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Time-Dependent Analysis of Reinforced and Prestressed Concrete Members



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A new step-by-step method is proposed that allows the determination of the history of strain and stress distribution in concrete sections with prestressed and nonprestressed reinforcement. It considers time-dependent effects on concrete and prestressed steel independently of the cracked or uncracked state. The analysis is performed at different time intervals satisfying the strain compatibility between concrete and all layers of steel, as well as the equilibrium equations in sections subjected to normal forces and moments due to external loads. The development along the time of the creep coefficient, shrinkage, and relaxation loss is taken into account on the basis of prediction models and equations that may be extracted from codes and recommendations. Numerical examples in reinforced, prestressed, and partially prestressed concrete sections, including different loading conditions, are used to demonstrate the applicability of the proposed method.

Keywords: compatibility; concretes; creep; curvature; equilibrium; interval; prestressed loss; relaxation; shrinkage; steels; strain; stress; time discretization; time-step method.

INTRODUCTION

Numerous methods have been developed to evaluate the strain and stress distribution in reinforced and prestressed concrete sections, taking into account the time-dependent effects of creep and shrinkage of concrete and the relaxation of prestressing steel.[1-5] Most of these include the age-adjusted effective modulus in the analysis.[6] The finite element method also has a wide acceptability for creep analysis of structures with sophisticated inelastic constitutive models.[7-10] Certain methods[5,11] are based on a time-step procedure and the aim of this paper is to present a new time-step method to determine the history of the strain and stress distribution in reinforced and prestressed concrete sections, with any degree of prestressing, in the cracked or uncracked state, when subjected to normal forces and bending moments enclosing the functions of creep and shrinkage of concrete in agreement with prediction models and the relaxation of the prestressing steel adjusted along the time.

The time period is subdivided into a number of time intervals within which a strain compatibility analysis is carried out and equilibrium equations are used to compute the values of stresses in the section.

The basic assumptions on which the proposed method is based are:

1. Plane sections before loading remain plane after loading and time-dependent effects.
2. A perfect bond exists between steel and concrete.
3. Tensile stresses in concrete are neglected after cracking.

Tensile stress, tensile force, and the corresponding deformation are assumed positive. A positive bending moment produces tension at the bottom fiber, and the corresponding curvature and slope of the stress diagram are also positive.

RESEARCH SIGNIFICANCE

The stress analysis in the sectional design (by checking serviceability in concrete sections with prestressed and nonprestressed steels when subjected to normal forces and bending moments) must take into account the time-dependent effects of creep and shrinkage of concrete and relaxation of prestressed steel. The transition from the uncracked to the cracked state usually represents a calculation problem reflected in most of the procedures of analysis. Furthermore, prestress loss due to stress relaxation is treated normally and is separated from the creep and shrinkage phenomena.

This research presents a unified step-by-step method for calculation of the strain and stress distribution in concrete sections based on strain compatibility and equilibrium equations, including the effects of cracking when the tensile strength of concrete is exceeded. The evolution of the prestress loss due to relaxation steel is computed together with the time development of creep and shrinkage in concrete and considers a relaxation reduction coefficient introduced by Ghali and Treviño.[12]

CONCRETE STRAIN AND STRESS AT TIME t_i

An assumption of linearity implies validity of the principle of superposition. This principle states that the strain response due to the sum of stress histories is the sum of the individual responses. Hence, if concrete is subjected to a uniaxial stress that changes its magnitude gradually, the strain of concrete at time t is described by the sum of the instantaneous and creep strain produced by an initial stress $\sigma_c(t_0)$, the strain produced by the continuous stress history $\sigma_c(t)$, and the shrinkage strain

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$$\varepsilon_c(t)\frac{\sigma_c(t_0)}{E_c}\cdot[1+\varphi(t,t_0)] + \int_{t_0}^{t}\frac{d\sigma_c(\tau)}{d\tau} \cdot \left[\frac{1+\varphi(t,\tau)}{E_c}\right]\cdot\ d\tau + \varepsilon_{sh}(t,t_s) \tag{1}$$

where

E_c = modulus of elasticity of concrete at 28 days (assumed constant)
$\varphi(t,t_0)$ = creep coefficient at time t for stress applied at time t_0
$\varepsilon_{sh}(t,t_s)$ = free shrinkage of concrete between time t and time t_s
t_0 = age of concrete at time when member was loaded
t_s = age of concrete at end of curing

The expressions of $\varphi(t,t_0)$ and $\varepsilon_{sh}(t,t_s)$ may be adopted on the basis of prediction models that may be extracted from Codes and Recommendations.[13-15]

If a time-discretization is performed and the concrete stress changes its magnitude abruptly at times t_1, t_2, etc., (Fig. 1) the integral of the previous equation can be replaced by a finite sum

$$\varepsilon_c(t_i) = \frac{\sigma_c(t_0)}{E_c}\cdot[1+\varphi(t_i,t_0)] + \sum_{k=1}^{i}\Delta\sigma_c(\tau_k)\cdot\frac{1+\varphi(t_i,\tau_k)}{E_c} + \varepsilon_{sh}(t_i,t_s) \tag{2}$$

with

$\Delta\sigma_c(\tau_k) = \sigma_c(t_k) - \sigma_c(t_{k-1})$

$\tau_k = \dfrac{t_k + t_{k-1}}{2}$

The concrete strains and stresses in the cross section are defined in terms of the top fiber strain $\varepsilon_{c2}(t)$ and stress $\sigma_{c2}(t)$, the curvature $\psi(t)$, and slope of the stress diagram $\gamma(t)$, as shown in Fig. 2

$$\varepsilon_c(t) = \varepsilon_{c2}(t) + \psi(t)\cdot y \tag{3}$$

$$\sigma_c(t) = \sigma_{c2}(t) + \gamma(t)\cdot y \tag{4}$$

Coordinate y of any fiber is measured positive downwards from the top fiber.

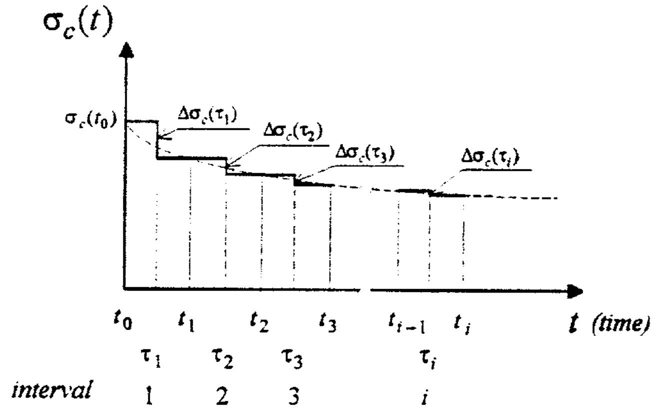


Fig. 1—Treatment of concrete stress according to time-discretization procedure.

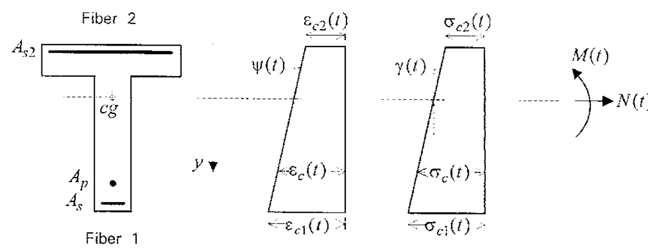


Fig. 2—Definitions of symbols: strain and stress distribution in concrete section.

Replacing Eq. (3) and (4) into Eq. (2), the following equations are given

$$\sigma_{c2}(t_i) = \frac{\sigma_{c2}t_0}{E_c}\cdot[1+\varphi(t_i,t_0)] + \sum_{k=1}^{i}\Delta\sigma_{c2}(\tau_k)\cdot\frac{1+\varphi(t_i,t_k)}{E_c} + \varepsilon_{sh}(t_i,t_s) \tag{5}$$

$$\psi(t) = \frac{\gamma(t_0)}{E_c}\cdot[1+\varphi(t_i,t_0)] + \sum_{k=1}^{i}\Delta\gamma(\tau_k)\cdot\frac{1+\varphi(t_i,\tau_k)}{E_c} \tag{6}$$

With these equations, the strain distribution in the cross section could be described if the stress history, the creep coefficient, and the free shrinkage in function of time are known. However, the concrete stress history depends on compatibility and equilibrium between the time-dependent changes in strain and stress of the three components of a concrete section reinforced with prestressed and nonprestressed steels. Therefore, the discussed time-discretization method is based on the condition that the magnitude of stress at interval (i) does not vary any more from the beginning with instant (i – 1), which has been determined in the preceding interval by taking into account compatibility and equilibrium conditions.

The concrete strain change due to creep and shrinkage at interval (i) can be calculated considering the stress values as far as t_{i-1} from Eq. (5) and (6) minus the elastic component

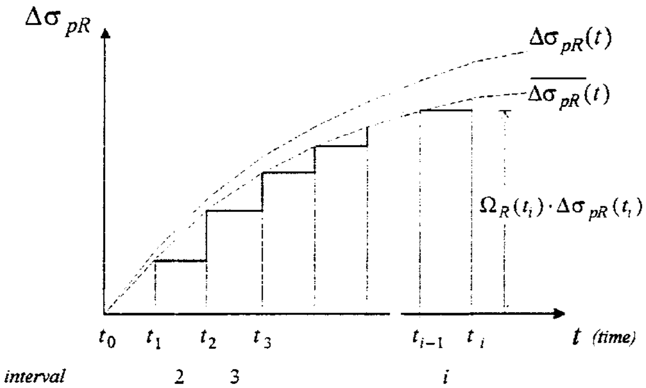


Fig. 3—Treatment of reduced relaxation loss in prestressed steel according to time-discretization procedure.

$$\Delta\varepsilon_{c2CS}(t_i) = \frac{\sigma_{c2}(t_0)}{E_c}\cdot\varphi(t_i, t_0) \tag{7}$$

$$+\sum_{k=1}^{i-1}\frac{\sigma_{c2}(t_k)-\sigma_{c2}(t_{k-1})}{E_c}\cdot\varphi(t_i,\tau_k)+\varepsilon_{sh}(t_i,t_s)$$

$$\Delta\psi_{CS}(t_i) = \frac{\gamma(t_0)}{E_c} \tag{8}$$

$$\cdot\,\varphi(t_i,t_0)+\sum_{k=1}^{i-1}\frac{\gamma(t_k)-\gamma(t_{k-1})}{E_c}\cdot\varphi(t_i,\tau_k)$$

The concrete stress distribution is determined by multiplying the modulus of elasticity of concrete with the difference between the total concrete strain and the change due to creep and shrinkage

$$\sigma_{c2}(t_i) = E_c\cdot[\varepsilon_{c2}(t_i)-\varepsilon_{c2CS}(t_i)] \tag{9}$$

$$\gamma(t_i) = E_c\cdot[\psi(t_i)-\Delta\psi_{CS}(t_i)] \tag{10}$$

PRESTRESS LOSS AT TIME t_i

When the stress in the prestressed reinforcement is considered, the stress loss $\Delta\sigma_{pR}$, due to the relaxation of steel, must be taken into account. The magnitude of $\Delta\sigma_{pR}$ depends on the initial stress σ_{p0} in prestressing steel immediately after stretching (at time t_p) under constant strain. In a prestressed concrete member, however, the initial tension of a tendon decreases its value due to the combined effect of creep and shrinkage of concrete and steel relaxation; the loss in tension is associated with shortening of the tendon. Thus, the value of relaxation to be used in predicting prestress loss should be reduced by a relaxation reduction coefficient that was introduced by Ghali and Treviño.[12] Hence, one can write

$$\overline{\Delta\sigma_{pR}} = \Omega_R\cdot\Delta\sigma_{pR} \tag{11}$$

where

$\Delta\sigma_{pR}$	=	relaxation loss that occurs under constant steel strain
$\overline{\Delta\sigma_{pR}}$	=	reduced loss relaxation value
Ω_R	=	relaxation reduction coefficient

If the relaxation loss in the prestressed steel at interval (i) is considered, the following equation at time t_i can be written

$$\overline{\Delta\sigma_{pR}}(t_i) = \Omega_R(t_i)\cdot\Delta\sigma_{pR}(t_i) \tag{12}$$

but the stress distribution at time t_i can be calculated only after the evaluation of compatibility and equilibrium at this interval; therefore, the value of $\overline{\Delta\sigma_{pR}}(t_i)$ is estimated considering the stress values as constant at time t_{i-1} (Fig. 3). The relaxation loss at any time t may be expressed as a product of the ultimate relaxation loss $\Delta\sigma_{pR\infty}$ and a dimensionless function of the period $(t - t_p)$ in hr[12]

$$\Delta\sigma_{pR}(t) = \Delta\sigma_{pR\infty}\cdot\left[\frac{1}{16}\cdot\ln\left(\frac{t-t_p}{10}+1\right)\right] \tag{13a}$$

$$0\le(t-t_p)\le 1000$$

$$\Delta\sigma_{pR}(t) = \Delta\sigma_{pR\infty}\cdot\left(\frac{t-t_p}{0.5\times 10^6}\right)^{0.2}$$

$$1000\le(t-t_p)\le 0.5\times 10^6 \tag{13b}$$

$$\Delta\sigma_{pR}(t) = \Delta\sigma_{pR\infty} \qquad (t-t_p)\ge 0.5\times 10^6 \tag{13c}$$

where t_p = instant of jacking and t is considered constant.

The ultimate relaxation loss $\Delta\sigma_{pR\infty}$ is adopted from the CEB-FIP Model Code for Concrete Structures 1990[14] and the following equation approximates closely its values (Fig. 4)

$$\Delta\sigma_{pR\infty} = -\eta\cdot\left(\frac{\sigma_{p0}}{f_{pk}}-0.4\right)^2\cdot\sigma_{p0} \qquad \text{when } \frac{\sigma_{p0}}{f_{pk}}\ge 0.4 \tag{14}$$

where

η	=	$^8/_3$ for steels of Group 1
η	=	$^2/_3$ for steels of Group 2
f_{pk}	=	characteristic strength of prestressing steel
σ_{p0}	=	initial prestressing stress

With pretensioning, the prestressing steel stress is just before releasing the tendon. Relaxation occurring during the period between jacking (t_p) and transfer (t_0) takes place without change in tendon length and thus, should not be subject to any reduction. With post-tensioning, the prestressing steel stress occurs after deducting the losses due to friction and anchor set.

The relaxation reduction coefficient may be calculated by the equation

$$\Omega_R(t_i) = e^{\left(-6.7+5.3\cdot\frac{\sigma_{p0}}{f_{pk}}\right)}\cdot\omega(t_i) \tag{15}$$

with

$$\omega(t_i) = -\frac{\Delta\sigma_{pCSR}(t_{i-1})-\Delta\sigma_{pR}(t_{i-1})}{\sigma_{p0}} \tag{16}$$

where

$\Delta\sigma_{pCSR}(t_{i-1})$	=	$\sigma_p(t_{i-1}) - \sigma_{p0}$ loss of stress in the prestressed steel due to the combined effect of creep, shrinkage, and relaxation

$\sigma_p(t_{i-1})$ = stress in prestressed steel at end of interval $(i-1)$

$\Delta\sigma_{pR}(t_{i-1})$ = value of relaxation loss at instant t_{i-1} from Eq. (13)

TIME-DEPENDENT ANALYSIS

The time period of analysis is subdivided into a number of time intervals within which the stresses in the component materials can be considered constant and change abruptly at the boundaries of these intervals. The stresses at the beginning of a time increment are used to calculate the time-dependent effects in concrete and prestressing steel. A strain compatibility analysis is carried out and equilibrium equations are used to compute the new values of stresses at the end of the increment. The procedure allows an application of load at each time interval. The numerical accuracy depends on the amplitude of time intervals and may be increased, reducing such an amplitude. For practical applications, it is recommended to use increasing time steps, which leads to adequate accuracy with a minimum of computation time for usual strain histories.

The calculation in interval (i) is very simple. Strain distribution in the cross section can be described in terms of the variables $\varepsilon_{c2}(t_i)$ and $\psi(t_i)$ through the strain compatibility

$$\varepsilon_c(t_i) = \varepsilon_{c2}(t_i) + \psi(t_i) \cdot y \qquad \text{concrete} \tag{17}$$

$$\varepsilon_{ns2}(t_i) = \varepsilon_{c2}(t_i) + \psi(t_i) \cdot d_2 \tag{18}$$
top nonprestressed steel

$$\varepsilon_{ns1}(t_i) = \varepsilon_{c2}(t_i) + \psi(t_i) \cdot d_1 \tag{19}$$
bottom nonprestressed steel

$$\varepsilon_{ps}(t_i) = \varepsilon_{c2}(t_i) + \psi(t_i) \cdot d_p \tag{20}$$
prestressed steel

The values of the concrete strain variation due to creep and shrinkage $\Delta\varepsilon_{c2CS}(t_i)$ and $\Delta\psi_{CS}(t_i)$ at this interval are calculated with Eq. (7) and (8) using the stress values as far as interval (i - 1), and the corresponding concrete stress distribution can be obtained using the stress-strain relationship of Eq. (9) and (10) into Eq. (4).

Stresses in nonprestressed and prestressed steel are given by the following equations

$$\sigma_{ns2}(t_i) = E_{ns} \cdot \varepsilon_{ns2}(t_i) \tag{21}$$

$$\sigma_{ns1}(t_i) = E_{ns} \cdot \varepsilon_{ns1}(t_i) \tag{22}$$

$$\sigma_{ps}(t_i) = \sigma_{p0} + \overline{E_{ps}} \cdot [\varepsilon_{ps}(t_i) - \beta \cdot \varepsilon_{ps}(t_0)] + \overline{\Delta\sigma_{pR}}(t_i) \tag{23}$$

and can be expressed in terms of the variables $\varepsilon_{c2}(t_i)$ and $\psi(t_i)$ using Eq. (18), (19), and (20). The prestress loss $\overline{\Delta\sigma_{pR}}(t_i)$ is determined using Eq. (12). The coefficient β is introduced to consider the difference between pretensioning and post-tensioning. The tendons in the post-tensioned section are bonded to the concrete after transfer and for that reason the strain in the bonded prestressed steel must be reduced with the subtraction of the initial concrete strain at time t_0, then

$\beta = 0$ for pretensioning

$\beta = 1$ for post-tensioning

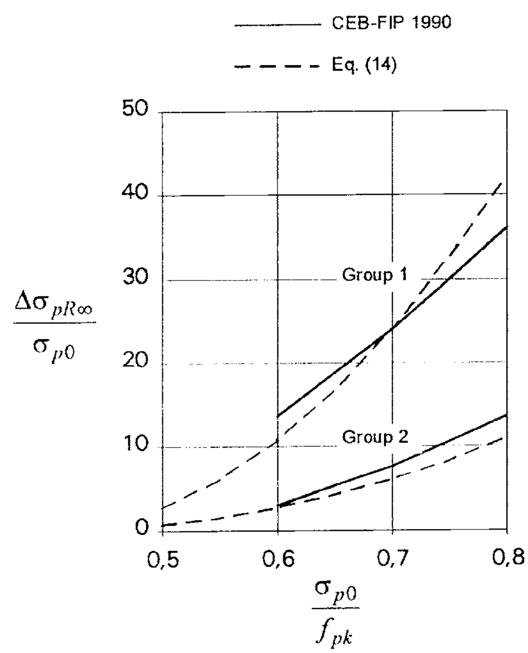


Fig. 4—Ultimate relaxation losses (percent) for different initial stress levels.

When the stress distribution in the component materials as a function of $\varepsilon_{c2}(t_i)$ and $\psi(t_i)$ is fixed, the values of these variables are calculated through an iterative method computing the equilibrium conditions for a normal force $N(t_i)$ and a bending moment $M(t_i)$ at the centroid of the transformed cross section

$$N(t_i) = \int \sigma_c(t_i) \cdot dA_c + \sigma_{ps}(t_i) \cdot A_{ps} + \sigma_{ns1}(t_i) \cdot A_{ns1} + \sigma_{ns2}(t_i) \cdot A_{ns2} \tag{24}$$

$$M(t_i) = \int \sigma_c(t_i) \cdot (y - y_{cg}) \cdot dA_c + \sigma_{ps}(t_i) \cdot A_{ps} \cdot (d_p - y_{cg}) + \sigma_{ns1}(t_i) \cdot A_{s1} \cdot (d_1 - y_{cg}) + \sigma_{ns2}(t_i) \cdot A_{ns2} \cdot (d_2 - y_{cg}) \tag{25}$$

Once these equations are solved and the values of $\varepsilon_{c2}(t_i)$ and $\psi(t_i)$ are determined, the strain and stress redistribution in the component materials, now at the end of interval (i), may be described using the previous equations. These values are then used for the calculation in the next interval (i + 1).

SUMMARY OF PROPOSED METHOD

The proposed method is best suited for a computerized solution. First initial stresses, strains, and curvature due to prestress force and dead load must be calculated. The total period of analysis should be divided into several time intervals and the steps of the analysis in each one of the intervals performed are as follows:

Step 1—Compute the strain and curvature increments due to creep and shrinkage $\Delta\varepsilon_{c2CS}(t_i)$ and $\Delta\psi_{CS}(t_i)$ using Eq. (7) and (8).

Step 2—Compute the time-dependent prestress loss $\overline{\Delta\sigma_{pR}}(t_i)$ using Eq. (12).

Step 3—Introduce normal force and bending moment increments if they are present at the beginning of time interval (i). Solve the equation system from Eq. (24) and (25)

$$g_1[\varepsilon_{c2}(t_i), \psi(t_i)] = N(t_i)$$

$$g_2[\varepsilon_{c2}(t_i), \psi(t_i)] = M(t_i)$$

Step 4—Compute the strain distribution at the end of time interval (i) using Eq. (17) to (20).

Step 5—Compute the final stress distribution at the end of time interval (i) using Eq. (9) and (10) into Eq. (4) and Eq. (21) to (23).

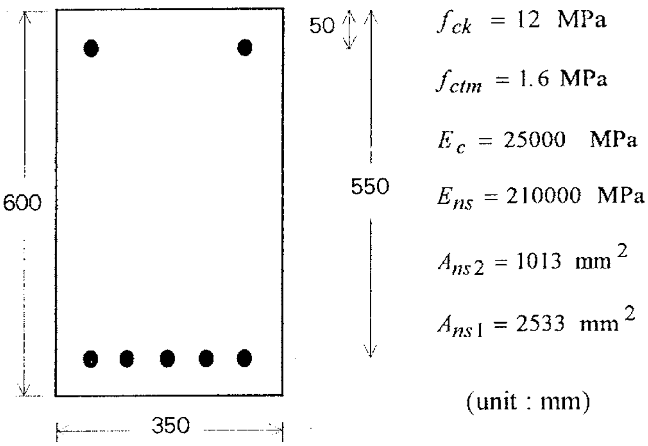


Fig. 5—Example 1: dimensions and material properties of reinforced concrete section.

Step 6—Considering these final values, begin the analysis in interval (i + 1), and then go back to Step 1.

EXAMPLES

To demonstrate the applicability of the proposed method, the following three numerical examples are presented herein. The values of $\varphi(t_i,t_0)$ and $\varepsilon_{sh}(t_i,t_s)$ in the time-discretization are calculated by means of the expressions of CEB-FIP (1990). The age of concrete at the time when the beam was prestressed is assumed t_0 = 28 days and the instant at the end of curing is supposed t_s = 3 days for steam-cured concrete. The relative humidity of the ambient environment is considered equal to 70 percent corresponding to outside conditions. The prestress loss $\overline{\Delta\sigma_{pR}}(t_i)$ is updated, taking into account the functions [Eq. (13a) to (13c)], including the ultimate relaxation loss values predicted by Eq. (14).

Example 1

The first example concerns a rectangular reinforced concrete section subjected to the bending moments M_g = 120 kNm and M_p = 100 kNm due to dead and live loads, respectively. The dimensions of the section, cross-sectional areas of the reinforcement, and the material properties are shown in Fig. 5. It is required to investigate the effects of creep and shrinkage of concrete on the section. Fig 6 shows the strain and stress distribution: a) immediately after application of the dead load; b) after the creep and shrinkage occurred at t = 30 years, $\varphi(t,t_0)$ = 2.70 and $\varepsilon_{sh}(t,t_s) = -0.48 \times 10^{-3}$; and c) when the live load is applied.

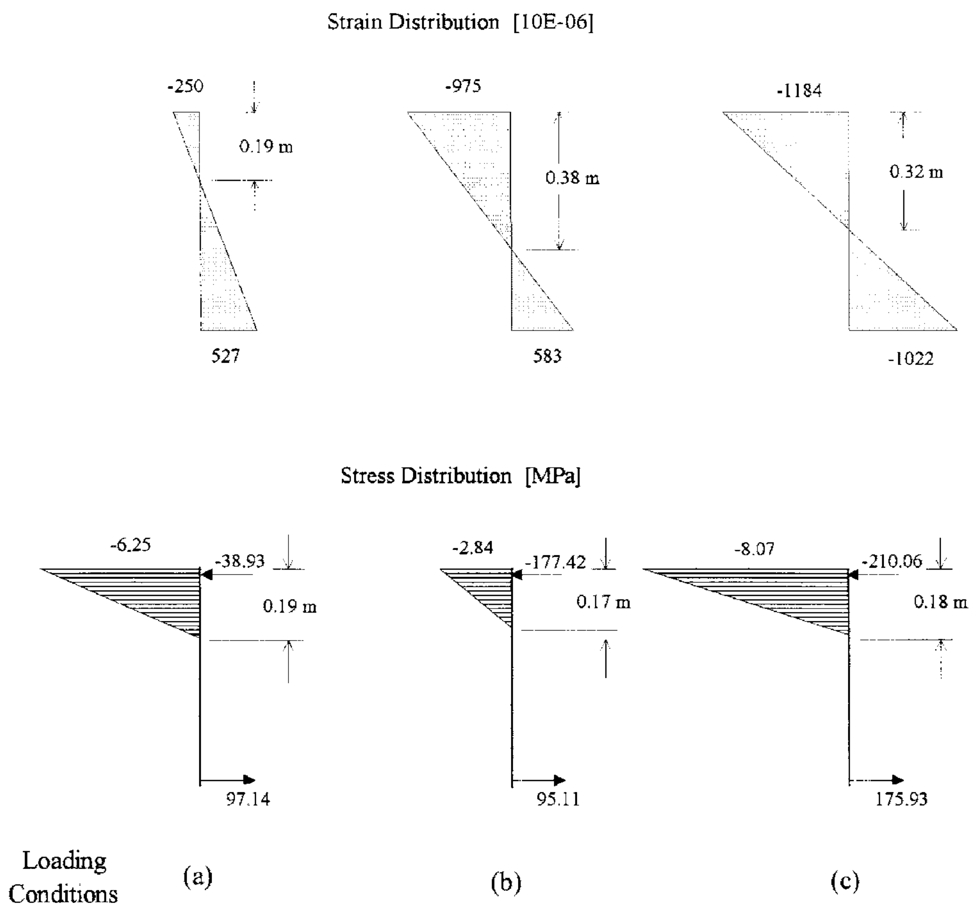


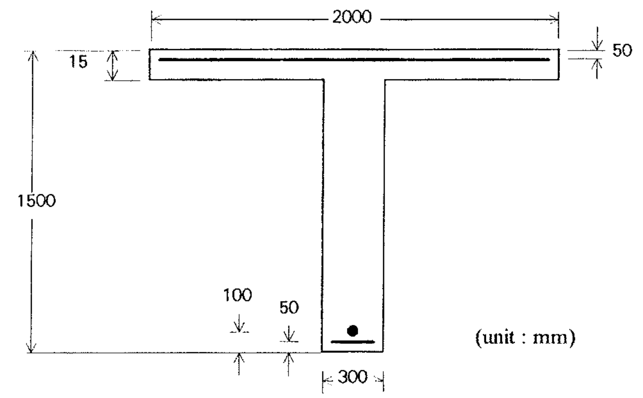
Fig. 6—Strain and stress distributions in concrete and steel under various loading conditions in Example 1.

Example 2

A fully prestressed concrete section is investigated. It corresponds to a midspan section of a 25-m simple span post-tensioned T-beam. The dimensions of the cross section, cross-sectional areas of the reinforcement and the tendon, and the material properties are given in Fig 7. The stress and strain distribution will be analyzed under the following loading conditions.

a. Uniform dead load of g = 13 kN/m and prestressing load at time t_0.

b. Application of uniform live load of $p = 0.3\ g$ = 3.9 kN/m.



f_{ck} = 35 MPa	f_{ctm} = 1.5 MPa	E_c = 33500 MPa
f_{pk} = 1570 MPa	E_{ps} = 205000 MPa	E_{ns} = 210000 MPa
A_{ps} = 1545 mm^2	A_{ns1} = 314 mm^2	A_{ns2} = 1006 mm^2

Fig. 7—Example 2: dimensions and material properties of prestressed concrete section.

c. Dead load plus the combined effects of creep and shrinkage of concrete and relaxation of prestressing steel at time t = 30 years, $\varphi(t,t_0) = 1.87$; $\varepsilon_{sh}(t, t_s) = -0.38 \times 10^{-3}$ and $\Delta\sigma_{pR\infty} = -66$ MPa.

d. Application of the live load after all time-dependent effects occurred. The results of the analysis are summarized in Fig. 8.

Example 3

The same T-section previously described is investigated now and is assumed as partially prestressed. The present cross-sectional areas of the reinforcement and the tendon are now A_{ns1} = 1570 mm^2, A_{ns2} = 1006 mm^2, and A_{ps} = 1100 mm^2. The results are shown in Fig. 9.

From Example 1 it can be noted that due to time-dependent effects the strain distribution changes the depth of the zero strain axis with time and this does not agree with the zero stress axis. After creep and shrinkage of concrete, the height of a crack does not agree with concrete stress distribution. The strain compatibility between concrete and steel and the elastic behavior of steel induces to attempt that the height of a crack coincides with the zero strain axis.

The increment of the strain in the top compression fiber produces a significant increment of the compression stress in the compression reinforcing bars that causes a reduction in the concrete compression stress. Nevertheless, the changes of stress in the tension reinforcing bars are not great. An increment of the curvature is also registered.

In Examples 2 and 3 the changes due to the time-dependent effects are very similar to the results of Example 1. The top and bottom reinforcing bars present a considerable compression stress increment that causes a reduction of the bottom concrete compression stress so that the concrete stress changes to tension in the partially prestressed section. The top compression concrete stress changes its value, but not significantly.

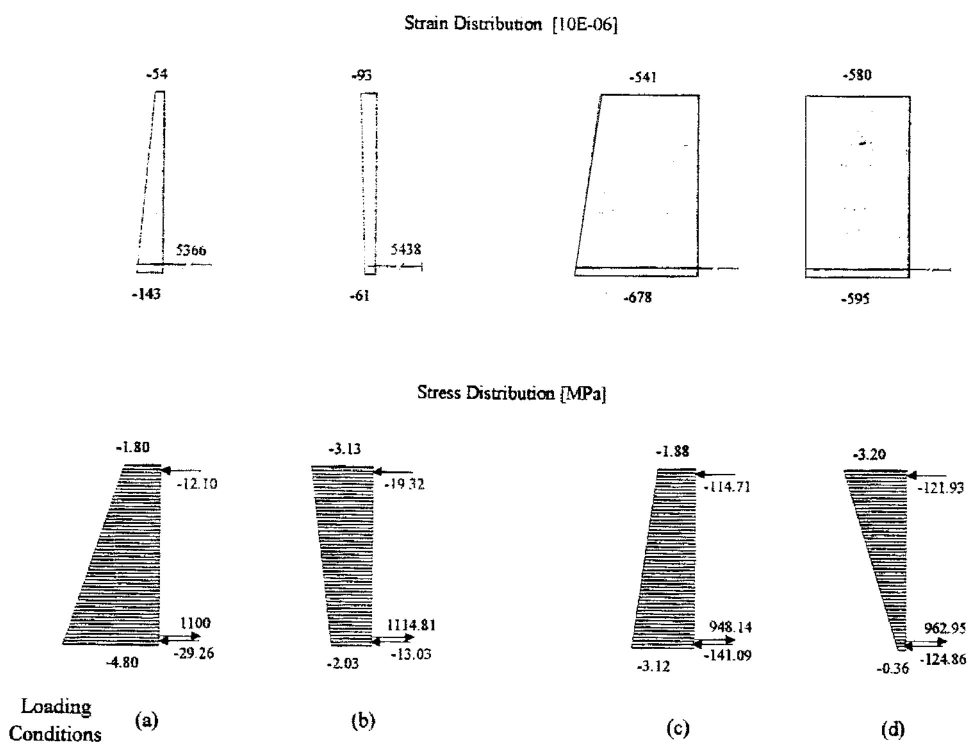


Fig. 8—Strain and stress distributions in concrete and steel under various loading conditions in Example 2.

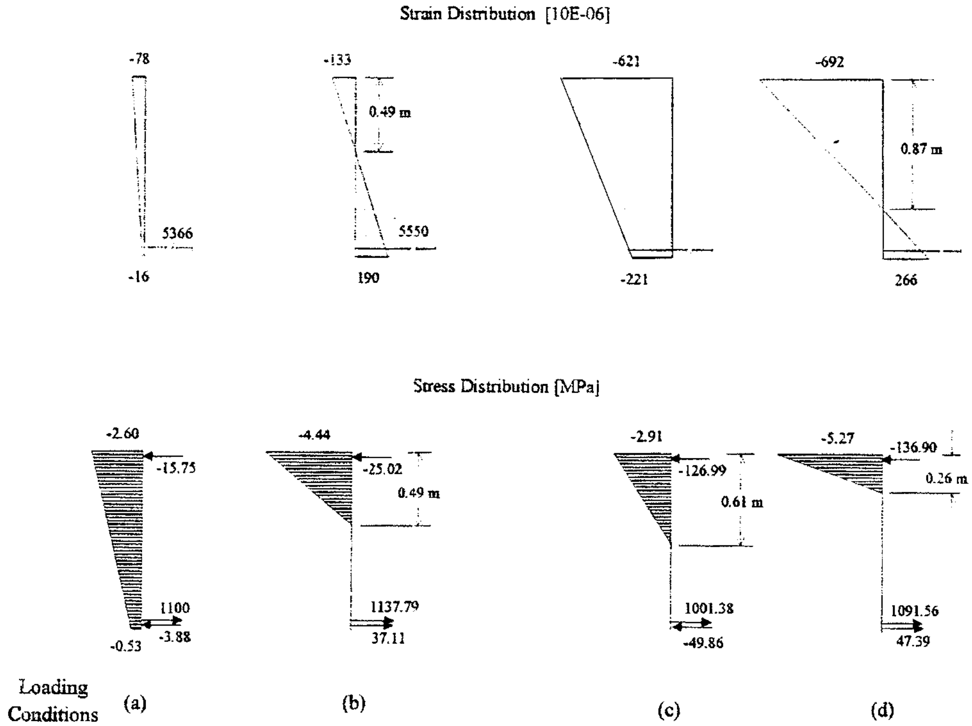


Fig. 9—Strain and stress distributions in concrete and steel under various loading conditions in Example 3.

The time-dependent variation of curvature produces a camber increment for the case of full prestressing, while in partial prestressing, it produces a deflection increment.

Considering that the influence of the time-dependent effects is lower due to a lower prestressing force, the prestress loss of the prestressing steel is lower for the case of the partially prestressed section of Example 3.

CONCLUSIONS

A time-step method for analyzing strain and stress distribution in reinforced and prestressed concrete members, taking into account the time-dependent effects, is presented. The analysis is based on requirements of equilibrium and compatibility of strain in concrete and nonprestressed and prestressed steel for uncracked or cracked structural concrete sections subjected to normal forces and bending moments that may change the magnitudes at the beginning of a time interval.

The proposed method introduced the development of the creep and shrinkage prediction models into the analysis, as well as the reduction of the relaxation loss in the prestressing steel as a function of the time.

A complete history of strains and stresses in the cross section is obtained through the step-by-step analysis over short time increments. Hence, the time-dependent change in stresses in concrete is continuously considered at each time interval and this eliminates the need to introduce a relaxation or aging coefficient in the creep analysis.

As the procedure of calculation involves an iteration for computing the equilibrium condition in each step of the analysis, a computer program has been implemented.

NOTATION

A = cross-sectional area
d = distance measured from top surface
E = modulus of elasticity
f_{pk} = characteristic strength of prestressing steel
M = bending moment
N = normal force
t_0 = age of concrete when member was loaded
t_p = instant of jacking
t_s = age of concrete at end of curing
y_{cg} = centroidal distance of concrete section measured from top surface
$\varepsilon(t)$ = strain in concrete or steel at time t
$\varepsilon_{sh}(t,t_s)$ = free shrinkage of concrete between time t and time t_s
$\Delta\varepsilon_{c2CS}$ = increment of concrete strain due to creep and shrinkage
$\varphi(t,t_0)$ = creep coefficient at time t for stress applied at time t_0
$\psi(t)$ = curvature of concrete section at time t
$\Delta\psi_{CS}$ = increment of curvature due to creep and shrinkage
$\sigma(t)$ = stress in concrete or steel at time t
σ_{p0} = initial stress in prestressing steel
$\Delta\sigma_{pR}$ = relaxation loss in prestressed steel that occurs under constant steel strain
$\overline{\Delta\sigma_{pR}}$ = reduced relaxation loss in prestressed steel
$\Delta\sigma_{pR\infty}$ = ultimate relaxation loss in prestressed steel
Ω_R = relaxation reduction coefficient in prestressed steel
$\gamma(t)$ = slope of concrete stress diagram

SUBSCRIPTS

1 = bottom surface or bottom steel level of concrete member
2 = top surface or top steel level of concrete member
c = concrete
sh = shrinkage
ps = prestressed steel
ns = nonprestressed steel
C = due to creep
S = due to shrinkage
R = due to relaxation in prestressed steel

REFERENCES

1. Ghali, A., "Stress and Strain Analysis in Prestressed Concrete: A Critical Review," *PCI Journal*, Nov.-Dec. 1989, pp. 80-97.

2. Inomata, S., "Unified Approach for Serviceability Design of Structural Concrete Members in Flexure," *Concrete Library of J.S.C.E.* (Japan Society of Civil Engineers), No. 14, Mar. 1990, pp. 213-230.

3. Frey, J., and Trost, H., “Serviceability Design of Partially Prestressed Concrete Structures” (Zur Berechnung von Teilweise Vorgespannten Betontragwerken in Gebrauchszustand), *Beton-und Stahlbetonbau* (Germany), H. 11, 1983, pp. 302-308; also H. 12, pp. 331-335. (in German)

4. Taerwe, L. R., “Simplified Analysis of Stress Redistribution in Partially Prestressed Concrete Sections,” *ACI Structural Journal*, V. 87, No. 1, Jan.-Feb. 1990, pp. 93-98.

5. Moustafa, S. E., “Nonlinear Analysis of Reinforced and Prestressed Concrete Members,” *PCI Journal*, Sept.-Oct. 1986, pp. 126-147.

6. Bažant, Z. P., “Prediction of Concrete Creep Effects Using Age-Adjusted Effective Modulus Method,” ACI JOURNAL, *Proceedings* V. 69, No. 4, Apr. 1972, pp. 212-217.

7. Anderson, C. A., “Finite Element Analysis of Creep and Shrinkage,” *Mathematical Modeling of Creep and Shrinkage of Concrete*, Z. P. Bažant, ed., John Wiley & Sons, Chichester, 1988, pp. 275-310.

8. Iglesias, C., “Formulation of Cracking Using the Multilayer Model (MLM),” *Fifth International RILEM Symposium on Creep and Shrinkage of Concrete*, Barcelona, Sept. 6-9, 1993, pp. 561-566.

9. Ulm, F. J., and Guggenberger, J. M., “3-D Nonlinear Time-Dependent Analysis of RC and PC Beams,” *Fifth International RILEM Symposium on Creep and Shrinkage of Concrete*, Barcelona, Sept. 6-9, 1993, pp. 573-578.

10. Timsah, Y., and Menezes, N. C., “Numerical Solution for the Time-Dependent Behavior of 3-D Concrete Structures,” *Fifth International RILEM Symposium on Creep and Shrinkage of Concrete*, Barcelona, Sept. 6-9, 1993, pp. 603-608.

11. Lambotte, H.; Van Nieuwenburg, D.; and Cocquyt, F., “Influence of Nonprestressed Steel on Shrinkage and Creep Deformations and on Steel-Concrete Stress Redistribution,” *Partial Prestressing—From Theory to Practice*, M. Z. Cohn, ed., Martinus Nijhoff Publishers, Dordrecht, 1986, pp. 45-56.

12. Ghali, A., and Treviño, J., “Relaxation of Steel in Prestressed Concrete,” *PCI Journal*, Sept-Oct. 1985, pp. 82-94.

13. *CEB Design Manual on Structural Effects of Time-Dependent Behavior of Concrete* (CEB Bulletin d’Information No. 142/142 bis), Georgi Publishing Co., Saint-Saphorin, Switzerland, 1984, 391 pp.

14. *CEB-FIP Model Code for Concrete Structures 1990*, Comité Euro-International du Béton/Fédération Internationale de la Précontrainte, Paris.

15. ACI Committee 209, “Prediction of Creep, Shrinkage, and Temperature Effects in Concrete Structures,“ *Designing for Effects of Creep, Shrinkage, and Temperature in Concrete Structures*, SP-27, American Concrete Institute, Detroit, 1971, pp. 51-93.

16. Trost, H., “Effects of the Principle of Superposition on Creep and Relaxation Problems in Reinforced and Prestressed Concrete,” (Auswirkungen des Superpositionsprinzips auf Kriech und Relaxationsprobleme bei Beton-und Spannbeton), Beton-und Stahlbetonbau (Germany), H. 10, 1967, pp. 230-238; also H. 11, pp. 261-269. (in German)

17. Rüsch, H.; Jungwirth, D.; and Hilsdorf, H. K., *Creep and Shrinkage—Their Effect on the Behavior of Concrete Structures*, Springer-Verlag, New York.

18. Moustafa, S. E., “Nonlinear Analysis of Reinforced and Prestressed Concrete Members,” *PCI Journal*, Sept.-Oct. 1987, pp. 122-130.

19. Teng, S., and Branson, D. E., “Initial and Time-Dependent Deformation of Progressively Cracking Nonprestressed and Partially Prestressed Concrete Beams,” *ACI Structural Journal*, V. 90, No. 5, Sept-Oct. 1993, pp. 480-488.

20. Gutiérrez, S. E., “Strain and Stress Time-Dependent Analysis of Partially Prestressed Concrete Sections” (Análisis Tiempo Dependiente de Deformación y Tensión en Secciones Parcialmente Pretensadas), thesis, Universidad Nacional de Tucumán, Tucumán, Argentina, Aug. 1994, 108 pp. (in Spanish)