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Summary

This first chapter illustrates how to use various elements of this text book template, such as definitions, theorems and exercises. You may want to start each chapter with a meta summary like this one, to explain to the reader what the chapter is all about, why it is important and how it fits into the bigger picture of the book. Another useful tip is to put the contents of each chapter into a separate LaTeX file and then use the command `\input{}` to include the chapter in the main document.

2.1 First section

Wo are 1234774567 1234567 Let's start out with the following theorem.

Theorem 2.1 (Logic algebra) Let P, Q and R be logical propositions (true or false). Then the following propositions are true:

$$
\begin{array}{lll}
P \wedge Q \Leftrightarrow Q \wedge P & P \vee Q \Leftrightarrow Q \vee P & \text{(commutative laws)} \\
(P \wedge Q) \wedge R \Leftrightarrow P \wedge (Q \wedge R) & (P \vee Q) \vee R \Leftrightarrow P \vee (Q \vee R) & \text{(associative laws)} \\
P \wedge (Q \vee R) \Leftrightarrow (P \wedge Q) \vee (P \wedge R) & P \vee (Q \wedge R) \Leftrightarrow (P \vee Q) \wedge (P \vee R) & \text{(distributive laws)} \\
\neg(P \wedge Q) \Leftrightarrow \neg P \vee \neg Q & \neg(P \vee Q) \Leftrightarrow \neg P \wedge \neg Q & \text{(De Morgan's laws)}
\end{array}
$$

Proof. We prove the first of De Morgan's laws and leave the proofs of the remaining propositions as exercises. To prove the statement, we create a truth table and fill in all possible values (true or false) for the propositions P and Q. Each of these propositions can be either true or false and we thus obtain the following truth table with four cases:

$\neg$	$(P$	$\wedge$	$Q)$	$\Leftrightarrow$	$\neg$	P	$\vee$	$\neg$	Q
	T		T			T			T
	T		F			T			F
	F		T			F			T
	F		F			F			F

By definition of the logical operators, we compete the table to obtain

$\neg$	$(P$	$\wedge$	$Q)$	$\Leftrightarrow$	$\neg$	P	$\vee$	$\neg$	Q
F	T	T	T	**T**	F	T	F	F	T
T	T	F	F	**T**	F	T	T	T	F
T	F	F	T	**T**	T	F	T	F	T
T	F	F	F	**T**	T	F	T	T	F

It follows that the statement we want to prove (the equivalence $\Leftrightarrow$) is always true (a *tautology*), which proves the statement. □

2.2 Second section

We begin our next section with the following central definition.

Definition 2.1 (Rational Cauchy sequence) A rational Cauchy sequence is a rational sequence $(x_n)_{n=0}^{\infty}$ such that

$$\forall \epsilon \in \mathbb{Q}_+ \; \exists N \in \mathbb{N} : m, n \geq N \Rightarrow |x_m - x_n| < \epsilon. \tag{2.1}$$

In other words, for each (small) rational number $\epsilon > 0$ there is a (big) number N such that the distance $|x_m - x_n|$ between x_m and x_n is less than ϵ if both m and n are larger than or equal to N.

A remark may be in order here. This definition is concerned with rational *Cauchy sequences. We will later encounter a similar definition of* real *Cauchy sequences.*

Exempel 2.1 (Solving the equation $x^2 = 2$) Consider the equation $x^2 = 2$. It is easy to prove that this equation does not have any rational solutions. However, consider the following iteration formula:

$$x_n = \frac{x_{n-1} + 2/x_{n-1}}{2}, \tag{2.2}$$

where $n = 1, 2, 3, \ldots$ and $x_0 = 1$. The resulting sequence of rational numbers quickly approaches a number in the vicinity of $x = 1.4142135623731$:

$$\begin{array}{rcl}
x_0 &=& 1 \\
x_1 &=& (x_0 + 2/x_0)/2 \;\;=\;\; 1.5 \\
x_2 &=& (x_1 + 2/x_1)/2 \;\;\approx\;\; 1.4166666666667 \\
x_3 &=& (x_2 + 2/x_2)/2 \;\;\approx\;\; 1.4142156862745 \\
x_4 &=& (x_3 + 2/x_3)/2 \;\;\approx\;\; 1.4142135623747 \\
x_5 &=& (x_4 + 2/x_4)/2 \;\;\approx\;\; 1.4142135623731 \\
x_6 &=& (x_5 + 2/x_5)/2 \;\;\approx\;\; 1.4142135623731 \\
x_7 &=& (x_6 + 2/x_6)/2 \;\;\approx\;\; 1.4142135623731 \\
x_8 &=& (x_7 + 2/x_7)/2 \;\;\approx\;\; 1.4142135623731 \\
x_9 &=& (x_8 + 2/x_8)/2 \;\;\approx\;\; 1.4142135623731 \\
x_{10} &=& (x_9 + 2/x_9)/2 \;\;\approx\;\; 1.4142135623731
\end{array}$$

We will later see that this iteration, or any other equivalent iteration, defines the real number $\sqrt{2}$.

2.3 Long Long Long Long Long Title

Now let's move on to the definition of the real number system. This may be defined in a multitude of ways, one of which is to think about a real number as a rational Cauchy sequence, or rather the equivalence class of Cauchy sequences "converging to" that number.

Definition 2.2 (The real numbers $\mathbb{R}$) The real numbers $\mathbb{R}$ is the set of all equivalence classes of rational Cauchy sequences.

Now that this is settled, lets prove the completeness of the real number system.

Theorem 2.2 (The completeness of the real numbers) Let $(x_n)_{n=0}^{\infty}$ be a sequence of real numbers. Then $(x_n)_{n=0}^{\infty}$ is convergent if and only if it is also a real Cauchy sequence.

Proof. Write $x_m = [(x_{mn})_{n=0}^{\infty}]$ where x_{mn} is the nth number in a rational Cauchy sequence representing the real number x_m. And so on.... □

For further reading, there are several excellent works that one could cite, such as (Tao 2006; Turing 1936–7).

Exercises

Exercise 2.1 Let $A = \{1, 2, 3\}$ and $B = \{2, 3, 4\}$. Determine the following sets.
(a) $A \cup B$ (b) $A \cap B$ (c) $A \setminus B$ (d) $A \times B$

Exercise 2.2 Let $A = \{1, 3, 5, 7, 9\}$ and $B = \{2, 4, 6, 8, 10\}$. Determine the following sets.
(a) $A \cup B$ (b) $A \cap B$ (c) $A \setminus B$ (d) $A \times B$

Exercise 2.3 Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$ and $C = \{3, 4, 5\}$. Determine the following sets.
(a) $A \cup B \cup C$ (b) $A \cap B \cap C$ (c) $(B \setminus A) \cap C$ (d) $(A \times B) \times C$

Problem

Problem 2.1 Interpret the following set definition (Russell's paradox) and discuss whether $X \in X$ or $X \notin X$:

$$X = \{x \mid x \notin x\}. \tag{2.3}$$

Computer exercises

Computer exercise 2.1 Write a program that generates the sequence $(x_n)_{n=0}^{100}$ for $x_n = n$.

Computer exercise 2.2 Write a program that generates the odd numbers between 1 and 100.

Computer exercise 2.3 Write a program that computes the sum $\sum_{n=0}^{100} x_n$ for $x_n = n$.

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3.1 First section

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4 Third chapter

Summary

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