Generalize: Class size N (and N-sided die)
sample of $n \leq N$ students

$$\mathbb{E}[T_{n\to 0}] = \sum_{i=1}^{n} \mathbb{E}[T_i] = \sum_{i=1}^{n} \frac{1}{i/N} = N \sum_{i=1}^{n} \frac{1}{i}$$

Each interval is geometrically distributed

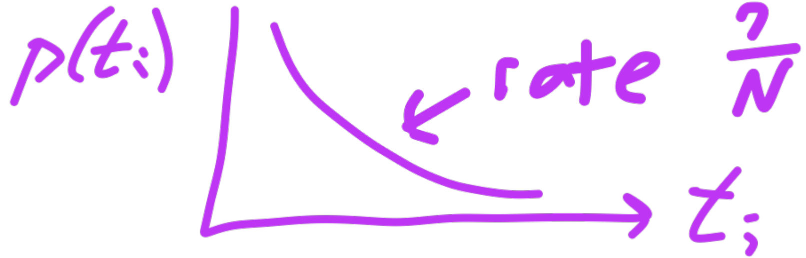
$$\mathbb{P}(T_i = \quad) = \frac{i}{N}\left(1 - \frac{i}{N}\right)^{t_i - 1}$$

t_i

$$p(t_i) \simeq \frac{i}{N} e^{-\frac{i}{N} t_i}$$

continuous

$\rightarrow$ For large N: approx. exponential dist.



Memoryless:

$$p(t_i \mid t_{i+1}) = p(t_i)$$

Cole-escent theory

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Coalescent theory

Only slightly fancier

	Cole-escent	coalescent
time	classes into the future	generations into the past
events	dice rolls, one student	coalescences, pairs of individuals
rate of events w/ i individuals	$\frac{i}{N}$ ← students uncalled ← class size	$\frac{\binom{i}{2}}{2N}$ ← $= \frac{1}{2} i(i-1)$, # pairs; ← population size; ↑ if diploid