Counting Subgraphs in Practice

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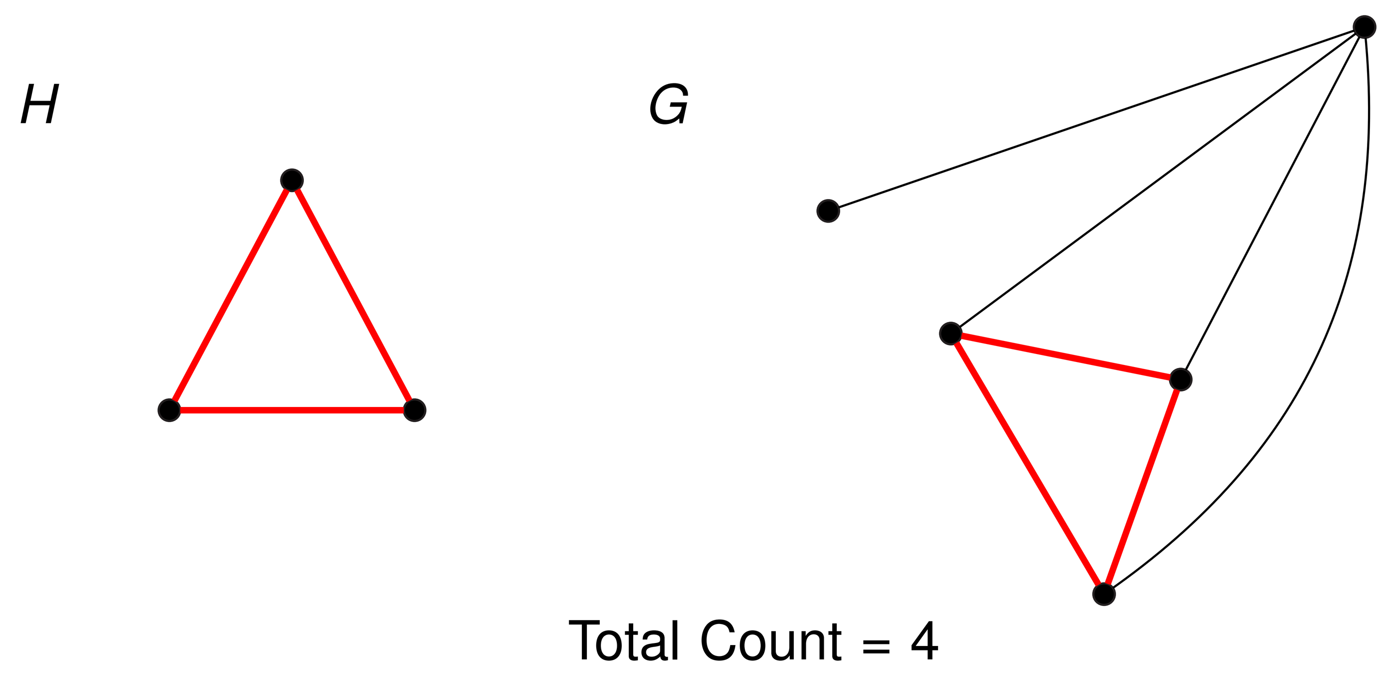
Key Definitions

- The **subgraph counting problem** asks how many unlabelled copies of the graph H are in the graph G.



- A graph has **almost bounded degree** k if it has at most k vertices with degree exceeding k for some constant k.
- A problem is **fixed-parameter tractable (FPT)** if it can be solved by an algorithm in time $f(k)n^{O(1)}$ where n is the size of the problem instance, k is some parameter associated with the problem instance, and f is a function of k which is independent of n.

Motivation

- Subgraph counting is a useful way of analysing and comparing real-world networks.
- Many real-world networks have almost bounded degree:

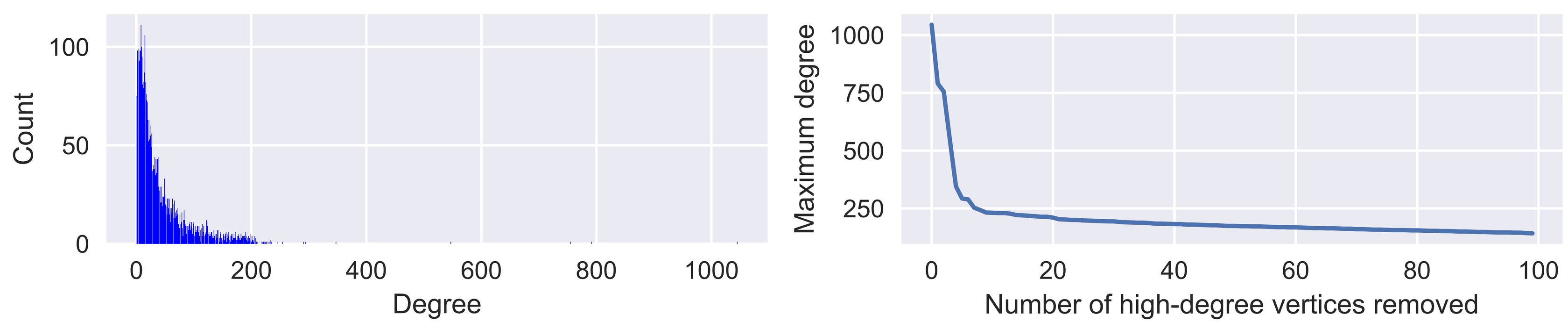


Fig. 1: A plot of the degree distribution of a graph derived from a Facebook data set (left), and the maximum degree of graphs derived from a variety of real-world data sets after greedily removing high-degree vertices (right) [2, 4].

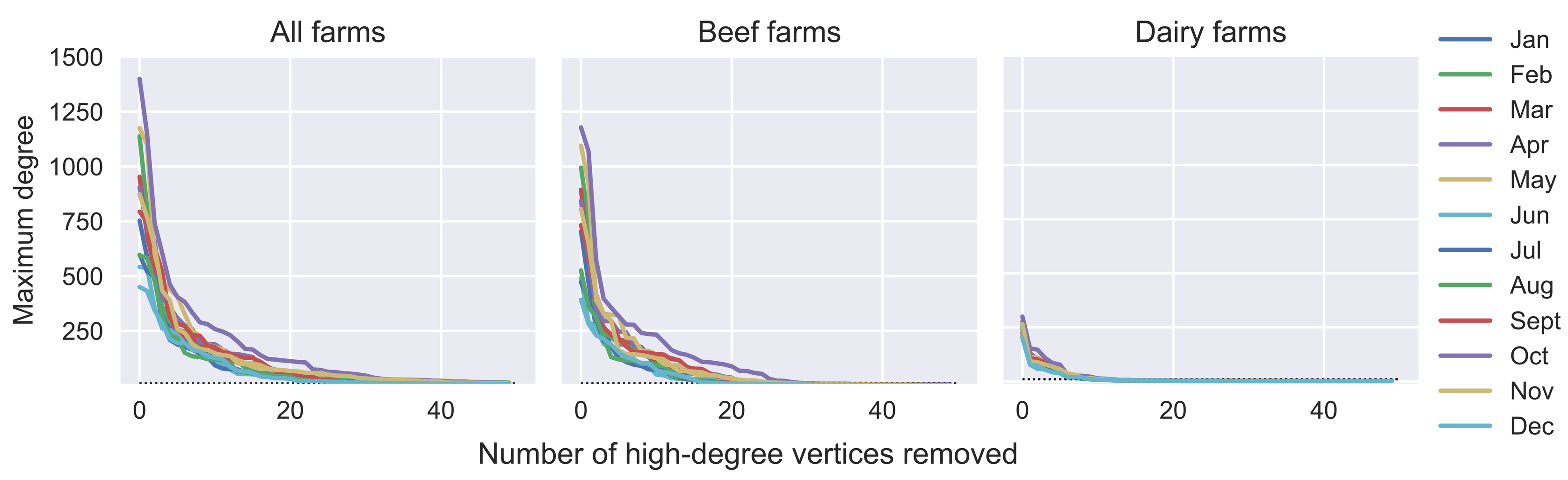


Fig. 2: Plots of the maximum degree of graphs derived from the Scottish cattle trading data [2].

Existing Complexity Results

- Subgraph counting is NP-complete in general.
- Assuming the Exponential Time Hypothesis, subgraph counting is not in FPT in general [1].
- Subgraph counting is in FPT for graphs with almost bounded degree [2].

Project Objectives

- Write and implement an FPT algorithm for subgraph counting in graphs with almost bounded degree.
- Test performance of FPT algorithm against more general constraint programming (CP) method [5] on large real-world data sets [3, 6].

Conclusions

- Significant preprocessing carried out by the CP algorithm lead to better performance of the FPT algorithm on "easy" instances (Figure 3).

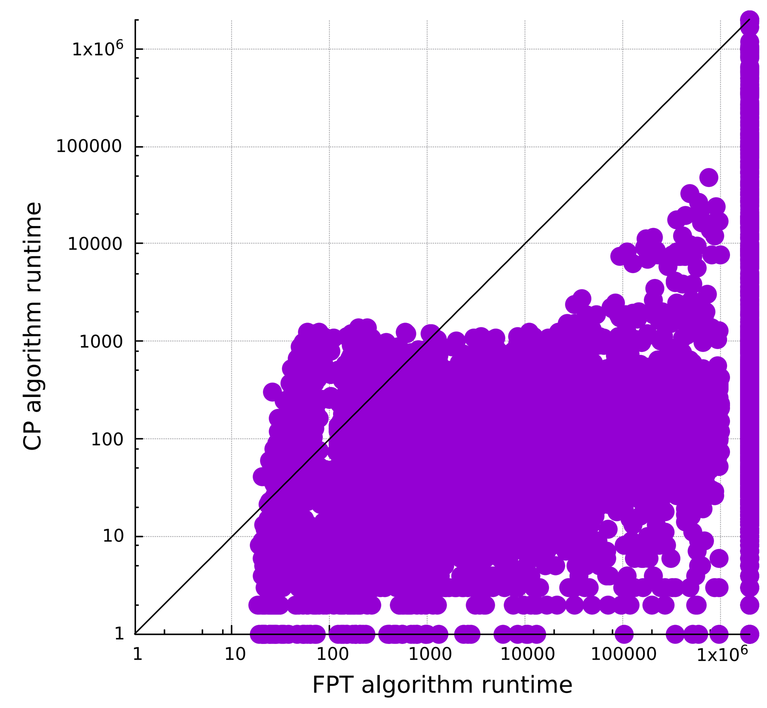


Fig. 3: A plot of the CP algorithm runtime against the FPT algorithm runtime for subgraph counting in real-world networks when the preprocessing time of the CP algorithm is included.

- When the preprocessing time is excluded, the CP approach outperforms the FPT approach on all instances (Figure 4).

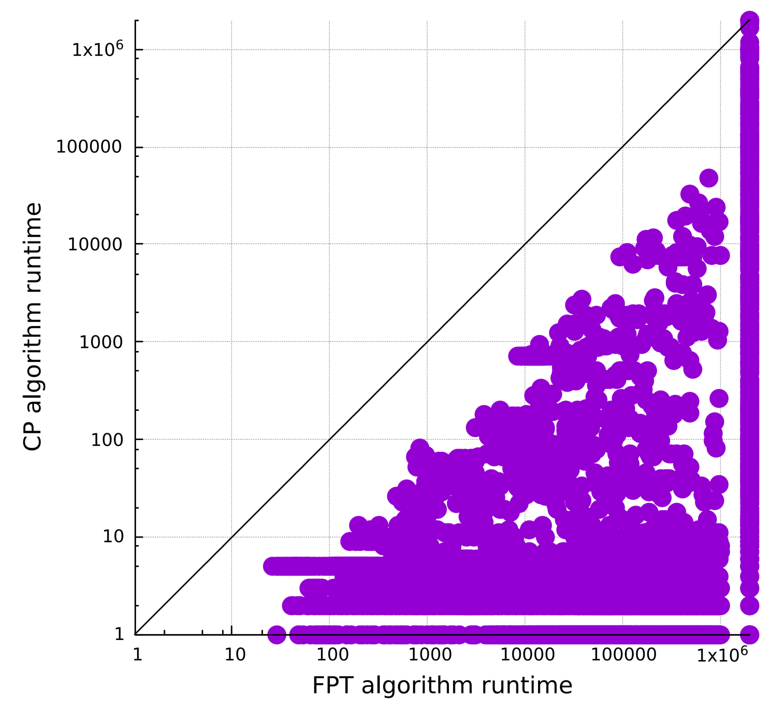


Fig. 4: A plot of the CP algorithm runtime against the FPT algorithm runtime for subgraph counting in real-world networks when the preprocessing time of the CP algorithm is excluded.

Future Work

- Combine FPT approach with improved search methods.
- Run experiments on further data sets known to have almost bounded degree.
- If the combined approach is successful, develop a per-instance algorithm selection model for subgraph counting.

References

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