Elec 4700 Assignment 2:
Finite Difference Method

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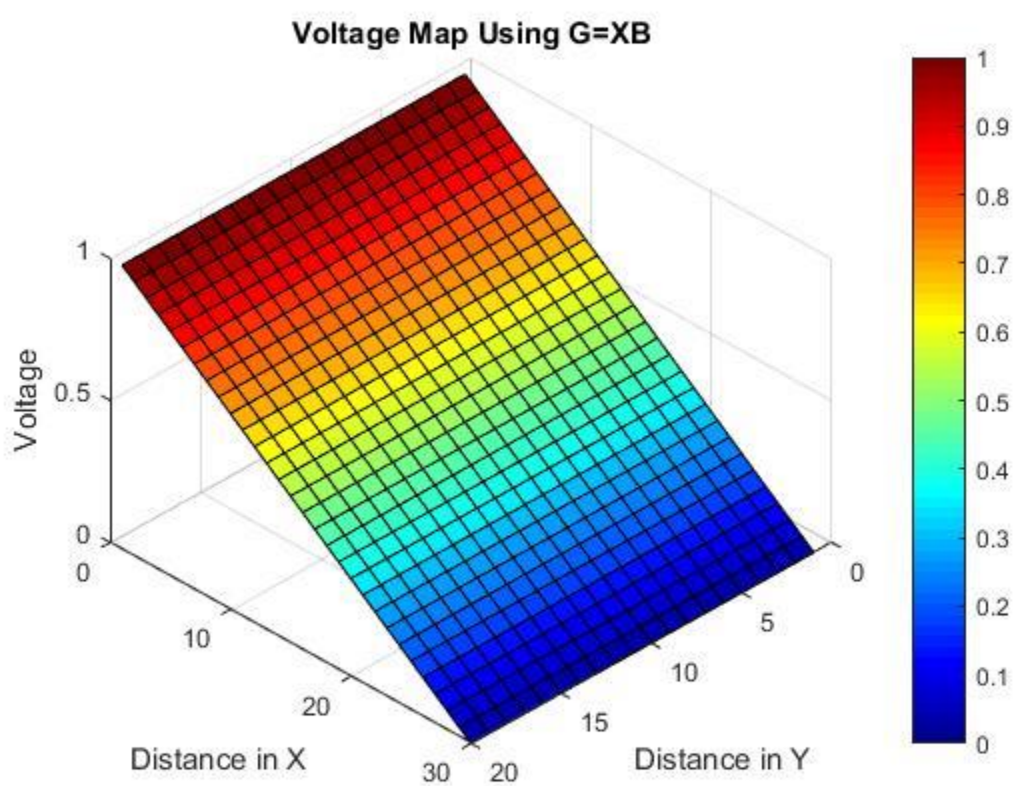
Introduction

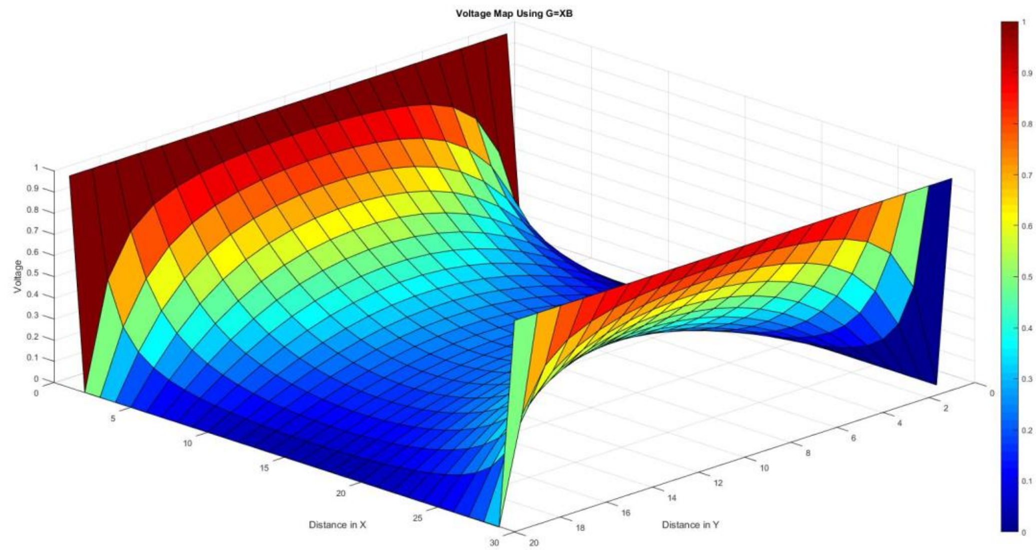
This assignment will focus on the finite difference method and how to use it to arrive at a solution for a given problem. To do this, first the voltage distribution in a 1D scenario will be calculated using FD method, then a 2D scenario solved. Finally the FD method will be used in a second 2D example with a bottle neck represented through two boxes with very low conductivity. From here the electric fields and current density will be found.

Voltage Distribution Using FD Method

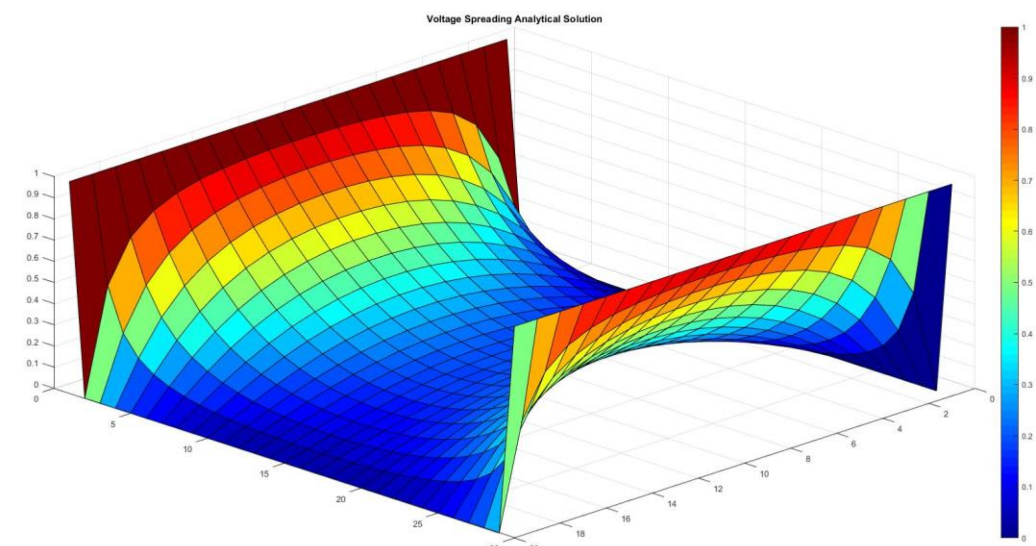
The first task of this assignment was to model a 1D scenario of voltage distribution through a region with equal conductivity. First the region was set to a specific size, and the G and boundary matrices were populated. The boundary matrix was populated with a 1 whenever the nested loops had a point on the left of the region, and a zero when a point was on the very right of the region. In the middle the G matrix was populated with a -4 at the location of n, and a 1 at the location of the min and max x and y values. The voltage matrix was then calculated by dividing the G matrix by the boundary vector. This new matrix was remapped to create the voltage matrix. This matrix was then plotted to produce the following plot.



The code was then copied, and modified to include two more boundary conditions to create the 2D problem and solution. This was done by populating the B matrix with a 1 when at the left and right edges, and a 0 at the top and bottom edges. The voltage matrix was then calculated the same way, and remapped back to get the final result. This yielded the following plot.



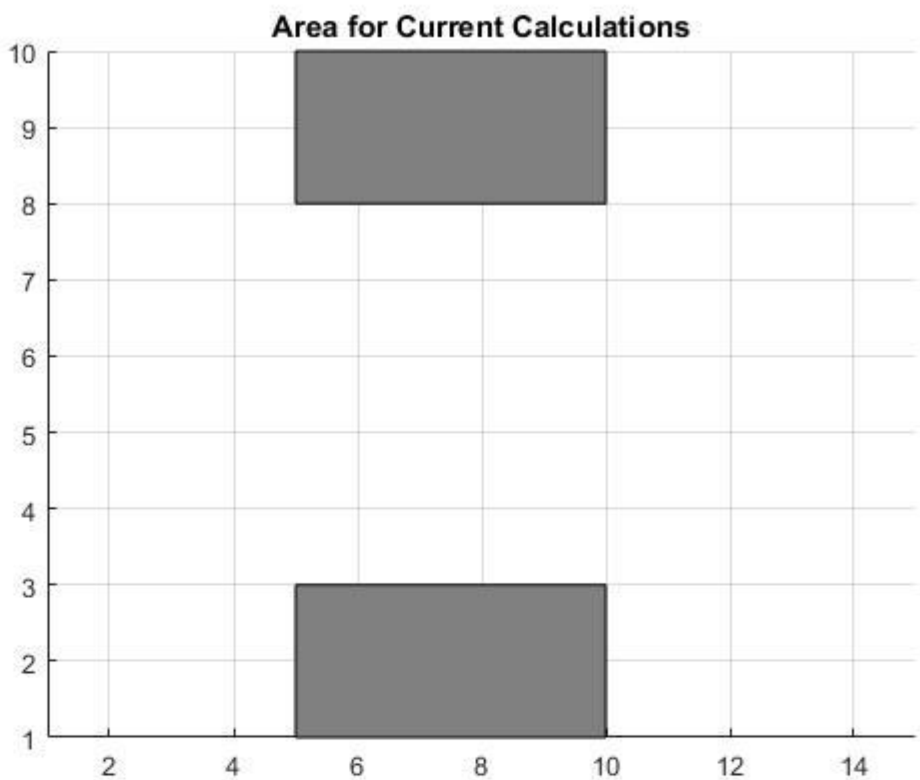
To double check that the solution obtained was correct, Laplace's equation was used with an analytical solution to double check the results. The resulting plot can be found below.



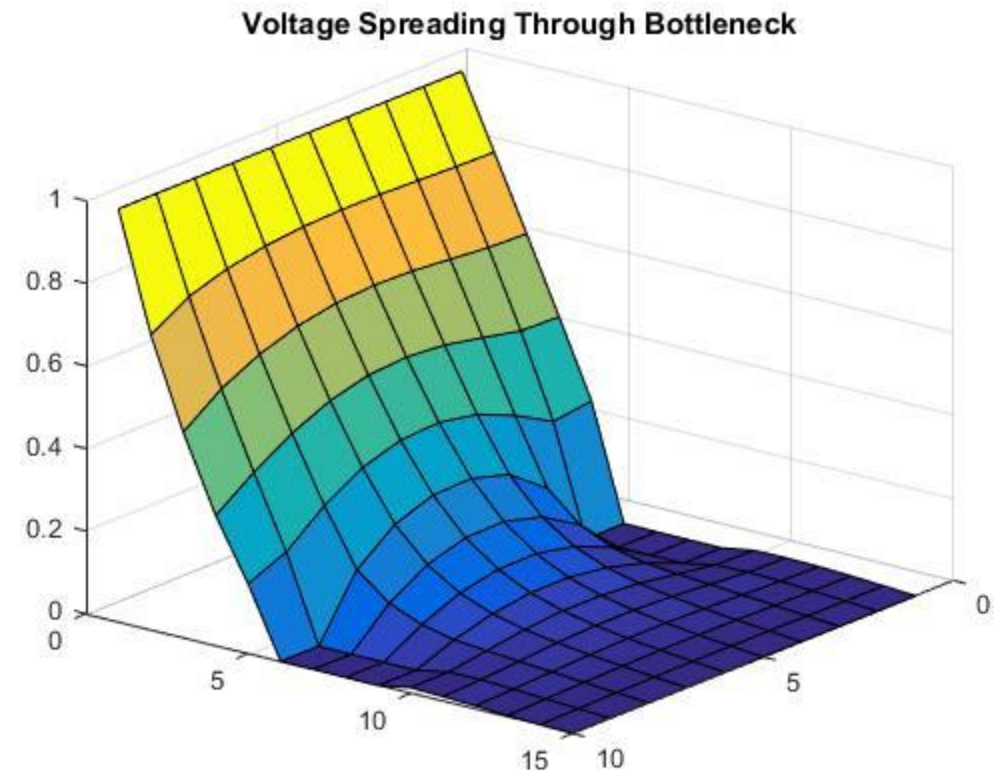
The plot obtained is almost identical to the one obtained through the FD method, proving that the method works. When plotting the analytical solution, a movie was used to see how from boundary conditions the voltage would spread in the region. The simulation will run for as many iterations as it is set too, but with this mesh size after 200 iterations the change in the voltage is so small that it does not affect the plot. If the mesh size is increased (ie more squares) then the number of iterations would have to be increased to obtain a plot. This is because the increment is so small between squares that the solution will need many more iterations to converge. The two methods both obtain a solution, but depending on what the purpose of the modelling is one method may be chosen over the other. If a final solution is needed, and it does not matter how the voltage spreads then the FD method would be preferred as it is fast. If you wanted to visualize how the voltage spreads through a region, then the analytical solution would be preferred to create a movie plot.

Bottleneck Region

Once the FD method was proven, it was time to model a bottleneck created by low conductivity squares. Unfortunately the final plot obtained for the voltage was not quite correct, but the error in the code could not be located. The same method was used as before to populate the G and boundary matrices. Along with these, a sigma matrix was also populated with the conductivity of the points as the code incremented its way through the area. The region tested can be seen below.

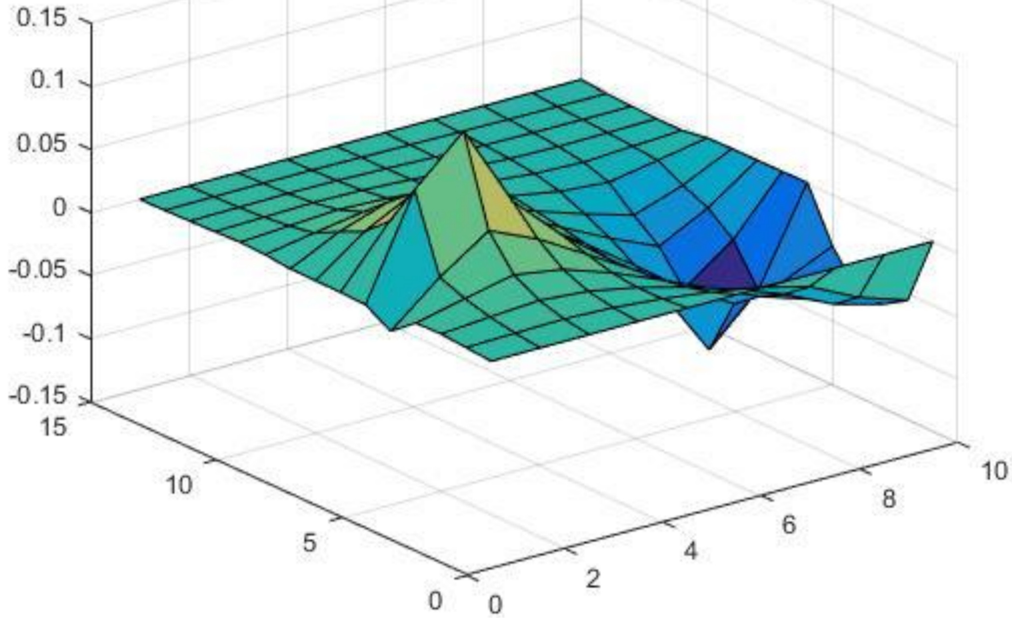


The voltage was then calculated the same as above, and remapped back to give the following.

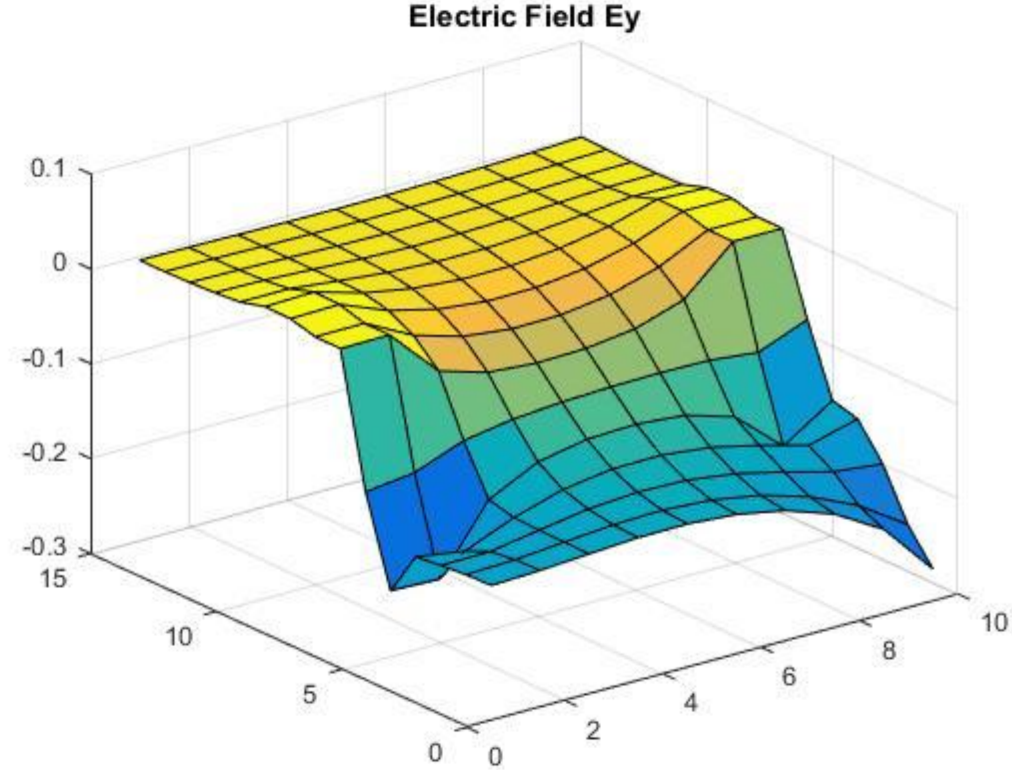


Once the voltage was calculated, the electric fields were calculated by taking the gradient of the remapped matrix. This provided and Ex and Ey as shown below.

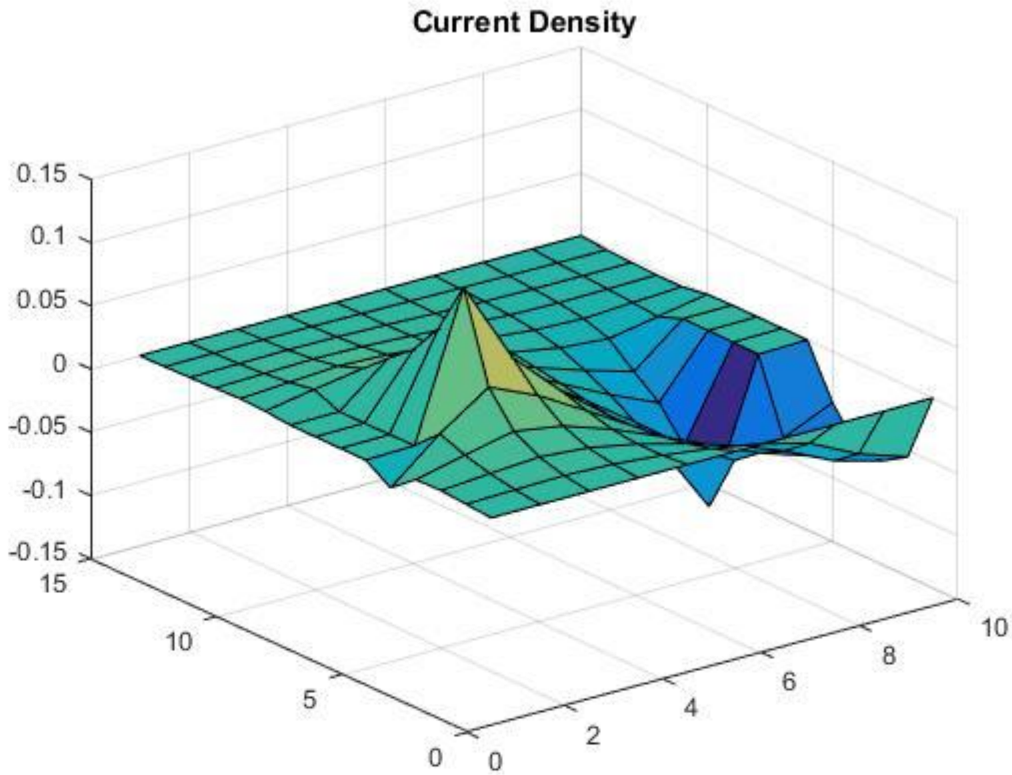
Electric Field Ex



Electric Field Ey



Finally the current density was calculated by multiplying the conductivity matrix with the gradient of the voltage to yield the following.



AS the mesh size is increased and decreased, the current remains relatively the same, but a more accurate value will be obtained with a higher mesh. When the gap in the bottle neck is increased, the current is less dense as there is more area for the electrons to pass through. When the bottle neck is narrowed, the current density increases as there is less space for the electrons to pass through.

Conclusion

This concluded the assignment, and showed how the Finite Difference method could be used to obtain a quicker solution for the voltage spreading through a given region.