TIME-DEPENDENT ANALYSIS OF PRESTRESSED CONCRETE COMPOSITE SECTIONS

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ABSTRACT: This paper presents a simple yet rational new model for time-dependent analysis of uncracked prestressed concrete composite sections in which plain or reinforced concrete slab is cast onto a precast prestressed concrete beam. The proposed model satisfies both the equilibrium and compatibility conditions considering the effects of creep and shrinkage of concrete as well as relaxation of prestressing steel. A closed form solution for the basic variab es determining the strain distribution over the composite section is obtained. The applicability of the model is illustrated by a numerical example.

1. INTRODUCTION

It has been well recognized that the evaluation of stresses and strains in prestressed concrete composite (PCC) structures can not be approached rationally without accounting for the redistribution of stresses due to shrinkage and creep of concrete and relaxation of prestressing steel [1]. Due to the differences in material properties, levels and stages of loading, and ages at loading between the precast beam (PB) and the cast-in-place slab (CIPS), a differential strain and hence a differential shearing force will develop at the interface of the two members. Furthermore, the restraint provided by the reinforcing steel will introduce compressive stresses in the steel whereas the concrete may be subjected to either tensile or compressive stresses depending on the position and amount of the steel.

Several methods have been proposed for analyzing the time dependent effects in uncracked prestressed concrete composite members [2,3,4 and 5]. Basically, two different approaches which give different results can be found in the literature. Their distinguishing features were discussed by Branson [6] and Saha [7]. In the first approach, the restraining forces provided by the steel were not accounted for in the formulation of equilibrium conditions. In the second approach, equal elastic curvatures in the PB and CIPS sections were assumed which is not the case.

2. THE PROPOSED MODEL

The proposed analytical model overcomes the simplifying assumption of equal elastic curvatures in the PB and CIPS sections considered in the previous models [3, 5]. The solutions for strains and stresses in the constituent materials are obtained based on the conditions of force and moment equilibrium, and the strain and deflection compatibilities between the two concrete elements as well as the strain compatibility between steel and concrete at the various levels of steel.

2. 1 General Formulation

The total strain at any time, t, is made out of two components; an elastic component, ϵ_e, and a nonrecoverable component, ϵ_t :

The nonrecoverable strain, ϵ_t, comprises of two components, the known shrinkage strain, ϵ_{sh}, and the creep strain, ϵ_υ.

$$\epsilon_t = \epsilon_{sh} + \epsilon_\upsilon \tag{2}$$

In turn, the creep strain is made out of two components, creep strain developed during the previous time intervals, $\epsilon_{\underline{\upsilon}}$, which is known, and the aging creep strain which is function of the current elastic strain :

$$\epsilon_\upsilon = \epsilon_{\underline{\upsilon}} + \chi \cdot \upsilon \cdot \epsilon_e \tag{3}$$

A fairly accurate estimate of shrinkage strain, ϵ_{sh}, and creep coefficient, υ, can be obtained from typical shrinkage and creep/time curves in Refs.[8,9]. The aging coefficient, χ, can be obtained from the set of curves provided in Ref.[3].

The total strains in prestressed and nonprestressed steel at any time, t, are, respectively, given by:

$$\epsilon_{ps_t} = -(\epsilon_e + \epsilon_{sh} + \epsilon_\upsilon + \epsilon_r)_t \tag{4}$$

$$\epsilon_{s_t} = -(\epsilon_e + \epsilon_{sh} + \epsilon_\upsilon)_t \tag{5}$$

where ϵ_r is the strain due to prestressing steel relaxation determined according to the PCI committee 209 on prestress losses [10].

2.2 Analytical Formulation

Consider the composite section shown in Fig. 1-a subjected to a total sustained bending moment M and initial prestressing force F_o. Fig.1-b shows a schematic illustration of the total elastic strains developed in the composite section due to total loads and time effects at any instant of time.

The elastic strains at any level in the PB and CIPS sections, $\epsilon_{ep}(y_1)$ and $\epsilon_{es}(y_2)$, are, respectively, given by:

$$\epsilon_{ep}(y_1) = \epsilon_{ep} - \phi_p \cdot y_1 \tag{6}$$

$$\epsilon_{es}(y_2) = \epsilon_{es} + \phi_s \cdot y_2 \tag{7}$$

Similarly, the creep strains at any level in the PB and CIPS sections, $\epsilon_{\upsilon p}(y_1)$ and $\epsilon_{\upsilon s}(y_2)$, are, respectively, given by:

$$\epsilon_{\upsilon p}(y_1) = \epsilon_{\upsilon p} - \phi_{\upsilon p} \cdot y_1 \tag{8}$$

$$\epsilon_{us}(y_2) = \epsilon_{us} + \phi_{us} \cdot y_2 \tag{9}$$

where ϵ_{vp} and ϵ_{vs} are the creep strains at the interface surface of the PB and CIPS , and ϕ_{vp} and ϕ_{vs} are the curvatures associated to creep in the PB and CIPS sections, respectively. These variables can be expressed according to Eq.(3) as:

$$\epsilon_{vp} = \epsilon_{\underline{vp}} + \chi_p \, v_p \, \epsilon_{ep} \tag{10}$$

$$\phi_{vp} = \phi_{\underline{vp}} + \chi_p \, v_p \, \phi_p \tag{11}$$

$$\epsilon_{us} = \epsilon_{\underline{us}} + \chi_s \, v_s \, \epsilon_{es} \tag{12}$$

$$\phi_{us} = \phi_{\underline{us}} + \chi_s \, v_s \, \phi_s \tag{13}$$

In each of Eqs.(10) through (13), the first term is known while the second term is a function of the current elastic strain and curvature.

2.3 Section Analysis

The proposed analytical model utilizes the following four conditions:

- Equilibrium of forces *($\Sigma F = 0$)*
- Equilibrium of Moments *($\Sigma M = 0$)*
- Strain compatibility at the interface of PB and CIPS
- Deformation compatibility of PB and CIPS

- *Equilibrium of forces leads to:*

$$F_o + F_{sp} + F_{ss} = E_{cp}[\epsilon_{ep} A_{tp} - \phi_p S_{tp}] + E_{cs}[\epsilon_{es} A_{ts} + \phi_s S_{ts}] \tag{14}$$

where:

F_{sp} is the total change of the force in the steel of PB due to shrinkage, relaxation in prestressing steel, and previous creep evaluated as:

$$F_{sp} = -\sum_{i=1}^{n_{rr}} E_{si} \, A_{si} \, \epsilon_{ri} - \sum_{i=1}^{n_p} E_{si} \, A_{si} \, [\epsilon_{sh\,p} + \epsilon_{\underline{vp}} - \phi_{\underline{vp}} d_{si}] \tag{15}$$

F_{ss} is the total change of the force in the steel of CIPS due to shrinkage and previous creep evaluated as:

$$F_{ss} = -\sum_{i=1}^{n_s} E_{si} \, A_{si} \, [\epsilon_{sh\,s} + \epsilon_{\underline{us}} + \phi_{\underline{us}} d_{si}] \tag{16}$$

A_{tp} and A_{ts} are the transformed areas of the PB and CIPS sections given by:
where the modular ratios n_{spi} and n_{ssi} are given by:
S_{tp} and S_{ts} are the first moments of area of the transformed PB and CIPS sections about their interface surface as given by:

$$A_{tp} = b.h + \sum_{i=1}^{n_p} (n_{spi} - 1) A_{si} \tag{17}$$

$$A_{ts} = b_1.h_1 + \sum_{i=1}^{n_s} (n_{ssi} - 1) A_{si} \tag{18}$$

$$n_{spi} = \frac{E_{si}}{E_{cp}} (1+\chi_p \, \upsilon_p) \tag{19}$$

$$n_{ssi} = \frac{E_{si}}{E_{cs}} (1+\chi_s \, \upsilon_s) \tag{20}$$

$$S_{tp} = \frac{b.h^2}{2} + \sum_{i=1}^{n_p} (n_{spi} - 1) A_{s_i} d_{s_i} \tag{21}$$

$$S_{ts} = \frac{b_1.h_1^2}{2} + \sum_{i=1}^{n_s} (n_{ssi} - 1) A_{s_i} d_{s_i} \tag{22}$$

■ *Equilibrium of Moments about the interface surface leads to*:

$$M - F_o d_p - M_{sp} + M_{ss} = - E_{cp}[\epsilon_{cp} S_{tp} - \phi_p I_{tp}] + E_{cs}[\epsilon_{cs} S_{ts} + \phi_s I_{ts}] \tag{23}$$

where:

M_{sp} is the moment of the forces developed in the steel of PB due to shrinkage, relaxation in prestressing steel, and previous creep evaluated as:

$$M_{sp} = -\sum_{i=1}^{n_{pp}} E_{si} \, A_{si} \, \epsilon_{ri} \, d_{si} - \sum_{i=1}^{n_p} E_{si} \, A_{si} \, d_{si} \, [\epsilon_{sh_p} + \epsilon_{\underline{vp}} - \phi_{\underline{vp}} \, d_{si}] \tag{24}$$

M_{ss} is the moment of the forces developed in the steel of CIPS due to shrinkage and previous creep evaluated as:

$$M_{ss} = -\sum_{i=1}^{n_s} E_{si} \, A_{si} \, d_{si} \, [\epsilon_{sh_s} + \epsilon_{\underline{vs}} + \phi_{\underline{vs}} \, d_{si}] \tag{25}$$

I_{tp} and I_{ts} are the second moments of area of the transformed PB and CIPS sections about their interface surface as given by:

$$I_{tp} = \frac{b.h^3}{3} + \sum_{i=1}^{n_p} (n_{sp_i} - 1) A_{s_i} d_{s_i}^2 \tag{26}$$

$$I_{ts} = \frac{b_1.h_1^3}{3} + \sum_{i=1}^{n_s} (n_{ssi} - 1) A_{si} \, d_{si}^2 \tag{27}$$

■ *Strain compatibility*
The condition of strain compatibility at the interface surface leads to :

$$(\epsilon_{cp} + \epsilon_{sh_p} + \epsilon_{vp}) - (\epsilon_{cs} + \epsilon_{sh_s} + \epsilon_{vs}) = \epsilon_{cpo} \qquad (28)$$

where ϵ_{cpo} is the total strain at top of the PB section just before composite action.

■ *Deformational compatibility*
The condition of deformational compatibility between the PB and CIPS leads to :

$$(\phi_p + \phi_{vp}) - (\phi_s + \phi_{vs}) = \phi_{cpo} \qquad (29)$$

where ϕ_{cpo} is the total curvature of the PB section just before composite action.

In Eqs. (14, 23, 28, and 29) there are four unknowns, namely, ϵ_{cp}, ϕ_p, ϵ_{cs}, and ϕ_s. These equations are arranged in a matrix form, Eq.(30). It is possible to find a closed form solution for each of the unknowns.

$$\begin{bmatrix} E_{cp}\,A_{tp} & -E_{cp}\,S_{tp} & E_{cs}\,A_{ts} & E_{cs}\,S_{ts} \\ E_{cp}\,S_{tp} & -E_{cp}\,I_{tp} & E_{cs}\,S_{ts} & E_{cs}\,I_{ts} \\ 1+\chi_p\upsilon_p & 0 & -1-\chi_s\upsilon_s & 0 \\ 0 & 1+\chi_p\upsilon_p & 0 & -1-\chi_s\upsilon_s \end{bmatrix} \cdot \begin{bmatrix} \epsilon_{cp} \\ \phi_p \\ \epsilon_{cs} \\ \phi_s \end{bmatrix} = \begin{bmatrix} F_o+F_{sp}+F_{ss} \\ M-F_o d_p-M_{sp}+M_{ss} \\ \epsilon_{cpo}-\epsilon_{sh\,p}-\epsilon_{\underline{vp}}+\epsilon_{sh\,s}+\epsilon_{\underline{vs}} \\ \phi_{cpo}-\phi_{\underline{vp}}+\phi_{\underline{vs}} \end{bmatrix} \qquad (30)$$

It should be noted that the other variables, namely, ϵ_{cpo}, ϕ_{cpo}, and F_o should first be determined by analyzing the PB just before composite action.

3. MODEL APPLICATION

The applicability of the proposed model is illustrated by considering a numerical example on the composite cross-section shown in Fig. 1. The input data is summarized in Table 1. Samples of the obtained results are presented in Figs. 2, 3, and 4.

Fig. 2 shows the variation of the stress at top of PB with time during the beam service life (50 years) for both shored and unshored construction. For the particular problem investigated, the figure shows that the stress at top of PB increases with time during the noncomposite stage while decreases with time during the composite stage. The rate of decrease is higher in the early life and for unshored beams. This trend of behavior indicates that the CIPS is holding on to the PB and that the holding is more in the case of unshored construction where the slab is lightly loaded and the beam is highly loaded.

Fig. 3 shows the variation of the stress at bottom of CIPS with time for both shored and unshored construction. The figure shows that the stress at bottom of CIPS follows an opposite trend of the stress at top of the PB as it increases with time, again, the rate of increase is higher in the early life of the CIPS and for unshored construction.

Fig. 4 shows the variation of the prestressing stress with time for both shored and unshored construction. The trend shows that the rate of prestressing loss is higher during the noncomposite stage and that the prestressing stress is higher in the unshored case.

4. CONCLUSIONS

The paper presented a solution for stresses and strains in the constituent materials of a PCC section formed by a cast-in-place slab on top of a precast prestressed concrete beam. The analytical model satisfies both the equilibrium and compatibility conditions. The model applicability was demonstrated by a numerical example from which the following conclusions are drawn:

1▪ Shrinkage and creep of concrete are of significant effects on stress redistribution in a PCC section. A relaxation in stresses on top of the investigated unshored PB of about 45% was observed.

2▪ Prestressed concrete composite sections are recommended for less prestress losses as compared to noncomposite sections.

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Appendix A:Notations

A_{si} Area of steel at level i.

d_{si} Distance of steel at level i from the interface surface.

E_{cp} Modulus of elasticity of PB concrete.

E_{cs} Modulus of elasticity of CIPS concrete.

E_{si} Modulus of elasticity of steel at level i.

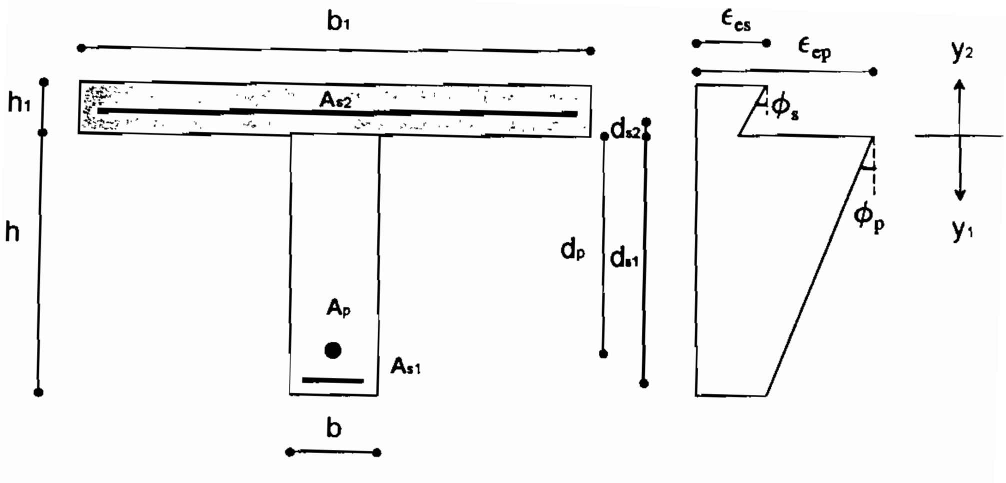
n_p Total number of steel layers in the P.B.

n_{pp} Number of prestressing steel layers.

n_s Number of steel layers in the CIPS.

$\epsilon_{sh\,p}$ Shrinkage strain component of the PB.

$\epsilon_{sh\,s}$ Shrinkage strain component of the CIP slab.



1-a *1-b*

Figure 1-a: ***Typical beam/cross section used in the model development and application***

Figure 1-b: ***Total elastic strains due to total loads and time effects at any instant of time after composite action.***

Table 1: Input data for model Application*

Precast Beam	Cast-in-Place Slab
Section Geometry:	*Section Geometry:*
$b=0.2$	$b_1=2.4$
$h=1.0$	$h_1=0.15$
Steel:	*Steel:*
$n_{pp}=1$	$n_s=1$
$n=2$	$A_{s2}=0.001$
$A_{s1}=0.001$	$d_{s2}=0.04$
$d_{s1}=0.9$	
$A_p=0.0015$	
$d_p=0.75$	
$F_o=180$	
Mechanical Properties:	*Mechanical Properties:*
$E_{cp}=30{,}000$	$E_{cs}=26{,}500$
$E_{s1}=200{,}000$	$E_{s2}=200{,}000$
$E_p=200{,}000$	$\upsilon_s=2.9$
$\upsilon_p=2.65$	$\chi_s=0.5$
$\chi_p=0.5$	$\epsilon_{shu}=600\times10^{-6}$
$\epsilon_{shu}=400\times10^{-6}$	

* *Units are combination of KN and meters.*

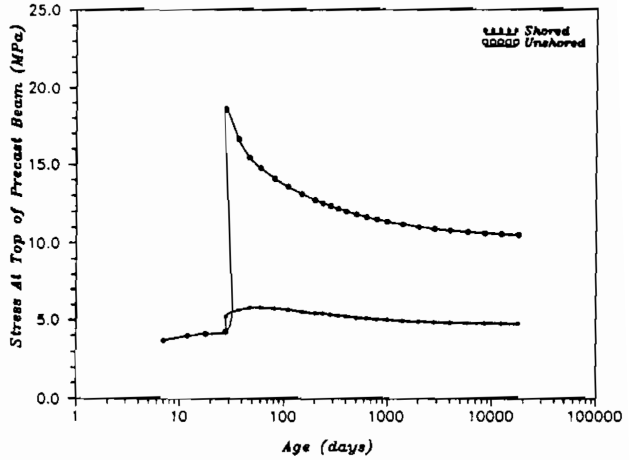


***Figure 2:* Variation of stress at top of PB with time.**

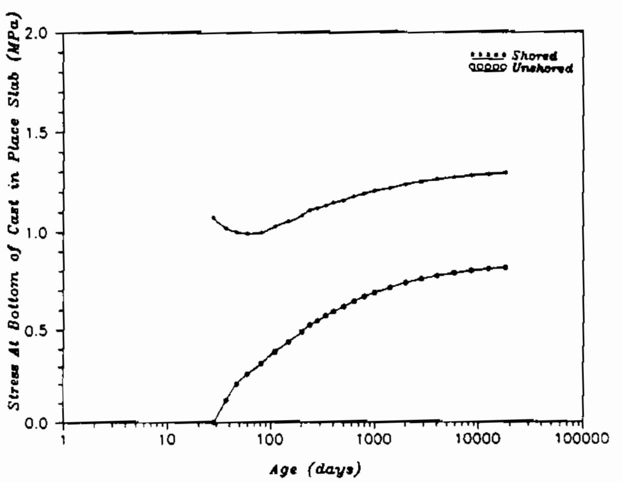


***Figure 3:* Variation of stress at bottom of CIPS with time.**

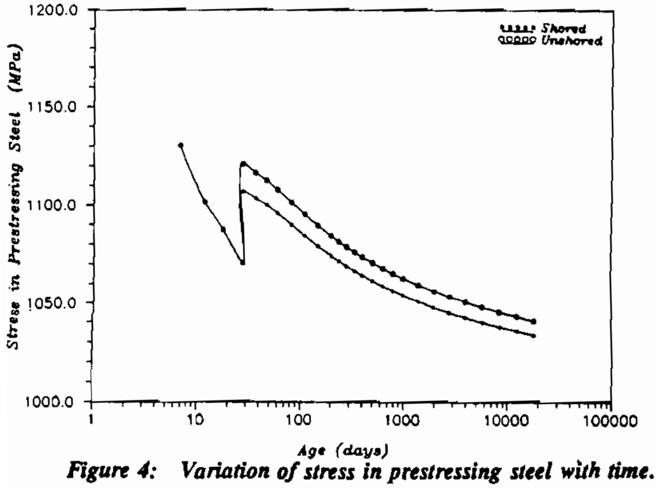


***Figure 4:* Variation of stress in prestressing steel with time.**