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# ABSTRACT

There has been an enigma regarding the singularity in the interior of a Black Hole since ages. This physical impossibility is assumed to be hidden by the existence of the so called Event Horizon of the Black Hole. The singularity gives birth to an extremely high curvature onto the space-time, near the Planck scale. In such a high curvature, it is known that General Relativity breaks down. This failure leads the physicists to think that this singularity might be somehow tamed by higher order corrections to General Relativity or Quantum Gravitational effects. Now, there is a possible way to escape this impossibility at the center of a Black Hole by utilizing the solution to the Einstein equation provided by the well known physicist de Sitter in 1917.

# Contents

# 1 INTRODUCTION

In 1915, Einstein published his famous field equation in the form of a tensor equation

$$G_{\mu\nu} = -8\pi G T \mu\nu \tag{1}$$

which relates the local space time curvature ( the Einstein Tensor $G_{\mu\nu}$) with the local distribution of energy and momentum ( the stress energy or energy momentum tensor $T_{\mu\nu}$ ) within that space time. Later, in 1917, he added an extra term to his field Equation ,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G T \mu\nu \tag{2}$$

where $\Lambda g_{\mu\nu}$ is the extra term with $\Lambda$, called the Cosmological Term. The earlier equation without the Cosmological Constant provides the Minkowski Geometry as the regular solution in the absence of energy momentum tensor. On the other hand if mass is present in the system, the solution outside that matter distribution, is given by Schwarzschild metric. This is a spherically symmetric solution with an inevitable singularity at the center. The other vacuum spherically symmetric solutions can also be reduced to Schwarzschild solution with the help of Birkhoff's theorem [1] in the case $T_{\mu\nu} = 0$. The non-singular modication of the Schwarzschild geometry can be obtained by replacing a singularity with a regular core asymptotically de Sitter as $r \to 0$.

The idea goes back to the 1966 papers of Sakharov [2] who considered $p = -\rho$ as the equation of state in the superhigh density and of Gliner who interpreted p = as corresponding to a vacuum and suggested that it could be a final state in a gravitational collapse[27]. Thus with the requirements of regularity conditions[4] it can be shown that there exists a family of global regular solution asymptotically Schwarzschild at infinity and with the de Sitter vacuum replacing the singularity at the center. In 1968 Bardeen presented the spherically symmetric metric of the same form as Schwarzschild and ReissnerNordstrom metric, describing a non-singular black hole (BH) without specifying the behavior at the center [3]. The very important point was noted in [3] for the first time: that the considered space-time exhibits the smooth changes of topology.

## 2 BASIC EQUATIONS

A static spherically symmetric line element in general can be written in the form[5]

$$ds^2 = e^{\mu(r)}dt^2 - e^{\nu(r)}dr^2 - r^2 d\Omega^2 \tag{3}$$

where $d\Omega^2$ is the metric of unit 2-sphere ie.

$$d\Omega^2 = r^2(d\theta^2 + \sin^2\theta d\phi^2)$$

This metric coefficients satisfy the Einstein Field Equation (1) and reduces to,

$$kT^t{}_t = k\rho(r) = e^{-\nu}(\frac{\nu'}{r} - \frac{1}{r^2}) + \frac{1}{r^2} \tag{4}$$

$$kT^r{}_r = -kp_r(r) = -e^{-\nu}(\frac{\mu'}{r} + \frac{1}{r^2}) + \frac{1}{r^2} \tag{5}$$

$$kT^\theta{}_\theta = kT^\phi{}_\phi = -kp_\perp = -e^{-\nu}(\frac{\mu''}{2} + \frac{\mu'^2}{4} + \frac{(\mu' - \nu')}{2r} - \frac{\mu'\nu'}{4}) \tag{6}$$

Here $k = 8\pi G$ and we have used $c = 1$ due to simplicity. $T^t{}_t = \rho(r)$ and $T^r{}_r = -p_r(r)$ are the energy density and radial pressure respectively. Again $T^\phi{}_\phi = T^\theta{}_\theta = -p_\perp(r)$ are the tangential components of pressure for an anisotropic ideal fluid[5]. The *prime* denotes the differentiation with respect to r. Integration of equation (4) gives,

$$e^{-\nu(r)} = g(r) = 1 - \frac{2GM}{r} \tag{7}$$

where,

$$M(r) = 4\pi \int_0^r \rho(r) x^2 dx \tag{8}$$

in asymptotic case for large r,

$$m = 4\pi \int_0^\infty \rho(x) x^2 dx \tag{9}$$

Now, from equation (4) to (6) we get the Oppenheimer equation[6],

$$k(T_t^t - T_r^r) = k(p_r + \rho) = \frac{e^{-\nu}}{r}(\mu' + \nu') \tag{10}$$

and hydrodynamic equation which generalizes the Tolman-Oppenheimer-Volkoff equation[7] to the case of dierent principal pressures

$$p_\perp = p_r + \frac{r}{2}p_r' + (\rho + p_r)\frac{GM + 4\pi G r^3 p_r}{2(r - 2GM)} \tag{11}$$

# 3 FORMATION OF THE METRIC

To investigate the system , the following requirements were imposed.
a) Regularity of density $\rho(r)$
b) Finiteness of mass $m$
c) Dominant Energy Condition on $T_\mu^\nu$

The Dominant Energy Condition ( DEC ) demands the matter density $\rho(r)$ to be non negative. So for timelike vector $X^a$ , the density $\rho(r) \geq T_{ab}X^aX^b$ , which is specifically the Weak Energy Condition ( WEC ) contained in DEC. So in our case, the condition $T^{00} \geq T^{ab}$[17] , where $a, b$ take $1, 2, 3$ , holds if and only if $\rho \geq 0$ and $-\rho \leq p_k \leq \rho$ , where $k$ takes $1, 2, 3$ ie. including the condition of non negativity , each principal pressures does not exceed the energy density. We can write this condition as

$$\rho \geq 0 \tag{12}$$

and

$$\rho + p_k \geq 0, \tag{13}$$

where $k$ takes $1, 2, 3$.

These conditions have following impacts on a system represented by equations $(4), (5), (6)$.

From equation (7) at $r \to \infty$ , finiteness of mass demands $\nu(r)$ to be 0. Now if m is finite at $r \to \infty$ then from the form of the mass we can say the density $\rho(r)$ has to fall faster than $r^{-3}$. Again DEC demands the pressure components $p_k$ has to fall equally or faster as density as $r \to \infty$. Now from equation (8) we can see that $(p_r + \rho)$ tends to vanish at $r \to \infty$ , so, the RHS part ie. $(\mu' + \nu')$ also tends to vanish. At $r \to \infty$ , $\nu$ vanishes so, $\mu'$ also vanishes at $r \to \infty$ ; which gives $\mu = constant$ at $r \to \infty$. This $\mu$ can be easily scaled to 0 at $r \to \infty$.

The regularity of density ie. $\rho(r) < \infty$ makes the mass term vanish at $r \to 0$. Now as $\rho(r)$ is finite at $r \to 0$,so the mass should fall as $r^{-3}$. From equation (7) , as $M$ is falling faster than the denominator at the second term at RHS so at $r \to 0$ , $e^{-\nu(r)} = 1$ . So, $\nu(r) = 0$ at $r \to 0$. Again from equation (8), the regularity of density and DEC demands that $(p_r + \rho) < \infty$at $r \to 0$ and so, $(\mu' + \nu') = 0$ at this limit. Thus we can say $(\mu + \nu) = constant = \mu(0)$ ; as $\nu(r = 0)$ turned out to be 0 earlier.

The WEC determines the sign of the function $(\mu' + \nu')$. As, for timelike vectors, WEC tells that the principal pressure can not exceed the matter density. So, from equation (8) , $T^t_t > T^r_r$ and that gives $(p_r + \rho) > 0$ where time is timelike and space is spacelike. Again inside the horizon $r$ is timelike and $T^t_t$ represents a tension, then , $(T^t_t - T^r_r) = -(p_r + \rho)$. So, basically the term $(p_r + \rho) > 0$ everywhere which gives $(\mu' + \nu') > 0$ everywhere ie. the slope of the function $(\mu + \nu)$ has a positive slope. So, this function is growing from $\mu(0)$ at $r = 0$ to $(\mu + \nu) = 0$ at $r = \infty$. Thus we can say $\mu(0) < 0$[8]. But again range of this parameter includes 0 as a value. So, $\mu(0) = 0$ at $r = 0$ and also at $r = \infty$. Therefore it is evident that the function $(\mu + \nu) = 0$ everywhere, as the derivative of the function is non negative. So we get the general form of the required metric as follows,

$$ds^2 = g(r)dt^2 - \frac{dr^2}{g(r)} - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{14}$$

where $g(r) = e^{\mu(r)} = e^{-\nu(r)}$

The fact that $\mu + \nu = 0$ is everywhere directly implies that

$$p_r = -\rho$$

Now from equation (9) we get, $p_\perp = p_r + \frac{r}{2}p'_r$ at the limit, $r \to 0$. Again at $r = 0$, $\rho + p_r = 0$, so,

$$p_\perp = -(\rho + \frac{r}{2}\rho')$$

Now, the DEC and the regularity of density requires $p_k + \rho < \infty$; so, the derivative of $\rho$ is also finite. This directly gives the Equation of State near the center as

$$p = -\rho \tag{15}$$

which corresponds to the solution of the Einstein Field equation $G_{\mu\nu} = -\Lambda g_{\mu\nu} - 8\pi GT\mu\nu$. Here the $\Lambda g_{\mu\nu}$ term serves as the stress energy tensor for a vacuum with the source term as $T_{\mu\nu} = \rho_{vac} g_{\mu\nu}$, where $\rho_{vac} = (8\pi G)^{-1}\Lambda = constant$. This solution is what is called de Sitter metric[9] , asymptotic at $r \to 0$ limit. This is as follows.

$$ds^2 = (1 - \frac{r^2}{{r_0}^2})dt^2 - \frac{dr^2}{1 - \frac{r^2}{{r_0}^2}} - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{16}$$

where $r_0^2 = 3/\Lambda$ with the Cosmological Constant $\Lambda$. We see that $\Lambda$ has appeared at the origin though it was not present in the basic equation. The weak energy condition demands the monotonic decrement of density function and a little analysis shows that this metric has not more than two horizons.

The equation (7) gives us the form of the coefficients in the general form of the metric in equation (3). Now the finiteness of mass makes $\rho$ vanish quicker than $r^-3$. Again from WEC we can see $\rho_\perp + p_r \geq 0$ which leads to $\rho' < 0$ ie. the density profile should decrease. Now, $\mu' = 0$ at $r \to \infty$ ie. $\mu = constant$ at infinity. Here we apply a boundary condition $\mu = 0$ at $r \to \infty$, which gives the Schwarzschild solution asymptotic at $r \to \infty$.

$$ds^2 = (1 - \frac{2Gm}{r})dt^2 - \frac{dr^2}{(1 - \frac{2Gm}{r})} - r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{17}$$

Summarizing, we can conclude that if we require asymptotic flatness, regularity of a density and metric at the center and finiteness of mass, then having imposed these conditions on the Einstein Field Equation, with the help of DEC we can define the family of asymptotically flat solution with a regular center whose behaviour is asymptotically de Sitter at the center and asymptotically Schwarzschild at $r \to \infty$. Thus with the monotonically decreasing density function we get a metric function $g(r)$ smoothly evolving between de Sitter and Schwarzschild metric function,

$$(1 - \frac{\Lambda}{3}r^2)[origin] \longleftarrow g(r) \longrightarrow (1 - \frac{2Gm}{r})[spatial - infinity]$$

and which has not more than two horizons.

# 4 SOURCE TERM AND COSMOLOGICAL TERM

The field equations were constructed so as to allow flat Minkowski space as a solution in the absence of all sources. If we drop this requirement, then a slightly more general symmetric and divergence-free tensor can be put on the LHS as[10],

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} + \Lambda g_{\mu\nu} = -kT_{\mu\nu} \tag{18}$$

where $\Lambda$ is a constant and can be negative positive or zero. Einstein introduced this term to his field equation to have a Static Universe. Though it belongs to the equation much as an integral constant. Now from the previous equation we get that

$$R_{\mu\nu} = -\Lambda g_{\mu\nu} + \frac{1}{2}g_{\mu\nu} - kT_{\mu\nu}$$

Now as $R = kT + 4\Lambda$, so we can write,

$$= -\Lambda g_{\mu\nu} + g_{\mu\nu}\frac{kT}{2} + 2\Lambda g_{\mu\nu} - kT_{\mu\nu} \tag{19}$$

$$= \Lambda g_{\mu\nu} - g_{\mu\nu}\frac{kT}{2} - kT_{\mu\nu} \tag{20}$$

Now, if we consider the source to be absent , then

$$R_{\mu\nu} = \Lambda g_{\mu\nu}$$

Thus it is evident that this form of Einstein equation does not permit flat space time in the absence of source term. Soon after the introduction of $\Lambda g_{\mu\nu}$, in the same year of 1917, de Sitter found the solution to the equation (2) with $T_{\mu\nu} = 0$

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 0$$

which made evident that a matter is not necessary to produce the property of inertia. In de Sitter geometry the metric reads as follows,

$$ds^2 = (1 - \frac{\Lambda}{3}r^2)dt^2 - \frac{dr^2}{(1 - \frac{\Lambda}{3}r^2)} - r^2 d\Omega^2$$

Here, $\Lambda$ has to be constant by virtue of the contracted Bianchi identities[1] $G^{\mu\nu}_{;\nu} = 0$. It plays the role of a universal repulsion whose physical manifestation remained obscure during several decades when de Sitter geometry was mainly used as a simple testing ground for developing quantum field technics in a curved space time.

Now the same analysis can be done for the modified Field Equation with cosmological term, what has been done for derivation of Schwarzschild metric from the unmodified Einstein Field equation. This leads to the *de Sitter-Schwarzschild* metric which reads as follows.

$$g(r) = 1 - \frac{2Gm}{r} - \frac{\Lambda r^3}{3} \tag{21}$$

This is so called as this turns into Schwarzschild metric near the origin and into de Sitter metric (if $\Lambda$ is positive ) for large $r$.

Though at first we have started the formulation without the background cosmological term but if that class of metrics is extended to the case of non zero background cosmological constant $\lambda$ ( Here the term $\lambda$ is used to denote the deviation in magnitude from the Cosmological term $\Lambda$, which appeared in the previous equation) by introducing[11]

$$T^t_t(r) = \rho(r) + (8\pi G)^{-1}\lambda$$

Then the metric function reads as equation ,

$$g(r) = 1 - \frac{2GM(r)}{r} - \frac{\lambda r^3}{3}$$

This is asymptotically de Sitter at limit $r \to 0$ and at $r \to \infty$ both and has not more than three horizons for the case of two lambda scales[12]. The source term thus evolve from $(\Lambda + \lambda)g_{\mu\nu}$ to $\lambda g_{\mu\nu}$. For the general form of the metric equation (12) the source term satisfies the equation $T^t_t = T^r_r$. Again from the equation (6) we got $T^\theta_\theta = T^\phi_\phi$. So, to make a smooth transition the source term need to satisfy these equations,

$$T^t_t = T^r_r; T^\theta_\theta = T^\phi_\phi \tag{22}$$

and the equation of states (15). So,it connects the de Sitter region $T_{\mu\nu} = \rho_0 g_{\mu\nu}$ with the Minkowski vacuum $T_{\mu\nu} = 0$ at infinity or two de Sitter vacua with two different values of cosmological constant.

$$k^{-1}\Lambda g_{\mu\nu} \longleftarrow T^{vac}_{\mu\nu} \longrightarrow k^{-1}\lambda g_{\mu\nu}$$

There are two basic approaches to $\Lambda g_{\mu\nu}$. At one approach the term is shifted on the right hand side of the Einstein equation and treated as the part of the matter content. On the second approach this is preferred to keep on the left hand side of the equation as a geometrical entity and is treated as the constant of nature. In any case direct association of a cosmological term $\Lambda g_{\mu\nu}$ with the vacuum stress tensor $\rho_{vac} g_{\mu\nu}; \rho_{vac} = k^{-1}\Lambda$ seems widely accepted today[13].

Here we started from the Einstein equation (1) and found the case when we have on the right-hand side a stress-energy tensor $T_{\mu\nu}$, describing a spherically symmetric anisotropic vacuum with variable density and pressures. Now this term can be shifted to the left hand side and be treated as an evolving geometrical entity. A variable cosmological constant (or we can say a cosmological tensor) is $\Lambda_{\mu\nu} = kT_{\mu\nu}$[14]. Now this satisfies the Einstein equation and from Bianchi identity we can say that covariant derivative of this cosmological tensor vanishes. This also satisfies the equation of state (15) and gives

$$k\rho^{\alpha} = \Lambda^t_t; kp^{\alpha}_r = -\Lambda^r_r; kp^{\alpha}_{\perp} = -\Lambda^{\theta}_{\theta} = -\Lambda^{\phi}_{\phi} \tag{23}$$

This $\Lambda_{\mu\nu}$ includes $\Lambda g_{\mu\nu}$ as a special case when $\Lambda = k\rho^{vac} = constant$.

# 5 GEOMETRY

The metric function of de Sitter-Schwarzschild geometry[15] reads as equation (21). It can be thought of as de Sitter space just disturbed by the presence of single spherical mass. Here we have used the regularity conditions. Imposing these on the metric of form (3) , leads to a specific kind of geometry which is asymptotically de Sitter at $r \to 0$ and asymptotically Schwarzschild at $r \to \infty$ without the background cosmological term. The key point of de Sitter-Schwarzschild geometry is the existence of two horizons, a black hole Event Horizon $r_+$ and an internal Cauchy Horizon $r_-$.

The exact analytical solution was found for the density profile[16]

$$\rho(r) = \rho_0 e^{\frac{-r^3}{r_0^2 r_g}} \tag{24}$$

where $r_0^2 = 3/\Lambda; r_g = \frac{2Gm}{c^2} = 2Gm$

Here we have used c = 1 for convenience. This can describe a smooth de Sitter Schwarzschild transition in a semi-classical model in a spherically symmetric gravitational field. We can show that it satisfies the condition of finiteness of mass at $r \to \infty$ limit.

From the form of $m$ we can see,

$$\begin{aligned} m &= 4\pi \int_0^\infty \rho(r) r^2 dr \\ &= \frac{4\pi}{3} \int_0^\infty \rho_0 e^{\frac{-r^3}{r_0^2 r_g}} d(r^3) \\ &= \frac{4\pi\rho_0}{3}(r_0^2 r_g)(1-0) \\ &= \frac{4\pi\rho_0}{3}(r_0^2 r_g) \\ &= Finite \end{aligned} \tag{25}$$

Again within the finite range of $r$ the form of mass function $M(r)$ gives , $M(r) = \int_0^r 4\pi\rho(x)x^2 dx$

$$\begin{aligned} M &= \int_0^r 4\pi x^2 \rho(x) dx \\ &= 4\pi\rho_0 \int_0^r e^{-Ax^3} x^2 dx; A = 1/(r_0^2 r_g) \\ &= \frac{4\pi\rho_0}{3A}(1 - e^{-Ar^3}) \\ &= \frac{4\pi\rho_0}{3}(r_0^2 r_g)(1 - e^{\frac{-r^3}{r_0^2 r_g}}) \end{aligned}$$

Thus from the value of $m$ in the previous calculation we can say that ,

$$M = m(1 - e^{\frac{-r^3}{r_0^2 r_g}}) \tag{26}$$

In the same way this density profile gives a particular form of metric function $g(r)$.

$$
\begin{aligned}
g(r) &= 1 - \frac{2GM(r)}{r} \\
&= 1 - \frac{2G}{r} m(1 - e^{\frac{-r^3}{r_0^2 r_g}}) \\
&= 1 - \frac{r_g}{r}(1 - e^{\frac{-r^3}{r_0^2 r_g}})
\end{aligned}
$$

Now if we equate this $g(r)$ function to 0 then, we get

$$
\begin{aligned}
\frac{r_g}{r}(1 - e^{\frac{-r^3}{r_0^2 r_g}}) &= 1 \\
or, e^{\frac{-r^3}{r_0^2 r_g}} &= 1 - r/r_g
\end{aligned}
$$

The proper plotting of this equation gives, that except for $r = 0$, for two values of $r$ , there are $g(r) = 0$, ie. this geometry has got two horizons. On the $figure 1$, $r_+$ is representing *Event Horizon* and $r_-$ is representing *Cauchy Horizon*. Along with these two surfaces there are another two surfaces called *Zero Gravity Surface* and *Zero Curvature Surface*. On the figure, $r_c$ represents the Zero Gravity Surface and $r_s$ represents the *Zero Curvature Surface*. At Zero Curvature Surface the value of $R(r)$ vanishes. This gives[15]

$$
r_s = (\frac{4}{3} r_0^2 r_g)^{1/3} = (\frac{m}{\pi \rho_0})^{1/3} \tag{27}
$$

from equation (24). From the plotting we can see that Horizons come together at the value of a mass parameter $m_{crit}$, which puts a lower limit on a Black Hole mass[15].

## 5.1 $m > m_{crit}$ Condition

For $m > m_{crit}$ de Sitter-Schwarzschild geometry describes the vacuum non singular black hole and global structure of space-time is shown in $figure 2$, contains an infinite sequence of black and white holes whose future and past singularities are replaced by regular cores asymptotically de Sitter at $r \to 0$. It resembles the Reissner-Nordstrom[18] case but differs at the surface $r = 0$

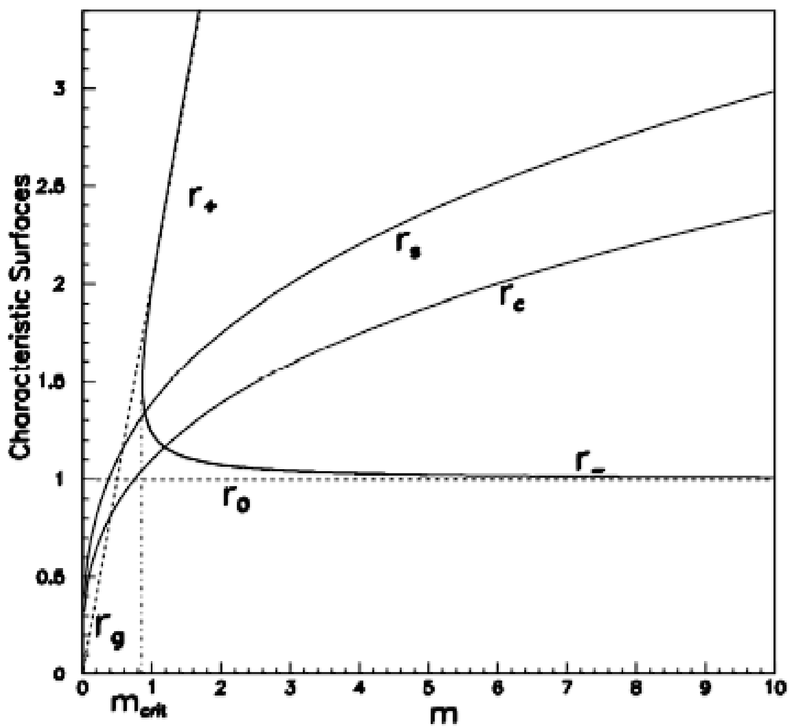


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Figure 1: Characteristics surfaces of de Sitter - Schwarzschild geometry

which are regular surfaces of de Sitter - Schwarzschild geometry in this case. Vacuum non singular black hole and white holes are represented by $\Lambda BH$ and $\Lambda WH$ respectively. In both cases, de Sitter vacuum appears instead of a singularity at approaching each regular surface $r = 0$.

### 5.1.1 Vacuum Non Singular White Hole

Replacing a Schwarzschild singularity with the regular core transforms the space-like singular surfaces r = 0 of Schwarzschild geometry into the time-like regular surfaces r = 0. This surface is in the future of a $\Lambda BH$ and in the past of a $\Lambda WH$. The regular core in the past of a $\Lambda WH$ models an early evolution of an expanding universe. The expansion starts from a regular surface $r = 0$ with a nonsingular nonsimultaneous de Sitter bang followed by a Kessner like stage of anisotropic expansion[19].

Though the existence of a White Hole in a singular version has been

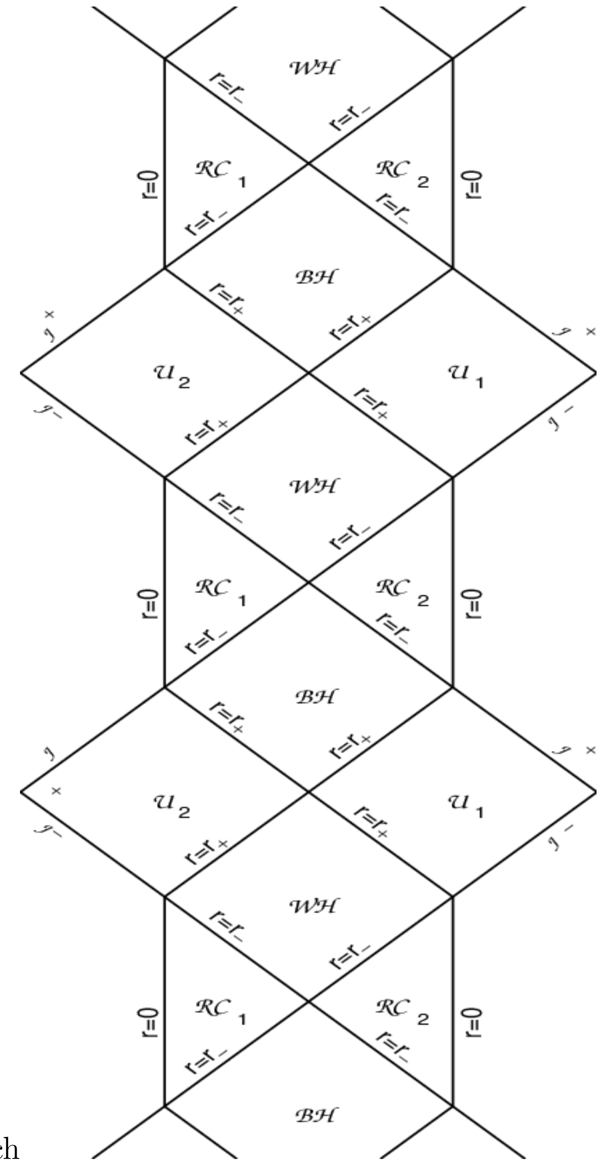


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Figure 2: Global structure for de Sitter Schwarzschild space time

forbidden since the singularity open into the future of a universe breaks the predictability in the universe[20]. In the case of $\Lambda WH$ predictability is only restricted by the existence of the Cauchy Horizon in de Sitter Schwarzschild geometry.

### 5.1.2 Vacuum Non Singular Black Hole

A $\Lambda BH$ emits Hawking radiation from both horizons with the Gibons - Hawking temperature[21]. The form of the temparature-mass diagram is generic for de Sitter Schwarzschild geometry. The temperature on the BH horizon drops to 0 at $m = m_{crit}$, while the Schwarzschild asymptotic requires $T_+ \to 0$ as $m \to \infty$. The temparature mass curve thus has a maximum between $m_{crit}$ and $m \to \infty$.

For particular form of density profile (24) the temperature is given by

$$T_h = \frac{\hbar c}{4\pi k r_0}[\frac{r_0}{r_h} - \frac{3r_h}{r_0}(1 - \frac{r_h}{r_g})]$$

The mass at the maximum and the temperature of the phase transition are

$$m_{tr} \simeq 0.38 m_{pl}\sqrt{\rho_{pl}/\rho_0}; T_{tr} \simeq 0.2 m_{pl}\sqrt{\rho_{pl}/\rho_0}$$

## 5.2 $m < m_{crit}$ Condition and G-Lump

For masses $m < m_{crit}$ de Sitter-Schwarzschild geometry describes a self-gravitating particle-like vacuum structure, globally regular and globally neutral. It resembles Colemans lumps - non-singular, non-dissipative solutions of finite energy, holding themselves together by their own self gravitation[**?**]. G-lump holds itself together by gravity due to balance between gravitational attraction outside and gravitational repulsion inside of zero-gravity surface $r = r_c$. For the case of density profile (20) it is perfectly localized. It is shown in figure (3).

Since de Sitter vacuum is trapped within a G Lump , it can be modelled by a spherical bubble with monotonically decreasing density. It's geometry is described by the metric

$$ds^2 = d\tau^2 - \frac{2GM(r)}{r(R,\tau)}dr^2 - r^2 d\Omega^2$$

The equation of motion $\dot{r}^2 - \frac{2GM}{r} = f(R)$. This resembles the equation of motion of a particle in a potential $V(r) = -\frac{GM}{r}$, with the constant of integration $f(R)$ playing the role of total energy $f = 2E$. A spherical bubble can be described by the minisuperspace model with a single degree of freedom[22]. Zero point vacuum energy for G-lump, which clearly represents an elementary spherically symmetric excitation of a vacuum defined macroscopically by its symmetry, is evaluated as its minimal quantized energy.

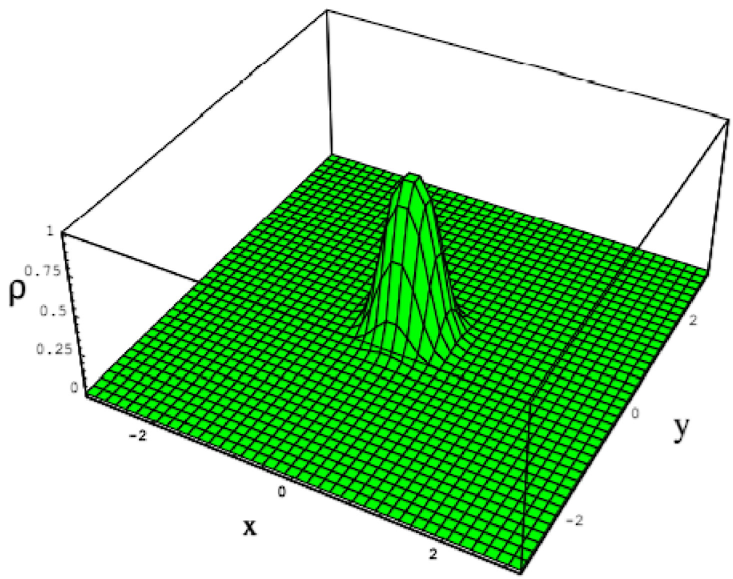


Figure 3: G-Lump for the case $r_g = 0.1 r_0 (m \simeq 0.06 m_{crit})$

# 6 COSMOLOGICAL TERM AS A SOURCE MASS

The mass of both G Lump and $\Lambda BH$ is directly connected to cosmological term $\Lambda_{\mu\nu}$ by the formula,

$$m = (2G)^{-1} \int_0^\infty \Lambda_t^t r^2 dr$$

and relates mass to the de Sitter vacuum at the origin. In de Sitter-Schwarzschild geometry the parameter $m$ is identified as a gravitational mass by flat asymptotic at infinity. A mass is related to cosmological term, since de Sitter vacuum appears as $r \to 0$. This fact does not depend on extension of a cosmological term from $\Lambda g_{\mu\nu}$ to $\Lambda_{\mu\nu}$. No matter what the matter of source is, the mass term is related to the de Sitter vacuum trapped inside an object and the breaking of space time symmetry. De Sitter vacuum supplies an object with mass via smooth breaking of space-time symmetry from the de Sitter group in its center to the Lorentz group at its infinity. As an application of this de Sitter Schwarzschild geometry, it is used to estimate limits on sizes of fundamental particles whose masses are related to de Sitter vacuum through the Higgs mechanism[23].

# 7 TWO - LAMBDA GEOMETRY

For finite value of cosmological constant at infinity, there are two vacuum scale, $\Lambda$ at the center and $\lambda$ at infinity. The geometry has maximum three horizons depending on the value of mass $m$ and the parameter $q = \sqrt{\frac{\Lambda}{\lambda}}$[24] ; the internal Cauchy Horizon $r_-$, Black Hole Event Horizon $r_+$ and the Cosmological Horizon $r_{++}$. From the figure we can see that in the range of masses $M_{cr1} < m < M_{cr2}$, geometry has three horizons and describes a vacuum cosmological non singular black hole. The global structure of space time contains an infinite sequence of $\Lambda BH$ and $\Lambda WH$, their future and past regular cores asymptotically de Sitter with $\Lambda g_{\mu\nu}$ as $r \to 0$ and with $\lambda g_{\mu\nu}$ as $r \to \infty$ in the region between cosmological horizons and space-like infinities[25]. The case $m = M_{cr1}(r_+ = r_-)$ is an extreme black hole state which appears due to replacing a singularity with a de Sitter core. The critical value of mass $M_{cr1}$ at which $r_- = r_+$, puts the lower limit on a black hole

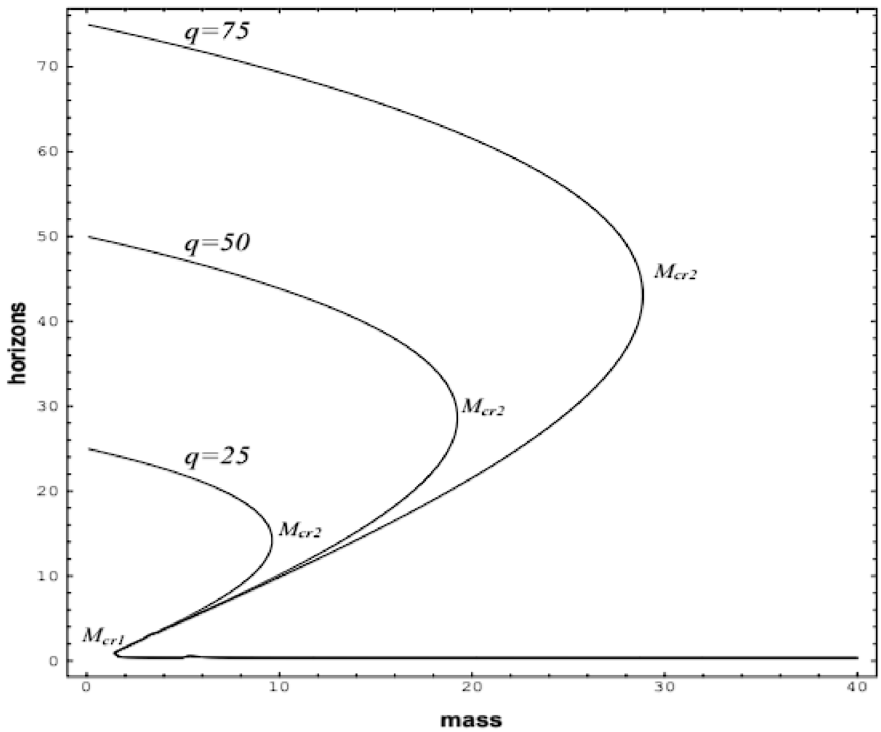


Figure 4: Horizon-Mass diagram of two lambda geometry

mass as discussed earlier. It practically does not depend on the parameter $q = \sqrt{\Lambda/\lambda}$.

# 8 DISCUSSION

Thus we have seen that there exists the class of globally regular solutions to the GR equations $(4)-(6)$, with the algebraic structure of a source term (18), interpreted as spherically symmetric vacuum with variable density and pressure $T_{\mu\nu}^{vac}$ associated with a variable cosmological term $\Lambda_{\mu\nu} = 8GT_{\mu\nu}^{vac}$. Moreover, the asymptotics of this $\Lambda_{\mu\nu}$ at the origin is the Cosmological term introduced by Einstein, $\Lambda g_{\mu\nu}$. For this class of metrics the mass term is defined by the ADM formula

$$m = 4\pi \int_0^\infty \rho(x) x^2 dx$$

This term is related to both de Sitter vacuum trapped in the origin and to breaking of space-time symmetry.

In a recent paper on -variability, Overduin and Cooperstock distinguished two basic approaches to $\Lambda g_{\mu\nu}$[26]. In the first approach $\Lambda g_{\mu\nu}$ is shifted onto the right-hand side of the field Einstein equations (2) and treated as a dynamical part of the matter content. This approach, characterized by Overduin and Cooperstock is similar to that of Gliner who interpreted $\Lambda g_{\mu\nu}$ as vacuum stress-energy tensor. In contrast, the idealistic approach prefers to keep $\Lambda$ on the left-hand side of equation (2) as geometrical entity and treat it as a constant of nature. This classification suggests that any variable  must be identified with a matter, in such a case the best fit for $T_{\mu\nu}^{vac} = (8G)^{-1}\Lambda_{\mu\nu}$ would be gravitational vacuum polarization. Again the term $T_{\mu\nu}$ can be shifted to the left hand side from the right hand side of equation (1) and have $\Lambda_{\mu\nu} = 8\pi G T_{\mu\nu}$ as the evolving geometrical entity. It was also shown that the connection between r-dependent cosmological term $\Lambda_{\mu\nu}$ and the ADM mass seems to satisfy Einsteins version of Machs principle - no matter, no inertia ; if we explicitly separate two aspects of the problem of inertia: existence of inertial frames and existence of inertial mass.

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