle R\rangle \sum_z \frac{m_{zi} n_z}{\tau_{zi}} u_{z,\varphi} - \left(U_{i,\varphi,imp} + \langle R\rangle \sum_z \frac{m_{zi} n_z}{\tau_{zi}} + \frac{\dot{\rho}_b}{\rho_b} m_i n_i \langle R\rangle \right) \cdot u_{i,\varphi} \right)
\end{aligned} \tag{7.24}
$$

Then, taking in account (6.8) and (6.9), coefficients for the interface with numerical solver can be used as:

$$
\begin{aligned}
& a(x) = V' \langle R\rangle m_i n_i \\
& b(x) = V'^{-} \langle R\rangle^{-} m_i n_i^{-} \\
& c(x) = \rho_b \\
& d(x) = \frac{V'\left\langle |\nabla\rho|^2 \right\rangle m_i n_i \langle R\rangle \chi_{u\varphi,i}}{\rho_b} \\
& e(\rho) = \left(V'\left\langle |\nabla\rho|^2 \right\rangle n_i V_{u\varphi,i}^{pinch} + \Gamma_i - \frac{\dot{\Phi}_b}{2\Phi_b} V' n_i \rho \right) \cdot \langle R\rangle m_i \\
& f(\rho) = V'\left(U_{i,\varphi,\exp} + \langle R\rangle \sum_z \frac{m_{zi} n_z}{\tau_{zi}} u_{z,\varphi} \right) \\
& g(\rho) = V'\left(U_{i,\varphi,imp} + \langle R\rangle \sum_z \frac{m_{zi} n_z}{\tau_{zi}} + \frac{\dot{\rho}_b}{\rho_b} m_i n_i \langle R\rangle \right) \\
& h = \tau \\
& Y^{t-1}(\rho) = u_{i,\varphi}^{-}
\end{aligned} \tag{7.25}
$$

boundary conditions:

at the axis, $x = 0$:

$$\left.\frac{\partial u_{i,\varphi}}{\partial x}\right|_{x=0} = 0 \tag{7.26}$$

$$v(1) = 1; \quad u(1) = 0; \quad w(1) = 0$$

at the edge, $x = 1$:

<u>type=1</u> (value)

$$\left.u_{i,\varphi}\right|_{x=1} = u_{\varphi,bnd}$$

$$v(2) = 0; \quad u(2) = 1; \quad w(2) = u_{\varphi,bnd}$$

<u>type=2</u> (gradient)

$$\left.\frac{\partial u_{i,\varphi}}{\partial x}\right|_{x=1} = -\nabla u_{\varphi,bnd} \cdot \rho_b$$

$$v(2) = 1; \quad u(2) = 0; \quad w(2) = -\nabla u_{\varphi,bnd} \cdot \rho_b$$

<u>type=3</u> (scale lenght)

$$\left.\frac{\rho_b}{\left(\partial \ln u_{i,\varphi} / \partial \rho\right)}\right|_{x=1} = -L_{u\varphi}$$

$$v(2) = 1; \quad u(2) = \frac{\rho_b}{L_{u\varphi}}; \quad w(2) = 0$$

<u>type=4</u> (flux)

$$\left.\Phi\right|_{x=1} = f_{u\varphi,bnd}$$

$$\Phi_i = V'\left\langle \left|\nabla\rho\right|^2 \right\rangle m_i n_i \left\langle R \right\rangle \cdot \left(-\frac{\chi_{u\varphi,i}}{\rho_b} \frac{\partial u_{i,\varphi}}{\partial x} + u_{i,\varphi} V_{u\varphi,i}^{pinch} \right) + m_i \left\langle R \right\rangle u_{i,\varphi} \Gamma_i$$

$$v(2) = -V'\left\langle \left|\nabla\rho\right|^2 \right\rangle \left\langle R \right\rangle m_i n_i \frac{\chi_{u\varphi,i}}{\rho_b}; \quad u(2) = V'\left\langle \left|\nabla\rho\right|^2 \right\rangle \left\langle R \right\rangle m_i n_i V_{u\varphi,i}^{pinch} + m_i \left\langle R \right\rangle \Gamma_i; \quad w(2) = f_{u\varphi,bnd}$$

<u>type=5</u> (generic)

$$\frac{v_{gen}}{\rho_b}\frac{\partial u_{i,\varphi,bnd}}{\partial x}\bigg|_{bnd} + u_{gen} u_{i,\varphi,bnd} = w_{gen}$$

$$v(2) = \frac{v_{gen}}{\rho_b}; \quad u(2) = u_{gen}; \quad w(2) = w_{gen}$$

Variables used inside fortran routines:

Variable	TYPE%NAME used in ETS data flow	Internal name used in ROTATION routine	Units
τ	EVOLUTION%TAU	TAU	s
$\dot{B}_0$	GEOMETRY%BTPRIME	BTPRIME	T/s
B_0	GEOMETRY%BT	BT	T
ρ	GEOMETRY%RHO	RHO	m
V'	GEOMETRY%VPR	VPR	m^2
V'^-	EVOLUTION%VPRM	VPRM	m^2
$\left\langle \lvert\nabla\rho\rvert^2 \right\rangle$	GEOMETRY%G1	G1	--
$\langle R \rangle$	GEOMETRY%G2	G2	m
$\langle R \rangle^-$	EVOLUTION%G2M	G2M	m
$u_{i,\varphi}$	PROFILES%VTOR	VTOR	m/s
$u_{i,\varphi}^-$	EVOLUTION%VTORM	VTORM	m/s
$\omega_{i,\varphi}$	PROFILES%WTOR	WTOR	1/s
M_i	PROFILES%MTOR	MTOR	kg*m/s
M_{tot}	PROFILES% MTOR_TOT	MTOR_TOT	kg*m/s
n_i	PROFILES%NI	NI	m^{-3}
n_i^-	EVOLUTION%NI	NIM	m^{-3}
Φ_i	PROFILES%FLUX_MTOR	FLUX_MTOR	$kg*m^2/s^2$
$\Phi_{i,conv}$	PROFILES%FLUX_MTOR_CONV	FLUX_MTOR_CONV	$kg*m^2/s^2$
$\Phi_{i,cond}$	PROFILES% FLUX_MTOR_COND	FLUX_MTOR_COND	$kg*m^2/s^2$
$\Phi_{i,\text{int}}$	PROFILES% INT_SOURCE_MTOR	INT_SOURCE	$kg*m^2/s^2$
Φ_{tot}	PROFILES% FLUX_MTOR_TOT	FLUX_MTOR_TOT	$kg*m^2/s^2$

$U_{i,\varphi,\exp}$	--	UI_EXP	kg/m/s^2
$U_{i,\varphi,imp}$	--	UI_IMP	kg/m^2/s
$U_{i,\varphi,isource,1}$	SOURCES%UI_EXP	--	kg/m/s^2
$U_{i,\varphi,isource,2}$	SOURCES%UI_IMP	--	kg/m^2/s
$\langle R \rangle \sum_z \frac{m_{zi} n_z}{\tau_{zi}}$	COLLISIONS %WZI	WZI	kg/m^2/s
$\langle R \rangle \sum_z \frac{m_{zi} n_z u_{z,\varphi}}{\tau_{zi}}$	COLLISIONS %UZI	UZI	kg/m/s^2
$\chi_{u\varphi,i}$	--	DIFF	m^2/s
$V_{u\varphi,i}^{pinch}$	--	VCONV	m/s
$\chi_{u\varphi,i,i\,\mathrm{mod}el}$	TRANSPORT%DIFF_VTOR	--	m^2/s
$V_{u\varphi,i,i\,\mathrm{mod}el}^{pinch}$	TRANSPORT%VCONV_VTOR	--	m/s
a	SOLVER%A	A	--
b	SOLVER%B	B	--
c	SOLVER%C	C	--
d	SOLVER%D	D	--
e	SOLVER%E	E	--
f	SOLVER%F	F	--
g	SOLVER%G	G	--
h	SOLVER%H	H	--
v(1:2)	SOLVER%V	V	--
u(1:2)	SOLVER%U	U	--
w(1:2)	SOLVER%W	W	--
Y^{t-1}	SOLVER%YM	W	--
solution	SOLVER%Y	Y	--
derivative of solution	SOLVER%DY	DY	--

Functions	**Internal name used in ROTATION routine**

$\frac{V'}{\rho}\left(U_{i,\varphi,\exp}+\langle R\rangle\sum_{z}\frac{m_{zi}n_{z}u_{z,\varphi}}{\tau_{zi}}+\frac{\langle R\rangle^{-}m_{i}n_{i}^{-}u_{i,\varphi,\mathrm{int}erpretative}^{-}}{\tau}\left(\frac{V'^{-}}{V'}\right)\right.$ $\left.-\left(U_{i,\varphi,imp}-\langle R\rangle\frac{m_{i}n_{i}}{\tau}-\langle R\rangle\sum_{z}\frac{m_{zi}n_{z}}{\tau_{zi}}\right)u_{i,\varphi,\mathrm{int}erpretative}\right)$	FUN1
$\int_{0}^{\rho}V'\left(U_{i,\varphi,\exp}+\langle R\rangle\sum_{z}\frac{m_{zi}n_{z}u_{z,\varphi}}{\tau_{zi}}+\frac{\langle R\rangle^{-}m_{i}n_{i}^{-}u_{i,\varphi,\mathrm{int}erpretative}^{-}}{\tau}\left(\frac{V'^{-}}{V'}\right)\right)\partial\rho$ $-\int_{0}^{\rho}V'\left(U_{i,\varphi,imp}-\langle R\rangle\frac{m_{i}n_{i}}{\tau}-\langle R\rangle\sum_{z}\frac{m_{zi}n_{z}}{\tau_{zi}}\right)u_{i,\varphi,\mathrm{int}erpretative}\partial\rho$	INTFUN1