0.1 常用 Taylor 级数

1. $e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}=1+x+\frac{x^2}{2!}+\frac{x^3}{6}+\cdots+\frac{x^k}{k!}+\cdots.$

2. $\ln(1+x)=\sum_{n=0}^{\infty}\frac{(-1)^n}{n+1}x^{n+1}=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots+\frac{(-1)^k}{k+1}x^{k+1}+\cdots,x\in(-1,1].$

3. $\sin x=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n+1)!}x^{2n+1}=x-\frac{x^3}{6}+\frac{x^5}{120}-\cdots+\frac{(-1)^k}{(2k+1)!}x^{2k+1}+\cdots.$

4. $\cos x=\sum_{n=0}^{\infty}\frac{(-1)^n}{(2n)!}x^{2n}=1-\frac{x^2}{2}+\frac{x^4}{24}-\cdots+\frac{(-1)^k}{(2k)!}x^{2k}+\cdots.$

5. $\tan x=\sum_{n=0}^{\infty}\frac{(-1)^{n-1}2^{2n}\left(2^{2n-1}-1\right)B_{2n}}{(2n)!}x^{2n-1}=2\sum_{n=0}^{\infty}\frac{(4^n-1)(2n)!}{(2n+1)!}x^{2n+1}=x+\frac{1}{3}x^3+\frac{2}{15}x^5+\frac{17}{315}x^7+\frac{62}{2835}x^9+\frac{1382}{155925}x^{11}+o(x^{11}),x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right).$

6. $\cot x=\sum_{n=0}^{\infty}\frac{(-1)^n2^{2n}B_{2n}}{(2n)!}x^{2n-1}=\frac{1}{x}-\frac{1}{3}x+\frac{1}{45}x^3-\frac{2}{945}x^5-\cdots,x\in(0,\pi).$

7. $\sec x=\sum_{n=0}^{\infty}\frac{(-1)^nE_{2n}}{(2n)!}x^{2n}=1+\frac{1}{2}x^2+\frac{5}{24}x^4+\frac{61}{720}x^6+\frac{277}{8064}x^8+\frac{50521}{3628800}x^{10}+o(x^{11}),x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right),x\in(0,\pi).$

8. $\csc x=\sum_{n=0}^{\infty}\frac{(-1)^{n+1}2\left(2^{2n-1}-1\right)B_{2n}}{(2n)!}x^{2n-1}=\frac{1}{x}+\frac{1}{6}x+\frac{7}{360}x^3+\frac{31}{15120}x^5+\cdots,x\in(0,\pi).$

9. $\arcsin x=\sum_{n=0}^{\infty}\frac{(2n)!}{4^n(2n+1)!}x^{2n+1}=x+\frac{1}{6}x^3+\frac{3}{40}x^5+\frac{5}{112}x^7+\frac{35}{1152}x^9+\frac{63}{2816}x^{11}+o(x^{11}),x\in(-1,1).$

10. $\arctan x=\sum_{n=0}^{\infty}\frac{(-1)^k}{2k+1}x^{2k+1}=x-\frac{1}{3}x^3+\frac{1}{5}x^5-\cdots+\frac{(-1)^k}{2k+1}x^{2k+1}+\cdots,x\in(-1,1).$

11. $\sinh x=\sum_{n=0}^{\infty}\frac{x^{2n+1}}{(2n+1)!}=x+\frac{1}{6}x^3+\frac{1}{120}x^5+\cdots+\frac{x^{2n+1}}{(2n+1)!}+\cdots.$

12. $\cosh x=\sum_{n=0}^{\infty}\frac{x^{2n}}{(2n)!}=1+\frac{1}{2}x^2+\frac{1}{24}x^4+\cdots+\frac{x^{2k}}{(2k)!}+\cdots.$

13. $\tanh x=\sum_{n=0}^{\infty}\frac{4^n(4^n-1)B_{2n}}{(2n)!}x^{2n-1}=x-\frac{1}{3}x^3+\frac{2}{15}x^5-\frac{17}{315}x^7+\frac{62}{2835}x^9-\frac{1382}{155925}x^{11}+o(x^{11}),x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right).$

14. $\operatorname{sech}x=\sum_{n=0}^{\infty}\frac{E_{2n}x^{2n}}{(2n)!}=1-\frac{1}{2}x^2+\frac{5}{24}x^4-\frac{61}{720}x^6+\frac{277}{8064}x^8-\frac{50521}{3628800}x^{10}+o(x^{11}),x\in\left(-\frac{\pi}{2},\frac{\pi}{2}\right).$

15. $\operatorname{arsinh}x=\sum_{n=0}^{\infty}\frac{(-1)^n(2n)!}{4^n(2n+1)!}x^{2n+1}=x-\frac{1}{6}x^3+\frac{3}{40}x^5-\frac{5}{112}x^7+\frac{35}{1152}x^9-\frac{63}{2816}x^{11}+o(x^{11}),x\in(-1,1).$

16. $\operatorname{artanh}x=\sum_{n=0}^{\infty}\frac{x^{2n+1}}{2n+1}=x+\frac{1}{3}x^3+\frac{1}{5}x^5+\frac{1}{7}x^7+\frac{1}{9}x^9+\frac{1}{11}x^{11}+o(x^{11}),x\in(-1,1).$

17. $e^{\sin x}=1+x+\frac{1}{2}x^2-\frac{1}{8}x^4-\frac{1}{15}x^5-\frac{1}{240}x^6+\frac{1}{90}x^7+\frac{31}{5760}x^8-\frac{1}{5670}x^9-\frac{2951}{3628800}x^{10}+o(x^{10}).$

18. $e^{\tan x}=1+x+\frac{1}{2}x^2+\frac{1}{2}x^3+\frac{3}{8}x^4+\frac{37}{120}x^5+\frac{59}{240}x^6+\frac{137}{720}x^7+\frac{871}{5760}x^8+\frac{41641}{3628800}x^9+o(x^9).$

19. $e^{\arcsin x}=1+x+\frac{1}{2}x^2+\frac{1}{3}x^3+\frac{5}{24}x^4+\frac{1}{6}x^5+\frac{17}{144}x^6+\frac{13}{126}x^7+\frac{629}{8064}x^8+\frac{325}{4536}x^9+o(x^9).$

20. $e^{\arctan x} = 1 + x + \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{7}{24}x^4 + \frac{1}{12}x^5 + \frac{29}{144}x^6 - \frac{1}{1008}x^7 - \frac{1249}{8064}x^8 - \frac{1163}{72576}x^9 + o(x^9).$

21. $\tan(\tan x) = x + \frac{2}{3}x^3 + \frac{3}{5}x^5 + \frac{181}{315}x^7 + \frac{59}{105}x^9 + \frac{3455}{6237}x^{11} + o(x^{11}).$

22. $\sin(\sin x) = x - \frac{1}{3}x^3 + \frac{1}{10}x^5 - \frac{8}{315}x^7 + \frac{13}{2830}x^9 - \frac{47}{49896}x^{11} + o(x^{11}).$

23. $\tan(\sin x) = x + \frac{1}{6}x^3 - \frac{1}{40}x^5 - \frac{107}{5040}x^7 - \frac{73}{24192}x^9 + \frac{41897}{39916800}x^{11} + o(x^{11}).$

24. $\sin(\tan x) = x + \frac{1}{6}x^3 + \frac{1}{40}x^5 - \frac{55}{846}x^7 - \frac{143}{3456}x^9 - \frac{968167}{39916800}x^{11} + o(x^{11}).$

25. $(1+x)^\alpha = 1+\sum_{n=1}^{\infty}\frac{\alpha(\alpha-1)\cdots(\alpha-n+1)}{n!}x^n = 1+\alpha x+\frac{\alpha(\alpha-1)}{2!}x^2+\cdots+\frac{\alpha(\alpha-1)\cdots(\alpha-k+1)}{k!}x^k+\cdots, x \in (-1,1).$

26. $(1+x)^{\frac{1}{x}} = e - \frac{e}{2}x + \frac{11e}{24}x^2 - \frac{7e}{16}x^3 + \frac{2447e}{5760}x^4 - \frac{959e}{2304}x^5 + \frac{281343e}{580608}x^6 - \frac{67223e}{168885}x^7 + o(x^7).$

27. $(1+x^2)^{\frac{1}{x}} = 1 + x + \frac{1}{2}x^2 - \frac{1}{3}x^3 - \frac{11}{24}x^4 + \frac{11}{120}x^5 + \frac{271}{720}x^6 + \frac{53}{2320}x^7 - \frac{4069}{13410}x^8 + o(x^8).$

28. $(1+\sin x)^{\frac{1}{x}} = e - \frac{e}{2}x + \frac{7e}{24}x^2 - \frac{3e}{16}x^3 + \frac{139e}{560}x^4 - \frac{899e}{11520}x^5 + \frac{29811e}{580608}x^6 - \frac{180617e}{580608}x^7 + o(x^7).$