

Audition CNRS - Concours 06/02

Complexity-Theoretic Foundations of Cryptography

Willy Quach

Area of research: Theory of cryptography

- September 2023 - : Postdoctoral Fellow at the Weizmann Institute of Science. Host: Zvika Brakerski
- September 2017 - August 2023: PhD student at Northeastern University. Advisor: Daniel Wichs
- September 2013 - August 2017: École Normale Supérieure de Lyon

Cryptography is a core backbone for security and privacy

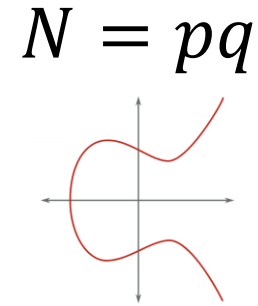
Cryptography is a core backbone for security and privacy

What makes modern cryptography reliable?



“Ancient” cryptography

VS



Modern cryptography

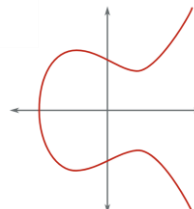
Cryptography is a core backbone for security and privacy

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VS

$$N = pq$$
A graph showing a red curve on a Cartesian coordinate system. The curve is symmetric about the y-axis and has a shape similar to a parabola opening upwards, but with a slight dip in the middle, resembling a W-shape. The x and y axes are represented by thin grey lines with arrows at the ends.

Modern cryptography

We have **abstractions** to reason about security and **paradigms** to achieve them.

Formalize and quantify security (!)

Techniques to tie security to **complexity theory**

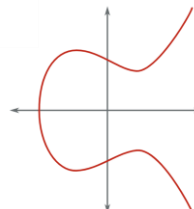
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$$N = pq$$
A diagram showing a red curve on a Cartesian coordinate system. The curve is a hyperbola, specifically the right branch of a hyperbola opening horizontally. It is centered on the origin and approaches the x and y axes as asymptotes. The curve is drawn in a vibrant red color.

Modern cryptography

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Techniques to tie security to **complexity theory**

What is there left to do?

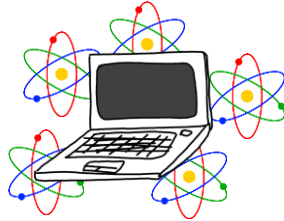
My Research in a Nutshell

My Research in a Nutshell

1

Address new threats

Devastating effects of
quantum attacks, side-channel attacks

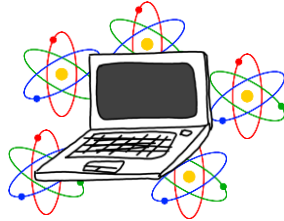


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- **Security against quantum computers**

How to argue security? [TCC '22, **Invited to the Journal of Cryptology**]

Alternate ``quantum-secure'' constructions [PKC '18, CRYPTO '19, TCC '21]

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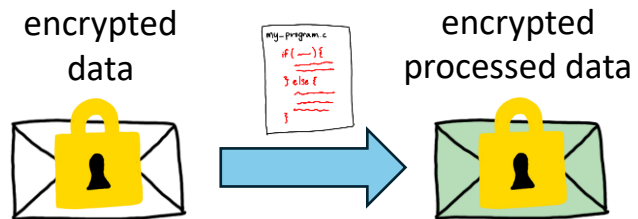
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Provide stronger functionalities

Private data used in **computation**, not just transit



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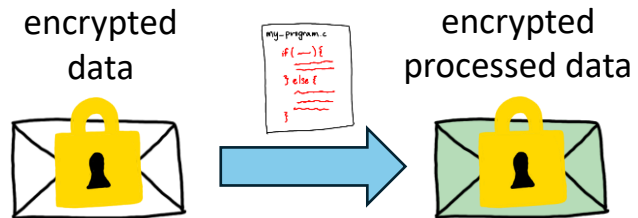
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- Tools to compute blindly over encrypted data

Introducing new cryptographic tools [FOCS '18]

Advanced encryption [CRYPTO '19], program obfuscation [TCC '21]

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Private data used in **computation**, not just transit

encrypted
data



encrypted
processed data



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Firmer foundations of cryptography

Foundations are still poorly understood



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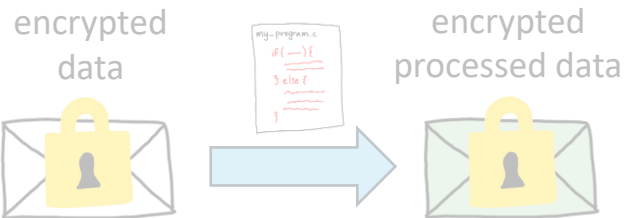
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- Refine ties with complexity theory

Cryptographic proof systems [EUROCRYPT '19, CRYPTO '19, '21, '23]

Alternate models of security [EUROCRYPT '22, '23, TCC '23]

A Global Lens: Computational Hardness

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Cryptographic security is proven under **computational assumptions**

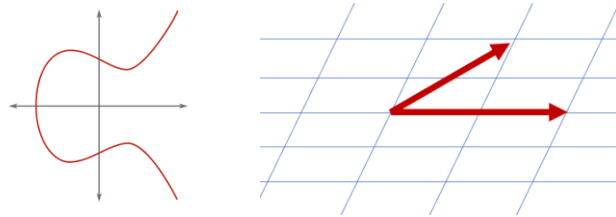
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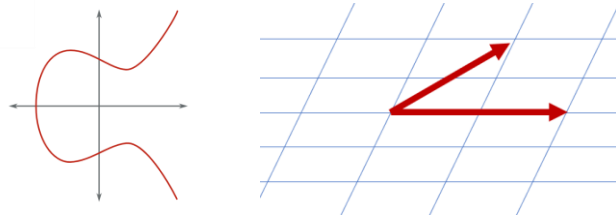
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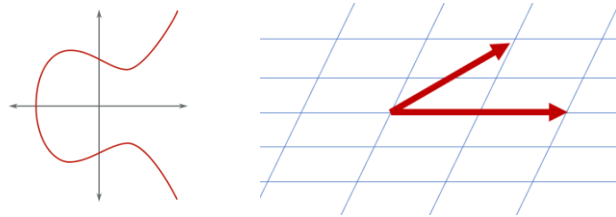
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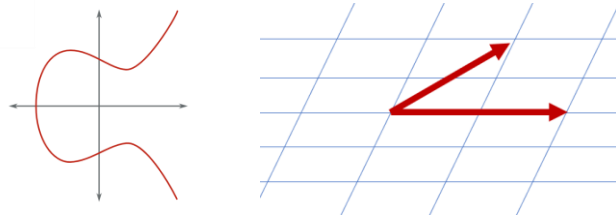
Property about the assumption:

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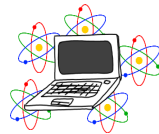
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Property about the assumption:

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Security against quantum attacks



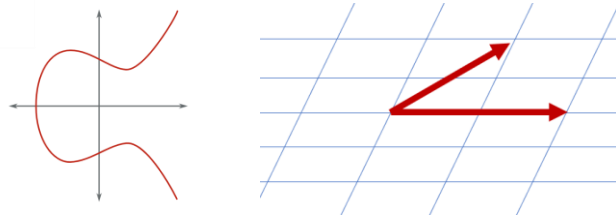
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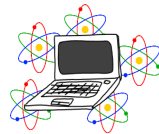
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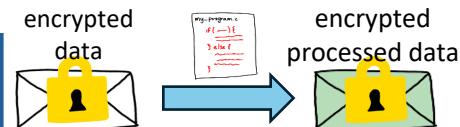
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Strong functionalities



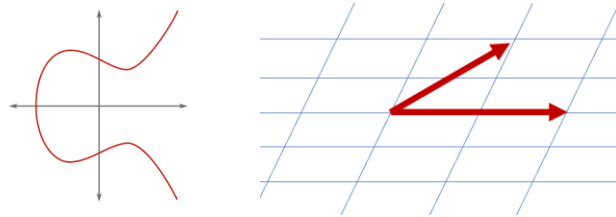
Exploitable algebraic structure

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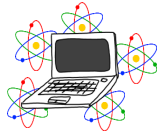


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Property about the assumption:


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Hardness against quantum computers
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Exploitable algebraic structure
- ③ Foundations of cryptography



Strength of assumption

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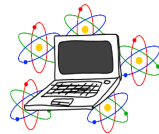
Focus for today:

How (not) to argue security against quantum attacks

[LMQW, TCC '22, *Invited to the Journal of Cryptology*]

Property about the assumption:

① Security against quantum attacks ——— Hardness against quantum computers



② Strong functionalities ——— Exploitable algebraic structure

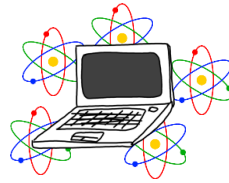


③ Foundations of cryptography ——— Strength of assumption



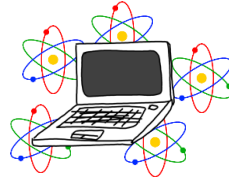
Post-Quantum Security

Quantum computers would **break** most public-key cryptography deployed.
via Shor's algorithm [Shor'94]



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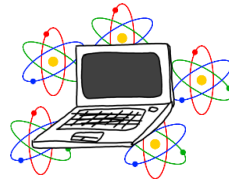


Need new cryptosystems that resist quantum attacks
a.k.a **post-quantum secure**

Data sensitive today might still be sensitive in 50 years!

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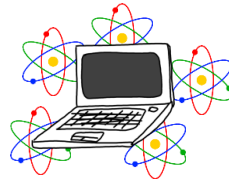
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How do we ensure security against quantum attacks?

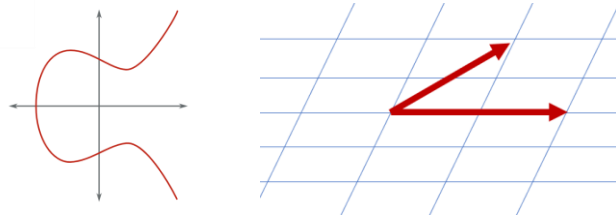
Surprisingly little attention given to this general question

Back to Computational Hardness

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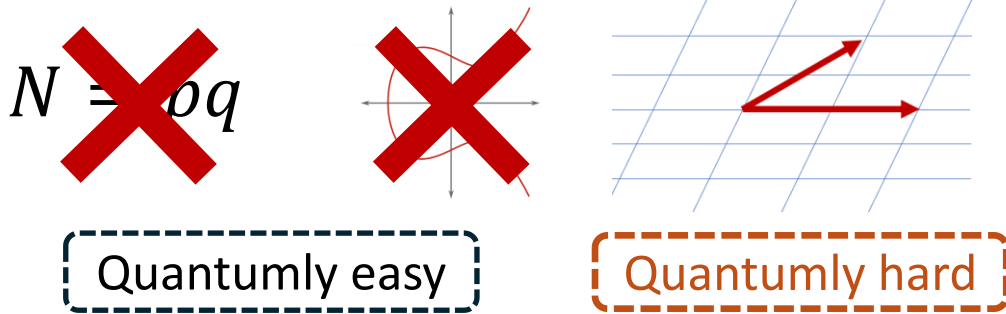
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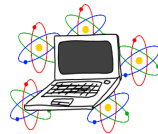
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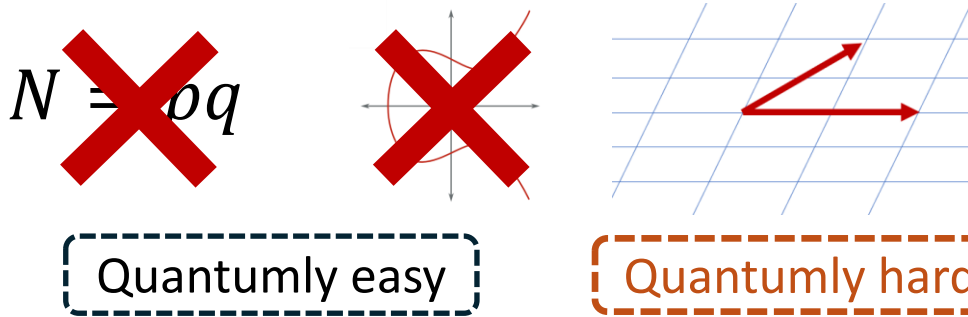
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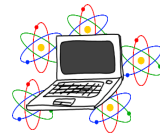
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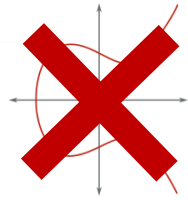
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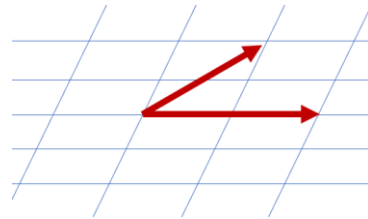
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Quantumly easy

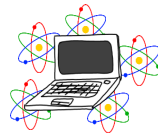


Quantumly hard

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Security against quantum attacks

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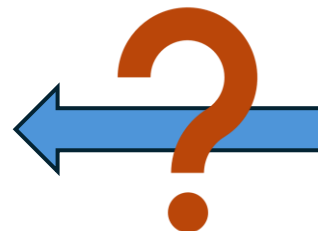


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Main Result

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Implicit in two decades of research

*“Non-interactive” cryptosystems such as **encryption schemes, signatures...**

As opposed to *interactive* or *heuristic* cryptosystems

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Main Theorem [LMQW TCC’22]: **NO**

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The folklore understanding is **wrong**!

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What Goes Wrong?

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Bob



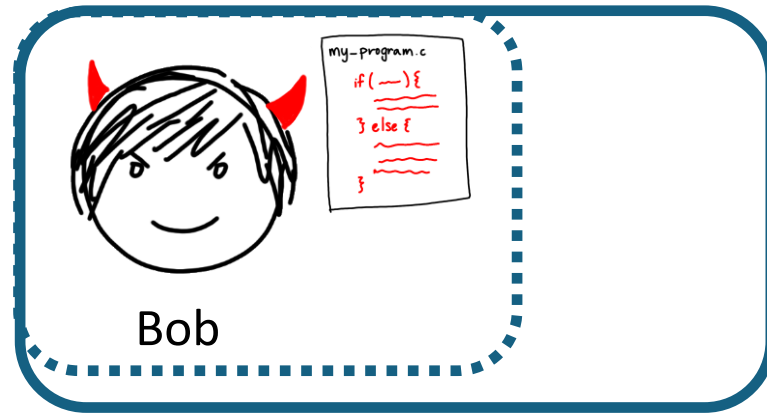
Alice



Security property
e.g. secrets keys are hidden

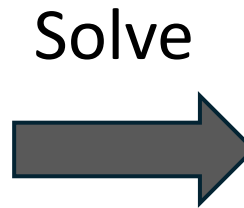
Successful attacks against  implicitly solve a hard problem

What Goes Wrong?

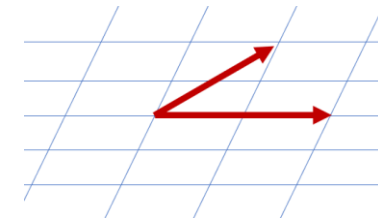
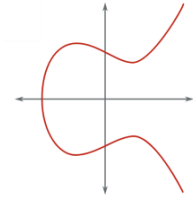


Bob

Reduction



$$N = pq$$

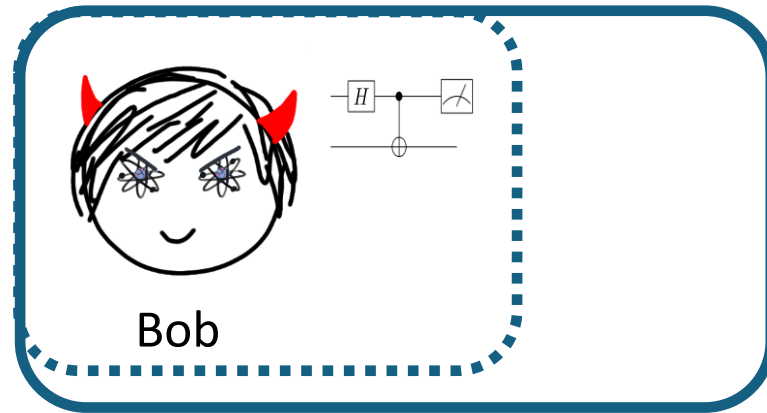


Hard algorithmic problems

Reduction turns successful attacks into efficient algorithms

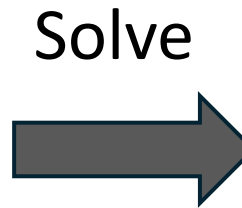
a.k.a **proof of security**

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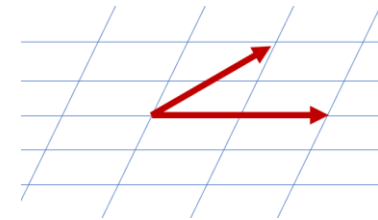
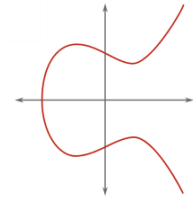


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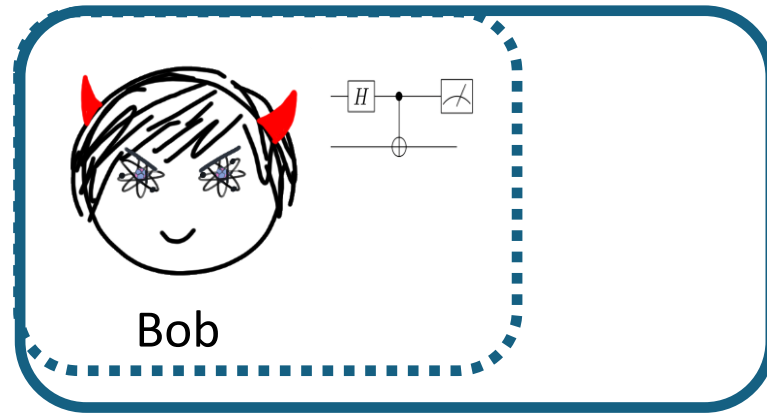


Quantumly hard problems

Reduction turns successful **quantum** attacks into efficient **quantum** algorithms

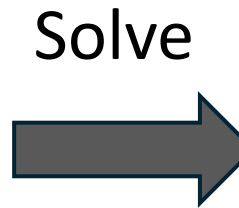
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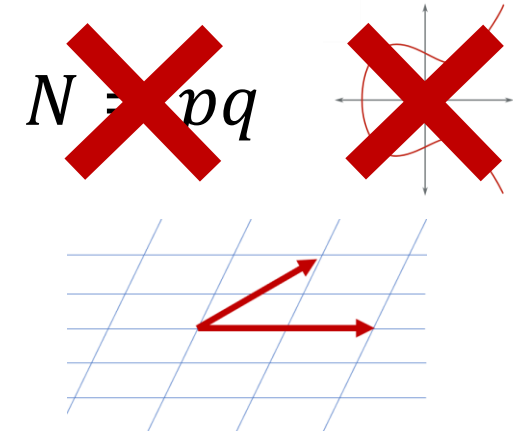


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Reduction



Solve

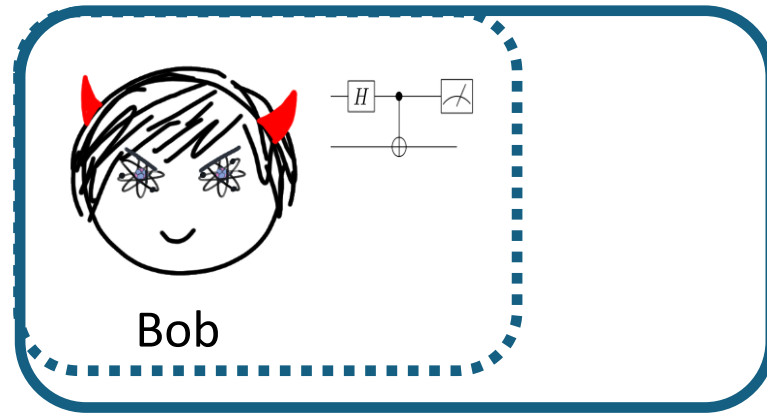


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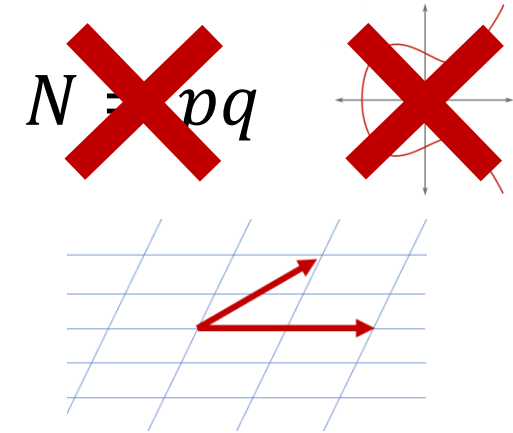
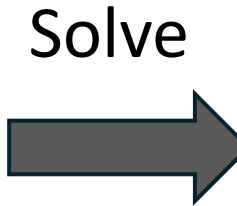
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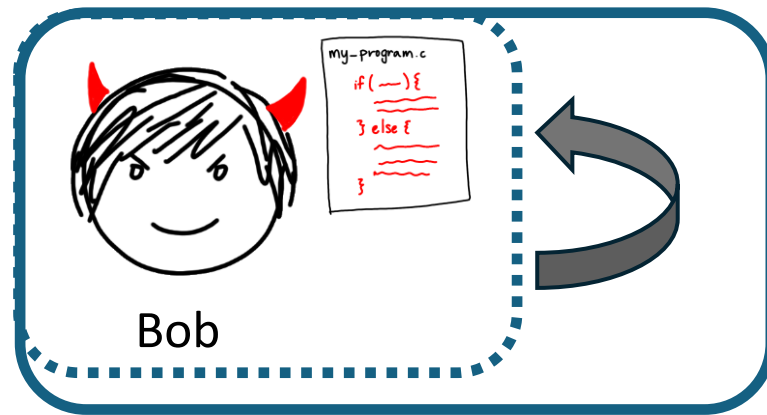
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Main issue: proofs of security *are not* proofs of post-quantum security

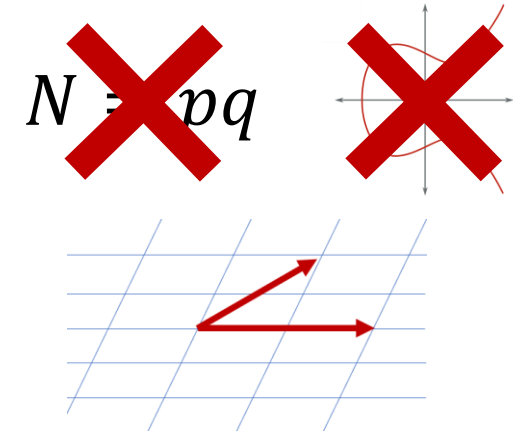
Quantum attacks behave very differently from classical attacks

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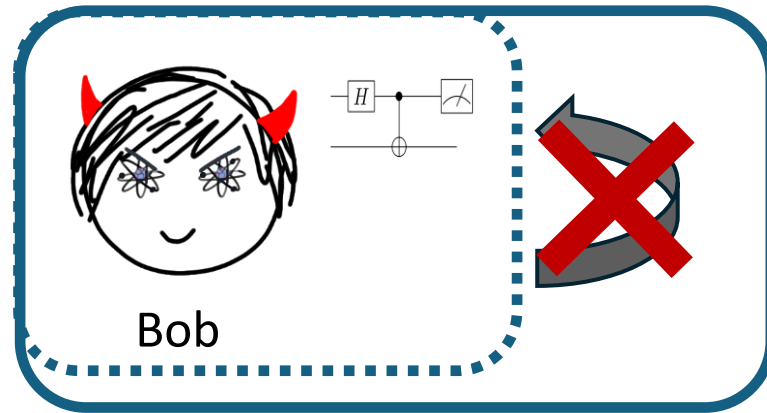
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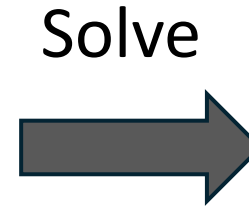
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- Stateful classical algorithms can be run several times (rewinding)

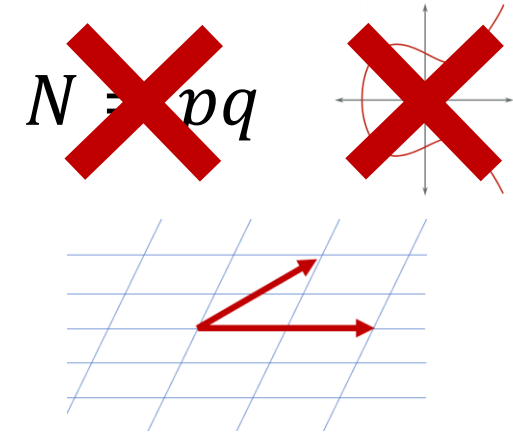
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Quantum attacks behave very differently from classical attacks

- Stateful classical algorithms can be run several times (rewinding)
- Stateful quantum algorithms **cannot** in general (measurements are destructive)

Main Result (2)

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Main Theorem [LMQW TCC'22] : **Explicit counter-examples:**

- Proven secure (classically) **under a post-quantum assumption**
- **Quantumly broken**

Includes symmetric-key encryption, digital signatures...

Main Result (2)

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Main observation: cryptographic attacks **can be stateful**
Reductions can interact with attacks in subtle ways

Reductions can run classical attacks twice, but not quantum attacks

Main Result (2)

Main Theorem [LMQW TCC'22] : **Explicit counter-examples:**

- Proven secure (classically) **under a post-quantum assumption**
- **Quantumly broken**

Includes symmetric-key encryption, digital signatures...

Main observation: cryptographic attacks **can be stateful**
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Conceptually: Proofs of quantumness \equiv Counter-examples

\Leftarrow breaking security “proves quantumness”

Takeaway: cannot simply plug-in post-quantum assumptions,
need special-purpose proofs of post-quantum security

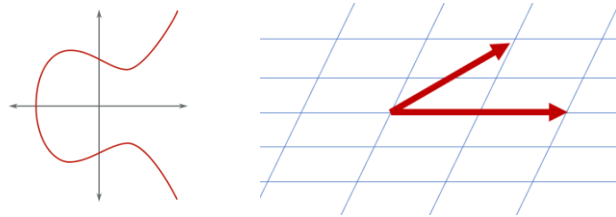
Research Project

Back to Computational Hardness (Again)

Cryptographic security is proven under **computational assumptions**

Most cryptography can be broken in $NP \Rightarrow$ need to assume algorithmic hardness / that $P \neq NP$

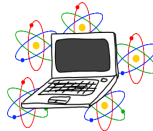


$$N = pq$$



Hard algorithmic problem

The choice of assumption matters a lot!

Property about the assumption:

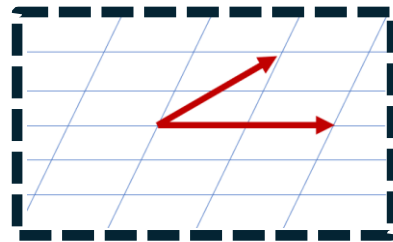
- 1 Security against quantum attacks  Hardness against quantum computers
- 2 Strong functionalities  Exploitable algebraic structure
- 3 Foundations of cryptography  Strength of assumption

Back to Computational Hardness (Again)

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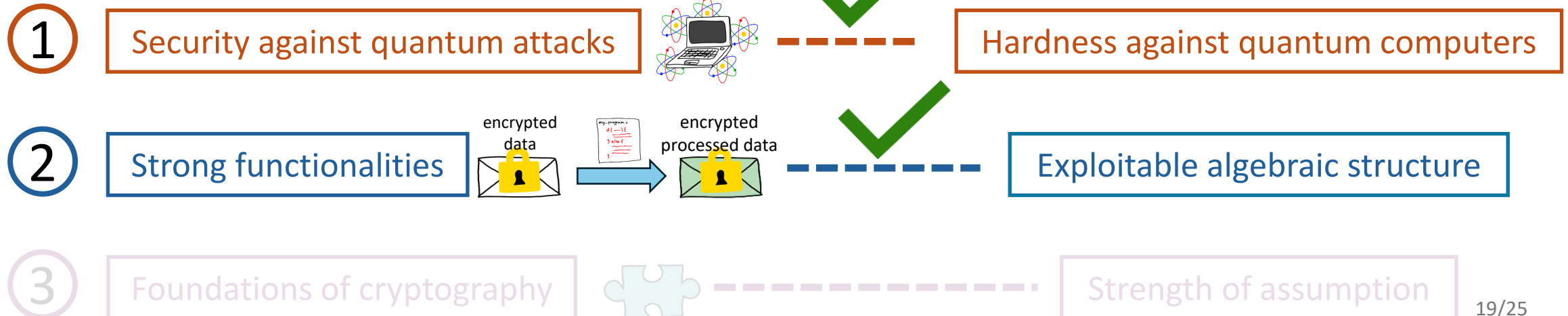
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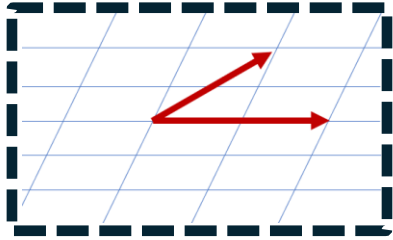
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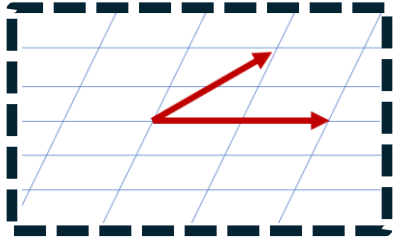
Back to Computational Hardness (Again)



Lattices are extremely powerful and convenient!

⇒ Main **post-quantum candidates**, only credible **homomorphic encryption**...

Back to Computational Hardness (Again)

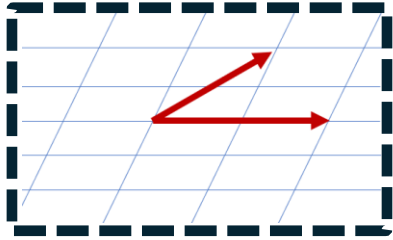


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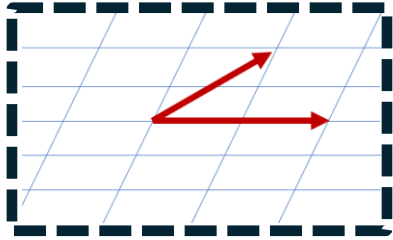
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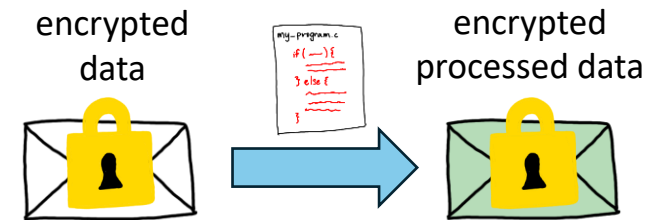
My goal: Leverage other sources of hardness in cryptography

Diversifying Assumptions in Cryptography

1

Strong functionalities from a **wide range of assumptions**

Can we build strong cryptography without lattices?

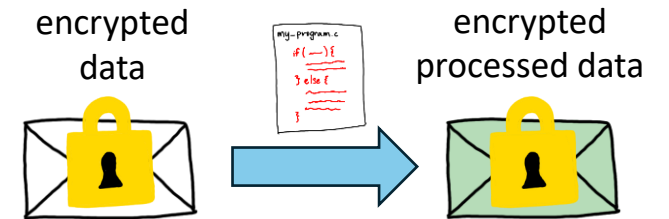


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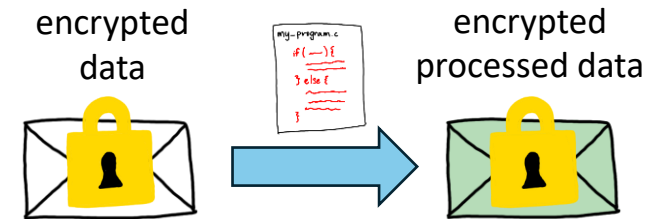
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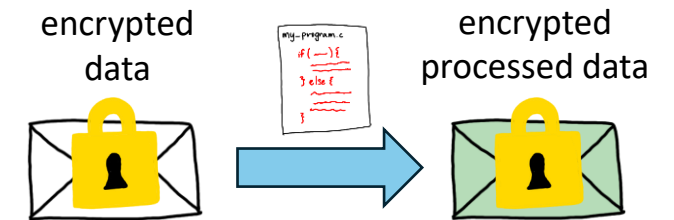
- Identify **technical barriers**, abstract out **concrete stepping stones** (e.g. relaxations)

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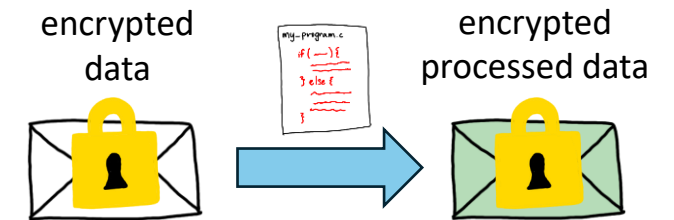
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- Develop techniques suited to **other assumptions**

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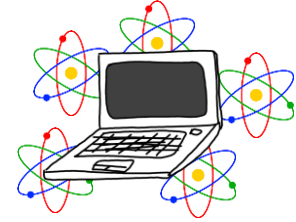


- ... or explain the absence of such techniques

Diversifying Assumptions in Cryptography (2)

2

Quantum computation and cryptography

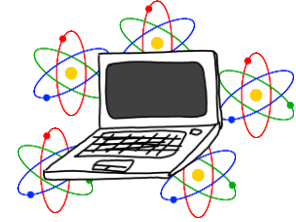


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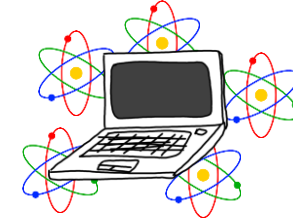
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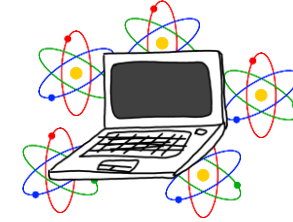
Can we use quantum computing for **stronger cryptography**?

a.k.a **quantum cryptography**, where honest users use quantum computers, ciphertexts are qubits, etc

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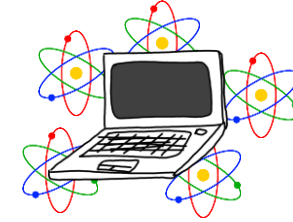
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Can we devise entirely new applications?

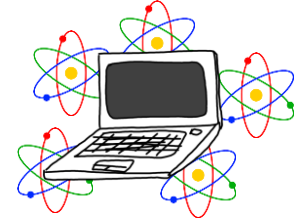
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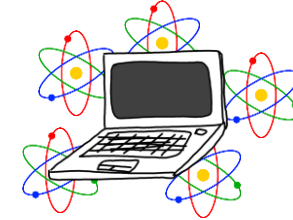
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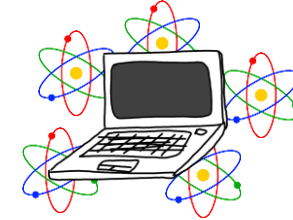
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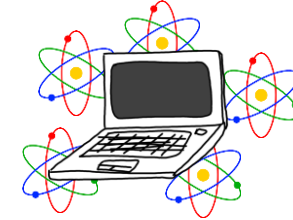
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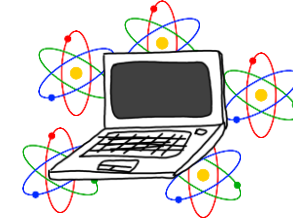
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e.g. find a Hamiltonian cycle in a graph, break a *classical* ciphertext...

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Need a new theory to reason about **inherently quantum problems**

e.g. breaking security of a *quantum* ciphertext

Integration in Teams

➤ DI-ENS, Paris, équipe CASCADE

David Pointcheval (elliptic curves, functional encryption...)

Phong Nguyen (lattices, quadratic forms...)

Brice Minaud (searchable encryption...)

Céline Chevalier (quantum uncloneable cryptography...)

➤ LIP6, Paris, équipe ALMASTY

Damien Vergnaud (randomness in cryptography, leakage-resilience...)

Charles Bouillaguet (alternate assumptions...)

QI team (e.g. Alex B. Grilo...) (foundations of quantum cryptography...)

Willy Quach

- Research area: **theory of cryptography**
Main axes: **post-quantum security**, **advanced cryptosystems**, **foundations**
Research project: **Diversifying sources of hardness in cryptography**
- 17 publications (“A* conferences”: CRYPTO x6, EUROCRYPT x3, FOCS)
- 25 co-authors
- Program committees (PKC ‘23, CRYPTO ‘24, TCC’24)