

Simplification for Lab 5

1. system model: $\left[V_A = \frac{V_R + V_L}{2} \right]$

$$\ddot{\theta}_x = \frac{G_T}{M_x} V_A - \frac{G_T G_V}{M_x} v_y + \frac{T_g}{M_x} \theta_x$$

$$\dot{v}_y = \frac{G_T}{M_y} V_A - \frac{G_T G_V}{M_y} v_y$$

↓

state space

$$\begin{bmatrix} \dot{\theta}_x \\ \ddot{\theta}_x \\ \dot{v}_y \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ \frac{T_g}{M_x} & 0 & -\frac{G_T G_V}{M_x} \\ 0 & 0 & -\frac{G_T G_V}{M_y} \end{bmatrix}}_A \begin{bmatrix} \theta_x \\ \dot{\theta}_x \\ v_y \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{G_T}{M_x} \\ \frac{G_T}{M_y} \end{bmatrix}}_B V_A$$

2. $K = \begin{bmatrix} K_1 & K_2 & K_3 \end{bmatrix}$

3. $V_A = G_V v_y^* - K_1 \dot{\theta}_x - K_2 \theta_x + K_3 (v_y^* - v_y)$

↓ \hookrightarrow reference speed ($= 0$)

$$V_A = -K_1 \dot{\theta}_x - K_2 \theta_x - K_3 v_y$$

$$4. \det(sI - A + BK)$$

$$= s^3 + \left[\frac{G_T}{M_y} (K_3 + G_v) + \frac{G_T}{M_x} K_1 \right] s^2 + \left(\frac{G_T}{M_x} K_2 - \frac{T_g}{M_x} \right) s$$

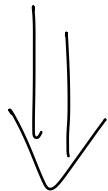
$$- \frac{G_T T_g}{M_x M_y} \cdot (K_3 + G_v)$$

↓ use triple root $-p_1$

$$\det(sI - A + BK)$$

$$= (s + p_1)^3$$

$$= s^3 + 3p_1 s^2 + 3p_1^2 s + p_1^3$$



$$K_1 = \frac{Mx^2}{G_T T_g} P_1^3 + \frac{3Mx}{G_T} P_1$$

$$K_2 = \frac{3Mx}{G_T} P_1^2 + \frac{T_g}{G_T}$$

$$K_3 = \frac{-MxM_y}{G_T T_g} P_1^3 - G_v$$