



So

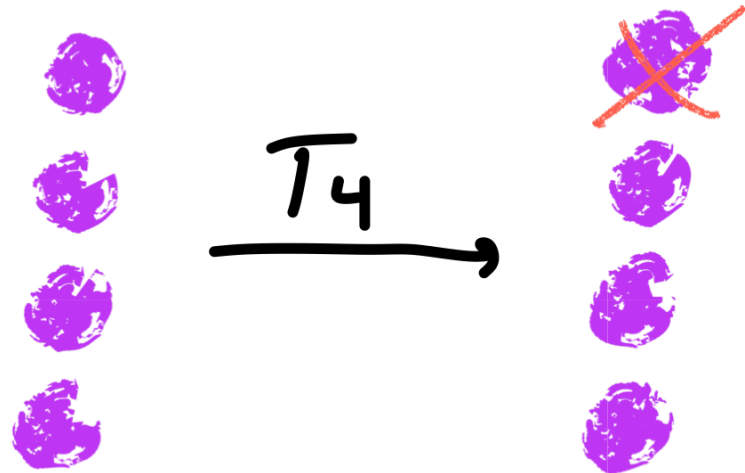
$$\begin{aligned} \mathbb{E}[T_{20 \rightarrow 0}] &= \mathbb{E}[T_{20}] + \mathbb{E}[T_{19}] + \dots + \mathbb{E}[T_1] \\ &= 1 + \frac{20}{19} + \dots + \frac{20}{1} = 1 + 20 \sum_{i=1}^{19} \frac{1}{i} \end{aligned}$$

Question: How many classes do you expect  
until all 20 students have been  
called on?

When there are  $i$  students left to call on,  
the prob of rolling one of these remaining  $i$

is  $\frac{i}{20}$

$$\mathbb{E}[T_i] = \frac{1}{i/20} = \frac{20}{i}$$



**Core-essential theory**

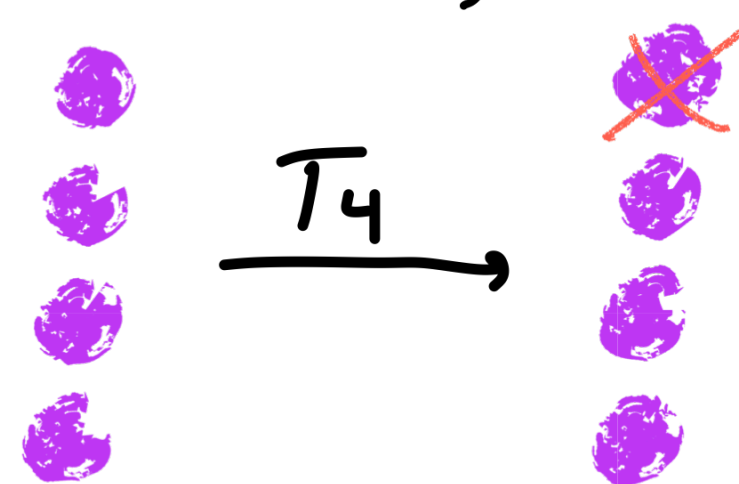


# Cole-escent theory

Question: How many classes do you expect until all 20 students have been called on?

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So

$$\begin{aligned}\mathbb{E}[T_{20 \rightarrow 0}] &= \mathbb{E}[T_{20}] + \mathbb{E}[T_{19}] + \dots + \mathbb{E}[T_1] \\ &= 1 + \frac{20}{19} + \dots + \frac{20}{1} = 1 + 20 \sum_{i=1}^{19} \frac{1}{i}\end{aligned}$$

# Cole-escent theory

Generalize: Class size  $N$  (and  $N$ -sided die)  
sample of  $n \leq N$  students