ES 3011 C-2019

Lab #3: Laplace Transforms

INTRODUCTION

In this lab, you and a partner will find the frequency-domain representations of examples and applied system models representing a mass-spring-damper, a RLC circuit, a car in motion, and a DC motor.

Please have the following outline for your report:

1. A few sentences of introduction of the topic of the lab.
2. Answers to each problem with concise explanations on your process in solving and outcome.
3. A paragraph concluding the report explaining the goals, what you learned, and any other conclusions.

LAPLACE AND INVERSE LAPLACE TRANSFORM PROBLEMS

Take the Laplace Transform by hand of the following equations: (5 points each)

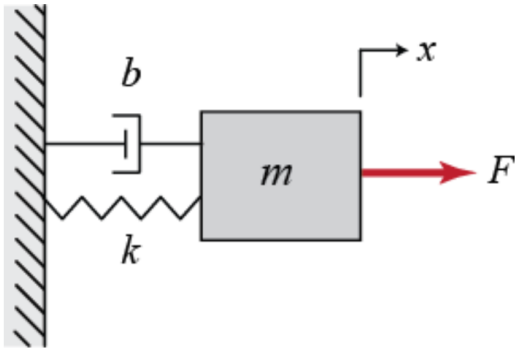
1. $f(t) = 5t + 12e^{4t}$
2. $f(t) = cos3t + sin3t$
3. 2y''+3y'-2y=t*exp(-2t), y(0)=0, y'(0) = -2
4. y''+16y=1+t, y(0)=-2,y'(0)=2
5. $y''' - y'' - 4y' + 4y = f(t),\ y(0) = y'(0) = 1,\ y''(0) = 0$ Hint: Convolution

Take the Inverse Laplace Transform by hand of the following equations: (5 points each)

6. $G(s) = \frac{7}{s^2-9}$
7. $Q(s) = \frac{s}{s^2+64}$
8. $P(s) = \frac{1}{s+42} - \frac{1}{(s+3)^4}$
9. $X(s) = \frac{s}{s^2+64}$
10. $F(s) = \frac{1}{(s^2+4)(s^2-4)}$ Hint: Use convolution

APPLICATIONS OF LAPLACE TRANSFORMS

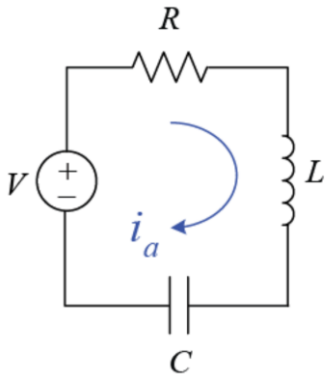
I) MASS-SPRING-DAMPER SYSTEM (10 POINTS)



$$F(t) - b\dot{x} - kx = m\ddot{x}$$

1. Take the Laplace transform of the system equation by hand. Assume zero initial conditions.

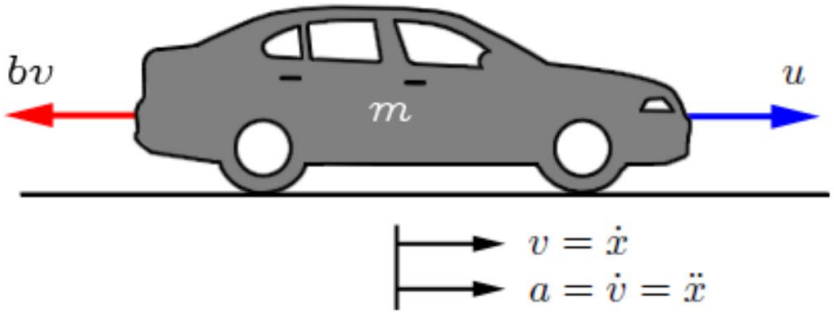
II) RLC CIRCUIT (10 POINTS)



$$V(t) - Ri - L\frac{di}{dt} - \frac{1}{C}\int idt = 0$$

1. Take the Laplace transform of the system equation by hand. Assume zero initial conditions.

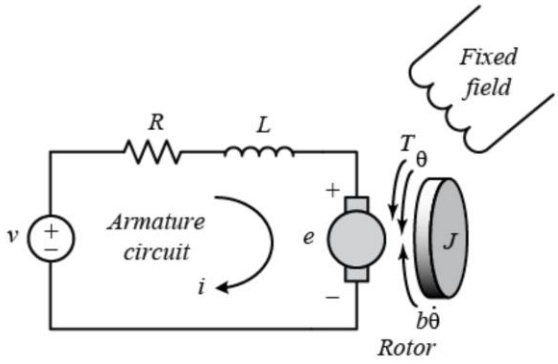
III) CRUISE-CONTROL CAR (10 POINTS)



$$m\dot{v} + bv = u$$

1. Take the Laplace transform of the system equation system by hand. Assume zero initial conditions.

IV) MOTOR POSITION (20 POINTS)



Newton's 2nd Law and Kirchhoff's voltage law gives us these equations:

$$J\ddot{\theta} + b\dot{\theta} = Ki$$

$$L\frac{di}{dt} + Ri = V - K\dot{\theta}$$

1. Take the Laplace transform of the system equations by hand. Assume zero initial conditions.