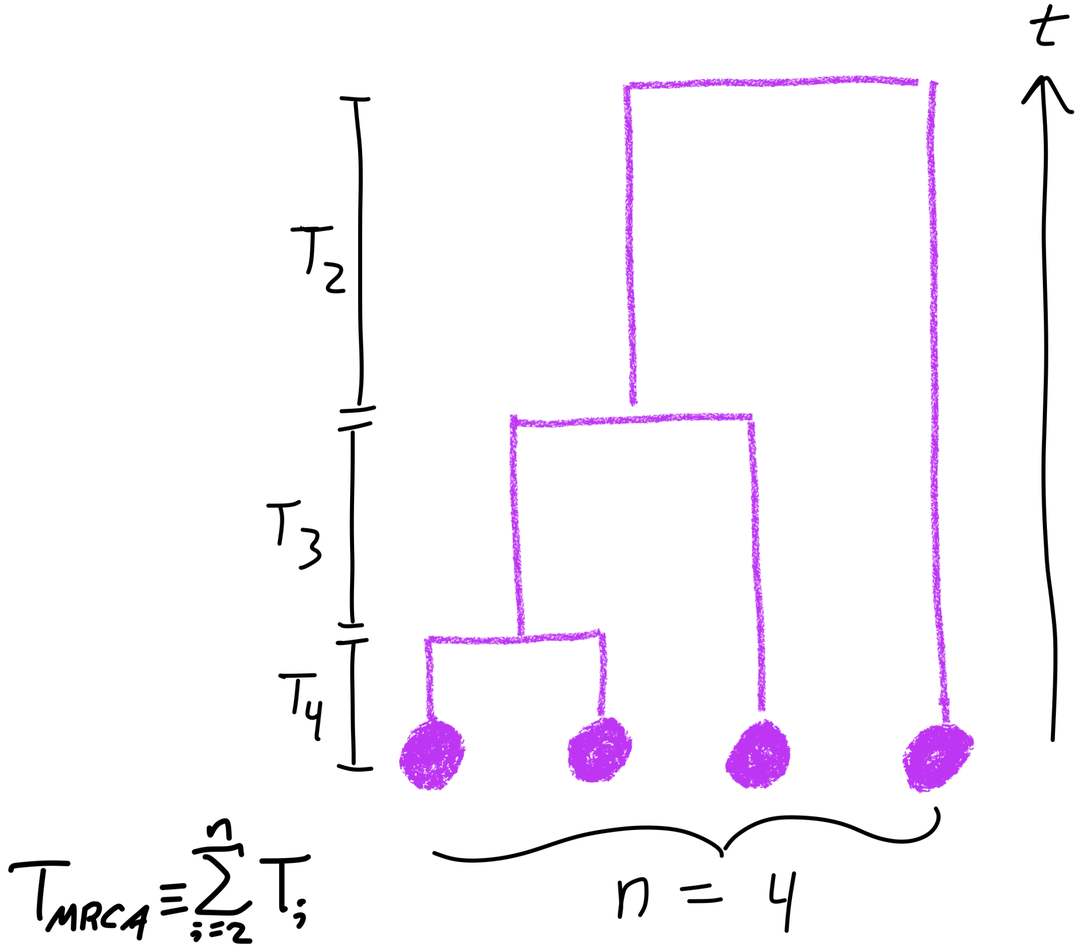
Coalescent theory

Updating the previous results



$$T_{MRCA} \equiv \sum_{i=2}^{n} T_i$$

"intercoalescence times"

$$\mathbb{E}[T_i] = \frac{2N}{\binom{i}{2}}$$

Exp. dist.

$$p(t_i) = \frac{\binom{i}{2}}{2N} e^{-\frac{\binom{i}{2}}{2N} t_i}$$

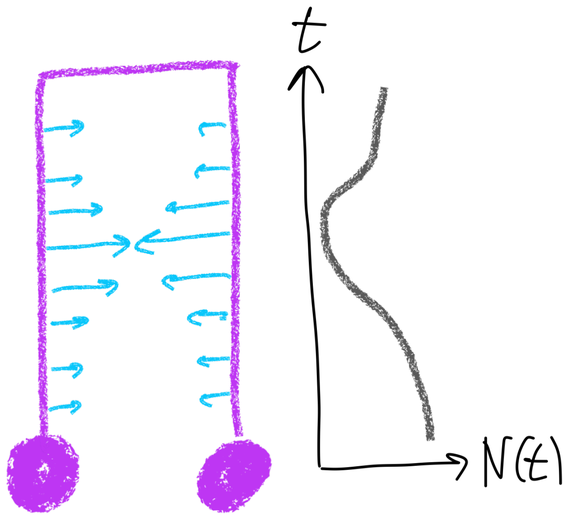
- Each pair is a process w/ rate $\frac{1}{2N}$
- The pairs race to coalesce

Coalescent theory

Population size determines coalescence rate

What if population size varies over time? $N(t)$

[Like students adding/dropping mid-quarter in Cole-escent theory]



$N(t)$ distorts time scale
From the standard coalescent

- time compressed when $N(t)$ is small
- time stretched when $N(t)$ is large

The details:

$$\mathbb{P}(T_i = t_i) = \frac{\binom{i}{2}}{2N_{t_i}} \prod_{j=2}^{i-1} \left(1 - \frac{\binom{i}{2}}{2N_j}\right)$$

big N $\rightarrow$ $$p(t_i) = \frac{\binom{i}{2}}{2N(t)} e^{-\binom{i}{2}\int_0^t \frac{ds}{2N(s)}}$$

inhomogeneous Poisson process