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Design Procedures for Computing Deflections

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Presents design procedures for computing short-time and long-time deflections of noncomposite and composite ordinary reinforced and prestressed concrete beams. The paper discusses phases of the subject indicated in the ACI Building Code but not specifically defined therein, and is, in part, based on recent work of ACI Committee 435, Deflections.

Keywords: beams (structural); camber; composite construction (concrete to concrete); cracking (fracturing); creep (materials); deflection; flexural strength; modulus of elasticity; moment of inertia; moments; precast concrete; prestressed concrete; reinforced concrete; shoring; shrinkage; structural design.

■ THE OBJECTIVES OF THIS PAPER are to present practical design procedures for computing deflections of ordinary reinforced and prestressed concrete flexural members. The paper is intended to discuss areas indicated in the ACI Building Code[1] but not specifically defined therein, and to supplement the "Ultimate Strength Design Handbook, V. 1," SP-17,[2] "Reinforced Concrete Design Handbook — Working Stress Method," SP-3,[3] ACI Committee 435 report, "Deflections of Reinforced Concrete Flexural Members,"[4] and the Subcommittee 5, ACI Committee 435 report, "Deflections of Prestressed Concrete Members."[5] It is also based in part on recent work of Subcommittees 1 — Allowable Deflections, 2 — Probability, 3 — Composite Beams, and 4 — Code Provisions, of ACI Committee 435.

Short-time and long-time deflections of noncomposite and composite ordinary reinforced and prestressed concrete beams are included. Also included are the effects of cracking, compression steel in ordinary reinforced beams, nontensioned steel in prestressed beams, and shrinkage and creep. Information on span-depth ratios is provided, and illustrative examples of the use of L/t curves to check deflections, and of the deflection calculations for an ordinary reinforced composite beam are presented.

NOTATION

1 = subscript denoting cast-in-place slab of composite beam

2 = subscript denoting precast beam

A = area of section

A_g = area of gross section, neglecting the steel

A_s = area of tension steel in ordinary reinforced beams and area of prestress steel in prestressed beams

A_s' = area of compression steel in ordinary reinforced beams and area of nontensioned steel in prestressed beams

a = distance from end of beam to harped points in two-point harping case of prestressed concrete beams

a = depth of rectangular stress block

b = width of compression face of flexural member

b' = width of web of T-beam

C_L = (live load)/(total sustained load) — see Eq. (17)

C_t = creep coefficient defined as ratio of creep strain to initial strain — see Table 1

C_{t_1} = creep coefficient of the composite beam under slab dead load

C_{t_2} = creep coefficient of the precast beam concrete

C_w' = E_c/f'_{cb}

c = subscript denoting composite section. Also used to designate concrete, such as E_c

D = differential shrinkage strain or coefficient — see Reference 10

d = effective depth of section

E_c = modulus of elasticity of concrete, such as at 28 days

E_{ci} = modulus of elasticity of concrete at the time of application of the precast beam dead load for ordinary reinforced beams, and at the time of transfer of prestress for prestressed beams

e = eccentricity of prestress steel

e_c = eccentricity of prestress steel at center of beam

e_o = eccentricity of prestress steel at end of beam

F = prestress force after losses

F_o = prestress force at transfer (after elastic losses)

f_c' = compressive strength of concrete

f'_{cb} = modulus of rupture of concrete — see Reference 4 for discussion and formulas. Average values suggested are $f'_{cb} = 7.5\sqrt{f_c'}$ for normal weight concrete and $f'_{cb} = 6.0\sqrt{f_c'}$ for lightweight concrete

f_y = yield strength of steel

I = moment of inertia (second moment of the area)

I_2 = moment of inertia of precast beam

I_c = moment of inertia of composite section with transformed slab. The slab is transformed into equivalent precast beam concrete by dividing the slab width by n, where $n = E_2/E_1$

I_{cr} = moment of inertia of cracked transformed section

I_{eff} = effective moment of inertia
I_g = moment of inertia of gross section, neglecting the steel
i = subscript denoting initial value
K = deflection constant. For example, for beams of constant section and uniformly loaded:
cantilever beam, $K = 1/4$
simple beam, $K = 5/48$
hinged-fixed beam (one end continuous), $K = 8/185$
fixed-fixed beam (both ends continuous), $K = 1/32$
K_1 = deflection constant for the slab dead load
K_2 = deflection constant for the precast beam dead load
k_r = reduction factor to take into account the effect of compression steel in ordinary reinforced precast beams and the effect of nontensioned steel in prestressed precast beams. See Eq. (4) and (5)
L = span length
M = bending moment. When used as the numerical maximum bending moment, for beams of constant section and uniformly loaded:
cantilever beam, $(-)\ M = wL^2/2$
simple beam, $(+)\ M = wL^2/8$
hinged-fixed beam (one end continuous), $(-)\ M = wL^2/8$
fixed-fixed beam (both ends continuous), $(-)\ M = wL^2/12$
M_1 = maximum bending moment under slab dead load
M_2 = maximum bending moment under precast beam dead load
M_{cr} = cracking moment. See Eq. (2) and (3)
M_{max} = maximum moment
m = parameter defined in Eq. (A2.2)
n = modular ratio
p = A_s/bd
Q = slab shrinkage force = DA_1E_1. See Ref. 10
T = multiplier for additional long-time deflections due to shrinkage and creep
t = total depth of section
t = subscript denoting time-dependent such as C_t. Also used to denote tension face as y_t
t' = flange thickness of T-beam
w = uniformly distributed load
w' = unit weight of concrete in pcf
y_{cs} = distance from centroid of composite section to centroid of cast-in-place slab
y_t = distance from centroid of gross section to the extreme fiber in tension
α = ratio of creep of precast beam concrete occurring before the slab has hardened to the total creep, which depends on the length of time between first loading of the precast beam and the time the slab is cast. See the percentages in Note 1 of Table 1. A suggested average value is $\alpha = 0.35$
ΔF = loss of prestress due to time-dependent effects only (such as shrinkage, creep, steel relaxation). The elastic loss is deducted from the tensioning force to obtain F_o
Δ = maximum deflection (positive downward)
$(\Delta_i)_1$ = initial deflection under slab dead load
$(\Delta_i)_2$ = initial deflection under precast beam dead load
$(\Delta_i)_{DL}$ = dead load deflection

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$(\Delta_i)_{F_o}$ = initial camber due to the initial prestress force F_o
$(\Delta)_{LL}$ = live load deflection

ACI CODE DEFLECTION PROVISIONS

With the use of ultimate strength design methods and higher strength steel and concrete, shallower structural members are the logical result. This has the effect of greatly increasing the importance of deflections and, correspondingly, the importance of the time-dependent deformational characteristics of reinforced concrete flexural members. Thus, for the first time the 1963 ACI Code contained a provision on the control of deflections under service load conditions.

In Section 1507 under ultimate strength design, the Code requires that deflections be checked under working loads by the provisions of Section 909, whenever p for singly reinforced beams, $(p - p')$ for doubly reinforced beams, or $(p_w - p_f)$ for T-beams, exceeds $0.18f_c'/f_y$, or whenever the specified yield strength, f_y, exceeds 40,000 psi. This is simply a "red flag" provision[6] to check deflections at or above the steel percentage of $0.18f_c'/f_y$, which is close to the balanced steel ratio by elastic theory and less than one-half the balanced steel ratio by ultimate strength theory (or a 2 plus load factor, since p_{bal} by Code Eq. (16-2) is $(0.50, 0.46, 0.43, \text{or } 0.39)f_c'/f_y$ for $f_y = 40, 50, 60, \text{or } 75$ ksi, respectively, and $k_1 = 0.85$ for $f_c' = 4000$ psi); also to check deflections when greater than 40 ksi yield steel is used. A companion requirement for checking deflections is also given by the minimum thicknesses of Table 909(b) in the Code.

Section 909 of the Code specifies that where short-time deflections are to be computed, the moment of inertia shall be based on the gross section when pf_y is equal to or less than 500 and on the cracked transformed section when pf_y is greater. This is an attempt to guard against underestimating deflections (using the gross section I) for shallow beams, among others, associated with pf_y values above 500, and also to provide a basis for determining approximate ranges of conditions in which to use I_g or I_{cr}. For $p = 0.18\ f_c'/f_y$ (previous paragraph), $pf_y = 0.18f_c' = 450$ and 540 when $f_c' = 2500$ and 3000 psi, respectively. Hence the Code value of $pf_y = 500$ was selected for these and higher strength concretes as a governing criterion.

For continuous spans the moment of inertia may be taken as the average of the values obtained for the positive and negative moment regions. The modulus of elasticity is specified in Section 1102.

Factors are provided in Section 909 (d) for computing the *additional* long-time deflections of singly and doubly reinforced beams, and allowable deflections are given in Section 909 (e) or Section 909 (f) for roofs and floors under live load only, and under live load plus the shrinkage and creep deflections under all sustained loads.

ADDITIONAL INFORMATION PERTAINING TO DEFLECTION CALCULATIONS

Effect of cracking

To include the effect of cracking in the computation of the effective moment of inertia or ordinary reinforced beams in Eq. (8), (9), (10), and (11), the following equation has been recently recommended by ACI Committee 435[4, 7].

$$I_{eff} = \left(\frac{M_{cr}}{M_{max}}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] I_{cr} \tag{1}$$

where M_{max} denotes the maximum moment in the span. Eq. (1) applies only when M_{max} is greater than M_{cr}; otherwise $I_{eff} = I_g$. For continuous beams, the average of the positive and negative moment region values in Eq. (1) was recommended. Eq. (1) has also been shown[8] to be applicable for computing deflections of cracked prestressed beams using Eq. (12), (13), and (14).

I_{eff} in Eq. (1) has the two limits of I_g and I_{cr}, and thus provides a transition expression between the two cases given in the ACI Code. The uncracked transformed section I might be more accurately used instead of the gross section I in Eq. (1), especially for heavily reinforced members.

In the case of composite beams, see the descriptions following Eq. (10), (11), (13), and (14) for reference to the loads to be used in the calculation of M_{max} in Eq. (1). With the exception of the live load deflection, $(\Delta)_{LL}$, it is suggested that the effective moment of inertia, I_{eff} in Eq. (1), be computed only for the precast beam, I_2. It follows for this case that I_g and y_t in $f'_{cb}I_g/y_t$ of Eq. (2) and (3) refer to the precast section. When I_{eff} is computed for live load deflections, I_g and y_t in $f'_{cb}I_g/y_t$ of Eq. (2) and (3) refer to the composite section.

Ordinary reinforced precast beams:

$$M_{cr} = \frac{f'_{cb}\, I_g}{y_t} \tag{2}$$

Prestressed precast beams:

$$M_{cr} = F\ (e)_{precast} + \frac{F\,(I_g)_{precast}}{(A_g\, y_t)_{precast}} + \frac{f'_{cb}\, I_g}{y_t} \tag{3}$$

Eq. (2) and (3) are also applicable to noncomposite construction.

Effect of compression steel in ordinary reinforced beams and nontensioned steel in prestressed beams

The reduced creep effect resulting from the presence of compression steel in ordinary reinforced beams and nontensioned steel (as specified below) in prestressed beams is given by Eq. (4) and (5).

Ordinary reinforced beams:

$$k_r = 1 - 0.6\ (A_s'/A_s),\ \text{but not less than } 0.40 \tag{4}$$

where A_s' is the area of compression steel and A_s is the area of the tension steel.

Prestressed beams:

$$k_r = \frac{1}{1 + \dfrac{A_s'}{A_s}} \tag{5}$$

where A_s' is the area of the nontensioned steel and A_s is the area of the prestressed steel. It is assumed in Eq. (5) that the nontensioned steel and the prestressed steel are on the same side of the section centroid and that the eccentricities of the two steels are approximately the same. See Reference 8 for case of different eccentricities.

Effect of shrinkage and creep

Table 1 has been recommended by ACI Committee 435[7] for general use, subject to the following qualifying statement: Table 1 may be considered satisfactory for computing additional long-time deflections in structures of common types and sizes. For unusual cases (particularly for very shallow members such as canopies) or when very early application of load is necessary, it is suggested that shrinkage and creep deflections be considered separately and that the choice

TABLE 1—MULTIPLIER, T, FOR ADDITIONAL LONG-TIME DEFLECTIONS DUE TO SHRINKAGE AND CREEP FOR BOTH NORMAL WEIGHT AND LIGHT-WEIGHT CONCRETE MEMBERS OF COMMON TYPES, SIZES, AND COMPOSITION

Concrete strength f_c'	Average relative humidity, age when loaded								
	100 percent			70 percent			50 percent		
at 28 days	≤7d	14d	≥28d	≤7d	14d	≥28d	≤7d	14d	≥28d
2500 to 4000 psi (176 to 281 kgf/cm²)	2.0	1.5	1.0	3.0	2.0	1.5	4.0	3.0	2.0
>4000 psi (281 kgf/cm²)	1.5	1.0	0.7	2.5	1.8	1.2	3.5	2.5	1.5

1. It is suggested that the following percentages of the values in the table be used for sustained loads that are maintained for the periods indicated: 25 percent for 1 month or less
50 percent for 3 months
75 percent for 1 year
100 pecent for 5 years or more

2. When shrinkage warping is not computed separately, which is the case throughout this paper, the time-dependent factor is lumped together, in which case C_t may be taken from this table, or $C_t = T$. A discussion of procedures for computing shrinkage warping is presented in Reference 4.

The 50-percent values may normally be used for average relative humidities lower than 50 percent, which might be the case in heated buildings, for example.

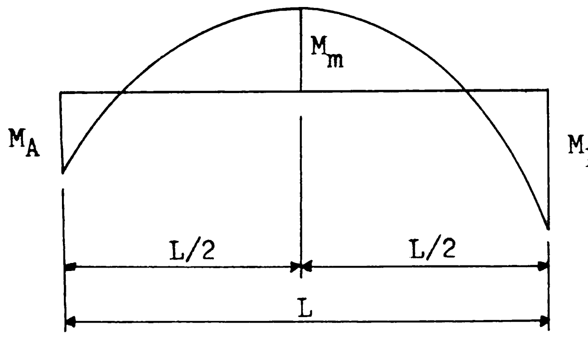


Fig. 1—General bending moment diagram for a uniformly loaded continuous beam

of the appropriate shrinkage and creep coefficients be made by the designer (preferably from local test data).

CALCULATION OF DEFLECTIONS

General equations

It is convenient to use the general deflection equation in the form of Eq. (6) for the case of assumed idealized end conditions.

$$\Delta = KML^2/EI \tag{6}$$

where K is a deflection constant for the beam and M is, for the constants given in the notation, the numerical maximum beam bending moment.

Deflections may also be computed for uniformly distributed loading using Eq. (7)[9].

$$\Delta_m = \frac{5L^2}{48EI}\left[M_m - \frac{1}{10}(M_A + M_B)\right] \tag{7}$$

where Δ_m is the deflection at midspan (approximate maximum deflection only), and the moments in the brackets of Eq. (7) are shown in Fig. 1.

Note that Eq. (6) rather than Eq. (7) is used in the remainder of this paper.

Ordinary reinforced concrete beams

Short-time deflections:

$$\Delta_i = KML^2/EI \tag{8}$$

Additional long-time deflections:

$$\Delta_t = k_r C_t \Delta_i \tag{9}$$

According to ACI 318-63, Section 909 (c) and (d), I is equal to I_g or I_{cr} depending on pf_y, and $(k_r C_t)$ is equal to 2.0 when $A_s' = 0$; 1.2 when $A_s' = 0.5A_s$; and 0.8 when $A_s' = A_s$. Using the method recommended by Committee 435,[7] k_r is given by Eq. (4), C_t may be selected from Table 1, and I is computed using Eq. (1). E is specified in the Code, Section 1102.

The results of a statistical study of these two methods is indicated in Table 2 for 107 tests for short-time deflections and 30 tests for long-time deflections, including T-beams and continuous beams:

TABLE 2—STATISTICAL STUDY RESULTS

	Mean value of $\frac{\text{meas defl}}{\text{calc defl}}$	
	ACI 318-63	Committee 435 method
Short-time with top steel	1.02	1.04
Short-time without top steel	0.95	1.01
Long-time with top steel	1.07	1.03
Long-time without top steel	0.90	1.02

In view of the limited amount of data, the actual deviations of the mean value are not too meaningful, but the trends appear correct. However, the variability of the deflection ratio is of more interest (see Table 3).

TABLE 3—VARIABILITY OF DEFLECTION RATIO

	Range of $\frac{\text{meas defl}}{\text{calc defl}}$ for probability	
	ACI 318-63	
	80 percent	90 percent
Short-time with top steel	0.73 to 1.32	0.65 to 1.41
Short-time without top steel	0.81 to 1.09	0.78 to 1.13
Long-time with top steel	0.84 to 1.30	0.78 to 1.36
Long-time without top steel	0.73 to 1.07	0.69 to 1.22
	Committee 435 method	
	80 percent	90 percent
Short-time with top steel	0.78 to 1.31	0.71 to 1.38
Short-time without top steel	0.86 to 1.17	0.82 to 1.21
Long-time with top steel	0.89 to 1.19	0.84 to 1.25
Long-time without top steel	0.87 to 1.17	0.82 to 1.22

For example, if the long-time deflection of a beam without compression steel is calculated by the Committee 435 method to be 1 in., the probability that the actual deflection will lie between 0.87 and 1.17 in. is 80 percent. Owing to the limited amount of data, one might be justified in lumping all the data and simply stating that the probability of the calculated deflection being within the range ± 0.2 is approximately 80 percent.

Composite beams—Ordinary reinforced precast

Unshored construction:

$$\Delta = \overbrace{(\Delta_i)_2}^{(1)} + \overbrace{\alpha k_r C_{t_2} (\Delta_i)_2}^{(2)} + \overbrace{(1-\alpha) k_r C_{t_2} (\Delta_i)_2 \frac{I_2}{I_c}}^{(3)}$$
$$+ \overbrace{(\Delta_i)_1}^{(4)} + \overbrace{k_r C_{t_1} (\Delta_i)_1 \frac{I_2}{I_c}}^{(5)} + \overbrace{\frac{Q y_{cs} L^2}{8 E_c I_c}}^{(6)} + \overbrace{(\Delta)_{LL}}^{(7)} \tag{10}$$

where:

Term (1) is the initial dead load deflection of the precast beam. $(\Delta_i)_2 = K_2 M_2 L^2/E_{ci} I_2$. For computing I_2 in Eq. (1), M_{max} refers to the precast beam dead load and M_{cr} to the precast beam*

Term (2) is the creep deflection of the precast beam up to the time of slab casting. k_r refers to any compression steel in the precast beam

Term (3) is the creep deflection of the composite beam for the period following slab casting due to the precast beam dead load. k_r refers to any compression steel in the precast beam. I_2 is the same as for Term (1)

Term (4) is the initial deflection of the precast beam under slab dead load. $(\Delta_i)_1 = K_1M_1L^2/E_cI_2$. For computing I_2 in Eq. (1), M_{max} refers to the precast beam plus slab dead load and M_{cr} to the precast beam

Term (5) is the creep deflection of the composite beam due to the slab dead load. k_r refers to any compression steel in the precast beam. I_2 is the same as for term (4)

Term (6) is the deflection due to differential shrinkage[10]

Term (7) is the live load deflection of the composite beam, in which the flexural rigidity, E_cI_c, is used. For computing I_c in Eq. (1), M_{max} refers to the precast beam plus slab dead load and the live load, and M_{cr} to the composite beam

In view of the complexities involved, it is suggested that the gross moment of inertia, I_g, be used in all cases for the composite moment of inertia, I_c, in Eq. (10); with the exception as noted in Term (7) for $(\Delta)_{LL}$. Note that shrinkage warping of the precast beam is not computed separately in Eq. *(10)* (see Note 2 of Table 1).

Shored construction:

Δ = Eq. (10), with Terms (4) and (5) modified as follows: (11)

Term (4) is the initial deflection of the composite beam under slab dead load. $(\Delta_i)_1 = K_1M_1L^2/E_cI_c$. It is suggested that the gross moment of inertia, I_g, be used also for the composite moment of inertia, I_c, in this term, as mentioned at the end of the discussion of Eq. (10).

Term (5) is the creep deflection of the composite beam due to the slab dead load $= k_rC_{t_1}(\Delta_i)_1$. k_r refers to any compression steel in the slab, since compression steel in the precast beam would normally be located near the neutral axis of the composite beam and thus have a very small effect. $(\Delta_i)_1$ is the same as Term (4).

Prestressed concrete beams

Eq. (12) was recommended by Committee 435,[5] with changes in notation and k_r added here.

$$\Delta = -\overbrace{(\Delta_i)_{F_o}}^{(1)} + \overbrace{(\Delta_i)_{DL}}^{(2)} - \overbrace{\left[-\frac{\Delta F}{F_o} + \left(1 - \frac{\Delta F}{2F_o}\right)k_rC_t\right](\Delta_i)_{F_o}}^{(3)} + \overbrace{k_rC_t(\Delta_i)_{DL}}^{(4)} + \overbrace{(\Delta)_{LL}}^{(5)} \qquad (12)$$

where:

Term (1) is the initial camber due to the initial prestress force, F_o. See Appendix 1 for common cases of prestress moment diagrams with formulas for computing camber, $(\Delta_i)_{F_o}$

Term (2) is the initial dead load deflection of the beam. $(\Delta_i)_{DL} = KML^2/E_{ci}I_g$

Term (3) is the creep (time-dependent) camber of the beam. This expression includes the effects of creep and loss of prestress. k_r is given by Eq. (5) and is equal to unity when the beam contains no nontensioned steel, or $A_s' = 0$. C_t may be selected from Table 1

Term (4) is the dead load creep deflection of the beam. k_r and C_t are the same as for Term (3)

Term (5) is the live load deflection of the beam. When prestressed beams are to be loaded into the cracking range, Eq. (1) and (3) should be used to determine the effective moment of inertia

Composite beams—Prestressed precast beams

Unshored construction:

$$\Delta = -\overbrace{(\Delta_i)_{F_o}}^{(1)} + \overbrace{(\Delta_i)_2}^{(2)}$$

$$\overbrace{-\alpha\left[-\frac{\Delta F}{F_o} + \left(1 - \frac{\Delta F}{2F_o}\right)k_r\,C_{t_2}\right](\Delta_i)_{F_o}}^{(3)}$$

$$\overbrace{-(1-\alpha)\left[-\frac{\Delta F}{F_o} + \left(1 - \frac{\Delta F}{2F_o}\right)k_r\,C_{t_2}\right](\Delta_i)_{F_o}\frac{I_2}{I_c}}^{(4)}$$

$$+\overbrace{\alpha k_rC_{t_2}(\Delta_i)_2}^{(5)} + \overbrace{(1-\alpha)k_r\,C_{t_2}(\Delta_i)_2\frac{I_2}{I_c}}^{(6)}$$

$$+\overbrace{(\Delta_i)_1}^{(7)} + \overbrace{k_rC_{t_1}(\Delta_i)_1\frac{I_2}{I_c}}^{(8)} + \overbrace{\frac{Q\,y_{cs}\,L^2}{8\,E_c\,I_c}}^{(9)} + \overbrace{(\Delta)_{LL}}^{(10)} \qquad (13)$$

where:

Term (1) —Same description as Term 1 of Eq. (12)

Term (2) is the initial dead load deflection of the precast beam. $(\Delta_i)_2 = K_2M_2L^2/E_{ci}I_g$

Term (3) is the creep (time-dependent) camber of the precast beam up to the time of slab casting. This expression includes the effects of creep and loss of prestress. k_r is given by Eq. (5) and is equal to unity when the beam contains no nontensioned steel [as specified for Eq. (5)], or $A_s' = 0$. C_{t_2} may be selected from Table 1

Term (4) is the creep camber of the composite beam for the period following slab casting. Note the same quantity in the bracket as in Term (3), which was taken from Reference 5

*Eq. (1) is suggested for determining the different moments of inertia involved in the calculation of composite beam deflections, rather than the Code method, Section 909(c). The steel design for a composite beam is based primarily on the live load and composite section (especially when the ultimate strength design method, Section 2504, is used), and thus the pf_y-parameter approach in the Code was not intended to apply to the precast beam alone, for example.

Term (5) is the creep deflection of the precast beam up to the time of slab casting. k_r and C_{t_2} are the same as for Term (3)

Term (6) is the creep deflection of the composite beam for the period following slab casting due to the precast beam dead load. k_r and C_{t_2} are the same as for Term (3)

Term (7) is the initial deflection of the precast beam under slab dead load. $(\Delta_i)_1 = K_1 M_1 L^2 / E_c I_g$

Term (8) is the creep deflection of the composite beam due to the slab dead load. k_r is the same as for Term (3)

Term (9) is the deflection due to differential shrinkage[10]

Term (10) is the live load deflection of the composite beam, in which the flexural rigidity, $E_c I_c$, is used. In cases where the beam is loaded into the cracking range, for computing I_c in Eq. (1), M_{max} refers to the precast beam plus slab dead load and the live load, and M_{cr} to the composite beam

In view of the complexities involved, it is suggested that the gross moment of inertia, I_g, be used in all cases for the composite moment of inertia, I_c, in Eq. (13), with the exception as noted in Term (10) for $(\Delta)_{LL}$.

Shored construction:

$\Delta =$ Eq. (13), with Terms (7) and (8) modified as follows: (14)

Term (7) is the initial deflection of the composite beam under slab dead load. $(\Delta_i)_1 = K_1 M_1 L^2 / E_c I_c$. It is suggested that the gross moment of inertia, I_g, be used also for the composite moment of inertia, I_c, in this term, as mentioned at the end of the discussion of Eq. (13)

Term (8) is the creep deflection of the composite beam due to the slab dead load $= k_r C_{t_1} (\Delta_i)_1$. k_r refers to any compression steel in the slab. $(\Delta_i)_1$ is the same as Term (7)

SPAN-DEPTH RATIOS

Using the average beam proportions indicated in Appendix 2, the L/t-curves of Fig. 2 may be used to check deflections in accordance with the method recommended by Committee 435,[7] Eq. (1), etc.

Span-depth ratios, L/t, are given in Fig. 2 for singly reinforced rectangular beams and one-way slabs, and for singly reinforced T-beams for the following conditions: $\Delta = L/360$, normal weight concrete, uniformly distributed short-time loading, and for different boundary conditions, steel percentages, and concrete strengths. Note that these refer to the conditions in ACI 318-63, Section 909(e)2: Based on immediate deflection due to live load for roofs which support plastered ceilings or for floors which do not support partitions — $L/360$.

For lightweight concrete:

$$\left(\frac{L}{t}\right)_{lightweight} = \left(\frac{L}{t}\right)_{normal\ weight} (0.72) \quad (15)$$

A conversion factor of 0.80 rather than 0.72 has been suggested in the past (Table A.3, etc.), but the factor of 0.72 is derived, generally, in Appendix 2. However, note that the factor 0.72 is obtained in conjunction with the abscissa of Fig. 2, or M_{max}/M_{cr}, which in turn reflects the reduced dead load of lightweight concrete and the change in cracking moment.

For other values of Δ/L; for example, for $\Delta = L/180$:

$$\left(\frac{L}{t}\right)_{\Delta = L/180} = \left(\frac{L}{t}\right)_{\Delta = L/360} \left(\frac{360}{180}\right) \quad (16)$$

Note that Eq. (16), along with Fig. 2, refers to the conditions in the Code, Section 909(e)1: Based on immediate deflection due to live load for roofs which do not support plastered ceilings — $L/180$.

For ACI 318-63, Section 909 (f): For a floor or roof construction intended to support or to be attached to partitions or other construction likely to be damaged by large deflections of the floor, the allowable limit for the sum of the *immediate deflection due to live load and the additional deflection due to shrinkage and creep under all sustained loads* shall not exceed $L/360$:

$$\left(\frac{L}{t}\right)_{\text{Sec. 909(f)}} = \left(\frac{L}{t}\right)_{\substack{\text{DL + LL} \\ \text{short-time}}} \left(\frac{C_L + 1}{C_L + k_r C_t}\right) \quad (17)$$

where $C_L =$ (live load)/(total sustained load). Although the results in Fig. 2 were developed for singly reinforced rectangular and T-beams, these procedures may be used for doubly reinforced beams as well, in which case, note the different values of $k_r C_t$ in Eq. (17), Table 1, and Eq. (4) corresponding to different percentages of compression steel as compared to tension steel.

Comments on Fig. 2:

1. The steel percentage, $p = A_s/bd$ in all cases except for the T-beam negative moment sections, where $p = A_s/b'd$. The same values for p were assumed for both positive and negative moment sections in all continuous beam solutions. In using these curves, it is suggested that an average of the positive and negative moment steel percentages be used.

2. M_{max}, M_{cr}, and y_t all refer to the numerical maximum moment region (positive for simple beams, negative for the other cases), except that an average of the I_{eff} (which include M_{cr}/M_{max} parameters) were used for the continuous beams.

3. The L/t-values are greater for the lower strength concrete because the n-value is greater. The compensator in this is the M_{cr} — abscissa parameter (M_{cr} is lower for lower strength concrete).

4. The L/t-values appear to be abnormally high for the case of the simple T-beam — higher than for the continuous beams. The reason for this is that M_{cr} in the abscissa refers to the positive moment section for the

simple beam (where $y_t = (4/5)t$ in the solution) and to the negative moment section (where $y_t = (1/5)t$ in the solution) for the other T-beam cases.

Committee 435[7] has also recommended a somewhat different table of maximum span-depth ratios, L/t, of flexural members unless deflections are computed, analogous to the Code Table 909(b). The table is included in Appendix 3.

ILLUSTRATIVE EXAMPLES

Attention is called to References 2, 3, 4, 5, 11, 12, and 13 for basic examples of deflection computations in accordance with ACI 318-63, and also to References 14, 15, and 16. The following examples illustrate additional information pertaining to the procedures in this paper.

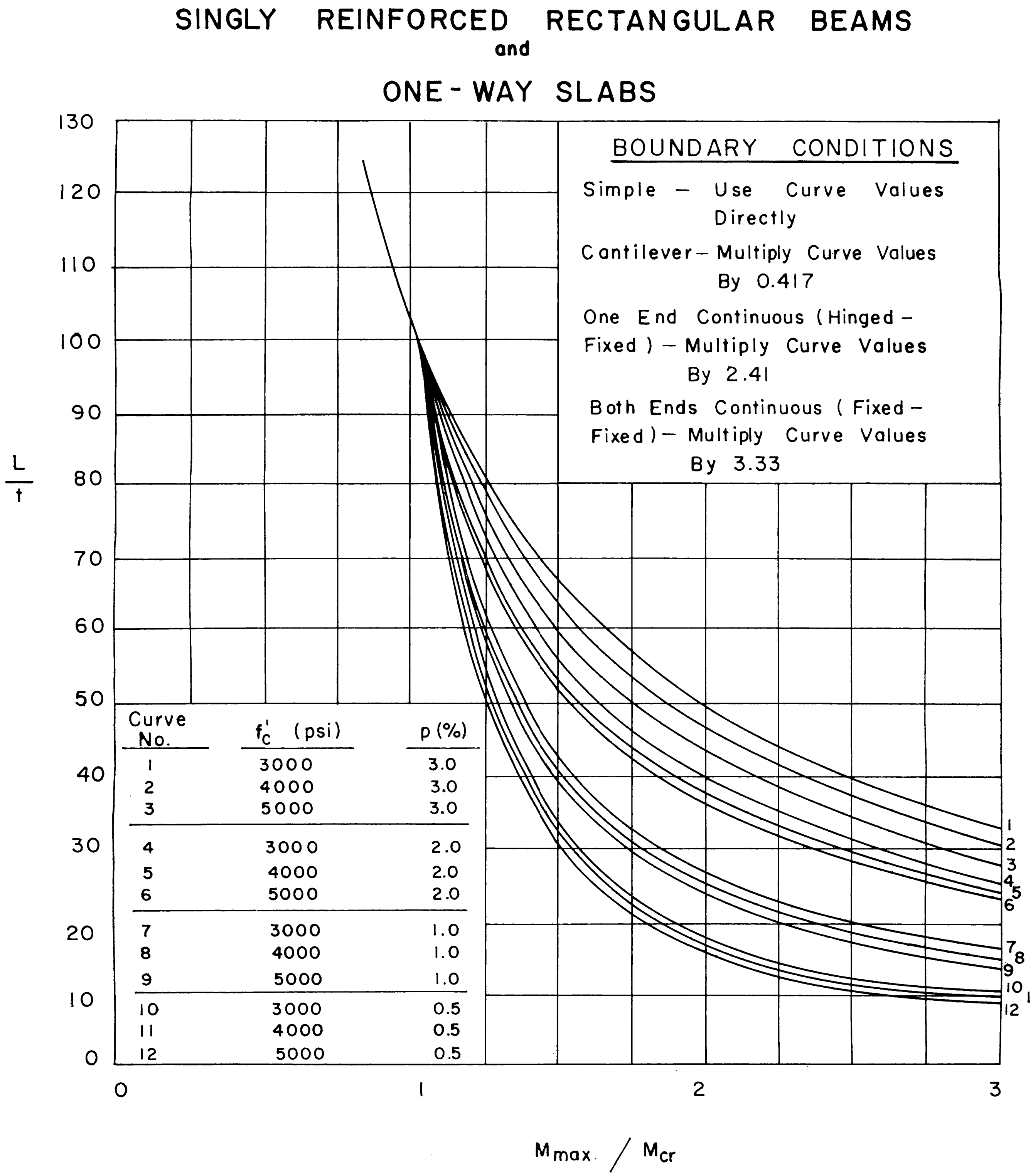


Fig. 2—L/t versus M_{max}/M_{cr} curves for the conditions: $\Delta = L/360$, normal weight concrete, and uniformly distributed short time loading; and for different boundary conditions, steel percentages, and concrete strengths

SINGLY REINFORCED T-BEAMS

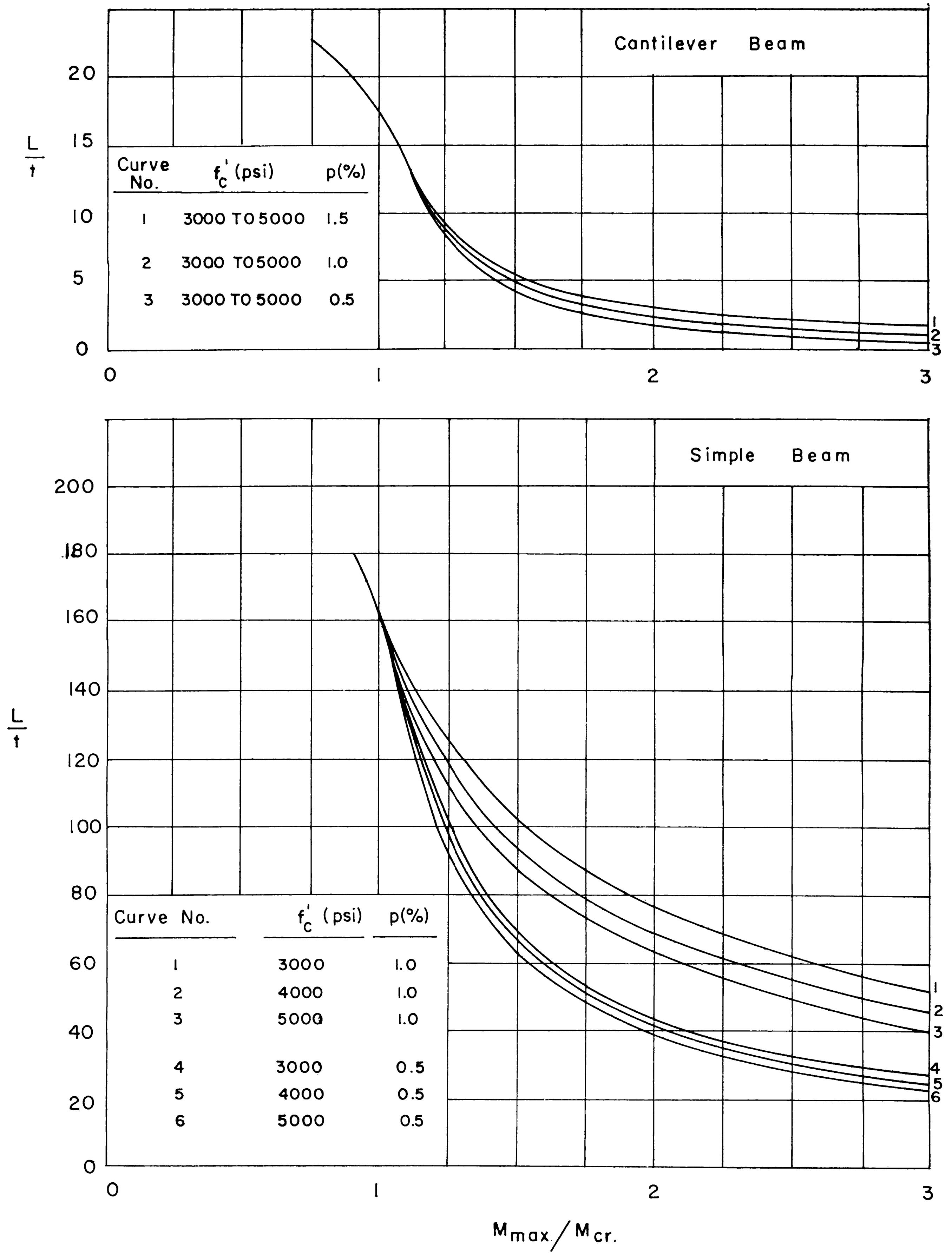
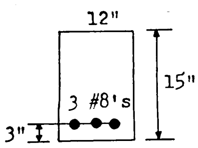


Fig. 2—Continued

Singly reinforced rectangular beam—Use of span-depth ratio curves to check deflections



Simple span = 25 ft
Live load = 300 lb per ft
$f_c' = 3000$ psi
$f_y = 40{,}000$ psi
$n = 9$
$A_s = 2.37$ sq in.
$p = 2.37/(12)(12) = 0.0165$

$$0.18 f_c'/f_y = (0.18)(3)/40 = 0.0135 < 0.0165$$

Hence, check deflections.

Section properties:

$$f'_{cb} = 7.5\sqrt{f_c'} = 411 \text{ psi}, \; I_g = (12)(15)^3/12 = 3380 \text{ in.}^4$$

$M_{cr} = f'_{cb} I_g / y_t = (411)(3380)/7.5 = 185{,}000$ in.-lb $= 15.5$ ft-kips

Loads and moments:

$$w_{DL} = (1)(1.25)(150) = 187 \text{ lb per ft},$$

$$M_{DL} = w_{DL} L^2/8 = (187)(25)^2/8 = 14{,}600 \text{ ft-lb},$$

$$M_{LL} = (300)(25)^2/8 = 23{,}500 \text{ ft-lb}$$

SINGLY REINFORCED T-BEAMS

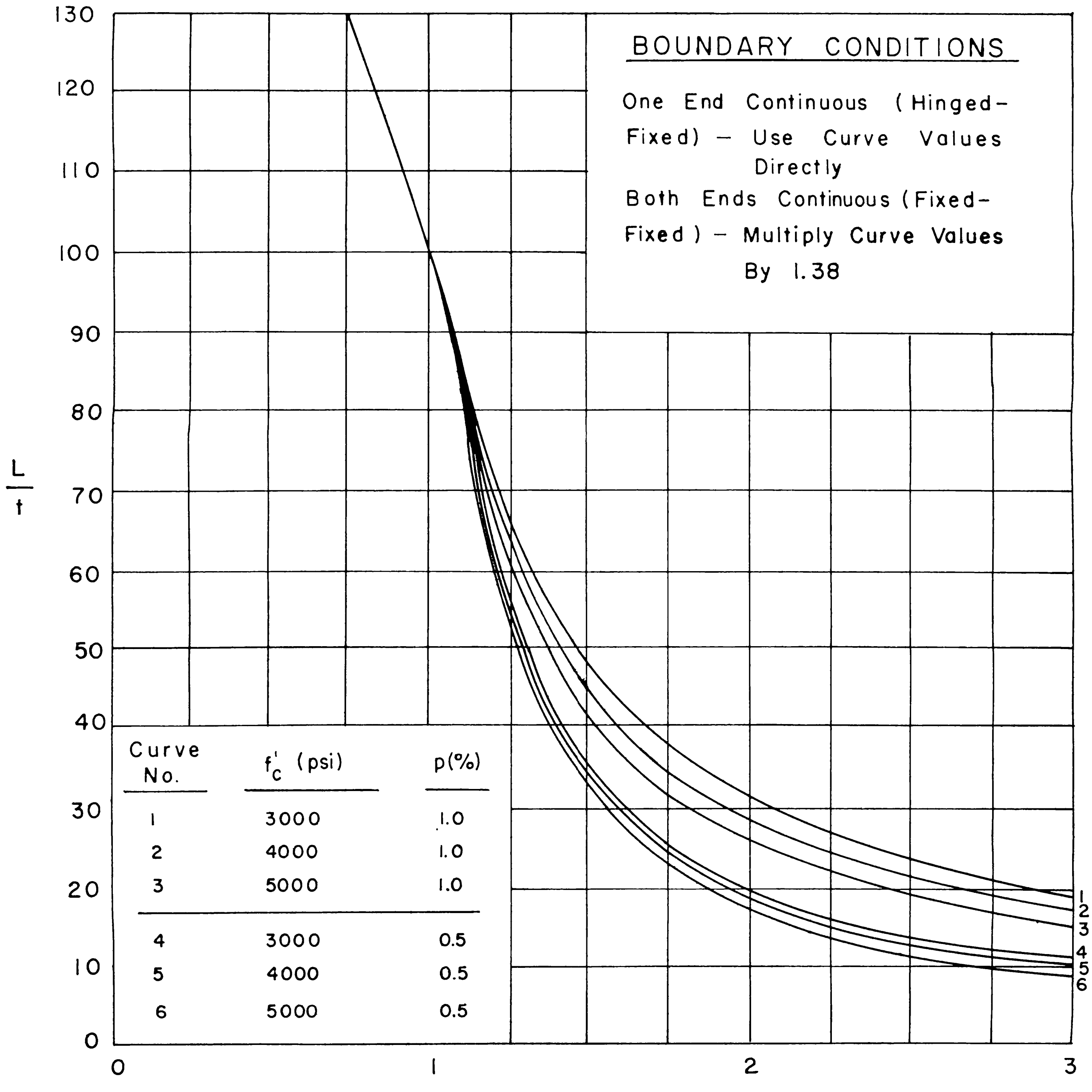


Fig. 2—Continued

Ultimate moment design:

$$M_u = 1.5M_{DL} + 1.8M_{LL} = (1.5)(14.6) + (1.8)(23.5) = 64.0 \text{ ft-kips (7880 m-kgf)}$$

Code Eq. (16-1): $a = 3.10$ in.,

$$M_u = (0.9)[(2.37)(40)(12 - 1.55)] = 71.1 \text{ ft-kips (9840 m-kgf) OK}$$

Check deflections—Code Section 909(f)—using L/t curves:

$$M_{max}/M_{cr} = (14.6 + 23.5)/15.5 = 2.46,$$
From Fig. 2: $L/t = 31$

$$C_L = M_{LL}/M_{DL} = 23.5/14.6 = 1.61,$$
Sec. 909(d): For $A_s' = 0$, $k_rC_t = 2.0$

Using Eq. (17):

$$\left(\frac{L}{t}\right)_{\text{Sec. 909(f)}} = \left(\frac{L}{t}\right)_{\substack{\text{DL+LL} \\ \text{short-time}}} \left(\frac{C_L + 1}{C_L + k_rC_t}\right)$$

$$= (31)(1.61) + 1)/(1.61 + 2) = 22$$

This is the maximum allowable L/t for the conditions of this problem and based on ACI 318-63, Section 909(f).

$$\text{Actual } L/t = (25)(12)/15 = 20 \quad \text{OK}$$

Illustration of deflection computation:
Additional quantities needed

$$E_c = 3.2 \times 10^6 \text{ psi}, I_{cr} = 1540 \text{ in.}^4$$

Note that different effective I's should be used for dead load and for total load.
Dead load

$$M_{DL} < M_{cr}$$

$$(\Delta_{DL})_i = \frac{5M_{DL}L^2}{48E_cI_g} = \frac{(5)(14.6)(25)^2(1728)}{(48)(3.2 \times 10^3)(3380)} = 0.152 \text{ in.}$$

Live load

$$I_{eff} = \left(\frac{M_{cr}}{M_{max}}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] I_{cr}$$

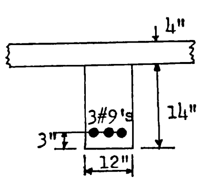
$$= \left(\frac{15.5}{34.1}\right)^3 (3380) + \left[1 - \left(\frac{15.5}{34.1}\right)^3\right](1540)$$

$$= 1700 \text{ in.}^4$$

$$\Delta_{LL} = \frac{5M_{LL}L^2}{48E_cI_{eff}} = \frac{(5)(19.5)(25)^2(1728)}{(48)(3.2 \times 10^3)(1700)} = 0.405 \text{ in.}$$

Deflection as per Code Section 909(f)
Total $\Delta = (0.152)(2) + 0.405 = 0.71$ in. (1.80 cm)
Allowable $\Delta = L/360 = (25)(12)/360 = 0.83$ in. (2.11 cm) OK

Composite beam—Ordinary reinforced precast beam, unshored construction



Simple span = 20 ft
Beam spacing = 10 ft
Live load = 100 psf
f_c' = 3000 psi for the slab concrete at 28 days, and for the precast beam concrete at the time of application of the precast beam DL
f_c' = 4000 psi for the precast beam concrete at 28 days
f_y = 50,000 psi
A_s = 3.00 sq in.

Code Section 906: Effective flange width (not including the effect of transformed slab) = 60 in.

Code Section 1507: $f_y > 40{,}000$ psi; hence, check deflections.

Use Eq. (10) to compute the deflection and check in accordance with the requirements of the Code, Section 909(f).

Material properties:

For $f_c' = 3000$ psi:

$$E_c = (145)^{1.5}(33)\sqrt{f_c'} = 3.2 \times 10^6 \text{ psi},$$
$$n = 9, f'_{cb} = 7.5\sqrt{f_c'} = 411 \text{ psi}$$

For $f_c' = 4000$ psi:
$$E_c = 3.6 \times 10^6 \text{ psi}, n = 8, f'_{cb} = 474 \text{ psi}$$

Section properties—Precast beam

$$I_g = (12)(14)^3/12 = 2740 \text{ in.},$$

$$p = 3.00/(12)(11) = 0.0227,$$

$$pn = (0.0227)(8) = 0.182,$$

$$k = \sqrt{(pn)^2 + 2pn} - pn = 0.448$$

$$kd = (0.448)(11) = 4.93 \text{ in.},$$

$$I_{cr} = (12)(4.93)^3/3 + (8)(3.00)(11 - 4.93)^2 = 1{,}360 \text{ in.}^4$$

Section Properties—Composite beam

$$(E_c)_{stem}/(E_c)_{slab} = 3.6/3.2 = 1.13,$$
$$\text{Flange width} = 60/1.13 = 53.1 \text{ in.}$$

$$\bar{y}_{bot} = \frac{(12)(14)(7) + (53.1)(4)(16)}{(12)(14) + (53.1)(4)} = 12.04 \text{ in.},$$

$$y_{cs} = 2 + 1.96 = 3.96 \text{ in.}$$

$$I_g = (12)(14)^3/12 + (12)(14)(5.04)^2 + (53.1)(4)^3/12 + (53.1)(4)(3.96)^2 = 10{,}620 \text{ in.}^4,$$

$$(53.1)(kd)^2/2 = (8)(3.00)(15 - kd), \; kd = 3.25$$

$$I_{cr} = (53.1)(3.25)^3/3 + (8)(3.00)(15 - 3.25)^2 = 3920 \text{ in.}^4$$

Loads and moments:

$$w_{LL} = (100)(10) = 1000 \text{ lb per ft},$$

$$w_{slab} = (150)(4 \times 10 \times 12)/144 = 500 \text{ lb per ft}$$

$$w_{precast\ beam} = (150)(12)(14)/144 = 175 \text{ lb per ft}$$

$$M_{LL} = w_{LL}L^2/8 = (1000)(20)^2/8 = 50{,}000 \text{ ft-lb}$$

$$M_1 = w_{slab}L^2/8 = (500)(20)^2/8 = 25{,}000 \text{ ft-lb}$$

$$M_2 = w_{precast\ beam}L^2/8 = (175)(20)^2/8 = 8800 \text{ ft-lb}$$

Ultimate moment design:

$$M_u = 1.5M_{DL} + 1.8M_{LL} = (1.5)(25 + 8.8) + (1.8)(50) = 140.7 \text{ ft-kips (19,450 m-kgf)}$$

After checking for T-beam, use Code Eq. (16-1):

$$a = (3)(50)/(0.85)(3)(60) = 0.98 \text{ in.}$$

$$M_u = (0.90)[(3.00)(50)(15 - 0.49)] = 1958 \text{ in.-kips} = 163 \text{ ft-kips (22,500 m-kgf) OK}$$

$$\left(1.15 + 0.24\frac{M_L}{M_D}\right) d_p = \left(1.15 + 0.24\frac{50}{33.8}\right)(11) = 16.5 \text{ in.}$$

which is greater than the actual effective depth of the composite section, or 15 in. Hence the design is OK without shores, by Code Section 2504(b).

Deflection calculations by terms in Eq. (10):

Term (1)

$$M_{max} = M_2 = 8.8 \text{ ft-kips}$$

Eq. (2):

$$M_{cr} = f'_{cb} I_g / y_t = (474)\ (2740)/7 = 185{,}000 \text{ in.-lb} = 15.4 \text{ ft-kips}$$

$$\text{Hence } I_2 = I_g = 2740 \text{ in.}^4$$

$$(\Delta_i)_2 = \frac{K_2 M_2 L^2}{E_{ci} I_2} = \frac{(5/48)\ (8.8)\ (20)^2 (12)^3}{(3.2 \times 10^3)\ (2740)} = 0.07 \text{ in. } (0.18 \text{ cm})$$

Term (2). From Notation, assume $\alpha = 0.35$, Code Section 909(d):

$$(k_r C_{t_2}) = 2.0$$

$$\alpha k_r C_{t_2} (\Delta_i)_2 = (0.35)\ (2.0)\ (0.07) = 0.05 \text{ in. } (0.13 \text{ cm})$$

Term (3)

$$(1 - \alpha) k_r C_{t_2} (\Delta_i)_2 \frac{I_2}{I_c} = (0.65)\ (2.0)\ (0.07) \frac{2740}{10{,}620} = 0.02 \text{ in. } (0.05 \text{ cm})$$

Term (4)

$$M_{max} = M_1 + M_2 = 33.8 \text{ ft-kips}$$

From above: $M_{cr} = 15.4$ ft-kips,

$$\frac{M_{cr}}{M_{max}} = \frac{15.4}{33.8} = 0.455$$

Eq. (1):

$$I_2 = I_{eff} = \left(\frac{M_{cr}}{M_{max}}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] I_{cr}$$

$$= (0.455)^3 (2740) + [1 - (0.455)^3]\ (1360)$$

$$= 1490 \text{ in.}^4$$

$$(\Delta_i)_1 = \frac{K_1 M_1 L^2}{E_c I_2} = \frac{(5/48)\ (25)\ (20)^2 (12)^3}{(3.6 \times 10^3)\ (1490)} = 0.33 \text{ in. } (0.84 \text{ cm})$$

Term (5). From Table 1, assume: $k_r C_{t_1} = 1.5$, (precast beam concrete may be a month or two old when the slab is cast). Since no nontensioned steel is used, or $A_s' = 0$: $k_r = 1.0$

$$k_r C_{t_1} (\Delta_i)_1 \frac{I_2}{I_c} = (1)\ (1.5)\ (0.33) \frac{1490}{10{,}620} = 0.07 \text{ in. } (0.84 \text{ cm})$$

Term (6). Assume $D = 300 \times 10^{-6}$ in./in.

$$Q = D A_1 E_1 = (300)\ (60 \times 4)\ (3.2) = 230{,}000 \text{ lb}$$

$$\frac{Q y_{cs} L^2}{8 E_c I_c} = \frac{(230)\ (3.96)\ (20)^2 (12)^2}{(8)\ (3.6 \times 10^3)\ (10{,}620)} = 0.17 \text{ in. } (0.43 \text{ cm})$$

Term (7)

$$M_{max} = M_1 + M_2 + M_{LL} = 83.8 \text{ ft-kips}$$

$$M_{cr} = \frac{f'_{cb} I_g}{y_t} = \frac{(474)\ (10{,}620)}{12.04} = 417{,}000 \text{ in.-lb} = 34.8 \text{ ft-kips}$$

$$\frac{M_{cr}}{M_{max}} = 34.8/83.8 = 0.415,\ (0.415)^3 = 0.072$$

Eq. (1):

$$I_{eff} = (0.072)\ (10{,}620) + (0.928)\ (3920) = 4390 \text{ in.}^4$$

$$(\Delta)_{LL} = \frac{K M_{LL} L^2}{E_c I_{eff}} = \frac{(5/48)\ (50)\ (20)^2 (12)^3}{(3.6 \times 10^3)\ (4390)} = 0.23 \text{ in.}$$

$$\text{Total } \Delta = \underset{(1)}{0.07} + \underset{(2)}{0.05} + \underset{(3)}{0.02} + \underset{(4)}{0.33} + \underset{(5)}{0.07} + \underset{(6)}{0.17} + \underset{(7)}{0.23} = 0.94 \text{ in. } (2.39 \text{ cm})$$

Deflections checked in accordance with Code Section 909(f):

$$\text{Computed } \Delta = 0.94 - 0.07 = 0.87 \text{ in. } (2.21 \text{ cm})$$

deleting Term (1) as per Section 909(f).

$$\text{Allowable } \Delta = L/360 = (20)\ (12)/360 = 0.67 \text{ in. } (1.70 \text{ cm}) \quad \text{NG}$$

The design is satisfactory based on ultimate moment but is unacceptable according to the Code deflection control provision, Section 909(f).

ACKNOWLEDGMENT

Special acknowledgment is made of the contribution of J. R. Benjamin, R. S. Fling, B. L. Meyers, and M. V. Pregnoff to the work of ACI Committee 435 as related to parts of this paper.

REFERENCES

1. ACI Committee 318, "Building Code Requirements for Reinforced Concrete (ACI 318-63)," American Concrete Institute, Detroit, 1963, 144 pp.

2. ACI Committee 340, "Ultimate Strength Design Handbook, V. 1," SP-17, American Concrete Institute, Detroit, 1967, 176 pp.

3. ACI Committee 317, "Reinforced Concrete Design Handbook-Working Stress Method," SP-3, 3rd Edition, American Concrete Institute, Detroit, 1965, 271 pp.

4. ACI Committee 435, "Deflections of Reinforced Concrete Flexural Members," ACI JOURNAL, *Proceedings* V. 63, No. 6, June 1966, pp. 637-674.

5. Subcommittee 5, ACI Committee 435, "Deflections of Prestressed Concrete Members," ACI JOURNAL, *Proceedings* V. 60, No. 12, Dec. 1963, pp. 1697-1728.

6. ACI-ASCE Committee 327, "Ultimate Strength Design," *Proceedings*, ASCE, Paper No. 809, Oct. 1955. The report without certain test data appears in ACI JOURNAL, *Proceedings* V. 52, No. 7, Jan. 1956, p. 505.

7. Subcommittee 4, ACI Committee 435, "Recommendation for ACI Building Code Provision on Deflections," Report approved by Committee 435 and submitted to ACI Committee 318, Oct. 1967.

8. Branson, D. E., and Shaikh, A. F., "Favorable and Unfavorable Effects of Non-Tensioned Steel in Prestressed Concrete Beams," Research Report No. HR-123, Iowa State Highway Commission, and Bureau of Public Roads No. HPR-1(3) (Iowa), Item No. 42, June 1967.

9. Hogan, John J., "Explanation of Control of Deflections—Section 909," *Bulletins* No. 63S-3-1 to 63S-3-14, Portland Cement Association, Skokie, Ill.

10. Branson, Dan E., "Time-Dependent Effects in Composite Concrete Beams," ACI JOURNAL, *Proceedings* V. 61, No. 2, Feb. 1964, pp. 213-230.

11. Wang, C. K., and Salmon, C. G., *Reinforced Concrete Design*, International Textbook Co., Scranton, Pa., 1965.

12. Winter, George; Urquhart, L. C.; O'Rourke, C. E.; and Nilson, A. H., *Design of Concrete Structures,* McGraw-Hill Book Co., New York, 7th Edition, 1964.

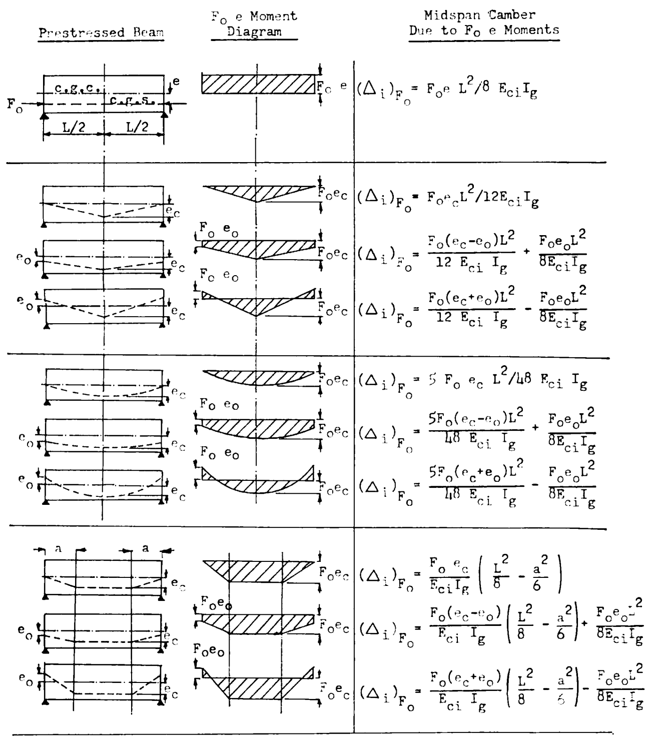
13. Ferguson, P. M., *Reinforced Concrete Fundamentals,* 2nd Edition, John Wiley & Sons, New York, 1965.

14. "Strength and Serviceability Criteria—Reinforced Concrete Bridge Members—Ultimate Design," Bureau of Public Roads, Aug. 1966.

15. *Symposium on Creep of Concrete,* American Concrete Institute, Detroit, 1964, 160 pp.

16. Corley, W. G., and Sozen, M. A., "Time-Dependent Deflections of Reinforced Concrete Beams," ACI JOURNAL, *Proceedings* V. 63, No. 3, Mar. 1966, pp. 373-386.

APPENDIX 1—COMMON CASES OF PRESTRESS MOMENT DIAGRAMS WITH FORMULAS FOR COMPUTING CAMBER



APPENDIX 2—DERIVATION OF SPAN-DEPTH RATIO, L/t, MATERIAL

General Formulas

Formulas for computing moments, moments of inertia, and deflections:

M_{max} = maximum moment due to all dead and live load. Positive and negative moments are considered separately for beams continuous at one or both ends in computing I_{eff}

M_{cr} = $f'_{cb} I_g / y_t$, where $f'_{cb} = 7.5\sqrt{f_c'}$ for normal weight concrete and $6.0\sqrt{f_c'}$ for lightweight concrete

When

$$\frac{M_{cr}}{M_{max}} \geqq 1, \ I_{eff} = I_g$$

$$\frac{M_{cr}}{M_{max}} < 1, \ I_{eff} = \left(\frac{M_{cr}}{M_{max}}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] I_{cr}$$

An average of the positive and negative moment region values for I_{eff} are used for beams continuous at one or both ends.

$$\Delta = K M_{max} L^2 / (E_c I_{eff})$$

Formulas for computing span-depth ratios:
For $M_{cr}/M_{max} \geqq 1$, or $M_{max}/M_{cr} \leqq 1$

$$\Delta = K \frac{M_{max} L^2}{E_c I_g} \frac{f'_{cb} I_g / y_t}{M_{cr}}$$

$$\frac{\Delta}{L} = K \frac{f'_{cb}}{E_c} \frac{M_{max}}{M_{cr}} \frac{L}{y_t}, \quad \frac{L}{y_t} = \frac{\Delta}{L} \frac{E_c}{f'_{cb} K} \frac{M_{cr}}{M_{max}}$$

or

$$\frac{L}{y_t} = \frac{\Delta}{L} \frac{C_w'}{K} \frac{M_{cr}}{M_{max}} \qquad \text{(A2.1)}$$

where $C_w' = E_c / f'_{cb}$. Using $33(w')^{1.5}\sqrt{f_c'}$, $w' = 145$ pcf and $f'_{cb} = 7.5\sqrt{f_c'}$ for normal weight concrete; and $w' = 100$ pcf and $f'_{cb} = 6.0\sqrt{f_c'}$ for lightweight concrete: $C_w' = 7680$ for normal weight concrete and $C_w' = 5550$ for lightweight concrete.

Note that the appropriate L/t conversion factor from normal weight to lightweight concrete is $5550/7680 = 0.72$, [see Eq. (15)], times the change in $(M_{cr})/(M_{max})$.

For $M_{cr}/M_{max} < 1$, or $M_{max}/M_{cr} > 1$

$$\frac{L}{y_t} = \frac{\Delta}{L} \frac{C_w'}{K} \frac{M_{cr}}{M_{max}} m$$

where

$$m = \left(\frac{M_{cr}}{M_{max}}\right)^3 + \left[1 - \left(\frac{M_{cr}}{M_{max}}\right)^3\right] \frac{I_{cr}}{I_g} \qquad \text{(A2.2)}$$

Note in Eq. (A2.1) and (A2.2) that M_{max}, M_{cr}, and y_t refer to the numerical maximum moment and corresponding section (which is negative moment, section for the continuous beams), but that in the formula for m in Eq. (A2.2) an average m-value is used for the positive and negative moment regions.

Once the value for y_t is defined (for a particular section; for example, $y_t = t/2$ for a rectangular section), Eq. (A2.1) and (A2.2) yield the appropriate values for L/t. Note that all ratios in Eq. (A2.1) and (A2.2) are dimensionless.

Lightweight concrete

Eq. (A2.1) refers to the uncracked case in which the ratio of lightweight to normal weight thicknesses for a given span are proportional to the cube root of the lightweight to normal weight total load ratio and the normal weight to lightweight modulus of elasticity ratio. Using a lightweight total load reduction of 20 percent and the ACI Code formula for the modulus of elasticity, the conversion factors for normal weight to lightweight thicknesses, for the same deflection and span are:

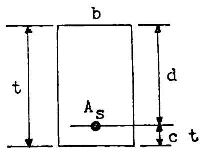
For 90 pcf — 1.20
For 100 pcf — 1.13
For 110 pcf — 1.08
For 120 pcf — 1.03

For the cracked case, Eq. (A2.2) applies in which the factor m involves the change in load, the change in section, the change in n-value, and the change in the modulus of rupture. However, it appears that the cracked condition for lightweight concrete should not greatly change the above factors (for the uncracked or gross section). In the cracked condition, the increased n-value for lightweight concrete tends to offset the reduced E. The physical reason for this is that as the steel becomes more effective, the section is penalized less for the lower modulus of elasticity of the lightweight concrete as compared to normal weight concrete.

Assuming approximately the same shrinkage plus creep factors for lightweight as normal weight concrete, a conversion factor of 1.15, or at most 1.20, for normal weight to lightweight concrete minimum thicknesses (or the reciprocal for L/t) appear to be reasonable.

Singly reinforced rectangular beams

Relationship between the gross and cracked transformed section moments of inertia:



$$n = E_s/E_c,\ p = A_s/bd,\ \text{Use } c = 0.1$$

$$k = \sqrt{(pn)^2 + 2pn} - pn$$

$$I_g = bt^3/12$$

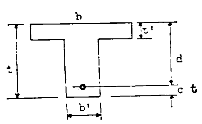
$$I_{cr} = b(kd)^3/3 + nA_s(d - kd)^2$$
$$= 0.729bt^3[k^3/3 + np(1-k)^2]$$

$$\frac{I_{cr}}{I_g} = 8.75[k^3/3 + np(1-k)^2]$$

Span-depth ratios:
$y_t = t/2$, See Eq. (A2.1) and (A2.2) for computing L/t.

Singly reinforced T-beams

Approximate relationship between the gross and cracked transformed section moments of inertia:



$n = E_s/E_c$, $p = A_s/bd$ for positive moment, and $p = A_s/b'd$ for negative moment, Use $c = 0.1$

Assume $\bar{y}_{bot} = (4/5)t$, $b' = (1/4)b$, $t' = (1/5)t$; hence the centroid of the cracked and gross sections are assumed to be at the bottom of the flange. The method is not used for $p > 1$ percent.

$$I_g = \frac{b'\left(\frac{4}{5}t\right)^3}{3} + \frac{b(t')^3}{3} = 0.181b'(t)^3$$

Positive moment region

$$I_{cr} = b(t')^3/3 + nA_s(d - t')^2 = b't^3(0.0107 + 1.764np)$$

$$\frac{I_{cr}}{I_g} = 5.51(0.0107 + 1.764np)$$

Negative moment region
See corresponding equation for rectangular beam, except

$$p = \frac{A_s}{b'd}$$

Get

$$I_{cr} = 0.729b't^3[k^3/3 + np(1-k)^2]$$

and

$$\frac{I_{cr}}{I_g} = 4.015[k^3/3 + np(1-k)^2]$$

Span-depth ratios:

Cantilever beams, $y_t = (1/5)t$
Simple beams, $y_t = (4/5)t$
Hinged-fixed beams, $y_t = (1/5)t$
Fixed-fixed beams, $y_t = (1/5)t$
Because M_{max} and M_{cr} refer to negative moment in Eq. (A2.1) and (A2.2); and y_t was introduced with M_{cr}.

See Eq. (A2.1) and (A2.2) for computing L/t.

APPENDIX 3

TABLE A.3—MAXIMUM SPAN-DEPTH RATIOS, L/t, OF FLEXURAL MEMBERS UNLESS DEFLECTIONS ARE COMPUTED*

Simply supported roof and floor construction *not intended* (first L/t-value in each case) or *intended* (second L/t-value in each case) to support or to be attached to partitions or other construction likely to be damaged by large deflections

Roof slab†	Floor slab†	Roof beam or ribbed roof slab	Floor beam or ribbed floor slab
24,14	18,12	18,12	14,10

NOTES:—
1. This table does not apply to prestressed concrete or composite members.
2. The L/t-values in the table apply to normal weight concrete. Use 0.8 of the tabulated values for lightweight concrete.
3. The L/t-values in the table apply to simply supported members. Multiply the tabulated values by 0.4 for cantilever members, 1.3 for members with one end continuous, and 1.6 for members with both ends continuous.
4. For square two-way slabs supported on all four sides by walls, or by beams at least 3 times as deep as the slab, multiply the tabulated values by 1.4. Span-depth ratios of rectangular two-way slabs should be between those of one-way slabs and two-way slabs.

*Table A.3 was recommended by ACI Committee 435[7].
†Refers to solid slabs only, either one-way slabs or flat plates. Also refers to two-way slabs when modified as described in Note 4 above.

Based on a paper presented at the 1967 ACI Fall Meeting, Des Moines, Iowa, Nov. 3, 1967.

Sinopsis — Résumé — Zusammenfassung

Procedimientos de Diseño para Calcular Deflexiones

Se presentan los procedimientos de diseño para calcular deflexiones a corto y a largo plazo de vigas de concreto reforzadas y presforzadas no compuestas y compuestas. Este artículo discute las fases de este tópico indicado en el Reglamento de las Construcciones del ACI pero que no está específicamente definido y que se basa, en parte, en el trabajo reciente del Comité ACI 435, Deflexiones.

Méthode de calcul des déformations de flexion

Les procédés de calculs actuels pour déterminer les déformations à court terme et à long terme de poutres en béton armé précontraint non composite et ordinairement composite sont présentés. Cet article traite des phases du sujet indiqué dans le code de construction ACI mais qui n'y sont pas spécifiquement définies. Il est en partie basé sur les travaux récents du comité ACI 435 sur les déformations.

Entwurf von Verfahren zur Errechnung von Abbiegungen

Entwurfsmethoden werden gezeigt, die es erlauben, Kurzzeit- und Langzeit-Durchbiegungen von Stahlbetonbalken, vorgespannten Balken und Verbundsystemen zu ermitteln. Die Arbeit diskutiert verschiedene Phasen dieses Problemes, die in der ACI Vorschrift zwar angedeutet jedoch nicht ausdrücklich definiert sind. Die Studie baut z.T. auf kürzlich durchgeführte Arbeiten des ACI Ausschusses 435, Durchbiegung, auf.