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Parameter Identification
of Robot Dynamics

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Abstract

This paper presents algorithms for identifying parameters of multi-degrees-of-freedom robotic arm. First, we outline the fundamental properties of the Newton-Euler formulation of robot dynamics from the view point of parameter identification. We then show that the Newton-Euler model which is nonlinear in some of dynamic parameters can be transformed into the equivalent model which is linear in dynamic parameters. We develop both on-line and off-line parameter estimation procedures. To illustrate our approach, we identify the dynamic parameters of the cylindrical robot, and the three degree-of-freedom positioning system of the CMU Direct-Drive Arm II.

1. Introduction

The robot control problem centers around the computation of the actuating torques/forces to produce the desired motion of the end-effector. The model-based control schemes such as the computed-torque [10] and resolved-acceleration [8] controllers accomplish this objective by incorporating the complete dynamic model of the manipulator in the control law. Since the fundamental assumption here is that the robot dynamics are modeled accurately, precise knowledge of the kinematic and dynamic parameters of the robot is required. In practice, it is also necessary to identify on-line the mass and inertial characteristics of the payload in order to achieve accurate trajectory tracking with varying payload.

Earlier work in identification of robot dynamics concentrated on estimating the mass of the payload. Paul [12] presented two techniques with the assumption that the manipulator is at rest. His first method used the joint torques/forces, and the second method a wrist torque/force sensor. Coiffet [4] extended this technique, for a manipulator at rest, to estimate also the center-of-mass of the payload. By using special test torques and moving only one degree-of-freedom at a time, the moments-of-inertia of the payload can also be estimated. Recently, Atkeson, An and Hollerbach [2] have proposed an approach which uses a wrist torque/force sensor to estimate the inertial characteristics of the payload. Their approach has also been extended to identify the inertial parameters of all the links of a robot [1].

This paper presents an algorithm to estimate the kinematic and dynamic parameters of a robot from the measurements of its inputs (actuating torques/forces) and outputs (joint positions, velocities and accelerations). To facilitate the identification procedure, we modify the Newton-Euler formulation so that it becomes linear in the dynamic parameters. We introduce the torque/force error model for parameter identification based on this modified Newton-Euler formulation of the robot dynamics.

Our general-purpose algorithm is suited for both on-line and off-line applications: in off-line identification only one link of the robot is commanded to move for the purpose of parameter estimation, whereas in on-line identification the parameters are estimated while the robot is in motion performing the task in hand. We can adopt the strategy of estimating off-line the kinematic and dynamic parameters of the robot and then estimating on-line the inertial characteristics of the payload and the dynamic frictional coefficients. This procedure improves the robustness of the estimation, decreases the computational requirements, and adapts to varying payloads. We demonstrate our identification algorithm through simulation experiments on a cylindrical robot, and the positioning system (i.e., the first three degrees-of-freedom) of the CMU Direct-Drive Arm II [6].

2. Properties of Robot Dynamics Model for Parameter Estimation

Robot dynamics describe the temporal interactions of the joint motions in response to the inertial, centrifugal, Coriolis, gravitational, and actuating torques/forces. Robot dynamics are highly coupled and nonlinear second-order differential equations. The identification problem is to estimate all of the kinematic and dynamic parameters that affect the link torques/forces. The Denavit-Hartenberg parameters constitute the kinematic parameters, and the link masses, link inertias, and center-of-mass vectors are the dynamic parameters.

Two formulations have been used to model robot dynamics [12, 3, 9]: closed-form Lagrange formulation and recursive Newton-Euler formulation. While the former leads to physical insight, the latter is computationally more efficient and suited for real-time control applications. We will base our development of parameter identification algorithms on the Newton-Euler formulation. We first summarize the O(N) recursive Newton-Euler dynamic equations, and observe their properties.

2.1. Newton-Euler Formulation

The Newton-Euler formulation shown in equations (1)-(9) computes the inverse dynamics (ie., joint torques/forces from joint positions, velocities, and accelerations) based on two sets of recursions: the forward and backward recursions. The forward recursions (1)-(4) transform the kinematic variables from the base to the end-effector. The backward recursions (5)-(9) transform the forces and moments from the end-effector to the

base, and culminate with the calculation of the joint torques/forces.

$$\omega_{i+1}=\begin{cases} \mathbf{A}_{i+1}^T[\,\omega_i + \mathbf{z}_o\dot{\theta}_{i+1}] & \text{rotational} \\ \mathbf{A}_{i+1}^T\,\omega_i & \text{translational} \end{cases} \quad (1)$$

$$\dot{\omega}_{i+1}=\begin{cases} \mathbf{A}_{i+1}^T[\dot{\omega}_i + \mathbf{z}_o\ddot{\theta}_{i+1} + \omega_i\times(\mathbf{z}_o\dot{\theta}_{i+1})] & \text{rotational} \\ \mathbf{A}_{i+1}^T\,\dot{\omega}_i & \text{translational} \end{cases} \quad (2)$$

$$\dot{\mathbf{v}}_{i+1}=\begin{cases} \mathbf{A}_{i+1}^T\dot{\mathbf{v}}_i + \dot{\omega}_{i+1}\times\mathbf{p}_{i+1} + \omega_{i+1}\times(\omega_{i+1}\times\mathbf{p}_{i+1}) & \text{rotational} \\ \mathbf{A}_{i+1}^T[\dot{\mathbf{v}}_i + \mathbf{z}_o\ddot{d}_{i+1} + 2\omega_i\times(\mathbf{z}_o\dot{d}_{i+1})] \\ \quad + \dot{\omega}_{i+1}\times\mathbf{p}_{i+1} + \omega_{i+1}\times(\omega_{i+1}\times\mathbf{p}_{i+1}) & \text{translational} \end{cases} \quad (3)$$

$$\omega_0 = \dot{\omega}_0 = \mathbf{v}_0 = \mathbf{0}$$

$$\dot{\mathbf{v}}_0 = [g_x\ g_y\ g_z]^T \quad \text{gravitational acceleration}$$

$$\dot{\mathbf{v}}_i^* = \dot{\omega}_i\times\mathbf{s}_i + \omega_i\times(\omega_i\times\mathbf{s}_i) + \dot{\mathbf{v}}_i \quad (4)$$

$$\mathbf{F}_i = m_i\dot{\mathbf{v}}_i^* \quad (5)$$

$$\mathbf{N}_i = \mathbf{I}_i\dot{\omega}_i + \omega_i\times(\mathbf{I}_i\omega_i) \quad (6)$$

$$\mathbf{f}_i = \mathbf{A}_{i+1}\mathbf{f}_{i+1} + \mathbf{F}_i \quad (7)$$

$$\mathbf{n}_i = \mathbf{A}_{i+1}\mathbf{n}_{i+1} + \mathbf{p}_i\times\mathbf{f}_i + \mathbf{N}_i + \mathbf{s}_i\times\mathbf{F}_i \quad (8)$$

$$\tau_i = \begin{cases} \mathbf{n}_i^T(\mathbf{A}_i^T\mathbf{z}_o) & \text{rotational} \\ \mathbf{f}_i^T(\mathbf{A}_i^T\mathbf{z}_o) & \text{translational} \end{cases} \quad (9)$$

$\mathbf{f}_{N+1}$: external force at the end-effector.
$\mathbf{n}_{N+1}$: external moment at the end-effector.

The computational requirements of the general-purpose and customized recursive Newton-Euler algorithms for various types of manipulators, such as manipulators with parallel/perpendicular joint axes, spherical wrists, or sparse center-of-mass vectors and inertia tensors, have been detailed in [6].

From equations (1)-(9), we note the following properties:

1. The Newton-Euler model is *linear* in the classical link inertia tensors $\mathbf{I}_i$.

This property follows directly from the backward recursions in (5)-(9). The joint torque/force τ_i in (9) is linear in the moment $\mathbf{n}_i$. In the recursion for the moment $\mathbf{n}_i$ in (8), the net moment $\mathbf{N}_i$ exerted on link i appears additively. Finally, the moment $\mathbf{N}_i$ in (6) is linear in the classical link inertia tensor $\mathbf{I}_i$.

2. For rotational joints, the Newton-Euler model is *nonlinear* in the center-of-mass vectors $\mathbf{s}_i$.

From (4) and (5), the net force $\mathbf{F}_i$ is linear in the center-of-mass vector $\mathbf{s}_i$. The vector cross product $\mathbf{s}_i\times\mathbf{F}_i$ in (8) is thereby nonlinear (quadratic) in $\mathbf{s}_i$. Hence, the torque τ_i for a rotational joint in (9) is nonlinear in the center-of-mass

Table 1: Kinematic and Dynamic Parameters

m_i	Total mass of link i
τ_i	Joint torque/force at joint i
ω_i and $\dot{\omega}_i$	Angular velocity and acceleration of the i-th coordinate frame
$\mathbf{v}_i$ and $\dot{\mathbf{v}}_i$	Linear velocity and acceleration of the i-th coordinate frame
$\mathbf{v}_i^*$ and $\dot{\mathbf{v}}_i^*$	Linear velocity and acceleration of the center-of-mass of link i
$\mathbf{F}_i$ and $\mathbf{N}_i$	Net force and moment exerted on link i
$\mathbf{f}_i$ and $\mathbf{n}_i$	Force and moment exerted on link i by link $i-1$
$\mathbf{p}_i$	Position of the i-th coordinate frame with respect to the $(i-1)$-th coordinate frame: $\mathbf{p}_i = [a_i\ \ d_i\ sin\alpha_i\ \ d_i\ cos\alpha_i]^T$
$\mathbf{s}_i$	Position of the center-of-mass of link i: $\mathbf{s}_i = [s_{ix}\ \ s_{iy}\ \ s_{iz}]^T$
$\mathbf{z}_o$	$= [0\ \ 0\ \ 1]^T$
$\mathbf{A}_i$	Orthogonal rotation matrix which transforms a vector in the i-th coordinate frame to a coordinate frame which is parallel to the $(i-1)$-th coordinate frame:

$$\mathbf{A}_i = \begin{matrix} cos\theta_i & -cos\alpha_i\ sin\theta_i & sin\alpha_i\ sin\theta_i \\ sin\theta_i & cos\alpha_i\ cos\theta_i & -sin\alpha_i\ cos\theta_i \\ 0 & sin\alpha_i & cos\alpha_i \end{matrix}$$

for $i = 1, 2, \cdots, N$, where $\mathbf{A}_{N+1} \triangleq \mathbf{E}$.

$\mathbf{I}_i$	Classical inertia tensor of link i about the center-of-mass of link i (and parallel to the i-th coordinate frame); with principal inertias I_{ixx}, I_{iyy} and I_{izz}; and cross-inertias I_{ixy}, I_{ixz} and I_{iyz}.

vector $\mathbf{s}_i$. It must be noted that for translational joints the center-of-mass vectors appear linearly.

3. The Newton-Euler model is *nonlinear* in the kinematic parameter vectors $\mathbf{p}_i$.

From (3)-(5) and (7), the link force $\mathbf{f}_i$ is linear in the vector, $\mathbf{p}_i$. The vector cross product $\mathbf{p}_i\times\mathbf{f}_i$ in (8) is thereby nonlinear in $\mathbf{p}_i$. Hence, the torque/force τ_i in (9) is nonlinear in the vector, $\mathbf{p}_i$.

4. The dynamic equations of links $i+1$ through N are independent of the mass m_i and the classical inertia tensor $\mathbf{I}_i$ of link i.

This physically intuitive property is an immediate consequence of the backward recursions.

In summary, the classical link inertia tensors $\mathbf{I}_i$ and the link masses m_i appear linearly in the Newton-Euler dynamics model, but the link masses are multiplied by linear and/or quadratic functions of the center-of-mass vectors $\mathbf{s}_i$ and nonlinear functions of the joint position variables θ_i. In contrast, the Lagrange formulation, which utilizes the pseudo-inertia

matrices, has been shown to be linear in the dynamic parameters [11]. The pseudo-inertia matrices are formed by first expressing the classical inertia tensors about the link coordinate frames and then combining their elements linearly. We thus infer that if the Newton-Euler model in (1)-(9) is reformulated such that the link inertia tensors are expressed about the link coordinate frames instead of the link center-of-mass coordinate frame, the modified Newton-Euler formulation will be also linear in the center of the mass vectors s_i.

2.2. Transformation of Inertia Tensor

Let $C_i = (x_i, y_i, z_i)$ be a Denavit-Hartenberg coordinate frame for link i and let $C_i^* = (x_i^*, y_i^*, z_i^*)$ be a coordinate frame which is fixed at the center-of-mass of link i and whose axes are parallel with those of C_i. From the definition, s_i is the translational vector from the origin of the link coordinate frame C_i to to the origin of the center-of-the-mass coordinate frame C_i^*.

If I_i is the classical link inertia tensor about the center-of-mass of link i, the corresponding inertia tensor I'_i about the link i coordinate frame C_i is computed according to the parallel-axis theorem or Steiner's law:

$$I'_i = I_i + m_i(s_i^T s_i E - s_i s_i^T) \tag{10}$$

where E is the 3×3 identity matrix. This transformation of the inertia tensor includes the quadratic terms of s_i. When we substitute this in (6) in the Newton-Euler formulation, the quadratic terms from $s_i \times F_i$ in (8) are absorbed in the transformation, thus resulting in the modified Newton-Euler dynamic formulation which is linear in center-of-mass vectors [7].

Properties 1 and 2 together with this transformation lay the foundation for our identification algorithms, and Property 4 will be used to derive our off-line identification algorithm.

2.3. Torque/Force Error Model

In general, identification of all the link masses, dynamics parameters and the kinematic parameters is a problem of nonlinear estimation. If we assume that we have nominal values of those parameters, say from engineering drawings, we can linearize the torque/force equation of each link about the nominal values of the kinematic and dynamic parameters to obtain the torque/force error model [11]:

$$\epsilon_i = \tau_i - \tau_i^0 = \phi_i^T \Delta\psi_i \quad for \quad i = 1, 2, \cdots, N \tag{11}$$

where τ_i is the applied torque/force to link i, τ_i^0 is the value of the torque/force, as computed by an inverse dynamics model using the nominal values [9], ϵ_i is the input torque/force error of link i, $\Delta\psi_i$ is the correction vector of unknown parameters that affect the torque/force of link i, and ϕ_i is the nonlinear vector function of the kinematic and dynamic parameters and the output measurements (joint positions, velocities and accelerations). The torque/force error model (11) relates the error torque/force of link i to corresponding modeling inaccuracies in the kinematic and dynamic parameters.

If we know the link masses and kinematic parameters (e.g., by measuring them), then identification of the remaining dynamic parameters is a problem of linear estimation from the beginning, or in (11) ϕ_i is a function of only known parameters and robot output measurement. Equation (11) is a linear equation with μ_i unknowns, ie., $\mu_i = dim(\Delta\psi_i)$. Because of the property 4 in Section 2.3, we can step sequentially through the links, from the tip to the base, and identify the dynamic parameters that affect the link torques/forces of each link. It must be noted, however, that all the parameters may not be always identifiable, nor need be identified. That is, the number of independent dynamic parameters is less than or equal to the total number of dynamic parameters of a robot.

2.4. Example: Cylindrical Robot

We will illustrate the approach presented so far by using a three degrees-of-freedom cylindrical robot. We first develop its Newton-Euler dynamics model. After observing its properties, we apply the transformation of inertia tensor to obtain the equivalent modified Newton-Euler model. Finally the torque/force error model is developed.

The cylindrical robot consists of three degrees-of-freedom: a rotation θ_1, a vertical translation d_2 and a radial translation d_3. The Denavit-Hartenberg parameters of the cylindrical robot are:

Link	θ	α	a	d
1	θ_1	0°	0	0
2	0	-90°	0	d_2
3	0	0°	0	d_3

The coordinate vector of the cylindrical robot is thus $q = [\theta_1 \ d_2 \ d_3]^T$. We assume that the classical link inertias of the three links are diagonal and that only the s_{1z}, s_{2y} and s_{3z} components of the center-of-mass vectors are non-zero. These assumptions do not suffer from any loss of generality, and the cylindrical robot preserves all of the inherent coupling and nonlinear characteristics of robot dynamics.

We expand the Newton-Euler recursions (1)-(9) to obtain the complete dynamics model of the cylindrical robot:

$$\tau_3 = m_3\ddot{d}_3 - m_3(s_{3z} + d_3)\dot{\theta}_1^2 \tag{12}$$

$$\tau_2 = (m_2 + m_3)\ddot{d}_2 + (m_2 + m_3)g \tag{13}$$

$$\tau_1 = [(I_{1zz} + I_{2yy} + I_{3yy}) + m_3(s_{3z} + d_3)^2]\ddot{\theta}_1 \tag{14}$$
$$+ [2m_3(s_{3z} + d_3)]\dot{d}_3\dot{\theta}_1$$

We observe that equations (12)-(14) are linear in the classical link inertia tensor components, but nonlinear in the link masses and the center-of-mass vectors due to the presence of $m_3 s_{3z}^2$ in (14). Also the dynamic parameters of link i do not affect the torques/forces of links $i+1$ through 3.

Now the transform (10) for this cylindrical robot is given as,

$$I'_{3yy} = I_{3yy} + m_3 s^2_{3z} \qquad (15)$$

$$I'_{2yy} = I_{2yy} \qquad (16)$$

$$I'_{1zz} = I_{1zz} \qquad (17)$$

Upon substituting (15)-(17) into (12)-(14), we obtain the modified dynamics model:

$$\tau_3 = m_3 \ddot{d}_3 - m_3 (s_{3z} + d_3) \dot{\theta}_1^2 \qquad (18)$$

$$\tau_2 = (m_2 + m_3) \ddot{d}_2 + (m_2 + m_3) g \qquad (19)$$

$$\tau_1 = [(I'_{1zz} + I'_{2yy} + I'_{3yy}) + m_3 (2s_{3z} + d_3^2)] \ddot{\theta}_1 \qquad (20)$$
$$+ [2m_3 (s_{3z} + d_3)] \dot{d}_3 \dot{\theta}_1$$

We note that the nonlinearity $m_3 s_{3z}^2$ in (14) has been absorbed by the transformation (15), and that I_{2yy} and I_{1zz} are retained. From this observation, we understand that once the nonlinearities associated with link i are identified in the dynamics model, we may proceed to transform the inertias of only link i.

Now the torque/force error models for the cylindrical robot are given by:

$$\epsilon_3 = (-m_3 \dot{\theta}_1^2) \Delta s_{3z} + [\ddot{d}_3 - (s_{3z} + d_3) \dot{\theta}_1^2] \Delta m_3 \qquad (21)$$

$$\epsilon_2 = (\ddot{d}_2 + g) \Delta m_2 + (\ddot{d}_2 + g) \Delta m_3 \qquad (22)$$

$$\epsilon_1 = \ddot{\theta}_1 \Delta (I'_{1zz} + I'_{2yy} + I'_{3yy}) \qquad (23)$$
$$+ (2m_3 \ddot{\theta}_1 + 2m_3 \dot{d}_3 \dot{\theta}_1) \Delta s_{3z} + (2s_{3z} + d^2_3) \ddot{\theta}_1 \Delta m_3$$

Therefore, if we assume the masses are known ($\Delta m_2 = \Delta m_3 = 0$), the number of independent dynamic parameters are, $\mu_3 = 1$ (s_{3z}: unknown), $\mu_2 = 0$, and $\mu_1 = 1$ (because we can only identify the sum of the three inertias in (23), and s_{3z} has already been identified in (21)). We measure the input torque/force to link i and the position, velocity and acceleration of link i at N sampling instants (where $N > \mu_i$), compute the nominal values of τ_i^0 (according to the inverse dynamics model in (18)-(20)), and apply the linear least-squares algorithm to estimate the dynamic parameters of link i.

3. Sequential Identification Procedures for an N-DOF Robot

In this section, we present the identification procedures for an N degree-of-freedom robot. We use the property 4 of the Newton-Euler formulation to simplify the derivation of the torque/force equations for the N links of the manipulator. We treat the parameter identification problem sequentially, starting from link N (the tip) and proceeding to link 0 (the base), and estimate the dynamic parameters of each link individually. The identified dynamic parameters of link i become known

parameters in the torque/force equation of link $i-1$. This sequential procedure reduces the number of dynamic parameters which must be estimated from the torque/force error model of each link, and thus results in a robust identfication procedure.

There are two situations in parameter identification: off-line and on-line. In the off-line procedure, we collect all the input—output data prior to analysis and do not impose any limits on the computation time. In contrast, the on-line procedure deals with real-time updates of the parameters during robot operations.

3.1. Off-Line Identification Procedure

Since the dynamic parameters of the all the links except the payload are constant, we can estimate these parameters off-line. If we lock the first $i-1$ joints mechanically (to set the velocities and accelerations of joints one through $i-1$ to zero), we reduce dramatically the complexity of the torque/force error model of link i. This simplification can only be achieved in off-line estimation and does not require the use of special trajectories.

Off-line identification procedure for an N degree-of-freedom robot:

1. Expand the Newton-Euler recursive equations to obtain the closed-form link torques/forces equations of the manipulator. In deriving the torque/force equation for link i, set the velocities and accelerations of links 1 through $i-1$ to zero.

2. Convert the Newton-Euler model into the modified (linear in dynamic parameters) Newton-Euler model, by applying the transformation (10) which transforms the classical inertias about the center-of-mass of link i to inertias about the i-th link frame.

3. Generate the torque/force error model (11) by incorporating the nominal values of the dynamic parameters to be estimated.

4. Calculate all of the Newton-Euler dynamic parameters in the torque/force error model that affect the torque/force of link i. Since the dynamic parameters affecting the torques/forces of links $i+1$ through N have already been identified, these dynamic parameters are known numerical quantities when working with link i.

3.2. On-line Identification Procedure

The on-line identification procedure for an N degree-of-freedom robot parallels steps 1 through 4 above, but in deriving the torque/force equations in Step 1 we allow positions, velocities and accelerations of links $i-1$ through 1 to respond to the actuating torques/forces instead of setting them to zero.

3.3. Identifiable Parameters

Each link of a robot is characterized by a maximum of ten dynamic parameters: the link mass, the six classical inertias and the three elements of the center-of-mass vector. In practice, only a fraction of the ten parameters of link i and a fraction of the $10(N-i)$ dynamic parameters of links $i+1$ through N affect the torque/force τ_i of link i. We emphasize that we can only estimate (and need to estimate) the dynamic parameters that

actually affect the joint torques/forces. For example, the I_{3zz} classical moment-of-inertia parameter of the cylindrical robot does not affect the dynamic robot model in (12)-(14) and hence cannot be identified.

The estimation of the classical inertias may not be unique and only their linear combinations may be identified. For example, equation (23) allows us to estimate only the sum $I'_{1zz}+I'_{2yy}+I'_{3yy}$ of the link inertias. Such an estimate is sufficient for computing the closed-form inverse dynamics in (18)-(20). If, however, the inverse dynamics are implemented by the original recursive Newton-Euler formulation in (1)-(9), the numerical values of *all* of the classical link inertias are required. For this purpose, we can set the values of first two of the three classical link inertias to zero, and set the third to the value of the estimated sum. Even though this particular assignment of numerical values may not have physical significance, it does lead to the arithmetically-correct computation of the torque/force.

The closed-form dynamic model of a six degree-of-freedom robot is in general very complex, and the corresponding torque/force error model for on-line estimation of all the robot parameters including payload characteristics is of comparable complexity. To facilitate the estimation process, a viable strategy is to identify the parameters of all the links first by the off-line procedure and to estimate on-line the mass and the inertial charateristics of the payload. This strategy requires the real-time identification of only the last link of the robot. Our identification algorithm is directly amenable to the real-time identification of the payload characteristics for dynamic feedback control. To achieve accurate trajectory control, our identification algorithm can also estimate on-line the dynamic frictional coefficients by modeling frictional torques/forces as linear functions of the joint velocities.

3.4. Parameter Identifiers

In secions 5.1 and 5.2, we have developed the torque/force error models. We now need to cast these models into parameter identifier structures. A parameter identifier consists of three components: the system to be identified, a postulated model, and an adaptation algorithm which updates the model based upon an error criterion. In our case, the system is the physical robot and the model is the Newton-Euler inverse arm dynamics model. The inputs to the robot are the joint torques/forces and the outputs are the joint positions, velocities, and accelerations. We adopt the so-called input-error structure for the identification: that is, as shown in Figure 1, the outputs of the robot are fed to the inverse arm dynamics model which computes the joint torques/forces. The error signal, which is the difference between the applied and the computed torques, drives the estimation algorithm.

4. Simulation Results

We have performed simulation experiments for identifying the dynamic parameters of the cylindrical robot and the first three degrees-of-freedom of the CMU DD Arm II. We commanded each joint to move in a sinusoidal trajectory, begining from the rest position to a ninety degree position and returning in one second. We sampled the trajectory at 20-ms intervals and used the first half of the trajectory (or 25 sample points) in our identification experiments. One data point consists of the measurements of the applied torque/force and the position, velocity and acceleration of a link. Since we estimate only the dynamics parameters, the problem is linear and we have applied the least-squares algorithm under the assumption that the input-output measurements were noise-free. This assumption is justified practically by the current availability of high resolution resolvers (16 bits/revolution) and tachometers [6].

4.1. Cylindrical Robot

In Table 2, we summarize our simulation results for the cylindrical robot. We implemented the least-squares algorithms for the torque/force error model in (20) of the third link using the 25 data sets and identified the z-component of the center-of-mass vector of the third link s_{3z}. We then identified the sum of the link inertias from the torque/force error model of the first link by again using the 25 data sets along the trajectory. The estimated values match exactly the true values.

4.2. CMU Direct-Drive Arm II

The configuration of the CMU DD Arm II is shown in Figure 2. Throughout this experiment, we assume that the Denavit-Hartenberg parameters and the link masses of the CMU DD Arm II are known.

The torque/force error model of the 3 DOF positioning system of the CMU DD Arm II is described the following equations:

$$\epsilon_3 = I_{3zz}[\dot\theta_1^{\,2} - 2C_3\dot\theta_1^{\,2} + 2\dot\theta_1\dot\theta_2 - 4C_3^{\,2}\dot\theta_1\dot\theta_2 + \dot\theta_2^{\,2} - 2C_3^{\,2}\dot\theta_2^{\,2}]$$
$$+ I_{3yz}[-C_3\ddot\theta_2 - C_3\ddot\theta_1] + I_{3yy}[\ddot\theta_3]$$
$$+ (I_{3zz} - I_{3xx})[2C_3S_3\dot\theta_1\dot\theta_2 + C_3S_3\dot\theta_1^{\,2} + C_3S_3\dot\theta_2^{\,2}] \quad (24)$$
$$+ I_{3xy}[-S_3\ddot\theta_2 - S_3\ddot\theta_1]$$
$$+ m_3s_{3z}[-d_3C_3\ddot\theta_2 - d_3C_3\ddot\theta_1 - gS_3 + a_1C_2C_3\dot\theta_1^{\,2}$$
$$-a_1C_3S_2\ddot\theta_1 + a_2C_3\dot\theta_2^{\,2} + 2a_2C_3\dot\theta_1\dot\theta_2 + a_2C_3\dot\theta_1^{\,2}]$$
$$+ m_3s_{3x}[-d_3S_3\ddot\theta_2 - d_3S_3\ddot\theta_1 + gC_3 + a_1C_2S_3\dot\theta_1^{\,2}$$
$$-a_1S_3S_2\ddot\theta_1 + a_2S_3\dot\theta_2^{\,2} + 2a_2S_3\dot\theta_1\dot\theta_2 + a_2S_3\dot\theta_1^{\,2}]$$

$$\epsilon_2 = I_{2yy}[\ddot\theta_1 + \ddot\theta_2] + I_{3zz}[C_3^{\,2}\ddot\theta_2 + C_3^{\,2}\ddot\theta_1]$$
$$+ I_{3xx}[S_3^{\,2}\ddot\theta_2 + S_3^{\,2}\ddot\theta_1]$$
$$+ m_2s_{2x}[2a_2\ddot\theta_1 + a_1C_2\ddot\theta_1 + a_1S_2\dot\theta_1^{\,2} + 2a_2\ddot\theta_2] \quad (25)$$
$$+ m_2s_{2z}[-a_1C_2\dot\theta_1^{\,2} + a_1S_2\ddot\theta_1]$$
$$+ m_3s_{3y}[a_1C_2\dot\theta_1^{\,2} - a_1S_2\ddot\theta_1 - 2d_3\ddot\theta_2 - 2d_3\ddot\theta_1]$$

$$\epsilon_1 = I_{1zz}\ddot\theta_1 + m_1s_{1x}2a_1\ddot\theta_1 \quad (26)$$

where $C_i = \cos\theta_i$ and $S_i = \sin\theta_i$.

From these questions, we observe that:

- The torque of the third link is affected by all the elements of the inertia matrix and also by the two elements, s_{3x} and s_{3z}, of the center-of-mass vector. However, we can only identify seven of the eight parameters in (26)-(24) because the inertia elements I_{3xx} and I_{3zz} occur as a linear combination.

- The torque of link 2 is affected by two inertia elements of the third link, one inertia element of the second link, two elements of the center-of-mass vector of the second link and one element of the center-of-mass vector of the third link. All these parameters occur independently and can be uniquely identified. Note that we could identify only the linear combination $I_{3zz} - I_{3xx}$ from the torque/force error model of the third link whereas we can identify I_{3zz} and I_{3xx} independently from the torque/force error model of the second link.

- The torque of the first link is affected by two independent dynamic parameters, I_{1zz} and s_{1x}, which can be identified uniquely.

In summary, for the three degree-of-freedom positioning system of the CMU DD Arm II, we can identify fourteen of the twenty-seven unknown dynamic parameters.

The results of our simulation experiments for the CMU DD Arm II are summarized in Table 3. We used the 25 data sets to identify successfully the dynamic parameters which affect the torques/forces of each of the three links.

5. Conclusion

In this paper, we have addressed the problem of robot dynamics parameter identification. In deriving our estimation algorithm we noted that some dynamic parameters do not appear in the joint torque/force equations and others appear in linear combinations. Since we don't sense the full torque/force vector at each joints, we can identify only those parameters which affect the joint torque/force. Based on this observation, we proceed to classify the parameters in three categories: those that can be uniquely identified, those that can be identified only in linear combinations, and those which cannot be identified. It is thus imperative that the parameters that can be identified only as linear combinations be singled out and this knowledge be incorporated in the identification procedure, so that the numerical procedure, such as the least-square error, to compute the parameter values, becomes stable and robust. Incorporating this knowledge in the inverse dynamics formulation also reduces its computational requirements. Since we derive the identification equations symbolically, we can easily establish the category to which each parameter belongs. This is an advantage over the alternative technique [1] where the estimation problem is formulated numerically and all the dynamic parameters are estimated at the same time. A systematic procedure for the parameter categorization is further developed in [5].
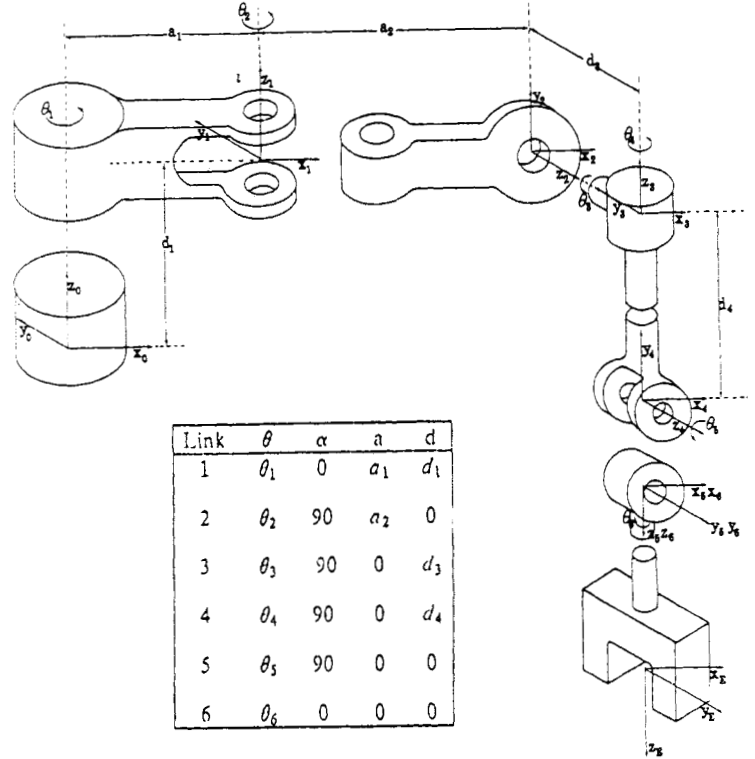
Link	θ	α	a	d
1	θ_1	0	a_1	d_1
2	θ_2	90	a_2	0
3	θ_3	90	0	d_3
4	θ_4	90	0	d_4
5	θ_5	90	0	0
6	θ_6	0	0	0

Figure 2: Kinematic configuration of the CMU Direct-Drive Arm II

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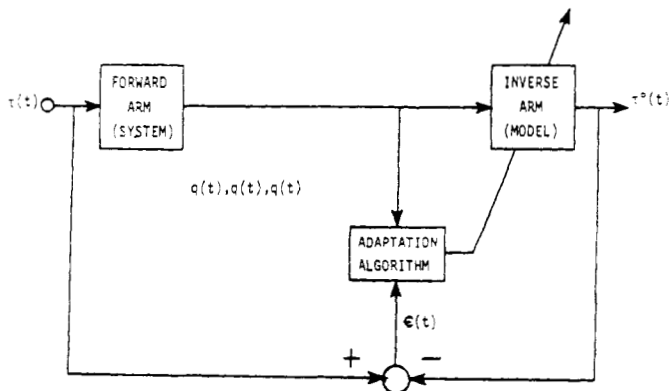
Figure 1: Input-error structure for parameter identification

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Table 2: Simulation Results for the Cylindrical Robot

Link	Dynamic Parameter (Dimensions)	Initial Value	True Value	Estimated Value
3	s_{3z} (m)	0.3	0.5	0.5
1	$I_{1zz}+I_{2yy}+I_{3yy}$ (kg-m^2)	2.0	2.5	2.5

Table 3: Simulation Results for the CMU DD Arm II

Link	Parameter (Dimensions)	Initial Value	True Value [*]	Estimated Value
3	I_{3xz} (kg-m^2)	0.0	0.1	0.1
	I_{3yz} (kg-m^2)	0.0	0.15	0.15
	I_{3yy} (kg-m^2)	0.0	0.2	0.1999
	I_{3xy} (kg-m^2)	0.0	0.15	0.15
	$I_{3zz}-I_{3xx}$ (kg-m^2)	0.0	0.1	0.1
	s_{3x} (m)	0.0	0.14	0.1399
	s_{3z} (m)	0.0	0.2	0.2
2	I_{2yy}	0.3	0.5	0.4999
	I_{3xx} (kg-m^2)	0.05	0.2	0.2001
	I_{3zz} (kg-m^2)	0.1	0.3	0.3001
	s_{2x} (m)	0.1	0.2	0.2
	s_{2z} (m)	0.0	0.10	0.10
	s_{3y} (m)	0.1	0.15	0.15
1	I_{1zz} (kg-m^2)	1.0	1.3	1.2999
	s_{1x} (m)	0.00	0.2	0.2001

(*) These values are hypothetical figures for simulation, and do not correspond to those of the real CMU DD Arm II.