On the Shoulders of Giants

A Collection of the Great Pedagogical Problems of Classical Physics

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1 Mechanics

1.1 Projectile Motion

Unless explicitly stated otherwise, assume that you can ignore any effects of air resistance.

1. **Throw or drop** $\star$

   Two identical balls are released at the same time: one ball is thrown horizontally at speed v, while the other is dropped straight down. Which one hits the ground first?

2. **Cannonball** $\star$

   A cannonball is fired into the air at an angle θ from the ground. What angle of launch gives the cannon the greatest horizontal range?

3. **Archer and apple** $\star\star$

   An archer fires an arrow at speed v at tree a distance d away, hoping to hit an apple a height h from the ground. He releases his arrow the moment the apple falls from the tree. At what angle should he shoot to hit the apple?

4. **Terminal velocity** $\star\star$

   An object of mass m falls from a great height. Assume that the force of air resistance is proportional to the square of its speed, $F_{air} = -kv^2$. Show that the speed of the falling object approaches a finite upper bound - its *terminal velocity* - and compute this value. Why can an insect fall from a tree unharmed, but a human cannot?

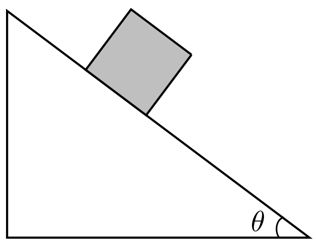
5. **Escape velocity** $\star\star\star$

   You are standing on the surface of a planet of mass M and radius R. How fast do you need to throw a ball horizontally so that it goes into orbit around the planet? How fast do you need to throw it vertically so that it never comes back? The latter is called the *escape velocity*. Would the escape velocity be lower if you threw it at and angle instead of straight up?

1.2 Newtonian Mechanics

6. **Block on an inclined plane** $\star$

   A block of mass m slides on a plane inclined at an angle θ from the horizontal.



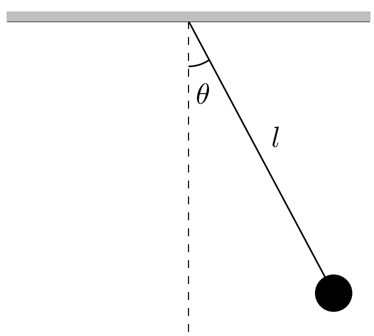
   (a) What is the acceleration of the block if there is no friction?

   (b) If the coefficient of static friction is μ_s, what is the maximum angle at which the block can remain at rest?

   (c) If the block is sliding with a coefficient of kinetic friction μ_k, what is its acceleration?

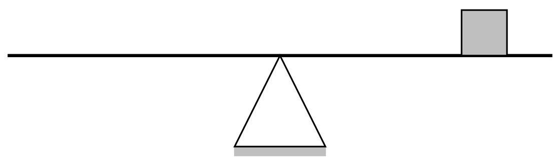
7. **The pendulum** $\star$

   A mass m hangs from a string of length l which can swing freely in the plane. Treat the string as having no mass. Find the angle of the string as a function of time. Assuming the angle is small, what is the period of the pendulum's motion?



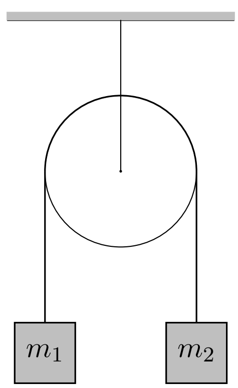
8. **Balance scale** $\star$

A mass sits on a scale a distance d from the pivot. Where can you place an object of half the mass in order to make the scale balance?



9. **Atwood's machine** $\star$

An *Atwood machine* consists of a rope on a pulley, with two masses m_1 and m_2 hanging from either end of the rope. Treating the rope and pulley as massless, find the acceleration of the system.

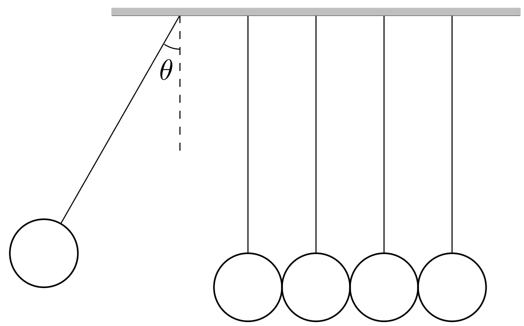


10. **Bouncing time** $\star\star$

A ball drops from an initial height h_0 and bounces up and down, steadily losing height with each bounce. Let the fraction of the ball's velocity that is retained after each bounce be e. How long does it take for the ball to come to rest on the floor?

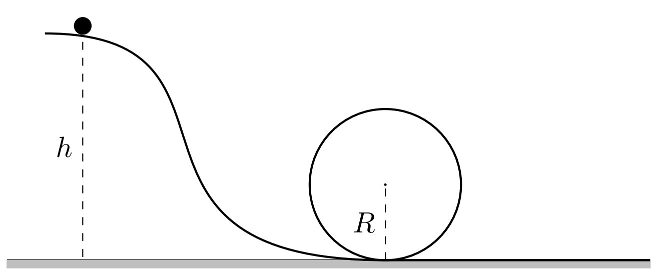
11. **Newton's cradle** $\star$

A series of identical pendulums hang in a line such that adjacent pairs of hanging masses are just barely touching. You lift up the leftmost pendulum and release. What happens? What if you lift up the leftmost two, three, or n?



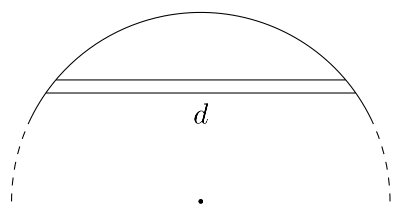
12. **Rollercoaster loop** $\star\star$

A cart slides starts at rest at the top of a track that has a circular loop of radius R at the bottom. What is the minimum starting height h that will permit the cart to maintain constant contact with the track and make it around the loop? What if it's a spherical ball (of radius $r \ll R$) rolling instead of sliding?



13. **Earth tunnel** $\star\star$

A tunnel is drilled through the Earth between two points on its surface, a straight-line distance d apart. How long does it take for an object to fall through the hole and reach the other side? How fast it is traveling when it arrives?

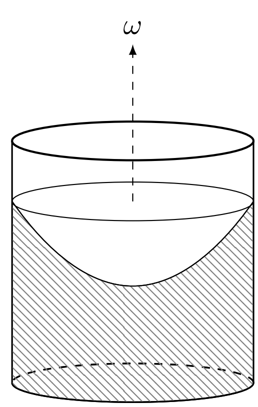


14. **Falling chain** $\star\star$

A chain of mass M and length L is suspended vertically with its lower end touching a scale. The chain is released and falls onto the scale. What is the reading on the scale when a length x of the chain has fallen?

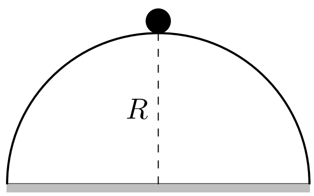
15. **Newton's bucket** $\star\star$

A bucket of water spins at a constant angular velocity about its symmetry axis. What is the shape of the water's surface?



16. **Slipping off a sphere** ⋆⋆

A particle of mass m slides down a frictionless, semi-circular dome of radius R. At what height does the particle leave the surface of the dome?

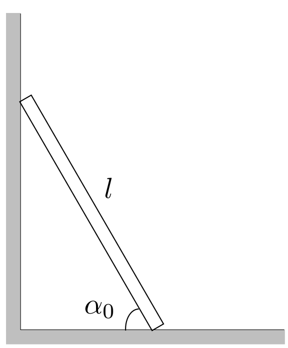


17. **Sliding ladder** ⋆⋆⋆

A ladder of mass M and length L leans against a frictionless wall with its feet on a frictionless floor.

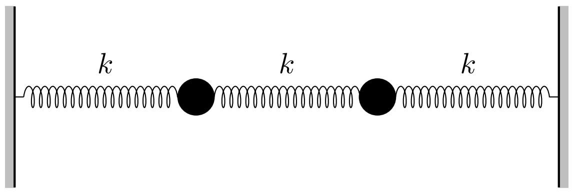
(a) If it is let go with initial angle α_0, at what angle will the ladder lose contact with the wall?

(b) Suppose the ladder was constrained to maintain contact with the wall and floor the whole way. As it falls, its envelope traces out a curve (the surface formed by the union of all locations of the ladder). What is the shape of this envelope?



18. **Coupled springs** $\star\star\star$

Two masses and three springs in a line are anchored by two walls. Find the normal modes of the system and their frequencies.



19. **The rocket equation** $\star\star\star$

A rocket ship of mass M loaded with an initial fuel supply of mass m_0 ejects fuel with a velocity v_e with respect to the the rocket at a fixed rate. Derive the *Tsiolkovsky equation* for the ship's final velocity in the absence of any other forces:

$$\Delta v = v_e \ln(\frac{M + m_0}{M}).$$

Now suppose the rocket takes off vertically in a constant gravitational field. The fuel is exhausted in a time T. Find the equation of motion of the rocket. What velocity does the rocket attain when all the fuel is expended?

20. **Deep space free-fall** $\star\star$

Two masses, m_1 and m_2, are released from rest in deep space a distance d apart, pulled only by their mutual gravitational attraction. How long do they take to collide, and what are their velocities at the moment of impact?

21. **Falling chimney** $\star\star\star\star$

A tall, slender chimney of height L is slightly perturbed from its vertical equilibrium position so that it topples over, rotating rigidly around its base until it breaks at a point P. Where along its length is the chimney most likely to break? (In other words, find the place where there is the greatest torque during the falling motion.)

22. **The Crab Pulsar** $\star$

A spherical object rotates with angular frequency ω. If the only force preventing it from flying apart is gravity, what is the minimum density the object can have? Use this to estimate the minimum density of the Crab pulsar, which rotates 30 times per second (Note: $G \approx 6 \times 10^{-11}\,\mathrm{m}^3{\cdot}\mathrm{kg}^{-1}{\cdot}\mathrm{s}^{-2}$). If the mass of the pulsar is about 1 solar mass ($\approx 2 \times 10^{30}\,\mathrm{kg}$), what is the maximum possible radius of the pulsar?

1.3 Collisions

23. **Collisions 101**

    (a) Two masses m_1 and m_2 move towards each other in one dimension, with velocities v_1 and v_2.

    i. They bounce elastically. What are their velocities after the bounce?

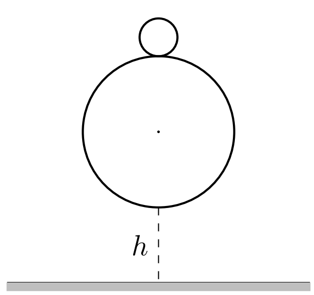
    ii. They bounce inelastically and stick together. What is the velocity of the conglomerate after the bounce?

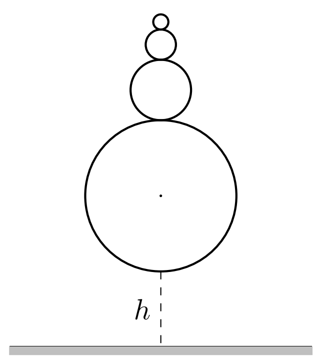
    (b) What if they collide in two dimensions at an angle θ? You can consider the case $m_1 = m_2$ for simplicity.

    (**Hint**: Work in the center-of-mass frame!)

24. **Bouncing tower** $\star\star\star$

    (a) A tennis ball of mass m_2 sits atop a much heavier basketball of mass m_1. The bottom of the basketball starts at a height h above the ground, and the bottom of tennis ball is at height $h + d$. The stack is dropped. To what height does the tennis ball bounce? Work in the approximation that $m_1 \gg m_2$, and assume the balls bounce elastically.





    (b) Now consider a vertical tower of n balls, with $m_{n+1} \ll m_n$. The bottom (heaviest) ball starts a height h above the ground, and the top (lightest) ball starts at height $h + l$. In terms of n, how high is top ball launched?

25. **Maximum deflection angle** ⋆⋆⋆

A mass M collides elastically with a stationary mass m. If $M \leq m$, then it is possible for M to bounce directly backwards. However, if $M \geq m$, there is a maximum possible angle of deflection of M. Find this angle.

26. **Mass-spring collision** ⋆⋆

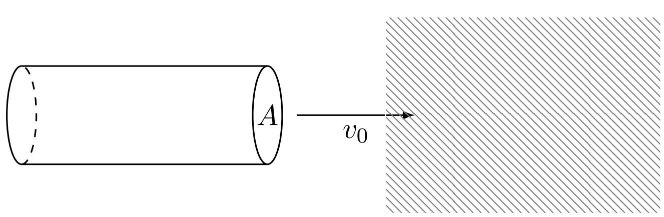
A mass m_1 with initial velocity v_0 collides with a mass-spring system m_2 initially at rest on a frictionless surface. The spring is massless with spring constant k.

(a) What is the maximum compression of the spring?

(b) Find the final velocities of the two masses (and assume they travel in the same direction after the collision).

27. **Raindrop** ⋆⋆⋆

(a) Imagine a spacecraft of mass m_0 and cross-sectional area A coasting at speed v_0. The craft encounters a stationary dust cloud of density ρ. As it moves through the cloud, the dust in its path sticks to the craft (thereby increasing its mass). Find the velocity of the spacecraft as a function of time.



(b) Now image a spherical raindrop falling through a cloud of water vapor, which condenses onto the drop as it falls. What is the acceleration of the raindrop?

1.4 Analytical Mechanics

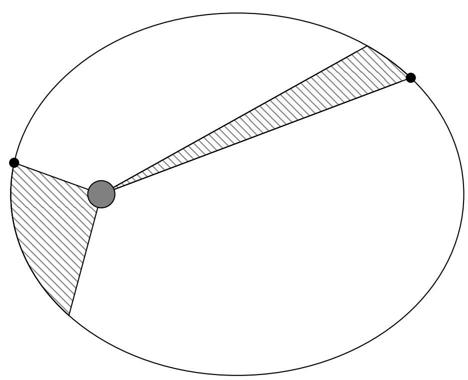
28. **Kepler's Laws** ⋆⋆⋆

    Show that:

    (a) The orbit of a planet is an ellipse with the Sun at one of the two foci;

    (b) A line segment joining a planet and the Sun sweeps out equal areas in equal intervals of time;

    (c) The square of the orbital period is proportional to the cube of the orbit's radius.



29. **Circular orbits** ⋆⋆⋆

    Two bodies orbit each other under a potential energy of interaction $U(r) = kr^n$.

    (a) What values of n allow for stable circular orbits?

    (b) Show that the frequency of small oscillations for $n = -1$ is the same as the orbital period itself. What does this imply about the shape of real orbits?

30. **Circular attractor** ⋆⋆⋆

    A particle orbits a fixed point with a potential $U(r) = kr^n$. The particle moves along a circular path which passes through the attracting point itself. Find n.

31. **Masses on a table** $\star\star$

A mass m is free to slide on a frictionless table and is connected by a string, which passes through a hole in the table, to a mass M which hangs below. Assume that M moves in a vertical line only, and assume that the string always remains taut.

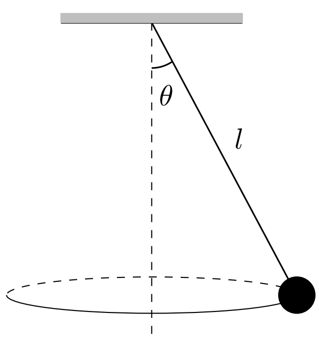
(a) Find the equations of motion for the variables r and θ.

(b) Under what condition does m undergo circular motion?

(c) What is the frequency of small oscillations (in the variable r) about this circular motion?

32. **Spherical pendulum** $\star\star$

Consider a pendulum of length l that is free to swing in two dimensions (or equivalently, a particle constrained to a spherical bowl). Find the equations of motion in terms of θ and ϕ. What is the frequency of a circular orbit of radius r?



33. **Particle in a cone**

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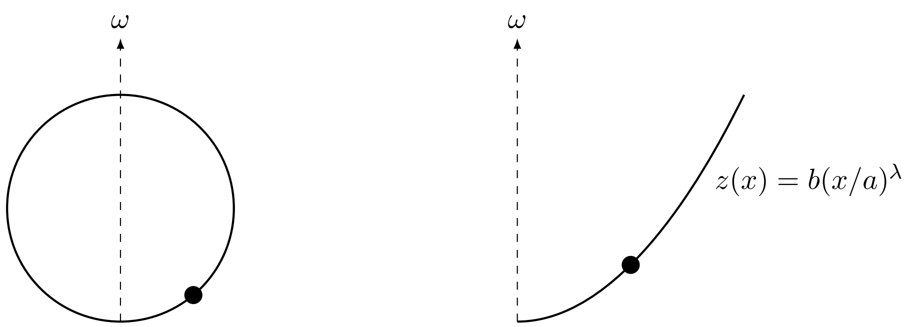
34. **Bead on a wire** ⋆⋆⋆

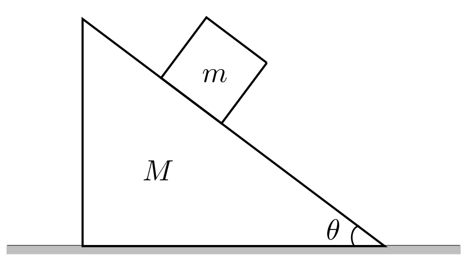
A bead slides frictionlessly along a circular hoop which starts in the x-z plane and rotates with a constant frequency ω about the z-axis. Find the frequency of small oscillations about the stable equilibrium. Find the unstable equilibrium that exists only for certain values of ω.



(b) Instead of a circular hoop, the wire is a curve $z(x) = b(x/a)^\lambda$, rotating about the z-axis. Under what conditions does a stable equilibrium exist?

35. **Inclined plane revisited** ⋆⋆

Consider a mass m on a frictionless inclined plane of mass M, which *itself* is free to slide along a frictionless horizontal surface. What is the motion of each object?



Revisit the "slipping off a sphere" problem in this context as well.

36. **Period as a function of Energy** ⋆⋆⋆

A particle of mass m moves in a one-dimensional potential $U(x) = A|x|^n$, where A is constant. For a given value of n, how does the period depend on the energy?

37. **Double pendulum** ⋆⋆⋆⋆

The double pendulum consists of a mass m suspended by a massless string or rod of length l, from which is suspended another such rod and mass.

Princeton/MIT version:

(a) Write the Lagrangian of the system for $\theta_1, \theta_2 \ll 1$;

(b) Derive the equations of motion;

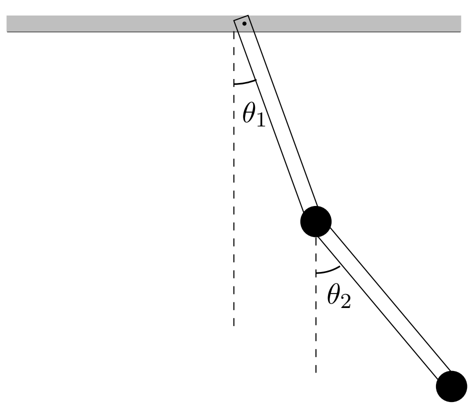
(c) Find the eigenfrequencies.

David Morin version:

(a) Find the equations of motion.

(b) For small oscillations, find the normal modes and their frequencies for the special case $l_1 = l_2$ (and consider the cases $m_1 = m_2$, $m_1 \gg m_2$, and $m_1 \ll m_2$).

(c) Do the same for the special case $m_1 = m_2$ (and consider the cases $l_1 = l_2$, $l_1 \gg l_2$, and $l_1 \ll l_2$).



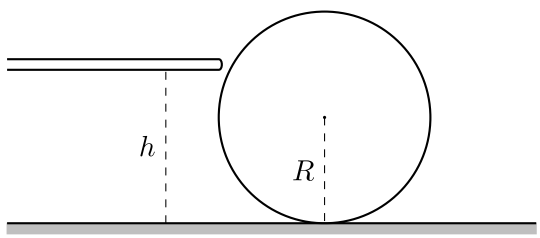
1.5 Rigid Bodies

38. **Moments of Inertia** ⋆

    Three objects of the same mass and radius roll down a hill: a uniform disk, a thin ring, and a sphere. Which one wins? What if you change the masses? What if you change the radii?

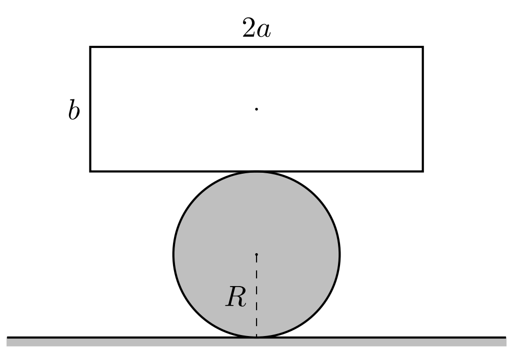
39. **Rolling without slipping** ⋆⋆⋆

    A billiard ball of radium R and mass M is struck horizontally by a cue stick at a point a height h above the table. Find the value of h for which the ball will roll without slipping.



40. **Balancing rocks** ⋆⋆⋆

    A uniform-density rectangle of width $2a$ and height b is balanced atop a fixed circle of radius R. Under what condition is the rectangle stable?



41. **Stable tops** ⋆⋆⋆⋆

Consider a curve $z(x)$, and the volume bounded by its surface of revolution about the z-axis. Let the resulting object have height h, and sit on a table balanced at the point $z = 0$. Under what circumstances is the object balanced stably, and what is the period of small oscillations?

42. **The Gyroscope**

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43. **Cone rolling on a table**

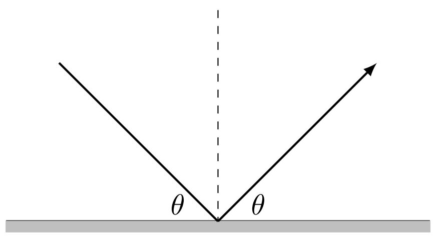
?

1.6 Minimization

44. **Fermat's principle** ⋆⋆

Fermat's principle of least time says that light traveling from point A to point B will always take the path with the shortest possible travel time.

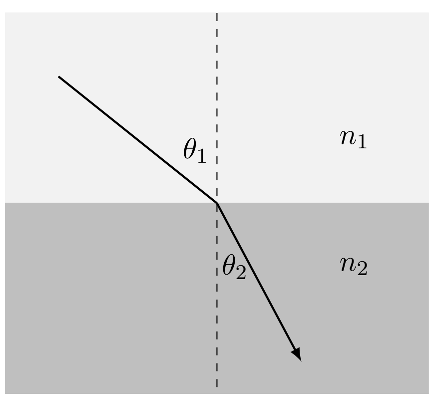
(a) Reflection: show that when light bounces off a surface, the angle of incidence is equal to the angle of reflection.



(b) Refraction: the *index of refraction* n of a material is the ratio of the speed of light in vacuum to its speed through the material: $n = c/v$. Consider a ray of light traveling across the interface of two materials with indices of refraction n_1 and n_2. Derive *Snell's Law*: that the angles of the incident and refracted rays are given by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2$$

where θ is the angle from the normal to the interface.

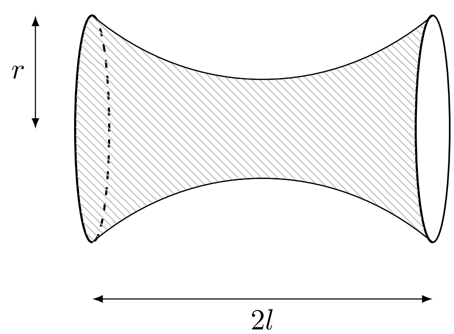


45. **Hanging chain** ⋆⋆⋆⋆

A chain of uniform mass density per unit length hangs between two given points. What is the shape of the chain? (This is known as the *catenary* curve.)

46. **Soap film** ⋆⋆⋆⋆

A soap bubble stretches between two identical circular rings of radius r. The planes of the rings are parallel, and the distance between them is $2l$. The soap will try to find the minimum total surface area. Find the shape of the soap bubble. Bonus (non-analytic): What is the largest value of l/r for which a stable soap bubble exists?



47. **The Brachistochrone** ⋆⋆⋆⋆

A bead is released from rest at the origin and slides down a frictionless wire that connects the origin to a given point (x, y). What shape should the wire take so that

(a) the bead reaches the endpoint in the shortest possible time? (This is known as the *brachistochrone* curve.)

(b) the time it takes to reach the end is the same no matter where on the curve you start? (This is known as the *tautochrone* curve.)

48. **Maximum gravity** ⋆⋆⋆

Given a point P in space, and given a lump of malleable material of constant density, how should you shape and place the material in order to create the largest possible gravitational field at P?

1.7 Misc

49. **Archimedes' principle** $\star$

A solid object of mass M and total volume V floats in a liquid of density ρ. What fraction of the object is above the water?

50. **Sea level** $\star$

(a) A fisherman rowing his boat on a small lake throws his anchor into the water. Does the water level of the lake rise, fall, or stay the same?

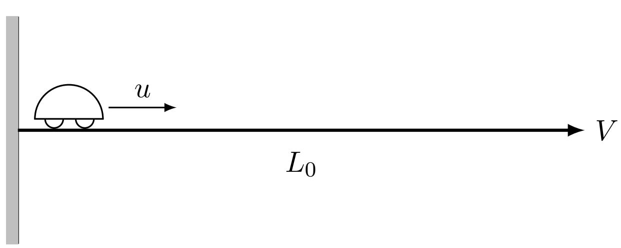
(b) A block of ice containing a small air bubble floats in a pond. After the ice melts, does the water level go up, down, or stay the same? What if the air bubble is replaced by a small piece of lead?

51. **Balancing a pencil** $\star\star$

How long does it take a pencil to tip over? Suppose the length of the pencil is 15cm and the initial angular displacement is one sixtieth of a degree. (Bonus: use the uncertainty principle to estimate the maximum possible time!)

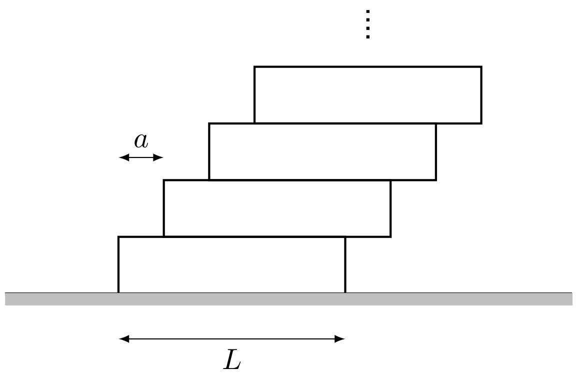
52. **Ant on a rubber band** $\star\star\star$

An infinitely stretchable rubber band with initial length L_0 is attached to a wall at one end. An ant starts at the wall and crawls along the rubber band at speed u relative to the band. At the same time, the other end of the rubber band is pulled away from the wall at speed V, causing the band to stretch *uniformly* at a constant rate. Does the ant ever reach the other end? If so, when?



53. **Stack of bricks** ⋆⋆

(a) Rectangular bricks of length L are placed in a stack so that the horizontal position of each brick is offset from the brick below it by a distance a. How many bricks can you stack in this way before the stack topples over?



(b) Suppose you have an infinite supply of bricks and can stack them any way you like. How far can you make your stack extend horizontally from the edge of the bottom brick?

54. **Olber's paradox**

Suppose each star in the sky has a surface brightness L (power per unit area), and that the stars are (on average) uniformly distributed in space forever, with spatial density ρ. Calculate the brightness of the night sky. Explain how this proves that if the universe is homogeneous and isotropic, then it either has a finite size or a finite age. (Bonus: if you're worried about stars blocking each other, suppose each star has a radius R and that if a star is in front of another star it will block its light. You should arrive at the same conclusion).

55. **Adiabatic change** ⋆⋆⋆

A pendulum, made up of a ball of mass M suspended from a pivot by a string of length L, swings freely in a plane. By what factor does the amplitude of oscillations change if the string is very slowly shortened by a factor of 2?

2 Electrodynamics

2.1 Electrostatics

1. **Basic shapes** $\star$

   Find the electric field and potential of the following configurations of charge everywhere in space:

   (a) A uniform spherical shell of radius R and total charge Q;

   (b) A uniform spherical ball of radius R and total charge Q;

   (c) An infinite one-dimensional line of charge density λ;

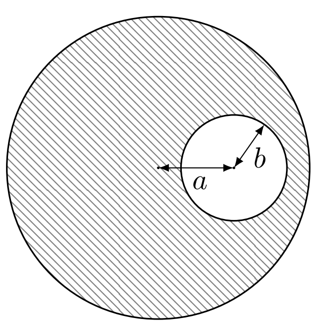
   (d) An infinite plane of surface charge density σ.

2. **Electric trap**

   Is it possible, by a clever arrangement of stationary charges, to construct a trap wherein a free charged particle placed inside cannot leak out? What if the trap is in a uniform gravitational field?

3. **Cylindrical cavity / Spherical bubble**

   Consider an infinite cylinder of radius R and charge density ρ, with a smaller cylindrical cavity inside it. The center of the cavity is a distance a from the center of the cylinder, and the cavity has radius b. What is the electric field inside the cavity? How about a charged ball with a spherical cavity?



4. **Capacitance** $\star$

   The capacitance of a pair of conductors is the ratio of the total charge on each the conductor to the potential difference between them: $C = Q/V$ (while all other conductors are grounded). Find the capacitance of the following capacitors:

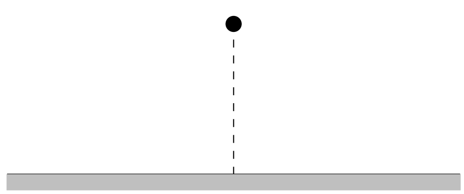
   (a) Parallel plates separated by a distance d;

   (b) Concentric spherical shells of radii a and b;

   (c) Concentric cylindrical shells of radii a and b.

Boundary Value Problems:

5. A point charge q is fixed a distance d above an infinite conducting plane which is grounded (i.e. held at zero potential). Find the surface charge density induced on the plane, and the force it exerts on the charge q.



6. (one side of a rectangle / cube held at fixed potential, the other sides zero)

7. A point charge is placed at a distance R from the center of a metallic sphere of radius a with $R > a$. The sphere is insulated and is electrically neutral. Find the electrostatic potential on the surface of the sphere and the force acting on the charge.

8. A conducting sphere of radius a on whose surface resides a total charge Q is placed in a uniform electric field. Find the potential at all points in space exterior to the sphere. What is the surface charge density?

2.2 Magnetostatics

9. **Straight wires** $\star$

   (a) Find the magnetic field of an infinite straight wire carrying a current J.

   (b) Two current-carrying wires run parallel, a short distance apart. If the currents are in the same direction, what happens? If they are in opposite directions what happens? What if they are perpendicular?

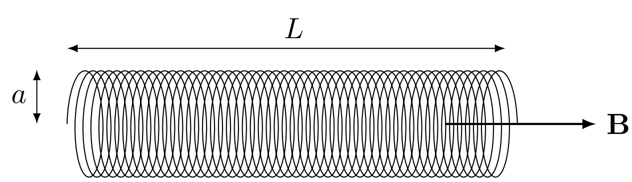
10. **Motion of a charge in a magnetic field** $\star$

    A particle with charge e moving with velocity $\mathbf{v_0}$ enters a region of constant magnetic field $B\hat{z}$. What is the subsequent motion of the particle?

11. **The Solenoid** $\star$

    (a) Imagine a loop of wire of radius R with a current J running through it. Find the magnetic field along the symmetry axis of the loop.

    (b) A *solenoid* is a cylindrical coil of wire with many windings. Consider a solenoid of length L and radius a, with n windings per unit length and a current J flowing through it. Assuming $L \gg a$, show that the magnetic field is negligible outside the solenoid, and find the strength of the field inside.



12. **Spinning sphere** $\star\star\star\star$

    A sphere with a uniform charge density on its surface rotates at a constant angular velocity. Find the magnetic field inside and outside the sphere.

13. **Force of a magnet**

A cylindrical permanent magnet of radius R, length L, and density ρ has a uniform magnetization M parallel to its axis. It is placed below an infinitely-permeable flat surface. What is the maximum length L such that the magnetic force is stronger than gravity and can prevent the magnet from falling?

2.3 Time-dependent fields

14. **Inductance** ⋆

    Inductance is the ratio of the voltage to the time-derivative of the current passing through an inductor. Find the inductance of the following systems:

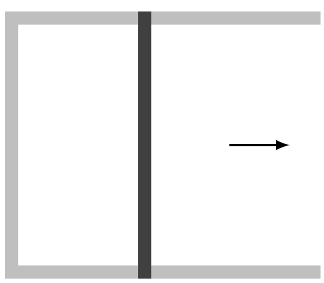
    (a) A solenoid of length L, cross-sectional area A, and winding density N;

    (b) Coaxial cable (explain);

    (c) Parallel rings?

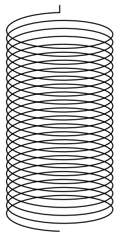
15. **The rail gun** ⋆⋆

    A conducting rod of length L is free to slide along frictionless rails. A current I flows through the rod and the U forming a circuit. (other info needed?) As the rod slides it changes the magnetic flux through the loop, which applies a force on the rod. How quickly does the rod accelerate? (with such and such current and time), how fast does the rail gun shoot?



16. **The magnet and the copper pipe**

    A cylindrical magnet falls through a copper pipe (the best physics demo ever). Find the terminal velocity of the magnet. (very hard) Or... A conducting circular loop of diameter D is made of wire of diameter d, resistivity ρ, and mass density ρ_m. The loop falls parallel to the ground from a great height h in a magnetic field with a component $B_z = B_0(1 + \kappa z)$ where κ is a constant. Disregarding air resistance, find the terminal velocity of the loop.



17. **Motor and Generator**

...

2.4 Electromagnetic waves

18. **The speed of light** $\star\star$

    Maxwell's equations in vacuum are:

$$\nabla \cdot \mathbf{E} = 0 \qquad\qquad \nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial B}{\partial t} \qquad\qquad \nabla \times \mathbf{B} = \mu_0 \epsilon_0 \frac{\partial E}{\partial t}$$

    Using the experimental value of $\epsilon_0 \approx 8.854\,\mathrm{A{\cdot}s/V{\cdot}m}$ and $\mu_0 = 4\pi \times 10^{-7}\,\mathrm{V{\cdot}s/A{\cdot}m}$, derive the speed of light.

2.5 Circuits

19. **Resistors & Capacitors** $\star$

    Resistors and Capacitors are two of the fundamental circuit elements.

    (a) Use Ohm's Law ($V = IR$) to derive the equivalent resistance of a pair of resistors in series and in parallel. Show that

    $$\text{In series:} \quad R_{eq} = R_1 + R_2; \qquad \text{In parallel:} \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

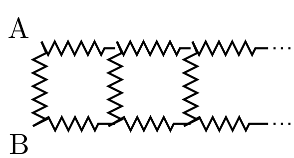
    (b) Use $Q = CV$ to derive the equivalent capacitance of a pair of capacitors in series and in parallel. Show that

    $$\text{In series:} \quad \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}. \qquad \text{In parallel:} \quad C_{eq} = C_1 + C_2$$

20. **Resistor puzzles** $\star\star\star$

    Consider a set of 1Ω resistors connected up in a graph. Find the equivalent resistance between the given pair of points in the following configurations:

    - opposite corners of a cube
    - adjacent corners of an icosahedron
    - base pair of an infinte ladder
    - adjacent points on an infinite planar grid



3 Relativity

1. **Time dilation & length contraction**

   A spaceship whose length at rest is L_0 is moving with speed v relative to you. Let $\gamma \equiv \frac{1}{\sqrt{1-v^2/c^2}}$. From the invariance of the interval, $\mathrm{d}s^2 = -\mathrm{d}t^2 + \mathrm{d}\mathbf{x}^2$, show that

   (a) an interval of time Δt on the ship is related to your clock by $\Delta t' = \gamma \Delta t$;

   (b) the length of the ship in your frame is $L' = L_0/\gamma$;

   (c) the width of the ship is unchanged.

2. **Addition of velocities**

   An observer moves with velocity v relative to you. A second observer moves at velocity w relative to the first. What is the velocity of the second observer in your frame of reference? Derive the relativistic law for the addition of velocities:

   $$u = \frac{v + w}{1 + vw/c^2}$$

3. **The Twin Paradox**

   Two twins perform an experiment to test the theory of relativity. Twin A stays on Earth, while Twin B travels in a spaceship at speed $v = (24/25)c$ for 7 years according to her watch. She then reverses direction and returns at the same speed. When they reunite, which twin is older, and by how much?

4. Paradox of fitting a beam through a door on a moving train, and the "pole through a barn" problem.

5. **Doppler shift**

   Show that the relativistic doppler shift is given by

   (a) $\nu/\nu_0 = \sqrt{\frac{1+\beta}{1-\beta}}$ when the source is approaching, and the inverse if it's receding;

   (b) $\nu/\nu_0 = \frac{1}{\sqrt{1-\beta^2}}$ when the motion is perpendicular to the line of sight.

4 Quantum Mechanics

4.1 Time-independent bound states

1. Infinite and finite square wells, spherical well
2. Attractive delta function - show there is only one bound solution
3. Gaussian wave packet. Show that it's the minimum-uncertainty wave packet.
4. Reflection / transmission / tunneling through a barrier
5. Harmonic oscillator

4.2 More

1. spin-1/2 rotation
2. Hydrogen!

5 Statistical Mechanics / Thermodynamics

5.1 Probability distributions

1. **The Gaussian distribution**

   A *Gaussian* distribution (or *Normal* distribution) $G(x)$ centered at zero has the form $A\mathrm{e}^{-x^2/2\sigma^2}$.

   (a) Find the variance (equivalent to one standard deviation);

   (b) Show that this lines up with the turning point where $G''(x) = 0$;

   (c) Find the area under the curve of one standard deviation.

2. **The binomial distribution**

   Flip N coins. The probability of obtaining k heads is $\binom{N}{k}$, known as the *binomial* distribution. Show that for $N \gg 1$ the binomial distribution becomes a Gaussian.

3. **The Poisson distribution**

   ...

4. **Stirling's approximation**

   Derive Stirling's formula for the logarithm of a factorial:

   $$\ln N! = N \ln N - N + \mathcal{O}(\ln N).$$

5.2 Classical Stat Mech

1. **Entropy**

   Assuming that the entropy S and the number of accessible states Ω of a physical system are related through an arbitrary functional form $S = f(\Omega)$, show that the additive character of S and the multiplicative character of Ω necessarily require that $S = k \ln \Omega$, where k is a constant.

2. **Pressure gradient**

   Consider an ideal gas at temperature T in a constant gravitational field. Find the pressure as a function of height, $\rho(z)$.

3. **Leaky spaceship**

   A spaceship (size) filled with air at 1 atm springs a tiny leak - a hole of (tiny size). How quickly does air rush out into space? (Note: at 1atm, the average speed of an air molecule is (?).)

4. that cool problem about the pressure gradient in a planet's atmosphere...

5. **Carnot engine**

   Derive the maximum theoretic efficiency η for a Carnot cycle.

5.3 Quantum Stat Mech

1. **Quantum statistics**

   Consider a large ensemble of particles all of the same species with a total energy E. Derive the Maxwell-Boltzmann, Fermi-Dirac, and Bose-Einstein distributions...

   (a) Maxwell-Boltzmann

   (b) Fermi-Dirac

   (c) Bose-Einstein

2. **The zipper**

   A zipper has N links, each with a binding energy of ϵ. A given link can only be open if all links to one side are open. What is the average number of open links at temperature T? What happens in the limits T $\to$ 0 and T $\to \infty$?

3. **Ensemble of oscillators**

   Consider N quantum oscillators all of frequency ω. The total energy of the system of N oscillators is E. (a) Compute the entropy of the system $S(N, E)$, using Stirling?s approximation to simplify your result. (b) Compute the temperature of the system.

4. **Blackbody Radiation**

   Given (knowns), derive the *Planck distribution*

$$EdE = \sigma \frac{\omega^3}{\mathrm{e}^{\hbar\omega/kT} - 1}.$$

   where $\sigma =$?

6 Math, Logic, & Miscellaneous

1. Line of people with two hat colors

2. N people and N hat colors

3. Green-eyed Dragons

4. **Two envelopes**

   (a) I give you an envelope containing a certain amount of money, and you open it. I then put into a second envelope either twice this amount or half this amount, with a fifty-fifty chance of each. You are given the opportunity to trade envelopes. Should you?

   (b) I put two sealed envelopes on a table. One contains twice as much money as the other. You pick an envelope and open it. You are then given the opportunity to trade envelopes. Should you?

5. Show that there is always at least one set of diametrically opposite points on the Earth's equator that at a given moment have the same temperature. Show that there are always two antipodal points that have the same temperature *and* pressure!