Calibration of Linearly Polarized Polarimetric SAR Data Subject to Faraday Rotation

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***Abstract*—A model for linearly polarized fully polarimetric backscatter measurements is used, incorporating the effects of system noise, channel amplitude, phase imbalance, crosstalk, and Faraday rotation. A step-by-step procedure is outlined for correction (or calibration) of fully polarimetric data subject to Faraday rotation, to recover the true scattering matrix. The procedure identifies steps for crosstalk removal and correction of channel imbalances that are robust in the presence of Faraday rotation. The final steps in the procedure involve a novel strategy for estimation and correction of Faraday rotation. Three approaches to estimate the (one-way) Faraday rotation angle $\boldsymbol{\Omega}$ directly from linear (quad-) polarized synthetic aperture radar (SAR) backscatter data obtained by a spaceborne SAR system are described. Each approach can initially be applied to the signature of *any* scatterer within the scene. Sensitivity analyses are presented that show that at least one of the measures can be used to estimate $\boldsymbol{\Omega}$ to within $\pm 3^\circ$ to 5°, with reasonable levels of residual crosstalk, noise floor, channel amplitude, and phase imbalance. Ambiguities may be present in the estimates of $\boldsymbol{\Omega}$ of $\pm n\pi/2$—the impact of this is discussed, and several approaches are suggested to deal with this possibility. The approach described in this paper is relevant for future L-band spaceborne SARs and removes one key obstacle to the deployment of even longer wavelength SARs (e.g., an ultrahigh frequency or P-band SAR) in earth orbit.**

***Index Terms*—Calibration, Faraday rotation, polarimetry, synthetic aperture radar (SAR).**

I. Introduction

THE EFFECT OF Faraday rotation on measurements of the polarization scattering matrix using monostatic radars was first addressed by Thompson and Moran [1], Evans and Hagfors [2], Bickel and Bates [3], who formulated the relation between measured and actual scattering matrices and showed that Faraday rotation is a nonreciprocal effect. The authors introduced an approach to calibrate polarimetric measurements subject to Faraday rotation using a calibration target such as a sphere or trihedral corner reflector, which has no cross-polarized return. They also described an approach to estimate and remove the effect of Faraday rotation from the measured scattering matrix of an arbitrary scatterer. This was done by referencing the measured scattering matrix to circular polarization: the result is an estimate for the Faraday rotation angle modulo $\pi/2$.

Gail [4] formulated a system model that combined the effects of Faraday rotation and system errors such as antenna crosstalk and transmit/receive gain and phase imbalances. Gail also described a numerical approach to calibration of measurements subject to such errors, but in [5] calls for a more robust approach to calibration of polarimetric radar measurements subject to Faraday rotation. The latter paper also addressed the effect of Faraday rotation on synthetic aperture radar (SAR) backscatter measurements at scale lengths smaller than the synthetic aperture length, concluding that, for most cases of interest, this could be ignored. This result was confirmed in [6] and [7]. the effects of the ionosphere on linearly polarized radar signals were also examined in [8]–[10]

Since 1988, the National Aeronautics and Space Administration Jet Propulsion Laboratory (NASA/JPL) have been operating a P-band SAR from a DC-8 airborne platform, as part of the Airborne SAR set of instruments. The calibrated, polarimetric measurements made by the P-band SAR have been used by earth scientists all over the world in a variety of disciplines. Amongst the most important results stemming from this activity are the noted strong correlation between linearly polarized backscatter measurements made at P-band and above-ground, dry woody biomass for a variety of forest types [11]–[13]. Others have pointed to the usefulness of linearly polarized P-band backscatter measurements in monitoring forest inundation, mapping subtropical forests and wetlands, measuring ice sheet thickness and bathymetry of coastal areas [14]. It is clear from these results that a P-band spaceborne SAR could provide valuable data for several earth science disciplines. Faraday rotation is a significant error source for longer wavelength (i.e., $\lambda > 32$ cm) spaceborne SARs for geophysical applications, and may even affect observations made by L-band SARs, such as the Japanese Space Agency's Japanese Earth Resources Satellite 1 or the Phased Array L-Band Synthetic Aperture Radar (PALSAR) system. Without a viable solution to this problem, linearly polarized backscatter measurements made by longer wavelength SARs will be difficult to calibrate, which will mean that they will not be useful in most geophysical algorithms that use radar measurements.

One approach, the adoption of a circularly polarized measurement system, does offer some benefits in that backscatter (i.e., RR, LL, LR, and RL σ°) measurements made in a circular basis are not altered by Faraday rotation. The relative phases of terms involving cross-polarized measurements, i.e., LR and RL circular polarizations, are affected, however. The overwhelming majority of algorithms available in the literature (e.g., [11]–[13]) relating SAR backscatter measurements to geophysical parameters, or to scattering models [22], [23], use

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linear polarizations, not circular. To successfully transform circular polarized measurements to a linear basis, one must first correct for the effects of Faraday rotation on the cross-pol terms. In other words, if measurements are made in a circular basis they must still be corrected for Faraday rotation for most applications. Thus, to exploit all the information contained in a coherent, fully polarimetric data Faraday rotations must be compensated *regardless of the choice of basis*. In addition, the deployment of SAR antennas with circular polarization offers some significant technical challenges and has, so far, not been adopted in future longer wavelength SAR designs such as TerraSAR-L and PALSAR. To obtain fully calibrated linearly polarized L-band or P-band SAR backscatter data from earth orbit, then, the potential problem of Faraday rotation must be resolved, which is the main thrust of this paper.

In a companion paper [15], the effects of Faraday rotation on linearly polarized SAR backscatter measurements were discussed. It was shown that acceptable levels of distortion of backscatter measurements are not exceeded provided $\boldsymbol{\Omega} \leq 5^\circ$ (or $\leq 3^\circ$ for some applications). In [15] the largest value of the one-way Faraday rotation angle $\boldsymbol{\Omega}$ at solar maximum was estimated to be 40° at L-band and 321° at P-band, clearly greater than the acceptable value of 5°. The main objective of this paper, then, is to develop a calibration strategy that yields acceptable backscatter signature distortion in the presence of Faraday rotation- *and* system-dependent effects. To achieve this, we seek to find a backscatter data-derived measure or measures for Faraday rotation that are accurate to within ±3° or 5° and which can be used in a full calibration procedure to correct fully polarimetric long-wavelength SAR backscatter signatures for Faraday rotation.

This paper begins with a discussion of the effects of Faraday rotation on linearly polarized scattering matrix measurements. A model is developed in Section II for such measurements, incorporating the effects of Faraday rotation, channel amplitude and phase imbalance, and antenna crosstalk. A calibration procedure that allows for correction of all of these effects is described. A careful approach is needed for correction of crosstalk and channel imbalance in the presence of Faraday rotation—such an approach is described in this paper. In addition, a novel strategy for estimation and correction for a Faraday rotation angle $\boldsymbol{\Omega}$ is addressed in Section III. The sensitivity of two measures of $\boldsymbol{\Omega}$ to varying levels of residual system noise, channel imbalance, and crosstalk is assessed in Section IV. Model results indicate that for reasonable levels of residual calibration errors in system noise, channel imbalance, and crosstalk, $\boldsymbol{\Omega}$ can be recovered to within the required ±3° or 5° using one of these measures. Finally, in Section V the results of the paper are summarized and their implications for future spaceborne SAR missions are discussed.

II. Polarimetric Calibration Procedure

In this section, a model is used for uncalibrated polarimetric SAR data that takes into account Faraday rotation. Then a procedure is outlined that will allow full calibration of the data.

For a SAR system measuring linear horizontal (H) and vertical (V) polarizations in the antenna coordinate system, the measured scattering matrix, $\mathbf{M}$, can be written, (after [4], [6], and [15]), as

$$\begin{pmatrix} M_{\rm hh} & M_{\rm vh} \\ M_{\rm hv} & M_{\rm vv} \end{pmatrix} = \mathrm{A}(\mathrm{r},\theta)e^{\mathrm{j}\phi} \begin{pmatrix} 1 & \delta_2 \\ \delta_1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & f_1 \end{pmatrix} \cdot \begin{pmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{pmatrix} \begin{pmatrix} S_{\rm hh} & S_{\rm vh} \\ S_{\rm hv} & S_{\rm vv} \end{pmatrix} \cdot \begin{pmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & f_2 \end{pmatrix} \cdot \begin{pmatrix} 1 & \delta_3 \\ \delta_4 & 1 \end{pmatrix} + \begin{pmatrix} N_{\rm hh} & N_{\rm vh} \\ N_{\rm hv} & N_{\rm vv} \end{pmatrix}$$

or

$$\mathbf{M} = \mathrm{A}e^{\boldsymbol{\phi}}\mathbf{R}^{\mathbf{T}}\mathbf{R_F}\mathbf{S}\mathbf{R_F}\mathbf{T} + \mathbf{N} \tag{1}$$

where $\mathbf{S}$ is the scattering matrix, $\mathbf{R_F}$ represents the one-way Faraday rotation matrix, $\mathbf{R}$ is the receive distortion matrix (of the radar system), and $\mathbf{T}$ is the transmit distortion matrix (of the radar system). Both $\mathbf{R}$ and $\mathbf{T}$ can include crosstalk (the δ terms) and channel amplitude and phase imbalance terms (f_1 and f_2). The (real) factor A represents the overall gain of the radar system, which is a function of range, r, and elevation angle, θ. The complex factor $e^{j\phi}$ represents the round-trip phase delay and system-dependent phase effects on the signal. The matrix $\mathbf{N}$ represents additive noise terms present in each measurement due to earth radiation, thermal fluctuations in the receiver, and digitization noise.

A procedure for calibration of polarimetric SAR data that takes into account all the terms in (1) is outlined in Fig. 1. Radiometric correction (Step 1) is straightforward. Numerous calibration techniques have been described in the literature (e.g., [16], [24], and [25]) that can accomplish Steps 2–4 in Fig. 1, i.e., successfully estimate and correct for the system-dependent terms $\mathbf{R}$ and $\mathbf{T}$. However, blind application of those techniques to polarimetric SAR data in the presence of significant Faraday rotation may lead to *very large* calibration errors. Techniques that estimate and remove crosstalk from polarimetric data based on the signatures of scatterers with reflection symmetry rely on the observation that any significant correlation between like- and cross-polarized measurements of such targets is due to system crosstalk. As shown in [15], in the presence of Faraday rotation significant correlations between like- and cross-polarized measurements can be caused by small Faraday rotation angles. These correlations are so large they will tend to dominate those due to system crosstalk. Similarly, techniques that aim to correct for channel imbalance through symmetrization of the HV and VH measurements rely on the reciprocity of those measurements. With reciprocal measurements any deviation in the ratio of HV and VH backscatter from unity is due to system-dependent gain and phase effects. The assumption of backscatter measurement reciprocity does *not* hold in the presence of Faraday rotation, as pointed out in [3] and [15].

Based on our experience with spaceborne SAR polarimetric data acquired during the SIR-C missions [17] it is reasonable to assume that a SAR system can be built with negligible levels

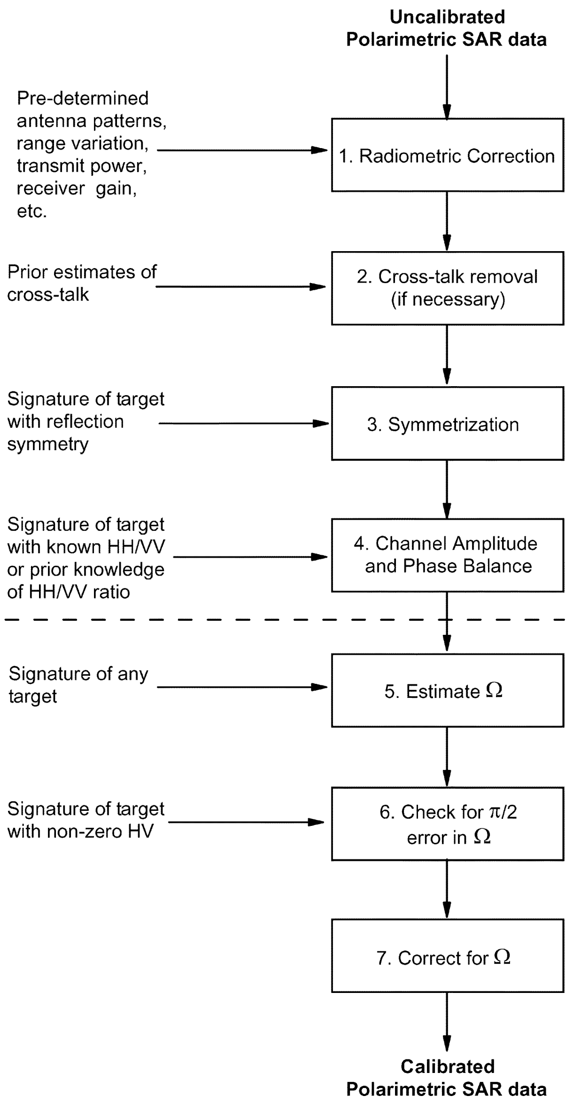

Fig. 1. Calibration procedure for polarimetric SAR data taking into account possible Faraday rotation. An additional absolute radiometric calibration step (not shown) may be necessary in practice.

of crosstalk (< -30 dB), or that the crosstalk amplitude and phase are stable and can be measured adequately, then removed (Step 2 in Fig. 1). This is the approach recommended by the author. Then, after radiometric correction (Step 1 in Fig. 1) and crosstalk removal (Step 2) a model for the polarimetric measurements is given by

$$\begin{pmatrix} M_{\rm hh} & M_{\rm vh} \\ M_{\rm hv} & M_{\rm vv} \end{pmatrix} \cong \begin{pmatrix} 1 & 0 \\ 0 & f_1 \end{pmatrix} \begin{pmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{pmatrix} \cdot \begin{pmatrix} S_{\rm hh} & S_{\rm vh} \\ S_{\rm hv} & S_{\rm vv} \end{pmatrix} \begin{pmatrix} \cos\Omega & \sin\Omega \\ -\sin\Omega & \cos\Omega \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & f_2 \end{pmatrix} + \begin{pmatrix} N_{\rm hh} & N_{\rm vh} \\ N_{\rm hv} & N_{\rm vv} \end{pmatrix} \tag{2}$$

where the complex term f_1 represents the receive H-V channel amplitude and phase imbalance and f_2 represents the transmit H-V channel amplitude and phase imbalance. SAR systems can be built with relatively good gain stability, but channel amplitude and phase imbalances may vary over time, or with the mode of operation. Thus, estimation and correction for f_1 and f_2 must be addressed.

To calibrate the measurements represented by (2) it is necessary to first determine the three unknowns: f_1, f_2, and $\mathbf{\Omega}$. Expanding (2), dropping the additive noise terms in $\mathbf{N}$ for the moment, and invoking backscatter reciprocity ($S_{\rm hv} = S_{\rm vh}$), we obtain

$$M_{\rm hh} = S_{\rm hh}\cos^2\Omega - S_{\rm vv}\sin^2\Omega \tag{3a}$$
$$M_{\rm vh} = f_2\left[S_{\rm hv} + (S_{\rm hh}+S_{\rm vv})\sin\Omega\cos\Omega\right] \tag{3b}$$
$$M_{\rm hv} = f_1\left[S_{\rm hv} - (S_{\rm hh}+S_{\rm vv})\sin\Omega\cos\Omega\right] \tag{3c}$$
$$M_{\rm vv} = f_1 f_2\left[S_{\rm vv}\cos^2\Omega - S_{\rm hh}\sin^2\Omega\right]. \tag{3d}$$

If the HV and VH backscatter measurements were reciprocal, which is the case when $\Omega = 0$, estimation of f_2/f_1 at this point would be trivial, involving a simple ratio of (3b) and (3c). Since the HV and VH backscatter measurements are *not* reciprocal, we must adopt an alternative approach. Forming the cross-product between the two cross-pol measurements in (3b) and (3c), and assuming reflection symmetry, we obtain,

$$M_{\rm hv}M_{\rm vh}^* = f_1 f_2^*\left[S_{\rm hv}S_{\rm vh}^* - (S_{\rm hh}S_{\rm hh}^* + S_{\rm vv}S_{\rm vv}^* + 2{\rm Re}\{S_{\rm hh}S_{\rm vv}^*\})\sin^2\Omega\cos^2\Omega\right]. \tag{4}$$

The term in square brackets on the right-hand side of (4) is a real number, which can take positive or negative values. Thus

$$\arg\{M_{\rm hv}M_{\rm vh}^*\} = \arg\{f_1 f_2^*\} \pm n\pi \tag{5}$$

where $n = 0$ if the term in square brackets is positive, and $n = 1$ if it is negative. Put another way, it is possible to recover the relative phase between f_1 and f_2 to within π, depending on the sign of the term in square brackets. A note of caution: not all targets within a scene will exhibit reflection symmetry, in particular those that are subject to azimuth slopes, but, as pointed out in [15], this latter effect an easily be separated from Faraday rotation in fully polarimetric data.

It is straightforward to show that, when reflection symmetry [18] holds, and the like- and cross-pol backscatter terms are uncorrelated

$$\frac{\langle M_{\rm hv}M_{\rm hv}^*\rangle}{\langle M_{\rm vh}M_{\rm vh}^*\rangle} = \frac{|f_1|^2}{|f_2|^2}. \tag{6}$$

Thus, the relative amplitude between f_1 and f_2 can be recovered using (6). Combining the two, the estimated value for the ratio is

$$\left(\frac{\hat{f}_1}{f_2}\right) = \pm\frac{f_1}{f_2}. \tag{7}$$

Next, if we form the symmetrized cross-pol measurement

$$\begin{aligned} M'_{\rm hv} &= 0.5\left(M_{\rm hv} + \left(\frac{\hat{f}_1}{f_2}\right)M_{\rm vh}\right) \equiv f_1 S_{\rm hv} \text{ if } \left(\frac{\hat{f}_1}{f_2}\right) = \frac{f_1}{f_2} \\ &\equiv -f_1(S_{\rm hh}+S_{\rm vv})\sin\Omega\,\cos\Omega \text{ if } \left(\frac{\hat{f}_1}{f_2}\right) = -\frac{f_1}{f_2}. \end{aligned} \tag{8}$$

If the sign of the estimated ratio using (7) is in error by π, then

$$\langle M_{\rm hh} M'^{*}_{\rm hv} \rangle \neq 0$$

unless $\mathbf{\Omega} = n\pi/2$, n integer, in which case it can also be seen that

$$\langle M'_{\rm hv} M'^{*}_{\rm hv} \rangle = 0.$$

Thus, provided the cross-pol backscatter is not zero, it should be possible to determine when the phase of the estimated ratio has an error of π. Our experience with SIR-C data also revealed that the argument of (f_1/f_2) varied very little, so the scenario outlined above should easily be detected as a sudden change in this value from previously determined values or by comparing with values obtained during night-time passes, when $\mathbf{\Omega}$ should be small. Thus, the procedure for determining (f_1/f_2) relies on ensemble averaging of the cross-pol measurements over a homogeneous area that exhibits reflection symmetry, or by using previously determined values for $|f_1|/|f_2|$. The argument of (f_1/f_2) can be determined for any target to within $\pm n\pi$.

Once (f_1/f_2) is completely determined, the VH cross-pol measurement can be symmetrized as in (8), and a factor (f_1/f_2) applied to (3d) to give

$$M_{\rm hh} = S_{\rm hh}\cos^2\Omega - S_{\rm vv}\sin^2\Omega \tag{9a}$$
$$M'_{\rm vh} = f_1\left[S_{\rm hv} + (S_{\rm hh}+S_{\rm vv})\sin\Omega\cos\Omega\right] \tag{9b}$$
$$M_{\rm hv} = f_1\left[S_{\rm hv} - (S_{\rm hh}+S_{\rm vv})\sin\Omega\cos\Omega\right] \tag{9c}$$
$$M'_{\rm vv} = (f_1)^2\left[S_{\rm vv}\cos^2\Omega - S_{\rm hh}\sin^2\Omega\right]. \tag{9d}$$

In a similar fashion to [19], the transmit channel imbalance f_2 has been removed to leave only f_1, the receive channel imbalance, in (9). This is easier to calibrate using prior measurements, reference signals fed into the system receive chain, or known calibration targets. A trihedral corner reflector, for example, (which has $S_{\rm hh} = S_{\rm vv}$) could be used to determine f_1 by dividing (9d) by (9a). Equation (9) consists of four complex equations in five complex unknowns—it is not possible to solve for any one of these without further knowledge of the scattering matrix values (as in the trihedral case) or f_1. If f_1 *is* known, then (9b)–(9d) can be corrected to leave

$$M_{\rm hh} = S_{\rm hh}\cos^2\Omega - S_{\rm vv}\sin^2\Omega \tag{10a}$$
$$M_{\rm vh} = S_{\rm hv} + (S_{\rm hh}+S_{\rm vv})\sin\Omega\cos\Omega \tag{10b}$$
$$M_{\rm hv} = S_{\rm hv} - (S_{\rm hh}+S_{\rm vv})\sin\Omega\cos\Omega \tag{10c}$$
$$M_{\rm vv} = S_{\rm vv}\cos^2\Omega - S_{\rm hh}\sin^2\Omega \tag{10d}$$

where the dashed notation has been dropped. Note that for cross-pol measurements, the presence of nonzero Faraday rotation means that they will not necessarily be reciprocal, i.e., $M_{\rm hv} \neq M_{\rm vh}$.

The above discussion shows how Steps 1–4 in the calibration procedure outlined in Fig. 1 may be accomplished. In the next section, Steps 5–7 are described.

III. Estimation and Correction of Faraday Rotation

This section addresses the problem of estimating the Faraday rotation $\mathbf{\Omega}$ from fully polarimetric data and then correcting measured data. We assume that any amplitude and phase calibration errors have been corrected as described in Section II. We will also assume that the effects of azimuth slopes on the backscatter measurements are negligible or that the data used in each measure has screened using some of the characteristics described in [15] so that the effects of azimuth slopes can be ignored.

For the case where Faraday rotation is the only remaining error source, and fully polarimetric (linear polarized) measurements are available, it is straightforward to start from (10) and estimate the Faraday rotation angle $\mathbf{\Omega}$ via

$$\Omega = \frac{1}{2}\tan^{-1}\left[\frac{(M_{\rm vh} - M_{\rm hv})}{(M_{\rm hh} + M_{\rm vv})}\right] \tag{11}$$

for any type of scatterer. Speckle and additive noise are usually present in backscatter signatures, however, which causes measures extracted straight from scattering matrix elements to have large errors. A more robust approach to estimate $\mathbf{\Omega}$ may use averaged second-order statistics, by first calculating

$$Z_{\rm hv} = 0.5(M_{\rm vh} - M_{\rm hv}) \tag{12}$$

and then estimating $\mathbf{\Omega}$ from

$$\mathbf{\Omega} = \pm\frac{1}{2}\tan^{-1} \cdot \sqrt{\frac{4\langle Z_{\rm hv}Z^*_{\rm hv}\rangle}{(\langle M_{\rm hh}M^*_{\rm hh}\rangle + \langle M_{\rm vv}M^*_{\rm vv}\rangle + 2{\rm Re}\{\langle M_{\rm hh}M^*_{\rm vv}\rangle\})}}. \tag{13}$$

Note that the HH and VV backscatter measurements in (13) may be corrected for additive noise if the noise powers in each measurement channel are sufficiently well known. Correction of additive noise contributions in ensemble averages of cross-products in polarimetric SAR is addressed in [20] and [21].

Another approach is due to Bickel and Bates [3], who applied a simple transformation of $\mathbf{M}$ to a circular basis, via the following operation:

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 1 & {\rm j} \\ {\rm j} & 1 \end{bmatrix}\begin{bmatrix} M_{\rm hh} & M_{\rm vh} \\ M_{\rm hv} & M_{\rm vv} \end{bmatrix}\begin{bmatrix} 1 & {\rm j} \\ {\rm j} & 1 \end{bmatrix}. \tag{14}$$

It is then straightforward to show that for any reciprocal scatterer with $S_{\rm hv} = S_{\rm vh}$

$$\mathbf{\Omega} = \frac{1}{4}\arg(Z_{12}Z^*_{21}). \tag{15}$$

Thus, estimation of the Faraday rotation angle becomes a (relative) phase estimation problem, which is a fairly well understood problem in radar polarimetry and interferometry and is susceptible to improved estimation by averaging the quantity $Z_{12}Z^*_{21}$.

To arrive at (11), (13), and (15) for $\mathbf{\Omega}$, the only assumptions necessary are that backscatter reciprocity holds, and that Faraday rotation is the only (remaining) source of calibration error. (The effects of residual levels of system noise, channel amplitude and phase imbalance and crosstalk are examined in Section IV.) These equations are also valid even when the cross-pol backscatter contribution goes to zero (e.g., for surface scattering only). Thus, $\mathbf{\Omega}$ can be estimated from any monostatic scattering signature or *for all scatterers within a strip map SAR image obtained over land or ocean*.

To complete the calibration and correct for a Faraday rotation of $\mathbf{\Omega}$, the following matrix multiplication should suffice:

$$\tilde{\mathbf{S}} = \mathbf{R_F^t M R_F^t} \tag{16}$$

where

$$\mathbf{R_F^t} \equiv \mathbf{R_F^{-1}}.$$

Expanding the matrix terms out, (16) can be written

$$\begin{bmatrix} \tilde{S}_{\rm hh} & \tilde{S}_{\rm vh} \\ \tilde{S}_{\rm hv} & \tilde{S}_{\rm vv} \end{bmatrix} = \begin{bmatrix} \cos\Omega & -\sin\Omega \\ \sin\Omega & \cos\Omega \end{bmatrix} \cdot \begin{bmatrix} M_{\rm hh} & M_{\rm vh} \\ M_{\rm hv} & M_{\rm vv} \end{bmatrix} \begin{bmatrix} \cos\Omega & -\sin\Omega \\ \sin\Omega & \cos\Omega \end{bmatrix}. \tag{17}$$

Since values of $\tan^{-1}$ are between $\pm\pi/2$, values of $\mathbf{\Omega}$ estimated using (11), (13), or (15) will lie between $\pm\pi/4$. Put another way, this means that $\mathbf{\Omega}$ can only be estimated modulo $\pi/2$, which is not sufficient. It is straightforward to show that, with an error in $\mathbf{\Omega}$ of $\pi/2$, the estimate for the corrected scattering matrix using (17) will give

$$\begin{pmatrix} \tilde{S}_{\rm hh} & \tilde{S}_{\rm vh} \\ \tilde{S}_{\rm hv} & \tilde{S}_{\rm vv} \end{pmatrix} = \begin{pmatrix} -S_{\rm vv} & S_{\rm hv} \\ S_{\rm vh} & -S_{\rm hh} \end{pmatrix} \tag{18}$$

which is clearly in error, since the HH and VV measurements are reversed. One approach to determine the presence of such an error is to deploy a calibration target (e.g., a transponder) within the scene, with polarization signature

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \text{ or } \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}. \tag{19}$$

This should readily reveal the presence of a $\pi/2$ error in $\mathbf{\Omega}$, since the reversal of the HH and VV measurements is easily detected. Another approach to detect a modulo $\pi/2$ error in $\mathbf{\Omega}$ is to examine the polarization signature of a flat, (slightly) rough surface, for which the VV backscatter term is typically greater than or equal to the HH. Another alternative is to constrain $\mathbf{\Omega}$ itself, by observing when the ionosphere is relatively quiet, at night during solar minimum for example, and $\mathbf{\Omega}$ may be less than 45° at both L- and P-band. Yet another approach is to correlate the dataset with an earlier one, obtained by the same radar under identical viewing conditions, but with no Faraday rotation. In this case, if there is a $\pi/2$ error in $\mathbf{\Omega}$, the cross correlation between nominally identical like-pol measurements will tend to be similar to the correlation between HH and VV measurements in the earlier dataset. Of course, temporal decorrelation would have to be factored in to the problem as well.

If an error of $\pi/2$ in $\mathbf{\Omega}$ is detected, the procedure to follow should be to add (or subtract) $\pi/2$ from $\mathbf{\Omega}$ (which means that the largest error in $\mathbf{\Omega}$ is now $\pm n\pi$), then apply the correction in (17) again. It is straightforward to show that with an error in $\mathbf{\Omega}$ of π, the estimate for the new corrected scattering matrix will be

$$\begin{bmatrix} \tilde{S}_{\rm hh} & \tilde{S}_{\rm vh} \\ \tilde{S}_{\rm hv} & \tilde{S}_{\rm vv} \end{bmatrix} = e^{\pm\tilde{\pi}} \begin{bmatrix} S_{\rm hh} & S_{\rm vh} \\ S_{\rm hv} & S_{\rm vv} \end{bmatrix} \tag{20}$$

i.e., all scattering matrix elements will be measured correctly, except for an overall phase error of $\pm\pi$. This is largely irrelevant for polarimetric SAR data analysis, in which the cross-products of (20) are of most interest. A phase error of $\pm\pi$ may be significant for analysis of repeat-pass interferometry data, however, in which relative phases between passes are of most interest. In this case, the problem can again be resolved by constraining measurements to night-time periods only, when it is expected that $\mathbf{\Omega} < \pi$. Alternatively, a digital elevation model of the area under observation or the use of phase-unwrapping techniques may help resolve this phase ambiguity.

TABLE I
P-BAND BACKSCATTER MEASUREMENTS FROM NASA/JPL AIRSAR DATA FOR A VARIETY OF LAND COVER TYPES

P-Band	HH σ^o (dB)	HV σ^o (dB)	VV σ^o (dB)	HH-VV Phase (deg.)	HH-VV Correlation
Bare Soil	-25.1	-34.6	-19.7	-8.8	0.75
Pasture	-20.3	-31.8	-18.3	-12.5	0.53
Upland Forest	-11.5	-17.9	-11.9	51.1	0.14
Swamp Forest	-13.8	-22.2	-13.2	149.5	0.10
Plantation	-9.2	-18	-10.5	137.3	0.40
Conifers	-5.5	-14.5	-9.8	78.5	0.29

TABLE II
L-BAND BACKSCATTER MEASUREMENTS FROM NASA/JPL AIRSAR DATA FOR A VARIETY OF LAND COVER TYPES

L-Band	HH σ^o (dB)	HV σ^o (dB)	VV σ^o (dB)	HH-VV Phase (deg.)	HH-VV Correlation
Bare Soil	-16.5	-26.9	-14.7	-23.7	0.75
Pasture	-13.3	-25	-11.8	-18.6	0.75
Upland Forest	-9.2	-14.3	-9.4	7.9	0.25
Swamp Forest	-6.9	-14.5	-7.3	165.4	0.06
Plantation	-8	-15.7	-9.7	52.1	0.12
Conifers	-6.2	-13.1	-8.9	36.9	0.21

IV. SENSITIVITY ANALYSIS

In this section, the effects of residual system noise, channel amplitude, phase imbalance, and crosstalk on estimates of $\mathbf{\Omega}$ obtained using (13) and (15) are addressed.

To model backscatter measurements affected by Faraday rotation, the full scattering matrix or covariance matrix should be known. The covariance matrix measurements in Tables I and II were extracted from L-band and P-band polarimetric SAR data collected by the NASA/JPL AIRSAR system over a tropical rain forest in Belize during 1991 and over a site in Raco, MI, in June 1991. For these data, Faraday rotation is definitely *not* a problem. They are presented here as typical of scattering from natural terrain with reflection symmetry (like- and cross-pol correlations are negligible). The L-band data in Table II were used to assess the effects of Faraday rotation on backscatter measurements in [15]. The behavior of the scattering from bare soil is representative of scattering observed from similar fields around the world. The scattering behavior for pasture is typical of grassland areas; the upland forest signature is representative of broadleaf forests worldwide; the swamp forest is an example of a forest with significant double-bounce as well as canopy scatter, because of flooding beneath the canopy; and the plantation and conifer areas are different examples of forests that exhibit a high degree of double-bounce as well as canopy scatter because of a relatively sparse canopy, which is more transparent to longer wavelength radar waves.

TABLE III
MAXIMUM ERROR IN ESTIMATES FOR $\mathbf{\Omega}_1$ [SEE (13)] AND $\mathbf{\Omega}_2$ [SEE (15)] AS A FUNCTION OF (a) NOISE-EQUIVALENT SIGMA-ZERO, (b) CHANNEL AMPLITUDE IMBALANCE $|f_1|^2$, (c) CHANNEL PHASE IMBALANCE $\arg(f_1)$, AND (d) CROSSTALK δ. DATA FROM TABLE I (P-BAND) WERE USED TO DERIVE THESE RESULTS

a) NE σ^o	-100 dB	-50 dB	-30 dB	-24dB	-18 dB
$\Delta\Omega_1$ (deg)	0	1.2	11	18	23.6
$\Delta\Omega_2$ (deg)	0	0	1.3	5.4	31.1

b) $\lvert f_1\rvert^2$	0.0 dB	0.1 dB	0.2 dB	0.3 dB	0.4 dB	0.5 dB	1.0 dB
$\Delta\Omega_1$ (deg)	0	0.4	0.9	1.3	1.8	2.2	4.4
$\Delta\Omega_2$ (deg)	0	0.1	0.3	0.4	0.6	0.7	1.4

c) Arg (f_1)	0 deg	2 deg	5 deg	10 deg	20 deg
$\Delta\Omega_1$ (deg)	0	1.3	3.4	6.6	12.4
$\Delta\Omega_2$ (deg)	0	0.4	1.0	2.1	5.1

d) $\lvert\delta\rvert^2$	-50 dB	-30 dB	-25 dB	-20dB	-15 dB
$\Delta\Omega_1$ (deg)	0	0.1	0.2	0.7	2.1
$\Delta\Omega_2$ (deg)	0.3	2.6	4.7	8.2	15.4

In what follows, we will use the P-band data given in Table I, unless otherwise stated. To assess the sensitivity of the two most promising methods for estimating $\mathbf{\Omega}$, i.e., those given in (13) and (15), we use (1), setting $A = 1$ and $\phi = 0$. In the first analysis, we set $f_1 = f_2$ and $\delta_1 = \delta_2 = \delta_3 = \delta_4 = 0$, then progressively increase the level of the additive noise terms $\langle N_{pq} N_{pq}^* \rangle$ in each measurement channel from a noise-equivalent sigma-naught of -100 to -18 dB. (The noise levels in each channel are assumed equal.) Then, we estimate $\mathbf{\Omega}$ using (13) and (15). The error in the estimate using (13) is denoted $\Delta\mathbf{\Omega}_1$, and the error in the estimate using (15) is denoted $\Delta\mathbf{\Omega}_2$. The results of this sensitivity analysis, assessed across the range of backscatter signatures in Table I, for values of $\mathbf{\Omega}$ between 0° and 90°, are presented in Table III(a). In a similar fashion, the effects of a residual channel amplitude imbalance between 0.0 and 1.0 dB are assessed in Table III(b), channel phase imbalance between 0° and 20° in Table III(c), and crosstalk levels from -50 to -15 dB in Table III(d). For simplicity, it is assumed that $\delta = \delta_1 = \delta_2 = \delta_3 = \delta_4$ throughout.

The results summarized in Table III indicate that the estimate for $\mathbf{\Omega}$ given in (13) is more sensitive than that given in (15) to residual levels of noise, channel amplitude, and phase imbalance. This situation is reversed for residual levels of crosstalk. Results obtained using L-band data from Table II are generally similar in behavior, though the errors in $\mathbf{\Omega}$ are usually less for both measures for a given level of noise, channel imbalance, or crosstalk.

Finally, in Table IV, the estimation errors for $\mathbf{\Omega}$ are summarized, allowing for reasonable levels of residual noise, channel imbalance, and crosstalk combined at both L-band and P-band. At both wavelengths, it is clear that the expression given in (15) at least allows $\mathbf{\Omega}$ to be estimated to within the required margin of error ($\pm 5°$ or $\pm 3°$).

TABLE IV
MAXIMUM ERROR IN ESTIMATES FOR DIFFERENT VALUES OF CROSSTALK FOR $\mathbf{\Omega}_1$ [SEE (13)] AND $\mathbf{\Omega}_2$ [SEE (15)] FOR (a) P-BAND, (b) L-BAND, WITH CHANNEL AMPLITUDE IMBALANCE OF 0.5 dB AND CHANNEL PHASE IMBALANCE OF 10° IN BOTH CASES. NOISE-EQUIVALENT SIGMA-ZERO WAS SET AT -30 dB FOR P-BAND DATA AND -24 dB FOR L-BAND DATA IN THE MODEL

a) P-Band	$\lvert\delta\rvert^2$= -30 dB	$\lvert\delta\rvert^2$= -25 dB
$\Delta\Omega_1$ (deg)	10.5	10.5
$\Delta\Omega_2$ (deg)	3.2	5.1

b) L-Band	$\lvert\delta\rvert^2$= -30 dB	$\lvert\delta\rvert^2$= -25 dB
$\Delta\Omega_1$ (deg)	10.6	10.5
$\Delta\Omega_2$ (deg)	2.9	5.2

V. SUMMARY AND DISCUSSION

A careful step-by-step procedure for calibration of polarimetric data subject to Faraday rotation has been outlined. For systems with negligible crosstalk, and stability in amplitude and phase, estimation and correction for Faraday rotation can be applied to the measured polarization signature of any scatterer. In practice, to minimize estimation errors in $\mathbf{\Omega}$, averaged backscatter measurements over large, homogeneous areas should be used. For nonstable systems, targets that exhibit reflection symmetry, combined with the signature of a known calibration target, such as a trihedral corner reflector, provide sufficient information to complete the calibration, except for the estimation and removal of the Faraday rotation. Targets subject to azimuth slopes will not exhibit reflection symmetry, but, as pointed out in [15], this effect can easily be separated from Faraday rotation. Care must be taken in applying data-based techniques for estimating and correcting for crosstalk and channel imbalance: many techniques available in the literature were *not* designed to cope with the presence of Faraday rotation and may lead to very large calibration errors if used blindly.

Three techniques for estimating $\mathbf{\Omega}$ and an approach for correcting fully polarimetric measurements made by a linearly polarized system have been described. Two new techniques and an earlier approach described in [3] are shown to be subject to $\pm\pi/2$ errors in the estimate for $\mathbf{\Omega}$. Techniques based on known target signatures offer an approach to estimate the presence of such an error. Alternatively, observation periods could be constrained to those times when $\mathbf{\Omega}$ is expected to be less than 45°. If a $\pi/2$ error in $\mathbf{\Omega}$ is identified, a correction procedure is described, and the resulting correction should be good to within a phase error of $\pm n\pi$.

The techniques for estimating $\mathbf{\Omega}$ and correcting fully polarimetric measurements presented here should work for any scatterer and be accurate to within a factor of $\pi/2$. To arrive at this conclusion, the reciprocal nature of monostatic radar scattering has been assumed.

Of the three techniques presented, the first [see (11)] is based on scattering matrix data. Such measures are not susceptible to initial averaging of the scattering matrix data and, therefore, have large errors in any one estimate. The author's experience with measurements derived by adding and subtracting (complex) scattering matrix terms is that subsequent averaging does not always lead to improved estimation, so this measure was not pursued further. The other two, in (13) and (15), are based on covariance matrix data that *can* be averaged over large areas (before estimating $\mathbf{\Omega}$). These two techniques were subjected to a sensitivity analysis, which showed that the technique derived from that given in [3], i.e., (15), was the more robust measure in the presence of residual calibration errors. An intuitive explanation for this can be found in the fact that (15) is a relative phase measure, while (13) is a relative amplitude measure. In a similar fashion, terrain height errors in data generated using the technique of radar interferometry (a relative phase method) tend to be smaller than errors in data generated using the technique of radar stereogrammetry (a relative amplitude method).

For L-band SARs, such as the Japanese Space Agency's planned PALSAR system or the European TerraSAR (both of which may be capable of fully polarimetric measurements), the techniques described in this paper offer a solution for the calibration of fully polarimetric data products. Since one-way Faraday rotation at L-band is expected to be $\leq 40^\circ$ [15], ambiguities of $\pm n\pi/2$ in the estimates of $\mathbf{\Omega}$ should not be a problem. For longer wavelength spaceborne SARs (e.g., ultrahigh frequency), ambiguities in $\mathbf{\Omega}$ may be resolved following the approach outlined in Section III. Alternatively, during periods of intense solar activity, data collection could be confined to night-time passes only, when the total electron content and, therefore, $\mathbf{\Omega}$ should be significantly less.

With the ability to correct for Faraday rotation, a longer wavelength earth-orbiting linearly polarized polarimetric SAR is more feasible. A further step forward in this direction was achieved with the decision in 2003 by the World Radiocommunication Conference to grant a secondary frequency allocation to the earth-exploration satellite services (active) in the band 432–438 MHz [26]. Our calculations show that a P-band SAR operating in this band, in a 600-km altitude polar orbit, with a simple linearly polarized reflector antenna and a low-power (200 W peak, 5 W average) solid-state amplifier with two receive channels, could generate fully polarimetric SAR strip map measurements at an incidence angle of $\sim 35^\circ$ over a swath width of 50 km at 50–100–m spatial resolution (on the ground). Such a low-power instrument would still provide a noise-equivalent sigma-naught of better than -30 dB and could easily operate continuously, providing a monitoring capability

TABLE V
PARAMETERS OF A MODERATE-RESOLUTION SPACEBORNE POLARIMETRIC P-BAND SAR SYSTEM

Parameter	**Value**
Wavelength	69 cm
Bandwidth	6 MHz
Polarizations	HH, HV, VH, VV
Incidence angle	35 degrees
Swath width	50 km
Spatial resolution	50 or 100 m
Number of looks	12
Orbit altitude	600 km
Orbit type	Polar, sun-synchronous
Antenna size	6 m diameter (reflector)
Peak (RF) power	200W
Average (DC) power	5W
Data rate	32 Mbps
Noise-equivalent sigma-zero	-30 dB
Cross-talk	-30 dB
Channel amplitude imbalance	±0.5 dB
Channel phase imbalance	±10 degrees

currently unavailable to earth scientists studying biomass and land cover changes, etc. With the proper choice of a midnight–midday sun-synchronous orbit, earth scientists studying these phenomena could remove uncertainties due to diurnal effects from their observations, and study seasonal effects with a guaranteed revisit period of, say, one month. The parameters of such a system are summarized in Table V.

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REFERENCES

[1] A. R. Thompson, J. M. Moran, and G. W. Swenson, *Interferometry and Synthesis in Radio Astronomy*. New York: Wiley, 1986.

[2] J. V. Evans and T. Hagfors, *Radar Astronomy*. New York: McGraw-Hill, 1968.

[3] S. H. Bickel and R. H. T. Bates, "Effects of magneto-ionic propagation on the polarization scattering matrix," *Proc. IRE*, vol. 53, pp. 1089–1091, 1965.

[4] W. B. Gail, "A simplified calibration technique for polarimetric radars," in *Proc. IGARSS*, vol. II, Tokyo, Japan, 1993, pp. 377–379.

[5] ——, "Effect of faraday rotation on polarimetric SAR," *IEEE Trans. Aerosp. Electron. Syst.*, vol. 34, Jan. 1998.

[6] P. Rosen, "Study of Faraday rotation effects on P-band polarimetric spaceborne SAR," JPL, Pasadena, CA, JPL Interoffice Memo. 3343–92–72, 1992.

[7] B. G. Kutuza, A. A. Kalinkevitch, A. I. Ephimov, E. A. Vostrov, and A. B. Dzenkevitch, "Application of SAR operating at P-band for space experiments," in *Proc. EUSAR '96, Eur. Conf. Synthetic Aperture Radar*, Konigswinter, Germany, 1996, pp. 309–313.

[8] T. S. Chu and H. F. Lenzing, "Ionosphere-induced cross polarization of circularly polarized UHF propagation," *IEEE Trans. Antennas Propagat.*, vol. 39, pp. 1644–1647, Nov. 1991.
[9] S. Quegan and J. Lamont, "Ionospheric and tropospheric effects on synthetic aperture radar performance," *Int. J. Remote Sensing*, vol. 7, no. 4, pp. 525–539, 1986.
[10] A. Ishimaru, Y. Kuga, and J. Liu, "Study of ionospheric/tropospheric effects on SAR operation for the JPL/NASA advanced radar technology program," JPL, Pasadena, CA, Dec. 1997.
[11] E. J. Rignot, R. Zimmermann, and J. J. van Zyl, "Spaceborne applications of P-band imaging radars for measuring forest biomass," *IEEE Trans. Geosci. Remote Sensing*, vol. 33, pp. 1162–1169, Sept. 1995.
[12] T. Le Toan, A. Beaudoin, J. Riom, and D. Guyon, "Relating forest biomass to SAR data," *IEEE Trans. Geosci. Remote Sensing*, vol. 30, pp. 403–411, Mar. 1992.
[13] M. C. Dobson, F. T. Ulaby, T. LeToan, A. Beaudoin, E. S. Kasischke, and Christiansen, "Dependence of radar backscatter on conifer forest biomass," *IEEE Trans. Geosci. Remote Sensing*, vol. 30, pp. 412–415, Mar. 1992.
[14] D. Evans, Ed., "Spaceborne synthetic aperture radar: Current status and future directions," Nat. Res. Council, Comm. Earth Sci., NASA Tech. Memo. 4679, 1995.
[15] A. Freeman and S. Saatchi, "On the detection of Faraday rotation in linearly polarized L-band SAR backscatter signatures," *IEEE Trans. Geosci. Remote Sensing*, vol. 42, Aug. 2004.
[16] A. Freeman, "SAR calibration: An overview," *IEEE Trans. Geosci. Remote Sensing*, vol. 30, pp. 1107–1121, Nov. 1992.
[17] A. Freeman, J. Cruz, M. Alves, B. Chapman, S. Shaffer, and E. Turner, "SIR-C data quality and calibration results," *IEEE Trans. Geosci. Remote Sensing*, vol. 33, pp. 848–857, July 1995.
[18] S. V. Nghiem, S. H. Yueh, R. Kwok, and F. K. Li, "Symmetry properties in polarimetric remote sensing," *Radio Sci.*, vol. 27, no. 5, pp. 693–711, Sept.-Oct. 1992.
[19] A. Freeman, J. J. van Zyl, J. D. Klein, H. A. Zebker, and Y. Shen, "Calibration of stokes and scattering matrix format polarimetric SAR data," *IEEE Trans. Geosci. Remote Sensing*, vol. 30, pp. 531–539, May 1992.
[20] A. Freeman, J. Villasenor, J. D. Klein, P. Hoogeboom, and J. S. Groot, "On the use of multi-frequency and polarimetric radar backscatter features for classification of agricultural crops," *Int. J. Remote Sens.*, vol. 15, no. 9, pp. 1719–1812, 1994.
[21] A. Freeman, "An algorithm for estimating the noise levels in polarimetric SAR image data," *Int. J. Remote Sens.*, vol. 15, no. 9, pp. 1791–1798, 1994.
[22] S. R. Cloude and E. Pottier, "A review of target decomposition theorems in radar polarimetry," *IEEE Trans. Geosci. Remote Sensing*, vol. 34, pp. 498–518, Mar. 1996.
[23] E. Krogager and Z. H. Czyz, "Properties of the sphere, diplane, helix decomposition," in *Proc. 3rd Int. Workshop Radar Polarimetry*, Nantes, France, Mar. 21–23, 1995, pp. 106–114.
[24] J. J. van Zyl, "Calibration of polarimetric radar images using only image parameters and trihedral corner reflector responses," *IEEE Trans. Geosci. Remote Sensing*, vol. GE-28, pp. 337–348, May 1990.
[25] J. D. Klein and A. Freeman, "Quadpolarization SAR calibration using target reciprocity," *J. Electromagn. Waves Applicat.*, vol. 5, no. 7, pp. 735–751, 1991.
[26] C. D. Wende, private communication, 2003.

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