ACI JOURNAL TECHNICAL PAPER

Title no. 79-13

Unified Procedures for Predicting the Deflection and Centroidal Axis Location of Partially Cracked Nonprestressed and Prestressed Concrete Members

by Dan E. Branson and Heinrich Trost

A unified I-effective procedure for predicting the short-time deflection of cracked nonprestressed and prestressed concrete members is formulated. The procedure also lends itself to a simple method for estimating the time-dependent effects on deflection of creep and cracking under a limited number of loading cycles.

Experimental and computed results were compared for four full-size beams, two under two-cycle short-time loading and two under a few hours' time-dependent two- and four-cycle loading. The agreement was found to be satisfactory to quite good. One nonprestressed, one bonded prestressed, and two unbonded prestressed concrete beams were tested.

In addition, one of the beams was used to study the effect of the cracking moment prediction on the deflection prediction, and another beam was used to study the deflection of an unbonded prestressed one-way slab-type member with no bonded steel for crack control.

A unified procedure was formulated for predicting the location of the centroidal axis of cracked nonprestressed and prestressed concrete members. The four beams described above, plus one additional full-sized bonded prestressed beam under two-cycle short-time loading, were used to compare experimental and computed results. The agreement ranged from reasonably satisfactory to quite good.

Both procedures empirically account for the effect of concrete tensile stresses, including in small moment regions, from the top of each crack to the neutral axis and also between cracks, referred to as tension stiffening.

Keywords: beams (supports); cracking (fracturing); creep properties; cyclic loads; **deflection;** flexural strength; loads (forces); modulus of elasticity; moments of inertia; prestressed concrete; reinforced concrete; structural members; unbonded prestressing.

For computing deflections of partially cracked members, the effective moment of inertia (I_e) method[1,2,3] provides a transition value between well-defined limits in the uncracked (I_{ucr} or I_g) and fully cracked (I_{cr}) states. The method is similar to the approach[4,*] of determining the partially cracked deflection as an intermediate value between the uncracked (State I) and fully cracked (State II) deflections.

The I-effective method was adopted for the 1971 and 1977 ACI Building Code ["Building Code Requirements for Reinforced Concrete (ACI 318-71 and 318-77)"],[5] the 1971 and 1978 PCI Design Handbooks,[6] the 1973 and 1977 AASHTO Highway Bridge Specifications,[7] and the 1977 Canadian Building Code.[8] However, its application to nonprestressed and prestressed concrete members has been somewhat different in the past, with the live load I_e for prestressed concrete members determined from the prestress plus dead load deflection point and not from the zero deflection point, as in the nonprestressed case using the dead load plus live load I_e.[3]

In this paper, unified procedures for predicting the deflection and centroidal axis location of partially cracked nonprestressed and prestressed (with or without nonprestressed tension steel) concrete members are formulated and compared with experimental results. A study is also made of several difficulties in predicting such behavior in general. The details of this project are presented in a research report[9] which includes a demonstration of all the initial, repeated loading, and time-dependent calculations in table footnotes.

PROCEDURE FOR NONPRESTRESSED CONCRETE MEMBER DEFLECTION

The I_e procedure was originally[1] applied to nonprestressed concrete members, as shown in Fig. 1. The procedure follows from basic mechanics in a typical problem as follows[1,3]

By definition of f_r

$$M_{cr} = f_r I_g / c_2 \qquad (1)$$

*Trost, H., "The Calculation of Deflections of Reinforced Concrete Beams," Presented at the Adrian Pauw Symposium on Designing for Creep and Shrinkage in Concrete Structures, ACI Fall Convention, Houston, 1978 (to be published).

Received Aug. 3, 1981, and reviewed under Institute publication policies. Pertinent discussion will be published in the January-February 1983 ACI JOURNAL if received by Oct. 1, 1982. 002-8061/82/02 0119-12 $2.50.

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When $M_D \leqslant M_{cr}$

$$(I_e)D = I_g \tag{2}$$

When $M_D > M_{cr}$, by definition

$$(I_e)_D = \left(\frac{M_{cr}}{M_D}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_D}\right)^3\right] I_{cr} \leqslant I_g \tag{3}$$

$$\Delta_D' = K_D M_D L^2 / E_c (I_e)_D \tag{4}$$

At the time the dead load is applied,

$$\Delta_D = \Delta_D' \, (E_c / E_{ci}) \tag{5}$$

By definition

$$(I_e)_{D+L} = \left(\frac{M_{cr}}{M_{D+L}}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{D+L}}\right)^3\right] I_{cr} \leqslant I_g \tag{6}$$

$$\Delta_{D+L} = (K_D M_D + K_L M_L) \, L^2 / E_c \, (I_e)_{D+L}$$
$$= K_{D+L} M_{D+L} L^2 / E_c (I_e)_{D+L} \tag{7}$$

where

$$K_{D+L} = (K_D M_D + K_L M_L) / M_{D+L} \tag{8}$$

Finally, from Fig. 1

$$\Delta_L = \Delta_{D+L} - \Delta_D' \tag{9}$$

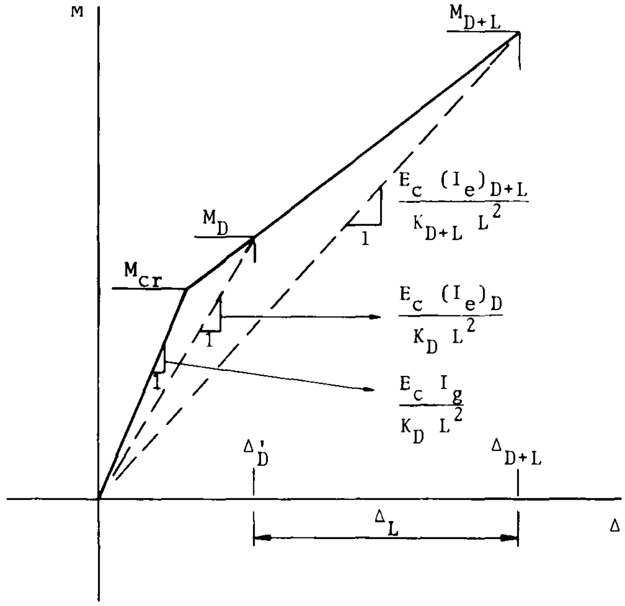

Fig. 1 — Idealized moment versus deflection for a nonprestressed concrete member loaded into the cracking range in which typical dead and live load effects are shown

UNIFIED PROCEDURE FOR PARTIALLY CRACKED NONPRESTRESSED AND PRESTRESSED CONCRETE MEMBER DEFLECTION

$(M_D + M_L - P_e e_p > M_{cr}')$

The stress distribution diagrams for uncracked, partially cracked, and fully cracked sections, including the location of the centroidal and neutral axes, are shown in Fig. 2 for a prestressed concrete member. In the prestressed case, which includes the effect of the normal force P_e in the calculation of the cracking moment, the fully cracked section is still taken in the limit to be the lower bound state, as shown in Fig. 2; that is, I_{cr} and α_{cr} are the lower limits of I_e and α_e, respectively, for both nonprestressed and prestressed cases. The effect of any nonprestressed tension steel in prestressed concrete members is included in the calculation of I_{cr}.

For the prestressed or general case of a member loaded into the cracking range, the strains and curva-

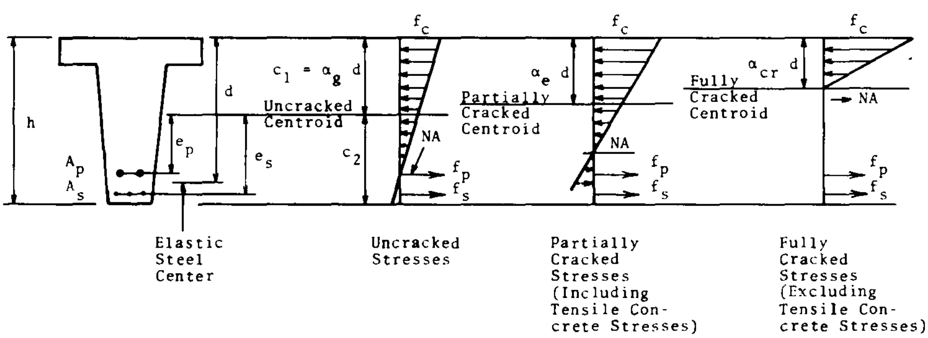

Fig. 2 — Basic stress distribution diagrams for uncracked, partially cracked, and fully cracked sections, with the location of the centroidal and neutral axes shown

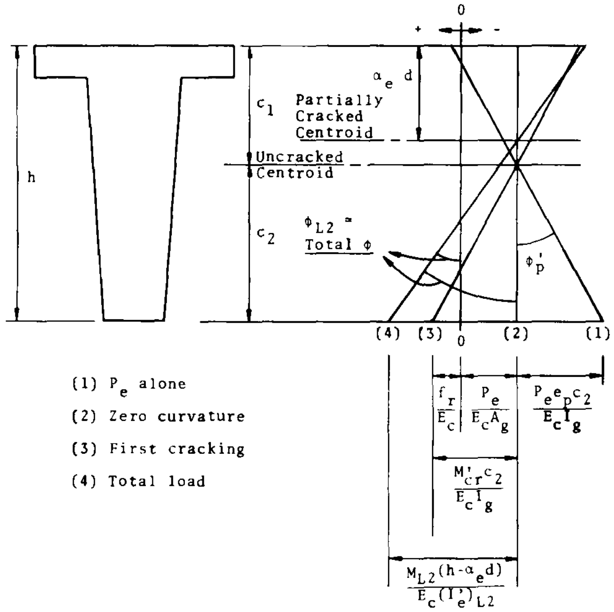

Fig. 3 — Strains and curvatures for the general case of a member loaded into the cracking range

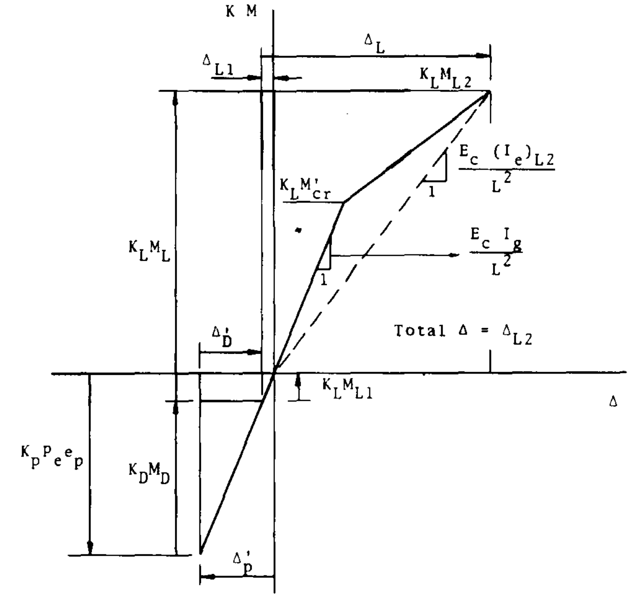

Fig. 4 — Idealized deflection coefficient, moment product versus deflection for a prestressed concrete member loaded into the cracking range, in which typical prestress, dead load, and live load effects are shown

tures are shown in Fig. 3, and the idealized moment-deflection diagram is shown in Fig. 4. This figure is presented in terms of KM versus Δ so the single line with slope $= E_c\, I_g/L^2$ can be applied to the deflection under different load distributions (different K's), such as those due to prestress, dead load, and live load in a typical problem.

From Distribution Line 1 in Fig. 3, the deflection corresponding to the prestress curvature ϕ_p' is given by Eq. (10) and shown in Fig. 4

$$\Delta_p' = K_p P_e e_p L^2/E_c I_g \tag{10}$$

At the time the initial prestress force P_i is applied

$$\Delta_p = \Delta_p' \;(E_c/E_{ci})\;(P_i/P_e) \tag{11}$$

The dead load deflection is given by Eq. (12) at the time under investigation (shown in Fig. 4), and initially by Eq. (5) (given above)

$$\Delta_D' = K_D\, M_D\, L^2/E_c\, I_g \tag{12}$$

Analogous to Distribution Line 2 in Fig. 3, corresponding to zero curvature, the condition of zero deflection is defined by Eq. (13), (14), and (15).
From Fig. 4

$$K_L\, M_{L1} = K_p\, P_e\, e_p - K_D\, M_D \tag{13}$$

or

$$M_{L1} = (K_p/K_L)\; P_e e_p - (K_D/K_L)\; M_D \tag{14}$$

In this uncracked region

$$\Delta_{L1} = K_L\, M_{L1}\, L^2/E_c\, I_g \tag{15}$$

where M_{L1} is the part of the live load moment necessary to produce the zero deflection.

From Distribution Line 3 in Fig. 3, corresponding to first cracking, Eq. (16) is obtained

$$\frac{M_{cr}' c_2}{E_c I_g} = \frac{f_r}{E_c} + \frac{P_e}{E_c A_g} \tag{16}$$

Solving

$$M_{cr}' = \frac{f_r I_g}{c_2} + \frac{P_e I_g}{A_g c_2} \tag{17}$$

The cracking moment M_{cr}' is also shown in Fig. 4 and refers to the moment above zero, or the net positive moment, required to crack the section.

Analogous to Distribution Line 4 in Fig. 3, corresponding to the total load curvature, the total load moment and deflection shown in Fig. 4 are obtained in Eq. (18) and (19).
From $K_L\, M_L = K_L\, M_{L1} + K_L\, M_{L2}$

$$M_{L2} = M_L - M_{L1} \tag{18}$$

$$\Delta_{L2} = K_L\, M_{L2}\, L^2/E_c\,(I_e)_{L2} \tag{19}$$

where M_{L2} is the total live load moment at the section M_L minus M_{L1} in Eq. (14). The effective moment of inertia in Eq. (19) is shown in Fig. 4 and computed by Eq. (20). By definition,[1,2,3]

$$(I_e)_{L2} = \left(\frac{M'_{cr}}{M_{L2}}\right) I_g + \left[1 - \left(\frac{M'_{cr}}{M_{L2}}\right)^3\right] I_{cr} \leq I_g \quad (20)$$

where M'_{cr} is computed in Eq. (17) and M_{L2} in Eq. (18). From Fig. 4

$$\text{Total } \Delta = -\Delta'_p + \Delta'_D + \Delta_{L1} + \Delta_{L2} = \Delta_{L2} \quad (21)$$

and

$$\Delta_L = \Delta_{L1} + \Delta_{L2} \quad (22)$$

Solution for nonprestressed case

The solution for nonprestressed concrete member live load deflection, for example, is obtained automatically in the above equations for prestressed concrete members by setting $P_e = 0$, $P_i/P_e = 1$, and $M_{L2} = M_{D+L}$ in the I_e equation, as the following results show. In Eq. (14) with $P_e = 0$

$$M_{L1} = -(K_D/K_L)\, M_D \quad (23)$$

In Eq. (15) and (23)

$$\Delta_{L1} = -K_D M_D L^2/E_c I_g = -\Delta'_D \text{ in Eq. (12)} \quad (24)$$

In Eq. (17) with $P_e = 0$

$$M'_{cr} = f_r I_g/c_2 = M_{cr} \text{ in Eq. (1)} \quad (25)$$

In Eq. (18) and (23)

$$M_{L2} = M_L + (K_D/K_L)\, M_D \quad (26)$$

In Eq. (6) and (20)

$$(I_e)_{L2} = (I_e)_{D+L} \quad (27)$$

since M_{L2} is set equal to M_{D+L} in the I_e equation, and $M'_{cr} = M_{cr}$ in Eq. (25). For many problems, $K_D = K_L$ (same type of load distribution), in which case the definition of M_{L2} in the I_e equation and in Eq. (26) is the same. This slight discrepancy (K values do not differ very much) between nonprestressed and prestressed cases, in general, is due to the fact that the zero point for applying I_e in the prestressed case in Fig. 4 is defined as the zero deflection point (which is not always the same as the zero curvature point) as in the nonprestressed case.

In Eq. (19), (26), and (27)

$$\Delta_{L2} = (K_D M_D + K_L M_L)\, L^2/E_c\, (I_e)_{D+L}$$
$$= \Delta_{D+L} \text{ in Eq. (7)} \quad (28)$$

Finally, in Eq. (22), (24), and (28)

$$\Delta_L = \Delta_{L1} + \Delta_{L2} = -\Delta'_D + \Delta_{D+L} \quad (29)$$

which is the same as Eq. (9) for nonprestressed concrete member live load deflection.

In all of the previous equations for computing deflections, the bending moments are usually the maximum moments for simple spans and the maximum moments between inflection points for continuous spans, with the deflection coefficients K defined accordingly.

SIMPLE METHOD FOR ESTIMATING THE TIME-DEPENDENT EFFECT ON DEFLECTIONS OF CREEP AND CRACKING UNDER A LIMITED NUMBER OF LOADING CYCLES

With the above method outlined for computing the total or net positive deflection (Δ_{L2} in Fig. 4), the following simple method for estimating the time-dependent effect of creep and cracking under a limited number of loading cycles on deflections is apparent.

For single or first loading cycle

$$\text{Time-dependent } \Delta_{L2} = k_r C_t \text{ (initial } \Delta_{L2}) \quad (30)$$

where k_r is a reduction factor for the effect of compression steel and downward movement of the neutral axis with time. For short-time creep, as used in this study, $k_r = 1.0/(1 + 50\,\varrho')$, and for long-time creep (from Reference 3), $k_r = 0.85/(1 + 50\,\varrho')$. C_t is the creep coefficient defined as the ratio of creep strain to initial strain. This approach was originally developed in Reference 10 (and later clarified in Reference 3) for nonprestressed concrete members. It has also been recommended by ACI Committee 435.[11] A more accurate method for evaluating k_r is given in Reference 4 and the footnote* (below), in which the effect of tension reinforcement on creep deformation is also included directly.

For a limited number of loading cycles

$$\text{Time-dependent } \Delta_{L2} = k_r C_t \text{ (initial } \Delta_{L2})$$
$$+ F_{rep} \text{ (initial } \Delta_{L2}) \quad (31)$$

where F_{rep} is a factor that takes into account the increased deflection due to the effect of cracking under repeated loading cycles, given as the ratio of this increased deflection to the initial deflection Δ_{L2} at the level of unloading (maximum load level in the cycle). Typical experimental results range from about 10 to 20 percent ($F_{rep} = 0.10$ to 0.20) for two to four loading cycles. Somewhat higher percentages or ratios were found for more cycles of loading.

Experimental and computed results using Eq. (30) and (31) are shown in Fig. 8 and 12 and in more detail in Reference 9.

*Trost, H., "The Calculation of Deflections of Reinforced Concrete Beams," Presented at the Adrian Pauw Symposium on Designing for Creep and Shrinkage in Concrete Structures, ACI Fall Convention, Houston, 1978 (to be published).

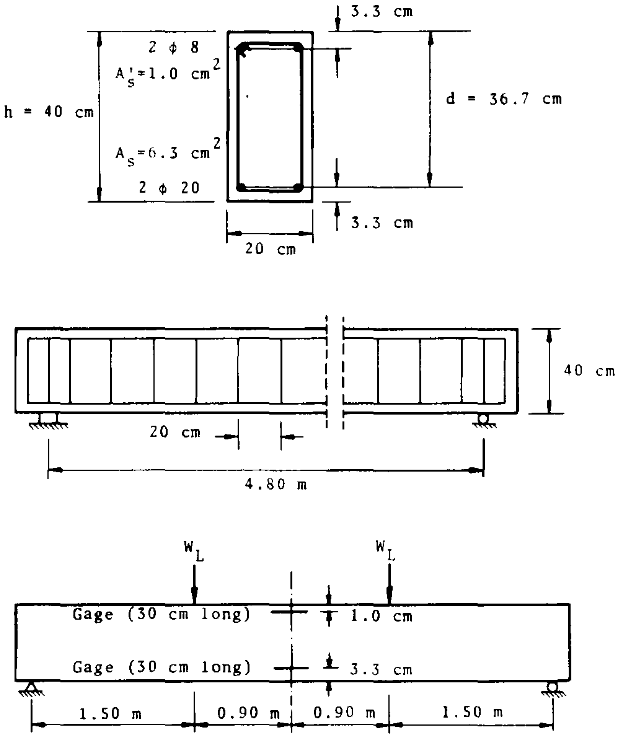

Fig. 5 — Beam 1 (nonprestressed, straight reinforcing steel)

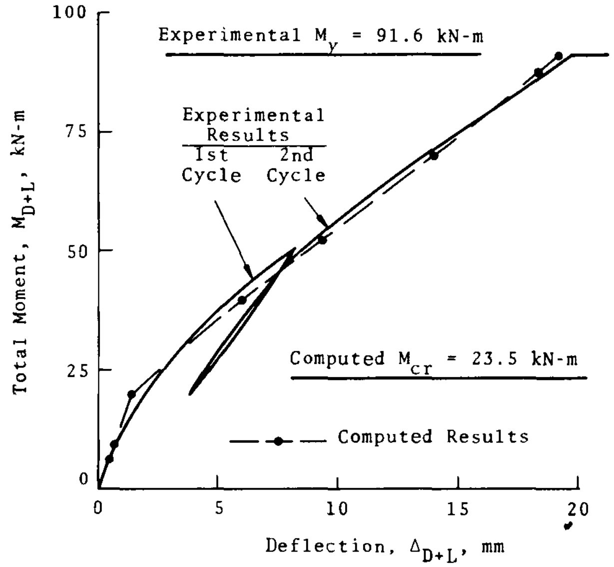

Fig. 6 — Total moment versus deflection results for Beam 1 (nonprestressed, under two-cycle, short-time loading)

EXPERIMENTAL RESULTS FOR PREDICTING DEFLECTIONS

Summary of previous results

The I_e procedure has been previously checked for at least 309 experimental beams and slabs by at least 20 different authors, using nonprestressed and both pretensioned and post-tensioned concrete members, as follows:

1. In 1963, the procedure was presented by Branson[1] based on 58 (34 separate evaluations after averaging) experimental nonprestressed concrete beams (simple-span and two-span continuous rectangular beams and simple-span T-beams). The agreement between computed and measured results was as follows: Using I_e in a 4th power equation for curvatures and in the Newmark numerical integration procedure, all results were within 18 percent except two, with a maximum error of 25 percent. Using I_e in the 3rd power equation for computing deflections directly, all results were within 19 percent except two, with a maximum error of 26 percent.

2. In 1968, Beeby[12] found that for 133 experimental nonprestressed concrete beams the mean error for the procedure was among the best for a number of different prediction methods. Also in 1973, from these same test results, Beeby[13] compared experimental and computed results for four commonly accepted methods and found the I_e procedure to be the best.

3. In 1970, Shaikh and Branson[2] found that for 12 rectangular pretensioned concrete beams the procedure predicted deflections up to 80 percent of the ultimate load within 19 percent of the measured values in all cases.

4. In 1970 and 1971, Nawy and Potyondy[14,15] found that for 22 I- and T- pretensioned concrete beams the procedure gives excellent results for evaluating the effective moment of inertia of the section for the short-term loading reported.

5. In 1972, Bennett and Veerasubramanian[16] found for 34 rectangular, I-, T-, and composite T- pretensioned concrete beams the procedure was typically very accurate for the I-section beam, but conservative for the monolithic and composite T-sections.

6. In 1974, Heiman[17] found that for panels in four (two using I_e) reinforced concrete buildings (beam and slab, flat slab, beam and slab, and flat plate types) the short-time deflections together with calculated time-dependent deflections differed with experimental results by 17 percent for the first building in which the short-time deflections were based on fully cracked conditions, by 14 percent for the second building in which the short-time deflections were based on partly cracked conditions not shown in the paper, and by a maximum of 13 percent for the other two buildings in which the short-time deflections were based on partly cracked conditions using the I_e procedure.

7. In 1974, Parameswaran, Annamalai, and Ramaswany[18] found that for 10 pretensioned beams the procedure computed results were in very close agreement with the experimental results.

8. In 1976, Kripanarayanan and Branson[19] found that for six experimental two-way slab systems the procedure, modified for two-way systems in accordance with the ACI Building Code Equivalent Frame Method of design, was satisfactory.

9. In 1977, Nawy and Huang[20] found that for 19 I- and T- pretensioned concrete beams the generally accepted I_e procedure provided results such that the degree of scatter is within a 20 percent band, which can be considered fully adequate.

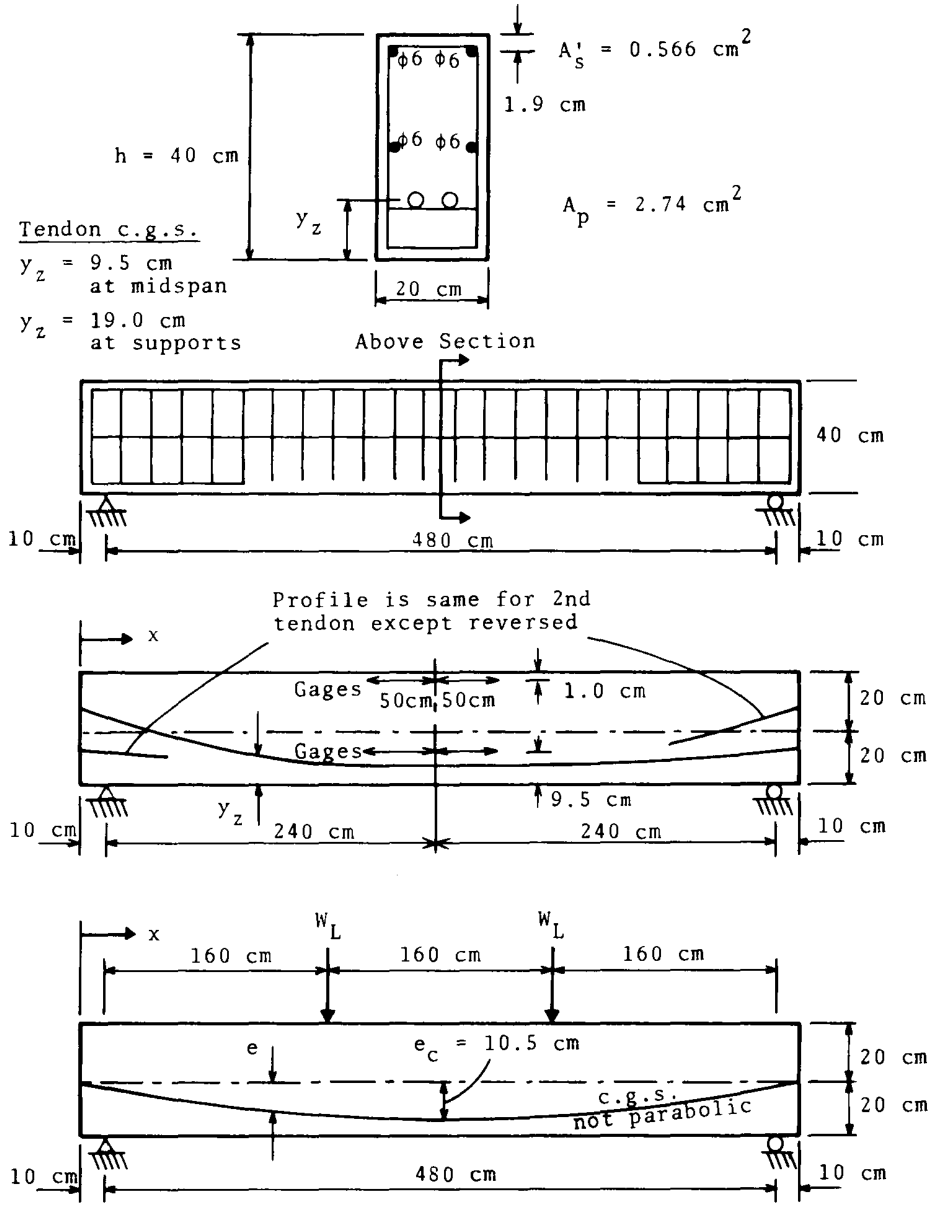

Fig. 7 — Beam 2 [unbonded prestressed, draped tendons (not parabolic)]

10. In 1978, Burns, Charney, and Vines[21] found that for two unbonded post-tensioned, half-size, three-span continuous, one-way slabs the procedure gave realistic though slightly conservative results as compared to the deflections measured in the tests.

11. In 1980, Sakai and Kakuta[22] found that for 10 nonprestressed rectangular concrete beams subjected to combined bending and axial loading the validity of the method could be confirmed experimentally.

12. In 1981, Moosecker and Grasser[23] found that for one nonprestressed concrete beam the procedure, together with others, agreed reasonably well with the test results, but somewhat overestimated the stiffness for first loading.

Experimental results of this study

In view of the experimental comparisons noted above, it is doubtful that the unified procedure herein should require extensive checks in addition. Four full-size beams with electronically recorded data from the Technical University Aachen Laboratory are used to study the principal aspects of the problem. The experimental beams, loading and other conditions, and midspan deflection results are described in the figure titles and shown in Fig. 5-12. In the case of Beam 2 (four-cycle, 7.5 hr loading) and Beam 4 (two-cycle, 2.9 hr loading), the following expression for C_t under short-time creep loading in Eq. (30) and (31) is used.[3] These and other calculations are shown in detail Reference 9.

$$C_t = \frac{t^{0.40}}{2.7 + 0.24t^{0.40}} \tag{32}$$

($C_{24\,hr} = 0.40$) (½ for first loading cycle and ¼ for two and four-cycle loading effects)
(0.81 for size correction factor)

where t is time in hr after loading.

The following observations pertain to the use of the unified I_e procedure for predicting short-time deflection of partially cracked nonprestressed and prestressed concrete members. They also pertain to a simple method for estimating the time-dependent deflection due to creep and cracking under a limited number of loading cycles:

1. Agreement between experimental and computed results tends to be satisfactory to quite good up to relatively high moment levels — well above the usual elastic range and working-load range for which deflections are normally computed. The principal reason for this seems to be that the calculated I_e approaches the fundamental value of I_{cr} in this range. The maximum differences between experimental and computed deflections are as follows:

Beam 1 (Fig. 6) — 10 percent from the cracking moment to the yield moment.

Beam 2 (Fig. 8) — 10 percent up to 78 percent of the ultimate moment.

Beam 3 (Fig. 10) — Up to 91 percent of the ultimate moment, 13 percent using the observed cracking moment and 37 percent using the computed cracking moment, as discussed in Observation 5.

Beam 4 (Fig. 12) — 13 percent up to 85 percent of the ultimate moment.

2. As can be seen in Fig. 6 and 10 under short-time loading, the effect of a limited number of cycles versus single-cycle loading on deflections may be of minor consequence.

3. Typical moment-deflection curves (Fig. 6, 8, 10, and 12) tend to "bend over" at higher moment levels because of the stress-strain behavior of the concrete and the effect of creep even for relatively short loading periods. Such behavior is caused by these material properties and is not predicted by the use of I_e which is primarily a section property. In fact, as can be seen in Fig. 6 and 10, and elsewhere,[3] the I_e prediction is characteristically an approximately bilinear curve which "bends back" gradually at higher moment levels to asymptotically approach the fully cracked (I_{cr}) moment-deflection line. Such a relationship is thought to be the ideally correct one for the case of a linear stress-strain material and a creep-free beam, which is the basic objective of the I_e procedure.

4. The moment-deflection curves in Fig. 8 and 12 demonstrate the complexity of the typical deflection problem in which the various effects due to elastic deformation, cracking, repeated loading cycles, and creep are included. When this complexity is coupled with the inability to know in advance such primary effects as the relative creep factor to be used for the structural concrete and the actual loading history (including construction loads) to which the structure is to be subjected, the difficulty in predicting deflections of actual structures can be appreciated.

Because of the variability in the deflection problem, it is thought to be most appropriate to use relatively simple prediction methods to guard against placing undue reliance on computed results.

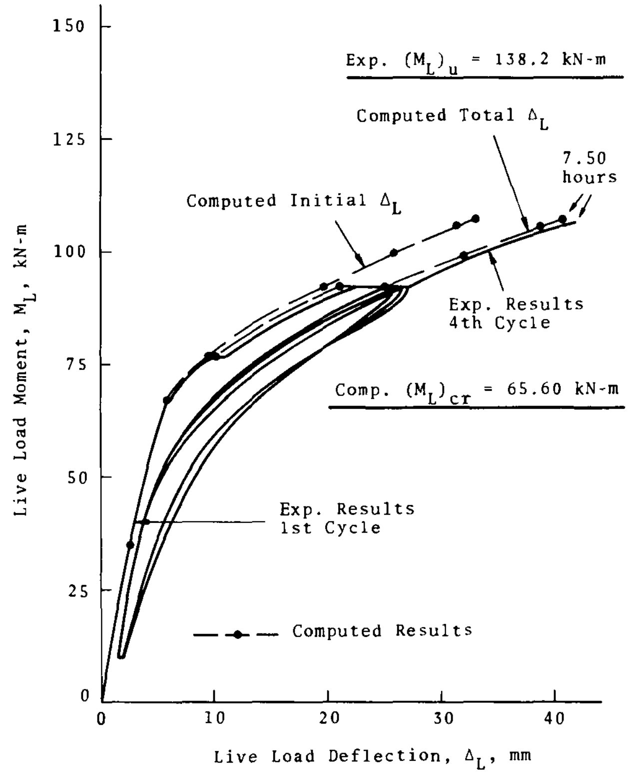

Fig. 8 — Live load moment versus deflection results for Beam 2 [unbonded prestressed, under four-cycle, 7.5 hr (creep) loading period]. Results show the complexity of the problem and a relatively simple solution

5. One problem that appears to be common to all deflection prediction methods is that of predicting the cracking moment. This is due primarily to the difficulty in accurately predicting the modulus of rupture f_r (standards tend to require the use of relatively low, conservative values) and, in the case of prestressed concrete members, the failure to include the effect of the increasing prestress force (above P_e) under applied loading.

This often results in the predicted deflection being less accurate near the cracking point than further below or above the point of first cracking. However, the effect of repeated load cycles tends to reduce the importance of this problem, as can be seen in Fig. 6, 8, 10, and 12.

Due to both the f_r and P_e effects described above, there is a tendency to underpredict the cracking moment. Also, after cracking, the rather sharp increase in P_e is normally not included in the calculations. These effects all contribute to computed downward deflections on the high (conservative) side, as has been observed by others as well, in Reference 12 for nonprestressed concrete members and in References 16 and 21 for prestressed concrete members. Such a case is also shown in Fig. 10, in which an optical instrument was used to carefully determine the moment at first cracking. For this beam it is seen that the I_e method

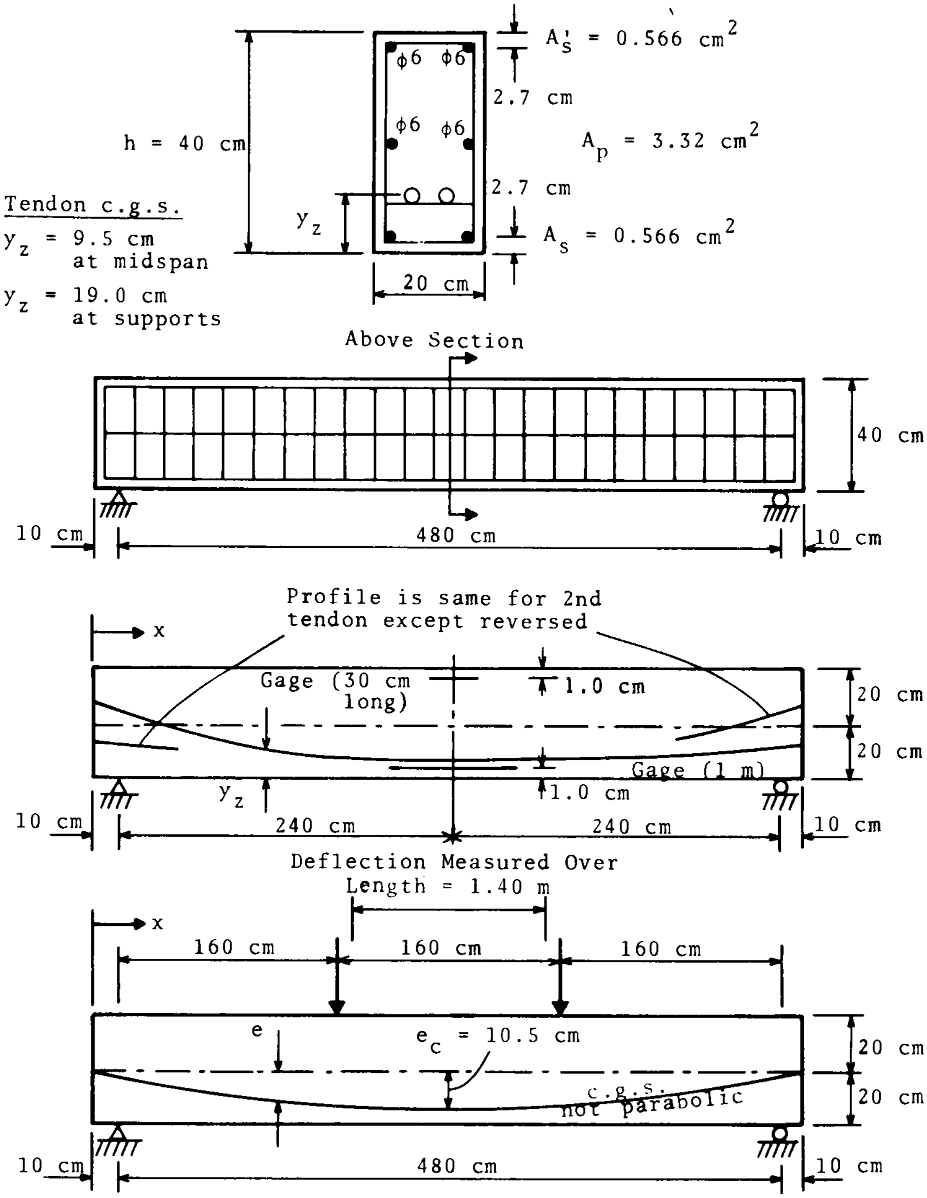

Fig. 9 — Beam 5 [bonded prestressed, draped tendons (not parabolic)]

still provides very good results when an appropriate value for the cracking moment is used. See Observation 1 (above), Beam 3 for a comparison of the experimental and computed deflections using both observed and computed cracking moments.

6. Beam 4 is considered to represent an extreme case — an unbonded prestressed concrete one-way slab-type member with no bonded steel for crack control. Typically such a beam produces relatively fewer cracks than the normal bonded crack distribution, but the cracks tend to be larger, resulting in greater tensile strains and deflections than in the case involving bonded steel. Because of such behavior, it was found that the usual I_e equation had to be modified by using the power of 4 instead of 3 in Eq. (20). The need for this was clearly evident in studying the slope of the moment-deflection curve in Fig. 12. This modification simply provides for a more rapid transition from I_g to I_{cr} as a result of the increased crack widths, tensile strains, and deflections.

UNIFIED PROCEDURE FOR LOCATING THE CENTROIDAL AXIS OF PARTIALLY CRACKED NONPRESTRESSED AND PRESTRESSED CONCRETE MEMBERS

The location of the centroidal axis of a partially cracked section is not required when computing short-time deflections, since the I_e equation is solved directly in terms of I_g and I_{cr}. However, when a strain analysis and not just curvatures or deflections is desired, such as when analyzing elastic and creep strains, the location of the centroidal and neutral axes becomes necessary. These axes can be located theoretically based on the fundamental equilibrium of forces and strain compatibility. However, such calculations are somewhat tedious since the moment equation for the internal forces that is used in locating the neutral axis results in a cubic equation that is normally solved by successive trials, as shown in Reference 24.

Eq. (33) empirically provides the best fit and applies generally to both partially cracked nonprestressed and prestressed concrete members.

$$\alpha_e = \left(\frac{M'_{cr}}{M_{L2}}\right)^{2.5} \alpha_g + \left[1 - \left(\frac{M'_{cr}}{M_{L2}}\right)^{2.5}\right] \alpha_{cr} \leqslant \alpha_g \quad (33)$$

where α_e, α_g, and α_{cr} are defined in Fig. 2; M'_{cr} is given by Eq. (17); M_{L2} is given by Eq. (18); and, based on zero curvature instead of deflection in Fig. 4 and Reference 9

$$M_{L1} = P_e e_p - M_D \quad (34)$$

For nonprestressed members in Eq. (33), $M'_{cr} = M_{cr}$, and M_{L2} becomes the applied moment, such as M_D for dead load and M_{D+L} for dead plus live load.

The power of 2.5 in Eq. (33) is consistent with a paper by Sakai and Kakuta[22] which found the appropriate power for nonprestressed concrete beams subjected to combined bending and axial loading to be between 2 and 3, with little difference when using either of these values. Their results were obtained indirectly by calculation based on the relationships between the location of the center of gravity and I_e. How-

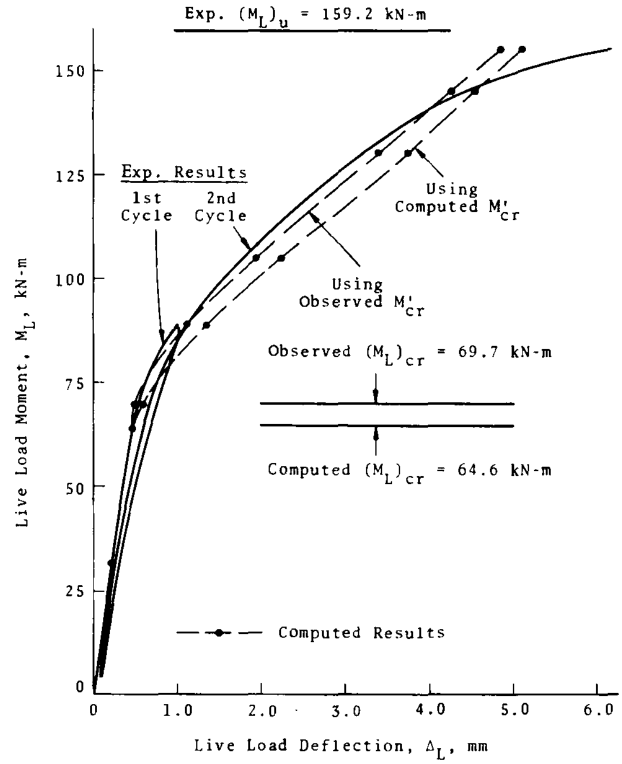

Fig. 10 — Live load moment versus deflection results for Beam 3 (bonded prestressed, under two-cycle, short-time loading). Results show the typical effect of using both predicted and observed cracking moments on the deflection prediction

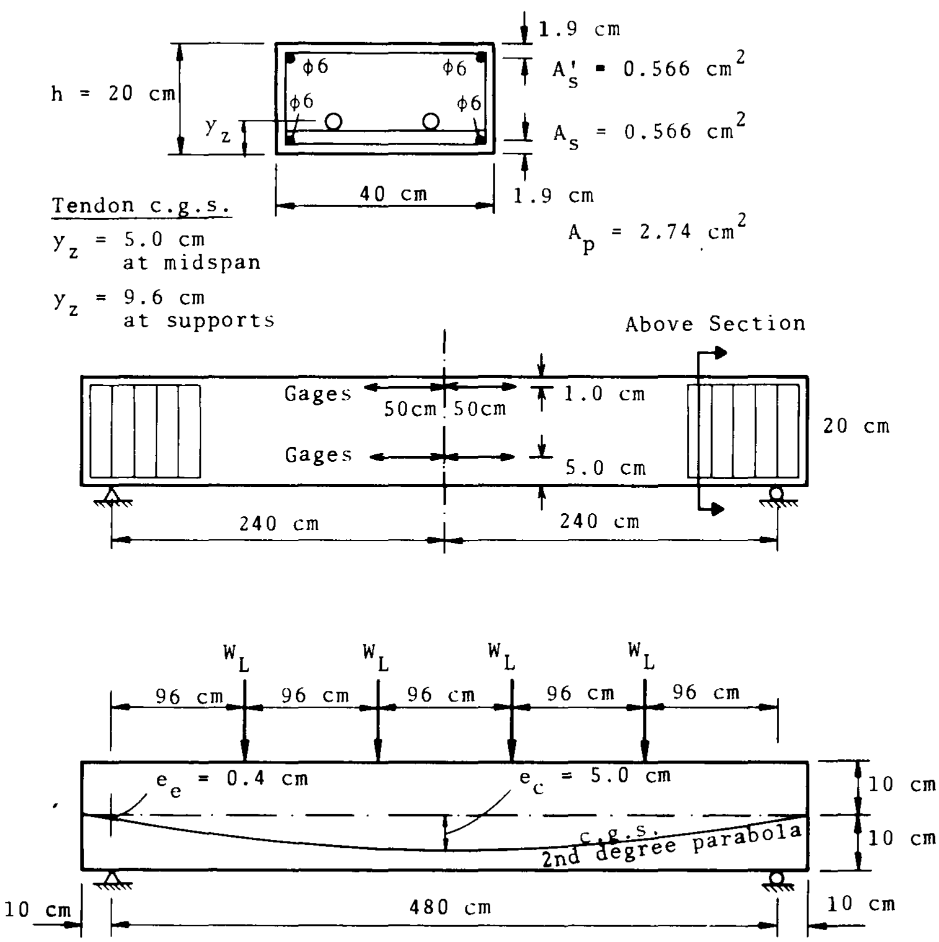

Fig. 11 — Beam 4 [unbonded prestressed one-way slab-type member (parabolically draped tendons)]

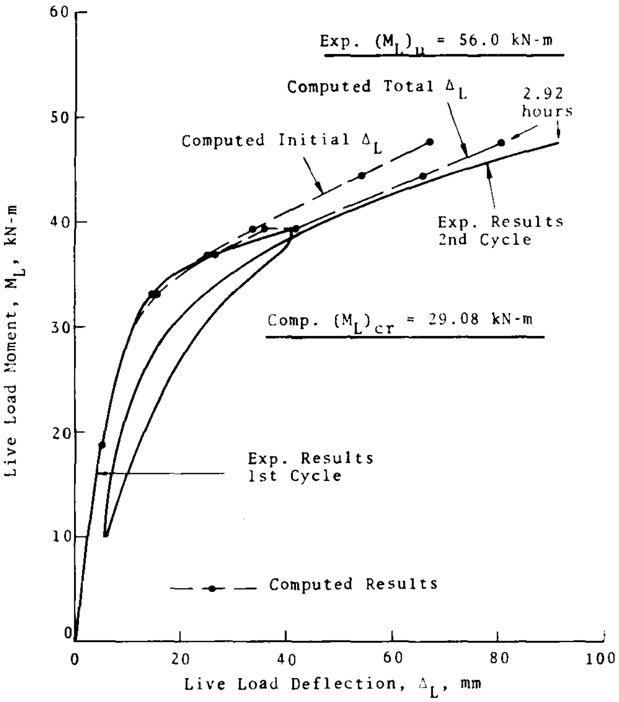

Fig. 12 — Live load moment versus deflection results for Beam 4 [unbonded prestressed one-way slab-type member, under two-cycle, 2.9 hr (creep) loading period]. Results show the effect on the deflection prediction of using no bonded steel for crack control for this type of member

ever, their results for curvatures were confirmed by experiments.

EXPERIMENTAL RESULTS FOR LOCATING THE CENTROIDAL AXIS

Centroidal and neutral axes are coincident in the case of nonprestressed concrete members in general, and for uncracked prestressed members under any transverse load. In the case of partially cracked prestressed concrete members, for example, these two axes are also coincident under the last (small) transverse load increment. In this way, values of α_e for locating the centroidal axis were determined empirically from strain measurements for both partially cracked nonprestressed and prestressed members, as shown in Fig. 13. The test beams and strain gage lengths and locations (an average of gages on both sides was used) are shown in Fig. 5, 7, 9, 11, and 14; and the experimental results are shown in Fig. 15.

The following observations pertain to the prediction of α_e:

1. Agreement between experimental and computed results is considered to be reasonably satisfactory to quite good, with the maximum differences shown in Fig. 15 as follows:

Beam 1 — 10 percent in both loading cycles up to just below the yield moment.

Beam 2 — 6 percent in the first loading cycle and 17 percent in the fourth loading cycle.

Beam 3 — 17 percent in both loading cycles.

Beam 4 — 35 percent in the first loading cycle and 7 percent in the second loading cycle.

Beam 5 — 16 percent in the first loading cycle and 26 percent in the second loading cycle.

2. The characteristic shape of the α_e curve can be seen in Fig. 15. The parameter proceeds as a vertical straight line in the uncracked range, diminishes rather sharply just above the cracking moment and approaches α_{cr} asymptotically at higher moment levels.

3. Since α_e approaches the well-defined α_{cr} at higher moment levels, Eq. (33) appears to be applicable up to and above a moment level that is normally considered to be within the working load range. However, some caution is in order when applying the procedure to higher moment levels, since α_{cr} is still an elastic concept based on the transformed section properties.

4. For all five beams, the results in Fig. 15 show that the prediction method is applicable to a few cycles of loading as well as to single loading cases.

5. Since incremental strains under transverse loading, and not total strains, were used in determining the experimental location of the neutral axes, the effect of creep did not seem to adversely affect the prediction of α_e for Beams 2 and 4. That is, the effect of tensile and compressive creep tended to be offsetting in determining the neutral axis location.

6. For the same reasons given in Observation 6 for deflection, the usual α_e equation for Beam 4 had to be modified by using the power of 5 instead of 2.5 in Eq.

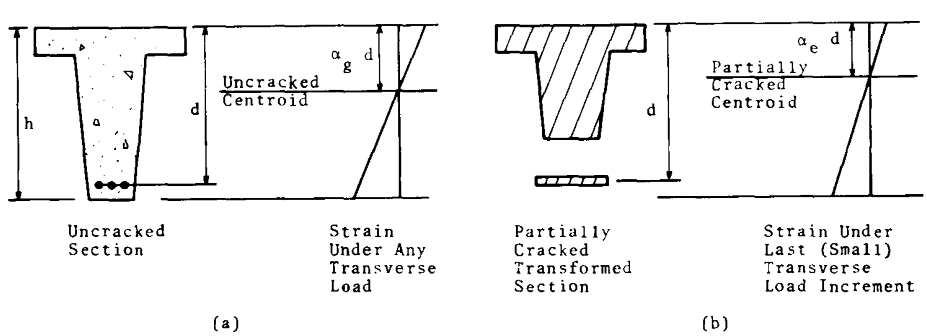

Fig. 13 — Location of centroidal axes (same as neutral axes under the transverse loading conditions shown) for both uncracked and partially cracked sections of either nonprestressed or prestressed concrete members

(33). This modification simply provides for a more rapid transition from α_g to α_{cr}, as a result of the increased crack widths and increased tensile strains.

ACKNOWLEDGMENTS

This paper summarizes part of a research project conducted as a result of a U.S. Senior Scientist Award by the Alexander von Humboldt Foundation of the Federal Republic of Germany. Support was also received from the University of Iowa, Iowa City, Iowa, and the Technical University, Aachen, West Germany. This assistance is greatly appreciated. Thanks is also extended to Dr.-Ing. H. Cordes, Dr.-Ing. J. Frey, and Dipl.-Ing. B. Weller for their assistance.

CONVERSION FACTORS

1 m = 3.28 ft
1 cm = 0.394 in.
1 mm = 0.0394 in.
1 cm² = 0.155 in.²
1 kN · m = 0.737 ft-kip

NOTATION

C_t = creep coefficient defined as the ratio of creep strain to initial strain

E_c = modulus of elasticity of concrete at the time the superimposed loading, such as live load, is applied; normally taken to be at age 28 days

E_{ci} = modulus of elasticity of concrete at the time of initial loading; normally at the time of prestressing for prestressed concrete members and the time the self weight is applied for nonprestressed members

e = eccentricity of steel

F_{rep} = factor in Eq. (31) that takes into account the increased deflection due to the effect of cracking under repeated loading cycles

f_r = modulus of rupture of concrete

I = moment of inertia (second moment of the area)

K = deflection coefficient

M_{cr} = cracking moment for a nonprestressed concrete member

M'_{cr} = cracking moment for a prestressed or general member (Eq. 18)

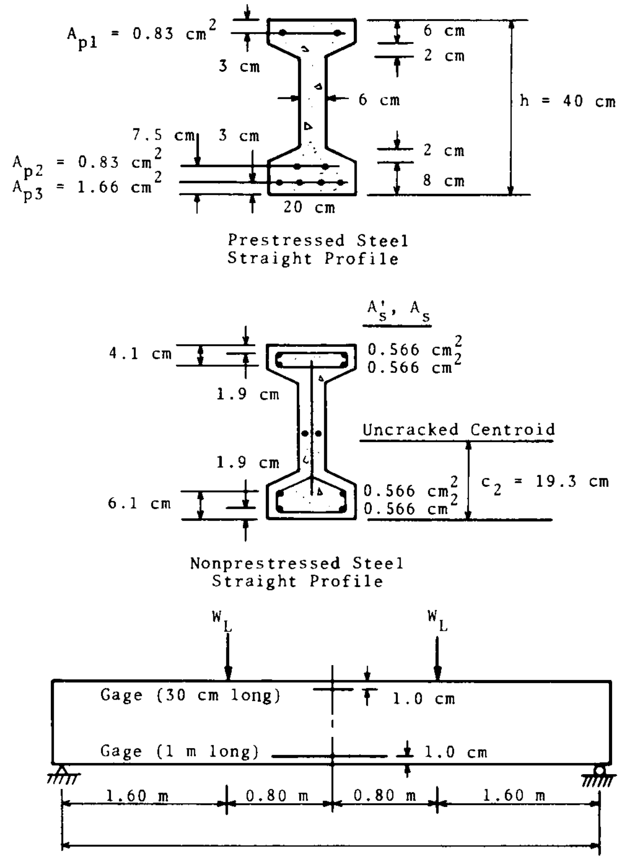

Fig. 14 — Beam 5 (bonded prestressed, straight pretensioned and nonprestressed steel, under two-cycle, short-time loading)

P_e = effective prestress force (after losses)

P_i = initial prestress force, or prestress force at transfer

α = ratio of distance from centroid to top surface — to the effective depth d

Δ = deflection

Δ'_D = fictitious dead load deflection using E_c rather than E_{ci}

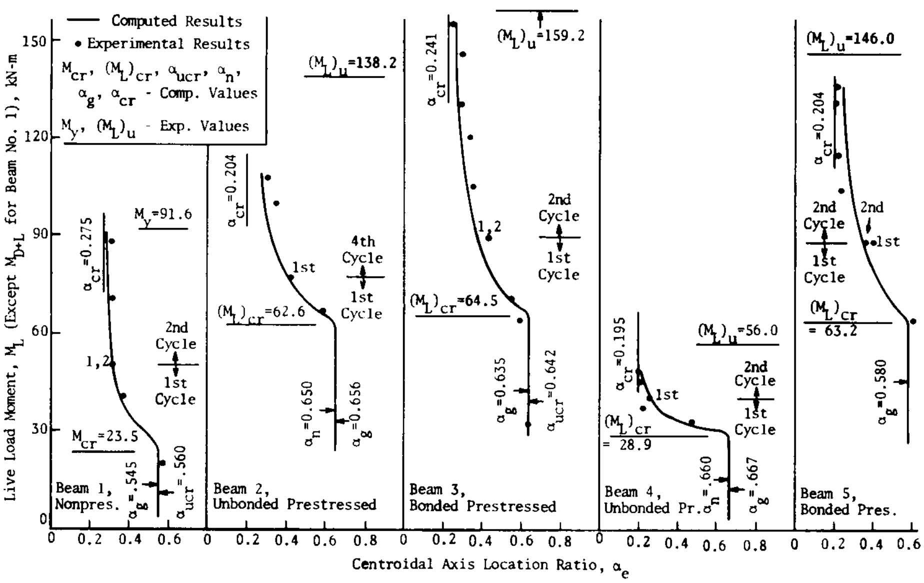

Fig. 15 — Experimental and computed results for moment versus centroidal axis location ratio

Δ_p' = fictitious deflection due to prestress using E_c rather than E_{ci}
ϱ' = compression steel area ratio = A_s'/bd
ϕ = curvature
ϕ_p = fictitious curvature due to prestress using E_c rather than E_{ci}

Subscripts

1 = top surface of concrete member
2 = bottom surface of concrete member
cr = cracking
D = dead load
D+L = dead plus live load
e = effective, end
e' = effective value for curvature in Fig. 3
g = gross section value (neglecting the steel)
i = initial value
L = live load
*L*1 = part of the live load effect corresponding to zero deflection
*L*2 = part of the live load effect corresponding to positive (downward) deflection
p = prestressing
s = nonprestressed steel
ucr = uncracked

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