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Displacements and Losses in Multistage Prestressed Members

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Introduction

An accurate estimation of prestress loss in concrete members is essential for the prediction of actual stresses and displacements. This paper examines the losses of prestress and the accompanying variation in deformation in multistage prestressed beams due to the effects of instantaneous deformations at the moment of prestressing, steel relaxation, creep, and shrinkage of concrete. The following parameters related to the materials properties and the environment are assumed to be known: (1) The modulus of elasticity of concrete, E_c; (2) the modulus of elasticity of steel, E_s, and its intrinsic relaxation $\bar{P}_r$; and (3) the free shrinkage of concrete, s, and its creep coefficient ϕ.

During the period of loss the concrete is subjected to a gradual stress reduction, while its modulus of elasticity increases due to aging. The reduction in concrete stress induces elastic strain and creep recoveries. The prestress steel, which is gradually shortened, exhibits smaller relaxation loss as compared to the "intrinsic" relaxation obtained from a constant strain relaxation test. In practice prestress loss is calculated by approximate methods ignoring the strain recoveries and reduction in relaxation and resulting in overestimation of losses. A closed-form solution that takes these effects into account is not possible. A numerical step-by-step procedure is developed in Refs. 2 and 4 for calculation of prestress loss. Graphs prepared by the method (3) can be used for rapid evaluation of loss when prestressing is applied in one stage. The present paper reviews the procedure in Ref. 2 and includes the following developments:

1. Use of a displacement method to calculate the loss due to instantaneous deformation in multistage multitendon prestress. This aims at a reduction of the computations.

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2. The procedure of calculation of losses is extended to compute deflections. A numerical example is presented for the calculation of losses and deflections of a prestressed concrete bridge built by the cantilever (segmental construction) method (consecutive segments cast and prestressed in multistages).

Step-by-Step Computation

Consider a concrete beam prestressed by a number of tendons, k, tensioned at different ages. It is required to find the following at a section: (1) The loss of prestress; (2) the strain at the centroid; and (3) the curvature occurring at any time after prestressing the first tendon. The period between the first prestressing and the age at which the loss (or deformation) is required is divided into n internals. Note that the ages at which other tendons are tensioned or

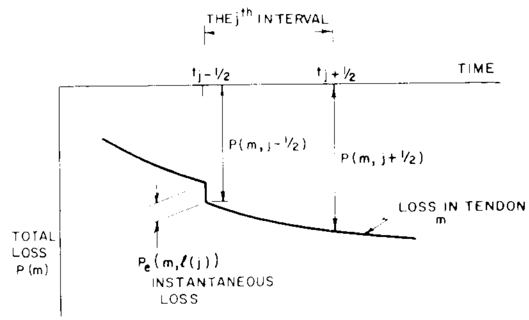

FIG. 1.—Variation of Loss in Tendon m During jth Interval at Beginning of Which Tendon $l(j)$ Is Tensioned

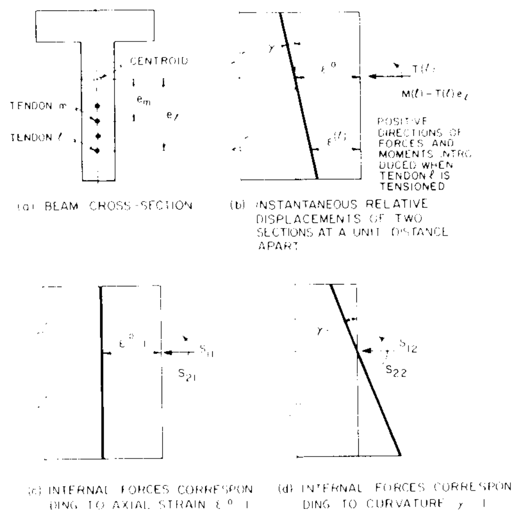

FIG. 2.—Instantaneous Deformation and Induced Forces (Symbols ϵ^o and γ Have Subscripts e, $j(l) - 1/2$ That Are Omitted for Clarity

external applied loads are applied coincide with interval limits.

The symbol, i (or j), represents the interval number and as a subscript it indicates the age at the middle of the i^{th} (or j^{th}) interval. The subscripts, $i - 1/2$ and $i + 1/2$, represent the ages at the beginning and the end of the interval, i, respectively.

The tendons are numbered following the order in which the prestress is applied. Thus the interval numbers and the tendon numbers are related. To represent this relation, define a vector $\{l(j)\}$ of n elements in which the element, $l(j)$, is the number of the last tendon tensioned prior or at the beginning of the j^{th} interval. Similarly, define a vector $\{j(l)\}$ of k elements in which the element, $j(l)$, is the number of the interval at the beginning of which tendon l is tensioned. For example, if there are seven intervals ($n = 7$), and the prestressing is applied at three stages at the beginning of the first, third, and fifth intervals, then $\{j(l)\} = \{1, 3, 5\}$ and $\{l(j)\} = \{1, 1, 2, 2, 3, 3, 3\}$.

In the step-by-step method the loss of prestress at the end of any j^{th} interval (age $j + 1/2$) can be calculated, provided that the loss at the beginning of the same interval (age $j - 1/2$) is known. The loss due to instantaneous deformation is introduced at the beginning of the interval (Fig. 1), while the other losses are assumed to act starting from the middle of the j^{th} interval. The contribution by different causes of losses is treated in the following sections. In these derivations, it is assumed that plane cross sections remain plane after bending. It is also assumed that the time-dependent change in strain in a tendon is compatible with the change in strain in concrete at the tendon level.

Note that this last assumption, while generally adopted in prestressed concrete, is not fully justified when the tendons are left unbonded for part or all the period for which the deformations are considered. The approximation resulting from this assumption in such a case is not examined in this paper.

Instantaneous Deformation

Tensioning of tendon $l, (l > 1)$, with a force, $T(l)$, accompanied by the application of the external bending moment, $M(l)$, produces instantaneous (elastic) deformations. At the section considered, these deformations are defined by the strain at the centroid, $\epsilon_e^o\,(j(l) - 1/2)$, positive for shortening, and curvature, $\gamma_e(j(l) - 1/2)$, positive for sagging). The subscript, e, indicates the cause of the deformation, while the index, $j(l) - 1/2$, indicates the beginning of the interval, $j(l)$, when the instantaneous deformation is produced.

From Fig. 2(c), the unit axial strain, $\epsilon_e^o = 1$, with $\gamma_e = 0$ corresponds to the following stress resultants (internal forces)

$$S_{11} = F\left(j(l) - \frac{1}{2}\right) + \sum_{m=1}^{l-1} A_s(m)\, E_s \quad \text{. . . . . . (1)}$$

$$S_{21} = -\sum_{m=1}^{l-1} A_s(m)\, E_s\, e_m \quad \text{. . . . . . (2)}$$

in which F is a time-dependent variable equal to the product of the modulus of elasticity of concrete, E_c, and the area of concrete A_c. At age t_i

$$F(t_i) = F(i) = E_c(i)\, A_c \quad \text{. . . . . . (3)}$$

in which E_s, $A_s(m)$, and e_m = the modulus of elasticity, area, and eccentricity of tendon m, respectively. For simplicity, E_c is assumed a constant average value when it is used in Eq. 3 to calculate the force, F. The effect of the presence of nontensioned steel is not accounted for in this analysis.

A curvature of $\gamma_e = 1$, with $\epsilon_e^o = 0$ [see Fig. 2(d)], corresponds to the stress resultants

$$S_{12} = -\sum_{m=1}^{l-1} A_s(m)\, E_s\, e_m \quad \text{(4)}$$

$$S_{22} = r^2 F\left(j(l) - \frac{1}{2}\right) + \sum_{m=1}^{l-1} A_s(m)\, E_s\, e_m^2 \quad \text{(5)}$$

in which r = the radius of gyration of the effective area. Equating the forces due to the deformation with the prestressing and external moments

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{Bmatrix} \epsilon^o \\ \gamma_e \end{Bmatrix}_{j(l)-1/2} = \begin{Bmatrix} T(l) \\ M(l) - T(l)\, e_l \end{Bmatrix} \quad \text{(6)}$$

Solution of Eq. 6 gives ϵ_e^o and γ_e. The instantaneous loss in tendon m occurring immediately after application of prestress force $T(l)$ and moment $M(l)$ is

$$P_e\left(m, j(l) - \frac{1}{2}\right) = A_s(m)\, E_s\left[\epsilon_e^o\left(j(l) - \frac{1}{2}\right) - e_m \gamma_e\left(j(l) - \frac{1}{2}\right)\right] \quad \text{(7)}$$

This loss represents a negative force that is assumed to act on the concrete section at the location of tendon m starting from the beginning of the interval, $j(l)$. At age $i + 1/2$, the loss in the m^{th} tendon due to instantaneous deformation is the sum caused by prestressing tendons $m+1$, $m+2$, ..., $l(i)$. Thus

$$\sum P_e\left(m, i - \frac{1}{2}\right) = \sum P_e\left(m, i + \frac{1}{2}\right) = \sum_{l=m+1}^{l(i)} P_e\left(m, j(l) - \frac{1}{2}\right) \quad \text{(8)}$$

The creep recovery due to the losses, $P_e(m)$, is accounted for in a subsequent sectin on Loss Due to Creep.

Expressions for Creep

The total strain at age t_i caused by a unit stress change introduced at an earlier age, t_j, is expressed as

$$\epsilon = \frac{1}{E_c(j)}\{1 + f(t_i, t_j)\} \quad \text{(9)}$$

in which $f(t_i, t_j)$ = a creep function equal to the ratio of the creep to the instantaneous strain.

For the practical range of stresses, creep is proportional to the applied stress; this is accepted here for both stress increments and decrements. A number of equations are available (7) to express the creep-time relation. The method of calculation of loss is independent of the choice of the function, f. In the examples in this paper, the creep function is expressed by (see Ref. 1)

$$f(i, j) = f(t_i, t_j) = \phi\, 1.35 \frac{\ln(t_i - t_j + 1)}{5 + \sqrt{t_j}} \quad \text{(10)}$$

which is derived from the graphs recommended by the joint Comité Européen du Béton-Federation Internationale de la Précontrainte (CEB-FIP) Committee (6). The coefficient, ϕ, depends on the concrete quality, the environment, and the effective thickness of the element. The value of ϕ is the ratio of the ultimate creep [assumed to occur at $(t_i - t_j) = 2{,}000$ days] to the instantaneous strain when $t_j = 28$ days.

Shrinkage

Assume that the free shrinkage, $s(i + 1/2)$, is known at the end of interval i and it is required to find its contribution to the loss in the tendons prestressed before the age, $t_{i+1/2}$. If shrinkage produces the loss, $P_s(m, i + 1/2)$, in the tendon, m, the corresponding change in strain in the prestressing steel is $P_s(m, i + 1/2)/[A_s(m)\, E_s]$. This change of strain is equal to the change in the strain of concrete at the level of the steel, the latter change being the combined effect of shrinkage and of the elastic and creep recoveries induced by the decrease in the stress on concrete due to shrinkage. Thus

$$\frac{P_s\left(m, i + \frac{1}{2}\right)}{A_s(m)\, E_s} = s\left(i + \frac{1}{2}\right) - \sum_{j=1}^{i} \sum_{l=1}^{l(j)} \left\{ \alpha_{lm} \frac{P_s\left(l, j + \frac{1}{2}\right) - P_s\left(l, j - \frac{1}{2}\right)}{F(j)} \left[1 + f\left(i + \frac{1}{2}, j\right)\right] \right\} \quad \text{(11)}$$

in which $\alpha_{lm} = \alpha_{ml} = 1 + \dfrac{e_m e_l}{r^2}$ (12)

The last term in Eq. 11 sums the recovery at the end of the i^{th} interval due to a loss of prestress $[P_s(l, j + 1/2) - P_s(l, j - 1/2)]$ in tendons $l = 1, 2, ..., l(j)$. These losses are assumed to occur at the middle of interval j.

Eq. 11 can be solved for $P_s(m, i + 1/2)$, with $m = 1, 2, ..., l(i)$. However, because the interest is in the total losses, the solution is made directly for these unknowns as explained in the section on Total Loss.

Relaxation of Steel

It is assumed that the intrinsic relaxation loss, $\bar{P}_{ro}$, is known for all tendons at any time. The value, $\bar{P}_{ro}$, represents the relaxed force under conditions existing in a constant-strain test in which the steel is tensioned between two fixed points with an initial stress equal to the stress at the time of application of prestress to the concrete beam. It is well known that the value, $\bar{P}_{ro}$, depends to a large extent on the initial stress value.

In a prestressed concrete structure, instantaneous deformation, shrinkage, and creep cause an additional decrease of steel stress, and therefore a smaller value of relaxation would be expected than indicated by the relaxation test described previously. For this reason, a "reduced" value, P_{ro}, is used in the step-by-step calculation. The reduced value is calculated from the constant-strain test values, $\bar{P}_{ro}$; the reduction accounts for the effect of the variation in stress levels on the relaxation behavior of steel. The method of calculation of the "reduced relaxation" is given in a separate section, while in the present section the elastic and creep recoveries induced by relaxation are considered.

Let the actual loss in tendon m induced by relaxation and the resulting recoveries be $P_r(m, i + 1/2)$ at the end of the i^{th} interval. Equating the strain corresponding to the difference between the reduced intrinsic relaxation loss and the actual loss in the beam to the change in strain in concrete at the level of the m^{th} tendon due to the elastic and creep recoveries induced by the relaxation loss gives

$$\frac{P_r\left(m, i+\frac{1}{2}\right) - P_{ro}\left(m, i+\frac{1}{2}\right)}{A_s(m)\,E_s} = -\sum_{j=1}^{i}\sum_{l=1}^{l(j)}\left\{\alpha_{lm}\frac{P_r\left(l, j+\frac{1}{2}\right) - P_r\left(l, j-\frac{1}{2}\right)}{F(j)}\left[1 + f\left(i+\frac{1}{2}, j\right)\right]\right\} \quad \ldots\ldots\ldots (13)$$

Loss Due to Creep

If creep at the end of the i^{th} interval introduces a loss of prestress $P_c(m, i + 1/2)$ in tendon m, then the change of strain in this tendon is $P_c(m, i + 1/2)/[A_s(m) E_s]$. This is equal to the creep of concrete at the level of the tendon from the prestressing forces and from the (dead-load) bending moments that come into action at the instance of prestressing. Thus

$$\frac{P_c\left(m, i+\frac{1}{2}\right)}{A_s(m)\,E_s} = \sum_{l=1}^{l(i)}\left[\frac{T(l)\,\alpha_{lm} - \frac{M(l)\,e_m}{r^2}}{F\left(j(l)-\frac{1}{2}\right)} f\left(i+\frac{1}{2}, j(l)-\frac{1}{2}\right)\right]$$

$$-\sum_{l=2}^{l(i)}\sum_{q=1}^{l-1}\left[\alpha_{qm}\frac{P_e\left(q, j(l)-\frac{1}{2}\right)}{F\left(j(l)-\frac{1}{2}\right)} f\left(i+\frac{1}{2}, j(l)-\frac{1}{2}\right)\right]$$

$$-\sum_{j=1}^{i-1}\sum_{l=1}^{l(j)}\left\{\alpha_{lm}\frac{P_c\left(l, j+\frac{1}{2}\right) - P_c\left(l, j-\frac{1}{2}\right)}{F(j)}\left[1 + f\left(i+\frac{1}{2}, j\right)\right]\right\} \quad (14)$$

in which the first summation term is the creep of concrete at age $i + 1/2$ induced by the forces, $T(l)$ and $M(l)$, with $l = 1, 2, \ldots, l(i)$, applied at age $j(l) - 1/2$; the second summation accounts for the creep recovery induced by the instantaneous loss in tendons $1, 2, \ldots, (l-1)$ when tendon l is tensioned; and the last term sums the recoveries (elastic and creep) at the end of the i^{th} interval due to a loss of prestress in the tendons during an earlier j^{th} interval.

Total Loss

At any age $t_{i+1/2}$ the total loss, $P(m, i + 1/2)$, in tendon m is the sum of the losses due to instantaneous (elastic) deformation, shrinkage, relaxation of steel, and creep:

$$P\left(m, i+\frac{1}{2}\right) = \sum P_e\left(m, i+\frac{1}{2}\right) + P_s\left(m, i+\frac{1}{2}\right) + P_r\left(m, i+\frac{1}{2}\right) + P_c\left(m, i+\frac{1}{2}\right) \quad \ldots\ldots (15)$$

n which the last three terms on the right-hand side increase gradually with ime, while $P_e(m, i + 1/2)$ varies in a step fashion at the interval limits (see ig. 1). Thus for any i^{th} interval

$$\sum P_e\left(m, i+\frac{1}{2}\right) = \sum P_e\left(m, i-\frac{1}{2}\right) \quad \ldots\ldots (16)$$

Substitution of Eqs. 8, 11, 13, and 14 into Eq. 15 and the use of Eq. 16, ives

$$\frac{P\left(m, i+\frac{1}{2}\right)}{A_s(m)\,E_s} - \frac{P_{ro}\left(m, i+\frac{1}{2}\right)}{A_s(m)\,E_s} = \sum_{l=m+1}^{l(i)}\frac{P_e\left(m, j(l)-\frac{1}{2}\right)}{A_s(m)\,E_s} + s\left(i+\frac{1}{2}\right)$$

$$\sum_{j=1}^{i}\sum_{l=1}^{l(j)}\left\{\alpha_{lm}\frac{P\left(l, j+\frac{1}{2}\right) - P\left(l, j-\frac{1}{2}\right)}{F(j)}\left[1 + f\left(i+\frac{1}{2}, j\right)\right]\right\}$$

$$\sum_{l=1}^{l(i)}\left[\frac{T(l)\,\alpha_{lm} - \frac{M(l)\,e_m}{r^2}}{F\left(j(l)-\frac{1}{2}\right)} f\left(i+\frac{1}{2}, j(l)-\frac{1}{2}\right)\right]$$

$$\sum_{l=2}^{l(i)}\sum_{q=1}^{l-1}\left[\alpha_{qm}\frac{P_e\left(q, j(l)-\frac{1}{2}\right)}{F\left(j(l)-\frac{1}{2}\right)} f\left(i+\frac{1}{2}, j(l)-\frac{1}{2}\right)\right] \quad \ldots\ldots (17)$$

Separating the last term of the second summation ($j = i$) of Eq. 17 and rearranging give

$$P\left(m, i+\frac{1}{2}\right) + \frac{A_s(m)\,E_s}{F(i)}\left[1 + f\left(i+\frac{1}{2}, i\right)\right]\sum_{l=1}^{l(i)}\left[\alpha_{lm}\,P\left(l, i+\frac{1}{2}\right)\right]$$

$$= \sum_{l=m+1}^{l(i)} P_e\left(m, j(l)-\frac{1}{2}\right) + P_{ro}\left(m, i+\frac{1}{2}\right) + A_s(m)\,E_s\left\{ s\left(i+\frac{1}{2}\right)\right.$$

$$-\sum_{j=1}^{i-1}\sum_{l=1}^{l(j)} \alpha_{lm}\frac{P\left(l, j+\frac{1}{2}\right) - P\left(l, j-\frac{1}{2}\right)}{F(j)}\left[1 + f\left(i+\frac{1}{2}, j\right)\right]$$

$$+\sum_{l=1}^{l(i)}\left[\frac{T(l)\,\alpha_{lm} - \dfrac{M(l)\,e_m}{r^2}}{F\left(j(l)-\dfrac{1}{2}\right)} f\left(i+\frac{1}{2}, j(l)-\frac{1}{2}\right)\right]$$

$$+\frac{1 + f\left(i+\frac{1}{2}, i\right)}{F(i)}\sum_{l=1}^{l(i)}\left[\alpha_{lm}\,P\left(l, i-\frac{1}{2}\right)\right]$$

$$\left.-\sum_{l=2}^{l(i)}\sum_{q=1}^{l-1}\left[\alpha_{qm}\frac{P_e\left(q, j(l)-\frac{1}{2}\right)}{F\left(j(l)-\frac{1}{2}\right)} f\left(i+\frac{1}{2}, j(l)-\frac{1}{2}\right)\right]\right\} \quad (18)$$

In the step-by-step calculations the P values on the left-hand side of Eq. 18 are the only unknown quantities.

Applying Eq. 18 for all the tendons tensioned before the age, $i + 1/2$ [putting $m = 1, 2, \ldots, l(i)$], a group of $l(i)$ simultaneous equations is obtained that may be written in matrix form as:

$$[L]_i\,\{P\}_{i+1/2} = \{Q\} \quad (19)$$

The elements of matrix $[L]_i$ are

$$L_{mm} = 1 + \frac{A_s(m)\,E_s}{F(i)}\left[1 + f\left(i+\frac{1}{2}, i\right)\right]\alpha_{mm} \quad (20)$$

and $$L_{ml} = \frac{A_s(m)\,E_s}{F(i)}\left[1 + f\left(i+\frac{1}{2}, i\right)\right]\alpha_{ml} \quad \text{for} \quad m \neq l \quad (21)$$

The elements of vector $\{Q\}$, are equal to the right-hand side of Eq. 18 with $m = 1, 2, \ldots, l(i)$. The solution of Eq. 19 in the step-by-step procedure gives the unknown vector, $\{P\}_{i+1/2}$, of the losses at the end of the i^{th} interval.

Reduced Steel Relaxation

In a constant-strain relaxation text, the loss in tension, $d\,\bar{P}_{ro}$, during a time interval, dt, is related to the initial stress (T/A_s). If this tendon has the same initial stress in a concrete beam, the difference, $P - \bar{P}_{ro}$, at any instant between the total loss and the intrinsic relaxation represents a reduction of stress caused by tendon shortening. It can thus be considered that the relaxation in the concrete beam varies at this instant as if the tendon had an initial tension, $T - (P - \bar{P}_{ro})$. The relaxation in the tendon in the concrete beam during time increment dt is assumed the same as the intrinsic relaxation measured in a constant-strain test of the same tendon with an initial tension equal to $T + \bar{P}_{ro} - P$.

The CEB-FIP report (6) suggests that the intrinsic relaxation at time infinity $\bar{P}_{ro\infty}$ is zero when the initial stress is $(T/A_s) \leq 0.5\,f_{su}$, in which f_{su} = the ultimate stress. The report suggests that the relaxation corresponding to higher initial stress ca be calculated from the intrinsic relaxation value, $\bar{P}_{ro\infty}$, obtained with initial tension $0.8\,f_{su}$ by assuming parabolic variation of the relaxation at time infinity versus the initial stress. The parabola has a zero ordinate and a horizontal tangent at $(T/A_s) = 0.5\,f_{su}$.

If, in addition, it is assumed that this variation is applicable for the relaxation increment during a small time interval, dt:

$$d\,P_{ro} = d\,\bar{P}_{ro}\left[\frac{\dfrac{T + \bar{P}_{ro} - P}{A_s} - 0.5\,f_{su}}{\dfrac{T}{A_s} - 0.5\,f_{su}}\right]^2 \quad \text{when} \quad \frac{T + \bar{P}_{ro} - P}{A_s f_{su}} > 0.5 \quad (22)$$

and $$d\,P_{ro} = 0 \quad \text{when} \quad \frac{T + \bar{P}_{ro} - P}{A_s f_{su}} \leq 0.5 \quad (23)$$

in which $\bar{P}_{ro}$ is the intrinsic relaxation in a constant-strain test with the initial tension, T; and P_{ro} is the relaxation in the same cable when tensioned in a concrete beam (reduced relaxation) with the same initial tension, T.

If the time is divided into intervals during which the values, $\bar{P}_{ro}$ and P, are assumed constant equal to the average of their values at the beginning and end of intervals, the value of the reduced relaxation in tendon m at the end of the i^{th} interval is

$$P_{ro}\left(m, i+\frac{1}{2}\right) = \sum_{j=j(m)}^{i}\left\{\left[\bar{P}_{ro}\left(m, j+\frac{1}{2}\right) - \bar{P}_{ro}\left(m, j-\frac{1}{2}\right)\right]\right.$$

$$\left.\times\left[\frac{T(m) + \frac{1}{2}\left[\bar{P}_{ro}\left(m, j+\frac{1}{2}\right) + \bar{P}_{ro}\left(m, j-\frac{1}{2}\right)\right] - \frac{1}{2}\left[P\left(m, j+\frac{1}{2}\right) + P\left(m, j-\frac{1}{2}\right)\right] - 0.5\,A_s(m)\,f_{su}}{T(m) - 0.5\,A_s(m)\,f_{su}}\right]^2\right\} \quad (24)$$

in which the term inside the square brackets must be replaced by zero if its numerator is negative. This means that the relaxation of steel may cease to contribute to the loss in a prestressed beam when the total loss exceeds a certain level.

In the step-by-step calculations of the previous sections, the values of the total loss, $\{P\}_{i+1/2}$ (or $\{P\}_{i-1/2}$), are not known; thus it becomes necessary to use an iterative procedure that may be done as follows.

Iterative Procedure

At the end of the first interval the reduced relaxation is assumed equal to the intrinsic relaxation, i.e., $\{P_{ro}\}^1_{1+1/2} = \{\bar{P}_{ro}\}_{1+1/2}$. Using Eq. 19, approximate values of the total loss, $\{P\}^1_{1+1/2}$, are derived. These approximate values are then used to calculate new values of $\{P_{ro}\}^2_{1+1/2}$ by Eq. 24. Eq. 19 is then used again to calculate more accurate values of $\{P\}^2_{1+1/2}$. Since we are dealing with a small correction of $\bar{P}_{ro}$, no further iterations are necessary.

For any other interval i, consider

$$\{P_{ro}\}^1_{i+1/2} = \{P_{ro}\}_{i-1/2} + \{\bar{P}_{ro}\}_{i+1/2} - \{\bar{P}_{ro}\}_{i-1/2} \quad \text{. . . . . (25)}$$

The procedure is repeated for better approximation as in the case of the first interval.

Note that in Eq. 24 the sum for $j = j(m)$ to $j = i - 1$ is already done and equals $\{P_{ro}\}_{i-1/2}$. Thus Eq. 24 becomes

$$P_{ro}\left(m, i+\frac{1}{2}\right) = P_{ro}\left(m, i-\frac{1}{2}\right) + \left[\bar{P}_{ro}\left(m, i+\frac{1}{2}\right) - \bar{P}_{ro}\left(m, i-\frac{1}{2}\right)\right]$$

$$\cdot \left[\frac{T(m) - \frac{1}{2}\left[P_{ro}\left(m, i+\frac{1}{2}\right) + P_{ro}\left(m, i-\frac{1}{2}\right)\right] - \frac{1}{2}\left[P\left(m, i+\frac{1}{2}\right) + P\left(m, i-\frac{1}{2}\right)\right] - 0.5\,A_s(m) f_{su}}{T(m) - 0.5\,A_s(m) f_{su}}\right]^2 \quad (26)$$

Strain and Curvature

For the beam considered in the step-by-step computation of losses, the concrete strain at the centroid, $\epsilon^o_{i+1/2}$, at age $i + 1/2$ and the corresponding curvature, $\gamma_{i+1/2}$, due to the multistage prestressing and the accompanying moments are given by

$$\epsilon^o_{i+1/2} = s\left(i+\frac{1}{2}\right) + \sum_{l=1}^{l(i)} \left\{ \frac{T(l)}{F\left(j(l) - \frac{1}{2}\right)} \left[1 + f\left(i+\frac{1}{2}, j(l) - \frac{1}{2}\right)\right] \right\}$$

$$- \sum_{l=2}^{l(i)} \sum_{q=1}^{l-1} \left\{ \frac{P_e\left(q, j(l) - \frac{1}{2}\right)}{F\left(j(l) - \frac{1}{2}\right)} \left[1 + f\left(i+\frac{1}{2}, j(l) - \frac{1}{2}\right)\right] \right\}$$

$$- \sum_{j=1}^{i} \sum_{l=1}^{l(j)} \left\{ \frac{\left[P\left(l, j+\frac{1}{2}\right) - P\left(l, j-\frac{1}{2}\right)\right]}{F(j)} \left[1 + f\left(i+\frac{1}{2}, j\right)\right] \right\} \quad (27)$$

$$\gamma\left(i+\frac{1}{2}\right) \sum_{l=1}^{l(i)} \left\{ \frac{M(l) - T(l)\,e_l}{r^2 F\left(j(l) - \frac{1}{2}\right)} \left[1 + f\left(i+\frac{1}{2}, j(l) - \frac{1}{2}\right)\right] \right\}$$

$$+ \sum_{l=2}^{l(i)} \sum_{q=1}^{l-1} \left\{ \frac{P_e\left(q, j(l) - \frac{1}{2}\right) e_q}{r^2 F\left(j(l) - \frac{1}{2}\right)} \left[1 + f\left(i+\frac{1}{2}, j(l) - \frac{1}{2}\right)\right] \right\}$$

$$+ \sum_{j=1}^{i} \sum_{l=1}^{l(j)} \left\{ \frac{\left[P\left(l, j+\frac{1}{2}\right) - P\left(l, j-\frac{1}{2}\right)\right] e_l}{r^2 F(j)} \left[1 + f\left(i+\frac{1}{2}, j\right)\right] \right\} \quad (28)$$

Once the centroidal strain and curvature are calculated by Eqs. 27 and 28 for various sections of a structure, the displacements can be obtained by numerical integration. This is done in the following example.

Bridge Example

Consider a box-girder bridge with the cross section shown in Fig. 3, which is built by the cantilever method. Segments 150 in. (3,800 mm) in length are cast in place and prestressed in stages to form a cantilever. For clarity the number of segments in this example is limited to three. Thus there are three stages of prestressing. The casting and prestressing schedule is as follows:

Segment number	Day of casting	Day of prestressing
1	0	7
2	17	21
3	24	28

The prestressing force in each stage is done by a tendon of area 4.71 sq in. (30.4 cm^2), eccentricity −4.74 in. (120 mm), and initial tension = 825 kips (3.7 MN). It is required to calculate at section 1 the losses in the tendons, the centroidal strain, and curvature at day 66. This example will also show how the vertical deflection at the free end (section 9) can be calculated.

The following data are given: A_c = 8,680 sq in. (55,700 cm^2); r^2 = 1910 sq in. (12,500 cm^2); E_s = 28,000 ksi (190 × 10^3 MN/m^2); and f_{su} = 250 ksi (1,700 MN/m^2). The modulus of elasticity of concrete at any age t_i is assumed (see Ref. 7)

$$E_c(i) = \frac{E_{c,28}}{\sqrt{0.875 + \frac{3.5}{t_i}}} \quad \text{. . . . . (29)}$$

in which t = the number of days after casting; and the modulus of elasticity at age 28 days in $E_{c,28}$ = 4,700 ksi (32×10^3 MN/m^2). Creep and creep recoveries are assumed to follow Eqs. 9 and 10 with $\phi = 1.8$. For tendon m, intrinsic relaxation is assumed to vary from zero at the day of prestressing, t_j, to its ultimate value (assumed at 2,000 days) according to

$$\bar{P}_{ro}(m, i) = \bar{P}_{ro,\infty} \times 0.1315 \ln (t_i - t_j + 1) \quad \text{. . . . . (30)}$$

in which $\bar{P}_{ro,\infty}$ = 57.75 kips (259 kN); and t_j = the age at prestressing.

TABLE 1.—Calculation of Losses in Section 1

Stage number (1)	T, in kips (2)	M, in kip-inches (3)	Age t, in days (4)
1	825	−9,000	7
2	0	−27,000	17
3	825	0	21
4	0	−45,000	24
5	825	0	28

Note: 1 kip = 4.48 kN; 1 kip-in. = 113 N·m.

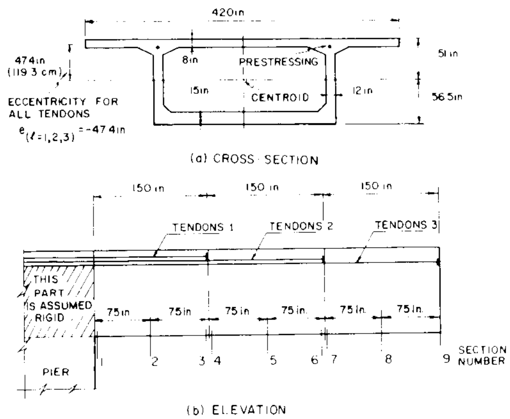

FIG. 3.—Cantilever Box-Girder Bridge Example (1 in. = 25.4 mm)

The free shrinkage from the age, t_j to t_i, is assumed to vary as

$$s(i) = s_\infty \times 0.1315 \ln \frac{t_i + 1}{t_j + 1} \quad \text{. . . . . (31)}$$

in which s_∞ = the shrinkage between the casting day and infinity (assumed 2,000 days). In the present example $s_\infty = 225 \times 10^{-6}$.

The dead-load effect of a new segment is assumed to come into action on the older segments on the day of casting. However, the dead load of a new segment is considered to act on this segment at the time of its prestressing. For the bridge considered the dead load = 0.8 kip/in. (140 kN/m). Thus for section 1 the moments are applied when the concrete at this particular section has the ages, 7 days, 17 days, and 24 days.

From the preceding assumptions it is seen that the time of application of T and M do not coincide for all stages. For the purpose of calculation of losses in section 1, the forces are considered to come into action in five stages as shown in Table 1.

The limits of the time intervals in the step-by-step computation are taken as follows: t = 7 days; 8 days; 10 days; 13 days; 17 days; 18 days; 19 days; 21 days; 22 days; 24 days; 25 days; 28 days; 29 days; 31 days; 35 days; 45 days; 54 days and 66 days.

Substitution into Eqs. 18, 25, and 26 gives the losses in the three tendons: $P(1, t = 66)$ = 53.9 kips (240 kN); $P(3, t = 66)$ = 47.8 kips (213 kN); and $P(5, t53 = 66)$ = 44.3 kips (197 kN).

The centroidal strain, $\epsilon^o(t = 66) = 1.9 \times 10^{-4}$, and the curvature, $\gamma(t = 66) = 9.65 \times 10^{-7}$.

Similar calculations repeated for the nine sections shown in Fig. 3 give the values of ϵ^o and γ from which the displacements at any point can be determined by numerical integration. The downward deflection at the free edge (section 9) is

$$D = \int \gamma\, m\, dl \quad \text{. . . . . (32)}$$

in which m = the moment due to a unit downward virtual load applied at section 9. The numerical integration Eq. 32 over the three segments gives $D(t = 66)$ = −0.17 in. (−4.3 mm) (upward deflection).

Special Cases

Eqs. 18, 27, and 28 give at any time the losses at all tendons, the centroidal strain, and the curvature in a concrete cross section subjected to prestressing forces and bending moments applied in stages. At any stage the prestress, $T(l)$ or $M(l)$, may be zero; thus the same equations may be used to find the effect of a bending moment only.

In the case when the prestress, T, and the moment, M, are applied in one stage at age $t_o = t_{1-1/2}$ with eccentricity of tendon e, Eqs. 18, 27 and 28 become

$$P_{i+1/2} = \frac{1}{1 + \alpha \dfrac{A_s E_s}{F(i)} \left[1 + f\left(i + \dfrac{1}{2}, i \right) \right]}$$

$$\left\{ \alpha \frac{A_s E_s}{F(i)} \left[1 + f\left(i + \frac{1}{2}, i \right) \right] P_{i-1/2} \right.$$

$$+ A_s E_s s_{i+1/2} + P_{ro,i+1/2} + \frac{A_s E_s}{F(o)} f\left(i + \frac{1}{2}, o \right) \left(T\alpha - \frac{M_e}{r^2} \right)$$

$$\left. - \sum^{i-1} \alpha \frac{A_s E_s}{F(j)} \left[1 + f\left(i + \frac{1}{2}, j \right) \right] (P_{j+1/2} - P_{j-1/2}) \right\} \quad \text{. . . . . (33)}$$

$$\epsilon^o\left(i+\frac{1}{2}\right) = s_{i+1/2} + \frac{T}{F(o)}\left[1+f\left(i+\frac{1}{2},o\right)\right]$$

$$-\sum_{j=1}^{i}\left\{\frac{(P_{j+1/2}-P_{j-1/2})}{F(j)}\left[1+f\left(i+\frac{1}{2},j\right)\right]\right\} \quad (34)$$

and $$\gamma\left(i+\frac{1}{2}\right) = \frac{M-Te}{r^2 F(o)}\left[1+f\left(i+1\frac{1}{2},o\right)\right]$$

$$+\sum_{j=1}^{i}\left\{\frac{(P_{j+1/2}-P_{j-1/2})\,e}{r^2 F(j)}\left[1+f\left(i+\frac{1}{2},j\right)\right]\right\} \quad (35)$$

in which $\alpha = 1 + (e^2/r^2)$.

Eqs. 33-35 are also applicable when the prestress is applied through several tendons tensioned at the same age. In this case T = the resultant force in the tendons (after deduction of loss caused by the instantaneous deformations); and e = the eccentricity of the resultant.

The calculations in Eqs. 18, 27, and 28 can be done by the computer program listed in FORTRAN IV in Ref. 5.

Conclusions

The method presented aims at an accurate prediction of prestress loss, axial strain, and curvature when the properties of concrete and steel are known. The step-by-step procedure presented accounts for the interdependence of the effect of the various contributories of prestress loss: instantaneous deformation; relaxation of steel; shrinkage; and creep of concrete. Prestress loss causes elastic and creep recoveries that are accounted for by considering the loss in any tendon as a negative force (reduction in compression) that starts to act at arbitrarily chosen time intervals. The values of steel relaxation that need be accounted for in the calculation of prestress loss in a concrete beam are "reduced" values of the intrinsic relaxation that would occur in a test in which the length is maintained constant. The amount of reduction is calculated as a function of the loss due to the other three causes.

The numerical procedure presented can be conveniently performed on a computer particularly when the prestress is applied in more than one stage. The computer program in Ref. 5 can be used in practice to give an accurate prediction of the time variation of loss, strain at the centroid, and curvature at sections of concrete beams due to the effect of prestressing or external moments applied in multistages.

Acknowledgment

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Appendix II.—Notation

The following symbols are used in this paper:

A_c = cross-sectional area of concrete;
$A_s(l)$ = area of prestressed steel tendon l;
E_c = modulus of elasticity of concrete—when followed by a subscript, it indicates age of concrete;
E_s = modulus of elasticity of steel;
e_l = eccentricity of tendon l (see Fig. 1);
f_s = steel stress;
$f(i, j)$ = ratio of creep at age i to instantaneous strain due to stress increment introduced at age j (see Eq. 10);
i, j = number of time interval—$i - (1/2)$, i, and $i + (1/2)$ refer to beginning, middle, and end of the i^{th} interval;
$j(l)$ = number of time interval at beginning of which tendon l is tensioned;
k = total number of loading stages (prestressing force or application of bending moment);
l, m = subscript or superscript indicating tendon number;
$l(j)$ = number of last tendon tensioned before or at beginning of j^{th} interval;
$M(l)$ = dead load moment applied at same time as prestressing of tendon l;
n = number of time intervals;
o = subscript indicating age of concrete at first loading stage;
P_e, P_s, P_r, P_c = loss of prestress induced by instantaneous deformation, shrinkage, relaxation of steel, and creep, respectively;
P_{ro} = relaxation of steel reduced to account for reduction of stress level compared with stress level in constant-strain test;
$\bar{P}_{ro}$ = intrinsic relaxation of steel obtained by constant-strain test;
$P(l, i + 1/2)$ = total loss of prestress in l^{th} cable at end of interval i;
r = radius of gyration of effective area of cross section;

s = free shrinkage;
$T(l)$ = initial tension in cable l;
t = age of concrete;
α_{lm} = dimensionless coefficients (see Eq. 12);
γ = curvature;
ϵ^o = concrete strain at centroid of area A_c; and
τ = age of concrete at time of application of stress.

STRU

Transient Wind Loads (
Cable R

By Paul Christiano,[1] M. ASCE, G
and Heinz Stefan

Introduction

Cable roof structures subjected to asy
deformations than most other structures.
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concave roofs subjected to typical dynam
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received considerable attention. However
of cable nets has been limited to the co
and shallow orthogonal nets represented a
to derive an approximate formula for the f
to the first symmetric mode of a shallow
Schleyer (9) applied Rayleigh's method to a
membrane and derived approximate formu
to the fundamental symmetric and antisym
frequencies obtained through a discrete an
of a membrane of equivalent uniform th
as the cable spacing approaches zero, the
those of the membrane. Soler and Afshari (
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was submitted for review for possible publicatio

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