Simplification for Lab 5

1. system model: $\left[V_A = \frac{V_R + V_L}{2} \right]$

$$\ddot{\theta}_x = \frac{G_T}{M_x} V_A - \frac{G_T G_V}{M_x} V_y + \frac{T_g}{M_x} \theta_x$$

$$\dot{V}_y = \frac{G_T}{M_y} V_A - \frac{G_T G_V}{M_y} V_y$$

$\Downarrow$

state space

$$\begin{bmatrix} \ddot{\theta}_x \\ \dot{\theta}_x \\ \dot{V}_y \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & \frac{T_g}{M_x} & -\frac{G_T G_V}{M_x} \\ 1 & 0 & 0 \\ 0 & 0 & -\frac{G_T G_V}{M_y} \end{bmatrix}}_{A} \begin{bmatrix} \dot{\theta}_x \\ \theta_x \\ V_y \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{G_T}{M_x} \\ 0 \\ \frac{G_T}{M_y} \end{bmatrix}}_{B} V_A$$

2. $K = \begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix}$

3. $V_A = G_V V_y^* - K_1 \dot{\theta}_x - K_2 \theta_x + K_3 (V_y^* - V_y)$

V_y^* $\rightarrow$ reference speed $(= 0)$

$\Downarrow$

$$V_A = -K_1 \dot{\theta}_x - K_2 \theta_x - K_3 V_y$$

4. $\det(sI - A + BK)$

$$= s^3 + \left[\frac{G_T}{M_y}(K_3 + G_V) + \frac{G_T}{M_x} K_1\right] s^2 + \left(\frac{G_T}{M_x} K_2 - \frac{T_g}{M_x}\right) s$$

$$- \frac{G_T T_g}{M_x M_y} \cdot (K_3 + G_V)$$

$$\Big\downarrow \text{ use triple root } -P_1$$

$\det(sI - A + BK)$

$$= (s + P_1)^3$$

$$= s^3 + 3P_1 s^2 + 3P_1^2 s + P_1^3$$

$$\Downarrow$$

$$K_1 = \frac{M_x^2}{G_T T_g} P_1^3 + \frac{3M_x}{G_T} P_1$$

$$K_2 = \frac{3M_x}{G_T} P_1^2 + \frac{T_g}{G_T}$$

$$K_3 = \frac{-M_x M_y}{G_T T_g} P_1^3 - G_V$$