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# ES 3011 Lab 6

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Department: RBE / ME

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## 1 Introduction

This lab is a continuation of Lab 5. You will set up your foundation for your PID controller and thus you can finally calculate the PID gains for your velocity.

The methodology is similar to Lab 5.

## 2 Yaw Velocity Controller

The simplified system model of the yaw controller is:

$$M_z \dot{\omega}_z = G_T(V_D - G_w \omega_z) \quad (1)$$

where  $\omega_z$  is the angular velocity rotating around z axis;  $M_z$ ,  $G_T$  and  $G_w$  are defined in Lab 4. In addition,  $V_D = V_R - V_L$ .

### 2.1 PI and PID controller

**Q2-1. Question:** What are the differences between a PI controller and a PID controller?

Hint: You may consider this question based on the following table.

Property	PI controller	PID controller
Overshoot		
Response time		
Response accuracy		
...		

Table 1: Difference between PI and PID

**Q2-2. Question:** Which controller would you choose and why?

## 3 Controller design

The general PID controller expression is:

$$output = K_p e(t) + K_I \int_0^t e(t) dt + K_D \frac{de(t)}{dt} + bias \quad (2)$$

Note: if you choose to use PI controller, please ignore the  $K_D$  term.

### 3.1 Substitute error and output

In this lab, we have  $e(t) = \text{your error} = \text{reference} - \text{actual} = \omega_z^* - \omega_z$ . Also, the output is  $V_D$  and the bias is  $G_w \omega_z^*$

**Q3-1. Question:** Please write down the equation of the controller based on your choice and given Equation 2 (PI or PID). [Just simply substitute the input, output and  $e(t)$  into the general equation of PID controller]

**Q3-2. Question:** Substitute the equation of PID controller into Equation 1. Rewrite the Equation 1.

Hint: From Q3-1, we should obtain an equation as  $V_D = \dots$ , simply substitute this equation into Equation 1.

**Q3-3. Question:** Let the input to be  $\int \omega_z^* dt$  and the output to be  $\int \omega_z dt$ , which represents the desired yaw angle and current yaw angles. Please write down the transfer function of the system based on the equation wrote in Q3-2.

Hint: Use Laplace Transform to get the transfer function and

$$\text{transfer function} = \frac{\mathcal{L}(\text{output})}{\mathcal{L}(\text{input})}$$

### 3.2 Find the gains of PI/PID controller

The system is based on the transfer function you wrote in Q3-3.

The procedures of finding gains are very similar to Lab 5. You firstly need to self-define the poles of the system (need to be in left-half plane), and then you can calculate the characteristic equations. After that, the characteristic equation can also be expressed by the denominator of the transfer function. Then you can solve and obtain the gains of PI/PID controller.

**Q4-1. Question:** Please follow the procedures to find the gains. Please report the figure of pzmap for the system based on the transfer function you wrote in Q3-3, the pole values, gain values and step response of transfer function  $\frac{\int \omega_z^* dt}{\int \omega_z dt}$ . Please attach your code.

Hint: If you choose to use PI controller, it would be a second-order system. If you choose to use PID controller, it would be a third-order system.

**Q4-2. Question:** Implement the PI/Pid controller on the robot, follow the instruction on the website and report the gif.

\*\*\* Note: If you have some difficulties on finding the suitable poles/ gains for the system. Please check the file in Hint on the Lab6 website, it talks about a method of simplification.