Introduction:

The First Order Predicate Logic(FOPL) is a powerful language to represent natural language statements and complex sentences in artificial intelligence. It is an extension of prepositional logic with objects, relations and functions. It uses symbols, connectives, quantifiers, and atomic sentences to express the logic.

For Example:

- Ram loves all dogs.

  $\forall x \; Dog(x) \Rightarrow Loves(ram, x)$

- Fido is a dog.

  Dog (Fido)

- Parent and child are inverse relation.

  $\forall x, y Parent(x, y) \Leftrightarrow Child(y, x)$

The key concepts of FOPL are as follow:

1. Objects:

In FOPL, there are objects or individuals about which statements can be made. These objects can represent real-world entities.

Example: {Alice, Bob, Charlie}.

2. Predicates:

Predicates express relationships between objects or properties of objects. They are used to form statements.

Example: the predicate "Likes(Alice, Bob)" expresses that Alice likes Bob.

3.Variables:

Variables are placeholders that can represent any object. They allow for the generalization of statements.

Example: The statement "Likes(x, Charlie)" expresses that there is a person who likes Charlie where x is a variable.

4. Quantifiers:

FOPL uses quantifiers to express the scope of variables. The two main quantifiers are the universal quantifier (∀), which means "for all," and the existential quantifier (∃), which means "there exists."

Example: Universal Quantifier (∀)   Existential Quantifier (∃)

Statement: ∀x Likes(x, Charlie)   ∃y Likes(Alice, y)

Meaning: "Everyone likes Charlie."   "There is someone whom Alice likes."

5. Connectives:

Logical connectives, such as AND (∧), OR (∨), NOT (¬), and IMPLIES (→), are used to form compound statement.

Example:

Statement: Likes(Alice, Bob) ∧ Likes(Bob, Charlie)

Meaning: "Alice likes Bob, and Bob likes Charlie."

6. Functions:

Functions can be used to represent relationships between objects and can take variables as arguments.

Example: Introduce a function "Age(x)" representing the age of a person.

Statement: Age(Alice) > Age(Bob)

Meaning: "Alice is older than Bob."

Prolog clauses can be directly translated into FOPL, except for a few exceptions like write, !, is, assert, retract, ...

The three simple rules for conversion are:

- "," corresponds to "&"
- ":-" corresponds to "<-"
- All variables are universally quantified.

PREMISES 1

1. Horses, cows, pigs are mammals.
2. An offspring of a horse is a horse.
3. Bluebeard is a horse.
4. Bluebeard is Charlie's parent.
5. Offspring and parent are inverse relations.
6. Every mammal has a parent.

To Prove: Is Charlie a horse?

FOPL:

1. $\forall \mathbf{x}$ (horse(x) $\lor$ cow(x) $\lor$ pig(x))$\rightarrow$ mammal(x)
2. $\forall$a $\forall$y [offspring (x, y) $\land$ horse (y)]$\rightarrow$horse (x)
3. horse (bluebeard)
4. parent (bluebeard, charlie)
5. $\forall$x $\forall$y offspring (x, y) $\leftrightarrow$ parent (y, x)
6. $\forall$x $\forall$y mammal (x) $\rightarrow$ parent (x, x)

To prove: horse(charlie)

PREMISES 2

1. All people who are not poor and are smart are happy.
2. Those people who read are not stupid.
3. John can read and is wealthy.
4. Happy people have exciting lives.

To Prove: Can anyone be found with an exciting life?"?

FOPL:

1. $\forall$x (rich(x) $\land$ smart(x)) $\rightarrow$ happy(x)
2. $\forall$x can_read(x) $\rightarrow$ smart(x)
3. rich(john) $\land$ can_read(john)

4. $\forall x$ happy(x) $\rightarrow$ HasExcitingLife(x)

To prove: $\exists x$ HasExcitingLife(x)

PREMISES 3

1. All pompeians are romans.
2. all romans were either loyal to Caesar or hated him.
3. everyone is loyal to someone.
4. people only try to assassinate rulers they are not loyal to.
5. marcus tried to assassinate Caesar.
6. marcus was Pompeian.

To Prove: did marcus hate Caesar?

FOPL:

1. $\forall x$ Pompeian(x) $\rightarrow$ roman(x)
2. $\forall x$ roman(x) $\rightarrow$ loyal(x,ceasar) $\vee$ hate(x,caeser)
3. $\forall x$ $\forall y$ loyal(x,y)
4. $\forall x$ $\forall y$ assassinate(x,y) $\leftrightarrow$ $\neg$loyal(y,x)
5. assassinate(marcus, ceasar)
6. pompeian(marcus)

To prove: $\neg$ hate(marcus,ceaser)

PREMISES 4

Bhogendra likes all kinds of food. Oranges are food. Chicken is food. Anything anyone eats and isn't killed by is food. If a person likes a food means that person has eaten it. Jogendra eats peanuts and is still alive. Shailendra eats everything Bhogendra eats.

To Prove: Does Shailendra like chicken?

FOPL:

1. $\forall x$ food(x) $\rightarrow$ like(bhogendra,x)
2. food(orange)

3. food(chicken)

4. ∀x ∀y [eats(y,x) ∧ ¬killed(y)] → food(x)

5. ∀x ∀y like(y,x) → eats(y,x)

6. eats(jogendra,peanuts) ∧ ¬killed(jogendra)

7. ∀x eats(bhogendra,x) → eats(shailendra,x)

To prove: like(shailendra,chicken)

PREMISES 5

Dave and Fred are members of a dancing club in which no member can both waltz and jive. Fred"s dad can"t waltz and Dave can do whatever fred can"t do. If a child can do something, then their parents can do it also.

To Prove: there is a member of the dancing club who can’t jive.

FOPL:

2. member(dave,dancingclub)

3. member(fred,dancingclub)

4. ∀x member(x,dancingclub) → ¬(can_do(x,waltz) ∧ can_do(x,jive))

5. parent(freddad,fred)

6. ¬can_do(freddad,waltz)

7. ∀y can_do(dave,y) → ¬can_do(fred,x)

8. ∀x ∀y can_do(x,y) →∀z (parent(z,x) ∧ can_do(z,y))

To prove: ∃x member(x, dancingclub) → ¬can_do(x,jive)