Audition CNRS - Concours 06/02

Complexity-Theoretic Foundations of Cryptography

Willy Quach

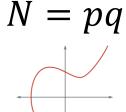
Area of research: Theory of cryptography

- > September 2023 : Postdoctoral Fellow at the Weizmann Institute of Science. Host: Zvika Brakerski
- September 2017 August 2023: PhD student at Northeastern University. Advisor: Daniel Wichs
- > September 2013 August 2017: École Normale Supérieure de Lyon

What makes modern cryptography reliable?



VS



"Ancient" cryptography

Modern cryptography

What makes modern cryptography reliable?



VS

$$N = pq$$

"Ancient" cryptography

Modern cryptography

We have abstractions to reason about security and paradigms to achieve them.

Formalize and quantify security (!)

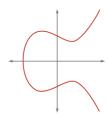
Techniques to tie security to complexity theory

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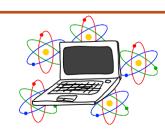
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What is there left to do?



Address new threats

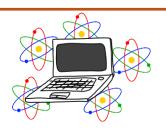
Devastating effects of **quantum attacks**, **side-channel attacks**





Address new threats

Devastating effects of quantum attacks, side-channel attacks



Security against quantum computers

How to argue security? [TCC '22, Invited to the Journal of Cryptology]
Alternate "quantum-secure" constructions [PKC '18, CRYPTO '19, TCC '21]



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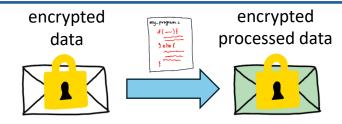
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Provide stronger functionalities

Private data used in **computation**, not just transit





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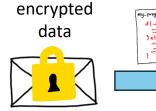
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encrypted processed data



Tools to compute blindly over encrypted data

Introducing new cryptographic tools [FOCS '18]
Advanced encryption [CRYPTO '19], program obfuscation [TCC '21]

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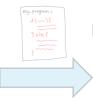
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Firmer foundations of cryptography

Foundations are still poorly understood



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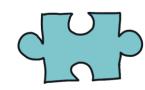
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Refine ties with complexity theory

Cryptographic proof systems [EUROCRYPT '19, CRYPTO '19, '21, '23] Alternate models of security [EUROCRYPT '22, '23, TCC '23]

Cryptographic security is proven under computational assumptions

Most cryptography can be broken in $NP \Longrightarrow$ need to assume algorithmic hardness / that $P \ne NP$

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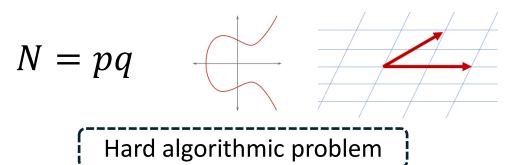
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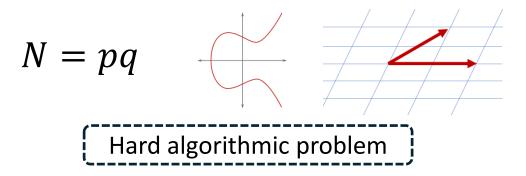
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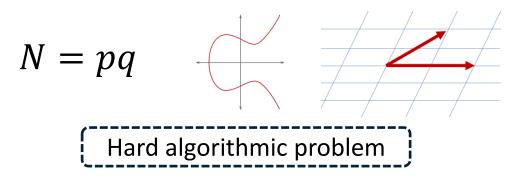
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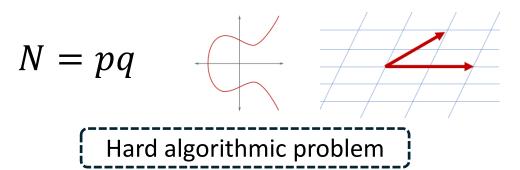




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Exploitable algebraic structure

(3) Foundations of cryptography



Strength of assumption

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Focus for today:

How (not) to argue security against quantum attacks [LMQW, TCC '22, Invited to the Journal of Cryptology]

lot!

Thata algorithmo problem

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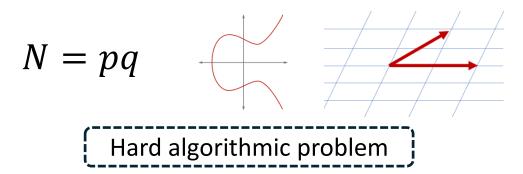
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How do we ensure security against quantum attacks?

Surprisingly little attention given to this general question

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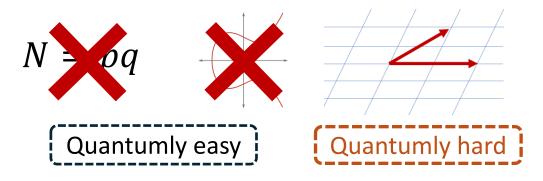
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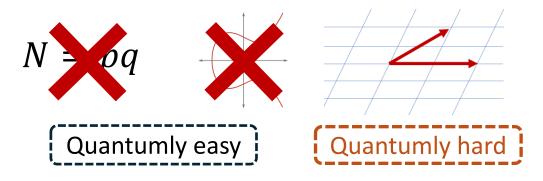
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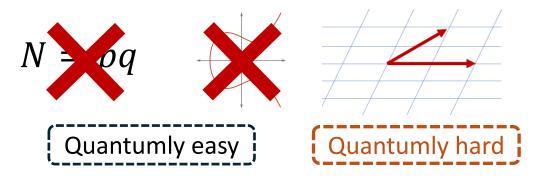
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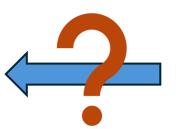
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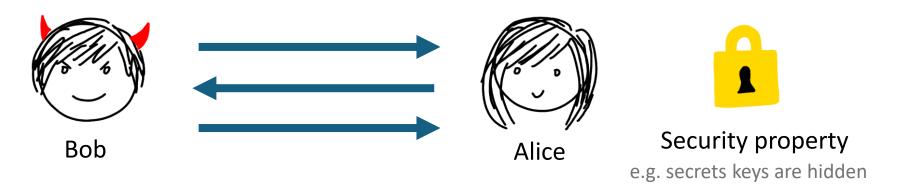
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The folklore understanding is wrong!

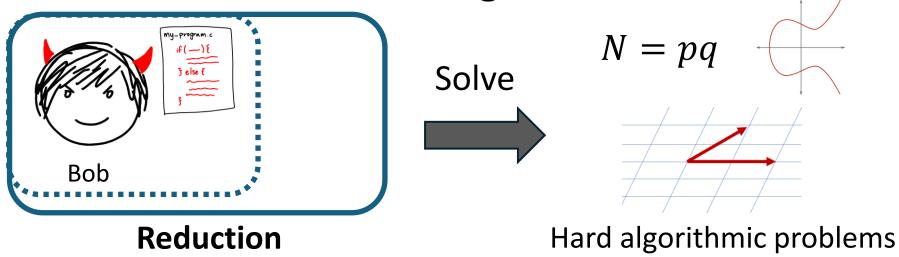
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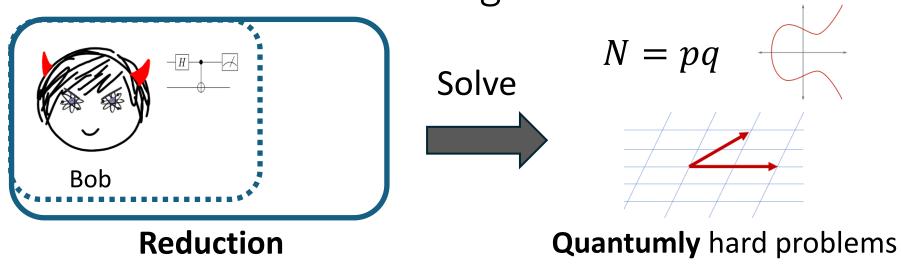
Successful attacks against implicitly solve a hard problem

What Goes Wrong?



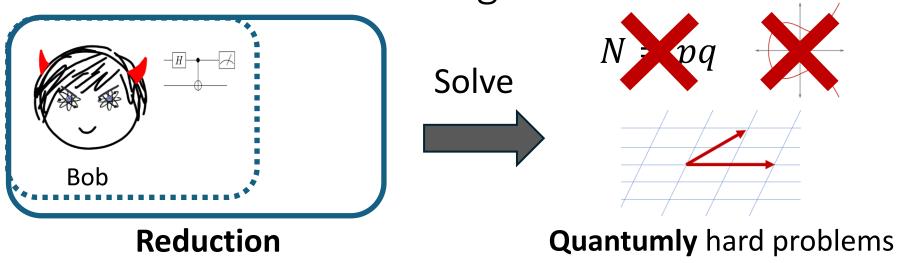
Reduction turns successful attacks into efficient algorithms

a.k.a proof of security



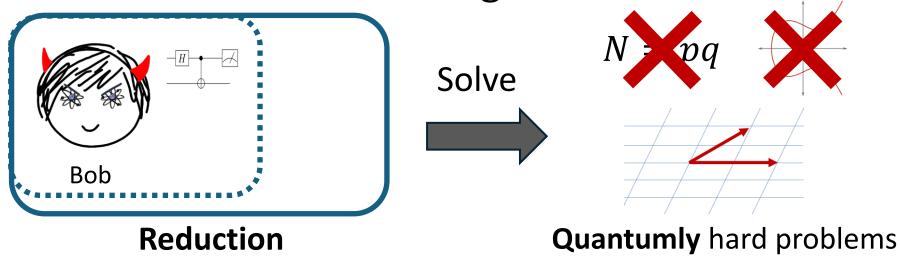
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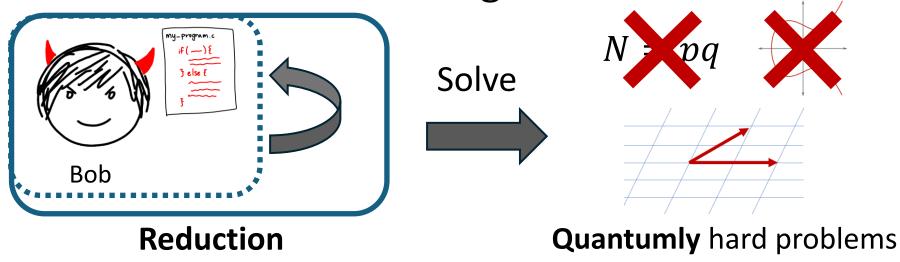
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Main issue: proofs of security are not proofs of post-quantum security Quantum attacks behave very differently from classical attacks

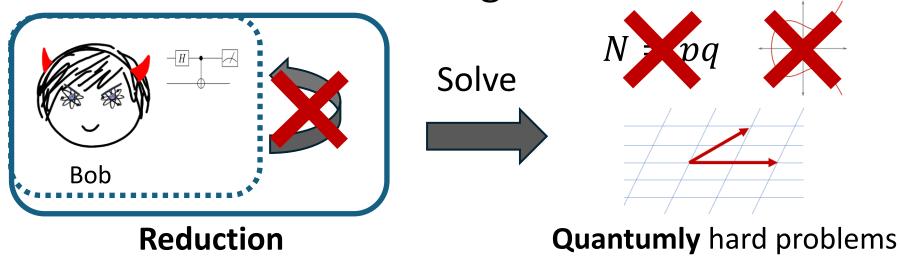


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> Stateful classical algorithms can be run several times (rewinding)



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Main issue: proofs of security are not proofs of post-quantum security Quantum attacks behave very differently from classical attacks

- > Stateful classical algorithms can be run several times (rewinding)
- > Stateful quantum algorithms *cannot* in general (measurements are destructive)

Main Theorem [LMQW TCC'22]: Explicit counter-examples:

- Proven secure (classically) under a post-quantum assumption
- Quantumly broken

Includes symmetric-key encryption, digital signatures...

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Conceptually: Proofs of quantumness \equiv Counter-examples

← breaking security "proves quantumness"

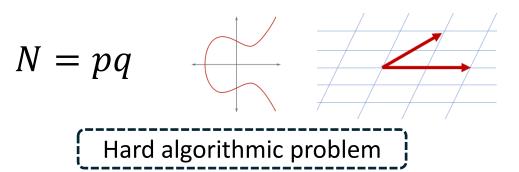
Takeaway of this work:

Cannot simply plug-in post-quantum assumptions, need special-purpose proofs of post-quantum security

Research Project

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Property about the assumption:

(1) | Security against quantum attacks



Hardness against quantum computers

(2) | Strong functionalities



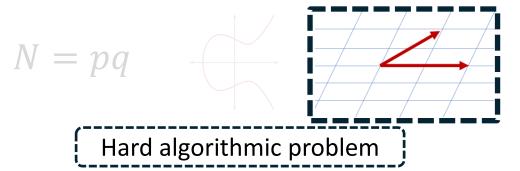


Exploitable algebraic structure

(3) Foundations of cryptography



Cryptographic security is proven under computational assumptions



Lattices are extremely convenient!

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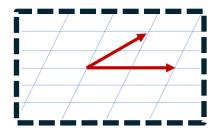
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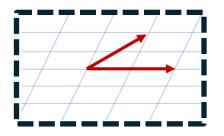
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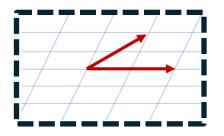
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... but we are starting to put all our eggs in the same basket

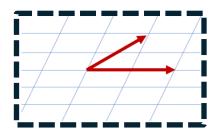


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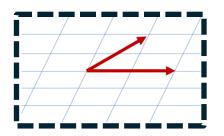


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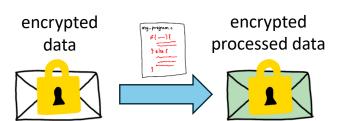
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My goal: Leverage other sources of hardness in cryptography

1

Strong functionalities from a wide range of assumptions

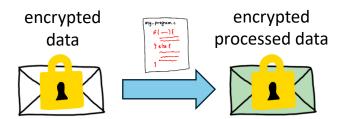
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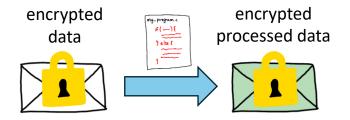


Long-term goal: develop new generic paradigms for cryptography

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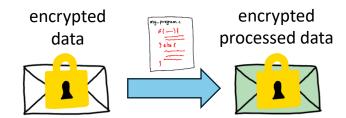
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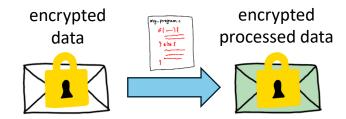
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Example: allowing a **single** computation, fixed in advance, on encrypted data suffices in applications, avoids complexity-theoretic barriers!



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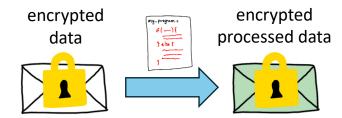
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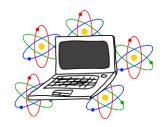
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New assumptions ignored by theory: lattice isomorphisms, isogenies, multivariate systems... But also old assumptions! (elliptic curves, coding theory...)

... or explain the absence of such techniques

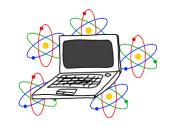


Quantum computation and cryptography





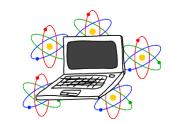
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Post-quantum cryptography: quantum as a **threat** to cryptography



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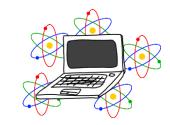
Quantum cryptography:

Use quantum computing for cryptographic constructions

where honest users use quantum computers, ciphertexts are qubits...



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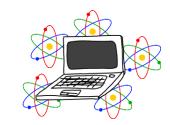
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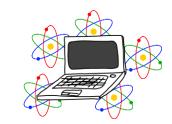
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What is the analogue of computational hardness in the quantum setting?



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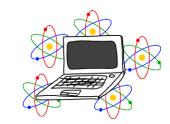
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- Cryptographic proofs of quantumness [BCMVV'18, Yamakawa-Zhandry'22]
- Classical verification of quantum computation [Mahadev'18]
- Black holes?? [Harlow-Hayden'13, Brakerski'23]

Integration in Teams

DI-ENS, Paris, équipe CASCADE David Pointcheval (elliptic curves, functional encryption...) Phong Nguyen (lattices, quadratic forms...) Brice Minaud (searchable encryption...) Céline Chevalier (quantum uncloneable cryptography...)

LIP6, Paris, équipe ALMASTY
Damien Vergnaud (randomness in cryptography, leakage-resilience...)
Charles Bouillaguet (alternate assumptions...)
QI team (e.g. Alex B. Grilo...) (foundations of quantum cryptography...)

Willy Quach

> Research area: theory of cryptography

Main axes: post-quantum security, advanced cryptosystems, foundations

Research project: Diversifying sources of hardness in cryptography

- > 17 publications ("A* conferences": CRYPTO x6, EUROCRYPT x3, FOCS)
- 22 co-authors
- Program committees (PKC '23, CRYPTO '24, TCC'24)