$A^{\mu} = (A^0, \vec{A}), \qquad x^{\mu} = (t, \vec{x})$ Contravariant four-vector  $A_{\mu} = q_{\mu\nu}A^{\nu} = (A^0, -\vec{A}), \qquad x_{\mu} = (t, -\vec{x})$ Covariant four-vector

Notations and conventions for four-vectors and the metric

Metric

Lorentz invariant

Antisymmetric tensor

Derivatives

Contractions

$$\Box \equiv \partial_{\mu} \partial^{\mu} = \frac{\partial^{2}}{\partial t^{2}} - \vec{\nabla}^{2}$$

$$\partial \cdot A = \partial_{\mu} A^{\mu} = \frac{\partial A^{0}}{\partial t} + \vec{\nabla} \cdot \vec{A}$$

$$\partial \cdot A = \partial_{\mu} A^{\mu} = \frac{\partial A^{0}}{\partial t} + \vec{\nabla} \cdot \vec{A}$$

$$a \overleftrightarrow{\partial}^{\mu} b = a \partial^{\mu} b - (\partial^{\mu} a) b$$

$$\epsilon^{\mu\nu\rho\sigma}, \text{ with } \epsilon_{0123} = +1 \text{ and } \epsilon^{0123} = -1$$

$$\partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}} = \left(\frac{\partial}{\partial t}, \nabla\right),$$
$$\Box \equiv \partial_{\mu} \partial^{\mu} = \frac{\partial^{2}}{\partial t^{2}} - \vec{\nabla}^{2}$$

$$A \cdot B \equiv A_{\mu}B^{\mu} = g_{\mu\nu}A^{\mu}B^{\nu} = A^{0}B^{0} - \vec{A} \cdot \vec{B}$$
$$\partial_{\mu} \equiv \frac{\partial}{\partial x^{\mu}} = \left(\frac{\partial}{\partial t}, \vec{\nabla}\right), \quad \partial^{\mu} \equiv \frac{\partial}{\partial x_{\mu}} = \left(\frac{\partial}{\partial t}, -\vec{\nabla}\right)$$

 $\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu\rho\sigma} = -24$   $\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu\rho\tau} = -6g_{\tau}^{\sigma}$ 

 $\epsilon^{\mu\nu\rho\sigma}\epsilon_{\mu\nu\tau\omega} = -2\left(q_{\tau}^{\rho}q_{\omega}^{\sigma} - q_{\tau}^{\rho}q_{\tau}^{\sigma}\right)$ 

 $g_{\mu\nu} = g^{\mu\nu} = \text{diag}(1, -1, -1, -1)$ 

$$\begin{cases} 0, \\ A^{\mu}B \end{cases}$$

$$g^{\nu}_{\mu} \equiv g^{\nu\sigma}g_{\mu\sigma} = \delta^{\nu}_{\mu} = \begin{cases} 1, & \mu = \nu \\ 0, & \mu \neq \nu \end{cases}$$
$$A \cdot B \equiv A_{\mu}B^{\mu} = g_{\mu\nu}A^{\mu}B^{\nu} = A^{0}$$

$$=A$$

$$A \cdot B \equiv A_{\mu}B^{\mu} = g_{\mu\nu}A^{\mu}B^{\nu} = A^{0}B^{0} - \vec{A} \cdot \vec{B}$$

$$\left(\frac{\partial}{\partial t}, -\right)$$