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Time-Dependent Effects in Composite Concrete Beams

By DAN E. BRANSON

The effects of direct shrinkage and creep deformation and differential shrinkage in composite concrete beams are discussed. Two different methods for determining differential shrinkage stresses and deflections are briefly summarized and compared. Procedures for predicting the total (initial plus time-dependent) deflection of shored and unshored composite beams, in which the precast beams are either reinforced or prestressed, are discussed. Also included is a discussion of existing experimental data dealing with the time-dependent behavior of composite concrete beams.

Key words: beam; composite beam; creep; deflection; design; differential shrinkage; precast beam; prestressed concrete; reinforced concrete; shrinkage.

■ BEFORE STUDYING THE history of shrinkage and creep effects in composite concrete beams, both before and after slab casting, it is helpful to note the effect of the bonded cast-in-place slab in restraining the creep strain of the precast beam. Consider the case of unshored construction in which the slab is unstressed under dead load. When the free shrinkage of the slab exceeds the "unbonded" shrinkage plus creep of the top fiber of the precast beam following slab casting (which is usually the case), the slab has the effect of restraining further creep curvature of the precast beam by virtue of its bending resistance or EI value.

Thus the analysis of the initial plus time-dependent behavior of composite concrete beams consists of the following considerations:

Unshored construction

1. Initial stresses and deflections.
2. For ordinary reinforced precast beams, shrinkage warping and creep deflection—and for prestressed precast beams, the loss of prestress and camber growth—of the precast beam up to the time of slab casting.
3. Same as Consideration 2 for the period following slab casting in which the hardened slab is effective in reducing the precast beam creep curvature.
4. Initial precast beam stresses and deflections under slab dead load. Here the age of the precast beam concrete should be considered in the choice of the concrete modulus of elasticity.
5. Creep deflection of the precast beam under the slab dead load and considering the age of the precast beam concrete when the slab was cast in choosing the creep coefficient or unit creep strain to be used. Here the slab is effective in reducing the precast beam creep curvature.
6. Differential shrinkage stresses and deflections.
7. Live load stresses and deflections.

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Shored construction

1. Same.
2. Same.
3. Same.
4. Initial transformed composite beam stresses and deflections under slab dead load.
5. Creep deflection of the transformed composite beam under slab dead load and considering the age of the precast beam concrete when the slab was cast. Here the problem is complicated by the different ages of the two concretes and their correspondingly different creep characteristics. However, some simplifying average creep coefficient or unit creep strain for the two concretes can be used.
6. Differential shrinkage stresses and deflections. Here the same procedures apply as for unshored construction. However, any creep differential at the interface under slab dead load (due to the fresher concrete slab) would tend to increase the differential shrinkage coefficient rather than decrease it as in the unshored construction case. For this same reason the differential shrinkage effect would also vary along the span but in an opposite sense and to a lesser degree than for unshored construction.
7. Live load stresses and deflections.

The aim of this paper is to briefly review and discuss these requirements for composite concrete beam design.

DIFFERENTIAL SHRINKAGE METHODS

There are at least two basic approaches that have been advanced for analyzing the differential shrinkage problem, along with several variations and extensions of these approaches. The distinguishing feature of these two approaches, insofar as their mechanics is concerned, is that the differential shrinkage force is applied to the slab alone and then to the composite section (both at the slab centroid) in one case and to the separate slab and beam sections (both at the interface) in the other case. These will henceforth be referred to as the "composite section method" and "separate section method."

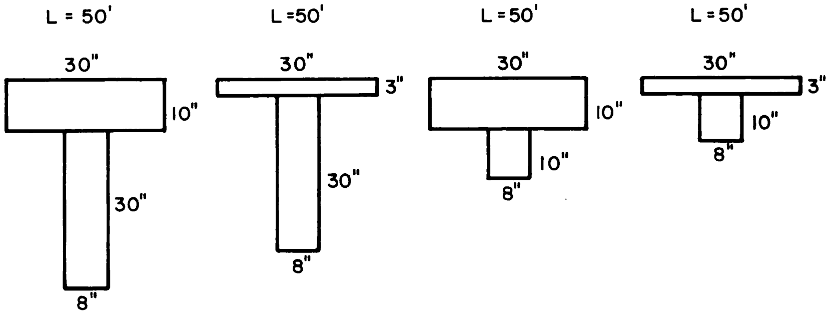
In the composite section method an eccentric column load is applied to the composite beam at the slab centroid. Thus the free shrinkage of the slab, after it has been bonded to the precast beam, is assumed to be restrained by the composite section. In the separate section method the decrease in volume (here shrinkage) of the slab induces an eccentric compressive force at the top of the precast beam, and, in turn, the restraint offered by the precast beam to free slab shrinkage induces an equal eccentric tensile force at the bottom of the slab. Four rather extreme examples are used in Tables 1 and 2 to show that the two methods lead to quite different results.

Contributors to the composite section method include Morsch;[1] Viest, Fountain and Singleton;[2] Bartlett;[3] and Birkeland.[4] The work of the latter two refers specifically to composite concrete beams, and both include an analysis of the interface normal and shear stresses that must be developed near the ends of the beam according to both methods. The fiber stresses and midspan deflection by the composite section method are given by Eq. (2) and (3) in which the transformed composite section properties are used.

Referring to Fig. 1 and 2a:

$$Q = D\,A_1\,E_1 \qquad (1)$$

TABLE 1—SECTION PROPERTIES AND DESIGN DATA



Sec. 1 Sec. 2 Sec. 3 Sec. 4

Sec.	Section Properties															
	d_1	d_2	A_1	A_2	I_1	I_2	y_{1T}	y_{1B}	y_{2T}	y_{2B}	A_c	I_c	y_{cT}	y_{cs}	y_{ci}	y_{cB}
	in.	in.	in.^{2}	in.^{2}	in.^{4}	in.^{4}	in.	in.	in.	in.	in.^{2}	in.^{4}	in.	in.	in.	in.
1	10	30	300	240	2500	18000	5	5	15	15	390	36350	17.3	12.3	7.3	22.7
2	3	30	90	240	76	18000	1.5	1.5	15	15	285	20240	15.4	13.9	12.4	17.6
3	10	10	300	80	2500	667	5	5	5	5	230	7075	8.5	3.5	-1.5	11.5
4	3	10	90	80	76	667	1.5	1.5	5	5	125	1357	5.7	4.2	2.7	7.3

$D = 300 \times 10^{-6}$ in./in.

$$E_1 = \frac{3 \times 10^6}{1 + C_t} = 1 \times 10^6 \text{ p.s.i., where } C_t = 2$$

$$E_2 = \frac{4 \times 10^6}{1 + C_t} = 2 \times 10^6 \text{ p.s.i., where } C_t = 1$$

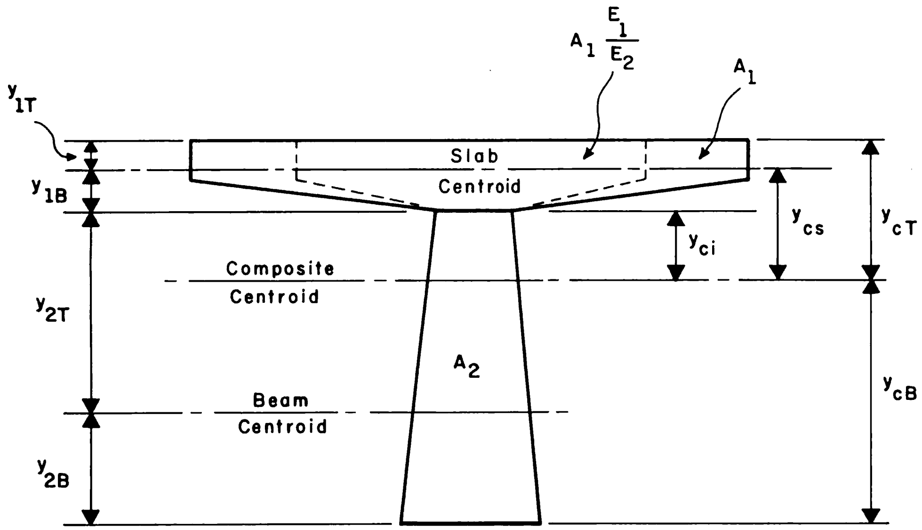
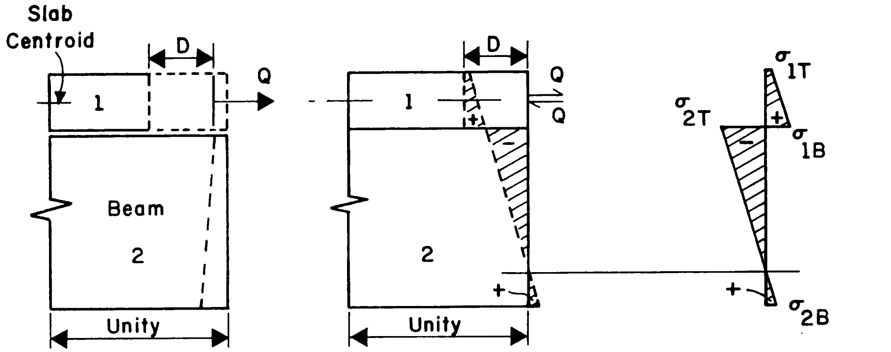


Fig. 1—Geometry of general section



a. Unbonded shrinkage and creep strain of both elements — and slab shrinkage force Q

b. Self-cancelling slab force Q applied as eccentric column load on composite section — and resulting differential shrinkage strains

c. Corresponding differential shrinkage stresses

Time-dependent concrete strains (all from time of slab casting)

Fig. 2—Shrinkage force Q applied to slab and to composite section

Applying this force to the composite section as in Fig. 2b, the resulting fiber stresses shown in Fig. 2c are obtained:

$$\left.\begin{aligned}
\sigma_{1t} &= \frac{Q}{A_1} + \left(- \frac{Q}{A_c} - \frac{Q y_{cs}\, y_{cT}}{I_c} \right) \frac{E_1}{E_2} \\
\sigma_{1B} &= \frac{Q}{A_1} + \left(- \frac{Q}{A_c} \pm \frac{Q y_{cs}\, y_{ci}}{I_c} \right) \frac{E_1}{E_2} \\
\sigma_{2T} &= - \frac{Q}{A_c} \pm \frac{Q y_{cs}\, y_{ci}}{I_c} \\
\sigma_{2B} &= - \frac{Q}{A_c} + \frac{Q y_{cs}\, y_{cB}}{I_c}
\end{aligned}\right\} \qquad (2)$$

The plus-minus term in the second and third equations is minus except when the composite centroid is within the slab. For the concentrated differential shrinkage force Q (See Fig. 4a) applied at the ends of the beam, the midspan deflection for simple beams is given by

$$\Delta = \frac{Q y_{cs} L^2}{8 E_2 I_c} \qquad (3)^*$$

Contributors to the separate section method include Frohlich,[5] Walter,[6] Evans and Parker,[7] Ozell and Diniz,[8] and Branson and Ozell.[9,10] Since there is no applied external force or couple resulting from differential shrinkage, the two forces F_1 and F_2 in Fig. 3b must be equal and collinear at any given section. These forces act on their respective sections and contribute both a direct stress and a bending stress. However, both

*See footnote with Eq. (13), p. 221.

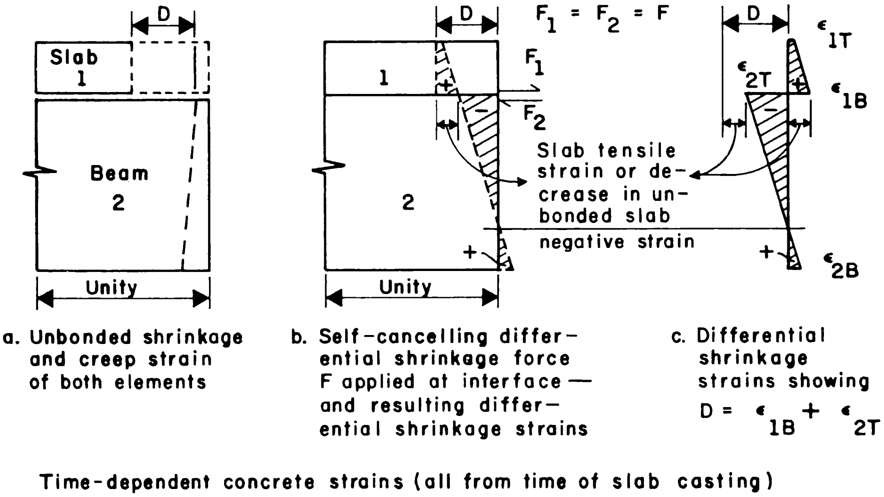


Fig. 3—Shrinkage force F applied to separate sections

produce a downward deflection that cannot act independently of each other since the two elements are bonded together. Hence the combined moment, $F_1 y_{1B} + F_2 y_{2T}$, must be distributed between the slab and beam sections in proportion to their respective stiffnesses or EI values.

Referring to Fig. 1 and 3:

$$\left.\begin{aligned} \varepsilon_{1B} &= \frac{1}{E_1}\left(\frac{F_1}{A_1} + \frac{M_1 y_{1B}}{I_1}\right) \\ \varepsilon_{2T} &= \frac{1}{E_2}\left(\frac{F_2}{A_2} + \frac{M_2 y_{2T}}{I_2}\right) \end{aligned}\right\} \quad \ldots\ldots (4)$$

where

$$\left.\begin{aligned} M_1 &= (F_1 y_{1B} + F_2 y_{2T}) \frac{E_1 I_1}{E_1 I_1 + E_2 I_2} \\ M_2 &= (F_1 y_{1B} + F_2 y_{2T} \frac{E_2 I_2}{E_1 I_1 + E_2 I_2} \end{aligned}\right\} \quad \ldots\ldots (5)$$

Note that

$$\left.\begin{aligned} F_1 &= F_2 = F \\ D &= \varepsilon_{1B} + \varepsilon_{2T} \end{aligned}\right\} \quad \ldots\ldots (6)$$

where D is the differential shrinkage coefficient defined as: (unbonded shrinkage strain of the bottom fiber of the slab) — (unbonded shrinkage plus creep of the top fiber of the beam); all strains measured from the time of slab-to-beam bonding. Solving Eq. (4), (5), and (6):

$$\left.\begin{aligned} F &= \frac{E_1 D}{\dfrac{1}{A_1} + \dfrac{1}{A_2\,(E_2/E_1)} + \dfrac{(y_{1B} + y_{2T})^2}{I_1 + I_2\,(E_2/E_1)}} \\ &= \frac{E_2 D}{\dfrac{1}{A_2} + \dfrac{1}{A_1\,(E_1/E_2)} + \dfrac{(y_{1B} + y_{2T})^2}{I_2 + I_1\,(E_1/E_2)}} \end{aligned}\right\} \quad \ldots\ldots (7)$$

and

$$\left.\begin{aligned} \sigma_{1T} &= \frac{F}{A_1} - \frac{M_1 y_{1T}}{I_1} \\ \sigma_{1B} &= \frac{F}{A_1} + \frac{M_1 y_{1B}}{I_1} \\ \sigma_{2T} &= -\frac{F}{A_2} - \frac{M_2 y_{2T}}{I_2} \\ \sigma_{2B} &= -\frac{F}{A_2} + \frac{M_2 y_{2B}}{I_2} \end{aligned}\right\} \quad \ldots\ldots (8)$$

TABLE 2—COMPARISAN OF DIFFERENTIAL SHRINKAGE STRESSES AND DEFLECTIONS BY TWO METHODS

Method (1)	Section (2)	Force, lb (3)	σ_{1T}, psi (4)	σ_{1B}, psi (5)	σ_{2T}, psi (6)	σ_{2B}, psi (7)	$\frac{\sigma_{1B}-\sigma_{1T}}{d_1}\frac{1}{E_1}$ 1/(in. $\times 10^{-6}$) (8)†	$\frac{\sigma_{2B}-\sigma_{2T}}{d_2}\frac{1}{E_2}$ 1/(in. $\times 10^{-6}$) (9)†	Δ,‡ in. (10)
Composite section method	1	$Q = 90{,}000$	− 79*	+ 74	−451	+461	15.3	15.3	0.48
	2	27,000	+110	+138	−325	+232	9.3	9.3	0.29
	3	90,000	− 86	+137	−326	+119	22.3	22.3	0.25
	4	27,000	− 47	+ 79	−443	+396	42.0	42.0	0.48
Separate section method	1	$F = 19{,}000$	+ 14	+112	−370	+214	9.8	9.8	0.22
	2	14,500	+151	+171	−258	+138	6.7	6.6	0.15
	3	8,400	− 81	+137	−325	+115	2.19	22.0	0.18
	4	6,300	+ 26	+114	−371	+213	29.3	29.2	0.23

*Negative sign denotes compression, positive sign tension.

†Slopes of strain distribution diagrams are equal in slab and precast beam.

‡In the deflection calculations, 50-ft spans were assumed for Sections 1 and 2 and 30 ft for Sections 3 and 4.

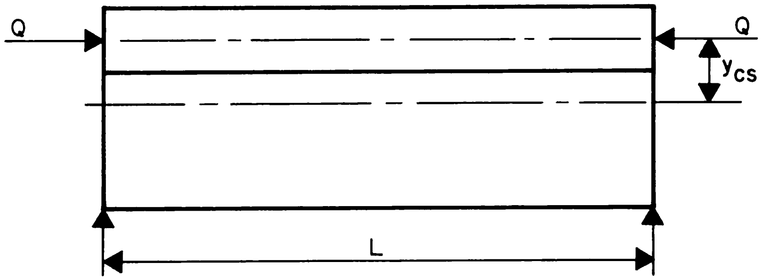
It is noted that a linear distribution across the composite section results from the differential shrinkage stresses as determined by both methods [Eq. (2) and (8)]. This is illustrated in Table 2.

The formulas presented up to now refer to the solution of differential shrinkage stresses in which the force Q or F is equal to the resultant internal force at a given section. In calculating stress, only this resultant force is needed so that any variation of the force along the span has not been considered. In calculating beam deflection, however, such variation requires attention. The composite section method assumes no distribution along the span and concentrates the force at the ends of the beam (at the slab centroid), as shown in Fig. 4a. The corresponding midspan deflection was given by Eq. (3).

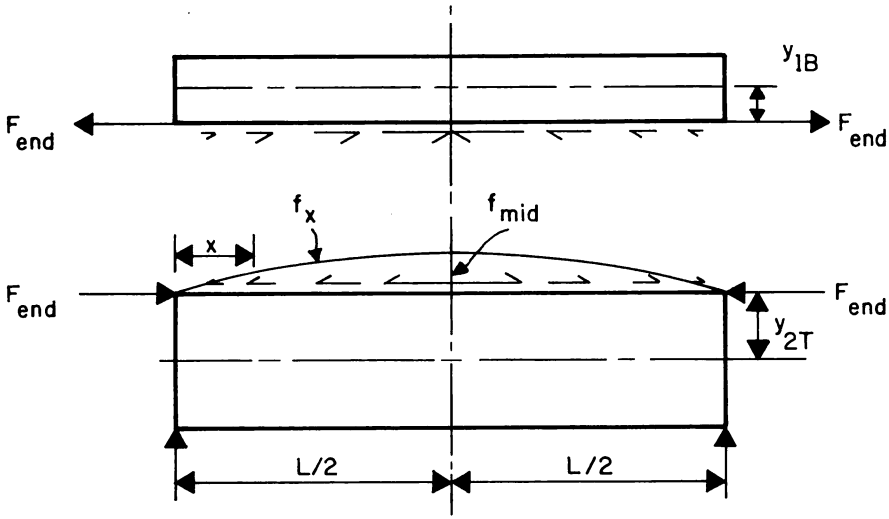
For unshored construction the creep strain of the top fiber of the precast beam (resulting primarily from slab dead load) varies along the span, with maximum creep at midspan. Thus the differential shrinkage force varies from a minimum at midspan to a maximum near the beam end. For shored construction this differential shrinkage force variation along the span is from maximum at midspan to minimum at the beam end, because of the differential creep at the interface (greater in the fresher slab concrete) resulting from slag dead load.

If each unit length underwent the same differential shrinkage, the resultant force along the beam would be constant, which would require the applied force to be concentrated at the end of the beam. But, as a result of the variation in differential shrinkage from end to midspan, the resultant force must also vary from end to midspan. It is the author's opinion that erroneous thinking results from the tendency to think of the slab as shrinking from the end of the beam toward the midspan. Actually each unit length of slab "squeezes" the precast beam in all directions in the horizontal interface plane as a result of the decrease in slab volume or shrinkage. Of course, there is an integrated effect along the beam.

Considering the resultant differential shrinkage force at the beam end and midspan respectively to be F_{end} and F_{mid}, and assuming a parabolic variation as shown in Fig. 4b (here the *change* in F or distributed force f varies parabolically), the solution for the midspan deflection of simple composite beams using unshored construction is obtained as follows:



a. Composite Section Method



b. Separate Section Method

Fig. 4—Differential shrinkage forces in the two different methods

For the parabolic variation:

$$f_x = \frac{4}{L} f_{mid} (x - x^2/L) \qquad (9)$$

and

$$(F_{end} - F_{mid}) = \int_0^{L/2} f_x \, dx = (2/3)(L/2) f_{mid} = (L/3) f_{mid} \qquad (10)$$

where f refers to the change in the resultant differential shrinkage force per unit length of span and F refers to the resultant differential shrinkage force at any section. The resultant force at any section x is

$$F_x = F_{end} - \int f_x dx$$

$$= F_{end} - (12/L^2)\,(F_{end} - F_{mid}) \left(\frac{x^2}{2} - \frac{x^3}{3L}\right) \qquad (11)$$

and the bending moment is

$$M_x = F_x \,(y_{1B} + y_{2T}) \qquad (12)$$

The corresponding curvature and midspan deflection expressions are thus:

$$\frac{d^2y}{dx^2} = (0.7) \frac{M_x}{E_1 I_1 + E_2 I_2} \qquad (13)^*$$

and

$$\Delta = (0.7) \frac{(y_{1B} + y_{2T})\, L^2}{(E_1 I_1 + E_2 I_2)\, 160} (9\, F_{end} + 11\, F_{mid}) \qquad (14)$$

For $F_{end} = F_{mid} = F$ (where the differential shrinkage variation along the span is neglected), Eq. (14) reduces to

$$\Delta = (0.7) \frac{(y_{1b} + y_{2T})\, L^2}{E_1 I_1 + E_2 I_2} \left(\frac{F}{8}\right) \qquad (15)$$

which is somewhat similar to Eq. (3) in the composite section method.

The variation between F_{end} and F_{mid} was found to be significant by Branson and Ozell[10] in which the different values for the differential

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*The calculated differential shrinkage force should be multiplied by some factor, less than 1, to account for the fact that the ultimate force F does not act over the entire time t' That is, the creep effect (incorporated in the reduced moduli E_1 and E_2) does not apply in the usual way to the ultimate force F. A factor of 0.7 is assumed because the ultimate differential shrinkage force is developed relatively early compared to the direct shrinkage and creep behavior.[15] Otherwise, a factor of 0.5 would be logical, as discussed in the next section. This 0.7 factor should also be used with Eq. (3). The factor, however, is not used in calculating differential shrinkage stresses.

shrinkage coefficients at the two locations were experimentally determined for different relative humidities and time periods between the casting of the two concretes.

In both methods the slab and precast beam moduli of elasticity should be reduced to allow for creep. Since the differential shrinkage stresses are induced largely at a time when the slab concrete is relatively more susceptible to creep, the creep effect for the slab concrete should be somewhat greater than that of the precast beam concrete. In the examples included herein, the values were assumed to be $C_t = 2$ for the slab and $C_t = 1$ for the precast beam.

The results of four examples are compared in Tables 1 and 2 using the composite section method and separate section method. The same section properties and other values were used, but the results by the two methods are seen to vary widely. For comparison purposes in the calculation of differential shrinkage deflection, F_{end} and F_{mid} were considered equal in the separate section method, and the 0.7 factor was used in both methods.

The following observations are made regarding the comparisons of the two methods and the four sections in Tables 1 and 2:

1. The composite section method predicts higher precast beam stresses, in general, and greater deflections (both as high as twice in some cases) than the separate section method.

2. Even the signs of the slab top fiber stresses are different in some cases when computed by the two methods.

3. The order of magnitude of the calculated tensile stresses for the precast beam in some cases approach a value of one-half to the full modulus of rupture of concrete for instantaneous loads. However, cracking loads up to 2.5 times the instantaneous cracking loads have been observed for loads applied over 2 to 3 months. That is, the extensibility of concrete reduces the cracking tendency under the gradually developed differential shrinkage load.

4. It is difficult to form general conclusions as to the relative effects of larger slabs, deeper stems, etc. For example, note the following:

(a) In comparing Sections 1 and 2 having the same size stem, the greater precast beam stresses occur in Section 1 which has the larger slab. The reverse is true for Sections 3 and 4. The two methods are consistent in this comparison.

(b) In comparing Sections 1 and 3 having the same size slab, the deeper precast beam in Section 1 results in higher precast beam stresses. The reverse is true for Sections 2 and 4. The two methods are consistent in this comparison.

(c) In comparing Sections 1 and 2 having the same size stem, the larger slab in Section 1 causes a greater deflection. The reverse is true for Sections 3 and 4. The two methods are consistent in this comparison.

ANALYSIS OF INITIAL PLUS TIME-DEPENDENT DEFLECTIONS

The effects of shrinkage and creep on the time-dependent deflection of prestressed concrete members can be expressed algebraically. For noncomposite prestressed concrete beams, the curvature expression corresponding to the total (initial plus time-dependent) deflection, exclusive of live load, is

$$\frac{d^2y}{dx^2} = \frac{1}{E_p I} \left[(1 + C_t)\ (M_x) - (1 + S_t)\ (P_i\, e_x) \right] \quad \text{(16)}$$

where

$$S_t = - \sum_{n=0}^{t} \frac{k_n - k_{n-1}}{E_n/E_p} + \sum_{n=0}^{t} (1 - k_n)\ (C_n - C_{n-1}) \quad \text{(17)}$$

In Eq. (16) the creep coefficient C_t modifies the constant sustained beam dead load moment M_x, and the summation coefficient S_t (a reduced creep coefficient to account for the loss of prestress and variable stress history) modifies the initial prestress moment $P_i\, e_x$.

The first term of Eq. (17) corresponds to the total "elastic" deflection resulting from the loss of prestress $k_n P_i$, for a given time $n = t$. If the variation in E_c with time is neglected so that $E_n = E_p$, this term is simply equal to the prestress loss coefficient k_t. Since E_p/E_n is less than 1, a constant value for this term might be approximated as $0.9\, k_t$. The second term in Eq. (17) corresponds to the creep camber resulting from the actual prestress force $(1 - k_n)\ P_i$ and the effective creep coefficient ΔC_t during a given increment of time. Assuming a linear variation of k_t with C_t (equivalent to saying that creep occurs under a constant prestress force equal to the mean of the initial and final prestress forces), this term can be approximated by $(1 - 0.5\, k_t)\ C_t$. Thus:

$$S_t = - 0.9\, k_t + (1 - 0.5\, k_t)\ C_t \quad \text{(18)}$$

For example, when $k_t = 0.20$ and $C_t = 2.0$, Eq. (18) yields $S_t = 1.6$. Eq. (18) agrees with the experimental data of Reference 11 within a few percent, where the k_t and C_t versus time curves were integrated numerically.

The general considerations that are necessary for the initial plus time-dependent analysis of composite beams were outlined at the beginning of the paper and will not be discussed in connection with deflection behavior.

Unshored construction

Prestressed precast beam—For composite beams the curvature expression corresponding to the total (initial plus time-dependent) deflection, exclusive of live load, is

$$\begin{aligned}\frac{d^2y}{dx^2} = &\Big[\overbrace{\frac{1}{E_p I_2}(1+C_q)\,(M_x'')}^{(1)} - \overbrace{\frac{1}{E_p I^2}(1+S_q)\,(P_i\, e_x)}^{(2)} + \overbrace{\frac{1}{E_q I_2}(M_x')}^{(3)} \Big] \\ &+ \Big[\overbrace{\frac{C'_{t'}}{E_p I_2}(M_x'')}^{(4)} - \overbrace{\frac{S'_{t'}}{E_p I_2}(P_i\, e_x)}^{(5)} + \overbrace{\frac{C_{t'}}{E_q I_2}(M_x')}^{(6)} \Big] \overbrace{\frac{E_2 I_2}{E_1 I_1 + E_2 I_2}}^{(7)} \\ &\overbrace{+\,0.7 \Big[F_{end} - (12/L^2)\,(F_{end} - F_{mid}) \Big(\frac{x^2}{2} - \frac{x^3}{3L}\Big)\Big] \frac{(y_{1B} + y_{2T})}{E_1 I_1 + E_2 I_2}}^{(8)} \ldots\ldots (19)\end{aligned}$$

[(1) + (2) + (3)] = total camber or deflection immediately following slab casting

[(4) + (5) + (6)](7) = total camber or deflection for the time t' (following slab casting), exclusive of differential shrinkage

(8) = total deflection resulting from differential shrinkage

Note that all E_p and E_q terms in Eq. (19) refer to the precast beam concrete and should not be reduced for any creep effect since the creep coefficients are included. Moduli E_1 and E_2 should be reduced for creep effects in terms (7) and (8).

where

(1) Initial plus time-dependent curvature resulting from the precast beam dead load up to the time of slab casting $t = q$.

(2) Initial plus time-dependent curvature resulting from the prestress force up to the time of slab casting $t = q$.

(3) Initial curvature resulting from the slab dead load.

(4) Time-dependent curvature resulting from the precast beam dead load after the slab was cast or time t' (Note $C_t = C_q + C'_{t'}$).

(5) Time-dependent curvature resulting from the prestress force after the slab was cast or t' (Note $S_t = S_q + S'_{t'}$).

(6) Time-dependent curvature resulting from slab dead load.

(7) Factor which includes the effect of the slab resistance to creep curvature of the precast beam.

(8) Curvature resulting from differential shrinkage.

For design purposes the value for S_t, as estimated by Eq. (18), is divided in a reasonable way between S_q and $S'_{t'}$ in Eq. (19). A similar division is required for C_t which, in both cases, depends on the relative time of slab casting. This simply divides the creep effect for the precast beam into that occurring before and after slab casting. The solution of Term (8) in Eq. (19) for midspan deflection is given by Eq. (14). When the differential shrinkage variation along the span is neglected, the bracket in Term (8) of Eq. (19) is replaced with the single force F, and the resulting solution for differential shrinkage deflection is given by Eq. (15).

Ordinary reinforced precast beam—The curvature expression here is the same as Eq. (19) [using the gross section properties for both] except that the prestress terms (2) and (5) are deleted and replaced by shrinkage warping terms for the precast beam with opposite signs. Shrinkage warping of reinforced concrete members is usually a problem only for slabs and shallow beams.

Procedures for predicting shrinkage warping are somewhat uncertain at present although at least two approaches have been advanced. One is the familiar application of a tensile force to the concrete section at the steel centroid and the use of a reduced concrete modulus to allow for creep. This is illustrated in the books of Peabody[12] and Large[13] among other places. A second approach which seems to have some merit is that of Miller.[14] He obtains warping curvature from the beam geometry, following the assumption that the top fiber of the section undergoes a shrinkage strain equal to the free shrinkage when the steel centroid is below the kern point. This procedure follows the idea that shrinkage warping is no function of f_c' and E_c or, for that matter, any reduced modulus E_{ct}.

Shored construction

Prestressed precast beam—For shored construction the terms in Eq. (19) are modified as follows:

1. Same
2. Same
3. $\dfrac{1}{E_q I_c} (M_x')$
4. Same
5. Same
6. $\dfrac{C_{t'}}{E_q I_c} (M_x')$

Here the creep coefficient $C_{t'}$ should be an average creep factor which includes the effect of the fresher slab concrete and correspondingly greater creep tendency.

7. Same
8. $\dfrac{0.7\, F\, (y_{1b} + y_{2T})}{E_1 I_1 + E_2 I_2}$

This assumes no differential creep at the interface and the force $0.7\,F$ to be concentrated at the ends of the beam.

Ordinary reinforced precast beam—The curvature expression here is the same as for prestressed shored construction except that the prestress terms (2) and (5) are deleted and replaced by shrinkage warping terms as discussed for unshored construction.

EXISTING EXPERIMENTAL DATA

Experimental data dealing with the time-dependent behavior of composite concrete beams has not been widely reported, although several studies have been published on the subject. Evans and Parker[7] calculated differential shrinkage stresses theoretically (using a theory similar to the separate section method presented herein and estimated differential shrinkage coefficients) and used these results to compare calculated and observed cracking loads for several types of composite beams. They concluded that differential shrinkage may affect cracking loads as much as 20 percent in normal design.

Birkeland[4] observed time-dependent deflections in two composite prestressed concrete beams. For the time following slab casting he compared the midspan deflection of one of the test beams with the calculated differential shrinkage deflection using the composite section method. As stated in his paper, he combined the effects of shrinkage and creep into the differential shrinkage solution. This required a very large assumed differential shrinkage coefficient of 450×10^{-6} in. per in. for 5 days between the casting dates of the two concretes. The portion of the precast beam creep resulting from the prestress force and both stem and slab dead load, for the time following slab casting, were thus combined into the differential shrinkage solution. Also note in Table 2 that the composite section method predicts differential shrinkage deflection in the neighborhood of twice that of the separate section method in some cases.

Branson and Ozell[9,10,11] reported experimental studies of differential shrinkage stresses and deflections using a number of test specimens. Differential shrinkage strains were determined by measuring the direct strains on the test composite beams and eliminating other strains through the use of companion noncomposite beams and shrinkage specimens. These studies were used to define the differential shrinkage coefficients for different relative humidities and time periods between the casting of the stem and slab concretes. The coefficients thus determined were used to compare calculated and measured deflections. The comparisons were favorable using a composite beam equation identical to Eq. (19) except for a slightly different solution for the differential shrinkage warping.

Mattock[15] investigated experimentally the influence of creep and differential shrinkage on the continuity behavior of two half scale prestressed concrete bridge girders. His data shows a leveling off of the differential shrinkage strain versus time curve at about 245×10^{-6} in. per in. The time period between the casting dates for the two concretes was 35 days (taken from graph) and the average relative humidity following the curing period was 50 percent. This value is close to the data of Reference 10 for the same relative humidity and relative casting times. Mattock pointed out that creep of the continuous composite beam induces positive restraint moments that can affect cracking but that shrinkage and creep will not affect the ultimate load capacity of the member. In this connection the restraint moments caused by differential shrinkage are of opposite sign to those caused by creep and would thus tend to reduce the positive restraint moment condition at the intermediate supports of continuous composite beams.

CONCLUSIONS

The following general statements can be made regarding the need for considering time-dependent effects in the design of composite concrete beams:

1. The effects of shrinkage and creep on design stresses in prestressed composite beams can be determined in the usual way; by considering the "final" prestress force (after losses) in combination with dead and live load and acknowledging that differential shrinkage may reduce somewhat the cracking load. However, the extensibility of concrete may practically eliminate any effect that differential shrinkage has on cracking loads in many cases. According to Troxell and Davis,[16] cracking loads up to 2.5 times the instantaneous cracking load have been observed for loads applied over 2 to 3 months. The increase in concrete strength with time (above the 28-day strength used in design) is also a factor which tends to permit differential shrinkage stresses to be neglected.

2. Differential shrinkage stresses can be estimated by either of the two methods summarized herein, although the separate section method [Eq. (8)] is recommended by this author.

3. In analyzing the time-dependent deflections of composite concrete beams, each of the terms in Eq. (19) should be considered. After reviewing the available experimental data relative to this equation, it appears that under certain conditions the smallest of these terms can contribute up to 15 to 20 percent of the numerical total. Hence, any simplification or generalization of this solution could easily lead to erroneous results.

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APPENDIX

NOTATION

1 = subscript denoting the cast-in-place slab of composite section

2 = subscript denoting the precast beam of composite section

A = area of section

B = subscript denoting bottom fiber of section

C = creep coefficient defined as ratio of creep strain to initial strain

$C'_{t'}$ = notation referring to the increase in the creep coefficient for the time t' (following slab casting), but where the sustained stress is applied at the beginning of time t, ($C_t = C_q + C'_{t'}$)

c = subscript denoting composite section with transformed slab

c = subscript denoting concrete as f_c' and E_c

D = differential shrinkage strain or coefficient

d = depth of beam section

E = modulus of elasticity

e = eccentricity of prestress steel from precast beam centroid

F = resultant differential shrinkage force at any section

f = change in differential shrinkage force per unit length (see Fig. 4 b)

I = moment of inertia (second moment of the area) of section

i = subscript denoting initial value
i = subscript denoting the slab and beam interface
k = prestress loss coefficient, $(P_i - P_t)/P_i$
L = beam span
M = bending moment
M' = slab dead load bending moment
M'' = precast beam dead load bending moment
P = prestress force
p = subscript denoting time of prestressing
Q = slab shrinkage force
q = subscript denoting time of slab casting
S = summation coefficient defined by Eq. (17). Actually a reduced creep coefficient to account for the loss of prestress and variable stress history
$S'_{t'}$ = notation referring to the increase in the summation coefficient for the time t' (following slab casting), but where the sustained stress is applied at the beginning of time t, $(S_t = S_q + S'_{t'})$
s = subscript denoting slab centroid
s = subscript denoting steel as E_s
T = subscript denoting top fiber of section
t = subscript denoting variable time measured from the time of application of prestress force and/or precast beam dead load (used for both ordinary reinforced and prestressed concrete members)
t' = subscript denoting variable time measured from time of slab casting
u = subscript denoting ultimate value
x = variable distance along beam
y = variable deflection of beam
y = distance from centroid to fiber under consideration
Δ = midspan deflection of simple beam
ε = unit strain
σ = unit stress

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Sinopsis — Résumés — Zusammenfassung

Efectos del Tiempo en Vigas de Concreto Compuestas

Se discuten los efectos de la contracción y deformaciónes lentas, así como la contracción diferencial en vigas de concreto compuestas. Dos métodos diferentes para determinar los esfuerzos diferenciales por contracción son brevemente resumidos y comparados. Se discuten los procedimientos para predecir la flecha total (inicial más la dependiente del tiempo) en vigas arriostradas y sin riostras, donde las vigas precoladas pueden ser reforzadas o presforzadas. También se incluye una discusión de los datas experimentales existentes referentes al comportamiento con el tiempo en vigas de concreto compuestas.

Effet du Temps sur Poutres en Béton Composées

On discute les effets de la retraite directe, la déformation par fluage, et la retraite différentielle des poutres en béton composées. Deux méthodes différentes pour déterminer les forces differentielles de retraite et de déflection sont bréve-ment resumées et comparées. De plus, on discute les méthodes de prédire les déflections totales (initiales et celles qui dépendent du temps) des poutres étayées et nonétayées dans lesquelles les poutres précoulées sont ou armées ou précontraintes. En outre, on ajoute une discussion des données experimentales existantes qui concernent le comportement des poutres en béton composées, sous l'influence du temps.

Zeitabhängige Wirkungen in Verbund-Betonträgern

Die Wirkungen von direkten Schwind- und Kriechdeformationen und differentialem Schwinden in Verbund-Betonträgern werden erörtert. Zwei verschiedene Methoden für die Bestimmung von differentialen Schwind- Spannungen und Durchbiegungen werden kurz zusammengefasst und verglichen. Methoden für die Berechnung der totalen Durchbiegung (elastisch plus sowie zeitbedingt) bei zwischenunterstützten und ungestützten Verbund-Betonträgern, bei denen die vorgefertigten Träger entweder schlaff bewehrt oder vorgespannt sind, werden erörtert. Ebenfalls angefügt ist eine Erörterung der vorhandenen experimentellen Ergebnisse hinsichtlich des zeitbedingten Verhaltens von Verbund-Betonträgern.