Bridget McLean
Homework 1

1 a) $O(g(n)) = f(n)$: there exist positive constants c and n_0 such that $0 \leq f(n) \leq cg(n)$ for all $n \geq n_0$

$f(n) = 2n^2 + n^3 \log n$
$\leq 2n^2 + n^4$
$\leq n^4 + n^4$
$= 2n^4 \leq O(n^4)$ for $n \geq 1$, $c = 2$ $n_0 = 1$ $K = 4$

b) $f(n) = 3n^5 + (\log n)^4$
$\leq 3n^5 + n^4$
$\leq 3n^5 + n^5$
$\leq 4n^5 \leq O(n^5)$ for $n \geq 1$, $c = 4$ $n_0 = 1$ $K = 5$

c) $f(n) = \frac{n^4 + n^2 + 1}{n^4 + 1}$
$\leq n^4/n^4$
$\leq 1 \leq O(1)$ for $n \geq 1$, $c = 1$ $n_0 = 1$ $K = 0$

d) $f(n) = \frac{n^3 + 5 \log n}{n^4 + 1}$
$= n^3/n^4$
$= 1/n \leq O(1)$ for $n \geq 1$, $c = 1$ $n_0 = 1$ $K = 0$

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2. a) $T(n) = 7T(n/2) + n^2$

$a=7,\ b=2,\ f(n)=n^2 \quad n^{\log_2 7} = n^{2.8074}$

Case 1: $n^2 \leq n^{\log_2 7} = n^{2.8+\epsilon}$

$T(n) = \Theta(n^{\log_2 7}) =$

b) $T(n) = T(n/2) + 1$

$a=1,\ b=2,\ f(n)=1 \quad n^{\log_2 1} = 1$

Case 2: ~~$f(n) = n^{\log_2 1}$~~ $1 = 1$

$T(n) = \Theta(\log n)$

c) $T(n) = 4T(n/2) + n^3$

$a=4,\ b=2,\ f(n)=n^3 \quad n^{\log_2 4} = n^2$

case 3: $f(n) > n^{\log_2 4} \quad n^3 > n^2 \quad T(n) = \Theta(n^3)$

Regularity Condition:

$4f(n/2) = 4(n/2)^3 = n^3/2 \leq cf(n), \quad c = 1/2$

3. a) - Choose the middle element to start, index $n/2 = i$
- Split A into subarrays $x[A_1 \ldots i-1]$ and $y[i+1 \ldots n]$
- If $A[i] = i$, return i (or true)
- If $A[i] > i$, search left subarray x
  - Because we know the index $A[i]$ won't have a matching value, because we found that value at i. However, we don't know this about the left subarray
- If $A[i] < i$, search right subarray y

b) $T(n) = T(n/2) + \Theta(1)$

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4. We can get an algorithm that runs in $O(n \log m)$ time if we assume array A is already sorted into a max heap structure. If so, then the algorithm would loop through each element of B, and for each element of B perform a binary search through A for a matching element.

5. Let A be the event that the i^{th} element is fixed

$$X_A = \begin{cases} 1 \text{ if the } i^{th} \text{ element is fixed} \\ 0 \text{ otherwise} \end{cases}$$

Let X be the random variable, which is the total number of fixed elements. Then X is the sum of the random indicator variables

$$X = \sum_{i=1}^{n} X_A$$

$$E(X) = E\left(\sum_{i=1}^{n} X_a\right) = \sum_{i=1}^{n} E(X_A) = \sum_{i=1}^{n} 1/n = 1$$