ES 3011 Lab 2

University: WPI
Department: RBE / ME

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1 Come up with the Motor Model

(Hint: simplify the torques on each motor separately. Generally, the torques on left and right motor should be different.)

Given equations:

$$\begin{aligned} T &= K_T \cdot i \ (N \cdot m) \\ i &= \frac{V - K_v \cdot \omega}{R} \ (A) \end{aligned} \tag{1}$$

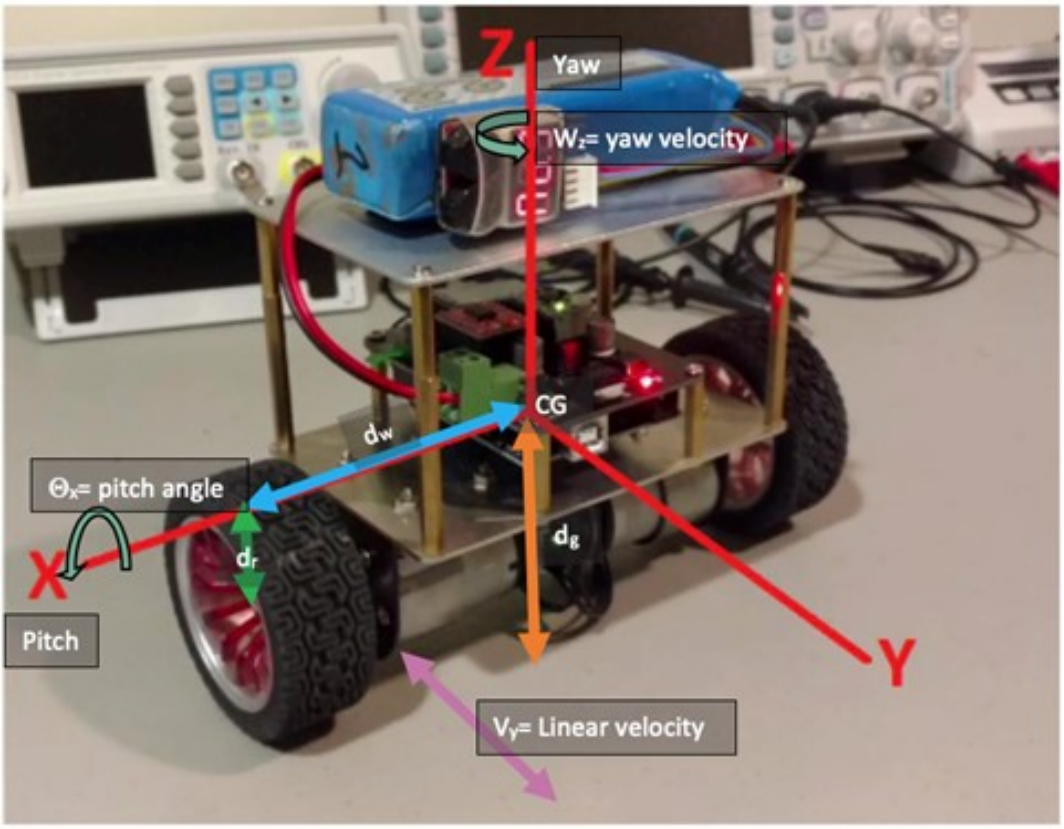
Please write down the equations of T_L and T_R, which are consist of the coefficient K_T and K_V, resistances $R_{R/L}$, voltages $V_{R/L}$, angular velocity $\omega_{R/L}$.

2 Parameter table

Please fill in the parameter table.

Symbol	Unit	Description
I_x		Pitch inertia
I_z		Yaw inertia
m		Robot mass
g		Gravitational constant
d_g		Ground to the center of gravity (CG)
d_w		Wheel to CG z-axis
d_r		Wheel radius
θ_x	rad	Pitch angle
ω_z		Yaw velocity
$\omega_{R/L}$		Angular velocity of the left and right wheel
v_y		Linear velocity

Tabel 1: Parameter table



Figuur 1: The self-balancing robot

3 Derive the mechanical model of the robot

This section is to build the simplified mathematical model for the robot.

3.1 Formula of torque using inertia

Please write down the equation for the torque correlated to the angular acceleration.

3.2 Formula of the torque due to the pitch angle relative to the center of gravity

$$\begin{aligned} T_x &= I_x \cdot \frac{d^2\theta_x}{dt^2} \\ T_x &= (\frac{d_g}{d_r})(T_R + T_L) + (m \cdot g \cdot d_g) \cdot \theta_x \end{aligned} \tag{2}$$

Please show how the above equations are derived, Free Body Diagrams required . (Hints, when θ is a small angle, $sin(\theta) = tan(\theta) = \theta$, $cos(\theta) = 1$)

3.3 Formula of the force applied to the robot due to the applied torque by the wheels

(Hint: friction force)

Please write down two equations of force F. (Hints: $F = f_1(m, v_y)$, $F = f_2(d_r, d_w, T_R, T_L)$)

3.4 Formula of the torque due to the yaw velocity relative to the center of gravity

(Hint: Please review section 3.2 and think about rotation about z axis.)

Please write down two equations of the torque T_z. (Hints: $T_z = f_3(I_z, \omega_z)$, $T_z = f_4(d_r, T_R, T_L)$)

4 Derive the motor and mechanical velocity relationship

4.1 Formula of the average yaw velocity

$$\omega_z = \frac{d_r \cdot (\omega_R - \omega_L)}{2 \cdot d_w} \tag{3}$$

Please show how the above equation is derived, conventional diagrams required.

4.2 Formula of the average linear velocity

Please write down the equation of linear velocity v_y in terms of d_r, ω_R, ω_L.

4.3 Derive the rotational velocities of the left and right wheel in terms of d_w, d_r, v_y and ω_z

(Hint: Solve for $\omega_{R/L}$ from section 4.1 and 4.2. And the solutions are equal to each other.)

5 Derive the Full system Dynamic Model

5.1 Rewrite the torque about the x-axis (section 3.2) to get expand the wheel torques and get rid of ω_L and ω_R

Please write down the equation of the torque about the x-axis following above requirement.

5.2 Rewrite the force applied to the robot (section 3.3) to get expand the wheel torques and get rid of ω_L and ω_R

Please write down the equation of the force applied to the robot following above requirement.

5.3 Rewrite the torque about the z-axis (section 3.4) to get expand the wheel torques and get rid of ω_L and ω_R

Please write down the equation of the torque about the z-axis following above requirement.