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Initial and Time-Dependent Deformation of Progressively Cracking Nonprestressed and Partially Prestressed Concrete Beams

by Susanto Teng and Dan E. Branson

An accurate time-step procedure for computing the initial and time-dependent deformation of progressively cracking nonprestressed and partially prestressed concrete flexural members is presented. The procedure is based on the I-effective method, which has been applied to many different cases and has been proven worldwide to be very reliable for predicting the initial deformation of cracked prestressed and nonprestressed members. The extension of the I-effective method to the time-dependent deformation of cracked nonprestressed and partially prestressed members has so far been achieved by using long-term multipliers.

In the proposed procedure, the I*-effective method is used in combination with the effective centroidal axis formula* Y_e *to perform the time-dependent analysis on the effective section, taking into account the effects of progressive cracking and tension stiffening. Experimental data from different groups of researchers have been used to verify the accuracy of the proposed procedure. From these comparisons, it can be seen that the proposed procedure gives accurate predictions of the initial and time-dependent deformation of progressively cracking concrete members for all prestressing range.*

*Discussions on the effect of prestress force on the value of the effective-*I, *and lower limits on the effective-*I, *etc., are also presented to clarify these concepts.*

Keywords: beams (supports); camber; computer programs; **cracking (fracturing);** creep properties; deflection; flexural strength; loads (forces); modulus of elasticity; **moments of inertia;** one-way slabs; **partial prestressing;** prestressed concrete; reinforced concrete; shrinkage; structural members.

The problem of predicting the initial and time-dependent deformation of progressively cracking concrete flexural members can be very complex. In addition to uncertainties involved in predicting the common time-dependent characteristics of concrete, such as compressive strength, modulus of elasticity, creep, shrinkage, etc., the effects of prestress losses, top reinforcement in nonprestressed members, and nonprestressed steel in prestressed members are very significant. Progressive cracking and tension stiffening are also very important factors.

When overloaded into the cracking range, a concrete flexural member undergoes progressive cracking as the load is increased further or as time passes. This progressive cracking nature is due to the presence of reinforcement and the ability of concrete to resist some tensile stress. For sectional design and stress analysis, it is common to neglect the small contribution of the concrete in tension. For deflection computations, however, the contribution of the concrete in tension (immediately below the neutral axis) should be taken into account, since it increases considerably the stiffness of the cracked member. The uncracked concrete sections between two cracks also increase the overall effective stiffness of the cracked member. This phenomenon and the contribution of progressive cracking have been termed collectively tension stiffening.

Despite the inherently complex nature of the deflection problem in progressively cracking nonprestressed and partially prestressed concrete flexural members, much has been contributed over the years by researchers worldwide to the understanding of this time-dependent and cracking behavior (see Reference 1 for complete review). However, a reliable method for detailed computations of the time-dependent deformations of cracked partially prestressed concrete flexural members is needed urgently by design engineers for them to be able to predict the long-term behavior of members with confidence.

RESEARCH SIGNIFICANCE

The purpose of this research is to present a new, unified procedure for deformation computation that is applicable to both cracked and uncracked concrete flexural members of all prestressing range. The proposed procedure is based on the familiar *I*-effective method, and it performs the time-dependent analysis directly on the effective section. The accuracy of the proposed procedure has been verified by experimental data from four different groups of researchers and has been found to be reliable and accurate.

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EFFECTIVE MOMENT OF INERTIA AND CENTROIDAL AXIS LOCATION

When a concrete flexural member is still uncracked, its deflection or camber can be computed on the basis of the uncracked moment of inertia I_{ucr}. Once cracking occurs, computations should be done on the basis of the effective moment of inertia I_e as a way to include the effect of tension stiffening and, therefore, also progressive cracking (Fig. 1).

The upper limit for I_e is the uncracked value I_{ucr}, while the lower limit is the fully cracked value I_{cr}. The I-effective method was originally proposed for nonprestressed members,[2] and its application to cracked prestressed members has been suggested by many researchers and has proved to be accurate and convenient.[3,4] However, there are quite a number of different opinions on how to best apply the I-effective method to cracked prestressed members. The difference in opinions are related mostly to the presence of prestressing. The term M_{cr}/M_a in the I-effective formula [see Eq. (1)] was derived without considering prestressing; thus, it may need to be modified. Since prestressing can be expected to increase the effect of progressive cracking, some suggest that I_{cr} needs also to be modified. Whether a major modification to the I_e equation is necessary or whether this modification will result in better prediction are questions that need to be investigated further.

The basic expression for the effective moment of inertia at a particular section can be written in the following general form as a function of time t_i

$$I_e(t_i) = \left\{\frac{M_{cr}(t_i)}{M_{L2}(t_i)}\right\}^4 I_{ucr}(t_i) + \left[1 - \left\{\frac{M_{cr}(t_i)}{M_{L2}(t_i)}\right\}^4\right] I_{cr}(t_i) \le I_{ucr}(t_i) \quad (1)$$

where

$$M_{cr}(t_i) = \frac{f_r(t_i) I_{ucr}(t_i)}{c_2(t_i)} + \frac{[P_{ps}(t_i) - P_{ns}(t_i)] I_{ucr}(t_i)}{A_{ucr}(t_i) c_2(t_i)} \quad (2)$$

This is the cracking moment of the member at time t_i. The first term corresponds to the cracking moment of nonprestressed members while the second term is the contribution of the prestress force and nonprestressed steel. The second term should go to zero for nonprestressed members.

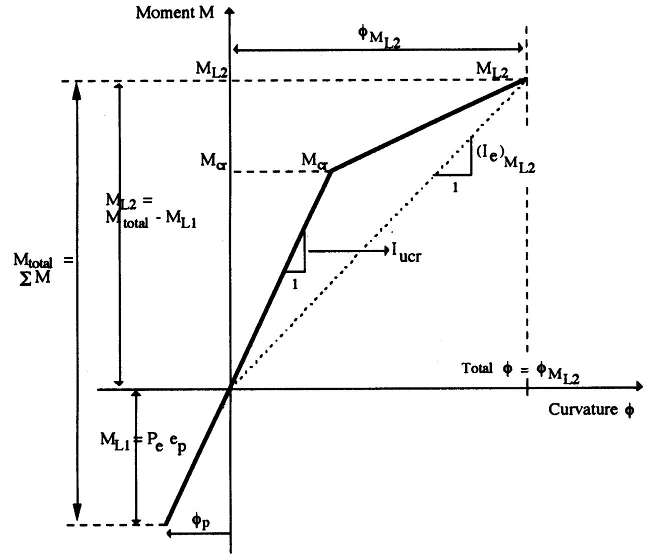


Fig. 1 — Moment-curvature relation for proposed procedure

$f_r(t_i)$ = concrete modulus of rupture
$c_2(t_i)$ = uncracked centroidal distance measured from bottom face
$P_{ps}(t_i)$ = prestress force transferred to concrete at time t_i (after losses)
$A_{ucr}(t_i)$ = uncracked transformed cross-sectional area
$P_{ns}(t_i)$ = force in bottom nonprestressed steel due to creep caused by prestress force

$$M_{L2}(t_i) = M_{L1}(t_i) + \sum_{j=1}^{i} M(t_j) \quad (3)$$

This is the total applied moment (including self-weight) at the section considered, where

$$M_{L1}(t_i) = -P_{ps}(t_i) e_{ps}(t_i) \quad (4)$$

where $e_{ps}(t_i)$ = eccentricity of prestressed steel on the basis of uncracked section. For nonprestressed members, $M_{L1}(t_i)$ is zero.

$I_{ucr}(t_i)$ = uncracked transformed moment of inertia at time t_i
$I_{cr}(t_i)$ = transformed fully cracked moment of inertia
$M(t_j)$ = individual sustained load moment applied at time t_j. $M(t_1)$ represents the self-weight moment plus any additional moment applied at the same time as the self-weight. $M(t_j)$, for computational purposes, equals zero when applied after time t_i

To perform a detailed time-dependent analysis on the effective section, the location of the effective centroidal axis is needed. The proposed time-dependent I_e formula given in Eq. (1) represents an interpolation of two values: the uncracked I and the fully cracked I. Thus, the value of I_e will be between I_{ucr} and I_{cr}. The logical consequence of this is that I_e should also be used with respect to the centroid of the effective section (Fig. 2). This will correctly take care of the effect of tension stiffening and progressive cracking.

Branson and Trost[3] have suggested an empirical equation

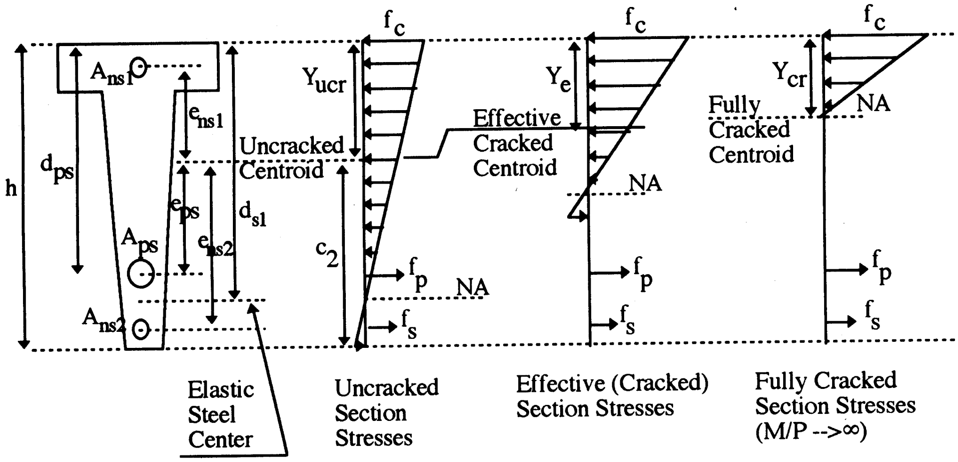


Fig. 2 — Stress distributions in uncracked, effective, and fully cracked sections

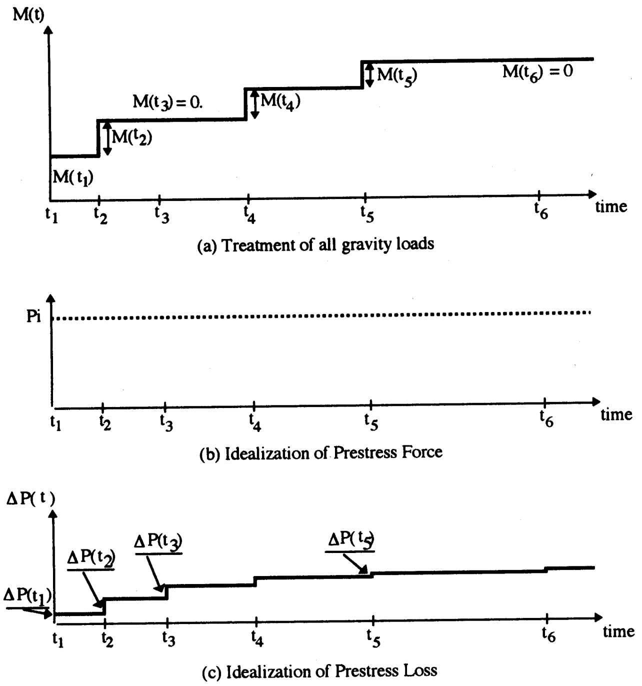


Fig. 3 — Treatment of loads, prestress, and prestress losses

which provides the best fit for the centroidal depth of the effective cracked section. The equation can be written in a modified form as a function of time as follows

$$Y_e(t_i) = \left\{\frac{M_{cr}(t_i)}{M_{L2}(t_i)}\right\}^{2.5} Y_{ucr}(t_i) + \left[1 - \left\{\frac{M_{cr}(t_i)}{M_{L2}(t_i)}\right\}^{2.5}\right] Y_{cr}(t_i) \le Y_{ucr}(t_i) \qquad (5)$$

where

$Y_{ucr}(t_i)$ = depth of centroid of uncracked transformed section at time t_i

$Y_{cr}(t_i)$ = depth of centroid of fully cracked transformed section at time t_i

By referring to the depth of the effective centroidal axis Y_e, all time-dependent stress and strain computations can be performed directly on the effective section.

TIME-DEPENDENT STRAINS AND CURVATURE

The exact computation of strains and curvature for a cracked concrete section is very complex and certain assumptions, such as plane sections remain plain and a perfect bond exists between the steel and concrete, have to be made to simplify the analysis. The proposed time-dependent strains and curvature[1] combine the formula of Neville and Dilger[5] with the I_e concept to extend its applicability to cover cracked members. For loading increasing in magnitude with time, the principle of superposition will be used, and for unloading or load decreasing with time, such as the prestress force, the corresponding creep recovery will be considered as a negative creep (Fig. 3). Creep and shrinkage in a prestressed member will cause prestress loss in the prestressed steel and possible stress gain in the nonprestressed steel. These effects will be treated collectively while the prestress loss due to stress relaxation will be treated separately.

In prestressed members, creep is caused by the prestress force and gravity loads, including self-weight. From the discussion that follows, it is found that the effect of the prestress force should always be considered with respect to the uncracked section while the effect of the gravity loads should be considered with respect to the effective section. Prestress loss due to relaxation can be represented by an additional gravity load, and thus should be considered with respect to the effective section.

For a cracked nonprestressed or partially prestressed section, the general curvature of the section at any time can be determined step by step from the strain distribution across the section. It is more convenient to use the strains at the top and bottom steel levels to obtain the curvature. When the top reinforcement is nonexistent, the top fiber strain can be used. The following are the final forms of the strain relations at any time at the level of steel reinforcements (all terms are time-dependent, except the constants).

The creep strains at the levels of top reinforcement (Subscript 1) and bottom reinforcement (Subscript 2), due to prestress force plus the effect of nonprestressed steel are (note that bottom reinforcement may contain both prestressed and nonprestressed steel; tensile stress and strain are positive; prestress force, downward deflection, and downward loads are positive)

$$\Delta\varepsilon_{1Pcr}(t_i) = \frac{(1+b_{22})f_{cs1} - b_{21}f_{cs2}}{E_c[(1+b_{11})(1+b_{22}) - b_{12}b_{21}]} C_t(t_i, t_1) \quad (6)$$

$$\Delta\varepsilon_{2Pcr}(t_i) = \frac{(1+b_{11})f_{cs2} - b_{12}f_{cs1}}{E_c[(1+b_{11})(1+b_{22}) - b_{12}b_{21}]} C_t(t_i, t_1) \quad (7)$$

where

$$b_{11} = \frac{A_{s1}E_s}{A_cE_c}\left(1 + \frac{e_{s1}e_{s1}}{r^2}\right)[1 + C_t(t_i, t_1)] \quad (8)$$

$$b_{12} = \frac{A_{s1}E_s}{A_cE_c}\left(1 + \frac{e_{s1}e_{s2}}{r^2}\right)[1 + C_t(t_i, t_1)] \quad (9)$$

$$b_{21} = \frac{A_pE_p + A_{s2}E_s}{A_cE_c}\left(1 + \frac{e_{s2}e_{s1}}{r^2}\right)[1 + C_t(t_i, t_1)] \quad (10)$$

$$b_{22} = \frac{A_pE_p + A_{s2}E_s}{A_cE_c}\left(1 + \frac{e_{s2}e_{s2}}{r^2}\right)[1 + C_t(t_i, t_1)] \quad (11)$$

$$e_{s1} = d_{s1} - Y_{ucr} \quad (12)$$

$$e_{s2} = \frac{A_pE_pe_{ps} + A_{s2}E_se_{ns2}}{A_pE_p + A_{s2}E_s} \quad (13)$$

where

$C_t(t_i,t_1)$ = creep coefficient at time t_i measured from the time of prestress release t_1 given by[6]

$$C_t(t_i, t_1) = \frac{(t_i - t_1)^{0.6}}{10 + (t_i - t_1)^{0.6}} C_u$$

f_{cs1} = stress in concrete due to initial prestress force at level of top reinforcement
f_{cs2} = stress in concrete due to initial prestress force at level of bottom reinforcement
e_{ps} = eccentricity of prestressed steel
e_{ns2} = eccentricity of bottom nonprestressed steel
e_{s1} = eccentricity of top reinforcement
e_{s2} = eccentricity of bottom reinforcement (combination of prestressed and nonprestressed steel)
r = uncracked radius of gyration
C_u = ultimate creep coefficient

All these calculations refer to the uncracked section. For shrinkage, the strains in concrete will be influenced by creep when reinforcement is present. Shrinkage strains at the levels of steel reinforcements are given in the following. They are also applied with respect to the uncracked section[5]

$$\Delta\varepsilon_{1sh}(t_i) = \frac{-(1+b_{22} - b_{21})}{[(1+b_{11})(1+b_{22}) - b_{12}b_{21}]} S_t(t_i, t_1) \quad (14)$$

$$\Delta\varepsilon_{2sh}(t_i) = \frac{-(1+b_{11} - b_{12})}{[(1+b_{11})(1+b_{22}) - b_{12}b_{21}]} S_t(t_i, t_1) \quad (15)$$

where

$S_t(t_i,t_1)$ = shrinkage strain at time t_i measured from time of prestress release t_1 given by[6]

$$S_t(t_i, t_1) = \frac{(t_i - t_1)}{f + (t_i - t_1)} S_u$$

f = 35 for moist-cured concrete with $t_1 \geq 7$ days
f = 55 for steam-cured concrete with $t_1 \geq 3$ days
S_u = ultimate shrinkage strain

The creep strain increment from time t_{i-1} to t_i due to gravity loads is then taken into account one at a time in the same manner as prestress force, except that it may have to be applied on the effective section at the time of load application. The procedure allows an application of load at each time increment. When the section is still uncracked, the immediate strain increments from time t_{i-1} to t_i at top and bottom steel levels are given by

$$\Delta\varepsilon_{1Mgr}(t_i) = \frac{M(t_i)e_{s1}(t_i)}{E_c(t_i)I_e(t_i)} \quad (16)$$

$$\Delta\varepsilon_{2Mgr}(t_i) = \frac{M(t_i)e_{s2}(t_i)}{E_c(t_i)I_e(t_i)} \quad (17)$$

When the section has already cracked, then the total moment applied up to time t_i is divided into M_{L1} and M_{L2} and the relation for immediate strain increment for time-step t_{i-1} to t_i becomes

$$\Delta\varepsilon_{1Mgr}(t_i) = \frac{M_{L2}e_{s1}(t_i)}{E_c(t_i)I_e(t_i)} + \frac{M_{L1}e_{s1u}(t_i)}{E_c(t_i)I_{ucr}(t_i)} - \sum_{j=1}^{i-1} \Delta\varepsilon_{1Mgr}(t_i) \quad (18)$$

$$\Delta\varepsilon_{2Mgr}(t_i) = \frac{M_{L2}e_{s2}(t_i)}{E_c(t_i)I_e(t_i)} + \frac{M_{L1}e_{s2u}(t_i)}{E_c(t_i)I_{ucr}(t_i)} - \sum_{j=1}^{i-1} \Delta\varepsilon_{2Mgr}(t_j) \quad (19)$$

where subscript ucr and additional subscript u refer to uncracked section.

The creep strain increments due to applied loads during the time-step t_{i-1} to t_i are then given by

$$\Delta\varepsilon_{1M}(t_i) = \sum_{j=1}^{i}\left\{\frac{(1+b_{22})\Delta\varepsilon_{1Mgr}(t_j) - b_{21}\Delta\varepsilon_{2Mgr}(t_j)}{[(1+b_{11})(1+b_{22}) - b_{12}b_{21}]} C_t(t_i, t_j)\right\} - \Delta\varepsilon_{1M}(t_{i-1}) \quad (20)$$

$$\Delta\varepsilon_{2M}(t_i) = \sum_{j=1}^{i}\left\{\frac{(1+b_{11})\Delta\varepsilon_{2Mgr}(t_j) - b_{12}\Delta\varepsilon_{1Mgr}(t_j)}{[(1+b_{11})(1+b_{22}) - b_{12}b_{21}]} C_t(t_i, t_j)\right\} - \Delta\varepsilon_{2M}(t_{i-1}) \quad (21)$$

where $C_t(t_i,t_j)$ is the creep coefficient at time t_i measured from time t_j.

The force picked up by the bottom nonprestressed steel can then be computed as

$$P_{ns}(t_i) = -\left\{\Delta\varepsilon_{2Pcr}(t_i) + \Delta\varepsilon_{2sh}(t_i) + \sum_{j=1}^{i}\Delta\varepsilon_{2M}(t_i)\right\}E_s \geq 0 \quad (22)$$

The total additional curvature at the section due to time-dependent prestress, creep, and shrinkage can then be computed by

$$\Delta\phi(i) = \frac{(\Delta\varepsilon_{2Pcr} - \Delta\varepsilon_{1Pcr}) + (\Delta\varepsilon_{2sh} - \Delta\varepsilon_{1sh})}{e_{s2} - e_{s1}} + \sum_{j=1}^{i}\left[\frac{[\Delta\varepsilon_{2M}(j) - \Delta\varepsilon_{1M}(j)]}{e_{s2}(j) - e_{s1}(j)} + \Delta\phi_{pLR}(j)\right] \quad (23)$$

The second term in the summation is the curvature due to prestress relaxation loss. It is to be applied to the effective section by assuming that a tensile force of the same magnitude as the relaxation loss is applied to the effective section at the level of prestressed steel.

TIME-DEPENDENT PRESTRESS LOSSES

To predict the time-dependent deflection of a prestressed member accurately, an accurate method for computing the prestress losses is needed. The method has to take into consideration the interdependent effects of creep, shrinkage, and stress relaxation on the total loss of prestress force. Several methods are available for prestress loss predictions; for example, those proposed by Branson,[6] Branson, Meyers, and Kripanarayanan,[7] and Tadros, Ghali, and Dilger.[8]

The general time-dependent prestress loss equation can be written in the following form

$$(\Delta f_p)t_i = \Delta f_{pE} + \Delta f_{pC} + \Delta f_{pS} + \Delta f_{pRR} - \Delta f_{pM} \quad (24)$$

where Δf_{pE} = elastic shortening loss.

$$\Delta f_{pE} = n_{ps} f_{ci} \quad \text{for pretensioned members} \quad (25a)$$

$$\Delta f_{pE} = 0.5 n_{ps} f_{ci} \quad \text{for post-tensioned members} \quad (25b)$$

$$\Delta f_{pE} = 0.0 \quad \text{for nonprestressed members} \quad (25c)$$

$\Delta f_{pC}(t_i)$ = loss due to creep of concrete

$$= -E_p\left\{\Delta\varepsilon_{2Pcr}(t_i) + \sum_{j=1}^{i}\Delta\varepsilon_{2M}(t_j)\right\} \quad (26)$$

$\Delta f_{pS}(t_i)$ = loss due shrinkage of concrete (including effect of creep, etc.)

$$= -E_p\Delta\varepsilon_{2sh}(t_i) \quad (27)$$

$\Delta f_{pRR}(t_i)$ = loss due stress relaxation

$$= X_R\Delta f_{pR}(t_i) \quad (28)$$

$\Delta f_{pM}(t_i)$ = increase in stress due to applied loads

$$= \sum_{j=1}^{i}\frac{n_{ps}M(t_j)e_{ps}}{I_e} \quad (29)$$

where

n_{ps} = modular ratio E_{ps}/E_c
n_{ns} = modular ratio E_{ns}/E_c
X_R = reduced relaxation coefficient
Δf_{pR} = PCI constant length relaxation loss at time (t_i)

$$= f_{pi}\left[\frac{\log t_i - \log t_1}{R_c}\left(\frac{f_{pi}}{f_{py}} - 0.55\right)\right]$$

f_{pi} = initial stress in prestressing steel
f_{py} = yield strength of prestressing steel
R_c = 10 for stress-relieved strands or 45 for low-relaxation strands

For computing prestress loss due to stress relaxation in actual prestressed beams, a reduced stress relaxation coefficient X_R is introduced to take into account the fact that the length of prestressed beams reduce a little over time, thus reducing the constant length relaxation loss. This phenomenon was recognized by Branson,[6] and Branson, Meyers, and Kripanarayanan,[7] who suggested that only 75 percent of the constant length relaxation loss can be used. Ghali and Trevino[9] later introduced the relaxation reduction factor to handle this problem in a more detailed manner.

SUMMARY OF PROPOSED TIME-STEP I_e PROCEDURE

Basically, the total period of analysis should be divided into several smaller time-steps (time-intervals) and the number of time-intervals selected for the computation of stresses and strains will determine the accuracy of the method. Since a numerical integration of the curvatures is used to obtain the deflection at any point along the member, the accuracy of the method also depends on how many sections are analyzed along the span. Following is the time-step procedure performed at a certain section along the span.

Step 1 — Preliminary calculations: set all variables to zero, and calculate initial stresses, strains, and curvature due to prestress force only.

Step 2 — Time-steps start here. For i = 1 to $n$steps (number of time-steps), calculate instantaneous stresses, strains, and curvature increments at time t_i due to

$$\sum_{j=1}^{i-1} M(t_j)_1$$

Table 1 — Details of experimental beams

Beam no.	b, mm	h, mm	Span, m	f'_c, N/mm²	C_u	S_u	A_{ns1}, mm²	A_{ns2}, mm²	A_{ps}, mm²	P_i, kN
B1*	152	203	6.10	24	4.8	0.0007	400	400	—	—
B2*	152	203	6.10	24	4.8	0.0007	200	400	—	—
B3*	152	203	6.10	24	4.8	0.0007	0	400	—	—
T1†	200	400	4.80	43	0.4	0.0008	100	630	—	—
T2†	200	400	4.80	39	0.4	0.0008	56.6	0	274	275
S1‡	152	203	4.57	45	2.5	0.0006	0	0	49.5	133
S2‡	152	203	4.57	45	2.5	0.0006	0	0	71.3	133
S3‡	152	203	4.57	45	2.5	0.0006	0	200	71.3	133

*Washa and Fluck experiments; †Branson and Trost experiments; ‡Branson and Shaikh experiments. (See Reference 1 for complete treatment of all experimental data.)
1 m = 3.28 ft; 1 mm = 0.0394 in; 1 mm² = 0.00155 in²; 1 kN = 0.2248 kips; 1 N/mm² = 145 psi.

Self-weight is included in $M(t_1)$. Effect of cracking is to be included. First perform uncracked analysis, and then use Eq. (1) when the section cracks.

Step 3 — Calculate strains and curvature increments due to creep effects caused by $P(t_1)$ and

$$\sum_{j=1}^{i-1} M(t_j)_1$$

For strains due to prestress force, use the uncracked section, but for strains due to gravity moments, use the effective section. Use Eq. (6), (7), and (16) through (21).

Step 4 — Calculate strains and curvature increments due to shrinkage, taking into account the time-dependent effects of creep, and prestressed and nonprestressed steel [Eq. (14) and (15)].

Step 5 — Calculate time-dependent prestress loss in prestressed steel and stress gain/loss in nonprestressed steel due to the effects of creep, shrinkage, and relaxation [Eq. (22) and (24)].

Step 6 — Calculate stress, strain, and curvature changes in concrete fibers due to gradual prestress relaxation loss [Eq. (28)].

Step 7 — Calculate stress changes in concrete fibers due to gradual change of stress in prestressed and nonprestressed steel as a result of creep and shrinkage of concrete.

Step 8 — Calculate final stresses, strains, and curvature at time t_i [Eq. (23)].

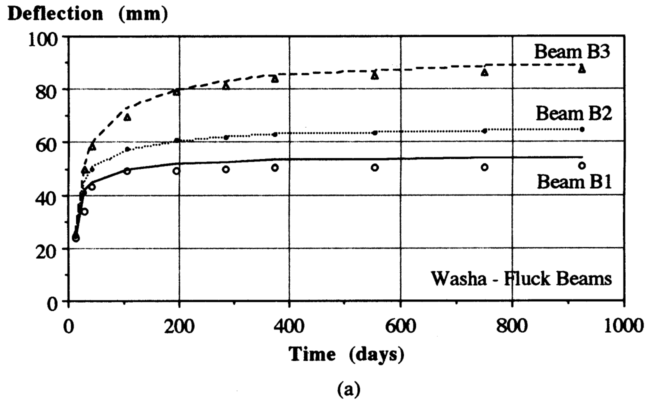
Step 9 — Check for limiting values.

Step 10 — For next section or next time-step, go to Step 2.

The type of loading considered is the step loading and is assumed to be applied at a certain time and sustained indefinitely. Varieties of different loadings can be achieved by combining several of these basic types of loadings. The procedure considers that the member may be prestressed or nonprestressed, may crack under self-weight or under additional loads, and may be uncracked throughout the analysis.

EXPERIMENTAL COMPARISONS AND DISCUSSION

To cover all range of prestressing, four groups of experimental data from four different sources were used for verifications,[1] namely, experiments by Branson and Trost,[10] Shaikh and Branson,[11] Jittawait,[12] and Washa and Fluck.[13] Table 1 shows details of some of the experimental beams. Complete treatment of all the experimental data, comparisons of theoretical and experimental results, and detailed step-by-step numerical examples are given in Reference 1.



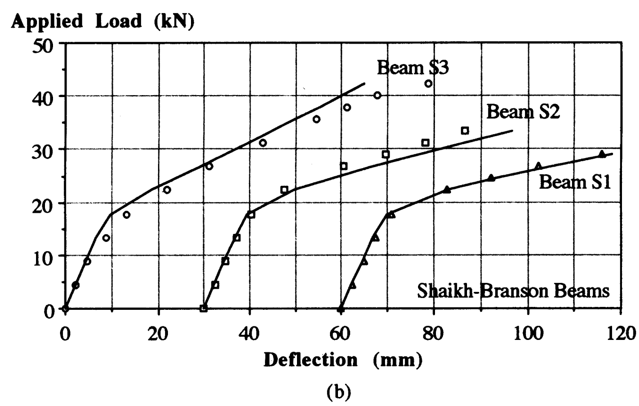


Fig. 4 — Some comparisons between experimental and computed results (1mm = 0.0394 in.; 1 kN = 0.2248 kip)

Comparisons of some of the theoretical and experimental results from Washa-Fluck[13] and Shaikh and Branson[11] experiments are shown in Fig. 4. It can be seen that, for the most part, the theoretical predictions deviate by only about 10 percent from the experimental values. In fact (see also Fig. 5), out of 19 sets of comparisons from a total of 25 beams, a total of 184 comparison data points were made and 80 percent of the data was predicted within 10 percent error, 94 percent within 15 percent error, and 97 percent within 20 percent error. This indicates that the proposed procedure is accurate and reliable.

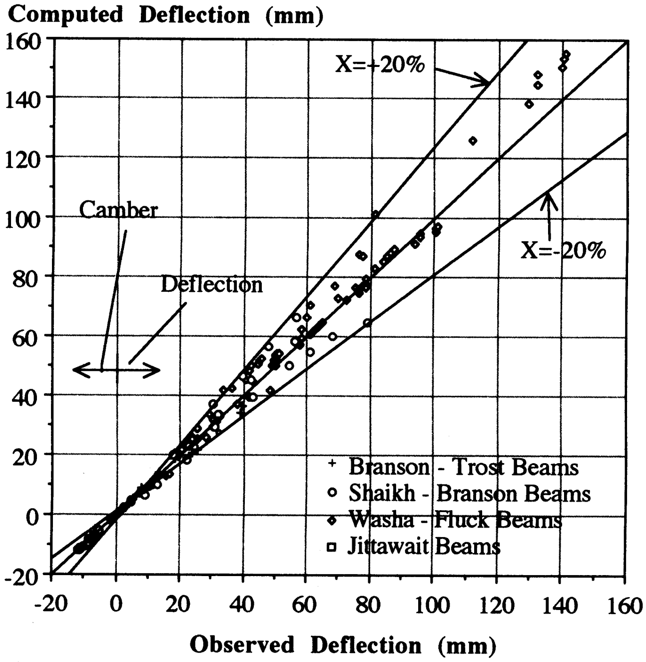


Fig. 5 — Comparison of all computed results to all observed data (1 mm = 0.0394 in.)

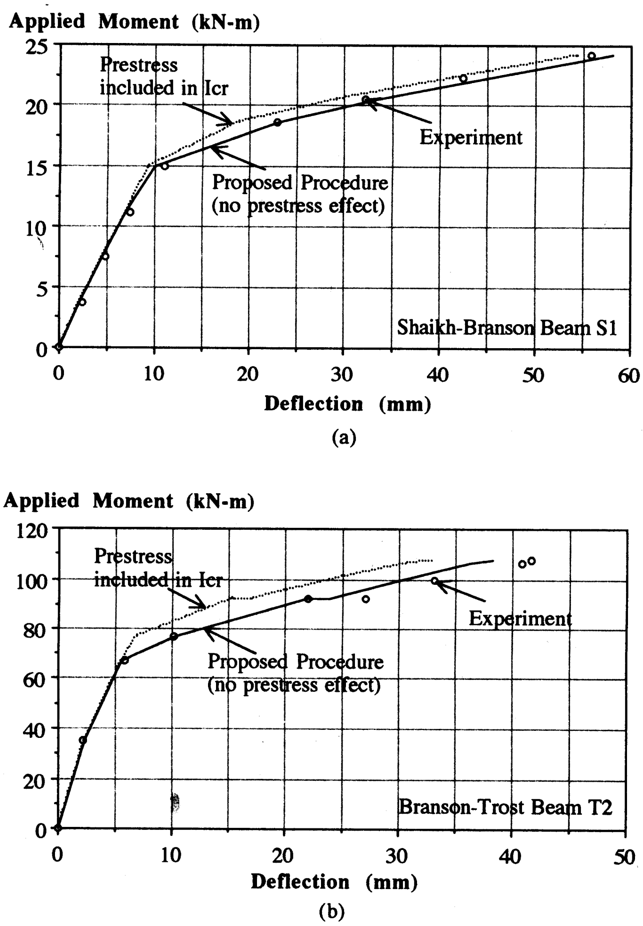


Fig. 6 — Underestimation of deflections due to incorrect lower limit for I_e value (1 mm = 0.0394 in.; 1 kN-m = 0.737 ft-kip)

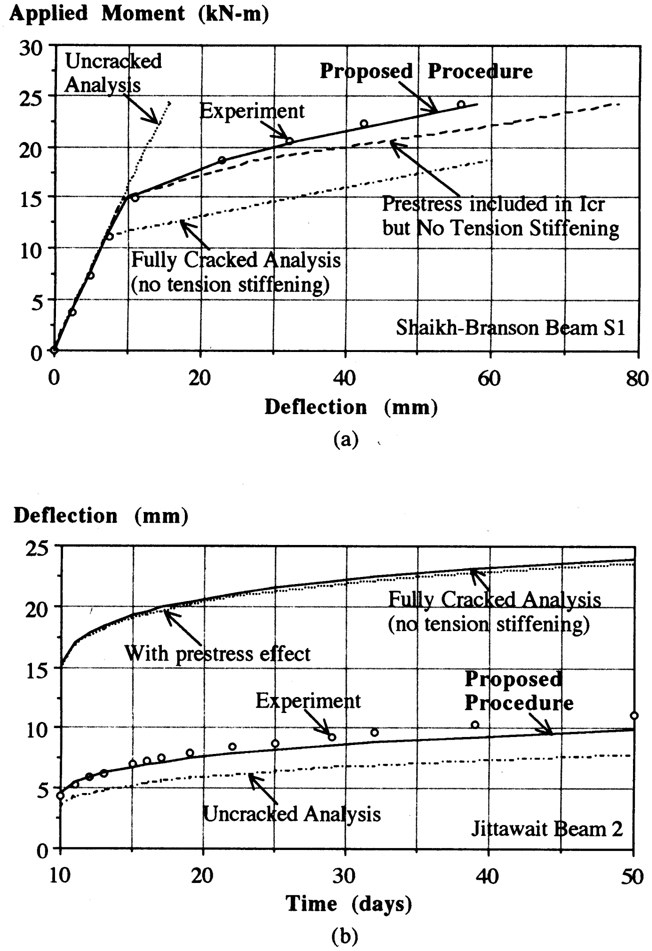


Fig. 7 — Effect of tension stiffening on deflections of cracked concrete beams (1mm = 0.0394 in.; 1 kN-m = 0.737 ft-kip)

The original I_e equation uses the fully cracked I (without considering the effect of prestress force) for the lower limit of I_e. Some researchers[3,4,11] later suggested the use of this fully cracked I as the lower limit for prestressed members as well. Naaman and Siriaksorn,[14] and Harajli and Alameh[15] suggested that the lower limit be the fully cracked I with the effect of prestress force included. Bennet and Lee[16] and Elzanaty[17] concluded from their experiments that the difference between the two lower limits was small. Fig. 6 shows comparisons between these two possible alternatives. It is clear that the differences between the pair of results for the three beams shown in the figure can be quite significant. The predicted results using the cracked I plus prestress as the lower limit of I_e seem to lead consistently to underestimations of the actual deflections. Another advantage of using the fully cracked I is that it requires a significantly less computational effort compared to using the cracked I plus prestress, which requires the use of a numerical scheme to determine the location of the neutral axis.

In nonprestressed members, the effect of tension stiffening and progressive nature of concrete cracking is well known. In prestressed members, the presence of the prestress force complicates the computations. However, several researchers[18-20] (see also Reference 1) have proposed time-step methods for the prediction of the time-dependent behavior of a cracked prestressed member. These methods, however, neglect the important contributions of tension stiffening and progressive cracking in reducing the deflection. Fig. 7 shows that neglecting the effect of tension stiffening and progressive cracking leads to significant overestimations of the actual deflection. Suggestions by Elbadry and Ghali,[21] and Joo and Tadros[22] to overcome this shortcoming leads to a somewhat lengthy procedure. The proposed time-step I_e procedure represents a reliable and accurate method for analyzing progressively cracking partially prestressed members in a time-step manner, taking into account directly the effect of tension stiffening by performing the time-dependent analysis on the effective section.

SUMMARY AND CONCLUSIONS

1. A time-step I_e procedure for computing the initial and time-dependent deformation of progressively cracking concrete flexural members of all prestressing range has been presented. The procedure has been verified by experimental data is found to be accurate and reliable. The proposed procedure recognizes the fact that a concrete member may crack at any time: immediately after the application of load or at a later time. Cracking in concrete may progress further under an increasing load or under constant load as time passes. The significant effect of tension stiffening and progressive nature of concrete cracking is thus taken into account.

2. The effects of both top and bottom nonprestressed steel on time-dependent deformation were taken into account accurately in the proposed procedure. Camber is reduced significantly when bottom nonprestressed steel is present. Downward deflection, on the other hand, is reduced when top nonprestressed steel is used. This effect is found to be similar for all range of prestressing.

3. The predictions of the proposed procedure are accurate even up to about 80 to 85 percent of the ultimate load, which is well above the service load range as well as the elastic range for which deflections are computed normally. The reason for this seems to be that the lower limit of I-effective value is the correctly defined fully cracked I. This premise is confirmed by the general understanding that the ultimate load of a prestressed member is the same as that of a nonprestressed member with the same quantity and quality of reinforcement.

4. The proposed procedure has been implemented on the computer and is named "Progressive_I_e". An extensive graphics capability has been added to the program as an aid in the interpretations of the predicted results. Almost all important quantities can be shown graphically in either 2-D or 3-D views.[1]

CONVERSION FACTORS

1 m = 3.28 ft
1 cm = 0.394 in.
1mm = 0.0394 in.
1 mm^2 = 0.00155 in.^{2}
1 kN = 0.2248 kip
1 kN - m = 0.737 ft - kip

NOTATION

A, A_c = cross-sectional area, cross-sectional area of concrete
$C_t(t_i,t_j)$ = creep coefficient at time t_i measured from time t_j, defined as the ratio of creep strain to initial strain
c_2 = centroidal distance of concrete section measured from bottom face
E_c, E_s = moduli of elasticity of concrete and steel
e = eccentricity of steel, force, etc.
f = stress in concrete or steel
f_{pi} = initial stress in prestressing steel
f_{py} = yield strength of prestressing steel
f_r = concrete modulus of rupture
I = moment of inertia
$M(t_j)$ = individual sustained load moment applied at time t_j
M_{cr} = cracking moment of concrete member
P = force transferred to concrete
r = uncracked radius of gyration
$S_t(t_i,t_j)$ = shrinkage strain at time t_i measured from time t_j
X_R = reduced relaxation coefficient
Y = depth of centroid of section measured from top surface
(t_i) = calculated at current time t_i
t_1 = start of analysis or time of prestress release
$\Delta\varepsilon$ = increment of strain
$\Delta\phi$ = increment in curvature
Δf_p = change in prestress or prestress loss
Δf_{pE} = elastic shortening loss

Subscripts

1 = top surface or top steel level of concrete member
2 = bottom surface or bottom steel level of concrete member
C = due to creep
cr = fully cracked condition
e = effective value
i = initial value
$L1$ = part of applied load effect corresponding to zero curvature
$L2$ = part of applied load effect corresponding to positive total curvature
M = due to applied moment
ns = nonprestressed
P = due to prestress force
ps = prestressed
R = due to stress relaxation
S = due to shrinkage
s = both prestressed and nonprestressed
sh = shrinkage
ucr = uncracked condition

REFERENCES

1. Teng, S., "Time-Step Analysis of the Initial and Time-Dependent Deformation of Progressively Cracking Nonprestressed and Partially Prestressed Concrete Flexural Members," PhD thesis, University of Iowa, 1991, 432 pp.

2. Branson, D. E., "Instantaneous and Time-Dependent Deflections of Simple and Continuous Reinforced Concrete Beams," *HPR Report* No. 7, Part 1, Alabama Highway Department, Bureau of Public Roads, Aug. 1963, 78 pp.

3. Branson, D. E., and Trost, H., "Unified Procedures for Predicting the Deflection and Centroidal Axis Location of Partially Cracked Nonprestressed and Prestressed Concrete Members," ACI JOURNAL, *Proceedings* V. 79, No. 13, Mar.-Apr. 1982, pp. 119-129.

4. Branson, D. E., and Shaikh, A. F., "Deflection of Partially Prestressed Members," Proceedings of the NATO — *Advanced Research Workshop on Partial Prestressing — From Theory to Practice,* M. Z. Cohn, ed., Paris, June 18-22, 1984, V. II, pp. 69-107.

5. Neville, A. M., and Dilger, W. H., *Creep of Concrete: Plain, Reinforced, and Prestressed,* North-Holland Publishing Company, Amsterdam, 1970, 622 pp.

6. Branson, D. E., *Deformation of Concrete Structures,* McGraw Hill Book Company, New York, 1977, 546 pp.

7. Branson, D. E; Meyers, B. L.; and Kripanarayanan, K. M., "Loss of Pre-

stress, Camber, and Deflection of Noncomposite and Composite Structures Using Different Weight Concretes," *Final Report* No. 70-6, Prepared under Iowa State Highway Commission Research Project HR-137, Aug. 1970.

8. Tadros, M. K.; Ghali, A.; and Dilger, W. H., "Time-Dependent Prestress Loss and Deflection in Prestressed Concrete Members," *PCI Journal,* May-June, 1975, pp. 86-98.

9. Ghali, A., and Trevino, J., "Relaxation of Steel in Prestressed Concrete," *PCI Journal,* Sept.-Oct. 1985, pp. 82-94.

10. Branson, D. E., and Trost, H., "Unified Procedures for Predicting the Deflection and Centroidal Axis Location of Nonprestressed and Partially Prestressed Members," *Lehrstuhl und Institut fur Massivbau,* Technische Hochschule (RWTH), Aachen, June 1981, 118 pp.

11. Shaikh, A. F., and Branson, D. E., "Non-Tensioned Steel in Prestressed Concrete Beams," *PCI Journal*, V. 15, No. 1, Feb. 1970, pp. 14-36.

12. Jittawait, V., "Deflection of Partially Prestressed Concrete Members," MS thesis, Department of Civil Engineering, University of West Virginia, 1979, 124 pp.

13. Washa, G. W., and Fluck, P. G., "Effect of Compressive Reinforcement on the Plastic Flow of Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 49, No. 8, Oct. 1952, pp. 89-108.

14. Naaman, A. E., and Siriaksorn, A., "Serviceability Based Design of Partially Prestressed Beams — Part 1: Analytical Formulation," *PCI Journal,* Mar.-Apr. 1979, pp. 64-89; also "Part 2: Computerized Design and Evaluation of Major Parameters," *PCI Journal*, May-June 1979, pp. 40-60.

15. Harajli, M. H., and Alameh, A. S., "Deflection of Progressively Cracking Partially Prestressed Concrete Flexural Members," *PCI Journal,* May-June 1989, pp. 94-128.

16. Bennett, E. W., and Lee, K. H., "Deflection of Partially Prestressed Beams under a Combination of Long-Time and Short-Time Loading," *Deflection of Concrete Structures,* SP-86, American Concrete Institute, Detroit, 1986, pp. 185-214.

17. Elzanaty, A., "Flexural Behavior of Unbonded Post-Tensioned Partially Prestressed Concrete Beams," MS thesis, Department of Structural Engineering, Cornell University, 1983, 105 pp.

18. Perisic, Z., and Alender, V., "Effects of Nonprestressed Reinforcement on Prestress Losses and Serviceability Limit of Prestressed Members," *International Symposium on Nonlinearity and Continuity in Prestressed Concrete,* University of Waterloo, Ontario, Canada, 1983, pp. 217-233.

19. Prasada Rao, A. S., and Jayaraman, R., "Creep and Shrinkage Analysis of Partially Prestressed Concrete Members," *Journal of Structural Engineering*, ASCE, V. 115, No. 5, May 1989, pp. 1169-1189.

20. Inomata, S., "Approach for Creep Analysis of Cracked Partially Prestressed Concrete Members," *PCI Journal,* Jan.-Feb. 1987, pp. 104-125.

21. Elbadry, M. M., and Ghali, A., "Serviceability Design of Continuous Prestressed Concrete Structures," *PCI Journal,* Jan.-Feb. 1989, pp. 54-91.

22. Joo, Y. S., and Tadros, M. K., "Time-Dependent Analysis of Partially Prestressed Composite Members," *Proceedings of the Session Related to Design, Analysis, and Testing, at Structures Congress '89 "Structural Design, Analysis, and Testing,"* ASCE, San Francisco, May 1-5 1989, pp. 241-250.