

# Prediction of Creep, Shrinkage, and Temperature Effects in Concrete Structures

## Reported by ACI Committee 209

James A. **Rhodes**†  
Chairman, Committee 209

Domingo J. **Carreira**\*†  
Chairman, Subcommittee II

James J. Beaudoin  
Dan E. **Branson**\*†  
Bruce R. Gamble  
H.G. Geymayer  
Brij B. **Goyal**†  
Brian B. Hope

John R. **Keeton**†  
Clyde E. Kesler  
William R. Lorman  
Jack A. **Means**†  
Bernard L. Meyers†  
R.H. Mills

K.W. Nasser  
A.M. Neville  
Frederic **Roll**†  
John Timus k  
Michael A. Ward

Corresponding Members: John W. Dougill, H.K. Hilsdorf

Committee members voting on the 1992 revisions:

Marwan A. Daye  
Chairman

Akthem Al-Manaseer  
James J. Beaudoin  
Dan E. Branson  
Domingo J. Carreira  
Jenn-Chuan Chem  
Menashi D. Cohen  
Robert L. Day

Chung C. Fu 1  
Satyendra K. Ghosh  
Brij B. Goyal  
Will Hansen  
Stacy K. Hirata  
Joe Huterer  
Hesham Marzouk

Bernard L. Meyers  
Karim W. Nasser  
Mikael P.J. Olsen  
Baldev R. Seth  
Kwok-Nam Shiu  
Lilia **Panula**†

\* Member of Subcommittee II, which prepared this report

† Member of Subcommittee II

‡ Deceased

*This report reviews the methods for predicting creep, shrinkage and temperature effects in concrete structures. It presents the designer with a unified and digested approach to the problem of volume changes in concrete. The individual chapters have been written in such a way that they can be used almost independently from the rest of the report.*

*The report is generally consistent with ACI 318 and includes material indicated in the Code, but not specifically defined therein.*

**Keywords:** beams (supports); buckling; camber; composite construction (concrete to concrete); compressive strength; concretes; concrete slabs; cracking (fracturing); creep properties; curing; deflection; flat concrete plates; flexural strength; girders; lightweight-aggregate concretes; modulus of elasticity; moments of inertia; precast concrete; prestressed concrete: prestress loss; reinforced concrete: shoring; shrinkage; strains; stress relaxation; structural design; temperature; thermal expansion; two-way slabs: volume change; warpage.

ACI Committee Reports, Guides, Standard Practices, and Commentaries are intended for guidance in designing, planning, executing, or inspecting construction and in preparing specifications. References to these documents shall not be made in the Project Documents. If items found in these documents are desired to be a part of the Project Documents, they should be phrased in mandatory language and incorporated into the Project Documents.

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## CHAPTER 1-GENERAL

### 1.1-Scope

This report presents a unified approach to predicting the effect of moisture changes, sustained loading, and temperature on reinforced and prestressed concrete structures. Material response, factors affecting the structural response, and the response of structures in which the time change of stress is either negligible or significant are discussed.

Simplified methods are used to predict the material response and to analyze the structural response under service conditions. While these methods yield reasonably good results, a close correlation between the predicted deflections, cambers, prestress losses, etc., and the measurements from field structures should not be expected. The degree of correlation can be improved if the prediction of the material response is based on test data for the actual materials used, under environmental and loading conditions similar to those expected in the field structures.

These direct solution methods predict the response behavior at an arbitrary time step with a computational effort corresponding to that of an elastic solution. They have been reasonably well substantiated for laboratory conditions and are intended for structures designed using the ACI 318 Code. They are not intended for the analysis of creep recovery due to unloading, and they apply primarily to an isothermal and relatively uniform environment.

Special structures, such as nuclear reactor vessels and containments, bridges or shells of record spans, or large ocean structures, may require further considerations which are not within the scope of this report. For structures in which considerable extrapolation of the state-of-the-art in design and construction techniques is achieved, long-term tests on models may be essential to provide a sound basis for analyzing serviceability response. **Reference 109** describes models and modeling techniques of concrete structures. For mass-produced concrete members, actual size tests and service inspection data will result in more accurate predictions. In every case, using test data to supplement the procedures in this report will result in an improved prediction of service performance.

## 1.2-Nature of the problem

Simplified methods for analyzing service performance are justified because the prediction and control of time-dependent deformations and their effects on concrete structures are exceedingly complex when compared with the methods for analysis and design of strength performance. Methods for predicting service performance involve a relatively large number of significant factors that are difficult to accurately evaluate. Factors such as the nonhomogeneous nature of concrete properties caused by the stages of construction, the histories of water content, temperature and loading on the structure and their effect on the material response are difficult to quantify even for structures that have been in service for years.

The problem is essentially a statistical one because most of the contributing factors and actual results are inherently random variables with coefficients of variations of the order of 15 to 20 percent at best. However, as in the case of strength analysis and design, the methods for predicting serviceability are primarily deterministic in nature. In some cases, and in spite of the simplifying assumptions, lengthy procedures are required to account for the most pertinent factors.

According to a survey by ACI Committee 209, most designers would be willing to check the deformations of their structures if a satisfactory correlation between computed results and the behavior of actual structures could be shown. Such correlations have been established for laboratory structures, but not for actual structures. Since concrete characteristics are strongly dependent on environmental conditions, load history, etc., a poorer correlation is normally found between laboratory and field service performances than between laboratory and field strength performances.

With the above limitations in mind, systematic design procedures are presented which lend themselves to a computer solution by providing continuous time functions for predicting the initial and time-dependent average response (including ultimate values in time) of structural members of different weight concretes.

The procedures in this report for predicting time-dependent material response and structural service performance represent a simplified approach for design purposes. They are not definitive or based on statistical results by any means. Probabilistic methods are needed to accurately estimate the variability of all factors involved.

## 1.3-Definitions of terms

The following terms are defined for general use in this report. It should be noted that separability of creep and shrinkage is considered to be strictly a matter of definition and convenience. The time-dependent deformations of concrete, either under load or in an unloaded specimen, should be considered as two aspects of a single complex physical phenomenon.<sup>88</sup>

### 1.3.1 Shrinkage

Shrinkage, after hardening of concrete, is the decrease

with time of concrete volume. The decrease is due to changes in the moisture content of the concrete and physico-chemical changes, which occur without stress attributable to actions external to the concrete. The converse of shrinkage is swelling which denotes volumetric increase due to moisture gain in the hardened concrete. Shrinkage is conveniently expressed as a dimensionless strain (in./in. or m/m) under steady conditions of relative humidity and temperature.

The above definition includes drying shrinkage, autogenous shrinkage, and carbonation shrinkage.

- a) Drying shrinkage is due to moisture loss in the concrete
- b) Autogenous shrinkage is caused by the hydration of cement
- c) Carbonation shrinkage results as the various cement hydration products are carbonated in the presence of CO<sub>2</sub>

Recommended values in [Chapter 2](#) for shrinkage strain ( $\epsilon_{sh}$ )<sub>t</sub> are consistent with the above definitions.

### 1.3.2 Creep

The time-dependent increase of strain in hardened concrete subjected to sustained stress is defined as creep. It is obtained by subtracting from the total measured strain in a loaded specimen, the sum of the initial instantaneous (usually considered elastic) strain due to the sustained stress, the shrinkage, and the eventual thermal strain in an identical load-free specimen which is subjected to the same history of relative humidity and temperature conditions. Creep is conveniently designated at a constant stress under conditions of steady relative humidity and temperature, assuming the strain at loading (nominal elastic strain) as the instantaneous strain at any time.

The above definition treats the initial instantaneous strain, the creep strain, and the shrinkage as additive, even though they affect each other. An instantaneous change in stress is most likely to produce both elastic and inelastic instantaneous changes in strain, as well as short-time creep strains (10 to 100 minutes of duration) which are conventionally included in the so-called instantaneous strain. Much controversy about the best form of "practical creep equations" stems from the fact that no clear separation exists between the instantaneous strain (elastic and inelastic strains) and the creep strain. Also, the creep definition lumps together the basic creep and the drying creep.

- a) Basic creep occurs under conditions of no moisture movement to or from the environment
- b) Drying creep is the additional creep caused by drying

In considering the effects of creep, the use of either a unit strain,  $\delta_c$  (creep per unit stress), or creep coefficient,  $\nu_c$  (ratio of creep strain to initial strain), yields the same

results, since the concrete initial modulus of elasticity,  $E_{ci}$ , must be included, that is:

$$v_i = \delta_i E_{ci} \quad (1-1)$$

This is seen from the relations:

$$\begin{aligned} \text{Creep strain} &= \sigma \delta_i \\ &= \epsilon_i v_i, \text{ and} \\ E_{ci} &= \sigma / \epsilon_i \end{aligned}$$

where,  $\sigma$  is the applied constant stress and  $\epsilon_i$  is the instantaneous strain.

The choice of either of  $\delta_i$  or  $v_i$  is a matter of convenience depending on whether it is desired to apply the creep factor to stress or strain. The use of  $v_i$  is usually more convenient for calculation of deflections and prestressing losses.

### 1.3.3 Relaxation

Relaxation is the gradual reduction of stress with time under sustained strain. A sustained strain produces an initial stress at time of application and a deferred negative (deductive) stress increasing with time at a decreasing rate.<sup>89</sup>

### 1.3.4 Modulus of elasticity

The static modulus of elasticity (secant modulus) is the linearized instantaneous (1 to 5 minutes) stress-strain relationship. It is determined as the slope of the secant drawn from the origin to a point corresponding to 0.45  $f_c'$  on the stress-strain curve, or as in ASTM C 469.

### 1.3.5 Contraction and expansion

Concrete contraction or expansion is the algebraic sum of volume changes occurring as the result of thermal variations caused by heat of hydration of cement and by ambient temperature change. The net volume change is a function of the constituents in the concrete.

## CHAPTER 2-MATERIAL RESPONSE

### 2.1-Introduction

The procedures used to predict the effects of time-dependent concrete volume changes in Chapters 3, 4, and 5 depend on the prediction of the material response parameters; i.e., strength, elastic modulus, creep, shrinkage and coefficient of thermal expansion.

The equations recommended in this chapter are simplified expressions representing average laboratory data obtained under steady environmental and loading conditions. They may be used if specific material response parameters are not available for local materials and environmental conditions.

Experimental determination of the response parameters using the standard referenced throughout this report and listed in Section 2.10 is recommended if an accurate prediction of structural service response is desired. No prediction method can yield better results than testing actual materials under environmental and

loading conditions similar to those expected in the field. It is difficult to test for most of the variables involved in one specific structure. Therefore, data from standard test conditions used in connection with the equations recommended in this chapter may be used to obtain a more accurate prediction of the material response in the structure than the one given by the parameters recommended in this chapter.

Occasionally, it is more desirable to use material parameters corresponding to a given probability or to use upper and lower bound parameters based on the expected loading and environmental conditions. This prediction will provide a range of expected variations in the response rather than an average response. However, probabilistic methods are not within the scope of this report.

The importance of considering appropriate water content, temperature, and loading histories in predicting concrete response parameters cannot be overemphasized. The differences between field measurements and the predicted deformations or stresses are mostly due to the lack of correlation between the assumed and the actual histories for water content, temperature, and loading.

### 2.2-Strength and elastic properties

#### 2.2.1 Concrete compressive strength versus time

A study of concrete strength versus time for the data of References 1-6 indicates an appropriate general equation in the form of Eq. (2-1) for predicting compressive strength at any time.<sup>6,7</sup>

$$(f_c')_t = \frac{t}{a + \beta t} (f_c')_{28} \quad (2-1)$$

where  $a$  in days and  $\beta$  are constants,  $(f_c')_{28}$  = 28-day strength and  $t$  in days is the age of concrete.

Compressive strength is determined in accordance with ASTM C 39 from 6 x 12 in. (152 x 305 mm) standard cylindrical specimens, made and cured in accordance with ASTM C 192.

Equation (2-1) can be transformed into

$$(f_c')_t = \frac{t}{a/\beta + t} (f_c')_u \quad (2-2)$$

where  $a/\beta$  is age of concrete in days at which one half of the ultimate (in time) compressive strength of concrete,  $(f_c')_u$  is reached.<sup>92</sup>

The ranges of  $a$  and  $\beta$  in Eqs. (2-1) and (2-2) for the normal weight, sand lightweight, and all lightweight concretes (using both moist and steam curing, and Types I and III cement) given in References 6 and 7 (some 88 specimens) are:  $a = 0.05$  to 9.25,  $\beta = 0.67$  to 0.98.

The constants  $a$  and  $\beta$  are functions of both the type of cement used and the type of curing employed. The use of normal weight, sand lightweight, or all-lightweight aggregate does not appear to affect these constants significantly. Typical values recommended in References 7 are given in Table 2.2.1. Values for the time-ratio,  $(f_c')_t / (f_c')_{28}$  or  $(f_c')_t / (f_c')_u$  in Eqs. (2-1) and (2-2) are given also in Table 2.2.1.



"Moist cured conditions" refer to those in ASTM C 132 and C 511. Temperatures other than  $73.4 \pm 3$  F ( $23 \pm 1.7$  C) and relative humidities less than 35 percent may result in values different than those predicted when using the constant on **Table 2.2.1** for moist curing. The effect of concrete temperature on the compressive and flexural strength development of normal weight concretes made with different types of cement with and without accelerating admixtures at various temperatures between 25 F (-3.9 C) and 120 F (48.9 C) were studied in **Reference 90**.

Constants in **Table 2.2.1** are not applicable to concretes, such as mass concrete, containing Type II or Type V cements or containing blends of portland cement and pozzolanic materials. In those cases, strength gains are slower and may continue over periods well beyond one year age.

"Steam cured" means curing with saturated steam at atmospheric pressure at temperatures below 212 F (100 C).

Experimental data from **References 1-6** are compared in **Reference 7** and all these data fall within about 20 percent of the average values given by **Eqs. (2-1) and (2-2)** for constants  $\alpha$  and  $\beta$  in **Table 2.2.1**. The temperature and cycle employed in steam curing may substantially affect the strength-time ratio in the early days following curing.<sup>107</sup>

### 2.2.2 Modulus of rupture, direct tensile strength and modulus of elasticity

Eqs. (2-3), (2-4), and (2-5) are considered satisfactory in most cases for computing average values for modulus of rupture,  $f_r$ , direct tensile strength,  $f'_t$ , and secant modulus of elasticity at  $0.4(f'_c)_t$ ,  $E_{ct}$  respectively of different weight concretes.<sup>1,4-12</sup>

$$f_r = g_r [w(f'_c)_t]^{1/2} \quad (2-3)$$

$$f'_t = g_t [w(f'_c)_t]^{1/2} \quad (2-4)$$

$$E_{ct} = g_{ct} [w^3(f'_c)_t]^{1/2} \quad (2-5)$$

For the unit weight of concrete,  $w$  in pcf and the compressive strength,  $(f'_c)_t$  in psi

$$g_r = 0.60 \text{ to } 1.00 \text{ (a conservative value of } g_r = 0.60 \text{ may be used, although a value } g_r = 0.60 \text{ to } 0.70 \text{ is more realistic in most cases)}$$

$$g_t = 1/3$$

$$g_{ct} = 33$$

For  $w$  in  $\text{Kg/m}^3$  and  $(f'_c)_t$  in MPa

$$g_r = 0.012 \text{ to } 0.021 \text{ (a conservative value of } g_r = 0.012 \text{ may be used, although a value of } g_r = 0.013 \text{ to } 0.014 \text{ is more realistic in most cases)}$$

$$g_t = 0.0069$$

$$g_{ct} = 0.043$$

The modulus of rupture depends on the shape of the tension zone and loading conditions Eq.(2-3) corresponds to a  $6 \times 6$  in. ( $150 \times 150$  mm) cross section as in ASTM C 78, Where much of the tension zone is remote from the neutral axis as in the case of large box girders or large I-beams, the modulus of rupture approaches the direct tensile strength.

Eq. (2-5) was developed by Pauw<sup>11</sup> and is used in Subsection 8.5.1 of **Reference 27**. The static modulus of elasticity is determined experimentally in accordance with ASTM C 649.

The modulus of elasticity of concrete, as commonly understood is not the truly instantaneous modulus, but a modulus which corresponds to loads of one to five minutes duration.<sup>86</sup>

### 2.3—Theory for predicting creep and shrinkage of concrete

The principal variables that affect creep and shrinkage are discussed in detail in **References 3, 6, 13-16**, and are summarized in **Table 2.2.2**. The design approach presented<sup>6,7</sup> for predicting creep and shrinkage: refers to "standard conditions" and correction factors for other than Standard conditions. This approach has also been used in **References 3, 7, 17, and 83**.

Based largely on information from **References 3-6, 13, 15, 18-21**, the following general procedure is suggested for predicting creep and shrinkage of concrete at any time.<sup>1</sup>

$$v_t = \frac{t^\psi}{d + t^\psi} v_u \quad (2-6)$$

$$(\epsilon_{sh})_t = \frac{t^\alpha}{f + t^\alpha} (\epsilon_{sh})_u \quad (2-7)$$

where  $d$  and  $f$  (in days),  $\psi$  and  $\alpha$  are considered constants for a given member shape and size which define the time-ratio part,  $v_u$  is the ultimate creep coefficient defined as ratio of creep strain to initial strain,  $(\epsilon_{sh})_u$  is the ultimate shrinkage strain, and  $t$  is the time after loading in Eq. (2-6) and time from the end of the initial curing in Eq. (2-7).

When  $\psi$  and  $\alpha$  are equal to 1.0, these equations are the familiar hyperbolic equations of Ross<sup>15</sup> and Lorman<sup>21</sup> in slightly different form.

The form of these equations is thought to be convenient for design purposes, in which the concept of the ultimate (in time) value is modified by the time-ratio to yield the desired result. The increase in creep after, say, 100 to 200 days is usually more pronounced than shrinkage. In percent of the ultimate value, shrinkage usually increases more rapidly during the first few months. Appropriate powers of  $t$  in Eqs. (2-6) and (2-7) were found in **References 6 and 7** to be 1.0 for shrinkage (flatter hyperbolic form) and 0.60 for creep (steeper curve for

larger values of  $t$ ). This can be seen in Fig. (2-3) and (2-4) of [Reference 7](#).

Values of  $\psi$ ,  $d$ ,  $v_u$ ,  $\alpha$ ,  $f$ , and  $(\epsilon_{sh})_u$  can be determined by fitting the data obtained from tests performed in accordance to ASTM C 512.

Normal ranges of the constants in [Eqs. \(2-6\) and \(2-7\)](#) were found to be:<sup>6,7</sup>

$$\begin{aligned}\psi &= 0.40 \text{ to } 0.80, \\ d &= 6 \text{ to } 30 \text{ days}, \\ v_u &= 1.30 \text{ to } 4.15, \\ \alpha &= 0.90 \text{ to } 1.10, \\ f &= 20 \text{ to } 130 \text{ days}, \\ (\epsilon_{sh})_u &= 415 \times 10^{-6} \text{ to } 1070 \times 10^{-6} \text{ in./in. (m/m)}\end{aligned}$$

These constants are based on the standard conditions in [Table 2.2.2](#) for the normal weight, sand lightweight, and all lightweight concretes, using both moist and steam curing, and Types I and III cement as in [References 3-6, 13, 15, 18-20, 23, 24](#).

Eqs. (2-8), (2-9), and (2-10) represent the average values for these data. These equations were compared with the data (120 creep and 95 shrinkage specimens) in [Reference 7](#). The constants in the equations were determined on the basis of the best fit for all data individually. The average-value curves were then determined by first obtaining the average of the normal weight, sand lightweight, and all lightweight concrete data separately, and then averaging these three curves. The constants  $v_u$  and  $(\epsilon_{sh})_u$  recommended in [References 7 and 96](#) were approximately the same as the overall numerical averages, that is  $v_u = 2.35$  was recommended versus 2.36;  $(\epsilon_{sh})_u = 800 \times 10^{-6}$  in./in. (m/m) versus  $803 \times 10^{-6}$  for moist cured concrete, and  $730 \times 10^{-6}$  versus  $788 \times 10^{-6}$  for steam cured concrete.

The creep and shrinkage data, based on 20-year measurements<sup>7,18</sup> for normal weight concrete with an initial time of 28 days, are roughly comparable with Eqs. (2-8) to (2-10). Some differences are to be found because of the different initial times, stress levels, curing conditions, and other variables.

However, subsequent work<sup>59</sup> with 479 creep data points and 356 shrinkage data points resulted in the same average for  $v_u = 2.35$ , but a new average for  $(\epsilon_{sh})_u = 780 \times 10^{-6}$  in./in. (m/m), for both moist and steam cured concrete. It was found that no consistent distinction in the ultimate shrinkage strain was apparent for moist and steam cured concrete, even though different time-ratio terms and starting times were used.

The procedure using Eqs. (2-8) to (2-10) has also been independently evaluated and recommended in [Reference 60](#), in which a comprehensive experimental study was made of the various parameters and correction factors for different weight concrete.

No consistent variation was found between the different weight concretes for either creep or shrinkage. It was noted in the development of Eq. (2-8) that more consistent results were found for the creep variable in the

form of the creep coefficient,  $v_t$  (ratio of creep strain to initial strain), as compared to creep strain per unit stress,  $\delta_t$ . This is because the effect of concrete stiffness is included by means of the initial strain.

#### 2.4-Recommended creep and shrinkage equations for standard conditions

Equations (2-8), (2-9), and (2-10) are recommended for predicting a creep coefficient and an unrestrained shrinkage strain at any time, including ultimate values.<sup>6-7</sup> They apply to normal weight, sand lightweight, and all lightweight concrete (using both moist and steam curing, and Types I and III cement) under the standard conditions summarized in [Table 2.2.2](#).

Values of  $v_u$  and  $(\epsilon_{sh})_u$  need to be modified by the correction factors in [Sections 2.5 and 2.6](#) for conditions other than the standard conditions.

Creep coefficient,  $v_t$  for a loading age of 7 days, for moist cured concrete and for 1-3 days steam cured concrete, is given by Eq. (2-8).

$$v_t = \frac{t^{0.60}}{10 + t^{0.60}} v_u \quad (2-8)$$

Shrinkage after age 7 days for moist cured concrete:

$$(\epsilon_{sh})_t = \frac{t}{35 + t} (\epsilon_{sh})_u \quad (2-9)$$

Shrinkage after age 1-3 days for steam cured concrete:

$$(\epsilon_{sh})_t = \frac{t}{55 + t} (\epsilon_{sh})_u \quad (2-10)$$

In Eq. (2-8),  $t$  is time in days after loading. In Eqs. (2-9) and (2-10),  $t$  is the time after shrinkage is considered, that is, after the end of the initial wet curing.

In the absence of specific creep and shrinkage data for local aggregates and conditions, the average values suggested for  $v_u$  and  $(\epsilon_{sh})_u$  are:

$$v_u = 2.35 \gamma_c \text{ and}$$

$$(\epsilon_{sh})_u = 780 \gamma_{sh} \times 10^{-6} \text{ in./in., (m/m)}$$

where  $\gamma_c$  and  $\gamma_{sh}$  represent the product of the applicable correction factors as defined in [Sections 2.5 and 2.6](#) by [Equations \(2-12\) through \(2-30\)](#).

These values correspond to reasonably well shaped aggregates graded within limits of ASTM C 33. Aggregates affect creep and shrinkage principally because they influence the total amount of cement-water paste in the concrete.

The time-ratio part, [right-hand side except for  $v_u$  and  $(\epsilon_{sh})_u$ ] of Eqs. (2-8), (2-9), and (2-10), appears to be applicable quite generally for design purposes. Values from the standard Eqs. (2-8) to (2-10) of  $v_t/v_u$  and

$(\epsilon_{sh})_t/(\epsilon_{sh})_u$  are shown in Table 2.4.1. Note that  $v$  is used in Eqs. (4-11), (4-20), and (4-22), hence,  $v_t/v_u = v_s/v_u$  for the age of the precast beam concrete at the slab casting.

It has also been shown<sup>24</sup> that the time-ratio part of Eqs. (2-8) and (2-10) can be used to extrapolate 28-day creep and shrinkage data determined experimentally in accordance with ASTM C 512, to complete time curves up to ultimate quite well for creep, and reasonably well for shrinkage for a wide variety of data. It should be noticed that the time-ratio in Eqs. (2-8) to (2-10) does not differentiate between basic and drying creep nor between drying autogenous and carbonation shrinkage. Also, it is independent of member shape and size, because  $d, f, \psi$ , and  $\alpha$  are considered as constant in Eqs. (2-8), (2-9), and (2-10).

The shape and size effect can be totally considered on the time-ratio, without the need for correction factors. That is, in terms of the shrinkage-half-time  $\tau_{sh}$ , as given by Eq. (2-35) by replacing  $t$  by  $t/\tau_{sh}$  in Eq. (2-9) and by  $0.1t/\tau_{sh}$  in Eq. (2-8) as shown in 2.8.1. Also by taking  $\psi = \alpha = 1.0$  and  $d = f = 26.0 [\exp 0.36(v/s)]$  in Eqs. (2-6) and (2-7) as in Reference 23, where  $v/s$  is the volume to surface ratio, in inches. For  $v/s$  in mm use  $d = f = 26.0 \exp [1.42 \times 10^{-2} (v/s)]$ .

References 61, 89, 92, 98 and 101 consider the effect of the shape and size on both the time-ratio (time-dependent development) and on the coefficients affecting the ultimate (in time) value of creep and shrinkage.

ACI Committee 209, Subcommittee I Report<sup>6</sup> is recommended for a detailed review of the effects of concrete constituents, environment and stress on time-dependent concrete deformations.

## 2.5-Correction factors for conditions other than the standard concrete composition<sup>7</sup>

All correction factors,  $y$ , are applied to ultimate values. However, since creep and shrinkage for any period in Eqs. (2-8) through (2-10) are linear functions of the ultimate values, the correction factors in this procedure may be applied to short-term creep and shrinkage as well.

Correction factors other than those for concrete composition in Eqs. (2-11) through (2-22) may be used in conjunction with the specific creep and shrinkage data from a concrete tested in accordance with ASTM C 512.

### 2.5.1 Loading age

For loading ages later than 7 days for moist cured concrete and later than 1-3 days for steam cured concrete, use Eqs. (2-11) and (2-12) for the creep correction factors.

$$\text{Creep } \gamma_{\ell a} = 1.25(t_{\ell a})^{-0.118} \text{ for moist cured concrete (2-11)}$$

$$\text{Creep } \gamma_{\ell a} = 1.13(t_{\ell a})^{-0.094} \text{ for steam cured concrete (2-12)}$$

where  $t_{\ell a}$  is the loading age in days. Representative values are shown in Table 2.51. Note that in Eqs. (4-11), (4-20), and (4-22), the Creep  $\gamma_{\ell a}$  correction factor must be used when computing the ultimate creep coefficient of the present beam corresponding to the age when slab is cast,  $v_{us}$ . That is:

$$v_{us} = v_u \times (\text{Creep } \gamma_{\ell a}) \quad (2-13)$$

### 2.5.2 Differential shrinkage

For shrinkage considered for other than 7 days for moist cured concrete and other than 1-3 days for steam cured concrete, determine the difference in Eqs. (2-9) and (2-10) for any period starting after this time.

That is, the shrinkage strain between 28 days and 1 year, would be equal to the 7 days to 1 year shrinkage minus the 7 days to 28 days shrinkage. In this example for moist cured concrete, the concrete is assumed to have been cured for 7 days. Shrinkage  $\gamma_{cp}$  factor as in 2.5.3 below, is applicable to Eq. (2-9) for concrete moist cured during a period other than 7 days.

### 2.5.3 Initial moist curing

For shrinkage of concrete moist cured during a period of time other than 7 days, use the Shrinkage  $\gamma_{cp}$  factor in Table 2.5.3. This factor can be used to estimate differential shrinkage in composite beams, for example.

Linear interpolation may be used between the values in Table 2.5.3.

### 2.5.4 Ambient relative humidity

For ambient relative humidity greater than 40 percent, use Eqs. (2-14) through (2-16) for the creep and shrinkage correction factors.<sup>7,20,22</sup>

$$\text{Creep } \gamma_{\lambda} = 1.27 - 0.0067\lambda, \text{ for } \lambda > 40 \quad (2-14)$$

$$\text{Shrinkage } \gamma_{\lambda} = 1.40 - 0.0102, \text{ for } 40 \leq \lambda \leq 80 \quad (2-15)$$

$$= 3.00 - 0.030\lambda, \text{ for } 80 > \lambda \leq 100 \quad (2-16)$$

where  $\lambda$  is relative humidity in percent. Representative values are shown in Table 2.5.4.

The average value suggested for  $\lambda = 40$  percent is  $(\epsilon_{sh})_u = 780 \times 10^{-6}$  in./in. (m/m) in both Eqs. (2-9) and (2-10). From Eq. (2-15) of Table 2.5.4, for  $\lambda = 70$  percent,  $(\epsilon_{sh})_u = 0.70(780 \times 10^{-6}) = 546 \times 10^{-6}$  in./in. (m/m), for example. For lower than 40 percent ambient relative humidity, values higher than 1.0 shall be used for Creep  $\gamma_{\lambda}$  and Shrinkage  $\gamma_{\lambda}$ .

### 2.5.5 Average thickness of member other than 6 in. (150 mm) or volume-surface ratio other than 1.5 in. (38 mm)

The member size effects on concrete creep and shrinkage is basically two-fold. First, it influences the time-ratio (see Equations 2-6, 2-7, 2-8, 2-9, 2-10 and 2-35). Secondly, it also affects the ultimate creep coefficient,  $v_u$  and the ultimate shrinkage strain,  $(\epsilon_{sh})_u$ .

Two methods are offered for estimating the effect of

member size on  $\nu_u$  and  $(\epsilon_{sh})_u$ . The average-thickness method tends to compute correction factor values that are higher, as compared to the volume-surface ratio method,<sup>59</sup> since Creep  $\gamma_h$  = Creep  $\gamma_{vs}$  = 1.00 for  $h = 6$  in. (150 mm) and  $\nu/s = 1.5$  in. (38 mm), respectively; that is, when  $h = 4\nu/s$ .

#### 2.5.5.a Average-thickness method

The method of treating the effect of member size in terms of the average thickness is based on information from [References 3, 6, 7, 23 and 61](#).

For average thickness of member less than 6 in. (150 mm), use the factors given in [Table 2.5.5.1](#). These correspond to the [CEB<sup>61</sup>](#) values for small members. For average thickness of members greater than 6 in. (150 mm) and up to about 12 to 15 in. (300 to 380 mm), use Eqs. (2-17) to (2-18) through (2-20).

During the first year after loading:

$$\text{Creep } \gamma_h = 1.14 - 0.023 h, \quad (2-17)$$

For ultimate values:

$$\text{Creep } \gamma_h = 1.10 - 0.017 h, \quad (2-18)$$

During the first year of drying:

$$\text{Shrinkage } \gamma_h = 1.23 - 0.038 h, \quad (2-19)$$

For ultimate values:

$$\text{Shrinkage } \gamma_h = 1.17 - 0.029 h, \quad (2-20)$$

where  $h$  is the average thickness in inches of the part of the member under consideration.

During the first year after loading:

$$\text{Creep } \gamma_h = 1.14 - 0.00092 h, \quad (2-17a)$$

For ultimate values:

$$\text{Creep } \gamma_h = 1.10 - 0.00067 h, \quad (2-18a)$$

During the first year after loading:

$$\text{Shrinkage } \gamma_h = 1.23 - 0.00015 h, \quad (2-19a)$$

For ultimate values:

$$\text{Shrinkage } \gamma_h = 1.17 - 0.00114 h, \quad (2-20a)$$

where  $h$  is in mm.

Representative values are shown in [Table 2.5.5.1](#).

#### 2.5.5.b Volume-surface ratio method

The volume-surface ratio equations (2-21) and (2-22) were adapted from [Reference 23](#).

$$\text{Creep } \gamma_{vs} = \frac{2}{3}[1 + 1.13 \exp(-0.54 \nu/s)] \quad (2-21)$$

$$\text{Shrinkage } \gamma_{vs} = 1.2 \exp(-0.12 \nu/s) \quad (2-22)$$

where  $\nu/s$  is the volume-surface ratio of the member in inches.

$$\text{Creep } \gamma_{vs} = \frac{2}{3}[1 + 1.13 \exp(-0.0213 \nu/s)] \quad (2-21a)$$

$$\text{Shrinkage } \gamma_{vs} = 1.2 \exp(-0.00472 \nu/s) \quad (2-22a)$$

where  $\nu/s$  in mm.

Representative values are shown in [Table 2.5.5.2](#).

However, for either method  $\gamma_{sh}$  should not be taken less than 0.2. Also, use  $\gamma_{sh}(\epsilon_{sh})_u \geq 100 \times 10^{-6}$  in./in., (m/m) if concrete is under seasonal wetting and drying cycles and  $\gamma_{sh}(\epsilon_{sh})_u \geq 150 \times 10^{-6}$  in./in. (m/m) if concrete is under sustained drying conditions.

#### 2.5.6 Temperature other than 70 F (21 C)

Temperature is the second major environmental factor in creep and shrinkage. This effect is usually considered to be less important than relative humidity since in most structures the range of operating temperatures is small,<sup>68</sup> and high temperatures seldom affect the structures during long periods of time.

The effect of temperature changes on concrete creep<sup>68</sup> and shrinkage is basically two-fold. First, they directly influence the time ratio rate. Second, they also affect the rate of aging of the concrete, i.e. the change of material properties due to progress of cement hydration. At 122 F (50 C), creep strain is approximately two to three times as great as at 68-75 F (19-24 C). From 122 to 212 F (50 to 100 C) creep strain continues to increase with temperature, reaching four to six times that experienced at room temperatures. Some studies have indicated an apparent creep rate maximum occurs between 122 and 176 F (50 and 80 C).<sup>86</sup> There is little data establishing creep rates above 212 F (100 C). Additional information on temperature effect on creep may be found in [References 68, 84, and 85](#).

#### 2.6-Correction factors for concrete composition

[Equations \(2-23\) through \(2-30\)](#) are recommended for use in obtaining correction factors for the effect of slump, percent of fine aggregate, cement and air content. It should be noted that for slump less than 5 in. (130 mm), fine aggregate percent between 40-60 percent, cement content of 470 to 750 lbs. per yd<sup>3</sup> (279 to 445 kg/m<sup>3</sup>) and air content less than 8 percent, these factors are approximately equal to 1.0.

These correction factors shall be used only in connection with the average values suggested for  $\nu_u = 2.35$  and  $(\epsilon_{sh})_u = 780 \times 10^{-6}$  in./in. (m/m). As recommended in [2.4](#), these average values for  $\nu_u$  and  $(\epsilon_{sh})_u$  should be used only in the absence of specific creep and shrinkage data for local aggregates and conditions determined in accordance with ASTM C 512.

If shrinkage is known for local aggregates and conditions, [Eq. \(2-31\)](#) as discussed in [2.6.5](#), is recommended.



The principal disadvantage of the concrete composition correction factors is that concrete mix characteristics are unknown at the design stage and have to be estimated. Since these correction factors are normally not excessive and tend to offset each other, in most cases, they may be neglected for design purposes.

### 2.6.1 Slump

$$\text{Creep } \gamma_s = 0.82 + 0.067s \quad (2-23)$$

$$\text{Shrinkage } \gamma_s = 0.89 + 0.041s \quad (2-24)$$

where  $s$  is the observed slump in inches. For slump in mm use:

$$\text{Creep } \gamma_s = 0.82 + 0.00264s \quad (2-23 \text{ a})$$

$$\text{Shrinkage } \gamma_s = 0.89 + 0.00161s \quad (2-24a)$$

### 2.6.2 Fine aggregate percentage

$$\text{Creep } \gamma_\psi = 0.88 + 0.0024\psi \quad (2-25)$$

For  $\psi \leq 50$  percent

$$\text{Shrinkage } \gamma_\psi = 0.30 + 0.014\psi \quad (2-26)$$

For  $\psi > 50$  percent

$$\text{Shrinkage} = 0.90 + 0.002\psi \quad (2-27)$$

where  $\psi$  is the ratio of the fine aggregate to total aggregate by weight expressed as percentage.

### 2.6.3 Cement content

Cement content has a negligible effect on creep coefficient. An increase in cement content causes a reduced creep strain if water content is kept constant; however, data indicate that a proportional increase in modulus of elasticity accompanies an increase in cement content.

If cement content is increased and water-cement ratio is kept constant, slump and creep will increase and Eq. (2-23) applies also.

$$\text{Shrinkage } \gamma_c = 0.75 + 0.00036c \quad (2-28)$$

where  $c$  is the cement content in pounds per cubic yard. For cement content in  $\text{Kg/m}^3$ , use:

$$\text{Shrinkage } \gamma_c = 0.75 + 0.00061c \quad (2-28a)$$

### 2.6.4 Air content

$$\text{Creep } \gamma_\alpha = 0.46 + 0.09\alpha, \quad \text{but not less than 1.0} \quad (2-29)$$

$$\text{Shrinkage } \gamma_\alpha = 0.95 + 0.008\alpha \quad (2-30)$$

where  $\alpha$  is the air content in percent.

### 2.6.5 Shrinkage ratio of concretes with equivalent paste quality<sup>91</sup>

Shrinkage strain is primarily a function of the shrinkage characteristics of the cement paste and of the aggregate volume concentration. If the shrinkage strain of a given mix has been determined, the ratio of shrinkage strain of two mixes  $(\epsilon_{sh})_1/(\epsilon_{sh})_2$ , with different content of paste but with equivalent paste quality is given in Eq. (2-31).

$$\frac{(\epsilon_{sh})_{u1}}{(\epsilon_{sh})_{u2}} = \frac{1 - (v_1)^{1/3}}{1 - (v_2)^{1/3}} \quad (2-31)$$

where  $v_1$  and  $v_2$  are the total aggregate solid volumes per unit volume of concrete for each one of the mixes.

### 2.7-Example

Find the creep coefficient and shrinkage strains at 28, 90, 180, and 365 days after the application of the load, assuming that the following information is known: 7 days moist cured concrete, age of loading  $t_{la} = 28$  days, 70 percent ambient relative humidity, shrinkage considered from 7 days, average thickness of member 8 in. (200 mm), 2.5 in. slump (63 mm), 60 percent fine aggregate, 752 lbs. of cement per  $\text{yd}^3$  ( $446 \text{ Kg/m}^3$ ), and 7 percent air content.<sup>7</sup> Also, find the differential shrinkage strain,  $(\epsilon_{sh})_\delta$  for the period starting at 28 days after the application of the load,  $t_{la} = 56$  days.

The applicable correction factors are summarized in Table 2.7.1. Therefore:

$$v_u = (2.35)(0.710) = 1.67$$

$$(\epsilon_{sh})_u = (780 \times 10^{-6})(0.68) = 530 \times 10^{-6}$$

The results from the use of Eqs. (2-8) and (2-9) or Table 2.4.1 are shown in Table 2.7.2.

Notice that if correction factors for the concrete composition are ignored for  $v_t$  and  $(\epsilon_{sh})_t$ , they will be 10 and 4 percent smaller, respectively.

### 2.8-Other methods for predictions of creep and shrinkage

Other methods for prediction of creep and shrinkage are discussed in Reference 61, 68, 86, 87, 89, 93, 94, 95, 97, and 98. Methods in References 97 and 98 subdivide creep strain into delayed elastic strain and plastic flow (two-component creep model). References 88, 89, 92, 99, 100, 102, and 104 discuss the conceptual differences between the current approaches to the formulation of the creep laws. However, in dealing with any method, it is important to recall what is discussed in Sections 1.2 and 2.1 of this report.

#### 2.8.1 Remark on refined creep formulas needed for special structures<sup>93,94,95</sup>

The preceding formulation represents a compromise between accuracy and generality of application. More accurate formulas are possible but they are inevitably not as general.

The time curve of creep given by Eq. (2-8) exhibits a decline of slope in log- $t$  scale for long times. This property is correct for structures which are allowed to lose their moisture and have cross sections which are not too massive (6 to 12 in., 150 to 300 mm). Structures which are insulated, or submerged in water, or are so massive they cannot lose much of their moisture during their lifetime, exhibit creep curves whose slope in log- $t$  scale is not decreasing at end, but steadily increasing. For example, if Eq. (2-8) were used for extrapolating short-time creep data for a nuclear reactor containment into long times, the long-term creep values would be seriously underestimated, possibly by as much as 50 percent as shown in Fig. 3 of Ref. 81.

It has been found that creep without moisture exchange (basic creep) for any loading age  $t_{\ell a}$  is better described by Equation (2-33).<sup>86,80,83,95</sup> This is called the double power law.

In Eq. (2-33)  $\psi_1$  is a constant, and strain  $\epsilon$  is the sum of the instantaneous strain and creep strain caused by unit stress.

$$\epsilon = \frac{1}{E_o} + \frac{\psi_1}{E_o} (t_{\ell a})^{-1/8} t^{1/8} \quad (2-33)$$

where  $1/E_o$  is a constant which indicates the lefthand asymptote of the creep curve when plotted in log  $t$ -scale (time  $t = 0$  is at  $-\infty$  in this plot). The asymptotic value  $1/E_o$  is beyond the range of validity of Eq. (2-33) and should not be confused with elastic modulus. Suitable values of constants are  $\psi_1 = 0.97\nu_u$  and  $1/E_o = 0.84/E_{ct}$ , being  $E_{ct}$  the modulus of concrete which does not undergo drying. With these values, Eq. (2-33) and Eq. (2-8) give the same creep for  $t_{\ell a} = 28$  days,  $t = 10,000$  days and 100 percent relative humidity ( $\gamma_A = 0.6$ ), all other correction factors being taken as one.

Eq. (2-33) has further the advantage that it describes not only the creep curves with their age dependence, but also the age dependence of the elastic modulus  $E_{ct}$  in absence of drying.  $E_{ct}$  is given by  $\epsilon = 1/E_{ct}$  for  $t \approx 0.001$  day, that is:

$$\frac{1}{E_{ct}} = \frac{1}{E_o} + \frac{\psi_1}{E_o} (0.001)^{1/8} (t_{\ell a})^{-1/8} \quad (2-34)$$

Eq. (2-33) also yields the values of the dynamic modulus, which is given by  $\epsilon = 1/E_{dyn}$  when  $t = 10^{-7}$  days is substituted. Since three constants are necessary to describe the age dependence of elastic modulus ( $E_o$ ,  $\psi$ , and  $1/8$ ), only one additional constant (i.e.,  $1/8$ ) is needed to describe creep.

In case of drying, more accurate, but also more complicated, formulas may be obtained<sup>94</sup> if the effect of cross section size is expressed in terms of the shrinkage half-time, as given in Eq. (2-35) for the age  $t_d$  at which concrete drying begins.

$$\tau_{sh} = 600 \left[ \frac{\lambda_s d_c^2}{150} \right] \frac{C_1}{(C_1)_a} \quad (2-35)$$

where:

- $d_c$  = characteristic thickness of the cross section, or twice the volume-surface ratio
- $(d_c) = 2 \sqrt{v/s}$  in mm
- $C_1$  = Drying diffusivity of the concrete (approx. 10 mm/day if measurements are unavailable)
- $(C_1)_a$  = age dependence coefficient
- $(C_1)_a = C_7 \lambda_T (0.05 + \sqrt{6.3/t_d})$
- $C_7 = \frac{w}{8} - 12$ , if  $C_7 < 7$ , set  $C_7 = 7$   
if  $C_7 > 21$ , set  $C_7 = 21$
- $\lambda_s$  = coefficient depending on the shape of cross section, that is:
- $\lambda_s = 1.00$  for an infinite long slab
- $\lambda_s = 1.15$  for an infinite long cylinder
- $\lambda_s = 1.25$  for an infinite long square prism
- $\lambda_s = 1.30$  for a sphere
- $\lambda_s = 1.55$  for a cube
- $\lambda_T$  = temperature coefficient
- $\lambda_T = \frac{T}{t_o} \exp \left( \frac{5000}{T_o} - \frac{5000}{T} \right)$
- $T$  = concrete temperature in kelvin
- $T_o$  = reference temperature in kelvin
- $w$  = water content in kg/m<sup>3</sup>

By replacing  $t$  in Eq. (2-9)  $t/\tau_{sh}$ , shrinkage is expressed without the need for the correction factor for size in Section 2.5.5.

The effect of drying on creep may then be expressed by adding two shrinkage-like functions  $\nu_d$  and  $\nu_p$  to the double power law for unit stress.<sup>96</sup> Function  $\nu_d$  expresses the additional creep during drying and function  $\nu_p$ , being negative, expresses the decrease of creep by loading after an initial drying. The increase of creep during drying arises about ten times slower than does shrinkage and so function  $\nu_d$  is similar to shrinkage curve in Eq. (2-9) with  $t$  replaced by  $0.1 t/\tau_{sh}$  in Eq. (2-8).

This automatically accounts also for the size effect, without the need for any size correction factor. The decrease of creep rate due to drying manifests itself only very late, after the end of moisture loss. This is apparent from the fact that function  $\tau_{sh}$  is similar to shrinkage curve in Eq. (2-9) with  $t$  replaced by  $0.01 t/\tau_{sh}$ . Both  $\nu_d$  and  $\nu_p$  include multiplicative correction factors for relative humidity, which are zero at 100 percent, and function  $\nu_d$  further includes a factor depending on the time lag from the beginning of drying exposure to the beginning of loading.

## 2.9-Thermal expansion coefficient of concrete

### 2.9.1 Factors affecting the expansion coefficient

The main factors affecting the value of the thermal coefficient of a concrete are the type and amount of aggregate and the moisture content. Other factors such as mix proportions, cement type and age influence its magnitude to a lesser extent.

The thermal coefficient of expansion of concrete usually reflects the weighted average of the various constituents. Since the total aggregate content in hardened concrete varies from 65 to 80 percent of its volume, and the elastic modulus of aggregate is generally five times that of the hardened cement component, the rock expansion dominates in determining the expansion of the composite concrete. Hence, for normal weight concrete with a steady water content (degree of saturation), the thermal coefficient of expansion for concrete can be regarded as directly proportional to that of the aggregate, modified to a limited extent by the higher expansion behavior of hardened cement.

Temperature changes affect concrete water content, environment relative humidity and consequently concrete creep and shrinkage as discussed in Section 2.5.6. If creep and shrinkage response to temperature changes are ignored and if complete histories for concrete water content, temperature and loading are not considered, the actual response to temperature changes may drastically differ from the predicted one.<sup>79</sup>

### 2.9.2 Prediction of thermal expansion coefficient

The thermal coefficients of expansion determined when using testing methods in ASTM C 531 and CRD 39 correspond to the oven-dry condition and the saturated conditions, respectively. Air-dried concrete has a higher coefficient than the oven-dry or saturated concrete, therefore, experimental values shall be corrected for the expected degree of saturation of the concrete member. Values of  $e_{mc}$  in Table 2.9.1 may be used as corrections to the coefficients determined from saturated concrete samples. In the absence of specific data from local materials and environmental conditions, the values given by Eq. (2-32) for the thermal coefficient of expansion  $e_{th}$  may be used.<sup>76</sup> Eq. (2-32) assumes that the thermal coefficient of expansion is linear within a temperature change over the range of 32 to 140 F (0 to 60 C) and applies only to a steady water content in the concrete.

For  $e_{th}$  in  $10^{-6}/F$ :

$$e_{th} = e_{mc} + 1.72 + 0.72 e_a \quad (2-32)$$

For  $e_{th}$  in  $10^{-6}/C$ :

$$e_{th} = e_{mc} + 3.1 + 0.72 e_a \quad (2-32a)$$

where:

$e_{mc}$  = the degree of saturation component as given in Table 2.9.1

1.72 = the hydrated cement past component (3.1)

$e_a$  = the average thermal coefficient of the total aggregate as given in Table 2.9.2

If thermal expansion of the sand differs markedly from that of the coarse aggregate, the weighted average by solid volume of the thermal coefficients of the sand and coarse aggregate shall be used.

A wide variation in the thermal expansion of the aggregate and related concrete can occur within a rock group. As an illustration, Table 2.9.3 summarizes the range of measured values for each rock group in the research data cited in Reference 76.

For ordinary thermal stress calculations, when the type of aggregate and concrete degree of saturation are unknown and an average thermal coefficient is desired,  $e_{th} = 5.5 \times 10^{-6}/F$  ( $e_{th} = 10.0 \times 10^{-6}/C$ ) may be sufficient. However, in estimating the range of thermal movements (e.g., highways, bridges, etc.), the use of lower and upper bound values such as  $4.7 \times 10^{-6}/F$  and  $6.5 \times 10^{-6}/F$  ( $8.5 \times 10^{-6}/C$  and  $11.7 \times 10^{-6}/C$ ) would be more appropriate.

### 2.10-Standards cited in this report

Standards of the American society for Testing and Materials (ASTM) referenced in this report are listed below with their serial designation:

|            |   |
|------------|---|
| ASTM A 416 | “Standard Specification for Uncoated Seven-Wire Stress-Relieved Strand for Prestressed Concrete”  |
| ASTM A 421 | “Standard Specification for Uncoated Stress-Relieved Wire for Prestressed Concrete”   |
| ASTM C 33  | “Standard Specifications for Concrete Aggregates”   |
| ASTM C 39  | “Standard Test Method for Compressive Strength of Cylindrical Concrete Specimens”   |
| ASTM C 78  | “Standard Test Method for Flexural Strength of Concrete (Using Simple Beam with Third-Point Loading)”                                   |
| ACI C 192  | “Standard Method of Making And Curing Concrete Test Specimens in the Laboratory”  |
| ASTM C 469 | “Standard Method for Static Modulus of Elasticity and Poisson’s Ratio of Concrete in Compression”                                       |
| ASTM C 511 | “Standard Specification for Moist Cabinets and Rooms Used in the Testing Hydraulic Cements and Concretes”                               |
| ASTM C 512 | “Standard Test Method for Creep of Concrete in Compression”   |
| ASTM C 531 | “Standard Method for Securing, Preparing, and Testing Specimens from Hardened Lightweight Insulating Concrete for Compressive Strength” |

ASTM E 328 “Standard Recommended Practice for Stress-Relaxation Tests for Materials and Structures”

The following standard of the U.S. Army Corps of Engineers (CRD) is referred in [Section 2.9](#) of this report:

CRD C39 “Method of Test for Coefficient of Linear Thermal Expansion of Concrete”

## CHAPTER 3-FACTORS AFFECTING THE STRUCTURAL RESPONSE-ASSUMPTIONS AND METHODS OF ANALYSIS

### 3.1-Introduction

Prediction of the structural response of reinforced concrete structures to time-dependent concrete volume changes is complicated by:

- a) The inherent nonelastic properties of the concrete
- b) The continuous redistribution of stress
- c) The nonhomogeneous nature of concrete properties caused by the stages of construction
- d) The effect of cracking on deflection
- e) The effect of external restraints
- f) The effect of the reinforcement and/or prestressing steel
- g) The interaction between the above factors and their dependence on past histories of loadings, water content and temperature

The complexity of the problem requires some simplifying assumptions and reliance on empirical observations.

### 3.2-Principal facts and assumptions

#### 3.2.1 Principal facts

- a) Each loading change produces a resulting deformation component continuous for an infinite period of time<sup>70</sup>
- b) Applied loads in homogeneous statically indeterminate structures cause no time-dependent change in stress and all deformations are proportional to creep coefficient  $\nu$ , as long as the support conditions remain unchanged<sup>70</sup>
- c) The secondary, statically indetermined moments due to prestressing are affected in the same proportion as prestressing force by time-dependent deformations, which is a relatively small effect that is usually neglected
- d) In a great many cases and except when instability is a factor, time-dependent strains due to actual loads do not significantly affect the load capacity of a member. Failure is controlled by very large

strains that develop at collapse, regardless of previous loading history.<sup>71</sup> In these cases, time-dependent strains only affect the structure serviceability. When instability is a factor, creep increment of the eccentricity in beam-columns under sustained load will decrease the member capacity with time

- e) Change in concrete properties with age, such as elastic, creep and shrinkage deformations, must be taken into account

#### 3.2.2 Assumptions

- a) Concrete members including their creep, shrinkage and thermal properties, are considered homogeneous
- b) Creep, shrinkage and elastic strains are mutually additive and independent
- c) For stresses less than about 40 to 50 percent of the concrete strength, creep strains are assumed to be approximately proportional to the sustained stress and obey the principle of superposition of strain histories.<sup>70,80</sup>  
However, tests in [References 105 and 106](#) have shown the nonlinearity of creep strain with stress can start at stresses as low as 30 to 35 percent of the concrete strength. Also, strain superposition is only a first approximation because the individual response histories affect each other as can be seen with recovery curves after unloading
- d) Shrinkage and thermal strains are linearly distributed over the depth of the cross section. This assumption is acceptable for thin and moderate sections, respectively, but may result in error for thick sections
- e) The complex dependence of strain upon the past histories of water content and temperature is neglected for the purpose of analyzing ordinary structures
- f) Restraint by reinforcement and/or prestressing steel is accounted for in the average sense without considering any gradual stress transfer between reinforcement and concrete
- g) The creep time-ratio for various environment humidity conditions and various sizes and shapes of cross section are assumed to have the same shape

Even with these simplifications, the theoretically exact analysis of creep effects according to the assumptions stated,<sup>66</sup> is still relatively complicated. However, more accurate analysis is not really necessary in most instances, except special structures, such as nuclear reactor vessels, bridges or shells of record spans, or special ocean structures. Therefore, simplified methods of analysis<sup>66,80</sup> are being used in conjunction with empirical methods to account for the effects of cracking and reinforcement restraint.



### 3.3-Simplified methods of creep analysis

In choosing the method of analysis, two kinds of cases are distinguished.

**3.3.1 Cases in which the gradual time change of stress due to creep and shrinkage is small and has little effect**

This usually occurs in long-time deflection and pre-stress loss calculations. In such cases the creep strain is accounted for with sufficient accuracy by an elastic analysis in which the actual concrete modulus at the time of initial loading, is replaced with the so-called effective modulus as given by Eq. (3-1).

$$E_e = E_{ci}/(1 + \nu_i) \quad (3-1)$$

This approach is implied in **Chapter 4**. To check if the assumption of small stress change is true, the stress computed on the basis of  $E_{ci}$  should be compared with the stress computed on the basis of  $E_e$ .

**3.3.2 Cases in which the gradual time change of stress due to creep and shrinkage is significant**

In such cases, the age-adjusted effective modulus method<sup>67,68,69</sup> is recommended as discussed in **Chapter 5**.

### 3.4-Effect of cracking in reinforced and prestressed members

To include the effect of cracking in the determination of an effective moment of inertia for reinforced beams and one-way slabs, Eq. (3-2)<sup>10,25,26</sup> has been adopted by the ACI Building Code (ACI 318).<sup>27</sup>

$$I_e = (M_{cr}/M_{max})^3 I_g + [1 - (M_{cr}/M_{max})^3] I_{cr} \quad (3-2)$$

where  $M_{cr}$  is the cracking moment,  $M_{max}$  denotes the maximum moment at the stage for which deflection is being computed,  $I_g$  is the moment of inertia of the gross section neglecting the steel and  $I_{cr}$  is the moment of inertia of the cracked transformed section.

Eq. (3-2) applied only when  $M_{max} \geq M_{cr}$ ; otherwise,  $I_e = I_g$ .

$I_e$  in Eq. (3-2) has limits of  $I_g$  and  $I_{cr}$ , and thus provides a transition expression between the two cases given in the ACI 318 Code.<sup>12,27</sup> The moment of inertia  $I$ , of the uncracked transformed section might be more accurately used instead of the moment inertia of the gross section  $I_g$  in Eq. (3-2), especially for heavily reinforced members and lightweight concrete members (low  $E_c$  and hence high modular ratio  $E_s/E_{ci}$ ).

Eq. (3-2) has also been shown<sup>28</sup> to apply in the deflection calculations of cracked prestressed beams.

For numerical analysis, in which the beam is divided into segments or finite elements, it has been shown<sup>25</sup> that  $I$ , values at individual sections can be determined by modifying Eq. (3-2). The power of 3 is changed to 4 and the moment ratio in both terms is changed to  $M_{cr}/M$ , where  $M$  is the moment at each section. Such a numerical procedure was used in the development of Eq. (3-2).<sup>25</sup>

The above cracking moment is given in Eqs. (3-3) and

(3-4).

For reinforced members:

$$M_{cr} = f_r I_g / y_t \quad (3-3)$$

For noncomposite prestressed members:

$$(M_L)_{cr} = Fe + (FI_g)/A_g y_t + (f_r I_g)/y_t - M_D \quad (3-4)$$

The cracking moment for unshored and shored composite prestressed beams is given in Eq. (41) and (42) of **Reference 63**.

Equation (3-2) refers to an average effective  $I$  for the variable cracking along the span, or between the inflection points of continuous beams. For continuous members (at one or both ends), a numerical procedure may be needed although the use of an average of the positive and negative moment region values from Eq. (3-2) as suggested in Section 9.5.2.4 of **Reference 27** should yield satisfactory results in most cases. For spans which have both ends continuous, an effective average moment of inertia  $I_{ea}$  is obtained by computing an average for the end region values,  $I_{e1}$  and  $I_{e2}$  and then averaging that result with the positive moment region value obtained for Eq. (3-2) as shown in Eq. (3-5).

$$I_{ea} = [(I_{e1} + I_{e2})/2 + I_{ep}]/2 \quad (3-5)$$

In other cases, a weighted average related to the positive and negative moments may be preferable. For example, the weighted average moment of inertia  $I_{ew}$  would be given by Eq. (3-6).<sup>72</sup>

$$I_{ew} = \left\{ I_{ep} \left( 1 - \left[ \frac{M_{e1} + M_{e2}}{2M} \right]^\beta \right) + \left[ \frac{I_{e1} + I_{e2}}{2} \right] \left[ \frac{M_{e1} + M_{e2}}{2M} \right]^\beta \right\} \quad (3-6)$$

where,  $I_{ep}$  is the effective moment of inertia for the positive zone of the beam and  $\beta$  is a positive integer that may be equal to unity for simplicity or equal to two, three or larger for a modest increase in accuracy.

For a span with one end continuous, the  $(I_{e1} + I_{e2})/2$  in Eqs. (3-5) and (3-6) shall be substituted for  $I$  for the negative end zone.

For a flat plate and two way slab interior panels, it has been shown<sup>29</sup> that Eq. (3-2) can be used along with an average of the positive and negative moment region values as follows:

Flat plate-both positive and negative values for the long direction column strip.

Two way slabs-both positive and negative values for the short direction middle strip.

The center of interior panels normally remains uncracked in common designs of these slabs.

For the effect of repeated load cycles on cracking range, see [Reference 63](#).

### 3.5-Effective compression steel in flexural members

Compression steel in reinforced flexural members and nontensioned steel in prestressed flexural members tend to offset the movement of the neutral axis caused by creep. The net movement of the neutral axis is the resultant of two movements. A movement towards the tensile reinforcement (increasing the concrete compression zone, which results in a reduction in the moment arm). This movement is caused by the effect of creep plus a reduction in the compression zone due to the progressive cracking in the tensile zone.

The second movement is produced by the increase in steel strains due to the reduction of the internal moment arm (plus the small effect, if any, of repeated live load cycles). As cracking progresses, steel strains increase further and reduce the moment arm.

The reduced creep effect resulting from the movement of the neutral axis and the presence of compression steel in reinforced members  $A_s'$ , and the inclusion of nontensioned high strength or mild steel (as specified below) in prestressed members is given by the reduction factor  $\xi_r$  in Eqs. (3-7) and (3-9).

The approximate effect of progressive cracking under creep loading and repeated load cycles is also included in the factor  $\xi_r$ . Eq. (3-8) refers to the combined creep and shrinkage effect in reinforced members.

For reinforced flexural members, creep effect only:<sup>10</sup>

$$\xi_r = 0.85 - 0.45 (A_s'/A_s), \text{ but not less than } 0.40 \quad (3-7)$$

For reinforced flexural members, creep and shrinkage effect:<sup>26,30</sup>

$$\xi_r = 1 - 0.60 (A_s'/A_s), \text{ but not less than } 0.30 \quad (3-8)$$

For prestressed flexural members:<sup>28,63</sup>

$$\xi_r = 1/[1 + A_s'/A_s] \quad (3-9)$$

Approximately the same results are obtained in Eqs. (3-7), (3-8), and (3-9) as shown in [Table 3.5.1](#). It is assumed in Eq. (3-9) that the nontensioned steel and the prestressed steel are on the same side of the section centroid and that the eccentricities of the two steels are approximately the same. See [Reference 28](#) when the eccentricities are substantially different.

Eqs. (3-8) and (4-3) are used in ACI 318<sup>27</sup> with a time-dependent factor for both creep and shrinkage,  $\tau_u = 2.0$ . As the ratio,  $A_s'/A_s$ , increases, these two sets of factors approach the same value, since shrinkage warping is negligible when the compression reinforcement is high.

The effects of creep plus shrinkage are arbitrarily lumped together in Eq. (3-8).

In [Reference 74](#), Branson notes that Eq. (3-8), as used in ACI 318<sup>27</sup> is likely to overestimate the effect of the

compression steel in restraining time-dependent deflections of members with low steel percentage (e.g. slabs) and recommends the alternate Eq. (3-10).

$$\xi_r \tau_u = \tau_u/[1 + 50 \rho'] \quad (3-10)$$

where  $\xi_r \tau_u$  is a long time deflection multiplier of the initial deflection and  $\rho'$  is the compressive steel ratio  $A_s'/bd$ . He further suggests that a factor,  $\tau_u = 2.5$  for beams and  $\tau_u = 3.0$  for slabs, rather than 2.0, would give improved results.

The calculation of creep deflection as  $\xi_r \tau_i$  times the initial deflection  $a_i$ , yields the same results as that obtained using the “reduced or sustained modulus of elasticity,  $E_{ci}$ , method,” provided the initial or short-time modular ratio,  $n$ , (at the time of loading) and the transformed section properties are used. This can be seen from the fact that  $E_{ci}$  used for computing the initial deflection, is replaced by  $E_e$  as given by [Eq. \(3-1\)](#), for computing the initial plus the creep deflection. The factor 1.0 in [Eq. \(3-1\)](#) corresponds to the initial deflection. Except for the calculation of  $I$  in the sustained modulus method (when using or not using an increased modular ratio) and  $I_e/\xi_r$  in the effective section method, the two methods are the same for computing long-time deflections, exclusive of shrinkage warping.

The reduction factor  $\xi_r$ , for creep only (not creep and shrinkage) in Eq. (3-7) is suggested as a means of taking into account the effect of compression steel and the offsetting effects of the neutral axis movement due to creep as shown in Figure 3 of [Ref. 10](#). These offsetting effects appear normally to result in a movement of the neutral axis toward the tensile reinforcement such that:

$$\frac{\epsilon_{cp}}{\epsilon_i} = \xi_r \frac{\phi_{cp}}{\phi_i} = \xi_r \frac{a_{cp}}{a_i} \quad (3-11)$$

in which  $\xi_r$  from Eq. 3-7 is less than unity. (See [Table 3.5.1](#)). Subscripts  $cp$  and  $i$  refer to the creep and initial strains, curvatures  $\phi$ , and deflections  $a$ , respectively.

The use of the long-time modular ratio,  $n_l = n(1 + \nu_l)$ , in computing the transformed section properties has also been shown<sup>31,32</sup> to accomplish these purposes and to provide satisfactory results in deflection calculations.

In all appropriate equations herein,  $\nu_p, \nu_w, \tau_p, \tau_u$ , are replaced by  $\xi_r \nu_p, \xi_r \nu_w, \xi_r \tau_p, \xi_r \tau_u$  respectively, when these effects are to be included.

### 3.6-Deflections due to warping

#### 3.6.1 Warping due to shrinkage

Deflections due to warping are frequently ignored in design calculation, when the effects of creep and warping are arbitrarily lumped together.<sup>27</sup> For thin members, such as canopies and thin slabs, it may be desirable to consider warping effects separately.

For the case in which the reinforcement and eccentricity are constant along the span and the same in the positive and negative moment regions of continuous

beams, shrinkage deflections for uniform beams are computed by Eq. (3-12).

$$a_{sh} = \xi_w \phi_{sh} \ell^2 \quad (3-12)$$

where  $\xi_w$  is a deflection coefficient defined in Table 4.2.1 for different boundary conditions, and  $\phi_{sh}$  is the curvature due to shrinkage warping. For more practical cases, some satisfactory compromise can usually be made with regard to variations in steel content and eccentricity, and for nonuniform temperature effects.

### 3.6.2-Methods of computing shrinkage curvature

Three methods for computing shrinkage curvature were compared in References 10 and 25 with experimental data: the equivalent tensile force method,<sup>31,33,36,37</sup> Miller's method<sup>38</sup> and an empirical method based on Miller's approach extended to include doubly reinforced beams.<sup>10,25</sup> The agreement between computed and measured results was reasonably good for all three of the methods.

The equivalent tensile force method (a fictitious elastic analysis), as modified in References 10 and 25 using  $E_c/2$  and the gross section properties for better results, is given by Eq. (3-13).

$$\phi_{sh} = (T_s e_g) / (E_c I_g / 2) \quad (3-13)$$

where  $T_s = (A_s + A_s') E_s e_g$ , and  $e_g$  and  $I_g$  refer to the gross section.

Miller's method<sup>38</sup> assumes that the extreme fiber of the beam furthest from the tension steel (method refers to singly reinforced members only) shrinks the same amount as the free shrinkage of the concrete,  $\epsilon_{sh}$ . Following this assumption, the curvature of the member is given by Eq. (3-14).

$$\phi_{sh} = \frac{\xi_{sh} - \epsilon_s}{d} = \frac{\epsilon_{sh}}{d} \left[ 1 - \frac{\epsilon_s}{\epsilon_{sh}} \right] \quad (3-14)$$

where  $\epsilon_s$  is the steel strain due to shrinkage. Miller suggested empirical values of  $(\epsilon_s / \epsilon_{sh}) = 0.1$  for heavily reinforced members and 0.3 for moderately reinforced members.

The empirical method represents a modification of Miller's method. The curvature of a member is given by Eqs. (3-15) and (3-16) which are applicable to both singly and doubly reinforced members. The steel percentage in these equations are expressed in percent ( $\rho = 3$  for 3 percent steel, for example).

For  $(\rho - \rho') \leq 3.0$  percent:

$$\phi_{sh} = (0.7) \frac{\epsilon_{sh}}{h} (\rho - \rho')^{1/2} \left[ \frac{\rho - \rho'}{\rho} \right]^{1/2} \quad (3-15)$$

For  $(\rho - \rho') > 3.0$  percent:

$$\phi_{sh} = \epsilon_{sh} / h \quad (3-16)$$

where  $h$  is the overall thickness of the section.

For singly reinforced members,  $\rho' = 0$ , and Eq. (3-15) reduces to Eq. (3-17).

$$\phi_{sh} = (0.7) \frac{\epsilon_{sh}}{h} \rho^{1/3} \quad (3-17)$$

which results in:

|   |     |
|---|-----|
| $\phi_{sh} = 0.56 (\epsilon_{sh})/h$ , when $\rho' = 0.5$ percent |     |
| 0.70  | 1.0 |
| 0.88  | 2.0 |
| 1.01  | 3.0 |

Eqs. (3-15), (3-16), and (3-17) were adapted from Miller's approach. For example, his method results in the following expression for singly reinforced members:

$$\phi_{sh} = 0.7 \epsilon_{sh} / d \text{ for "moderately" reinforced beams}$$

$$\phi_{sh} = 0.9 \epsilon_{sh} / d \text{ for "heavily" reinforced beams}$$

which approximately correspond to  $\rho = 1.0$  and  $\rho = 2.0$  in Eq. 3-17.

The use of the more convenient thickness,  $h$ , instead of the effective depth,  $d$ , in Eqs. (3-15), (3-16), and (3-17) was found to provide closer agreement with the test data.

### 3.6.3 Warping due to temperature change

Since concrete and steel reinforcement have similar thermal coefficients of expansion (i.e.,  $4.7$  to  $6.5 \times 10^{-6}/^\circ\text{F}$  for concrete and  $6.5 \times 10^{-6}/^\circ\text{F}$  for steel), the stresses produced by normal temperature range are usually negligible.

When the temperature change is constant along with the span, thermal deflections for uniform beams are given by Eq. (3-18).

$$a_T = \xi_w \phi_{th} \ell^2 \quad (3-18)$$

where  $\xi_w$  is the deflection coefficient (Table 4.2.1). The curvature  $\phi_{th}$  due to temperature warping is given by Eq. (3-19).

$$\phi_{th} = (e_{th} t_h) / h \quad (3-19)$$

where  $e_{th}$  is the thermal coefficient of expansion and  $t_h$  is the difference in temperature across the overall thickness  $h$ .

The values of  $v_t$ ,  $\epsilon_{sh}$ , and  $e_{th}$  usually correspond to steady state conditions. A sustained nonuniform change in temperature will influence creep and shrinkage. As a result, significant redistribution of stresses in statically indeterminate structures may occur to such an extent that the thermal effects caused by heating may be completely nullified. A nonuniform temperature reversal may cause a stress reversal.<sup>79</sup>

### 3.7-Interdependency between steel relaxation, creep and shrinkage of concrete

The loss of stress in a wire or strand that occurs at constant strain is the intrinsic relaxation  $(f_{sr})_t$ . Stress loss due to steel relaxation as shown in **Table 3.7.1** and as supplied by the steel manufacturers (ASTM designations A 416, A 421, and E 328) are examples of the intrinsic relaxation. In actual prestressed concrete members, a constant strain condition does not exist and the use of the intrinsic relaxation loss will result in an overestimation of the relaxation loss. The use of  $(f_{sr})_t$  and  $(f_{sr})_w$ , as in **Table 4.4.1.3**, is a good approximation for most design calculations because of the approximate nature of creep and shrinkage calculations. In **Reference 78**, a relaxation reduction factor,  $\psi$ , is recommended to account for conditions different than the constant strain. Values of  $\psi$  in **Table 3.7.2** are entered by the  $f_{si}/f_{py}$  ratio and the parameter  $\bar{\omega}$  given in Eq. (3-20).

$$\bar{\omega} = (\lambda)_t / 100 - (f_{sr})_t / f_{si} \quad (3-20)$$

where  $(\lambda)_t$  is the total prestress loss in percent for a time period  $(t_1 - t)$  excluding the instantaneous loss at transfer.

Prestress losses due to steel relaxation and concrete creep and shrinkage are inter-dependent and also time-dependent.<sup>103</sup> To account for changes of these effects with time, a step-by-step procedure in which the time interval increases with age of the concrete is recommended in **Ref. 78**. Differential shrinkage from the time curing stops until the time the concrete is prestressed should be deducted from the total calculated shrinkage for post-tensioned construction. It is recommended that a minimum of four time intervals be used as shown in **Table 3.7.3**.<sup>78</sup>

When significant changes in loading are expected, time intervals other than those recommended should be used. It is neither necessary nor always desirable to assume that the design live load is continually present. The four time intervals in **Table 3.7.3** are recommended for minimum noncomputerized calculations.

## CHAPTER 4-RESPONSE OF STRUCTURES IN WHICH TIME-CHANGE OF STRESSES DUE TO CREEP, SHRINKAGE AND TEMPERATURE IS NEGLIGIBLE

### 4.1-Introduction

#### 4.1.1 Assumptions

For most cases of long-time deflection and loss of prestress in statically determinate structures, the gradual time-change of stresses due to creep, shrinkage and temperature is negligible; only time changes of strains are significant. In some continuous structures, the effects of creep and shrinkage may be approximately lumped together as discussed in this chapter. Shrinkage induced time-change of stresses in statically indeterminate structures is discussed in **Chapter 5**.

While deflections and loss of prestress have essentially no effect on the ultimate capacity of reinforced and prestress members, significant over-prediction or under-prediction of losses can adversely affect such serviceability aspects as camber, deflection, cracking and connection performance.<sup>63</sup> The procedures in this chapter are reviewed in detail in **Reference 83**.

#### 4.1.2 Presentation of equations

It should be noted that **Eqs. (4-8) through (4-24)** can be greatly shortened by combining terms and substituting the approximate parameters given herein. These equations are presented in the form of separate terms in order to show the separate effects or contributions, such as prestress force, dead load, creep, shrinkage, etc., that occur both before and after slab casting in composite construction.

### 4.2-Deflections of reinforced concrete beam and slab

#### 4.2.1 Deflection of noncomposite reinforced concrete beams and one-way slab

Deflections in general may be computed for uniformly distributed loadings on prismatic members using Eq. (4-1).<sup>33,34</sup>

$$a_m = \frac{5\ell^2}{48EI} \left[ M_m + \frac{1}{10} (M_A + M_B) \right] \quad (4-1)$$

where  $a_m$  is the deflection at midspan (approximate maximum deflection in unsymmetrical cases), and the moments  $M_m$ ,  $M_A$ , and  $M_B$ , refer to the midspan and two ends respectively. This is a general equation in which the appropriate signs must be included for the moments, usually (+) for  $M_m$  and (-) for  $M_A$  and  $M_B$ .

When idealized end conditions can be assumed, it is convenient to use the deflection equation in the form of Eq. (4-2), where  $\xi$  and  $M$  are the deflection coefficients given in **Table 4.2.1** for the numerically maximum bending moment. Eqs. (4-2) and (4-3), which describe an " $I_e - \xi_r - \tau_i$ " or " $I_e - \xi_r - \tau$ " procedure for computing deflections, are used in this chapter.

Short-time deflections:

$$a_i = \xi M \ell^2 / E_{ci} I_e \quad (4-2)$$

Additional long-time deflections due to creep or creep plus shrinkage:

$$a_c = \xi_r v_i a_i \text{ or } a_s = \xi_r \tau a_i, \quad (4-3)$$

when the creep and shrinkage effect is lumped together.

Equations for  $\xi$ ,  $M$ ,  $E_{ci}$ ,  $I_e$ ,  $\xi_r$ , and  $v_i$  are as given in this report. The ACI 318 Code<sup>27</sup> specifies  $\xi_r$  as in **Eq. (3-8)**, but not less than 0.3 and an ultimate value of  $\tau = 2.0$ .

Since live load does not act in the absence of dead load, the following procedure must be used to determine the various deflection components:

$$(a_i)_D = \xi M_D \ell^2 / E_{ci} (I_e) \quad (4-4)$$



frequently  $(I_e)$  for  $M_D$  equals  $I_g$ ,

$$(a_i)_D = \xi_r v_i (a_i)_D \quad (4-5)$$

a fictitious value

$$(a_i)_{D+L} = \xi M_{D+L} \ell^2 / E_c (I_e) \text{ for } M_{D+L} \quad (4-6)$$

and then for live load,

$$(a_i)_L = (a_i)_{D+L} - (a_i)_D \frac{E_{ci}}{E_c} \quad (4-7)$$

The ACI-318 Codes<sup>12,27</sup> refer to  $(a_i)_D + (a_i)_L$  in certain cases for example.

In general, the deflection of a noncomposite reinforced concrete member at any time and including ultimate value in time is given by Eqs. (4-8) and (4-9) respectively."

$$a_t = \underbrace{(a_i)_D}_{(1)} + \underbrace{(a_i)_D}_{(2)} + \underbrace{a_{sh}}_{(3)} + \underbrace{(a_i)_L}_{(4)} \quad (4-8)$$

$a_t$  = [Eq. (4-8) except that  $v_u$  and  $(\epsilon_{sh})_u$  shall be used in lieu of  $v_t$  and  $\epsilon_{sh}$  when computing terms (2) and (3) respectively.] (4-9)

where:

- Term (1) is the initial dead load deflection as given by Eq. (4-4)
- Term (2) is the dead load creep deflection as given by Eq. (P-5)
- Term (3) is the deflection due to shrinkage warping as given by Eq. (3-12)
- Term (4) is the live load deflection as given by Eq. (4-7)

#### 4.3-Deflection of composite precast reinforced beams in shored and unshored construction<sup>48,49,77</sup>

For composite beams, subscripts 1 and 2 are used to refer to the slab or the effect of the slab dead load and the precast beam, respectively. The effect of compression steel in the beam (with use of  $\xi_r$ ) should be neglected when it is located near the neutral axis of the composite section.

It is suggested that the 28-day moduli of elasticity for both slab and precast beam concretes, and the gross  $I$  (neglecting steel and cracking), be used in computing the composite moment of inertia,  $I_c$ , in Eqs. (4-10) and (4-12), with the exception as noted in term (7) for live load deflection. Note that shrinkage warping of the precast beam is not computed separately in Eqs. (4-10) and (4-12).

##### 4.3.1 Deflection of unshored composite beams

The deflection of unshored composite beams at any time and including ultimate values, is given by Eqs. (4-10) and (4-11) respectively.

$$a_t = \underbrace{(a_i)_2}_{(1)} + \underbrace{v_s(a_i)_2}_{(2)} + \underbrace{(v_{i2} - v_s)(a_i)_2}_{(3)} \frac{I_2}{I_c} + \underbrace{(a_i)_1}_{(4)} + \underbrace{v_{i1}(a_i)_1}_{(5)} \frac{I_2}{I_c} + \underbrace{a_\delta}_{(6)} + \underbrace{a_L}_{(7)} \quad (4-10)$$

$$a_u = \underbrace{(a_i)_2}_{(1)} + \underbrace{v_s(a_i)_2}_{(2)} + \underbrace{(v_u - v_s)(a_i)_2}_{(3)} \frac{I_2}{I_c} + \underbrace{(a_i)_1}_{(4)} + \underbrace{v_{us}(a_i)_1}_{(5)} \frac{I_2}{I_c} + \underbrace{a_\delta}_{(6)} + \underbrace{a_L}_{(7)} \quad (4-11)$$

where:

Term (1) is the initial dead load deflection of the precast beam,  $(a_i)_2 = \xi M_2 \ell^2 / E_{ci} I_2$ . See Table 4.2.1 for  $\xi$  and  $M$  values. For computing  $I_2$  in Eq. (3-2),  $M_{max}$  refers to the precast beam dead load and  $M_{cr}$  to the precast beam.

Term (2) is the creep deflection of the precast beam up to the time of slab casting.  $v_s$  is the creep coefficient of the precast beam concrete at the time of slab casting. Multiply  $v_s$  and  $v_u$  by  $\xi_r$  (from Eq. 3-8) for the effect of compression steel in the precast beam. Values of  $v_t/v_u = v_s/v_u$  from Eq. (2-8) are given in Table 2.4.1.

Term (3) is the creep deflection of the composite beam for any period following slab casting due to the precast beam dead load.  $v_{i2}$  is the creep coefficient of the precast beam concrete at any time after slab casting. Multiply this term by  $\xi_r$  (from Eq. 3-8) for the effect of compression steel in the precast beam. The expression,  $I_2/I_c$ , modifies the initial value, in this case  $(a_i)_2$ , and accounts for the effect of the composite section in restraining additional creep curvature after slab casting.

Term (4) is the initial deflection of the precast beam under slab dead load,  $(a_i)_1 = \xi M_1 \ell^2 / E_{cs} I_2$ . See Table 4.2.1 for  $\xi$  and  $M$  values. For computing  $I$  in Eq. (3-2),  $M_{max}$  refers to the precast beam plus slab dead load and  $M_{cr}$  to the precast beam.

Term (5) is the creep deflection of the composite beam due to slab dead load.  $v_{i1}$  is the creep coefficient for the slab loading, where the age of the precast beam concrete at the time of slab casting is considered. Multiply  $v_{i1}$  and  $v_u$  by  $\xi_r$  (from Eq. 3-8) for the effect of compression steel in the precast beam. See Term (3) for comment on  $I_2/I_c \cdot v_{us}$  is given by Eq. (2-13).

Term (6) is the deflection due to differential shrinkage. For simple spans,  $a_\delta = Q_{cs} \ell^2 / 8 E_{cs} I_c$ , where  $Q = \Delta A_1 E_{c1} / 3$ . The factor 3 provides for the gradual increase in the shrinkage force from day 1, and also approximates the creep and varying stiffness effects.<sup>6,48</sup> In the case of

continuous members, differential shrinkage produces secondary moments (similar to the effect of prestressing but opposite in sign, normally) that should be included.<sup>58</sup>

Term (7) is the live load deflection of the composite beam, which should be computed in accordance with Eq. (4-7), using  $E_c I_c$ . For computing  $I_c$  in Eq. (3-2),  $M_{max}$  refers to the precast beam plus slab dead load and the live load, and  $M_{cr}$  to the composite beam.

Additional information on deflection due to shrinkage warping of composite reinforced concrete beams of unshored construction is given by Eq. (2) in Ref. 77.

#### 4.3.2 Deflection of shored composite beams

The deflection of shored composite beams at any time and including ultimate values is given by Eqs. (4-12) and (4-13), respectively.

$a_i$  = Eq. (4-10) with Terms (4) and (5) modified as follows. (4-12)

$a_u$  = Eq. (4-11) except that the composite moment of inertia is used in Term (4) to compute  $(a_i)_I$ , and the ratio,  $I_2/I_c$ , is eliminated in Term (5). (4-13)

Term (4) is the initial deflection of the composite beam under slab dead load,  $(a_i)_I = \xi M_I \ell^2 / E_{cs} I_c$ .

Term (5) is the creep deflection of the composite beam under slab dead load,  $v_{II}(a_i)_I$ . The composite section effect is already included in Term (4).

### 4.4-Loss of prestress and camber in noncomposite prestressed beams<sup>6,49-58,63</sup>

#### 4.4.1 Loss of prestress in prestressed concrete beams

Loss of prestress at any time and including ultimate values, in percent of initial tensioning stress, is given by Eqs. (4-14) and (4-15).

$$\lambda_i = \underbrace{[(nf_c) + (nf_c) v_i (1 - \frac{F_i}{2F_0})]}_{(3)} + \underbrace{(\epsilon_{sh})_i E_s / (1 + n \rho \xi_s) + (f_{sr})_i]}_{(4)} \frac{100}{f_{si}} \quad (4-14)$$

$$\lambda_u = \underbrace{[(nf_c) + (nf_c) v_u (1 - \frac{F_u}{2F_0})]}_{(3)} + \underbrace{(\epsilon_{sh})_u E_s / (1 + n \rho \xi_s) + (f_{sr})_u]}_{(4)} \frac{100}{f_{si}} \quad (4-15)$$

where:

Term (1) is the prestress loss due to elastic shortening, in which

$f_c = \frac{F_i}{A} + \frac{F_i e^2}{I_i} - \frac{M_{de}}{I_i}$ , and  $n$  is the modular ratio at the time of prestressing. Frequently  $F_o$ ,  $A_g$ , and  $I_g$  are used as an approximation instead of  $F_i$ ,  $A_p$  and  $I_i$ , being  $F_o = F_i(1 - np)$ . Only the first two terms for  $f_c$  apply at the ends of simple beams. For continuous members, the effect of secondary moments due to prestressing should also be included. Suggested values for  $n$  in are given in Table 4.4.1.1.

Term (2) is the prestress loss due to the concrete creep. The expression,  $v_i(1 - F_i/2FJ)$ , was used in References 50 and 53 to approximate the creep effect resulting from the variable stress history. Approximate values of  $F_i/F_o$  (in the form of  $F_s/F_o$  and  $F_u/F_o$ ) for this secondary effect as given in Table 4.4.1.2. To consider the effect of nontensioned steel in the member, multiply  $v_i$ ,  $v_u$ ,  $(\epsilon_{sh})_i$  and  $(\epsilon_{sh})_u$  by  $\xi_r$  (from Eq. 3-9).

Term (3) is the prestress loss due to shrinkage.<sup>56</sup> The expression,  $(\epsilon_{sh})_i E_s$ , somewhat overestimates this loss. The denominator represents the stiffening effect of the steel and the effect of concrete creep. Additional information on Term (3) is given in Ref. 63.

Term (4) is the prestress loss due to steel relaxation. Values of  $(f_{sr})_i$  and  $(f_{sr})_u$  for wire and strand are given in Table 4.4.1.3,<sup>63</sup> where  $t$  is the time after initial stressing in hours and  $f_y$  is the 0.1 percent offset yield stress. Values in Table 4.4.1.3 are recommended for most design calculations because they are consistent with the approximate nature of creep and shrinkage calculations. Relaxation of other types of steel should be based on manufacturer's recommendations supported by adequate test data. For a more detailed analysis of the interdependency between steel relaxation, creep and shrinkage of concrete see Section 3.7 of this report.

#### 4.4.2 Camber of noncomposite prestressed concrete beams

The camber at any time, and including ultimate values, is given by Eqs. (4-16) and (4-17) respectively. It is suggested that an average of the end and midspan loss be used for straight tendons and 1-pt. harping, and the midspan loss for 2-pt harping.

$$a_i = - \underbrace{(a_i)_{F_0}}_{(1)} + \underbrace{(a_i)_D}_{(2)} \cdot \left[ - \underbrace{\frac{F_i}{F_0}}_{(4)} + \underbrace{(1 - \frac{F_i}{2F_0}) v_i}_{(5)} \right] \underbrace{(a_i)_{F_0}}_{(3)} + \underbrace{v_i(a_i)_D}_{(4)} + \underbrace{a_L}_{(5)} \quad (4-16)$$

$$a_i = \underbrace{-(a_i)_{F_0}}_{(1)} + \underbrace{(a_i)_D}_{(2)} + \underbrace{\left[ -\frac{F_u}{F_0} + \left(1 - \frac{F_u}{2F_0}\right) v_u \right] (a_i)_{F_0}}_{(3)} + \underbrace{v_u (a_i)_D}_{(4)} + \underbrace{a_L}_{(5)} \quad (4-17)$$

where:

Term (1) is the initial camber due to the initial prestress force after elastic loss,  $F_0$ . See Table 4.4.2.1 for common cases of prestress moment diagrams with formulas for computing camber,  $(a_i)_{F_0}$ .

Here,  $F_0 = F_i(1 - nf_c/f_{si})$ , where  $f_c$  is determined as in Term (1) of Eq. (4-14). For continuous members, the effect of secondary moments due to prestressing should also be included.

Term (2) is the initial dead load deflection of the beam,  $(a_i)_D = \xi M \ell^2 / E_{ci} I_g$ .  $I_g$  is used instead of  $I_t$  for practical reasons. See Table 4.2.1 for  $\xi$  and  $M$  values.

Term (3) is the creep (time-dependent) camber of the beam due to the prestress force. This expression includes the effects of creep and loss of prestress; that is, the creep effect under variable stress.  $F_t$  refers to the total loss at any time minus the elastic loss. It is noted that the term,  $F_t/F_0$ , refers to the steel stress or force after elastic loss, and the prestress loss in percent,  $\lambda$  as used herein, refers to the initial tensioning stress or force. The two are related as:

$$\frac{F_t}{F_0} = \frac{i}{100} (\lambda_t - \lambda_{el}) \frac{f_{si}}{f_c} \quad (4-18)$$

and can be approximated by:

$$\frac{F_t}{F_0} = \frac{1}{100} (\lambda_t - \lambda_{el}) \frac{1}{1 - n\rho} \quad (4-18a)$$

Term (4) is the dead load creep deflection of the beam. Multiply  $v_t$  and  $v_u$  by  $\xi$  (from Eq. 3-9) for the effect of compression steel (under dead load) in the member.

Term (5) is the live load deflection of the beam.

Additional information on the effect of sustained loads other than a composite slab or topping applied some time after the transfer of prestress is given by Terms (6) and (7) in Eqs. (29) and (30) in Ref. 63.

#### 4.5-Loss of prestress and camber of composite precast and prestressed beams, unshored and shored constructions<sup>6,49-58,63,77</sup>

##### 4.5.1 Loss of prestress of composite precast-beams and prestressed beams

The loss of prestress at any time and including ultimate values, in percent of initial tensioning stress, is

given by Eqs. (4-19) and (4-20) respectively for unshored and shored composite beams with both prestressed steel and nonprestressed steel.

$$\lambda_t = \underbrace{[(nf_c) + (nf_c)v_s(1 - \frac{F_s}{2F_0}) + (nf_c)(v_{t2} - v_s)(1 - \frac{F_s + F_t}{2F_0})]}_{(1)} \underbrace{\frac{I_2}{I_c}}_{(2)} + \underbrace{(\epsilon_{sh})_t E_s / (1 + n\rho\xi_s) + (f_{sr})_t - (mf_{cs}) - (mf_{cs})v_{t1} \frac{I_2}{I_c} - mf_{cd} \frac{100}{f_{si}}}_{(3)} \quad (4-19)$$

$$\lambda_u = \underbrace{[(nf_c) + (nf_c)v_s(1 - \frac{F_s}{2F_0})]}_{(1)} + \underbrace{(nf_c)(v_u - v_s)(1 - \frac{F_s + F_u}{2F_0})}_{(2)} \underbrace{\frac{I_2}{I_c}}_{(3)} + \underbrace{(\epsilon_{sh})_u E_s / (1 + n\rho\xi_s) + (f_{sr})_u - (mf_{cs}) - (mf_{cs})v_{us} \frac{I_2}{I_c} - mf_{cd} \frac{100}{f_{si}}}_{(4)} \quad (4-20)$$

where:

Term (1) is the prestress loss due to elastic shortening. See Term (1) of Eq. (4-14) for the calculation of  $f_c$ .

Term (2) is the prestress loss due to concrete creep up to the time of slab casting.  $v_s$  is the creep coefficient of the precast beam concrete at the time of slab casting. See Term (2) of Eq. (4-14) for comments concerning the reduction factor,  $(1 - \frac{F_s}{2F_0})$ . Multiply  $v_s$  and  $v_u$  by  $\xi_r$  (from Eq. 3-9) for the effect of nontensioned steel in the member. Values of  $v_t/v_u = v_s/v_u$  from Eq. (2-8) are given in Table 2.4.1.

Term (3) is the prestress loss due to concrete creep for any period following slab casting.  $v_{t2}$  is the creep coefficient of the precast beam concrete at any time after slab casting. The reduction factor,  $(1 - \frac{F_s + F_t}{2F_0})$ , with the incremental creep coefficient,  $(v_{t2} - v_s)$ , estimates the

effect of creep under the variable prestress force that occurs after slab casting. Multiply this term by  $\xi_r$  (from Eq. 3-9) for the effect of nontensioned steel in the precast beam. See Term (3) of Eq. (4-10) for comment on  $I_2/I_c$ .

Term (4) is the prestress loss due to shrinkage. See Term (3) of Eqs. (4-14) and (4-15) for comment.

Term (5) is the prestress loss due to steel relaxation. In this term  $t$  is time after initial stressing in hours. See Term (4) of Eqs. (4-14) and (4-15) for comments.

Term (6) is the elastic prestress gain due to slab dead load, and  $m$  is the modular ratio at the time of slab casting.

$f_{cs} = \frac{(M_{s,D})e}{I_g}$ ,  $M_{s,D}$  refers to slab or slab plus diaphragm dead load;  $e$  and  $I_g$  refer to the precast beam section properties for unshored construction and the composite section properties for shored construction. Suggested values for  $n$  and  $m$  are given in Table 4.4.1.1.

Term (7) is the prestress gain due to creep under slab dead load.  $v_{II}$  is the creep coefficient for the slab loading, where the age of the precast beam concrete at the time of slab casting is considered. See Term (5) of Eq. (4-10) for comments on  $\xi_r$  and  $I_2/I_c$ . For shored construction, drop the term,  $I_2/I_c v_{us}$  is given by Eq. (2-13).

Term (8) is the prestress gain due to differential shrinkage, where  $f_{cd} = Qy_{cs}e_c/I_c$  is the concrete stress at the steel c.g.s. and  $Q = (\delta A_{gI} E_{cI})/3$  in which  $A_{gI}$  and  $E_{cI}$  refer to the cast in-place slab. See Notation for additional descriptions of terms. Since this effect results in a prestress gain, not loss, and is normally small, it may usually be neglected."

#### 4.5.2 Camber of composite beams—precast beams prestressed unshored and shored construction

The camber at any time, including ultimate values, is given by Eqs. (4-21), (4-22), (4-23), and (4-24) for unshored and shored composite beams, respectively. It is suggested that an average of the end and midspan loss of prestress be used for straight tendons and 1-pt. harping, and the midspan loss for 2-pt. harping.<sup>6</sup>

It is suggested that the 28-day moduli of elasticity for both slab and precast beam concretes be used. For the composite moment of inertia,  $I$ , in Eqs. (4-21) through (4-24), use the gross section  $I_g$  except in Term (10) for the live load deflection.

##### a) Unshored construction

$$a_u = \underbrace{-(a_i)_{F_0}}_{(1)} + \underbrace{(a_i)_2}_{(2)} \cdot \left[ -\frac{F_s}{F_0} + \left( 1 - \frac{F_s}{2F_0} \right) v_s \right] \underbrace{(a_i)_{F_0}}_{(3)} \quad (4-21)$$

$$- \left[ -\frac{F_t - F_s}{F_0} + \left( 1 - \frac{F_s + F_t}{2F_0} \right) (v_{I2} - v_s) \right] \underbrace{(a_i)_{F_0}}_{(4)} \frac{I_2}{I_c} + \underbrace{v_s(a_i)_2}_{(5)} \quad (4-22)$$

$$+ \underbrace{(v_{I2} - v_s)(a_i)_2}_{(6)} \frac{I_2}{I_c} + \underbrace{(a_i)_1}_{(7)} + \underbrace{v_{II}(a_i)_1}_{(8)} \frac{I_2}{I_c} + \underbrace{a_\delta}_{(9)} + \underbrace{a_L}_{(10)} \quad (4-21)$$

$$a_u = \underbrace{-(a_i)_{F_0}}_{(1)} + \underbrace{(a_i)_2}_{(2)} \cdot \left[ -\frac{F_s}{F_0} + \left( 1 - \frac{F_s}{2F_0} \right) v_s \right] \underbrace{(a_i)_{F_0}}_{(3)} \quad (4)$$

$$- \left[ -\frac{F_u - F_s}{F_0} + \left( 1 - \frac{F_s + F_u}{2F_0} \right) (v_u - v_s) \right] \underbrace{(a_i)_{F_0}}_{(4)} \frac{I_2}{I_c} + \underbrace{v_s(a_i)_2}_{(5)} \quad (5)$$

$$+ \underbrace{(v_u - v_s)(a_i)_2}_{(6)} \frac{I_2}{I_c} + \underbrace{(a_i)_1}_{(7)} + \underbrace{v_{us}(a_i)_1}_{(8)} \frac{I_2}{I_c} + \underbrace{a_\delta}_{(9)} + \underbrace{a_L}_{(10)} \quad (4-22)$$

where:

Term (1) See Term (1) of Eq. (4-16).

Term (2) is the initial dead load deflection of the precast beam,  $(a_i)_2 = \xi M_2 \ell^2 / E_{ci} I_g$ . See Term (2) of Eq. (4-16) for additional comments.

Term (3) is the creep (time-dependent) camber of the beam, due to the prestress force, up to the time of slab casting. See Term (3) of Eq. (4-16) and Terms (2) and (3) of Eq. (4-19) for additional comments.

Term (4) is the creep camber of the composite beam, due to the prestress force, for any period following slab casting. See Term (3) of Eq. (4-16) and Terms (2) and (3) of Eq. (4-19) for additional comments.

Term (5) is the creep deflection of the precast beam up to the time of slab casting due to the precast beam dead load. See Term (2) of Eq. (4-10) for additional comments.

Term (6) is the creep deflection of the composite beam for any period following slab casting due to the precast beam dead load. See Term (3) of Eq. (4-10) for additional comments.

Term (7) is the initial deflection of the precast beam under slab dead load,  $(a_i)_1 = \xi M_1 \ell^2 / E_{cs} I_g$ . See Table 4.2.1 for  $\xi$  and  $M$  values. When diaphragms are used, for example, add to this term:

$$(a_i)_{1D} = \frac{M_{1D}}{E_{cs} I_g} \left[ \frac{\ell^2}{8} - \frac{a^2}{6} \right],$$



where  $M_{ID}$  is the moment between two symmetrical diaphragms, and  $a = \ell/4, \ell/3$ , etc., for the diaphragms at the quarter points, third points, etc., respectively.

Term (8) is the creep deflection of the composite beam due to slab dead load.  $v_{II}$  is the creep coefficient for the slab loading, where the age of the precast beam concrete at the time of slab casting is considered. See Term (5) of Eq. (4-10) for additional comments.  $v_{us}$  is given by Eq. (2-13).

Term (9) is the deflection due to differential shrinkage. See Term (6) of Eq. (4-10) for additional comments.

Term (10) is the live load deflection of the composite beam, in which the gross section flexural rigidity,  $E_c I_c$ , is normally used. For partially prestressed members which are cracked under live load, see Term (7) of Eq. (4-10) for additional comments.

#### b) Shored construction

$a_i$  = Eq. (4-21), with terms (7) and (8) modified as follows: (4-23)

Term (7) is the initial deflection of the composite beam under slab dead load,  $(a_i)_I = \xi M_I \ell^2 / E_{cs} I_c$ . See Table 4.2.1 for  $\xi$  and M values.

Term (8) is the creep deflection of the composite beam under slab dead load =  $v_{II}(a_i)_I$ . The composite-section effect is already included in Term (7). See Term (5) of Eq. (4-10) for additional comments.

$a_u$  = Eq. (4-22) with Terms (7) and (8) modified as follows: (4-24)

Term (7), use composite moment of inertia to compute  $(a_i)_I$ .

Term (8), eliminate the ratio  $I_2/I_c$ .

For additional information on composite concrete members partially or fully prestressed, see Refs. 62 to 64.

### 4.6-Example: Ultimate midspan loss of prestress and camber for an unshored composite AASHTO Type IV girder with prestressing steel only, normal weight concrete<sup>63</sup>

Material and section properties, parameters and conditions of the problem are given in Tables 4.6.1 and 4.6.2. The ultimate loss of prestress is computed by the (Eq. 4-20) and the ultimate camber by (Eq. 4-22). Results are tabulated term by term in Tables 4.6.3 and 4.6.4.

The loss percentages in Table 4.6.3 show the elastic loss to be about 7.5 percent. The creep loss before slab casting about 6 percent and about 2 percent following slab casting. The total shrinkage loss about 6 percent. The relaxation loss about 7.5 percent and the gain in prestress due to the elastic and creep effect of the slab dead load plus the differential shrinkage and creep of about 4.5 percent. The total loss is 24.3 percent.

The following is shown in Table 4.6.4 for the midspan camber:

Initial Camber =  $1.93 - 0.80 = 1.13$  in (28.7 mm)

Residual Camber = 0.13 in (3.3 mm), Total in Table 4.6.4

Live Load Plus Impact Deflection = -0.50 in (-12.7 mm), (Girder is uncracked)

Residual Camber + Live Load Plus Impact Deflection =  $0.13 - 0.50 = -0.37$  in, (3.3 - 12.7 = -9.4 mm)

AASHTO (1978) Check:

Live Load Plus Impact Deflection = -0.50 in, (-12.7 mm)

$\ell/800 = (80)(12)/800 = 1.20 > 0.50$  in, (30.5 > 12.7mm), OK.

The detailed calculations for the results in this example can be seen in Ref. 83.

### 4.7-Deflection of reinforced concrete flat plates and two-way slabs

A state of the art report on practical methods for calculating deflection of the reinforced concrete floor systems, including that of plates, beam-supported slabs, and wall-supported slabs is given in Ref. 74.

Although creep and shrinkage effects may be higher in thin slabs than in beams (time-dependent deflections as large as 5 to 7 times the initial deflections have been noted,<sup>29,39</sup> the same approach for predicting time-dependent beam deflections may, in most cases, be used with caution for flat plates and two-way slabs. These include Eqs. (3-7), (3-8) and (3-10) for the effect of compression steel, etc., and Eq. (4-3) for additional long-time deflections. The effect of cracking on the effective moment of inertia  $I_e$ , for flat plates and two-way slabs is discussed in Section 3.4 of this report.

The initial deflection for uniformly loaded flat plates and two-way slabs are given by Eqs. (4-25) and (4-26).<sup>29,40-47</sup>

$$\text{Flat plates} \quad a_i = \xi_{fp} q \ell^4 / E_{ci} I_e \quad (4-25)$$

$$\text{Two-way slabs} \quad a_i = \xi_{tws} q \ell^4 / E_{ci} I_e \quad (4-26)$$

where  $I_e$  and  $q$  refer to a unit width of the slab. The Poisson-ratio effect is neglected in the flexural rigidity of the slab. Deflection coefficients  $\xi_{fp}$  and  $\xi_{tws}$  are given in Table 4.7.1 for interior panels. Note that these coefficients are dimensionless, so that  $q$  must be in load/length (e.g. lb/ft or kN/m). These equations provide for the approximate calculation of slab initial deflections in which the effect of cracking is included.

Reference 44 presents a direct rational procedure for computing slab deflections, in which the effect of cracking and long-term deformation can be included.

An approximate method based on the equivalent

frame method is presented in [Reference 75](#). This method accounts for the effect of cracking and long-term deformations, is compatible in approach and terminology with the two alternate methods of analysis in Chapter 13 of ACI 318<sup>27</sup> and requires very few additional calculations to obtain deflections.

#### 4.8-Time-dependent shear deflection of reinforced concrete beams

Shear deformations are normally ignored when computing the deflections of reinforced concrete members; however, with deep beams, shear walls and T-beams under high load, the shear deformation can contribute substantially to the total deflection.

Test results on beams with shear reinforcement and a span-to-depth ratio equal to 8.7 in [Ref. 73](#) show that:

Shear deformation contributes up to 23 percent of the total deflection, although the shear stresses in the webs of most test beams were not very high.

Shear deflections increase with time much more rapidly than flexural deflections.

Shear deflection due to shrinkage of the concrete webs is of importance.

##### 4.8.1 Shear deflection due to creep<sup>73</sup>

The time-dependent shear stiffness  $G_{cr}$  for the, initial plus creep deformation of a cracked web with vertical stirrups can be expressed as given by Eq. (4-27).

$$G_{cr} = \frac{b_w j d E_s}{(1-1.1 v_c/v_x)/\rho_w + 4n (1 + v_t)} \quad (4-27)$$

where:

- $v_x$  = nominal shear stress acting on section
- $v_c$  = nominal permissible shear stress carried by concrete as given in Chapter 11 of ACI 318<sup>27</sup>
- $b_w$  = web width
- $\rho_w$  =  $A_v/b_w s$
- $A_v$  = area of shear reinforcement within a distance  $s$
- $s$  = spacing of stirrups

Eq. (4-27) is based on a modified truss analogy assuming that the shear cracks have formed at an angle of 45 deg to the beam axis, that the stirrups have to carry the shear not resisted by concrete and that the concrete stress in the 45 deg struts are equal to twice the nominal shear stresses  $v_x$ .

##### 4.8.2 Shear deflection due to shrinkage<sup>73</sup>

In a truss with vertical hangers and 45 deg diagonals, a shrinkage strain  $\epsilon_{sh}$  results in a shear angle of  $2 \epsilon_{sh}$  radians. The shear deflection due to shrinkage of a member with a symmetrical crack pattern is given by Eq. (4-28).

$$(a_{sh})_s = 2 (\epsilon_{sh}) \ell/2 = (\epsilon_{sh}) \ell \quad (4-28)$$

Eq. (4-28) may overestimate the shrinkage deflection because the length of the zone between the inclined cracks is shorter than  $\ell$ .

#### 4.9-Comparison of measured and computed deflections, cambers and prestress losses using procedures in this chapter

The method presented in [4.2,4.3,4.4,4.5,4.7](#), and 4.8 for predicting structural response has been reasonably well substantiated for laboratory specimens in the references cited in the above sections.

The correlation that can be expected between the actual service performance and the predicted one is reasonably good but not accurate. This is primarily due to the strong influence of environmental conditions, load history, etc., on the concrete response.

In analyzing the expected correlation between the predicted service response (i.e., deflections, cambers and losses) and the actual measurements from field structures, two situations shall be differentiated: (1) The prediction of their elastic, creep, shrinkage, temperature, and relaxation components; and (2) the resultant response obtained by algebraically adding the components.

In the committee's opinion, the predicted values of the deflection, camber, and loss components will normally agree with the actual results within  $\pm 15$  percent when using experimentally determined material parameters. Using average material parameters given in [Chapter 2](#) will generally yield results which agree with actual measurements in the range of  $\pm 30$  percent. With some knowledge of the time-dependent behavior of concrete using local concrete materials and under local conditions, deflection, camber, and loss of prestress can normally be predicted within about  $\pm 20$  percent.

If the predicted resultant is expressed in percent, wider scatter may result; however, the correlation between the dimensional values is reasonably good.

Most of the results in the references are far more accurate than the above limits because a better correlation exists between the assumed and the actual laboratory histories for water content, temperature and loading histories.

## CHAPTER 5-RESPONSE OF STRUCTURES WITH SIGNIFICANT TIME CHANGE OF STRESS

### 5.1-Scope

In statically indeterminate structures, significant redistribution of internal forces may arise. This may be caused by an imposed deformation, as in the case of a differential settlement, or by a change in the statical system during construction, as in the case of beams placed first as simply supported spans and then subsequently made continuous.

Another cause may be the nonhomogeneity of creep

properties, which may be due to differences in age, thickness, in other concrete parameters, or due to interaction of concrete and steel parts and temperature reversal. Large time changes of stress are also produced by shrinkage in certain types of statically indeterminate structures. These changes are relaxed by creep. In columns, the bending moment increases as deflections grow due to creep and this further augments the creep buckling deflections.

As stated in Chapter 3, creep in homogeneous statically indeterminate structures causes no change in stress due to sustained loads and all time deformations are proportional to  $v_t$ .

## 5.2-Concrete aging and the age-adjusted effective modulus method

In the type of problems discussed in Section 5.1 above, the prediction of deformation by the effective modulus method is often grossly in error as compared with theoretically exact solutions.<sup>66</sup> The main source of error is aging of concrete, which is expressed by the correction factor Creep  $\gamma_{\ell a}$  in Eqs. (2-11) or (2-12), and by the time variation of  $E_{ci}$  given by Eqs. (2-1) and (2-5). Gradual stress changes during the service life of the structure produce additional instantaneous and creep strains, which are superimposed on the creep strains due to initial stresses and to all previous stress changes. Because of concrete aging, these additional strains are much less than those which would arise if the same stress changes occurred right after the instant of first loading,  $t_{\ell a}$ . This effect can be accounted for by using the age-adjusted effective modulus method, originated by Trost<sup>67,68</sup> and rigorously formulated in Ref. 65 and Ref. 69. Further applications are given in References 66, 81, and 82. References 66 and 82 indicate that this method is better in theoretical accuracy than other simplified methods of creep analysis and is, at the same time, the simplest one among them. In similarity to the effective modulus method, this method consists of an elastic analysis with a modified elastic modulus,  $E_{ca}$  which is defined by Eq. (5-1), and is called the age-adjusted modulus.

$$E_{ca} = E_{ci} / (1 + X v_t) \quad (5-1)$$

The aging coefficient,  $X$ , depends on age at the time  $t_{\ell a}$ , when the structure begins carrying the load and on the load duration  $t - t_{\ell a}$ . Notice that  $t - t_{\ell a}$ , as used in Chapter 5, represents the  $\underline{t}$  used in Eq. (2-8) and in Chapter 4.

In Table 5.1.1, the  $X$  values are presented for the creep function in Eq. (2-8). For interpolation in the table, it is better to assume linear dependence on  $\log t_{\ell a}$  and  $\log (t - t_{\ell a})$ .

The values in Table 5.1.1 are applicable to creep functions for different humidities and member sizes that have the same time shapes as Eq. (2-8) when plotted as functions of  $t - t_{\ell a}$ , that is, mutually proportional to Eq. (2-8). An empirical equation for the approximation of the age-

adjusted effective modulus  $E_{ca}$  that is generally applicable to any given creep function is given by Eq. (16) in Reference 108. The percent error in  $E_{ca}$  is usually below 1 percent when compared with the exact calculations by solving the integral equations.

The analysis is based on the following quasi-elastic strain law for stress and strain changes after load application:

$$(\epsilon_c)_\delta = \frac{(f_c)_\delta}{E_{ca}} + (\epsilon_{ie})_\delta \quad (5-2)$$

where:

$$(\epsilon_c)_\delta = (\epsilon_c)_t - (\epsilon_c)_{t_{\ell a}} \quad (5-3)$$

$$(f_c)_\delta = (f_c)_t - (f_c)_{t_{\ell a}} \quad (5-4)$$

$$(\epsilon_{ie})_\delta = \frac{f_{ci}}{E_{ci}} v_t + \delta_{sh} \quad (5-4)$$

$$\delta_{sh} = (\epsilon_{sh})_t - (\epsilon_{sh})_{t_{\ell a}} \quad (5-5)$$

Here  $(\epsilon_{ie})_\delta$  represents a known inelastic strain change due to creep and shrinkage and is treated in the analysis in the same manner as thermal strain.  $\delta_{sh}$  in Eqs. (5-4) and (5-5) represents shrinkage differential strain. If gradual thermal strain occurs, it may be included under  $(\epsilon_{sh})_\delta$ .

Some applications of the age-adjusted effective method are discussed in the following sections. Equations (5-6) through (5-13) are theoretically exact for a given linear creep law, only if the creep properties are the same in all cross sections, i.e., the structure is homogenous. In most practical situations, the error inherent to this assumption is not serious.

## 5.3-Stress relaxation after a sudden imposed deformation<sup>68,69</sup>

Let  $(S)_i$  be the stress, internal force or moment produced by a sudden imposed deformation at time  $t_{\ell a}$  (such as short-time differential settlement or jacking of structure). Then the stress, internal force or moment  $(S)_t$  at any time  $t > t_{\ell a}$  is given by Eq. (5-6).

$$(S)_t = (S)_i \left[ 1 - \frac{v_t}{1 + X v_t} \right] \quad (5-6)$$

The creep coefficient  $v_t$  in this equation must include the correction by factor  $\xi_r$  in Section 3.5 of this report.

## 5.4-Stress relaxation after a slowly imposed deformation<sup>69,65,82</sup>

Let  $(S)_{sh}$  be the statically indeterminate internal force, moment or stress that would arise if a slowly imposed deformation (e.g., shrinkage strain or slow differential set-

tlement) would occur in a perfectly elastic structure of modulus  $E_{cl}$  (at no creep). Then the actual statically indeterminate internal force, moment or stress,  $(S)_t$  at time  $t$  caused by a slowly imposed deformation including the relaxation due to creep is given by Eq. (5-7).

$$(S)_t = \frac{(S)_{st}}{1 + X v_t} \quad (5-7)$$

### 5.5-Effect of a change in static system<sup>69</sup>

#### 5.5.1 Stress relaxation after a change in static system

Consider that static System (1) is changed at time  $t_1$  to static System (2).

If Subscripts 1 and 2 refer to the stress, internal force or moment computed according to the theory of elasticity for static Systems (1) and (2), respectively, the actual stress, internal force or moment after a sudden change in the static system at time  $t > t_1$  is given by Eq. (5-8).

$$(S)_t = S_1 + (S_2 - S_1) \left[ \frac{v_t - (v_t)_1}{1 + X v_t} \right] \quad (5-8)$$

and by Eq. (5-9), after a progressive change in the static system.

$$(S)_t = S_1 + (S_2 - S_1) \left[ \frac{(v_t)_1}{1 + X (v_t)_1} \right] \quad (5-9)$$

It is assumed that the structure begins carrying the load at time  $t_{ea} \leq t_1$  and  $v_t$  and  $(v_t)_1$  are creep coefficients at time  $t$  and  $t_1$ , respectively.

The value of  $X$  is to be read from Table 5.1.1 for arguments  $t_{ea}$  and  $t - t_{ea}$ . Equation (5-8) is exact only if the load is applied just before time  $t_1$  that is, for  $t_1 \approx t_{ea}$ , and  $(v_t)_1 \approx 0$ , but, in most other cases, it is good approximation.

#### 5.5.2 Long-time deflection due to creep after a change in static system

The long-time deflection due to creep  $a_p$  after static system (1) is changed into static system (2) at time  $t_1$  is given by Eq. (5-10).

$$a_t = v_t a_1 + (v_t - v_{t1})(a_2 - a_1) \quad (5-10)$$

where  $a_1$  and  $a_2$  are the elastic (short-time) deflections corresponding to static systems (1) and (2). Term  $v_t a$ , represents the usual creep deflection without the effect of the change of static system (1). The second term is the creep deflection (positive or negative) due to the change in the static system at time  $t_1 \geq t_{ea}$ .

Typical examples are beams which are first cast as simply supported spans and carry part of the dead load before time  $t$  at which the ends of the beams are rigidly connected, without changing the stress and strain state at time  $t_1$ . Also, a cantilever which carries the load before its free end is placed on a support. This is a typical situation in segmental bridge construction.

### 5.6-Creep buckling deflections of an eccentrically compressed member<sup>69,66</sup>

The creep deflection in excess of the elastic (short-time) deflection for a symmetric cross section is given by Eq. (5-11).

$$a_t = a_0 \left[ \frac{(1 - I_s/I_t)}{(p_{ca}/p - 1)} \right] \left[ \frac{v_t}{(1 + X v_t)} \right] \quad (5-11)$$

where

$$a_0 = \frac{y_p}{1 - (p/p_{ca})}$$

where  $y_p$  is the maximum distance of the cross-section centroid from the axis of axial load  $P$  prior to its application.  $P_{ca}$  or  $P_{ca}$  is the buckling load of an elastic column with concrete modulus  $E_{ci}$  or  $E_{ca}$  respectively.  $I_s$  is the moment of inertia of steel and  $I_t$  is the moment of inertia of the whole transformed cross section with concrete modulus  $E_{ci}$ . Coefficient  $v_t$  in this equation must include correction by factor  $\xi_t$  in Section 3.5 of this report.

Equation (5-11) is theoretically exact if creep properties are the same in all cross sections and if the column has initially a sinusoidal curvature. The error is usually small for cases other than sinusoidal curvature. Similar equations hold for creep buckling deflections of arches, shells, plates, and for lateral creep buckling of concrete beams or arches.

### 5.7-Two cantilevers of unequal age connected at time $t_1$ by a hinge<sup>66,69</sup>

The statically indeterminate shear force  $S_1$  in the hinge at time  $t > t_1$  is computed from the compatibility relation in Eq. (5-12).

$$\{[1 + X_1 (v_t)_1] (f)_1 + [1 + X_2 (v_t)_2] (f)_s\} S_t = [(v_t)_2 - (v_{t1})_2] a_2 - [(v_t)_1 - (v_{t1})_1] a_1 \quad (5-12)$$

Subscripts 1 and 2 refer to cantilevers (1) and (2) respectively.  $(v_t)_1$ ,  $(v_t)_2$ , and  $X_1$  are determined using  $t_{ea} = (t_{ea})_1$  in which  $(t_{ea})_1$  is the age of cantilever (1) when it starts carrying its dead load or prestress.  $a_1$  is the elastic deflection at the end of cantilever (1) due to its dead load or prestress, considering concrete modulus as  $E_c$  at age  $(t_{ea})_1$ .  $(f)_1$  is the elastic flexibility coefficient of cantilever (1) which is the relative displacement of cantilever end in the sense of  $S_1$  due to load  $S_t = 1$ , using modulus  $E_c$  at age  $(t_{ea})_1$ .

### 5.8-Loss of compression in slab and deflection of a steel-concrete composite beam<sup>69</sup>

Compression loss  $(N_c)_t$  in a steel-concrete composite statically determinate simple supported beam is given in Eq. (5-13).

$$(N_c)_t = - \frac{v_t N_{cl} + \delta_{sh} E_{cl} A_c}{1 + X v_t + n \frac{A_c}{A_s} \left( 1 + \frac{e^2 A_s}{z_{st}} \right)} \quad (5-13)$$



where  $N_{ci}$  is the initial compressive force carried by the slab at the time  $t_{la}$  of dead load, application and  $\delta_{sh}$  as given by Eq. (5-5).  $A_s$  and  $I_s$  are the area and the moment of inertia of the steel girder about its centroidal axis,  $e$  is the eccentricity of slab centroid with regard to steel girder centroid,  $n = E_s/E_{ci}$  and  $A_c$  is the area of a concrete slab.  $I_c$  is assumed negligible.

The moment change in the steel girder equals  $e(N_c)_t$ . The creep deflection of a composite girder can be computed from the moment in the steel girder  $e(N_c)_t$ .

### 5.9-Other cases

Similar equations of greater theoretical accuracy are possible  $f_{cr}$  prestress loss,<sup>66</sup> but here the difference between the results using such equations and those of this chapter is normally less than 2 percent and thus negligible.

For a general creep analysis of nonhomogeneous cross sections and nonhomogeneous structures, see Ref. 66, for example. An application of the age-adjusted effective modulus method to the creep effects due to the nonuniform drying of shells has been made in Reference 81. Bruegger, in Reference 82, presented a number of other applications.

### 5.10-Example: Effect of creep on a two-span beam coupled afterloading<sup>82</sup>

Find the maximum negative moment at the support of a two-span beam made continuous by coupling two 90 ft (27.43 m) simple supported beams.

Data:  $\ell_1 = \ell_2 = \ell = 90$  ft (27.43 m)  
 $q = 4$  k/ft (58.4 kN/m) (sustained load applied before coupling)

For coupling at  $t_{la} = 30$  days  
 Average thickness = 8 in. (200 mm)  
 Relative humidity,  $\lambda = 60$  percent  
 $v_u = 2.35$

Since the rotation at the support resulting from creep is prevented after coupling of the two single span beam, Eq. (5-9) applies.

$$\gamma_{la} = 0.83 \quad \gamma_{\lambda} = 0.87 \quad \gamma_h = 0.96$$

hence,  $v_t = 2.35 \times 0.83 \times 0.96 = 1.63$

in which,  $X_{30} = 0.83$ , (for  $t_{la} = 30$  and  $(v_u)_{30} = 2.35$ )

$$\begin{aligned} \text{since } S_1 &= 0 \quad \text{and } S_2 = -\frac{q \ell^2}{8} \\ &= -4050 \text{ ft-kips, } (-5492 \text{ kNm}) \end{aligned}$$

therefore,

$$S = -4050 \left( \frac{1.63}{1 + 0.83 \times 1.63} \right)$$

$$= -2805 \text{ ft-kips, } (-3804 \text{ kN/M}).$$

The effect of creep is to induce a negative moment at support equal to about 69 percent of that obtained for the continuous system that is, whole structure constructed in one operation.

In a similar way, the induced negative moment at support would be 78, 62, and 53 percent of that obtained for the continuous system if coupling time  $t_{la}$  equals to 10, 90, and 1000 days respectively.

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In the balloting of the nine members of Subcommittee II, ACI Committee 209, all nine voted affirmatively. In the balloting of the entire Committee 209 consisting of twenty voting members, fifteen returned their ballot, of whom fifteen voted affirmatively.

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## NOTATION

- 1 = subscript denoting cast-in-place slab of a composite beam or the effect of the slab due to slab dead load
- 2 = subscript denoting precast beam
- $A_g$  = area of gross section, neglecting the steel
- $A_s$  = area of tension steel in reinforced members and area of prestressed steel in prestressed members
- $A_s'$  = area of compression steel in reinforced

members and area of nontensional steel on prestressed members

- $A_t$  = area of transformed section
- $a$  = deflection in general. Also used as distance from end of beam to the nearest of 2 symmetrical diaphragms, or as the distance from end to harped point in 2-point harping
- $(a_i)_1$  = initial deflection under slab dead load
- $(a_i)_{1D}$  = initial deflection due to diaphragm dead load
- $(a_i)_2$  = initial deflection under precast beam dead load
- $(a_i)_D$  = initial dead load deflection
- $(a_i)_{F_o}$  = initial camber due to the initial prestress force,  $F_o$
- $a_{DS}$  = live load deflection
- $a_u$  = ultimate (in time) deflection, camber
- $a_\delta$  = deflection due to differential shrinkage
- $a_{sh}$  = shrinkage deflection
- $(a_{sh})_s$  = shear deflection due to shrinkage
- $a_t$  = total deflection, camber, at any time
- $c$  = subscript denoting composite section. Also used to denote concrete, as  $E_c$ ; cement content and initial curing
- $cp$  = subscript denoting creep or curing period
- $D$  = subscript denoting dead load
- $d$  = effective depth of section
- $E_{ca}$  = age-adjusted effective modulus
- $E_{ci}$  = modulus of elasticity of concrete at the time of initial load, such as at transfer of prestress, etc., or of a sudden enforced deformation at time  $t_{\ell a}$
- $E_{cs}$  = modulus of elasticity of concrete at the time of slab casting
- $E_{ct}$  = modulus of elasticity of concrete at any time  $t$
- $E_s$  = modulus of elasticity of steel
- $e$  = eccentricity, also eccentricity of steel
- $e_c$  = eccentricity of steel at center of beam. Also used, as indicated, to denote eccentricity of steel in composite section
- $e_o$  = eccentricity of steel at end of beam
- $e_{th}$  = thermal coefficient of expansion of concrete
- $F$  = prestress force after losses
- $F_i$  = initial prestress force
- $F_\ell$  = loss of prestress due to time dependent effects only such as creep, shrinkage, steel relaxation. The elastic loss is deducted from the tensioning force,  $F_i$ , to obtain  $F_o$
- $F_o$  = prestress force at transfer, after elastic loss
- $F_s$  = total loss of prestress at slab casting minus the initial elastic loss that occurred at the time of prestressing
- $F_t$  = total loss of prestress at any time minus the initial elastic loss
- $F_u$  = total ultimate (in time) loss of prestress minus the initial elastic loss

|                  |   |                  |  |  |   |
|------------------|---|------------------|--|--|---|
| $f_c$            | = concrete stress such as at steel c.g.s. due to prestress and precast beam dead load in the prestress loss equations   |                  |  |  | such as an uncracked prestressed concrete section |
| $f_r$            | = modulus of rupture of concrete  | $i$              | = subscript denoting initial value   |  |   |
| $f_{cd}$         | = concrete stress at steel c.g.s. due to differential shrinkage   | $\ell$           | = span length in general and longer span for rectangular slabs   |  |   |
| $f_{ci}$         | = concrete stress at the time of initial loading, such as at transfer of prestress  | $\ell_a$         | = subscript denoting loading age   |  |   |
| $f_{cs}$         | = concrete stress at steel c.g.s. due to slab dead load plus diaphragm, etc., dead load when applicable   | $L$              | = subscript to denote live load  |  |   |
| $(f_c')_7$       | = compressive strength of concrete at 7 days; similarly, for subscript 2 for the avg. of 1 to 3 days, subscript 28, for 28 days, etc  | $M$              | = total moment. Also bending moment, used as the numerical maximum bending moment, for prismatic beams uniformly loaded  |  |   |
| $(f_c')_t$       | = compressive strength of concrete at any time $t$  | $M_D$            | = bending moment due to dead load  |  |   |
| $(f_c')_u$       | = ultimate (in time) compressive strength of concrete   | $M_1$            | = maximum bending moment under slab dead load for composite beams  |  |   |
| $f_o$            | = stress in prestressing steel at transfer, after elastic loss  | $M_2$            | = maximum bending moment under precast beam dead load  |  |   |
| $f_{pu}$         | = ultimate strength of prestressing steel   | $M_{ID}$         | = bending moment between symmetrically placed diaphragms   |  |   |
| $f_{py}$         | = steel stress at 0.1 percent strain  | $M_{S,Di}$       | = bending moment due to slab or slab plus diaphragm, etc., dead load   |  |   |
| $f_r$            | = modulus of rupture of concrete  | $M_{e1}, M_{e2}$ | = end bending moments  |  |   |
| $f_{si}$         | = initial or tensioning stress in prestressing steel  | $m$              | = modular ratio of the precast beam concrete at the time of additional sustained load application $E_s/E_{cs}$ (e.g. at the time of slab casting). Also subscript to denote mid-span |  |   |
| $(f_{sr})_t$     | = stress loss due to steel relaxation under constant strain at any time or intrinsic relaxation   | $n$              | = modular ratio, $E_s/E_{ci}$ , at the time of loading, such as at release of prestress for prestressed concrete members. Also usually used as $E_s/E_c$ for reinforced members      |  |   |
| $(f_{sr})_t$     | = stress loss due to steel relaxation in prestressed members at any time  | $n_t$            | = modular ratio due to creep, defined as $E_s/E_{ci}$  |  |   |
| $(f_{sr})_u$     | = ultimate (in time) stress loss due to steel relaxation on prestressed members   | $Q$              | = differential shrinkage force   |  |   |
| $f'_t$           | = tensile strength of concrete  | $q$              | = uniformly distributed load   |  |   |
| $f_y$            | = yield strength of steel, defined herein as 0.1 percent offset   | $q^2$            | = $I_g/A_g$  |  |   |
| $h$              | = average thickness of the part of the member under consideration. Also, overall thickness of the section   | $r^2$            | = $I_g/A_g$  |  |   |
| $I_1$            | = moment of inertia of slab   | $s$              | = subscript denoting time of slab casting referred to the precast beam concrete. Also used to denote steel, slump and spacing of stirrups  |  |   |
| $I_2$            | = moment of inertia of precast beam   | $sh$             | = subscript denoting shrinkage   |  |   |
| $I_c$            | = moment of inertia of composite section with transformed slab. The slab is transformed into equivalent precast beam concrete by dividing the slab width by $E_{c2}/E_{c1}$ | $t$              | = time in general, time in hours in the steel relaxation equations, and time in days in other equations herein. Also subscript to denote time-dependent                              |  |   |
| $I_{cr}$         | = moment of inertia of cracked transformed section  | $t_h$            | = temperature difference across the overall thickness  |  |   |
| $I_e$            | = effective moment of inertia   | $th$             | = subscript to denote temperature  |  |   |
| $I_{ea}$         | = average effective moment of inertia   | $t_{\ell a}$     | = age of concrete at first load application in days  |  |   |
| $I_{ep}$         | = effective moment of inertia for the positive zone of a beam   | $u$              | = subscript denoting ultimate value in time  |  |   |
| $I_{ew}$         | = weighted (average) effective moment of inertia  | $w$              | = unit weight of concrete in pcf or $\text{Kg/m}^3$  |  |   |
| $I_{e1}, I_{e2}$ | = $I_e$ for each one of the negative moment end zones of a beam   | $y_{cs}$         | = distance from centroid of composite section to centroid of slab  |  |   |
| $I_g$            | = moment of inertia of gross section, neglecting the steel  | $y_t$            | = distance from centroid of gross section to extreme fiber in tension  |  |   |
| $I_s$            | = moment of inertia of reinforcing steel  | $Z_b$            | = Section modulus with respect to the bottom fiber of a cross section  |  |   |
| $I_t$            | = moment of inertia of transformed section,   | $Z_t$            | = Section modulus with respect to the top fiber of a cross section   |  |   |
|                  |   | $y$              | = shrinkage or creep correction factor, also   |  |   |

|                     |  |             |  |
|---------------------|--|-------------|--|
|                     | used as the product of all applicable correction factors   | $v_u$       | = ultimate (in time) creep coefficient   |
| $\delta$            | = differential shrinkage strain, also subscript denoting differential strain or differential stress  | $v_{us}$    | = ultimate (in time) creep coefficient of the precast beam concrete corresponding to the age when the slab is cast for composite beams                     |
| $(\epsilon_{sh})_t$ | = shrinkage strain in in./in. or mm/mm at any time   | $\xi$       | = deflection coefficient   |
| $(\epsilon_{sh})_u$ | = ultimate (in time) shrinkage strain in in./in. or mm/mm  | $\xi_{fp}$  | = deflection coefficient for flat plates   |
| $\lambda$           | = relative humidity in percent   | $\xi_r$     | = reduction factor to take into account the effect of compression steel, movement of neutral axis, and progressive cracking in reinforced flexural members |
| $\lambda_{el}$      | = prestress loss due to elastic shortening in percent of initial tensioning stress or force          | $\xi_s$     | = cross section shape coefficient  |
| $\lambda_r$         | = prestress loss due to steel relaxation in percent  | $\xi_{tws}$ | = deflection coefficient for two-way slabs   |
| $\lambda_t$         | = total prestress loss in percent at any time  | $\xi_w$     | = deflection coefficient for warping $w$ due to shrinkage or temperature change  |
| $\lambda_u$         | = ultimate (in time) prestress loss in percent   | $\rho$      | = reinforcement ratio, $A_s/bd$ for cracked members, and $A_s/A_g$ for uncracked members. Used in percent in shrinkage warping equations                   |
| $v_s$               | = creep coefficient of precast beam concrete at time of slab casting                                 | $\tau$      | = multiplier for additional long time deflections due to creep and shrinkage   |
| $v_t$               | = creep coefficient at any time  | $\phi$      | = curvature  |
| $v_{t1}$            | = creep coefficient of the composite beam under slab dead load, also creep coefficient at time $t_1$ | $X$         | = aging coefficient  |
| $v_{t2}$            | = creep coefficient due to precast beam dead load  |             |  |

**Table 2.2.1 Values of the Constants  $a$ ,  $\beta$  and  $a/B$  and the Time Ratio  
From Eqs. (2-1) and (2-2).**

| Time Ratio                          | Type of Curing | Cement Type | Constants $a, \beta$ and $a/\beta$ | Concrete Age |     |     |     |     |      |      |       |      | Ultimate (in time) |
|-------------------------------------|----------------|-------------|------------------------------------|--------------|-----|-----|-----|-----|------|------|-------|------|--------------------|
|                                     |                |             |                                    | Days         |     |     |     |     |      |      | Years |      |                    |
|                                     |                |             |                                    | 3            | 7   | 14  | 21  | 28  | 56   | 91   | 1     | 10   |                    |
| $(f'_c)_t/(f'_c)_{28}$<br>Eq. (2-1) | Mbist Cured    | I           | $a= 4.0$<br>$\beta = .85$          | .46          | .70 | .88 | .96 | 1.0 | 1.08 | 1.12 | 1.16  | 1.17 | 1.18               |
|                                     |                | III         | $a = 2.3$<br>$\beta = .92$         | .59          | .80 | .92 | .97 | 1.0 | 1.04 | 1.06 | 1.08  | 1.09 | 1.09               |
|                                     | Steam Cured    | I           | $a = 1.0$<br>$\beta = .95$         | .78          | .91 | .98 | 1.0 | 1.0 | 1.03 | 1.04 | 1.05  | 1.05 | 1.05               |
|                                     |                | III         | $a=.70$<br>$\beta = .98$           | .82          | .93 | .97 | .99 | 1.0 | 1.0  | 1.01 | 1.01  | 1.02 | 1.02               |
| $(f'_c)_t/(f'_c)_u$ ,<br>Eq. (2-2)  | Mbist Cured    | I           | $a/\beta = 4.71$                   | .39          | .60 | .75 | .82 | .86 | .92  | .95  | .99   | 1.0  | 1.0                |
|                                     |                | III         | $a/\beta = 2.5$                    | .54          | .74 | .85 | .89 | .92 | .96  | .97  | .99   | 1.0  | 1.0                |
|                                     | Steam Cured    | I           | $a/\beta = 1.05$                   | .74          | .87 | .93 | .95 | .96 | .98  | .99  | 1.0   | 1.0  | 1.0                |
|                                     |                | III         | $a/\beta = 0.71$                   | .81          | .91 | .95 | .97 | .97 | .99  | .99  | 1.0   | 1.0  | 1.0                |



**Table 2.2.2 Factors Affecting Concrete Creep and Shrinkage and Variables Considered in the Recommended Prediction Method.**

| Factors  |                      |   | Variables Considered                                   | Standard Conditions  |
|--|----------------------|---|--|--|
| Concrete<br>(Creep & Shrinkage)                      | Concrete Composition | Cement Paste Content<br>Water-Cement Ratio<br>Mix Proportions<br>Aggregate Characteristics<br>Degree of Compaction      | Type of cement   | Type I and III   |
|  |                      |   | Slump  | 2.7 in, (70 mm)  |
|  |                      |   | Air Content  | ≤ 6 percent  |
|  |                      |   | Fine Aggregate Percentage                              | 50 percent   |
|  |                      |   | Cement Content   | 470 to 752 lb/cu <sub>3</sub> yd<br>(279 to 446' kg/m <sup>3</sup> ) |
|  | Initial Curing       | Length of Initial Curing<br><br>Curing Temperature<br><br>Curing Humidity   | Moist Cured  | 7 days   |
|  |                      |   | Steam Cured  | 1-3 days   |
|  |                      |   | Moist Cured  | 73.4 ± 4°F (23 ± 2°C)  |
|  |                      |   | Steam Cured  | ≤ 212°F. (≤ 100°C)   |
|  |                      |   | Relative Humidity                                      | ≥ 95 percent   |
| Member Geometry & Environment<br>(Creep & Shrinkage) | Environment          | Concrete Temperature<br>Concrete Water Content  | Concrete Temperature                                   | 73.4 ± 4°F, (23 ± 2°C)   |
|  |                      |   | Ambient Relative Humidity                              | 40%  |
|  | Geometry             | Size and Shape  | Volume-Surface Ratio, (v/s)<br>or<br>Minimum Thickness | v/s = 1.5 in<br>(v/s = 38 mm)  |
|  |                      |   |  | 6 in, (150 mm)   |
| Loading<br>(Only Creep)                              | Loading History      | Concrete Age at Load Application<br>Duration of Loading Period<br>Duration of Unloadins Period<br>Number of Load Cycles | Moist Cured  | 7 days   |
|  |                      |   | Steam Cured  | 1-3 days   |
|  |                      |   | Sustained Load   | Sustained Load   |
|  |                      |   | -  | -  |
|  | Stress Conditions    | Type of Stress and Distribution Across the Section  | Compressive Stress                                     | Axial Compression  |
|  |                      | Stress/Strength Ratio   | Stress/Strength Ratio                                  | ≤ 0.50   |

Table 2.4.1 Time-Ratio Values for Creep and Shrinkage

| Creep and Shrinkage<br>Time Ratios               | Time |       |       |      |      |      |      |      |      |
|--|------|-------|-------|------|------|------|------|------|------|
|  | 28 d | 3 mth | 6 mth | 1 yr | 2 yr | 5 yr | 10yr | 20yr | 30yr |
| $v_t/v_u$ Eq. (2-8)                              | 0.42 | 0.60  | 0.69  | 0.78 | 0.84 | 0.90 | 0.93 | 0.95 | 0.96 |
| $(\epsilon_{sh})_t/(\epsilon_{sh})_u$ Eq. (2-9)  | 0.44 | 0.72  | 0.84  | 0.91 | 0.95 | 0.98 | 0.99 | 1.00 | 1.00 |
| $(\epsilon_{sh})_t/(\epsilon_{sh})_u$ Eq. (2-10) | 0.34 | 0.62  | 0.77  | 0.87 | 0.93 | 0.97 | 0.99 | 0.99 | 1.00 |

Table 2.5.1 Correction Factors for  
Loading Age, from  
Eqs. (2-11) and (2-12)

| Loading<br>Age,<br>days | Creep<br>$\gamma_{la}$<br>moist cured | Creep<br>$\gamma_{la}$<br>steam cured |
|-------------------------|---------------------------------------|---------------------------------------|
| 7                       | 1.00                                  | 0.94                                  |
| 10                      | 0.95                                  | 0.90                                  |
| 20                      | 0.87                                  | 0.85                                  |
| 28                      | 0.84                                  | 0.83                                  |
| 60                      | 0.77                                  | 0.76                                  |
| 90                      | 0.74                                  | 0.74                                  |

**Table 2.5.3 Shrinkage Correction Factors  
for Initial Moist Curing**

| Mist curing duration,<br>days | Shrinkage $\gamma_{cp}$ |
|-------------------------------|-------------------------|
| 1                             | 1.2                     |
| 3                             | 1.1                     |
| 7                             | 1.0                     |
| 14                            | 0.93                    |
| 28                            | 0.86                    |
| 90                            | 0.75                    |

**Table 2.5.4 Correction Factors for  
Relative Humidity, from  
Eqs. (2-14), (2-15), and  
(2-16).**

| Relative<br>Humidity,<br>percent | Creep<br>$\gamma_{\lambda}$ | Shrinkage<br>$\gamma_{\lambda}$ |
|----------------------------------|-----------------------------|---------------------------------|
| < 40                             | 1.00                        | >1.00                           |
| 40                               | 1.00                        | 1.00                            |
| 50                               | 0.94                        | 0.90                            |
| 60                               | 0.87                        | 0.80                            |
| 70                               | 0.80                        | 0.70                            |
| 80                               | 0.73                        | 0.60                            |
| 90                               | 0.67                        | 0.30                            |
| 100                              | 0.60                        | 0.00                            |

**Table 2.5.5.1 Correction Factors for Average  
Thickness of Members, from  
Eqs. (2-17) to (2-20)**

| Average<br>Thickness<br>of Member*  |     | Creep<br>$\gamma_h$ |               | Shrinkage<br>$\gamma_h$ |               |
|---|-----|---------------------|---------------|-------------------------|---------------|
| in.   | mm  | $\leq 1$ yr.        | ult.<br>value | $\leq 1$ yr.            | ult.<br>value |
| 2   | 51  | 1.30                | 1.30          | 1.35                    | 1.35          |
| 3   | 76  | 1.17                | 1.17          | 1.25                    | 1.25          |
| 4   | 104 | 1.11                | 1.11          | 1.17                    | 1.17          |
| 5   | 127 | 1.04                | 1.04          | 1.08                    | 1.08          |
| Eqs.  |     | (2-17)              | (2-18)        | (2-19)                  | (2-20)        |
| 6   | 152 | 1.00                | 1.00          | 1.00                    | 1.00          |
| 8   | 203 | 0.96                | 0.96          | 0.93                    | 0.94          |
| 10  | 254 | 0.91                | 0.93          | 0.85                    | 0.88          |
| 12  | 305 | 0.86                | 0.90          | 0.77                    | 0.82          |
| 15  | 381 | 0.80                | 0.85          | 0.66                    | 0.74          |
| *This method is recommended for average thicknesses (part being considered) up to about 12" to 15", (305 to 38 mm). |     |                     |               |                         |               |

**Table 2.5.5.2 Correction Factors  
for Volume-Surface  
Ratios, from Eqs.  
(2-21) and (2-22)**

| Volume-Surface Ratio  |     | Creep<br>$\gamma_{v/s}$ | Shrinkage<br>$\gamma_{v/s}$ |
|---|-----|-------------------------|-----------------------------|
| in.   | mm  | (2-21)                  | (2-22)                      |
| 1.0   | 25  | 1.09                    | 1.06                        |
| 1.5   | 38  | 1.00                    | 1.00                        |
| 2   | 51  | 0.92                    | 0.94                        |
| 3   | 76  | 0.81                    | 0.84                        |
| 4   | 102 | 0.75                    | 0.74                        |
| 5   | 127 | 0.72                    | 0.66                        |
| 6   | 152 | 0.70                    | 0.58                        |
| 8   | 203 | 0.68                    | 0.46                        |
| 10  | 254 | 0.67                    | 0.36                        |
| <b>Examples:</b><br><b>For a rectangular section</b><br><b>6"x 12" (150 x 350mm), <math>v/s =</math></b><br><b>2.0" (51 mm). For the</b><br><b>Standard AASHTO I-Beams,</b><br><b><math>v/s</math> varies from 3.0" to 4.7",</b><br><b>(76 to 120mm).</b> |     |                         |                             |

**Table 2.7.1 Correction Factors Used in Example 2.7**

| Conditions                                     | Creep             |        | Shrinkage            |        |
|--|-------------------|--------|----------------------|--------|
|  | Eq.               | Factor | Eq.                  | Factor |
| $t_{la} = 28$ days                             | (2-11)            | 0.84   | -                    | -      |
| $A = 70\%$                                     | (2-14)            | 0.80   | (2-15)               | 0.70   |
| $h = 8$ in (200 mm)                            | (2-17)            | 0.96   | (2-19)               | 0.93   |
| $s = 2.5$ in (63 mm)                           | (2-23)            | 0.99   | (2-24)               | 0.99   |
| $\psi = 60\%$                                  | (2-25)            | 1.02   | (2-27)               | 1.02   |
| $c = 752$ lbs/cu yd<br>(446kg/m <sup>3</sup> ) | -                 |        | (2-28)               | 1.02   |
| $\alpha = 7\%$                                 | (2-29)            | 1.09   | (2-30)               | 1.01   |
| <b>Factors' product</b>                        | $\gamma_c = 0.71$ |        | $\gamma_{sh} = 0.68$ |        |



**Table 2.7.2 Creep Factors and Shrinkage Strains in  
Example 2.7**

|  |             |             |             |             |
|--|-------------|-------------|-------------|-------------|
| <b>Concrete age, days</b>                                | <b>56</b>   | <b>118</b>  | <b>208</b>  | <b>393</b>  |
| <b>Time after initial curing, days</b>                   | <b>49</b>   | <b>111</b>  | <b>201</b>  | <b>386</b>  |
| <b>Time after load application, days</b>                 | <b>28</b>   | <b>90</b>   | <b>180</b>  | <b>365</b>  |
| $\nu_t$ , <b>Eq. (2-8)</b>                               | <b>0.72</b> | <b>1.02</b> | <b>1.18</b> | <b>1.32</b> |
| $(\epsilon_{sh})_t \times 10^{-6}$ , <b>Eq. (2.9)</b>    | <b>309</b>  | <b>403</b>  | <b>451</b>  | <b>486</b>  |
| $\epsilon_{sh} \times 10^{-6}$ for $t_{da} \geq 56$ days | <b>0</b>    | <b>93</b>   | <b>142</b>  | <b>176</b>  |

**Table 2.9.1 Suggested Values for the Degree of Saturation**

| Concrete Member Environmental Conditions   | Degree of Saturation  | $\epsilon_{mc}$     |                     |
|--|---|---------------------|---------------------|
|  |   | $10^{-6}/^{\circ}F$ | $10^{-6}/^{\circ}C$ |
| <b>Immersed structures, high humidity conditions.</b>  | <b>Saturated</b>  | <b>0</b>            | <b>0</b>            |
| <b>Mass concrete pours, thick walls, beams, columns and slabs, particularly where surface is sealed.</b>   | <b>Between partially saturated and saturated</b>                        | <b>0.72</b>         | <b>1.3</b>          |
| <b>External slabs, walls, beams, columns, and roofs allowed to dry out or internal walls, columns slabs, not sealed (e.g. by mosaic or tiling) and where underfloor heating or central heating exists.</b> | <b>Partially saturated decreasing with time to the dryer conditions</b> | <b>0.83</b>         | <b>1.5</b>          |
|  |   | <b>to</b>           | <b>to</b>           |
|  |   | <b>1.11</b>         | <b>2.0</b>          |

**Table 2.9.2 Average Thermal  
Coefficient of  
Expansion of Aggregate**

| Rock Group             | $\epsilon_a$         |                      |
|------------------------|----------------------|----------------------|
|                        | 10 <sup>-6</sup> /°F | 10 <sup>-6</sup> /°C |
| Chert                  | 6.6                  | 11.8                 |
| Quartzite              | 5.7                  | 10.3                 |
| Quartz                 | 6.2                  | 11.1                 |
| Sandstone              | 5.2                  | 9.3                  |
| Marble                 | 4.6                  | 8.3                  |
| Siliceous<br>limestone | 4.6                  | 8.3                  |
| Granite                | 3.8                  | 6.8                  |
| Dolerite               | 3.8                  | 6.8                  |
| Basalt                 | 3.6                  | 6.4                  |
| Limestone              | 3.1                  | 5.5                  |

**Table 2.9.3 Range of the Concrete Thermal Coefficient  
of Expansion**

| Rock Group             | Aggregate, $\epsilon_a$ |                      | Concrete, $\epsilon_{th}$ |                      |
|------------------------|-------------------------|----------------------|---------------------------|----------------------|
|                        | 10 <sup>-6</sup> /°F    | 10 <sup>-6</sup> /°C | 10 <sup>-6</sup> /°F      | 10 <sup>-6</sup> /°C |
| Chert                  | 4.1-7.2                 | 7.4-13.0             | 6.3-6.8                   | 11.4-12.2            |
| Quartzite              | 3.9-7.3                 | 7.0-13.2             | 6.5-8.1                   | 11.7-14.6            |
| Quartz                 |                         |                      | 5.0-7.3                   | 9.0-13.2             |
| Sandstone              | 2.4-6.7                 | 4.3-12.1             | 5.1-7.4                   | 9.2-13.3             |
| Marble                 | 1.20-8.9                | 2.2-16.0             | 2.4-4.1                   | 4.4-7.4              |
| Siliceous<br>limestone | 2.0-5.4                 | 3.6-9.7              | 4.5-6.1                   | 8.1-11.0             |
| Granite                | 1.0-6.6                 | 1.8-11.9             | 4.5-5.7                   | 8.1-10.3             |
| Dolerite               | 2.5-4.7                 | 4.5-8.5              | - -                       | - -                  |
| Basalt                 | 2.2-5.4                 | 4.0-9.7              | 4.4-5.8                   | 7.9-10.4             |
| Limestone              | 1.0-6.5                 | 1.8-11.7             | 2.0-5.7                   | 4.3-10.3             |

\*Test data for the concrete does not necessarily correspond to test data for the aggregate in Table 2.9.2. These ranges are limited to the research work compiled in **Reference 76**.

**Table 3.5.1 Reduction Factors  $\xi_r$ ,  $\xi_r v_u$  and  $\xi_r \tau_u$  from Eqs. (3-7), (3-8), and (3-9).**

| $A'_S/A_S$ | Eq.(3-7)<br>$\xi_r$ | $\xi_r v_u$<br>for<br>$v_u=2.0$ | Eq.(3-8)<br>$\xi_r$ | $\xi_r \tau_u$<br>for<br>$\tau_u=2.0$ | Eq.(3-9)<br>$\xi_r$ | $\xi_r \tau_u$<br>for<br>$\tau_u=2.0$ |
|------------|---------------------|---------------------------------|---------------------|---------------------------------------|---------------------|---------------------------------------|
| <b>0</b>   | 0.85                | 1.7                             | 1.0                 | 2.0                                   | 1.0                 | 2.0                                   |
| 0.5        | 0.625               | 1.25                            | 0.70                | 1.4                                   | 0.667               | 1.3                                   |
| <b>1.0</b> | 0.40                | 0.8                             | 0.40                | 0.8                                   | 0.5                 | 1.0                                   |

**Table 3.7.1 Intrinsic Relaxation Stress Loss (Steel Relaxation Under Constant Strain).**

| Wire or Strand |                                | $(f_{sir})_t$ for $f_{si}/f_{py}$ 0.60                                       | $(f_{sir})_u/f_{si}$<br>at $t=10^5$ hours<br>and $t_l = 1$ hour | $f_{py}$<br>at 0.1% strain |
|----------------|--------------------------------|--|---|----------------------------|
| Steel          | Stress<br>relieved             | $0.1 f_{si} \left[ \frac{f_{s1}}{f_{py}} - 0.55 \right] \log_{10}(t/t_l)$    | (0.025 to 0.175)  | $f_{py} = 0.85 f_{pu}$     |
|                | Stabilized<br>(Low-relaxation) | $0.0222 f_{si} \left[ \frac{f_{si}}{f_{py}} - 0.55 \right] \log_{10}(t/t_l)$ | (0.0055 to 0.39)  | $f_{py} = 0.90 f_{pu}$     |

**Table 3.7.2 Relaxation Reduction Factor**

| $f_{si}/f_{py}$ |      | 0.50  | 0.55  | 0.60  | 0.65  | 0.70  | 0.75  | 0.80  |
|-----------------|------|-------|-------|-------|-------|-------|-------|-------|
| $\omega$        | 0.00 |       | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
|                 | 0.05 | 0.000 | 0.547 | 0.729 | 0.798 | 0.835 | 0.857 | 0.872 |
|                 | 0.10 | 0.000 | 0.289 | 0.516 | 0.627 | 0.689 | 0.729 | 0.756 |
|                 | 0.15 | 0.000 | 0.172 | 0.361 | 0.486 | 0.564 | 0.615 | 0.652 |
|                 | 0.20 | 0.000 | 0.099 | 0.262 | 0.375 | 0.458 | 0.516 | 0.558 |
|                 | 0.30 | 0.000 | 0.013 | 0.150 | 0.238 | 0.305 | 0.361 | 0.406 |
|                 | 0.40 | 0.000 | 0.000 | 0.077 | 0.159 | 0.216 | 0.262 | 0.300 |
|                 | 0.50 | 0.000 | 0.000 | 0.029 | 0.102 | 0.157 | 0.197 | 0.230 |

**Table 3.7.3 Minimum Time Intervals  
to Compute Steel Relaxation**

| Step | Beginning Time, $t_1$   | End time, $t$  |
|------|---|--|
| 1    | Pretensioned:<br>anchorage of<br>prestressing<br>steel.<br>Post-tensioned:<br>end of curing<br>of concrete. | Age at pre-<br>stressing of<br>concrete  |
| 2    | End of Step 1   | Age = 30 days,<br>or time when a<br>member is sub-<br>jected to load<br>in addition to<br>its own weight |
| 3    | End of Step 2   | Age = 1 year   |
| 4    | End of Step 3   | End of service<br>life   |

**Table 4.2.1 Values of  $M$ ,  $\xi$  and  $\xi_w$  for Beams of Uniform Section  
and Uniform Load**

| Boundary Conditions                     | $M$            | $\xi$ | $\xi_w$ |
|---|----------------|-------|---------|
| Cantilever beam                         | $-q \ell^2/2$  | 1/4   | 1/2     |
| Simple beam                             | $+q \ell^2/8$  | 5/48  | 1/8     |
| Hinged-fixed beam (one end continuous)  | $-q \ell^2/8$  | 8/185 | 11/128  |
| Fixed-fixed beam (both ends continuous) | $-q \ell^2/12$ | 1/32  | 1/16    |



**Table 4.4.1.1 Suggested Modular Ratios for Prestressed Beams**

| Modular Ratio | Type of Concrete  | Normal Weight            |                          | Sand-light Weight        |                          | All-light Weight             |                              |
|---------------|---|--------------------------|--------------------------|--------------------------|--------------------------|------------------------------|------------------------------|
|               | w in pcf, (kg/m <sup>3</sup> )  | 145<br>(2323)            |                          | 120<br>(1922)            |                          | 100<br>(1602)                |                              |
|               | Curing  | M C.                     | S. C.                    | M C.                     | S. C.                    | M C.                         | S. C.                        |
| n             | At release of prestress   | 7.3                      | 7.3                      | 9.8                      | 9.8                      | 12.9                         | 12.9                         |
| m             | For the time between pre-stressing and slab casting<br>= 3 weeks,<br>1 month,<br>2 months,<br>3 months, | 6.1<br>6.0<br>5.9<br>5.8 | 6.2<br>6.2<br>7.9<br>6.0 | 8.1<br>8.0<br>8.2<br>7.7 | 8.3<br>8.2<br>8.2<br>8.0 | 10.7<br>10.5<br>10.3<br>10.2 | 10.9<br>10.7<br>10.6<br>10.5 |

The above average modular ratios are based on  $f'_{ci} = 4000$  to  $4500$  psi ( $27.6$  to  $31.0$  MPa) for both moist cured and steam cured concrete and type I cement; up to 3-mths  $f'_c = 6360$  to  $7150$  psi ( $43.9$  to  $49.3$  MPa), using **Eq. (2-1)** for moist cured, and 3-mths  $f'_c = 6050$  to  $6800$  psi ( $41.7$  to  $46.9$  MPa), using **Eq. (2-1)** for steam cured concrete.  $E_s = 27 \times 10^6$  psi ( $18.62 \times 10^4$  MPa) for ASTM A-416 Grade 250 ( $1725$  MPa) strands and  $E_s = 28 \times 10^6$  psi ( $19.3 \times 10^4$  MPa) for Grade 270 ( $1860$  MPa) prestressing strands.

M.C. = Moist Cured, S.C. = Steam Cured

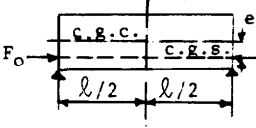
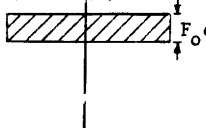
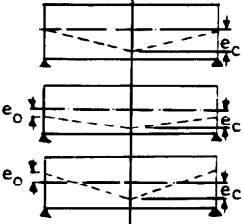
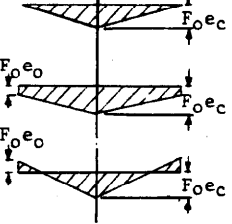
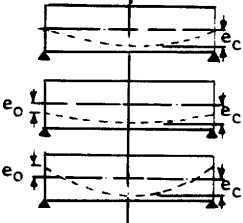
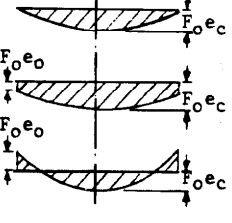
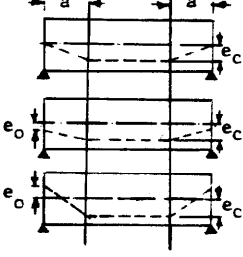
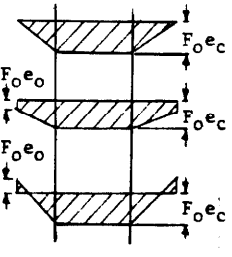
Table 4.4.1.2 Typical Loss of Prestress Ratios for Different Concretes

| Type of Concrete  | Normal weight concrete | Sand-light weight concrete | All-light weight concrete |
|---|------------------------|----------------------------|---------------------------|
| w in pcf, (kg/m <sup>3</sup> )  | 145<br>(2323)          | 120<br>(1922)              | 100<br>(1602)             |
| $F_s/F_o$ for 3 weeks to 1 month between prestressing and sustained load application, including composite slab. | 0.10                   | 0.12                       | 0.14                      |
| $F_s/F_o$ -- for 2 to 3 months between prestressing and sustained load application, including composite slab.   | 0.14                   | 0.16                       | 0.18                      |
| $F_u/F_o$   | 0.18                   | 0.21                       | 0.23                      |

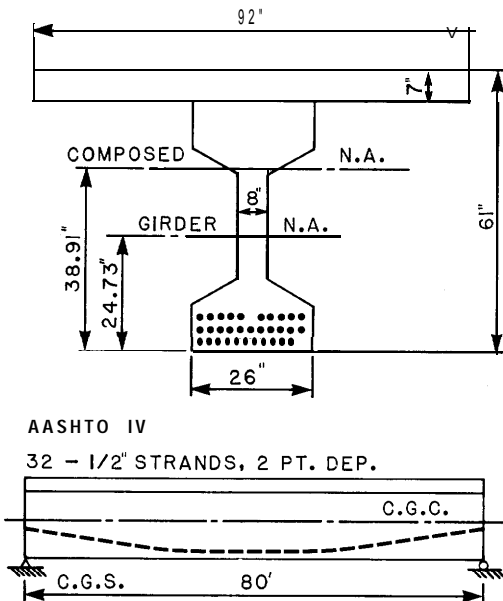
Table 4.4.1.3 Values of  $(f_{sr})_t$  and  $(f_{sr})_u$  for Wires and Strands

| Wire or Strand |                             | $(f_{sr})_t$<br>for $f_{si}/f_{py}$ from 0.65 to .80 | $(f_{sr})_u$<br>at $t = 10^5$ hours |
|----------------|-----------------------------|--|-------------------------------------|
| Steel          | Stress Relieved             | $0.015 f_{si} (\log_{10} t)$                         | $0.075 f_{si}$                      |
|                | Stabilized (Low relaxation) | $0.005 f_{si} (\log_{10} t)$                         | $0.025 f_{si}$                      |

**Table 4.4.2.1 Common Cases of Prestress Moment Diagrams and Equations for Computing Camber**

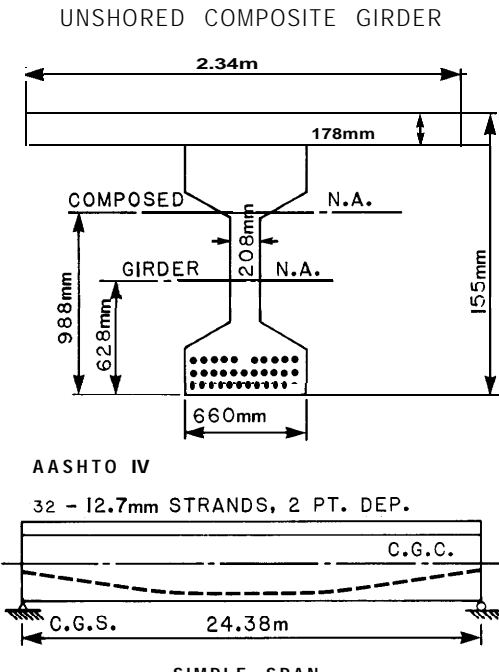
| Prestress Beam  | $F_0 e$ Moment Diagram  | Midspan Camber Due to $F_0 e$ Moments  |
|---|---|--|
|    |    | $(a_1)_{F_0} = F_0 e \ell^2 / 8 E_{ci} I_g$  |
|    |    | $(a_1)_{F_0} = F_0 e_c \ell^2 / 12 E_{ci} I_g$<br>$(a_1)_{F_0} = \frac{F_0 (e_c - e_o) \ell^2}{12 E_{ci} I_g} + \frac{F_0 e_o \ell^2}{8 E_{ci} I_g}$<br>$(a_1)_{F_0} = \frac{F_0 (e_c + e_o) \ell^2}{12 E_{ci} I_g} - \frac{F_0 e_o \ell^2}{8 E_{ci} I_g}$   |
|  |  | $(a_1)_{F_0} = 5 F_0 e_c \ell^2 / 48 E_{ci} I_g$<br>$(a_1)_{F_0} = \frac{5 F_0 (e_c - e_o) \ell^2}{48 E_{ci} I_g} + \frac{F_0 e_o \ell^2}{8 E_{ci} I_g}$<br>$(a_1)_{F_0} = \frac{5 F_0 (e_c + e_o) \ell^2}{48 E_{ci} I_g} - \frac{F_0 e_o \ell^2}{8 E_{ci} I_g}$   |
|  |  | $(a_1)_{F_0} = \frac{F_0 e_c}{E_{ci} I_g} \left[ \frac{\ell^2}{8} - \frac{a^2}{6} \right]$<br>$(a_1)_{F_0} = \frac{F_0 (e_c - e_o)}{E_{ci} I_g} \left[ \frac{\ell^2}{8} - \frac{a^2}{6} \right] + \frac{F_0 e_o \ell^2}{8 E_{ci} I_g}$<br>$(a_1)_{F_0} = \frac{F_0 (e_c + e_o)}{E_{ci} I_g} \left[ \frac{\ell^2}{8} - \frac{a^2}{6} \right] - \frac{F_0 e_o \ell^2}{8 E_{ci} I_g}$ |

**Table 4.6.1 Material and Section Properties, Parameters and Conditions for Example 4.6, (U.S. Customary Units)**

|   |   |
|---|---|
| <p style="text-align: center;">UNSHORED COMPOSITE GIRDER</p>  <p style="text-align: center;">AASHTO IV<br/>32 - 1/2" STRANDS, 2 PT. DEP.</p> <p style="text-align: center;">C.G.C.</p> <p style="text-align: center;">C.G.S. 80'</p> <p style="text-align: center;">SIMPLE SPAN</p>  | <p><b>Material Properties:</b></p> <p><b>Steam Cured Normal Weight Concrete</b></p> <p><math>f'_c = 270</math> ksi</p> <p><math>f_{ci}^{pu} = 4000</math> psi, <math>f'_c = 5000</math> psi</p> <p>Deck <math>f'_c = 4000</math> psi</p> <p><math>E_{Girder}/E_{Slab} = E_2/E_1 = 3.89/3.64 = 1.07</math></p> <p><math>E_s = 3.64 \times 10^6</math> psi, <math>E_c = 3.89 \times 10^6</math> psi</p> <p><b>Section Properties and Loading from PCI Handbook</b></p> <p><b>Girder:</b></p> <p>Spacing = 7'-8", <math>A_g = 789</math> in<sup>2</sup></p> <p><math>y_b = 24.73</math>", <math>I_g = 260,740</math> in<sup>4</sup></p> <p><math>Z_t = 8,908</math> in<sup>3</sup>, <math>Z_b = 10,544</math> in<sup>3</sup></p> <p><math>e_o = 2.86</math>", <math>e_c = 21.26</math>", <math>w_D = 822</math> #/ft</p> |
| <p><b>Calculated Section Properties and Loading -- Composite Section:</b></p> <p>Modified Slab Area = <math>7 \times 92/1.07 = 602</math>     <math>y_b = (789 \times 24.73 + 602 \times 57.50)/(789 + 602) = 38.91</math>"</p> <p><math>I_g = 260,740 + 789(38.91 - 24.73)^2 + 92 \times 7^3/(12 \times 1.07) + 602(57.50 - 38.91)^2 = 629,890</math> in<sup>4</sup></p> <p><math>Z_t = 629,890/(61.00 - 38.91) = 28,510</math> in<sup>3</sup>, <math>Z_b = 629,890/38.91 = 16,190</math> in<sup>3</sup></p> <p>Including 1/2" w.s., Slab D.L. = <math>w_s = (7.5 \times 92)(150/144) = 719</math> lb/ft</p> <p>HS 20-44 AASHTO Loading, Impact = <math>50/(80 + 125) = 0.25</math></p> <p>Assume Deck Slab Cast 2 Mnth's After Prestressing</p> <p>Area of One 1/2" Strand = <math>0.153</math> in<sup>2</sup> (Fig. 11.3.3, PCI Handbook)</p> <p><math>A_{ps} = (32)(0.153) = 4.90</math> in<sup>2</sup>, <math>f_{si} = (0.70)(270) = 189</math> ksi</p> <p><math>M_D = w_D L^2/8 = (822)(80)^2/8 = 657,600</math> ft-lb, <math>M_{S, Di} = (719)(80)^2/8 + 50,000</math></p> <p><math>= 625,200</math> ft-lb, <math>M_D + M_{S, Di} = 1,282,800</math> ft-lb, Interior Girder <math>M_L + I = (1165 - -</math></p> <p>AASHTO Table) (1/2 -- Single Wheels)(1.25 -- L + 1)(7.67/5.5 -- AASHTO, S/5.5)</p> <p><math>= 1,015,400</math> ft-lb</p> |   |
| <p>Assume 60% Ambient Humidity</p> <p><b>Other Parameters:</b> <math>n = 7.3</math>, <math>m = 6.1</math> (Table 4.4.1.1), <math>v_u = 1.64</math>, <math>v_s = v_t</math>,</p> <p><math>\gamma_{la} = (0.54)(1.64)(0.78) = 0.69</math>, where <math>v_t/v_u = 0.54</math> (Eq. 2-8)</p> <p>and creep <math>\gamma_{la} = 0.78</math> (Eq. 2-12), <math>(\epsilon_{sh})_u = 487 \times 10^{-6}</math> in/in, (m/m)</p> <p><math>f_s/F_o = 0.14</math>, <math>F_u/F_o = 0.18</math> (Table 4.4.1.2),</p> <p>and <math>(1 + n \rho \xi_s) = 1.25</math> (Design simplification)</p>   |   |



**Table 4.6.2 Material and Section Properties, Parameters and Conditions for Example 4.6 (SI Units)**

|   |  |
|---|--|
| <p style="text-align: center;">UNSHORED COMPOSITE GIRDER</p>  <p style="text-align: center;">SIMPLE SPAN</p> | <p><b>Material Properties:</b></p> <p><b>Steam Cured Normal Weight Concrete</b></p> $f_{pu} = 1862 \text{ MPa}$<br>$f_{ci} = 27.6 \text{ MPa}, f'_c = 34.5 \text{ MPa}$<br>Deck $f'_c = 27.6 \text{ MPa}$<br>$E_{\text{Girder}}/E_{\text{Slab}} = E_2/E_1 = 2.68/2.51 = 1.07$<br>$E_{ci} = 2.51 \times 10^4 \text{ MPa}, E_c = 2.68 \times 10^4 \text{ MPa}$ <p><b>Section Properties and Loading from PCI Handbook</b></p> <p><b>Girder:</b></p> Spacing = 2.34m, $A_g = 0.509 \text{ m}^2$<br>$y_b = 623\text{mm}, I_g = 0.1085\text{m}^4$<br>$Z_t = 0.1460\text{m}^3, Z_b = 0.1728\text{m}^3$<br>$e_o = 73\text{mm}, e_c = 540\text{mm}, w_D = 12\text{KN/m}$ |
|---|--|

**Calculated Section Properties and Loading -- Composite Section:**

$$\text{Modified Slab Area} = 0.178 \times 2.34 / 1.07 = 0.389 \text{ m}^2, y_h = (0.509 \times 0.628 + 0.389 \times 1.46) / (0.509 + 0.389) = 0.988 \text{ m}$$

$$I_g = 0.1085 + 0.509(0.988 - 0.623)^2 + 2.34 \times 0.178^3 / (12 \times 1.07) + 0.389(1.46 - 0.988)^2 = 0.2622 \text{ m}^4$$

$$Z_t = 0.2622 / (1.549 - 0.988) = 0.4674 \text{ m}^3, Z_b = 0.2622 / 0.988 = 0.2653 \text{ m}^3$$

$$\text{Including 12.7mm w.s., Slab D.L.} = w_s = (0.191 \times 2.34) \times 2.4 \times 9.807 = 10.5 \text{ KN/m}$$

$$\text{HS20-44 AASHTO Loading, Impact} = 50 / (80 + 125) = 0.25$$

Assume Deck Slab Cast 2 Mnth After Prestressing

$$\text{Area of One 12.7mm Strand} = 9.87 \times 10^{-5} \text{ m}^2 \text{ (Fig. 11.3.3, PCI Handbook)}$$

$$A_{ps} = (32)(9.87 \times 10^{-5}) = 3.16 \times 10^{-3} \text{ m}^2, f_{si} = (0.70)1862 - 1303 \text{ MPa}$$

$$M_D = w_D L^2 / 8 = 12(24.38)^2 / 8 = 891.6 \text{ KNm}, f_{si} = (0.70)1862 - 1303 \text{ MPa}$$

$$= 847.9 \text{ KNm}, M_D + M_S D_i = 1739.5 \text{ KNm}, \text{ Interior Girder } M_L + I = (1579.5 -$$

$$\text{AASHTO Table) (1/2 - Single Wheels) (1.25 - L + I) (2.34 \times 3.28 / 5.5 - \text{AASHTO, S/5.5}) = 13.76.7 \text{ KNm}$$

Assume 60% Ambient Relative Humidity

**Other Parameters:**  $n = 7.3, m = 6.1$  (Table 4.4.1.1),  $v_u = 1.64, v_s = v_t$ ,

$$\gamma_{la} = (0.54)(1.64)(0.78) = 0.69, \text{ where } v_t/v_u = 0.54 \text{ (Eq. 2-8)}$$

$$\text{and creep } \gamma_{la} = 0.78 \text{ (Eq. 2-12), } (\epsilon_{sh})_u = 487 \times 10^{-6} \text{ in/in, (m/m)}$$

$$F_s/F_o = 0.14, F_u/F_o = 0.18 \text{ (Table 4.4.1.2),}$$

$$\text{and } (1 + n\rho\epsilon_s) = 1.25 \text{ (Design simplification)}$$

**Table 4.6.3      Term by Term Loss of Prestress and Ultimate (in time) Midspan  
Loss for Example in 4.6, Composite AASHTO Type IV Girder,  
Normal Weight Concrete**

| Units | Elastic | Losses                 |                       |           |            |                      | Gains              |                           | Total Loss |
|-------|---------|------------------------|-----------------------|-----------|------------|----------------------|--------------------|---------------------------|------------|
|       |         | Creep Before Slab Cast | Creep After Slab Cast | Shrinkage | Relaxation | Elastic, Due to Slab | Creep, Due to Slab | Diff. Shrinkage and Creep |            |
| ksi   | 14.10   | 11.61                  | 3.93                  | 10.91     | 14.18      | -3.73                | -1.58              | -3.44                     | 45.98      |
| MPa   | 97.22   | 80.05                  | 27.10                 | 75.22     | 97.77      | -25.72               | -10.89             | -23.72                    | 317.02     |
| %     | 7.46    | 6.14                   | 2.08                  | 5.77      | 7.50       | -1.97                | -0.84              | -1.82                     | 24.3       |

**Table 4.6.4      Term by Term Camber, Deflection and Ultimate Midspan Values  
for Example in Composite AASHTO Type IV Girder, Normal  
Weight Concrete**

[illegible]

**Table 4.7.1 Elastic Deflection Coefficients  $\xi_{fp}$  and  $\xi_{tws}$  ( $1 \times 10^{-5}$ ) for Interior Panel.**

| Type   | Interior Panel Support   |   |            | $\ell/s$   |            |            |            |            |     |
|--|--|---|------------|------------|------------|------------|------------|------------|-----|
|  |  |   |            | 1.0        | 1.1        | 1.7        | 1.3        | 1.4        | 1.5 |
| $\xi_{fp}$<br>(Flat Plates)  | Zero edge beam stiffness   | $c/\ell$  | 0.0        | 581        | 487        | 428        | 387        | 358        | 337 |
|  |  |   | 0.1*       | 441        | 372        | 320        | 283        | 260        | 243 |
| $\xi_{tws}$<br>(Two-way slabs)   | Elastically supported edges. The appropriate coefficient is in between the case of zero edge beams | Relatively flexible edge beams (total depth about $2t$ )* | 380 to 250 | 330 to 230 | 290 to 210 | 260 to 190 | 240 to 170 | 230 to 160 |     |
|  | stiffness (flat plate) and infinitely stiff edge beams (rigid supports).                           | Relatively stiff edge beams (total depth about $3t$ )*    | 290 to 170 | 260 to 140 | 230 to 120 | 210 to 105 | 190 to 90  | 180 to 80  |     |
|  | Rigid supports. Built-in edges on infinitely stiff edge beams                                      |   |            | 126        | 102        | 83         | 67         | 54         | 43  |
| *approximate values<br>$c/\ell$ = column/span ratio<br>$\ell/s$ = longer span/shorter span ratio |  |   |            |            |            |            |            |            |     |

**Table 5.1.1 Aging Coefficient**

| $t-t_{la}$<br>days | $v_u$ | $t_0$ in days |        |        |        |
|--------------------|-------|---------------|--------|--------|--------|
|                    |       | $10^1$        | $10^2$ | $10^3$ | $10^4$ |
| $10^1$             | 0.5   | .525          | .804   | .811   | .809   |
|                    | 1.5   | .728          | .826   | .825   | .820   |
|                    | 2.5   | .774          | .842   | .837   | .830   |
|                    | 3.5   | .806          | .856   | .848   | .839   |
| $10^2$             | 0.5   | .505          | .888   | .916   | .915   |
|                    | 1.5   | .739          | .919   | .932   | .928   |
|                    | 2.5   | .804          | .935   | .943   | .938   |
|                    | 3.5   | .839          | .946   | .951   | .946   |
| $10^3$             | 0.5   | .511          | .912   | .973   | .981   |
|                    | 1.5   | .732          | .943   | .981   | .985   |
|                    | 2.5   | .795          | .956   | .985   | .988   |
|                    | 3.5   | .830          | .964   | .987   | .990   |
| $10^4$             | 0.5   | .501          | .899   | .976   | .994   |
|                    | 1.5   | .717          | .934   | .983   | .995   |
|                    | 2.5   | .781          | .949   | .986   | .996   |
|                    | 3.5   | .818          | .958   | .989   | .997   |