R74-22, Department of Civil Engineering, Massachusetts Institute of Technology, Cambridge, Mass., 1974.

Appendix II.—Notation

The following symbols are used in this paper:

C_I, C_F = initial cost and failure cost;
C_T, C_T^o, C_T^{PL} = total cost and optimal (minimum) total cost, total cost including proof load test;
c, c_{PL}, c_{PF} = relative cost of failure, of proof load and of failure due to proof load test;
D, D_n = dead load, nominal dead load;
$F(\ ), f(\ )$ = probability distribution function, probability density function;
k = proof-load level;
L, L_n = live load and nominal live load;
P_F = probability of failure;
Q, Q^* = design point before and after proof-load test;
R, r, r^* = strength, reduced strength variable, and reduced Gaussian strength variable;
S, s = load, reduced load variable;
X_p, X_p^c = proof load, proof load defined by Eq. 36;
V = coefficient of variation;
α_D, α_L = proof-load factor on dead and live load;
α_{D1}, α_{D2} = calibrated proof-load factor on dead load;
α_{L1}, α_{L2} = calibrated proof-load factor on live load;
β, β^*, β_c = safety index, proof-load test safety index, and target safety index;
$\bar{\theta}, \bar{\theta}^o$ = central safety factor, optimal central safety factor;
λ = ratio of (nominal) live to dead load;
μ = mean;
ν = ratio of mean to nominal;
σ^2 = variance; and
$\Phi(\ )$ = standard normal (Gaussian) distribution function.

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Time-Dependent Analysis of Composite Frames

By Maher K. Tadros,[1] Amin Ghali,[2] M. ASCE, and Walter H. Dilger,[3] M. ASCE

Introduction

In a simple prestressed concrete beam, the stresses in concrete and prestressing steel vary continually with time due to the effects of creep and shrinkage of concrete and stress relaxation of steel. The result is a stress reduction in the prestressing steel and concrete. The continuous shortening of the prestressing tendons results in a stress relaxation that is smaller than that occurring in a constant-length laboratory test (the "intrinsic" relaxation).

The term composite is used here to mean that the cross section is made up of two or more concrete parts with different concrete properties or of steel girder and concrete deck (Fig. 1). In addition to stress variation in the concrete and the reinforcement, continuous stress redistribution occurs between the parts of the section such that the compatibility of displacement is maintained at their interface.

If the deformation of a simple beam is restrained at its ends by making it a part of a continuous beam or frame, statically indeterminate end forces develop. In turn, these time-dependent forces cause change of stresses in the various cross sections.

The objective of this paper is to evaluate the time development of the displacements and the stresses in concrete and steel of any cross section in a statically indeterminate composite beam or plane frame due to the effects of shrinkage and creep of concrete and stress relaxation of steel. The loads considered are self-weight, prestress, and any other superimposed sustained loads. Data related to material and geometric properties and construction and loading schedules are assumed to be available.

The method adopted is a step-by-step computer procedure in which the time is divided into intervals; the stresses and deformations at the end of each time

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interval are calculated in terms of the stress applied in the first interval and the stress increments that occurred in preceding intervals.

Constitutive Relations for Materials

Three materials are involved: (1) Concrete; (2) prestressing steel; and (3) nonprestressed reinforcing bars. Because of creep, the first two materials have

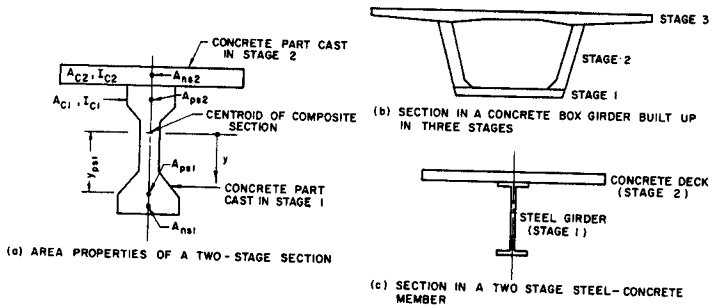


FIG. 1.—Typical Cross Section of Composite Structures

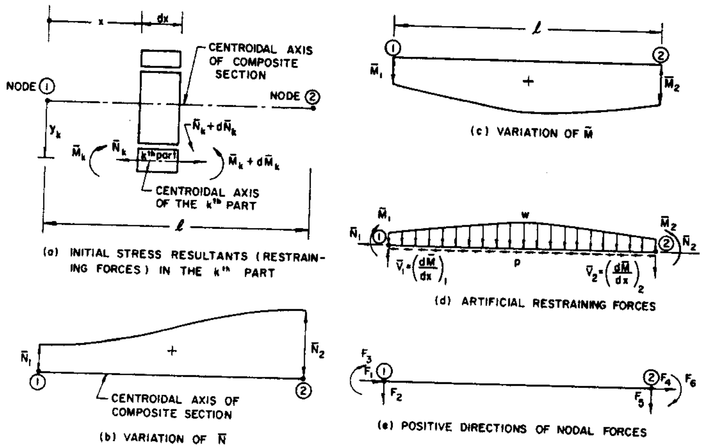


FIG. 2.—External Forces Required to Restrain Initial Strain in all Parts of Composite Member

a time-dependent stress-strain relation. Nonprestressed steel is considered to be a linear elastic material obeying Hooke's law.

For a constant sustained stress in the service range, the instantaneous as well as the creep strains in concrete are assumed to be linearly proportional to the applied stress. This implies that the principle of superposition is applicable to the stresses and strains. For a variable stress, the time in which the stress varies is divided into discrete time intervals. Stress variation in any interval is assumed to occur at its middle. Instantaneous applied loads such as the prestressing or the self-weight are assumed, for consistency, to occur at the

middle of an interval of length zero. The total concrete strain, instantaneous plus creep plus shrinkage at the end of any interval i, is

$$\epsilon_c\left(i+\frac{1}{2},0\right)=\sum_{j=1}^{i}\frac{\Delta f_c(j)}{E_c(j)}\left[1+\upsilon\left(i+\frac{1}{2},j\right)\right]+s\left(i+\frac{1}{2},0\right)\quad\ldots\ldots(1)$$

in which i and j refer to the time at the middle of the ith and jth intervals; $i + (1/2)$ refers to the time at the end of the ith interval; and 0 refers to the time at the beginning of the first interval, in which the first stress increment is applied to the concrete. Thus, $\Delta f_c(j)$ = concrete stress increment introduced at the middle of the jth interval; $E_c(j)$ = modulus of elasticity of concrete at the middle of the jth interval; $s[i + (1/2), 0]$ = the free shrinkage of concrete between the beginning of the first interval and the end of the ith interval; and $\upsilon[i + (1/2), j]$ = creep coefficient = ratio of creep at the end of interval i to the instantaneous strain caused by a sustained stress introduced at the middle of interval j. Creep is the difference between the total strain caused by stress and the instantaneous strain.

Eq. 1 is a simple superposition of strain caused by the stress increments at various intervals. This equation is valid with any assumed form of time variation adopted by the analyst for s, υ, or E_c (see, e.g., Refs. 7 or 10). If the time period in which the prestressed steel strain changes is divided into discrete time intervals, the strain change occurring between the beginning of the first interval and the end of the ith interval is

$$\epsilon_{ps}\left(i+\frac{1}{2},0\right)=\frac{1}{E_{ps}}\sum_{j=1}^{i}\left[\Delta f_{ps}(j)-\Delta f_r(j)\right]\quad\ldots\ldots(2)$$

The symbols in this equation have the same meaning as in Eq. 1; here the subscript ps refers to the prestressing steel; and $\Delta f_r(j)$ is a "reduced" stress relaxation. The reduction is due to the fact that in a prestressed concrete member, relaxation in tendons is smaller than the "intrinsic" relaxation, Δf_{ro} that occur in a test in which the tendon is stretched between two fixed points. The reduced value, Δf_r, is calculated from the intrinsic value, Δf_{ro}, as considered in a separate section.

The strain in a tendon calculated by Eq. 2 does not include the extension immediately before transfer. The equation is applicable for post-tensioned as well as pretensioned tendons. The first term in the summation will be zero for the former and equal to the loss at transfer in the latter.

Step-by-Step Analysis

With the usual assumption that plane cross sections remain plane, the axial strain, ϵ, and the curvature, ϕ at any cross section can be related to the axial force, N, and the bending moment, M. During any interval i, an incremental axial strain, $\Delta\epsilon(i)$, occurs

$$\Delta\epsilon(i)=\frac{\Delta N(i)}{AE(i)}\left[1+\upsilon\left(i+\frac{1}{2},i\right)\right]$$

$$+\left\{\sum_{j=1}^{i-1}\frac{\Delta N(j)}{AE(j)}\left[\upsilon\left(i+\frac{1}{2},j\right)-\upsilon\left(i-\frac{1}{2},j\right)\right]+\Delta s(i)\right\}\quad\ldots\ldots(3)$$

Eq. 3 can be derived from Eq. 1 noting that $\Delta f(i) = \Delta N(i)/A$ and expressing the increment of strain in the ith interval as the difference of the total strain at the end of the intervals i and $i-1$. Similarly, the incremental curvature during the ith interval

$$\Delta\phi(i) = \frac{\Delta M(i)}{IE(i)}\left[1 + \upsilon\left(i + \frac{1}{2}, i\right)\right] \left\{\sum_{j=1}^{i-1} \frac{\Delta M(j)}{IE(j)}\left[\upsilon\left(i + \frac{1}{2}, j\right) - \upsilon\left(i - \frac{1}{2}, j\right)\right]\right. \quad (4)$$

Each of the two previous equations can be put into pseudolinear form by expressing the right-hand side as the sum of two terms. The first represents deformation proportional to the incremental forces, $\Delta N(i)$ or $\Delta M(i)$, with an effective modulus

$$E_e = \frac{E(i)}{1 + \upsilon\left(i + \frac{1}{2}, i\right)} \quad (5)$$

The term between braces in each of Eq. 3 and 4 represents "initial" deformation, independent of the stress introduced in interval i. For these, the symbols $\Delta\bar{\epsilon}(i)$ and $\Delta\bar{\phi}(i)$ are used. The term "initial" (often used in conjunction with stress analysis for the effect of temperature variation) should not be confused with the instantaneous elastic deformations.

Thus the analysis can be transformed into an incremental linear analysis. During any interval i the following linear relations apply

$$\Delta\epsilon(i) = \frac{\Delta N(i)}{AE_e(i)} + \Delta\bar{\epsilon}(i) \quad (6)$$

$$\Delta\phi(i) = \frac{\Delta M(i)}{IE_e(i)} + \Delta\bar{\phi}(i) \quad (7)$$

Eqs. 3 and 4 can be applied for any part of the composite section. If considered part is a steel girder [Fig. 1(c)], the shrinkage and creep parameters, s and υ, are zero. Nonprestressed steel A_{ns} [Fig. 1(a)] is treated as a separate part with zero flexural rigidity.

From Eq. 2, the incremental strain in the prestressing steel during the ith interval is

$$\Delta\epsilon_{ps}(i) = \frac{\Delta N_{ps}(i)}{A_{ps}E_{ps}} + \left[-\frac{\Delta f_r(i)}{E_{ps}}\right] \quad (8)$$

The term between square brackets represents an initial strain in the prestressed steel, $\Delta\bar{\epsilon}_{ps}(i)$. Thus the incremental strain in the ith interval is expressed in pseudolinear form:

$$\Delta\epsilon_{ps}(i) = \frac{\Delta N_{ps}(i)}{A_{ps}E_{ps}} + \Delta\bar{\epsilon}_{ps}(i) \quad (9)$$

In a step-by-step analysis the initial deformations, $\Delta\bar{\epsilon}(i)$ and $\Delta\bar{\phi}(i)$, must be calculated from the results of the preceding intervals, by summation.

Effects of Initial Strains

The analysis for the effects of the initial strains, $\bar{\epsilon}$ and $\bar{\phi}$, can be performed by the standard displacement method (5,11) in the same way as solving for the effect of temperature change.

Fig. 2(a) represents a straight composite member of a frame that is free to deform. If the initial axial strain, $\bar{\epsilon}_k$, and curvature, $\bar{\phi}_k$, of the kth part are prevented by application of appropriate forces, initial stresses will develop. At any section of the kth part, the initial stress resultants are

$$\bar{N}_k = -(EA\,\bar{\epsilon})_k \quad (10)$$

$$\bar{M}_k = -(EI\bar{\phi})_k \quad (11)$$

For the whole composite section the initial stress resultants are

$$\bar{N} = \sum_{k=1}^{q} \bar{N}_k \quad (12)$$

$$\bar{M} = \sum_{k=1}^{q} \bar{M}_k + \sum_{k=1}^{q} \bar{N}_k\, y_k \quad (13)$$

in which q = the number of parts in the composite section; and y_k = the distance between the centroid of the kth part and the centroid of the composite section. These stress resultants vary along the member [Figs. 2(b) and 2(c)].

Fig. 2(d) shows a set of externally applied forces that produce the same internal forces $\bar{N}$ and $\bar{M}$ at any section of the composite member. The intensities of the distributed forces in this figure are

$$p = -\frac{d\bar{N}}{dx} \quad (14)$$

$$w = -\frac{d^2\bar{M}}{dx^2} \quad (15)$$

To eliminate this artificial restraint, a set of forces equal and opposite to those shown in Fig. 2(d) must be applied to the structure analyzed by the displacement method. The method gives the final displacement of the composite frame caused by the initial strains, as well as internal forces at each section. The stress resultants, in the kth part, corresponding to these internal forces are to be added to the initial stress resultants given by Eqs. 10 and 11 to give the final stress resultants.

If $\bar{N}$ and $\bar{M}$ are assumed to vary linearly along the length, l, between two nodes, p will be constant and $w = 0$. The forces to eliminate the initial restraint will have the following equivalent (5) nodal forces [Fig. 2(e)]: $F_1 = 0.5(\bar{N}_1 + \bar{N}_2)$, $F_2 = (\bar{M}_2 - \bar{M}_1)/l$, $F_3 = \bar{M}_1$, $F_4 = -F_1$, $F_5 = -F_2$, and $F_6 = \bar{M}_2$.

In the step-by-step procedure described in the previous sections, the aforementioned analysis must be done for each interval separately using the initial strain increments.

Reduced Relaxation

The value of intrinsic relaxation, f_{ro}, is determined from tests in which the steel tendon is stretched between two fixed points; thus the length is maintained constant. Various empirical equations expressing f_{ro} in terms of the time and the initial stress, f_{pso}, are available. The equation adopted by the Prestressed Concrete Institute Committee on prestress loss (13) is given in Appendix I. The reduction in steel stress due to shortening of the tendon in a concrete member results in a reduced relaxation, f_r, compared to the intrinsic value, f_{ro}.

The change in strain of the prestressing steel in the period preceding interval i is (see Eq. 2)

$$\epsilon_{ps}\left(i - \frac{1}{2}, 0\right) = \frac{1}{E_{ps}} \sum_{j=1}^{i-1} [\Delta f_{ps}(j) - \Delta f_r(j)] \quad \ldots \ldots \ldots \ldots \ldots \ldots \quad (16)$$

The actual (reduced) relaxation occurring in the interval i can be obtained by substituting for the initial stress in the intrinsic relaxation equation a fictitious stress $= f_{pso} + E_{ps}\epsilon[(i - 1/2, 0]$.

In the previously mentioned procedure the effect of the shortening of the tendon in interval i on the relaxation increment in the same interval is not accounted for. Since this shortening is a function of $\Delta f_{ps}(i)$ and $\Delta f_r(i)$, which are yet to be determined, iteration within the interval would be necessary (6). The approximation involved is adopted here to avoid a substantial increase in computation time. Only a small error is involved especially when the interval length is small.

Scope of Application

The method described previously is general from two points of view: (1) It can be used to analyze any statically determinate or indeterminate composite plane frame, regardless of the number of different materials used; and (2) it can be used with any chosen function expressing the time variation of creep and shrinkage of concrete, and relaxation of steel. For creep and shrinkage one can use prediction methods suggested by the American Concrete Institute-209 (10) Committee, the Comite European de Béton-Fédération-Internationale de la Précontrainte (7) Committee, the rate of flow method, (14) or any future development.

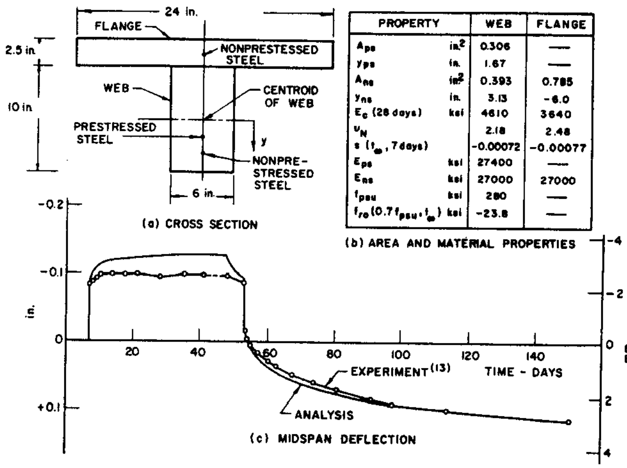
When the frame is statically indeterminate, the stresses and strains in one section are not independent of those in other sections. The method of the present paper should give, in the special case of a statically determinate frame, the same results as the step-by-step procedures used in Refs. 6, 12, and 16, which treat each cross section separately.

Considerations in Analysis of Frames

The procedure of analysis presented is equally applicable to continuous beams and frames. The computer program in Ref. 15 can be used for any continuous

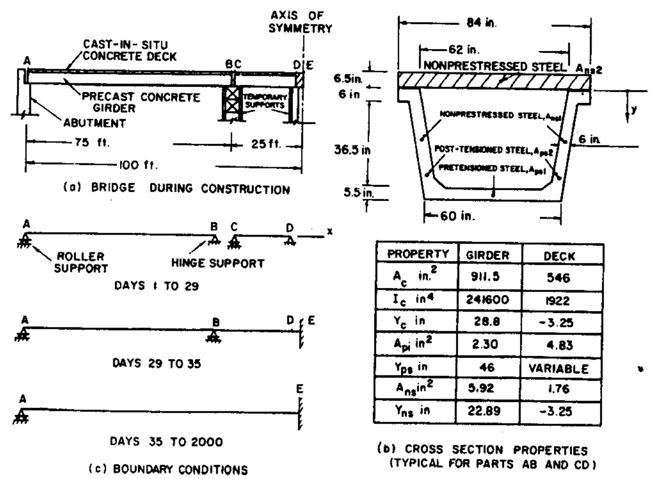
plane frame for which some or all members have composite sections.

An example of this type of structure is a bridge composite deck monolithically connected to vertical or inclined legs. In such a case, the frame is idealized



PROPERTY		WEB	FLANGE
A_{ps}	in.^{2}	0.306	—
y_{ps}	in.	1.67	—
A_{ns}	in.^{2}	0.393	0.785
y_{ns}	in.	3.13	-6.0
E_c (28 days)	ksi	4610	3640
ν_N		2.18	2.48
s (t_∞, 7 days)		-0.00072	-0.00077
E_{ps}	ksi	27400	—
E_{ns}	ksi	27000	27000
f_{psu}	ksi	280	—
$f_{ro}(0.7 f_{psu}, t_\infty)$	ksi	-23.8	—

FIG. 3.—Verification Example: Simple Beam, Span 12 ft (3.66 m) (1 in. = 25.4 mm; 1 sq in. = 6.45 cm²; 1 ksi = 6.89 MN/m²)



PROPERTY	GIRDER	DECK
A_c in.^{2}	911.5	546
I_c in^4	241600	1922
Y_c in	28.8	-3.25
A_{pi} in^2	2.30	4.83
Y_{ps} in	46	VARIABLE
A_{ns} in^2	5.92	1.76
Y_{ns} in	22.89	-3.25

FIG. 4.—Two-Span Continuous Bridge Considered in "Application" (1 ft = 0.305 m; 1 in. = 25.4 mm; 1 sq in. = 6.45 cm²; 1 in.⁴ = 41.6 cm²)

as assemblage of linear prismatic elements connected at the nodes. The nodes are on the centroidal axis of the composite section of the members. The time-dependent change in the position of the nodes is ignored when generating the stiffness matrix of the assembled structure.

Verification

The computer program based on the method presented in this paper is verified by applying it to a composite beam and comparing the deformations with experimental results (12).

The cross section of the beam as well as the area and material properties are given in Figs. 3(*a*) and 3(*b*).

The time schedule for casting and loading was: on day zero, casting of the web; on day 7, curing terminated, prestressing force equals 65.8 kip (293 kN) transferred, and the web starting to carry its own weight, 146 pcf (2,340 kg/m^3); on day 41, the flange cast on a propped web; on day 48, curing of flange ended, props removed causing the introduction of the flange self-weight [142.5 pcf (2,280 kg/m^3), and the beam mounted on a simple span of 12 ft (3.7 m); on day 53, two superimposed concentrated loads of 5.83 kip (25.9 kN) each were applied and sustained at the third points.

The time variation of the concrete modulus of eleasticity, E_c, the creep coefficient, υ, the shrinkage, s, and the stress relaxation, f_{ro}, were taken as given in Appendix I.

In the analysis, the concrete prestress is assumed, according to Kaar, et al. (9), to be zero at the beam ends and transferred from steel at a constant rate, reaching its full value in a transfer length = $110(d - 0.1)$, in which d = the strand diameter, in inches. The beam is assumed to be simply supported in the period between day 7 and day 41, prevented from deflection between day 41 and day 48, and simply supported thereafter. The limits of the time intervals are 7, 7, 8, 10, 15, 25, 41, 43, 48, 48, 50, 53, 53, 58, 65, 75, 90, 110 and 147 days.

Fig. 3(*c*) shows the measured and the calculated values of the midspan deflection of the beam.

Further verification of the method of analysis was made (15) using published experimental results (1,2,3). These were for non-composite and composite simple beams. Also, the verification included a continuous non-composite prestressed concrete beam subjected to gradual support settlement (4). A limited number of experiments on composite continuous structures is reported in the literature, but none of these included all the data necessary for the calculations. Application of the method to a continuous bridge example is given below.

Application

The method of analysis will be applied to a two-span symmetrical bridge shown in Fig. 4. The results of the analysis are given to indicate the importance of the time-dependent effects.

The bridge is made up of pretensioned precast girders of an open trapezoidal cross section [Fig. 4(*b*)] and of cast-in-situ concrete deck and joints. The dimensions of the cross section and the construction sequence conform to bridges built in various regions in North America.

The construction schedule is as follows. Precast girders with lengths of 75 ft (23 m) and 22.5 ft (6.9 m) are pretensioned at age 1 day and consequently transported to site and erected on props [Fig. 4(*a*)]. Twenty-eight days after casting the girders, the deck slab and the joints between the girders were cast in situ. The structure is thus made composite and continuous. At 35 days, post-tensioned continuity cables are stressed and the props removed. The cable ducts are grouted shortly thereafter. On day 50, superimposed dead load representing added finishes is applied and the bridge opened to traffic.

The cross-sectional dimensions and properties are given in Fig. 4(*b*). For a length of 5 ft (1.5 m) above the central pier, the bridge cross section is solid, forming a diaphragm.

The profile of the centroid of the post-tensioned cables is parabolic with positive curvature over 78.5 ft (24 m) from the abutment and a negative curvature

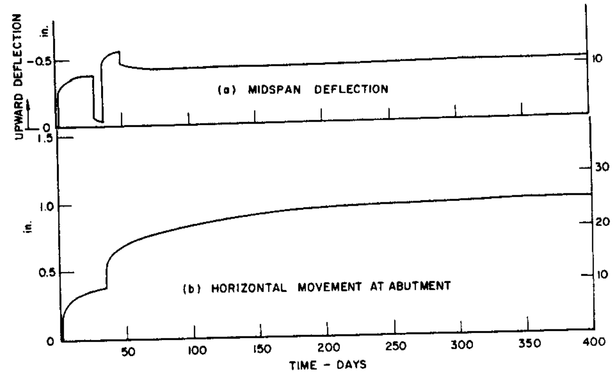


FIG. 5.—Bridge Deformation

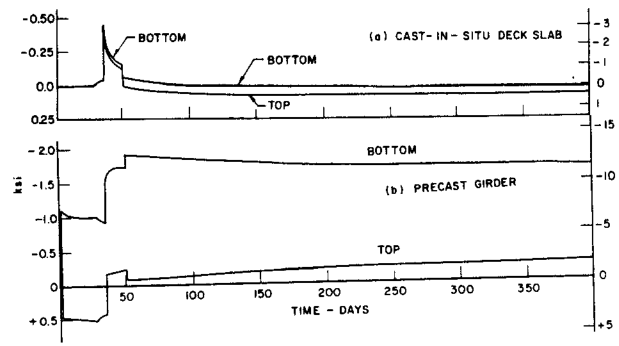


FIG. 6.—Stresses in Concrete in Section 2.5 ft (0.76 m) from Pier Center Line

for the remainder of the span. The coordinates y_{ps} defining the two parabolas are: (1) x, in feet (meters): 0.0 (0.0), 39.25 (11.96), 78.50 (23.93), and 100.00 (30); and (2) y_{ps}, in inches (millimeters): 24 (610), 42 (1,100), 14 (360), and 7 (180).

Material Properties.—Prestressing steel properties: E_{ps} 27,800 ksi (192,000 MN/m^2); the intrinsic relaxation at time infinity f_{ro} = -10.9 ksi (75 MN/m^2) for an initial stress, f_{pso} = 189 ksi (1,300 MN/m^2), equals 82.5% of the yield stress. The precast and cast-in-situ concretes weigh 150 pcf (2,400 kg/m^3);

other properties for precast and cast in situ, respectively, include f'_c (28 days), in kips per square inch (Meganewtons per square meter): 6.0 (41) and 4.0 (28); E_c (28 days), in kips per square inch (Meganewtons per square meter): 4,700 (32,000) and 3,800 (26,000); υ_N: 1.7 and 2.7; and $s(t_\infty, t_d)$: -3.0×10^{-4} and -3.3×10^{-4}.

The solid section above the pier has a different value of the parameter surface/volume and thus different creep and shrinkage values from the hollow

TABLE 1.—Stress Comparison, Section 2.5 feet (0.76 m) from Bridge Center Line

Stress, in kips per square inch (1)	Precast Part		Cast-In-Situ Part		Pre-tensioned steel (6)	Post-tensioned steel (7)	Nonprestressed Steel	
	Top (2)	Bottom (3)	Top (4)	Bottom (5)			Precast part (8)	Cast-in-situ part (9)
At t = 2,000	−0.39	−1.50	+0.04	−0.01	153	161	−26.4	−11.6
Instantaneous	−0.03	−1.93	−0.29	−0.32	187	176	−7.1	−2.9

Note: 1 ksi = 6.89 MN/m².

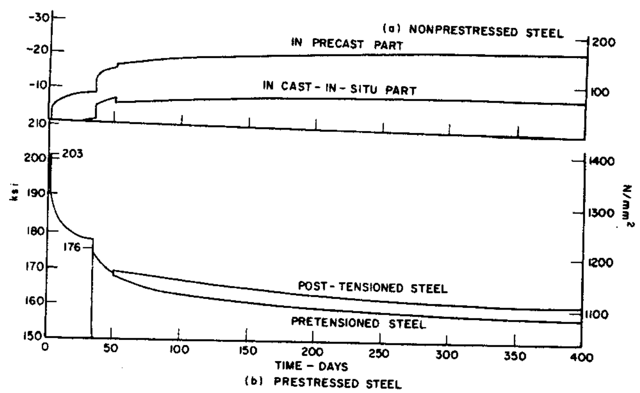


FIG. 7.—Stresses in Steel in Section 2.5 ft (0.76 m) from Pier Center Line

section: $\upsilon_N = 2.1$ and $s(t_\infty, t_d) = -1.9 \times 10^{-4}$.

The expressions in Appendix I for the time variation of E_c, υ, s, and f_{ro} are used in the analysis.

Time Intervals.—The chosen interval limits are, in days: 1, 1, 3, 7, 15, 28, 28, 30, 35, 35, 35, 38, 43, 50, 50, 60, 100, 300, 800, and 2,000. This makes 19 intervals of which several have zero length. These are the intervals at the "middle" of which sudden loads are applied or support conditions changed [see Fig. 4(c)].

Loading.—The forces applied at the various intervals are: (1) Interval 1—pretensioning force and the girder self-weight, the steel stress immediately before transfer = 203 ksi (1,400 MN/m²); (2) interval 6—deck self-weight; (3) interval 9—forces equal and opposite to the reaction of props; (4) interval 10—post-tensioning force of 848 kip (3,770 kN), the corresponding steel stress = 176 ksi (1,210 MN/m²); and (5) interval 14—superimposed dead load = 263 lb/ft (3.83 kN/m).

Results.—A computer program based on the method presented in this paper was used to give the time variation of the stresses in steel and concrete and the displacements at various sections of the bridge. Figs. 5-7 are plots of some results.

Comments.—The instantaneous camber [Fig. 5(a)] caused by pretensioning increases gradually until a sudden deflection due to deck weight occurs. The other two vertical parts of the curve correspond to the post-tensioning and the superimposed dead load. The sudden increases in the horizontal movement at the abutment [Fig. 5(b)] are at time 1 and 35 days when the prestressing is applied. Note that the final values of the displacement (at t = 2,000) in each of Figs. 5(a) and 5(b) are −0.46 in. and 1.08 in. (−12 mm and 27 mm). As would be expected, these values differ considerably from the instantaneous displacement increments which sum up to −0.26 in. and 0.32 in. (−7 mm and 8 mm).

The concrete and steel stresses at t = 2,000 days at 2.5 ft (0.76 m) from the bridge center line are compared in Table 1 with the instantaneous values calculated by summing the effect of loads and ignoring creep, shrinkage, and steel relaxation.

The preceding comparisons show the importance of the time-dependent effects. These effects are reduction (relief) of stress at some points and increase of stress at others. Examples of substantial stress increases in the section considered previously are at the top of the precast part and the nonprestressed steel. The comparison introduced in the deck slab by post-tensioning is largely reduced by the time-dependent effects (Fig. 6).

Conclusions

A method is presented for the analysis of the effects of creep and shrinkage of concrete and relaxation of prestressing steel in composite plane frames. Examples are bridges built of precast prestressed concrete or steel girders with cast-in-situ concrete decks. The approach is general; it can be used regardless of the number of different materials in the cross section or whether the structure is statically indeterminate or not. The loading, prestressing forces, or prescribed support displacements may be introduced in arbitrary stages. Also the boundary conditions may be changed at any stage, as would be the case when precast segments are assembled and made continuous by prestressing or by cast-in-situ concrete, or both.

The method assumes that concrete creep is proportional to stress, and thus superposition of stresses and strains is assumed valid. It can be used with any chosen functions expressing the time variation of creep and shrinkage of concrete and relaxation of steel.

The bridge example analyzed show that accurate evaluation of stresses and deformations of continuous composite prestressed concrete structures cannot be done without rationally accounting for the effects of creep and shrinkage of concrete and relaxation of steel.

Appendix I.—Time Variation of E_c, υ, s, and f_{ro}

In the applications included in the paper the following time functions are adopted.

The modulus of elasticity of precast concrete at age t (10)

$$E_c(t) = E_c(28)\sqrt{\frac{t}{2.3 + 0.92t}} \quad (17)$$

and for cast-in-situ concrete

$$E_c(t) = E_c(28)\sqrt{\frac{t}{4.0 + 0.85t}} \quad (18)$$

Creep coefficient for concrete at age t when the age at loading is τ

$$\upsilon(t, \tau) = \upsilon_N\left[\frac{E_c(\tau)}{E_c(28)}\right] k_1(\tau)k_2(t-\tau) \quad (19)$$

in which $k_1(\tau) = \dfrac{10.29}{5 + \sqrt{\tau}}$ (20)

$$k_2(t-\tau) = 1 - \exp\left[-0.1564\,(t-\tau)^{0.4} + 0.0555\right] \quad (21)$$

Shrinkage at age t

$$s(t, t_d) = s(t_\infty, t_d)\,k_3(t - t_d) \quad (22)$$

in which t_d = the age at which curing is terminated, and

$$k_3(t - t_d) = 1 - \exp\left[-0.0887(t - t_d + 0.075)^{0.59} + 0.0192\right] \quad (23)$$

Eqs. 19-23 are derived from graphs given in Ref. 7.

The relaxation at time t in a steel tendon stretched at time τ between two fixed points (intrinsic relaxation) (9,13)

$$f_{ro}(t, \tau, f_{pso}) = K f_{pso}\left(\frac{f_{pso}}{f_{psy}} - 0.55\right)\log\left(\frac{24t+1}{24\tau+1}\right); \quad f_{pso} \geq 0.55 f_{py} \quad (24)$$

in which f_{pso} = the initial stress; f_{psy} = the yield stress (= 0.1% offset stress); K is a constant depending on the relaxation properties of steel. It can be derived from the data given for the two examples considered by substituting in Eq. 24 the values of f_{ro} at time infinity and the initial stress, f_{pso}.

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Appendix III.—Notation

The following symbols are used in this paper:

A = cross-sectional area;
E = modulus of elasticity;
$F_1, \ldots, F_6$ = element nodal forces [Fig. 2(e)];
f = normal stress, positive when tensile—subscripts *pso*, *psy*, and *psu* indicate initial, 0.1% offset, and ultimate stress in prestressed steel, respectively;

I = moment of inertia of section;
i, j = time interval number, referred to in step-by-step calculation—when used with time-dependent parameters, they indicate time at middle of interval—$i - 1/2$ and $i + 1/2$ refer to beginning and end of interval i, respectively;
l = element length (see Fig. 2);
M = bending moment in section, positive when causing sagging;
m = total number of time intervals;
N = normal force in section, positive when tensile;
p, w = distributed load intensities (Eqs. 14 and 5);
q = number of section parts in a composite cross section;
$s[i + 1/2, j - 1/2]$ = shrinkage of concrete in period $\{t[i + 1/2] - t[j - 1/2]\}$;
t = time, in days, measured from reference date;
x, y = coordinates along and normal to centroidal line of composite member, positive directions are shown in Fig. 2;
Δ = used as prefix to indicate incremental value;
ϵ = normal strain, positive when corresponds to extension;
$\upsilon[i + 1/2, j]$ = creep coefficient ratio of creep strain at time $t[i + 1/2]$ to instantaneous strain caused by constant sustained stress applied at $t(j)$—subscript N is used to indicate normal creep, equals υ for $t[i + 1/2]$ = infinity and t_j = 28 days; and
ϕ = curvature, positive when corresponds to sagging; and
$\bar{}$ = superscript to indicate initial strains (Eqs. 6, 7, and 9) and corresponding stresses.

Subscripts

c, ns, ps = concrete, nonprestressed steel, and prestressed steel, respectively;
e = effective modulus of elasticity (Eq. 5);
k = part number of cross section;
r = reduced relaxation in shortened prestressed strand; and
ro = intrinsic relaxation in constant-length strand.

12889 APRIL 1977 ST4

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Economic Feasibility of Concrete Recycling

By Stamatia Frondistou-Yannas[1] and Taichi Itoh[2]

Introduction

The successful recycling of concrete from demolition wastes as a substitute aggregate for new concrete can make a contribution to the solution of two problems of increasing magnitude.

There exists, first, an availability problem: Concrete aggregates are locally unavailable in many metropolitan areas (10), both because urban expansion has led to closing of several aggregate plants and because enforcement of environmental laws has led to closing of still others. Consequently, it becomes necessary to transport concrete aggregates from increasingly longer distances. This creates a serious economic problem since concrete aggregates are bulky and heavy, and the cost of their transportation is correspondingly high.

Secondly, there is a waste disposal problem: Recent studies (14) indicate that demolition wastes in this country reach the substantial annual figure of 3×10^7 tons (27×10^9 kg). Since concrete accounts for about 75%, by mass, of all construction materials used (8), it follows that concrete will account for three-quarters, by mass, of all demolition wastes. The disposal of such massive quantities of concrete waste poses a difficult problem. Indeed, while torn-up pavements and demolished buildings have found use as landfill for many years, the need for fill today, especially in the highly developed metropolitan centers, suffices to take care of only a small fraction of the rubble generated to make way for new construction. It has become increasingly difficult and expensive to dispose of construction debris within the bounds of the increasingly critical environmental requirements. In short, contractors are running out of dumps.

Under the pressure of the problems previously outlined, highway contractors have already started recycling concrete highway debris and using it as base material for pavement construction (3,11). However, since concrete used in highways and streets accounts only for 15%-20% of total concrete consumption, it would appear desirable to consider recycling concrete waste generated from

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