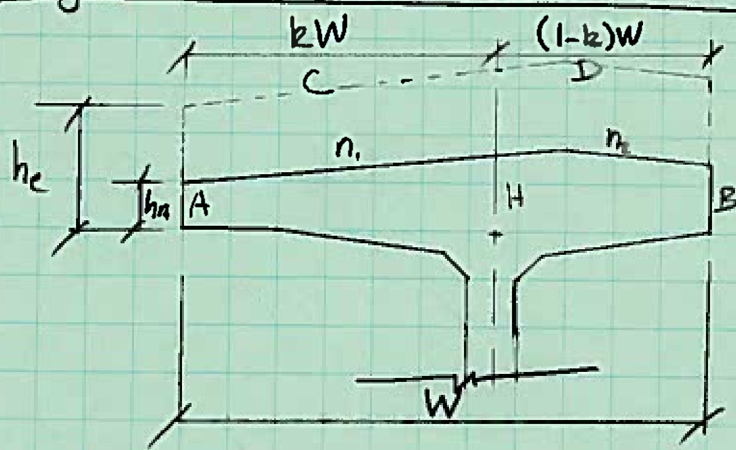
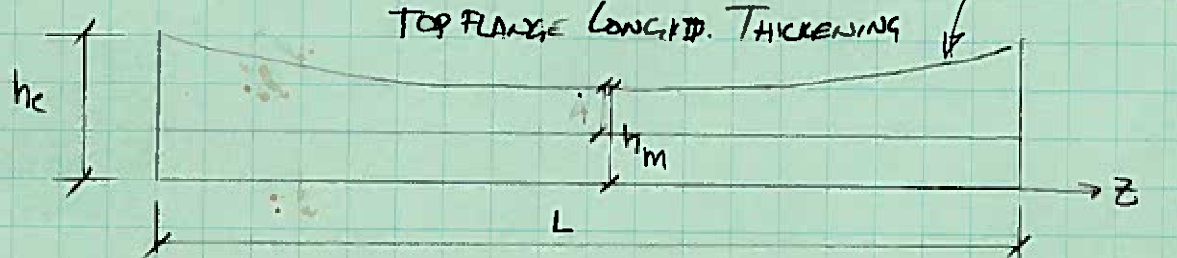
Washington State Department of Transportation

Bridge & Structures Design Calculations

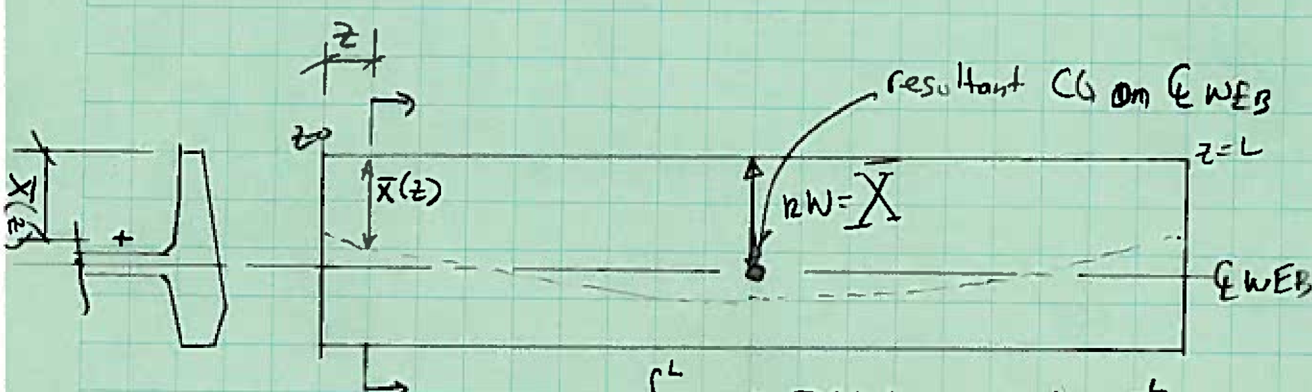
Project WF-DG Development			Sheet No. of Sheets	
S.R.	Made By R. Brice	Check by	Date	Supv

Top Flange Geometry such that CG is on ℄ WEB





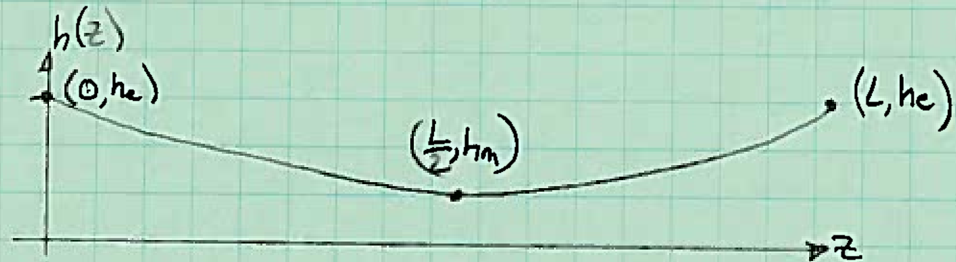
ELEVATION



FIND k for $kW = \bar{X} = \dfrac{\int_0^L w(z)\,\bar{x}(z)\,dz}{\int_0^L w(z)\,dz} = \dfrac{1}{W_g}\int_0^L w(z)\,\bar{x}(z)\,dz$

W_g = girder weight

Top flange height w/ parabolic flange thickening



$$h(z) = az^2 + bz + c \qquad \frac{dh}{dz} = 2az + b$$

$$h(0) = h_e \quad h(L) = h_e \quad h\left(\frac{L}{2}\right) = h_m \quad \frac{dh}{dz}(L/2) = 0$$

$$2a\left(\frac{L}{2}\right) + b = 0 \qquad b = -aL$$

$$h(0) = a(0)^2 + b(0) + c = h_e \rightarrow c = h_e$$

$$h\left(\frac{L}{2}\right) = a\left(\frac{L}{2}\right)^2 - aL\left(\frac{L}{2}\right) + h_e = h_m$$

$$\frac{a}{4}L^2 - \frac{a}{2}L^2 = h_m - h_e$$

$$-\frac{a}{4}L^2 = h_m - h_e$$

$$a = \frac{4(h_e - h_m)}{L^2}$$

$$h(z) = \frac{4(h_e - h_m)}{L^2} z^2 - \frac{4(h_e - h_m)}{L} z + h_e \qquad h_e - h_m = \bar{h}$$

$$h(z) = \frac{4\bar{h}}{L^2} z^2 - \frac{4\bar{h}}{L} z + h_e$$

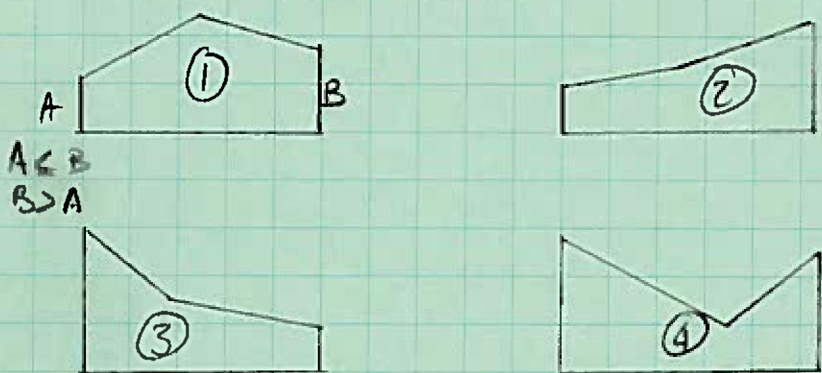
FIND CENTROID OF SECTION

FOR SOME VALUE OF k

Given

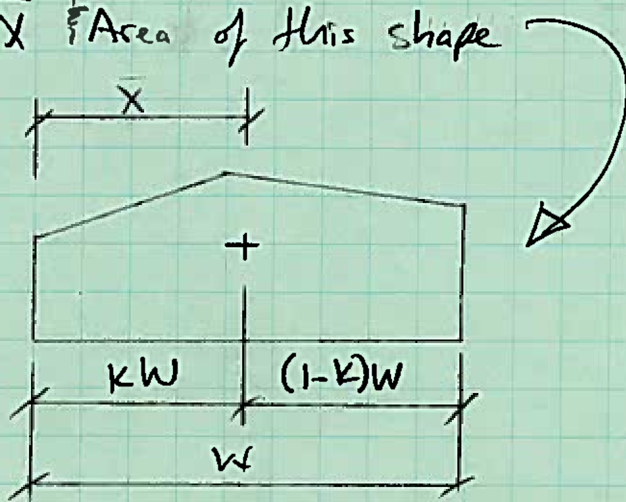
- left to right upward slopes >0

- 4 cases shown here



1a A≤B
1b B>A

Find $\bar{X}$ & Area of this shape



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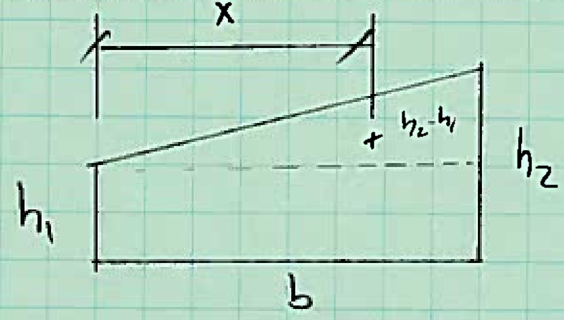
C.G of trapezoid - case A



A	$\bar{x}$	$A\bar{x}$
$h_1 b$	$\frac{b}{2}$	$\frac{1}{2} h_1 b^2$
$\frac{1}{2} b(h_2 - h_1)$	$\frac{2}{3} b$	$\frac{1}{3}(h_2 - h_1) b^2$

$$\bar{x} = \frac{\frac{1}{2} h_1 b^2 + \frac{1}{3}(h_2 - h_1) b^2}{h_1 b + \frac{1}{2} b(h_2 - h_1)}$$

$$= \frac{\frac{3}{6} h_1 b^2 + \frac{2}{6}(h_2 - h_1) b^2}{\frac{2}{2} h_1 b + \frac{1}{2} b(h_2 - h_1)}$$

$$= \frac{\frac{1}{6} b^2 (3h_1 + 2(h_2 - h_1))}{\frac{1}{2} b(2h_1 + h_2 - h_1)}$$

$$= \frac{1}{3} b \frac{(3h_1 + 2h_2 - 2h_1)}{(2h_1 + h_2 - h_1)}$$

$$= \frac{1}{3} b \frac{(h_1 + 2h_2)}{(h_1 + h_2)}$$

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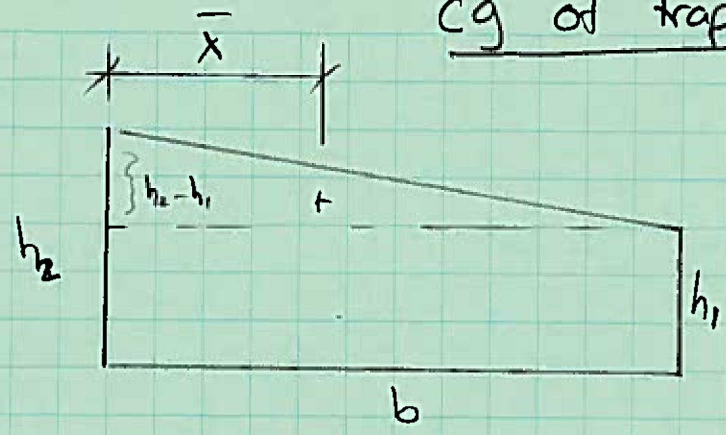
cg of trapezoid - Case B



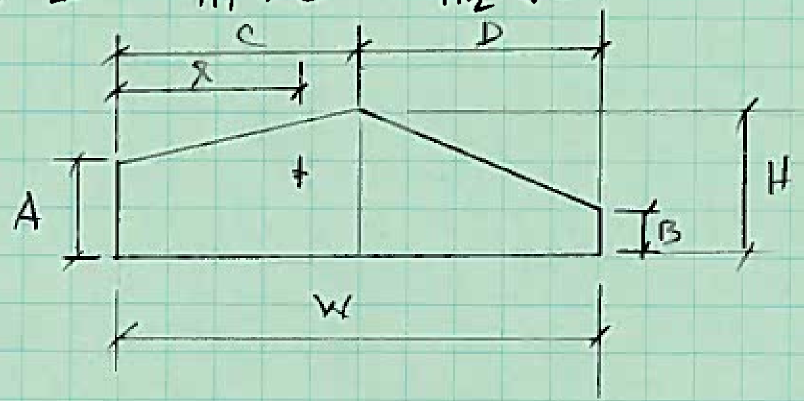
A	$\bar{x}$	$A\bar{x}$
$h_1 b$	$b/2$	$\frac{1}{2} h_1 b^2$
$\frac{1}{2} b (h_2 - h_1)$	$\frac{1}{3} b$	$\frac{1}{6} (h_2 - h_1) b^2$

$$\bar{x} = \frac{\frac{3}{6} h_1 b^2 + \frac{1}{6} (h_2 - h_1) b^2}{\frac{2}{2} h_1 b + \frac{1}{2} b (h_2 - h_1)}$$

$$= \frac{\frac{1}{6} b^2 (3h_1 + h_2 - h_1)}{\frac{1}{2} b (2h_1 + h_2 - h_1)}$$

$$\bar{x} = \frac{1}{3} b \frac{(2h_1 + h_2)}{(h_1 + h_2)}$$

CASE 1 $n_1 > 0$ $n_2 < 0$



$$\bar{x} = \frac{\Sigma(\text{Area})\bar{x}}{\Sigma(\text{Area})}$$

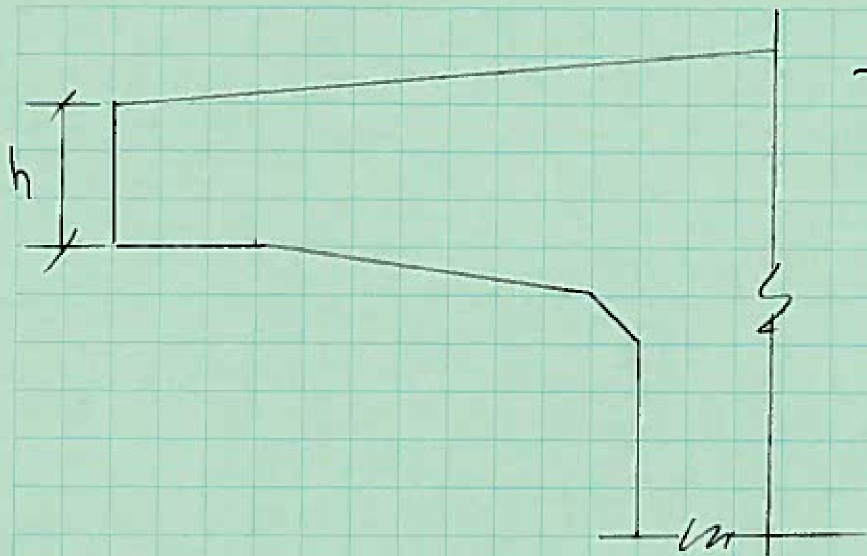
Area	$\bar{x}$	(Area) $\bar{x}$
$\frac{1}{2}(A+H)C$	$\frac{1}{3}C\left(\frac{A+2H}{A+H}\right)$	$\frac{1}{6}C^2(A+2H)$
$\frac{1}{2}(B+H)D$	$C + \frac{1}{3}D\left(\frac{H+2B}{B+H}\right)$	$\frac{1}{2}CD(B+H) + \frac{1}{6}D^2(H+2B)$

$$\bar{x} = \frac{\frac{1}{6}C^2(A+2H) + \frac{3}{6}CD(B+H) + \frac{1}{6}D^2(2B+H)}{\frac{1}{2}\left[(A+H)C + (B+H)D\right]}$$

$$\bar{x} = \frac{1}{3}\,\frac{C^2(A+2H) + 3CD(B+H) + D^2(2B+H)}{\left[(A+H)C + (B+H)D\right]}$$

$$K = \bar{x}/W$$

$$K = \frac{1}{3W} \cdot \frac{C^2(A+2H) + 3CD(B+H) + D^2(2B+H)}{\left[(A+H)C + (B+H)D\right]}$$



Top flange of WF-DG

$n_1 > 0$

$n_2 < 0$

if $(n_1 C \le -n_2 D)$

$A = h$

$B = h + n_1 C + n_2 D$

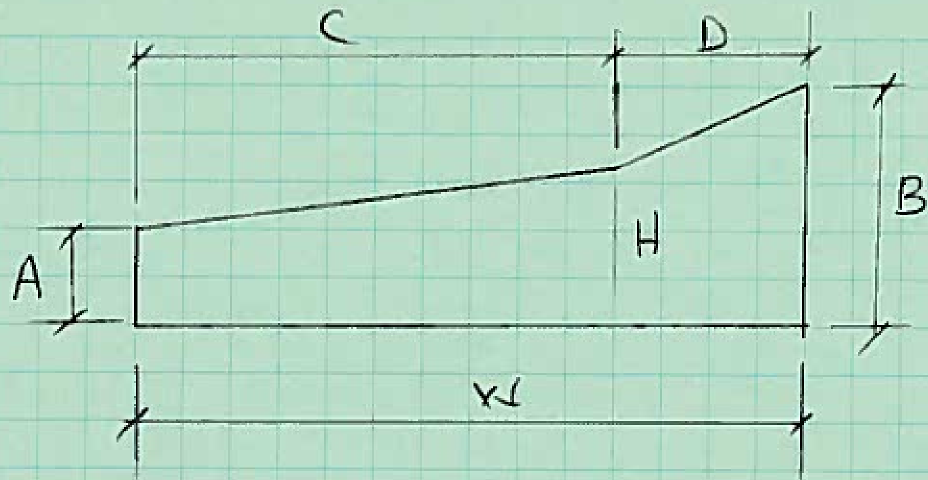
$H = h + n_1 C$

otherwise

$B = h$

$A = h - n_1 C - n_2 D$

$H = h - n_2 D$



A	$\bar{x}$	$A\bar{x}$
$\frac{1}{2}(A+H)C$	$\frac{1}{3}C\left(\frac{A+2H}{A+H}\right)$	$\frac{1}{6}C^2(A+2H)$
$\frac{1}{2}(B+H)D$	$C+\frac{1}{3}D\left(\frac{H+2B}{H+B}\right)$	$\frac{1}{2}(B+H)DC+\frac{1}{6}D^2(H+2B)$

$$\bar{x} = \frac{\frac{1}{6}C^2(A+2H)+\frac{1}{6}D^2(H+2B)+\frac{1}{2}(B+H)DC}{\frac{1}{2}(A+H)C+\frac{1}{2}(B+H)D}$$

$$\bar{x} = \frac{\frac{1}{3}C^2(A+2H)+\frac{1}{3}D^2(H+2B)+DC(B+H)}{(A+H)C+(B+H)D}$$

$$\bar{x} = \frac{1}{3}\cdot\frac{C^2(A+2H)+D^2(H+2B)+3DC(B+H)}{(A+H)C+(B+H)D}$$

$$k = \frac{\bar{x}}{W}$$

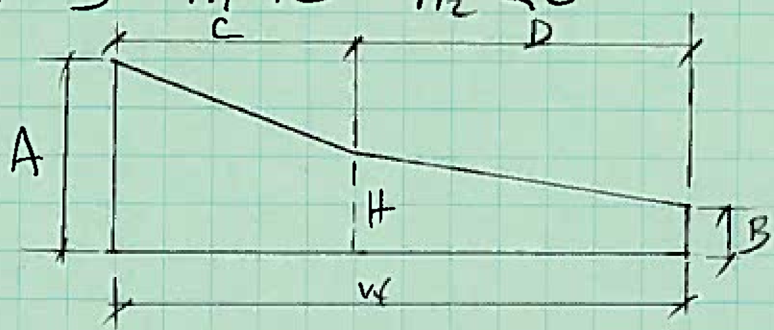
$$K = \frac{1}{3W} \cdot \frac{C^2(A+2H) + D^2(H+2B) + 3DC(B+H)}{(A+H)C + (B+H)D}$$

$$A = h$$

$$H = h + n_1 C$$

$$B = h + n_1 C + n_2 D$$

CASE 3 $n_1 < 0$ $n_2 < 0$



Area	$\bar{x}$	(Area) $\bar{x}$
$\frac{1}{2}(A+H)C$	$\frac{1}{3}C\frac{(2H+A)}{(A+H)}$	$\frac{1}{6}C^2(A+2H)$
$\frac{1}{2}(B+H)D$	$C+\frac{1}{3}D\frac{(H+2B)}{(B+H)}$	$\frac{1}{2}CD(B+H)+\frac{1}{6}D^2(2B+H)$

$$\bar{x} = \frac{\frac{1}{6}C^2(A+2H) + \frac{3}{6}CD(B+H) + \frac{1}{6}D^2(2B+H)}{\frac{1}{2}\left[(A+H)C + (B+H)D\right]}$$

$$K= \; = \frac{1}{3W}\,\frac{C^2(A+2H) + 3CD(B+H) + D^2(2B+H)}{\left[(A+H)C + (B+H)D\right]}$$

$B = h$

$A = h - n_2 D - n_1 C$

$H = h - n_2 D$

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Project CASE 4 $n_1 < 0$ $n_2 > 0$				Sheet No. of Sheets
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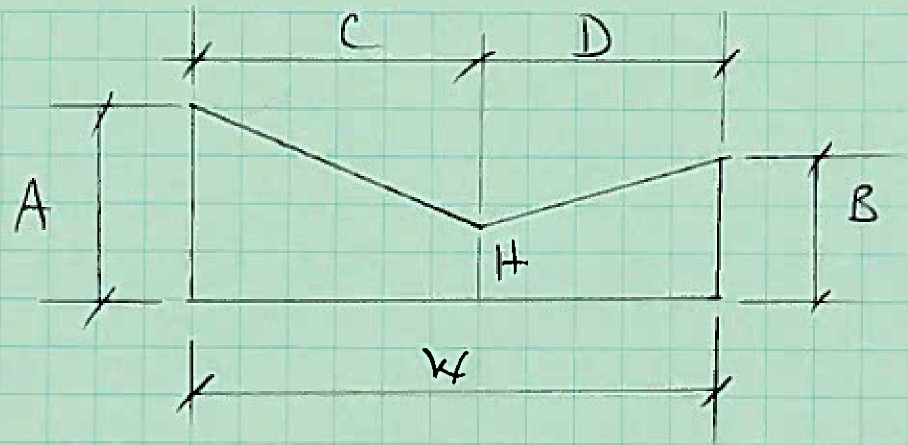

Area	$\bar{x}$	(Area) $\bar{x}$
$\frac{1}{2}(A+H)C$	$\frac{1}{3}C\frac{(2H+A)}{(A+H)}$	$\frac{1}{6}C^2(A+2H)$
$\frac{1}{2}(B+H)D$	$C+\frac{1}{3}D\left(\frac{H+2B}{H+B}\right)$	$\frac{1}{2}(B+H)DC+\frac{1}{6}D^2(2B+H)$

$$\bar{x} = \frac{\frac{1}{6}C^2(A+2H)+\frac{3}{6}DC(B+H)+\frac{1}{6}D^2(2B+H)}{\frac{1}{2}(A+H)C \quad +\frac{1}{2}(B+H)D}$$

$$k = \frac{1}{3W}\frac{C^2(A+2H)+3DC(B+H)+D^2(2B+H)}{\left[(A+H)C+(B+H)D\right]}$$

$A = h - n_1 C$

$B = h + n_1 D$

$H = h$

FOR ALL CASES

$$\bar{x} = \frac{1}{3} \frac{C^2(A+2H) + 3DC(B+H) + D^2(2B+H)}{(A+H)C + (B+H)D}$$

$$H = A + n_1 C \qquad B = A + n_1 C + n_2 D$$

$$\bar{x} = \frac{1}{3} \frac{C^2(A+2A+2n_1C) + 3DC(A+n_1C+n_2D+A+n_1C) + D^2(2A+2n_1C+2n_2D+A+n_1C)}{(A+A+n_1C)C + (A+n_1C+n_2D+A+n_1C)D}$$

$$\bar{x} = \frac{1}{3} \frac{C^2(3A+2n_1C) + 3DC(2A+2n_1C+n_2D) + D^2(3A+3n_1C+2n_2D)}{(2A+n_1C)C + (2A+2n_1C+n_2D)D}$$

$$\bar{x} = \frac{1}{3} \frac{C^2(3A+2n_1C) + 3DC(2A+2n_1C+n_2D) + D^2(3A+3n_1C+2n_2D)}{(2A)(C+D) + n_1C^2 + 2n_1CD + n_2D^2}$$

$$\bar{x} = \frac{1}{3} \frac{C^2(3A+2n_1C) + 3DC(2A+2n_1C+n_2D) + D^2(3A+3n_1C+2n_2D)}{2AW + n_1C^2 + 2n_1CD + n_2D^2}$$

Let $\bar{x} = \frac{1}{3} \frac{K_1}{K_2}$

$$K_1 = C^2(A+2H) + 3DC(B+H) + D^2(2B+H)$$

$$K_2 = (A+H)C + (B+H)D$$

First, some simplifications

$$C^2(A+2H) = C^2(3A+2n_1C) = 3AC^2 + 2n_1C^3$$

$$3DC(B+H) = 3(WC-C^2)(2A+2n_1C+n_2W-n_2C)$$

$$= 3(2AWC + 2n_1WC^2 + n_2W^2C - n_2WC^2 - 2AC^2 - 2n_1C^3 - n_2WC^2 + n_2C^3)$$

$$= 6AWC + 6n_1WC^2 + 3n_2W^2C - 3n_2WC^2 - 6AC^2 - 6n_1C^3 - 3n_2WC^2 + 3n_2C^3$$

$$D^2(2B+H) = (W^2 - 2WC + C^2)(3A + 3n_1C - 2n_2C + 2n_2W)$$

$$= 3AW^2 + 3n_1W^2C - 2n_2W^2C + 2n_2W^3$$
$$- 6AWC - 6n_1WC^2 + 4n_2WC^2 - 4n_2W^2C$$
$$+ 3AC^2 + 3n_1C^3 - 2n_2C^3 + 2n_2WC^2$$

$$= 3AW^2 + (3n_1 - 2n_2 - 4n_2)W^2C + (-6n_1 + 4n_2 + 2n_2)WC^2$$
$$+ 3AC^2 + (3n_1 - 2n_2)C^3 + 2n_2W^3 - 6AWC$$

$$= 3AW^2 + 3AC^2 - 6AWC + (3n_1 - 6n_2)W^2C - 6(n_1 - n_2)WC^2$$
$$+ (3n_1 - 2n_2)C^3 + 2n_2W^3$$

$$C^2(A+2H)+3DC(B+H)+D^2(2B+H)=$$

$$\cancel{3AC^2}+2n_1C^3+\cancel{6AWC}+\cancel{6n_1WC^2}+3n_2W^2C-\cancel{3n_2WC^2}-\cancel{6AC^2}$$

$$-6n_1C^3-\cancel{3n_2WC^2}+3n_2C^3+3AW^2+\cancel{3AC^2}-\cancel{6AWC}+3n_1W^2C$$

$$-6n_2W^2C-\cancel{6n_1WC^2}+\cancel{6n_1WC^2}+3n_1C^3-2n_2C^3+2n_2W^3$$

$$=2n_1C^3+3n_2W^2C-6n_1C^3+3n_2C^3+3AW^2+3n_1W^2C$$

$$-6n_2W^2C+3n_1C^3-2n_2C^3+2n_2W^3$$

$$=-n_1C^3+n_2C^3+3n_1W^2C+3n_2W^2C-6n_2W^2C+3AW^2+2n_2W^3$$

$$=-(n_1-n_2)C^3+3(n_1-n_2)W^2C+3AW^2+2n_2W^3$$

$$A+H = 2A + n_1 C$$

$$C(A+H) = 2AC + n_1 C^2$$

$$B+H = 2A + 2n_1 C + n_2 W - n_2 C$$

$$D(B+H) = (W-C)(B+H) = (W-C)(2A + 2n_1 C + n_2 W - n_2 C)$$

$$= 2AW + 2n_1 WC + n_2 W^2 - n_2 WC$$
$$-2AC - 2n_1 C^2 - n_2 WC + n_2 C^2$$

$$= 2AW - 2AC + 2(n_1 - n_2)WC - 2n_1 C^2 + n_2 C^2 + n_2 W^2$$

$$C(A+H) + D(B+H) =$$

$$\cancel{2AC} + n_1 C^2 + 2AW - \cancel{2AC} + 2(n_1 - n_2)WC - 2n_1 C^2 + n_2 C^2 + n_2 W^2$$

$$= 2AW + 2(n_1 - n_2)WC - (n_1 - n_2)C^2 + n_2 W^2$$

Substitute & Simplify

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$$\bar{X} = \frac{1}{3}\,\frac{2n_2W^3 - (n_1-n_2)C^3 + 3(n_1-n_2)W^2C + 3AW^2}{2AW + 2(n_1-n_2)WC - (n_1-n_2)C^2 + n_2W^2}$$

$$\text{Area} = AW + \tfrac{1}{2} n_1 C^2 + n_1 CD + \tfrac{1}{2} n_2 D^2$$

$$D = W - c$$

$$= AW + \tfrac{1}{2} n_1 C^2 + n_1 C(W-C) + \tfrac{1}{2} n_2 (W-C)^2$$

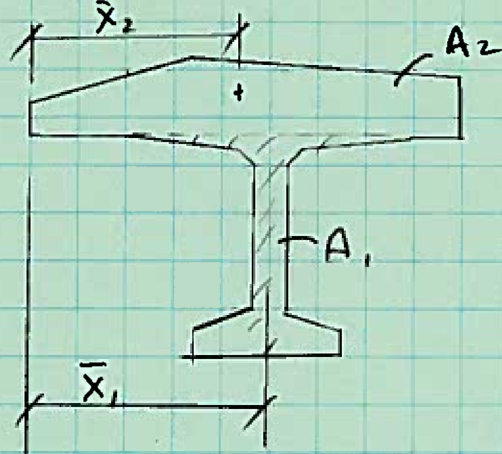
$$= AW + \tfrac{1}{2} n_1 C^2 - n_1 C^2 + n_1 CW + \tfrac{1}{2} n_2 (w^2 - 2wc + c^2)$$

$$= AW + \tfrac{1}{2} n_1 C^2 - n_1 c^2 + n_1 CW + \tfrac{1}{2} n_2 W^2 - n_2 CW + \tfrac{1}{2} n_2 c^2$$

$$= AW - \tfrac{1}{2} n_1 C^2 + \tfrac{1}{2} n_2 C^2 + (n_1 - n_2) CW + \tfrac{1}{2} n_2 W^2$$

$$\boxed{\text{Area} = AW + \tfrac{1}{2}(n_2 - n_1) C^2 - (n_2 - n_1) CW + \tfrac{1}{2} n_2 W^2}$$

$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{A_1\bar{x}_1 + A_2\bar{x}_2}{A_1 + A_2}$$



We want $\bar{x}$ such that $\bar{x} = \bar{x}_1$

$$\bar{X} = \frac{A_1\bar{x} + A_2\bar{x}_2}{A_1 + A_2}$$

$$A_1\bar{X} + A_2\bar{x} = A_1\bar{x} + A_2\bar{x}_2$$

$$A_2\bar{X} = A_2\bar{x}_2$$

$$\bar{X} = \bar{x}_2 = \frac{1}{3}\,\frac{2n_2W^3 - (n_1 - n_2)C^3 + 3(n_1 - n_2)W^2C + 3AW^2}{2AW + 2(n_1 - n_2)WC - (n_1 - n_2)C^2 + n_2W^2}$$

FIND k FOR

$$kW = \bar{x} = \frac{\int_0^L w(z)\,\hat{x}(z)\,dz}{\int_0^L w(z)\,dz}$$

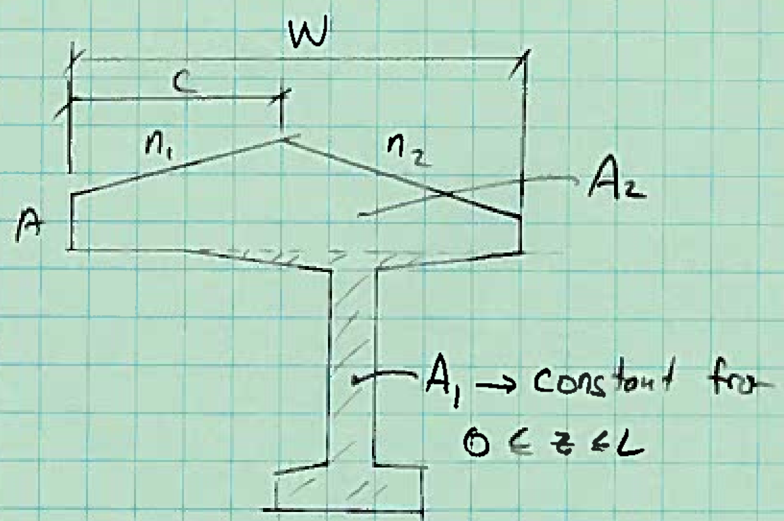
Recognize that

$$\int_0^L w(z)\,dz = W_g$$

Find W_g

Weight of Girder

$$W_g = \gamma \int_0^L A(z)\,dz$$



$$W_g = \gamma A_1 L + \gamma \int_0^L A_2(z)\,dz$$

$$A_2(z) = AW - \tfrac{1}{2}(n_1 - n_2)C^2 + (n_1 - n_2)CW + \tfrac{1}{2}n_2W^2$$

$$= AW + K \qquad K = -\tfrac{1}{2}(n_1 - n_2)C^2 + (n_1 - n_2)CW + \tfrac{1}{2}n_2W^2$$

Case 1a $\quad A = h(z)$

1b $\quad A = h(z) - n_2(W - C) - n_1C$

$\quad = h(z) - (n_1 - n_2)C - n_2W$

Case 2 $\quad A = h(z)$

Case 3 $\quad A = h(z) - (n_1 - n_2)C - n_2W$

Case 4 $\quad A = h(z) - n_1C$

$$h(z) = \frac{4(h_e - h_m)}{L^2}z^2 - \frac{4(h_e - h_m)}{L}z + h_e$$

$$= \frac{4h}{L^2}z^2 - \frac{4h}{L}z + h_e \qquad h = h_e - h_m$$

Case 1a & 2

$$W_g = \gamma \int_0^L w\left(\frac{4h}{L^2}z^2 - \frac{4h}{L}z + h_e\right) + k \; dz + \gamma A_1$$

$$W_g = \gamma A_1 L + \gamma\left[w\left(\frac{4h}{L^2}\cdot\frac{L^3}{3} - \frac{4h}{L}\cdot\frac{L^2}{2} + h_e L\right) + kL\right]$$

$$= \gamma A_1 L + \gamma\left[\left(\frac{4}{3}hL - \frac{6}{3}hL\right)w + L(wh_e + k)\right]$$

$$= \gamma A_1 L + \gamma\left[-\frac{2}{3}hLw + L(wh_e + k)\right]$$

$$= \gamma A_1 L + \gamma\left[w\left(h_e L - \frac{2}{3}hL\right) + kL\right]$$

$$= \gamma A_1 L + \gamma\left[w\left(h_e L - \frac{2}{3}(h_e - h_m)L\right) + kL\right]$$

$$= \gamma A_1 L + \gamma\left[w\left(\frac{1}{3}h_e L + \frac{2}{3}h_m L\right) + kL\right]$$

$$= \gamma A_1 L + \gamma L\left[w\left(\frac{1}{3}h_e + \frac{2}{3}h_m\right) + k\right]$$

$$W_g = \gamma A_1 L + \gamma L\left[w\left(\frac{1}{3}h_e + \frac{2}{3}h_m\right) - \frac{1}{2}(n_1 - n_2)C^2 + (n_1 - n_2)Cw + \frac{1}{2}n_2 w^2\right]$$

Case 1b & 3

$$W_g = \gamma A_1 L + \gamma \int_0^L W\left(\frac{4h}{L^2} z^2 - \frac{4h}{L} z + h_e - (n_1 - n_2)C - n_2 W\right) + K \quad dz$$

$$= \gamma A_1 L + \gamma \left[W\left(\frac{4h}{L^2}\cdot\frac{L^3}{3} - \frac{4h}{L}\frac{L^2}{2} + \left(h_e - (n_1-n_2)C - n_2 W\right)L\right) + KL \right.$$

$$= \gamma A_1 L + \gamma L\left[W\left(\frac{1}{3}h_e + \frac{2}{3}h_m\right) + K - (n_1-n_2)CW - n_2 W^2\right]$$

$$W_g = \gamma A_1 L + \gamma L\left[W\left(\frac{1}{3}h_e + \frac{2}{3}h_m\right) - \frac{1}{2}(n_1-n_2)C^2 + (n_1-n_2)CW + \frac{1}{2}n_2 W^2 - (n_1-n_2)CW - n_2 W^2\right]$$

$$W_g = \gamma A_1 L + \gamma L\left[W\left(\frac{1}{3}h_e + \frac{2}{3}h_m\right) - \frac{1}{2}(n_1-n_2)C^2 - \frac{1}{2}n_2 W^2\right]$$

Case 4

$$W_g = \gamma A_1 L + \gamma \int_0^L W\left(\frac{4h}{L^2}z^2 - \frac{4h}{L}z + h_e - n_1 c\right) + K \; dz$$

$$= \gamma A_1 L + \gamma \left[W\left(\frac{4h}{L^2}\cdot\frac{L^3}{3} - \frac{4h}{L}\cdot\frac{L^2}{2} + h_e L\right) - n_1 C W L + K L \right]$$

$$= \gamma A_1 L + \gamma L \left[W\left(\tfrac{1}{3}h_e + \tfrac{2}{3}h_m\right) - n_1 C W - \tfrac{1}{2}(n_1 - n_2)C^2 + (n_1 - n_2)CW + \tfrac{1}{2}n_2 W^2 \right]$$

$$W_g = \gamma A_1 L + \gamma L \left[W\left(\tfrac{1}{3}h_e + \tfrac{2}{3}h_m\right) - \tfrac{1}{2}(n_1 - n_2)C^2 - n_2 C W + \tfrac{1}{2}n_2 W^2 \right]$$

FINALLY, FIND k from

$$k\,W = \frac{1}{Wg}\int_0^L w(z)\,\bar{x}(z)\,dz$$

$$\int_0^L w(z)\,\bar{x}(z)\,dz = \int_0^L \gamma A(z)\,\bar{x}(z)\,dz$$

$$\frac{\Sigma \overline{Ax}}{\Sigma A}$$

$$= \int_0^L \gamma A(z) \frac{\overline{Ax}(z)}{A(z)} \, dz$$

$$= \gamma \int_0^L \overline{Ax}(z)\,dz$$

$$\overline{Ax}(z) = \frac{1}{6}C^2(A+2H) + \frac{1}{2}(CD)(B+H) + \frac{1}{6}D^2(2B+H) + A_1\bar{x}_1$$

$$H = A + n_1 C \qquad B = A + n_1 C + n_2 D$$

$$\overline{Ax} = \frac{1}{6}C^2(A+2A+2n_1C) + \frac{1}{2}(CD)(A+n_1C+n_2D+A+n_1C)$$
$$+ \frac{1}{6}D^2(2A+2n_1C+2n_2D+A+n_1C) + A_1\bar{x}_1$$

$$\overline{Ax} = \frac{1}{6}C^2(3A+2n_1C) + \frac{1}{2}(CD)(2A+2n_1C+n_2D) + \frac{1}{6}D^2(3A+3n_1C+2n_2D) + A_1\bar{x}_1$$

$$\overline{Ax} = \left(\frac{1}{2}C^2\right)A + \frac{1}{3}n_1C^3 + (CD)A + n_1C^2D + \frac{1}{2}n_2CD^2 + \left(\frac{1}{2}D^2\right)A + \frac{1}{2}n_1CD^2 + \frac{1}{3}n_2D^3 + A_1\bar{x}_1$$

$$\overline{Ax} = \left(\frac{1}{2}C^2 + CD + \frac{1}{2}D^2\right)A + \frac{1}{3}n_1C^3 + n_1C^2D + \frac{1}{2}n_2CD^2 + \frac{1}{2}n_1CD^2 + \frac{1}{3}n_2D^3 + A_1\bar{x}_1$$

$$\overline{Ax} = \underbrace{\left(\frac{1}{2}C^2 + CD + \frac{1}{2}D^2\right)}_{M}A + \underbrace{\frac{1}{3}n_1C^3 + \frac{1}{3}n_2D^3 + n_1C^2D + \frac{1}{2}(n_1+n_2)CD^2}_{N} + A_1\bar{x}_1$$

$$\overline{Ax} = MA + N + A_1\bar{x}_1$$

Case 1a $A = h(z)$

1b $A = h(z) - n_1 C - n_2 D = h(z) - n_1 C - n_2 W + n_2 C$

$= h(z) + (n_2 - n_1) C - n_2 W$

Case 2 $A = h(z)$

Case 3 $A = h(z) - n_1 C - n_2 D$

Case 4 $A = h(z) - n_1 C$

Case 1a & 2 $\quad A = h(z)$

$$\int_0^L w(z)\bar{x}(z)\,dz = \gamma\int_0^L \bar{A}x(z)\,dz = \gamma\int_0^L MA + N + A_1\bar{x}_1\,dz$$

$$= \gamma\int_0^L Mh(z) + N + A_1\bar{x}_1\,dz$$

$$= \gamma\int_0^L M\left(\frac{4\bar{h}}{L^2}z^2 - \frac{4\bar{h}}{L}z + h_e\right) + N + A_1\bar{x}_1\,dz$$

$$= \gamma\left[M\left(\frac{4\bar{h}}{L^2}\cdot\frac{L^3}{3} - \frac{4\bar{h}}{L}\frac{L^2}{2} + h_eL\right) + NL + A_1\bar{x}_1L\right]$$

$$= \gamma\left[M\left(\frac{4}{3}\bar{h}L - 2\bar{h}L + h_eL\right) + NL + A_1\bar{x}_1L\right]$$

$$= \gamma L\left[M\left(h_e - \frac{2}{3}\bar{h}\right) + N + A_1\bar{x}_1\right]$$

$\bar{h} = h_e - h_m$

$h_e - \frac{2}{3}h_e + \frac{2}{3}h_m$

$= \frac{1}{3}h_e + \frac{2}{3}h_m = \frac{1}{3}(h_e + 2h_m)$

$$= \gamma L\left[\frac{M}{3}(h_e + 2h_m) + N + A_1\bar{x}_1\right]$$

$$kW = \frac{1}{W_g}\int_0^L w(z)\bar{x}(z)\,dz = \frac{\gamma L}{W_g}\left[\frac{M}{3}(h_e + 2h_m) + N + A_1\bar{x}_1\right]$$

$$kW = \frac{\gamma L\left[\frac{M}{3}(h_e+2h_m)+N+A_1\bar{x}_1\right]}{\gamma A_1 L+\gamma L\left[\frac{W}{3}(h_e+2h_m)+\frac{1}{2}(n_1C^2+2n_1CD+n_2D^2)\right)}$$

γL cancels & $\bar{x}_1 = kW$

$$kWA_1 + kW\left[\frac{W}{3}(h_e+2h_m)+\frac{1}{2}(n_1C^2+2n_1CD+n_2D^2)\right] = \frac{M}{3}(h_e+2h_m)+N+A_1kW$$

subtract kWA_1

solve for k

$$k=\frac{\frac{M}{3}(h_e+2h_m)+N}{\left[\frac{W}{3}(h_e+2h_m)+\frac{1}{2}(n_1C^2+2n_1CD+n_2D^2)\right]W}$$

$$k=\frac{\frac{1}{3}(\frac{1}{2}C^2+CD+\frac{1}{2}D^2)(h_e+2h_m)+\frac{1}{3}n_1C^3+\frac{1}{3}n_2D^3+n_1C^2D+\frac{1}{2}(n_1+n_2)CD^2}{W\left[\frac{W}{3}(h_e+2h_m)+\frac{1}{2}(n_1C^2+2n_1CD+n_2D^2)\right]}$$

$$k=\frac{\frac{2}{6}(\frac{1}{2}C^2+CD+\frac{1}{2}D^2)(h_e+2h_m)+\frac{2}{6}n_1C^3+\frac{2}{6}n_2D^3+\frac{6}{6}n_1C^2D+\frac{3}{6}(n_1+n_2)CD^2}{\left[\frac{2}{6}W(h_e+2h_m)+\frac{3}{6}(n_1C^2+2n_1CD+n_2D^2)\right]W}$$

$$k=\frac{2}{W}\cdot\frac{(\frac{1}{2}C^2+CD+\frac{1}{2}D^2)(h_e+2h_m)+2n_1C^3+2n_2D^3+6n_1C^2D+3(n_1+n_2)CD^2}{2W(h_e+2h_m)+3(n_1C^2+2n_1CD+n_2D^2)}$$

Case 1 & 2

$$k = \frac{\frac{M}{3}\left(h_e + 2h_m\right) + N}{W\left[\frac{W}{3}\left(h_e + 2h_m\right) + \frac{1}{2}\left(n_1 C^2 + 2n_1 CD + n_2 D^2\right)\right]}$$

$$k = \frac{1}{W} \cdot \frac{\frac{6}{1}\left(\frac{1}{3}\right)\left(M\left(h_e + 2h_m\right) + 3N\right)}{\frac{6}{1}\left(\frac{1}{6}\right)\left(2W\left(h_e + 2h_m\right) + 3\left(n_2 C^2 + 2n_1 CD + n_2 D^2\right)\right)}$$

$$k = \frac{2}{W} \cdot \frac{M\left(h_e + 2h_m\right) + 3N}{2W\left(h_e + 2h_m\right) + 3\left(n_1 C^2 + 2n_1 CD + n_2 D^2\right)}$$

$$k = \frac{2}{W} \cdot \frac{\left(\frac{1}{2}C^2 + CD + \frac{1}{2}D^2\right)\left(h_e + 2h_m\right) + 3\left(\frac{1}{3}n_1 C^3 + \frac{1}{3}n_2 D^3 + n_1 C^2 D + \frac{1}{2}\left(n_1 + n_2\right) CD^2\right)}{2W\left(h_e + 2h_m\right) + 3\left(n_1 C^2 + 2n_1 CD + n_2 D^2\right)}$$

$D = W - C$

$CD = C(W-C) = WC - C^2$

$D^2 = (W-C)^2 = W^2 - 2WC + C^2$

$\frac{1}{2}C^2 + CD + \frac{1}{2}D^2 = \frac{1}{2}C^2 + WC - C^2 + \frac{1}{2}W^2 - WC + \frac{1}{2}C^2 = \frac{1}{2}W^2$

$C^2D = C^2(W-C) = WC^2 - C^3$

$CD^2 = W^2C - 2WC^2 + C^3$

$D^3 = (W-C)(W^2 - 2WC + C^2) = W^3 - 2W^2C + WC^2 - W^2C + 2WC^2 - C^3$

$= W^3 - 3W^2C + 3WC^2 - C^3$

$3\left(\frac{1}{3}n_1C^3 + \frac{1}{3}n_2D^3 + n_1C^2D + \frac{1}{2}(n_1+n_2)CD^2\right)$

$= n_1C^3 + n_2D^3 + 3n_1C^2D + \frac{3}{2}(n_1+n_2)CD^2$

$= n_1C^3 + n_2(W^3 - 3W^2C + 3WC^2 - C^3) + 3n_1(WC^2 - C^3) + \frac{3}{2}(n_1+n_2)(W^2C - 2WC^2 + C^3)$

$= n_1C^3 + n_2W^3 - 3n_2W^2C + 3n_2WC^2 - n_2C^3 + 3n_1WC^2 - 3n_1C^3$

$+ \frac{3}{2}n_1W^2C - 3n_1WC^2 + \frac{3}{2}n_1C^3$

$+ \frac{3}{2}n_2W^2C - 3n_2WC^2 + \frac{3}{2}n_2C^3$

$$= n_1 c^3 - n_2 c^3 - 3n_1 c^3 + \tfrac{3}{2} n_1 c^3 + \tfrac{3}{2} n_2 c^3$$

$$+ 3n_2 Wc^2 + 3n_1 Wc^2 - 3n_1 Wc^2 - 3n_2 Wc^2$$

$$- 3n_2 W^2 c + \tfrac{3}{2} n_1 W^2 c + \tfrac{3}{2} n_2 W^2 c$$

$$+ n_2 W^3$$

$$= -\tfrac{1}{2} n_1 c^3 + \tfrac{1}{2} n_2 c^3$$

$$+ 0$$

$$+ \tfrac{3}{2} n_1 W^2 c - \tfrac{3}{2} n_2 W^2 c$$

$$+ n_2 W^3$$

$$= \tfrac{1}{2}(n_2 - n_1) c^3 - \tfrac{3}{2}(n_2 - n_1) W^2 c + n_2 W^3$$

$$3(n_1 C^2 + 2n_1 CD + n_2 D^2)$$

$$= 3n_1 C^2 + 6n_1 CD + 3n_2 D^2$$

$$= 3n_1 C^2 + 6n_1 (WC - C^2) + 3n_2 (W^2 - 2WC + C^2)$$

$$= 3n_1 C^2 + 6n_1 WC - 6n_1 C^2 + 3n_2 W^2 - 6n_2 WC + 3n_2 C^2$$

$$= 3n_1 C^2 - 6n_1 C^2 + 3n_2 C^2$$

$$+ 6n_1 WC - 6n_2 WC$$

$$+ 3n_2 W^2$$

$$= -3n_1 C^2 + 3n_2 C^2 + 6(n_1 - n_2) WC + 3n_2 W^2$$

$$= 3(n_2 - n_1) C^2 - 6(n_2 - n_1) WC + 3n_2 W^2$$

$$h = \frac{2}{W} \cdot \frac{\frac{1}{2} W^2 (h_e + 2h_m) + \frac{1}{2}(n_2 - n_1) C^3 - \frac{3}{2}(n_2 - n_1) W^2 C + n_2 W^3}{2W(h_e + 2h_m) + 3(n_2 - n_1) C^2 - 6(n_2 - n_1) WC + 3n_2 W^2}$$

Case 1a & 2

Case 1b & 3 $\quad A = h(z) - n_1 C - n_2 D$

$$\int_0^L w(z)\,\bar{x}(z)\,dz = \gamma \int_0^L M(h(z) - n_1 C - n_2 D) + N + A_1 \bar{x}_1 \; dz$$

$$= \gamma \int_0^L M\left(\frac{4\bar{h}}{L^2} z^2 - \frac{4\bar{h}}{L} z + h_e - n_1 C - n_2 D\right) + N + A_1 \bar{x}_1 \; dz$$

$$= \gamma \left[M\left(\frac{4\bar{h}}{L^2} \cdot \frac{L^3}{3} - \frac{4\bar{h}}{L} \frac{L^2}{2} + h_e L - n_1 C L - n_2 D L\right) + N L + A_1 \bar{x}_1 L \right]$$

$$= \gamma \left[M\left(\frac{4}{3}\bar{h} L - 2\bar{h} L + h_e L - n_1 C L - n_2 D L\right) + N L + A_1 \bar{x}_1 L \right]$$

$$= \gamma \left[M\left(h_e L - \frac{2}{3}\bar{h} L - n_1 C L - n_2 D L\right) + N L + A_1 \bar{x}_1 L \right]$$

$$\bar{h} = h_e - h_m \qquad h_e - \frac{2}{3}\bar{h} = h_e - \frac{2}{3}h_e + \frac{2}{3}h_m = \frac{1}{3}h_e + \frac{2}{3}h_m$$

$$= \frac{1}{3}(h_e + 2h_m)$$

$$= \gamma L \left[M\left(\frac{1}{3}(h_e + 2h_m) - n_1 C - n_2 D\right) + N + A_1 \bar{x}_1 \right]$$

$$= \gamma L \left[\frac{M}{3}\left(h_e + 2h_m - 3n_1 C - 3n_2 D\right) + N + A_1 \bar{x}_1 \right]$$

$$\bar{x}W = \frac{1}{W_g}\int_0^L w(z)\bar{x}(z)\,dz = \frac{\gamma L}{W_g}\left[\frac{M}{3}\left(h_e + 2h_m - 3n_1C - 3n_2D\right) + N + A_1\bar{x}\right]$$

$$\bar{x}W = \frac{\gamma L\left[\frac{M}{3}\left(h_e + 2h_m - 3n_1C - 3n_2D\right) + N + A_1\bar{x}\right]}{\gamma A_1 L + \gamma L\left[\frac{W}{3}\left(h_e + 2h_m\right) - \frac{1}{2}(n_1 - n_2)C^2 - \frac{1}{2}n_2W^2\right]}$$

divide out γL

$\bar{x}_1 = \bar{x}W$ subtract $\bar{x}A_1W$

$$\bar{x} = \frac{\frac{M}{3}\left(h_e + 2h_m - 3n_1C - 3n_2D\right) + N}{W\left[\frac{W}{3}\left(h_e + 2h_m\right) - \frac{1}{2}(n_1 - n_2)C^2 - \frac{1}{2}n_2W^2\right]}$$

$$\bar{x} = \frac{1}{W}\,\frac{6\left(\frac{M}{3}\right)\left(h_e + 2h_m - 3n_1C - 3n_2D\right) + 3N}{6\left(\frac{1}{6}\right)\left[2W\left(h_e + 2h_m\right) - 3(n_1 - n_2)C^2 - 3n_2W^2\right]}$$

$$\bar{x} = \frac{2}{W}\,\frac{M\left(h_e + 2h_m - 3n_1C - 3n_2D\right) + 3N}{2W\left(h_e + 2h_m\right) - 3(n_1 - n_2)C^2 - 3n_2W^2}$$

$$\bar{x} = \frac{2}{W}\,\frac{\left(\frac{1}{2}C^2 + CD + \frac{1}{2}D^2\right)\left(h_e + 2h_m - 3n_1C - 3n_2D\right) + 3\left(\frac{1}{3}n_1C^3 + \frac{1}{3}n_2D^3 + n_1C^2D + \frac{1}{2}(n_1 + n_2)\ldots\right.}{2W\left(h_e + 2h_m\right) - 3(n_1 - n_2)C^2 - 3n_2W^2}$$

$$k = \frac{2}{W}\,\frac{\frac{1}{2}W^2(h_e+2h_m) + \frac{1}{2}W^2\left(-3n_1C - 3n_2(W-C)\right) + \frac{1}{2}(n_2-n_1)C^3 - \frac{3}{2}(n_2-n_1)W^2C + n_2W^3}{2W(h_e+2h_m) - 3(n_1-n_2)C^2 - 3n_2W^2}$$

$$= \frac{2}{W}\,\frac{\frac{1}{2}W^2(h_e+2h_m) - \frac{3}{2}W^2\left(n_1C - n_2C + n_2W\right) + \frac{1}{2}(n_2-n_1)C^3 - \frac{3}{2}(n_2-n_1)W^2C + n_2W^3}{2W(h_e+2h_m) + 3(n_2-n_1)C^2 - 3n_2W^2}$$

$$= \frac{2}{W}\,\frac{\frac{1}{2}W^2(h_e+2h_m) + \cancel{\frac{3}{2}(n_2-n_1)W^2C} - \frac{3}{2}n_2W^3 + \frac{1}{2}(n_2-n_1)C^3 - \cancel{\frac{3}{2}(n_2-n_1)W^2C} + n_2W^3}{2W(h_e+2h_m) + 3(n_2-n_1)C^2 - 3n_2W^2}$$

$$k = \frac{2}{W}\,\frac{\frac{1}{2}W^2(h_e+2h_m) + \frac{1}{2}(n_2-n_1)C^3 - \frac{1}{2}n_2W^3}{2W(h_e+2h_m) + 3(n_2-n_1)C^2 - 3n_2W^2}$$

* Case 1b & 3

$$\cancel{k = \frac{2}{W}\,\frac{W^2(h_e+2h_m) + (n_2-n_1)C^3 - n_2W^3}{2W^2(h_e+2h_m) + 3(n_2-n_1)WC^2 - 3n_2W^3}}$$

~~Case 1b & 3~~

*

Case 4 $\quad A = h(z) - n_1 C$

$$\int_0^L w(z)\bar{x}(z)\,dz = \gamma \int_0^L A\bar{x}(z)\,dz = \gamma \int_0^L MA + N + A_1\bar{x}_1 \,dz$$

$$= \gamma \int_0^L M(h(z) - n_1 C) + N + A_1\bar{x}_1 \,dz$$

$$= \gamma \int_0^L M\left(\frac{4\bar{h}}{L^2}z^2 - \frac{4\bar{h}}{L}z + h_e - n_1 C\right) + N + A_1\bar{x}_1 \,dz$$

$$= \gamma \left[M\left(\frac{4\bar{h}}{L^2}\frac{L^3}{3} - \frac{4\bar{h}}{L}\frac{L^2}{2} + h_e L - n_1 C L\right) + NL + A_1\bar{x}_1 L\right]$$

$$= \gamma \left[M\left(\frac{4}{3}\bar{h}L - 2\bar{h}L + h_e L - n_1 C L\right) + NL + A_1\bar{x}_1 L\right]$$

$$= \gamma \left[M(h_e L - \tfrac{2}{3}\bar{h}L - n_1 C L) + NL + A_1\bar{x}_1 L \right]$$

$$h_e - \tfrac{2}{3}\bar{h} = \tfrac{1}{3}(h_e + 2h_m)$$

$$= \gamma L\left[\frac{M}{3}(h_e + 2h_m - 3n_1 C) + N + A_1\bar{x}_1\right]$$

$$kW = \frac{1}{W_g}\int_0^L w(z)\,\bar{x}(z)\,dz = \frac{\gamma L}{W_g}\left[\frac{M}{3}(h_e + 2h_m - 3n_1 C) + N + A_1\bar{x}_1\right]$$

$$kW = \frac{\gamma L\left[\frac{M}{3}(h_e + 2h_m - 3n_1 C) + N + A_1\bar{x}_1\right]}{\gamma A_1 L + \gamma L\left[\frac{W}{3}(h_e + 2h_m) - Wn_1 C + \frac{n_1 C^2}{2} + n_1 CD + \frac{n_2 D^2}{2}\right]}$$

divide out γL

$\bar{X}_1 = kW$ subtract out kA_1W

$$k = \frac{\frac{M}{3}(h_e + 2h_m - 3n_1 C) + N}{W\left[\frac{W}{3}(h_e + 2h_m) - Wn_1 C + \frac{n_1 C^2}{2} + n_1 CD + \frac{n_2 D^2}{2}\right]}$$

$$k = \frac{1}{W}\cdot\frac{\frac{6}{1}\left(\frac{1}{3}\right)\left[M(h_e + 2h_m - 3n_1 C) + 3N\right]}{\frac{6}{1}\left(\frac{1}{6}\right)\left[2W(h_e + 2h_m) - 6Wn_1 C + 3n_1 C^2 + 6n_1 CD + 3n_2 D^2\right]}$$

$$k = \frac{2}{W}\cdot\frac{M(h_e + 2h_m - 3n_1 C) + 3N}{2W(h_e + 2h_m) - 6Wn_1 C + 3n_1 C^2 + 6n_1 CD + 3n_2 D^2}$$

$$k = \frac{2}{W}\cdot\frac{\left(\frac{1}{2}C^2 + CD + \frac{1}{2}D^2\right)(h_e + 2h_m - 3n_1 C) + 3\left[\frac{1}{3}n_1 C^3 + \frac{1}{3}n_2 D^3 + n_1 C^2 D + \frac{1}{2}(n_1 + n_2)CD^2\right]}{2W(h_e + 2h_m) - 6Wn_1 C + 3(n_1 C^2 + 2n_1 CD + n_2 D^2)}$$

$$k = \frac{2}{W}\,\frac{\left(\frac{1}{2}C^2 + CD + \frac{1}{2}D^2\right)(h_c + 2h_m) - \left(\frac{1}{2}C^2 + CD + \frac{1}{2}D^2\right)(3n_1 C) + 3\left[\frac{1}{3}n_1 C^3 + \frac{1}{3}n_2 D^3 + n_1 C^2 D + \frac{1}{2}(n_1 + n_2)CD^2\right]}{2W(h_c + 2h_m) - 6Wn_1 C + 3(n_1 C^2 + 2n_1 CD + n_2 D^2)}$$

$$k = \frac{2}{W}\cdot\frac{\frac{1}{2}W^2(h_c + 2h_m) - \frac{3}{2}n_1 W^2 C + \frac{1}{2}(n_2 - n_1)C^3 - \frac{3}{2}(n_2 - n_1)W^2 C + n_2 W^3}{2W(h_c + 2h_m) - 6n_1 WC + 3(n_2 - n_1)C^2 - 6(n_2 - n_1)WC + 3n_2 W^2}$$

$$-\frac{3}{2}n_1 W^2 C + \frac{1}{2}(n_2 - n_1)C^3 - \frac{3}{2}(n_2 - n_1)W^2 C + n_2 W^3$$

$$= -\frac{3}{2}n_1 W^2 C - \frac{3}{2}n_2 W^2 C + \frac{3}{2}n_1 W^2 C + \frac{1}{2}(n_2 - n_1)C^3 + n_2 W^3$$

$$= -\frac{3}{2}n_2 W^2 C + \frac{1}{2}(n_2 - n_1)C^3 + n_2 W^3$$

$$-6n_1 WC - 6(n_2 - n_1)WC = -6n_2 WC$$

$$k = \frac{2}{W}\cdot\frac{\frac{1}{2}W^2(h_c + 2h_m) - \frac{3}{2}n_2 W^2 C + \frac{1}{2}(n_2 - n_1)C^3 + n_2 W^3}{2W(h_c + 2h_m) - 6n_2 WC + 3(n_2 - n_1)C^2 + 3n_2 W^2}$$

Case 4

Summary

Case 1a & 2

$$k = \frac{W^2(h_e + 2h_m) + (n_2 - n_1)C^3 - 3(n_2 - n_1)W^2C + 2n_2W^3}{2W^2(h_e + 2h_m) + 3(n_2 - n_1)WC^2 - 6(n_2 - n_1)W^2C + 3n_2W^3}$$

Case 1b & 3

$$k = \frac{W^2(h_e + 2h_m) + (n_2 - n_1)C^3 - n_2W^3}{2W^2(h_e + 2h_m) + 3(n_2 - n_1)WC^2 - 3n_2W^3}$$

Case 4

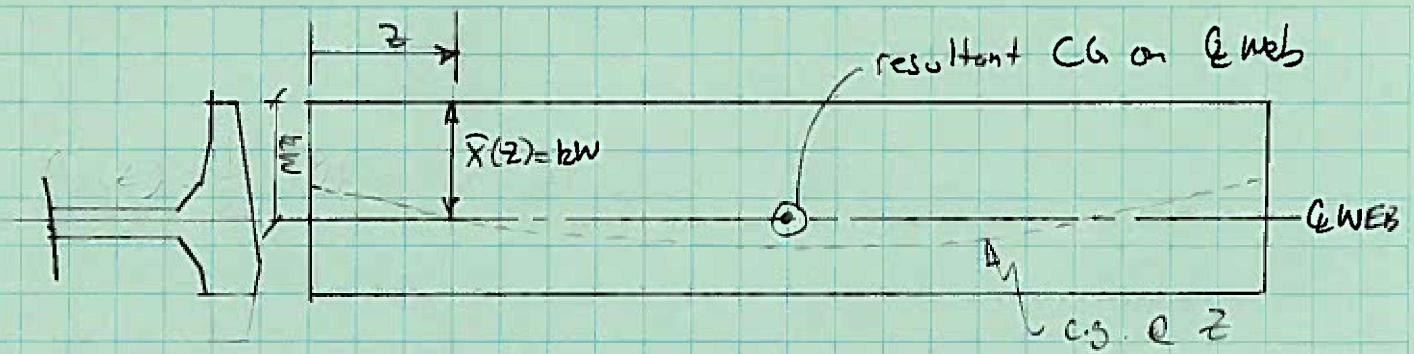
$$k = \frac{W^2(h_e + 2h_m) + (n_2 - n_1)C^3 - 3(n_2 - n_1)W^2C + 2n_2W^3}{2W^2(h_e + 2h_m) + 3(n_2 - n_1)WC^2 - 6n_2W^2C + 3n_2W^3}$$

FIND THE VALUE OF Z

WHEN $\bar{x}(z) = kW$

- not really useful for anything but checking results

- because of top flange thickening, h_e & h_m, the CG moves transversely along the length of the girder



What is the value of z when $\bar{x}(z) = kW$?

$$\bar{X} = \frac{\sum A\bar{x}}{\sum A} = \frac{A_1\bar{x}_1 + A_2\bar{x}_2}{A_1 + A_2} = kW$$

$$(A_1 + A_2)kW = A_1\bar{x}_1 + A_2\bar{x}_2$$

$$A_1 kW + A_2 kW = A_1\bar{x}_1 + A_2\bar{x}_2$$

$$\bar{x}_1 = kW$$

$$\cancel{A_1 kW} + A_2 kW = \cancel{A_1 kW} + A_2\bar{x}_2$$

$$kW = \frac{A_2\bar{x}_2}{A_2}$$

$$kW = \bar{x}_2$$

$$KW = \frac{1}{3}\,\frac{C^2(3A+2n_1C) + 3DC(2A+2n_1C+n_2D) + D^2(3A+3n_1C+2n_2D)}{2AW + n_1C^2 + 2n_1CD + n_2D^2}$$

$A = f_n(h(t))$ – isolate A

$$3KW(2AW + n_1C^2 + 2n_1CD + n_2D^2) = C^2(3A+2n_1C) + 3DC(2A+2n_1C+n_2D) + D^2(3A+3n_1C+2n_2D)$$

$$6KW^2A - 3C^2A - 6CDA - 3D^2A =$$

$$2n_1C^3 + 3DC(2n_1C+n_2D) + D^2(3n_1C+2n_2D) - 3KW(n_1C^2+2n_1CD+n_2D^2)$$

$$(6KW^2 - 3C^2 - 6CD - 3D^2)A =$$

$$2n_1C^3 + 3DC(2n_1C+n_2D) + D^2(3n_1C+2n_2D) - 3KW(n_1C^2+2n_1CD+n_2D^2)$$

$$A = \frac{2n_1C^3 + 3DC(2n_1C+n_2D) + D^2(3n_1C+2n_2D) - 3KW(n_1C^2+2n_1CD+n_2D^2)}{6KW^2 - 3(C^2+2CD+D^2)}$$

$D = C - W$

$$3DC(2n_1C + n_2D)$$

$$= 3(WC - C^2)(2n_1C + n_2W - n_2C)$$

$$= 3(2n_1WC^2 + n_2W^2C - n_2WC^2 - 2n_1C^3 - n_2WC^2 + n_2C^3)$$

$$= 6n_1WC^2 + 3n_2W^2C - 3n_2WC^2 - 6n_1C^3 - 3n_2WC^2 + 3n_2C^3$$

$$D^2(3n_1C + 2n_2D)$$

$$= (W^2 - 2WC + C^2)(3n_1C + 2n_2W - 2n_2C)$$

$$= 3n_1W^2C + 2n_2W^3 - 2n_2W^2C - 6n_1WC^2 - 4n_2W^2C + 4n_2WC^2$$
$$+ 3n_1C^3 + 2n_2WC^2 - 2n_2C^3$$

$$3DC(2n_1C + n_2D) + D^2(3n_1C + 2n_2D)$$

$$= 6n_1WC^2 + 3n_2W^2C - 3n_2WC^2 - 6n_1C^3 - 3n_2WC^2 + 3n_2C^3$$
$$+ 3n_1W^2C + 2n_2W^3 - 2n_2W^2C - 6n_1WC^2 - 4n_2W^2C + 4n_2WC^2$$
$$+ 3n_1C^3 + 2n_2WC^2 - 2n_2C^3$$

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Bridge & Structures Design Calculations

Project				Sheet No. of Sheets
S.R.	Made By	Check by	Date	Supv

$$= -6n_1C^3 + 3n_2C^3 + 3n_1C^3 - 2n_2C^3$$

$$+ \cancel{6n_1WC^2} - \cancel{3n_2WC^2} - \cancel{3n_2WC^2} - \cancel{6n_1WC^2} + \cancel{4n_2WC^2} + \cancel{2n_2WC^2}$$

$$+ 3n_2W^2C + 3n_1W^2C - 2n_2W^2C - 4n_2W^2C$$

$$+ 2n_2W^3$$

$$= -3n_1C^3 + n_2C^3$$

$$+ 0$$

$$+ 3n_1W^2C - 3n_2W^2C$$

$$+ 2n_2W^3$$

$$= -3n_1C^3 + n_2C^3 + 3(n_1 - n_2)W^2C + 2n_2W^3$$

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Bridge & Structures Design Calculations

Project				Sheet No. of Sheets
S.R.	Made By	Check by	Date	Supv

$$3KW(n_1C^2 + 2n_1CD + n_2D^2)$$

$$= KW[3(n_2-n_1)C^2 - 6(n_2-n_1)WC + 3n_2W^2]$$

$$C^2 + 2CD + D^2 = 2\left(\tfrac{1}{2}C^2 + CD + \tfrac{1}{2}D^2\right) = 2\left(\tfrac{1}{2}W^2\right) = W^2$$

$$A = \frac{2n_1C^3 - 3n_1C^3 + n_2C^3 - 3(n_2-n_1)W^2C + 2n_2W^3 - KW[3(n_2-n_1)C^2 - 6(n_2-n_1)WC + 3n_2W^2]}{6KW^2 - 3W^2}$$

$$A = \frac{(n_2-n_1)C^3 - 3(n_2-n_1)W^2C + 2n_2W^3 - KW[3(n_2-n_1)C^2 - 6(n_2-n_1)WC + 3n_2W^2]}{3W^2(2K-1)}$$

$$(n_2-n_1)C^3 - 3(n_2-n_1)W^2C + 2n_2W^3 - 3KW(n_2-n_1)C^2 + 6K(n_2-n_1)W^2C - 3n_2KW^3$$

$$= (n_2-n_1)C^3 - 3KW(n_2-n_1)C^2 + 3(2K-1)(n_2-n_1)W^2C + n_2(2-3K)W^3$$

$$A = \frac{(n_2-n_1)C^3 - 3KW(n_2-n_1)C^2 + 3(2K-1)(n_2-n_1)W^2C + n_2(2-3K)W^3}{3W^2(2K-1)}$$

$$A = R$$

Case 1a & 2 $\quad A = h(z)$

$$\frac{4\bar{h}}{L^2} z^2 - \frac{4\bar{h}}{L} z + h_e = R \qquad \bar{h} = h_e - h_m$$

Solve for z

$$\frac{4(h_e - h_m)}{L^2} z^2 - \frac{4(h_e - h_m)}{L} z + h_e - R = 0$$

$$z = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{4(h_e - h_m)}{L^2} \qquad b = -\frac{4(h_e - h_m)}{L} \qquad c = h_e - R$$

Case 1b & 3

$$A = h(z) - n_1 C - n_2 D$$

$$= h(z) - n_1 C - n_2 W + n_2 C$$

$$= h(z) + (n_2 - n_1)C - n_2 W$$

$$\frac{4(h_e - h_m)}{L^2} z^2 - \frac{4(h_e - h_m)}{L} z + h_e + (n_2 - n_1)C - n_2 W - R = 0$$

Solve for z

Case 4

$$A = h(z) - n_1 C$$

$$\frac{4}{L^2}(h_e - h_m) z^2 - \frac{4}{L}(h_e - h_m) z + h_e - n_1 C - R = 0$$

Solve for z