1 INTRODUCTION

The Reissner-Nordstrom[1] solution of the Einstein equation denotes a black hole shielding a singularity. In nature, the black hole is formed as an end state of the collapse of a star of a particular mass range. But, it is widely believed that these singularities do not exist in nature, but that they are the creation of General Relativity. The existence of a singularity means at that very point spacetime fails to exist, signaling a breakdown of laws of physics. So, it has become extremely necessary to substitute the singularity by any other *object* in any other more suitable theory.

The earliest idea in mid 1960's, due to Sakharov [2] and Gliner [3] suggests that the singularity could be avoided by matter specifically with a de Sitter core, with the equation of state $p = -\rho$. This equation of state is obeyed by cosmological vacuum and hence, the energy-stress tensor $T_{\mu\nu}$ takes on a false vacuum of the form $T_{\mu\nu} = \Lambda g_{\mu\nu}$, where Λ is the cosmological constant.

Thus spacetime filled with a vacuum could provide a proper discrimination at the final stage of the gravitational collapse, replacing the future singularity[6] . The first regular black hole solution based on this idea was proposed by *Bardeen*[4], according to whom there are horizons but no singularity.

2 ABSTRACT

Reissner Nordstrom solution[1] of Einstein equation is given by the following metric,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \tag{1}$$

where μ , $\nu = 0, 1, 2, 3$
with

$$g_{\mu\nu} = diag(-f(r), f(r), r^2, r^2 \sin^2\theta)$$

and

$$f(r) = 1 - \frac{2m}{r} + \frac{q^2}{r^2}$$

where m and q are the mass and charge of the system.

Here, the solution represents a black hole shielding a singularity. Now this solution is modified by incorporating a new metric function $f(r)$, where $f(r)$ is as follows,

$$\begin{aligned} f(r) &= 1 - \frac{2mr^2}{(r^2+g^2)^{3/2}} \\ &= 1 - (\frac{m}{g})\frac{2(r/g)^2}{[1+(r/g)^2]^{3/2}} \\ \text{and } r \geq 0 \end{aligned}$$

Bardeen solution is regular everywhere. This can be realized from its scalar invariant $R_{ab}R^{ab}$. This black hole is asymptotically flat and near the origin it behaves like de Sitter region, since

$$f(r) \approx 1 - \frac{2m}{g}r^2 \ , \ r \approx 0^+$$

whereas, for large r it is behaves as Schwarzschild solution.

Now the rotating counterpart of regular Bardeen black hole can be written in Boyer-Lindquist coordinates which is in a similar form as Kerr coordinate. These black holes are often referred to as *Kerr-like* Black Holes. In this paper, the horizon structure and ergosphere of rotating Bardeen regular black hole will be investigated.

3 SUBJECT MATTER

3.1 ROTATING BARDEEN REGULAR BLACK HOLE

Bambi[5] , starting from a regular Bardeen metric using the Newman-Janis algorithm[6] constructed a Kerr-like regular black hole solution. In Boyer-Lindquist coordinate[5] it looks like

$$\begin{aligned} ds^2 = -(1-\frac{2mr}{\Sigma})dt^2 - \frac{4amr\sin^2\theta}{\Sigma}dtd\phi + \frac{\Sigma}{\Delta}dr^2 \\ + \Sigma d\theta^2 + (r^2+a^2+\frac{2a^2mr\sin^2\theta}{\Sigma})\sin^2\theta d\phi^2 \end{aligned} \quad (2)$$

where

$$\Sigma = r^2 + a^2 \cos^2 \theta \ , \ \Delta = r^2 - 2mr + a^2 \ , \tag{3}$$

and

$$m \rightarrow m_{\zeta,\lambda}(r,\theta) = M\left(\frac{r^{2+\zeta}\Sigma^{-\zeta/2}}{r^{2+\zeta}\Sigma^{-\zeta/2} + g^2 r^{\lambda}\Sigma^{-\lambda/2}}\right)^{3/2} \tag{4}$$

where $m = m_{\lambda,\zeta}(r,\theta)$ is a function of both r and θ. Here ζ and λ are two real numbers and a is the rotation parameter. The g is called the *Magnetic Charge* of non-linear electrodynamics. This metric (2) will be referred as the *Rotating Bardeen Regular Black Hole Metric.*

3.2 HORIZONS AND ERGOSPHERE

From the previously mentioned rotating Bardeen regular black hole metric, we can directly see that at $\Delta = 0$, there is a singularity. Now if this equation has double roots that implies the black hole has two horizons which coincides at a point. The structure of the Horizons at various θ will depend on the values of ζ and λ (except for the case when $\theta = \pi/2$).

Now the rotating Bardeen regular black hole metric in Boyer-Lindquist coordinates does not depend on t and ϕ, which means that the Killing Vectors[7] and Killing Symmetries[7] are given by $\eta^\mu = \delta^\mu_t$ and $\xi^\mu = \delta^\mu_\phi$, with δ^μ_a be the *Kronecker delta*. Now when the Killing Vectors become Null we get the *Static Limit Surface* or *Infinite Redshift Surface* or sometimes referred as *Null Killing Surface*. Setting g_{tt} of rotating Bardeen regular black hole metric equal to 0 gives the Null Killing Surface for a rotating Bardeen black hole. Observing the outer Event Horizon and the Null Killing Surface for these black hole, it is verified that the Null Killing Surface always lies outside the Event Horizon. This region between these two surfaces is called *Ergosphere*, which naturally lies outside the black hole. This Ergosphere region has various scientifically valuable aspects. The shape of this region depends mainly on the parameter g and a.

4 CONCLUSION

Einstein's well established theory to explain gravity which happens to be the best theory available to explain facts in large scale, even got questioned when it comes to the center of a Black Hole. And it has been a long period of time since when people are trying to remove this problem. Following some very early ideas ultimately such solution of black hole spacetime has been found where the central singularity is absent.

Such a solution without the central singularity is the Bardeen blackhole solution. In this paper the horizon structure and the ergosphere behaviour of a rotating Bardeen black hole are investigated. It has been well observed that how the two horizons and Ergosphere behave depending on the parameters and the Magnetic Charge g. Further, a couple of applications were studied using the properties of the Ergosphere.

5 REFERENCE

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