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A Learning Scheme for the Parameter Identification of Robot Dynamics

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Abstract— — **In this paper, a new identification method, called learning identification algorithm, of the model parameters of robot dynamics is proposed. This method has the following features: it does not require the exciting trajectory, and the measurements of joint accelerations and velocities. The method is demonstrated experimentally by estimating the dynamic parameters of a shaped glass cutting robot.**

I. INTRODUCTION

For the most control schemes of robot manipulators, such as feedforward control scheme, computed torque control scheme and adaptive control scheme, the accuracy of the model parameters of robot dynamics has a great effect on the precision, performance, stability, and robustness of robot control system. How to accurately estimate the dynamic parameters of robot manipulators is an important problem in the field of robot[1] ~ [5]. The existing identification algorithms in the literature are all based on the least square technique and the accuracy of the estimated parameters depend, to a certain extent, on the motion trajectory of robots, which must be exciting as indicated by Armstrong[4]. However, the exciting trajectory is complicated and easy to cause the vibration of robot arms, which is undesired.

In this paper, according to the learning control scheme proposed by Arimoto[6], a learning identification scheme, which is used to progressively estimate the model parameters of robot dynamics through several learning, is proposed. This scheme does not require that the motion trajectory used to estimate parameters is sufficiently exciting and can accurately estimate the dynamic parameters along a smooth trajectory. It is demonstrated to be effective by a shaped glass cutting robot with two arms.

This paper is organized as follows. Section 2 presents the description of parameter identification of robot. A learning identification scheme is derived in Section 3. The design and implementation of a filter used to estimate the joint accelerations and velocities are discussed in Section 4. The simulation and experiments are included in Section 5 and the conclusions in Section 6.

II. THE PROBLEM DESCRIPTION

In accordance with the Langrange-Euler(LE) formulation, the dynamic model of a robot arm can be expressed as a compact state-space formulation:

$$M(\mathbf{q}(t))\ddot{\mathbf{q}}(t)+C(\mathbf{q}(t),\dot{\mathbf{q}}(t))+g(\mathbf{q}(t))=\tau(t) \tag{1}$$

where $\tau(t)\in R^n$ is the input vector, i.e. applied force/torque vector for joint actuators. $\mathbf{q}(t)$, $\dot{\mathbf{q}}(t)$ and $\ddot{\mathbf{q}}(t)\in R^n$ are the output vectors, which denote the positions, velocities and accelerations respectively. $M(\mathbf{q}(t))\in R^{n\times n}$ is the effective and coupling inertia matrix. $C(\mathbf{q}(t),\dot{\mathbf{q}}(t))\in R^n$ denotes the Coriolis and centripetal effects vector. $g(\mathbf{q}(t))\in R^n$ denotes the gravitational force vector.

As we know, in eqn. (1), $M(\mathbf{q}(t))$, $C(\mathbf{q}(t),\dot{\mathbf{q}}(t))$ and $g(\mathbf{q}(t))$ are the nonlinear functions of $\mathbf{q}(t)$ and $\dot{\mathbf{q}}(t)$, which contain a constant unknown parameter vector $\Phi\in R^m$, determined by the mass, inertia and geometrical shape of each joints. The exact mathematical model of robot, which is essential to design the manipulator controller and to make the manipulators complete the specified tasks, will be available when Φ is determined. The precise determination of the dynamic parameter vector is very useful to most control schemes. There are two possible methods used to determine the dynamic parameters of robot: (1) using theoretical calculation, that is, from the mechanical data provided by the manufacturer; (2) from the data obtained from planned manipulator motions. Based on the assumption that the dynamic equation for any robot with n joints has the property of being linear in the parameter vector Φ, the dynamic equation can be written as[5]

$$\tau=W(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})\Phi \tag{2}$$

where, $W(\mathbf{q},\dot{\mathbf{q}},\ddot{\mathbf{q}})$ is a n × m kinematic information matrix that describes the motion of the arms, τ is a n × 1 vector of forces and torques at joints of robot, and Φ is a m × 1 vector of unknown dynamic parameters. ϕ_i denotes the ith element of Φ, which can be an independent parameter or a linear combination of several parameters. By the least square technique, the unknown dynamic parameter vector Φ can be determined as

$$\Phi=W^{+}\tau \tag{3}$$

where W^+ denotes the generalized inverse of W. In eqn. (3), each element of W is the function of joint angles, velocities and accelerations of a robot. For a precise estimate, eqn. (3) is augmented by using N sample data points as

$$\tau=\begin{bmatrix}\tau(1)\\ \vdots\\ \tau(N)\end{bmatrix},\quad \mathbf{W}=\begin{bmatrix}\mathrm{W}(1)\\ \vdots\\ \mathrm{W}(N)\end{bmatrix} \tag{4}$$

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thus, once τ and W were obtained, one could calculate Φ as follows:

$$\hat{\Phi} = (\mathbf{W}^T\mathbf{W})^{-1}\mathbf{W}^T\tau \tag{5}$$

However, the matrix $W^{\mathrm{T}}W$ is not invertible. Therefore, Armstrong[4] proposed a sufficiently exciting motion trajectory scheme, which is difficult to calculate the exciting trajectory and may be unrealizable due to no consideration of the torque limits applied on joints. Lu et al. [5] excite the system by superposing a certain number of different frequencies to the motion trajectory to precisely estimate the dynamic parameters and eliminate the joint acceleration measurements.

In reality, the accuracy of the estimated dynamic parameters of a robot is always affected by the unmodelling dynamics and external disturbances. To obtain the exact model parameters is impractical. We know, once the model parameter vector Φ is inside a certain range of parameter

where τ_f is the feedforward compensation torque vector and $\hat{\Phi}$ is the estimated parameter vector. Define

$$\Delta\tau = [W(\mathbf{q}_d, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d) - W(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})]\hat{\Phi} \tag{9}$$

then

$$W(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})(\Phi - \hat{\Phi}) = \tau - \tau_f + \Delta\tau + v \tag{10}$$

Similar to equation (5), we obtain the estimation scheme as

$$\Delta\hat{\Phi} = \Phi - \hat{\Phi} = (\mathbf{W}^T\mathbf{W})^{-1}\mathbf{W}\eta \tag{11}$$

where

$$\mathbf{W} = \begin{bmatrix} \mathrm{W}_1(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \\ \vdots \\ \mathrm{W}_N(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) \end{bmatrix}, \quad \eta = \begin{bmatrix} [\tau - \tau_f + \Delta\tau]_1 \\ \vdots \\ [\tau - \tau_f + \Delta\tau]_N \end{bmatrix} \tag{12}$$

thus, in accordance with the learning control proposed by Arimoto, the learning identification scheme is shown as the following Fig. 1.

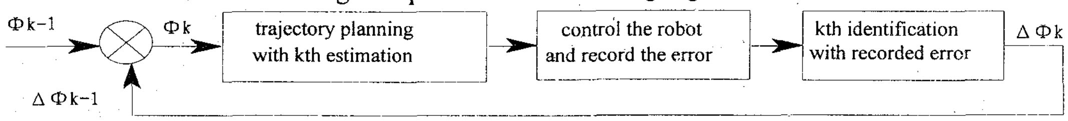

Fig. 1 Diagram of the learning identification scheme

space Ω, the system error caused by the difference between estimated parameters and the real parameters could be overcome by the closed control loop. Based on the above concept, a new parameter identification scheme, called learning identification, for dynamic equation of a robot will be derived in the following section.

III. LEARNING IDENTIFICATION SCHEME

Up to 1980, Arimoto et al. [6] proposed a learning control scheme, the basic concept of which is to obtain the desired control signal for the specified control purpose by gradually learning. The learning law is as follows:

$$u_{k+1}(t) = u_k(t) + \Delta u_k(t) \tag{6a}$$

$$\Delta u_k(t) = F(u_k, e_k, r, t) \tag{6b}$$

that is, the correcting signal $\Delta u_k(t)$ used to calculate the control signal $u_{k+1}(t)$ is obtained by the previous control, and the desired control signal $u(t)$ can be obtained by several learning, not by one. The learning identification scheme based on this concept is derived as below.

Consider the equation (2) and assume that the joint angles, velocities and accelerations of robot are all measurable, we have

$$W(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\Phi = \tau + v \tag{7}$$

where $\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}$ and τ represent corresponding measurements respectively, v is the high frequency residual. Let $\mathbf{q}_d, \dot{\mathbf{q}}_d$ and $\ddot{\mathbf{q}}_d$ be the desired motion trajectory of the robot respectively, then

$$\tau_f = W(\mathbf{q}_d, \dot{\mathbf{q}}_d, \ddot{\mathbf{q}}_d)\hat{\Phi} \tag{8}$$

At kth learning, similar to eqn. (6), the model parameters can be calculated as follows:

$$\hat{\Phi}_k = \hat{\Phi}_{k-1} + \Delta\hat{\Phi}_{k-1} \tag{13}$$

$$\Delta\hat{\Phi}_{k-1} = (\mathbf{W}_{k-1}^T\mathbf{W}_{k-1})^{-1}\mathbf{W}_{k-1}\eta_{k-1} \tag{14}$$

where, W_{k-1} and η_{k-1} present the W and η along the planned motion trajectory corresponding to the estimated $\hat{\Phi}_{k-1}$ respectively. Thus, at first using the initial parameter estimation $\hat{\Phi}_0$ of dynamic model to plan the trajectory and calculate the feedforward compensation torque of each joint, and then to control the robot to move along the planned trajectory and record the error of each joint angle along the trajectory. After the 1st learning, one can obtain the $\hat{\Phi}_1$, and then to plan the motion trajectory with the estimation $\hat{\Phi}_1$ and to calculate $\hat{\Phi}_2$, and so on, until the model error is within the desired error range by successively learning. Finally, one get the desired model parameter vector Φ.

IV. THE DESIGN AND IMPLEMENTATION OF FILTERING ESTIMATOR

The learning identification algorithm, described by eqn. (9), (12) ~ (14) in the above section, still requires the measurements of joint angles, velocities and accelerations of a robot. The estimation of the joint velocities and accelerations through a filtering estimator will be discussed below.

Define the joint angle error vector **e**, angle velocity error vector $\mathbf{e}_v$ and angle acceleration error vector $\mathbf{e}_a$ respectively as follows:

$$\mathbf{e} = \mathbf{q}_d - \mathbf{q},\ \mathbf{e}_v = \dot{\mathbf{q}}_d - \dot{\mathbf{q}},\ \mathbf{e}_a = \ddot{\mathbf{q}}_d - \ddot{\mathbf{q}} \tag{15}$$

where $\mathbf{e}_a = \dot{\mathbf{e}}_v = \ddot{\mathbf{e}}$. Let $\hat{\mathbf{e}}_v$ and $\hat{\mathbf{e}}_a$ denote the estimation of $\mathbf{e}_v$ and $\mathbf{e}_a$ respectively, then the estimation of joint angle velocities and accelerations can be calculated by

$$\hat{\dot{\mathbf{q}}} = \dot{\mathbf{q}}_d - \hat{\mathbf{e}}_v,\ \hat{\ddot{\mathbf{q}}} = \ddot{\mathbf{q}}_d - \hat{\mathbf{e}}_a \tag{16}$$

where $\hat{\mathbf{e}}_a$ and $\hat{\mathbf{e}}_v$ are obtained by the following filtering estimator

$$\hat{\mathbf{e}}_a = F(s)\ddot{\mathbf{e}} = F(s)s^2\mathbf{e}, \tag{17a}$$

$$\hat{\mathbf{e}}_v = F(s)\dot{\mathbf{e}} = F(s)s\mathbf{e} = \frac{1}{s}\hat{\mathbf{e}}_a \tag{17b}$$

where *F(s)* is the transfer function of filter. The filter function used in this paper is a low-pass filter with frequency bandwidth ρ:

$$F(s) = \rho^3 \big/ (s+\rho)^3 \tag{18}$$

and its minimal realization is

$$\frac{d}{dt}\begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \end{bmatrix} = \begin{bmatrix} O & I & O & O \\ O & O & I & O \\ O & O & O & O \\ O & O & I & O \end{bmatrix} \bullet \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \end{bmatrix} + \begin{bmatrix} k_1 I \\ k_2 I \\ k_3 I \\ O \end{bmatrix} (\mathbf{e} - \mathbf{x}_1)$$

$$\hat{\mathbf{e}}_a = \mathbf{x}_3,\ \hat{\mathbf{e}}_v = \mathbf{x}_4 \tag{19}$$

where $k_1 = 3\rho$, $k_2 = 3\rho^2$ and $k_3 = \rho^3$. Define the filtered estimation of the control signal τ as $\hat{\tau}$, we have

$$\mathbf{W}_{k-1} = \begin{bmatrix} \mathbf{W}_1(\mathbf{q}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}}) \\ \vdots \\ \mathbf{W}_N(\mathbf{q}, \hat{\dot{\mathbf{q}}}, \hat{\ddot{\mathbf{q}}}) \end{bmatrix}_{k-1},\ \eta_{k-1} = \begin{bmatrix} [\hat{\tau} - \tau_f + \Delta\tau]_1 \\ \vdots \\ [\hat{\tau} - \tau_f + \Delta\tau]_N \end{bmatrix}_{k-1} \tag{20}$$

where $\mathbf{q}$ is the measurement of joint angles, and $\hat{\dot{\mathbf{q}}}$ and $\hat{\ddot{\mathbf{q}}}$ are the estimation of joint velocities and accelerations.

V. SIMULATION AND EXPERIMENTS

Consider the shaped glass cutting robot shown in Fig. 2, which is a direct drive robot with two links and whose working range reaches 1.5m by 2.8m. From LE formulation, the dynamic equation can be written as

$$\begin{bmatrix} a & b\cos(\theta_2 - \theta_1) \\ b\cos(\theta_2 - \theta_1) & c \end{bmatrix} \bullet \begin{bmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} + \begin{bmatrix} -b\dot{\theta}_2^2 \sin(\theta_2 - \theta_1) \\ b\dot{\theta}_1^2 \sin(\theta_2 - \theta_1) \end{bmatrix} = \begin{bmatrix} \tau_1 \\ \tau_2 \end{bmatrix} \tag{21}$$

where, $a = m_1 l^2/3 + m_2 l^2$, $b = m_2 l^2/3$, $c = m_2 l^2/2$. Rewrite the above equation as the parameter form similar to eqn. (2), that is

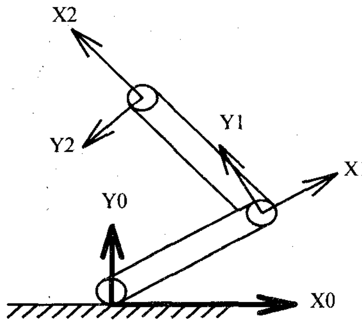

Fig. 2 Schemantic of the shaped glass cutting robot

$$\mathbf{W}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) = \begin{bmatrix} \ddot{\theta}_1 & \ddot{\theta}_2\cos(\theta_2 - \theta_1) - \dot{\theta}_2^2 \sin(\theta_2 - \theta_1) & 0 \\ 0 & \ddot{\theta}_1\cos(\theta_2 - \theta_1) + \dot{\theta}_1^2 \sin(\theta_2 - \theta_1) & \ddot{\theta}_2 \end{bmatrix} \tag{22}$$

$$\Phi = [a \quad b \quad c]^T,\ \tau = [\tau_1 \quad \tau_2]^T \tag{23}$$

and the system schematic diagram used to complete the simulation of the learning identification for the model parameters of robot is shown as Fig. 3.

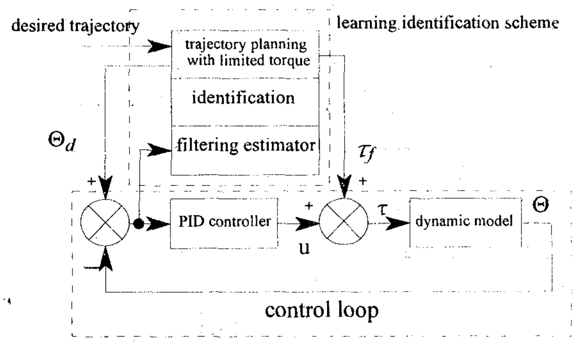

Fig. 3 Schemantic of learning identification

Before simulation, the first is to determine the coefficients of each joint controller and the parameters of the filtering estimator (18). Each joint controller is specified to a PID controller, that is

$$\mathbf{u} = \mathbf{k}_p\mathbf{e} + \int \mathbf{k}_i\mathbf{e}dt + \mathbf{k}_d\hat{\dot{\mathbf{e}}}$$

where

$$\mathbf{e} = \mathbf{q}_d - \mathbf{q},\ \hat{\dot{\mathbf{e}}} = \frac{\lambda^2 s}{(s+\lambda)^2}\mathbf{e} \tag{24}$$

and the choice for λ is related to the bandwidth of closed loop. We have chosen λ=40. The values of coefficient vectors, i.e. $\mathbf{k}_p$, $\mathbf{k}_i$ and $\mathbf{k}_d$, of each joint PID controller are gradually determined from the end joint to the base joint, and were chosen as

$$\mathbf{k}_p = [4500 \quad 5000]^T,\ \mathbf{k}_i = [30 \quad 30]^T,\ \mathbf{k}_d = [600 \quad 600]^T$$

and the coefficient ρ was also chosen the same value as the λ, that is ρ=40. The total output torques for the arms are 8Nm and 4Nm respectively. Assume that the real values of the dynamic parameters Φ of robot are $[20\ 5\ 8]^T$. The motion trajectory used to the simulation of learning identification is a straight line with the starting point (500,500) and the length 500mm. The digital simulation results are shown in Table 1, the maximum torque used in the trajectory planning is the 80% of the total output torques. The error curves for the first 3 learning identifications are shown as Fig. 4, Fig.5 and Fig. 6 respectively. The each graduation for the vertical axis in figures is 0.02°.

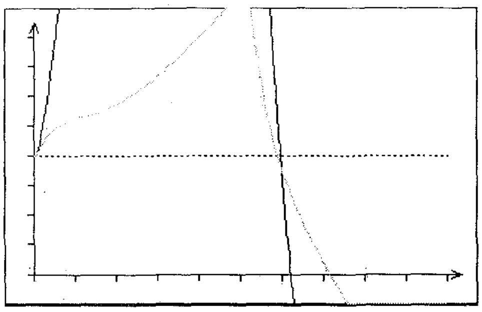

Fig. 4 The tracking error curves of the 1st learning along the straight line

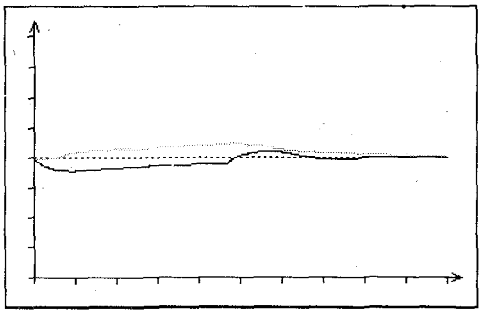

Fig. 5 The tracking error curves of the 2nd learning along the straight line

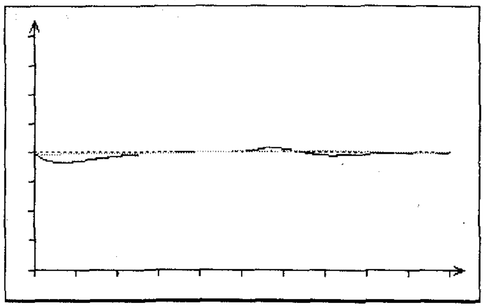

Fig. 6 The tracking error curves of the 3rd learning along the straight line

Table 1 shows that the average errors of the estimated parameters of dynamic model of the robot are less than 2% through 3 learning identifications. Fig. 6 shows the trajectory errors are less than 0.002° after the third learning identification, which also shows that once the model parameters are within a certain range of parameter space, the robot can move along the planned trajectory accurately and the precise determination of the dynamic parameters is not of very importance.

Table 1

Times	a	b	c
0	40.0	20.0	30.0
1	19.78	3.76	7.10
2	20.23	5.00	8.15
3	20.23	5.04	8.17
4	20.23	5.04	8.18
5	20.23	5.05	8.18
6	20.23	5.05	8.18

For the practical shaped glass cutting robot, the experimental system is shown as Fig. 7. The sample period is taken as 2ms and the straight line defined as the above is chosen as the motion trajectory, and the values of the estimated parameters by the learning identification scheme detailed in the above sections is

$\hat{\Phi}=[36.0\ \ 12.1\ \ 26.2]^T$

and using the values as the dynamic parameters of the cutting robot, the robot can move along various motion trajectories accurately.

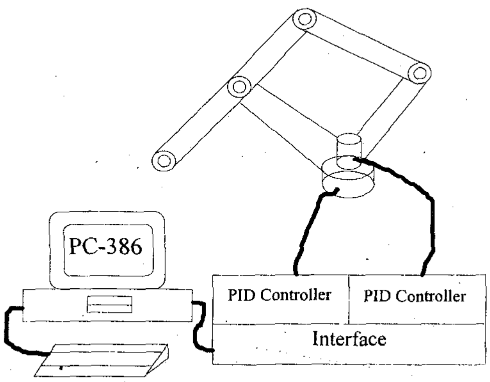

Fig. 7 Schemantic of the experiment with the shaped glass cutting robot

VI. CONCLUSIONS

The learning identification scheme, proposed in this paper, for dynamic parameters of a robot have been successfully used in a shaped glass cutting robot. This scheme can obtain a set of most approximating model parameters by successively learning along a specified motion trajectory, which does not need to be exciting. In accordance with the simulation in the above section, at most by 3 learning the trajectory errors will be satisfactory. This scheme eliminate the excitation condition, which is necessary to other identification schemes, on the planned motion trajectory of a robot. And the remarkable feature of this learning identification scheme is that only the measurements of the joint angles are required and the cost is reduced. This is a very practical and efficient method.

REFERENCES

[1]P.K. Khosla, T.Kanade, **Parameter identification of**

robot dynamics, *Proc. 24th IEEE Conf. on Decision & Control,* 1985

[2]C.H. An, Atkeson, C.G. and Hollerbach, J.M., **Estimation of inertial parameters of rigid body links of manipulators**, *Proc. of Conf. on Decision & Control*, 990-1002, 1985

[3]I.-J. Ha, M.-S.Ko, S.K.Kwon, **An efficient estimation algorithm for the model parameters of robotic manipulators**, *IEEE Trans.*, vol.RA-5(3), 386-394, 1989

[4]B. Armstrong, **On finding exciting trajectories for identification experiments involving systems with nonlinearing dynamics**, *Int.J.Robotics Research*, Vol.8(6), 28-40, 1989

[5]Z. Lu, Shimoga,K.B. and et al, **Experimental Determination of Dynamic Parameters of Robotic Arms**, *J. Robotic Systems*, Vol. 10(8), 1009-1029, 1993

[6]S. Arimoto and et al, **Learning Control Theory for Dynamic Systems**, *Proc. 24th Conference on Decision & Control,* 1985