Revision - 1st internals MODULE 1

Class	SS
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Introduction and Classification of Signals

Signal

A signal is defined mathematically as a function of one or more independent variables, which conveys information about the nature of natural phenomenon.

→ F{x1, x2, x3, x4, ... }

Where F is the function and x1,x2,.. etc are the independent variables of the signal.

- Anything that is variable is called a signal. Things that are not variable are not called signals.

  eg : AC and DC current respectively.
- Signals that depend on a single variable are called one dimensional signals, and signals that depend on two or more variable`s are known as multvariable signals.

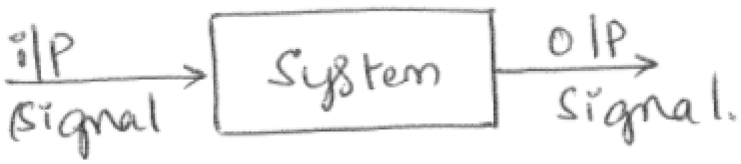
System

A system is defined as an entity that manipulates one or more signals to accomplish a function, thereby yielding a new signal.

- System can also be defined as the meaningful interconnection of physical devices, components and operations, which transform or modify one or more input signals to accomplish a function and produce output signal.

eg: Filters. Filters are reduced to reduce the noise in the signal

→ The operation performed by the system on the signal is known as 'processing' which involves elimination of noise and interference from the signal.

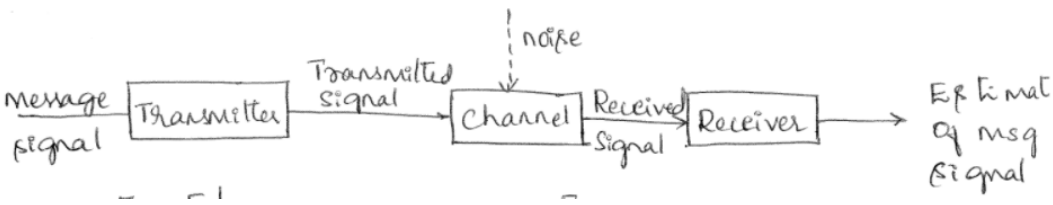


- Signals that enter the system from some external source are called input signals or excitation signals.
- The signals produced by the system through processing of the input signal are called output signals or response signals.
- Signals that occur within a system but are neither input nor output signals are known as internal signals. These signals are functions of an independent variable such as time, distance etc. The responses are always more desireable than the excitation.

Real life examples of Systems

1. Communication Systems

- The three basic elements of every communication system are: Transmitter, Channel, Receiver.

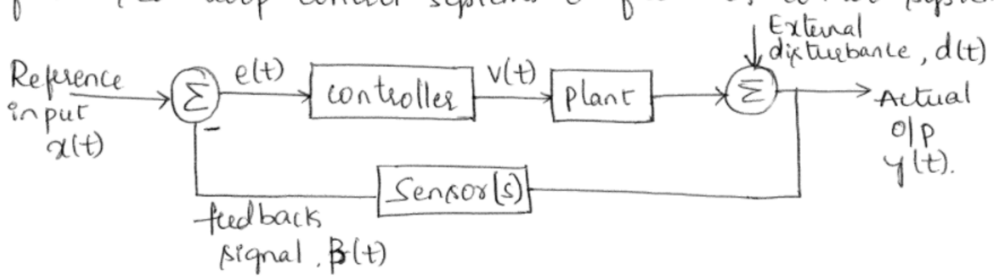


- The transmitter and receiver are placed at a distance apart and are connected by a physical medium called the channel. The channel may be free space, optical fiber, co-axial cable etc.

- The purpose of the transmitter is to convert the message signal produced by a source of information into a form suitable for transmission over the channel. The message could be speech signal, television signal, computer data etc.
- When the transmitted signal travels through the channel, it can get distorted due to the physical characteristics of the channel, as well as have noise and interfering signals from other sources contaminate it.
- The receiver then reconstructs the distorted signal into recognizable form or into an extended form of the original message signal, essentially reversing the process of the transmitter and compensating for or removing noise, weak signal etc.

2. Control Systems

The below fig shows the block diagram representation of a closed loop control systems or feedback control system



The control of physical systems like automobiles, machine tools, mills etc. are all examples of the application of control systems.

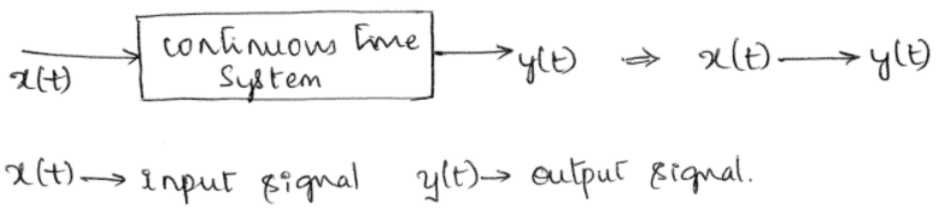
- The object to be controlled is referred to as the 'plant'
- In any control systems, the plant is defined by the mathematical operations that generate output y(t) in response to the input x(t) and the external disturbance d(t.
- The sensors in the feedback loop measures the plant output y(t) and converts it into $B(t)$ known as feedback. This feedback is compared against the reference input x(t) to produce an error signal e(t).

- The error signal is applied to a controller which in turn produces the actuating signal v(t) that performs the controlling action of the plant.

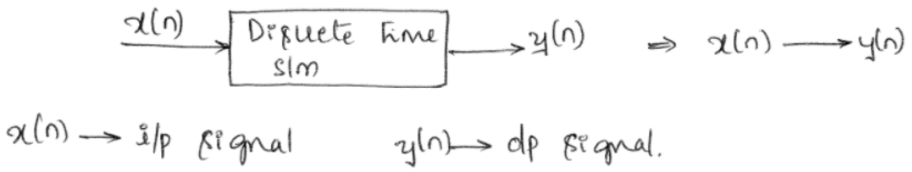
There are basically two types of systems:

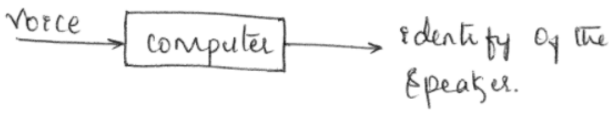
1. Continuous time systems
2. Discrete time systems

A continuous time system is one where continuous time input signals are applied which in turn result in continuous time output signals. It can be represented as shown:



Similarly, a discrete time signal is one where discrete time signals are applied which results in discrete time outputs.





Classification of Signals

A One dimensional signal is said to be single valued signal or function of time. The term single-valued means that at every instance of time there is a unique value of the function. This may be a real number in which case it is a 'Real Valued signal' or it may be a complex number in which case it is a 'complex valued singal'

Single-valued functions can be classified into five categories:

1. Continuous and Discrete Time signals
2. Even and Odd signals
3. Periodic and Non-periodic signals
4. Energy and Power signals
5. Determinate and Random signals

1. Continuous and Discrete Time signals

- A signal x(t) is said to be continuous time signal if it has a value of amplitude for all time 't' i.e. it is defined for all values of t. It is also called an **Analog signal**.

  eg: Speed signal as a function of time

- A signal x(n) is said to be a discrete time signal if it is defined only at specific instants of time. These are also known as digital signals.

A discrete time signal is often derived from continuous time signal by a process called **sampling**. It is the process of converting continuous time signal into a discrete time signal by sampling at a uniform rate.

eg : x(n) = x(nTs); n = +/- 1, 2, 3, ...

2. Even and Odd signals

- A signal is said to be an even signal if it is identical to its time reversed counterpart, or with its reflection about the origin, or symmetrical about the vertical axis.

  → x(t) is even if x(-t) = x(t)

- Likewise a signal is said to be odd if it is asymmetrical about the origin.

  → x(t) is odd if x(-t) = -x(t)

Any arbitrary signal x(t) can be decomposed into an even and odd signal by applying the corresponding definitions.

Let $x(t)$ be expressed as a sum of two components $x_e(t)$ and $x_o(t)$

i.e $x(t) = x_e(t) + x_o(t) \longrightarrow (1)$

$x_e(t)$ = even component

$x_o(t)$ = odd component

put $t = -t$ in eq -(1)

$x(-t) = x_e(-t) + x_o(-t)$

$x(-t) = x_e(t) - x_o(t) \longrightarrow (2)$

Solving for $x_e(t)$ and $x_o(t)$ in eq eq(1) & (2)

we get,

$$\boxed{\begin{aligned} x_e(t) &= \frac{x(t) + x(-t)}{2} \\ x_o(t) &= \frac{1}{2}[x(t) - x(-t)] \end{aligned}}$$

lllly for D.T signal

$$\boxed{\begin{aligned} x_e(n) &= \frac{1}{2}[x(n) + x(-n)] \\ x_o(n) &= \frac{1}{2}[x(n) - x(-n)] \end{aligned}}$$

If the signal is complex-valued signal then a complex valued signal $x(t)$ is said to be conjugate symmetric if $x(-t) = x^*(t) \longrightarrow (*)$

Let $x(t) = a(t) + jb(t)$

$a(t)$ is the real part of $x(t)$ and

$b(t)$ is the imaginary part of $x(t)$

$j = \sqrt{-1}$ then complex conjugate of $x(t)$ is

$$x^*(t) = a(t) - j\,b(t)$$

Substituting for $x(t)$ and $x^*(t)$ in $(*)$ yields

$$a(t) + jb(t) = a(t) - jb(t)$$

A complex valued signal $x(t)$ is said to be conjugate symmetric if its real part is even and its imaginary part is odd.

3. Periodic and Non-periodic signals

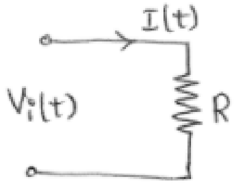
- A continuous time signal x(t) is said to be periodic if there exists a positive, non-zero value T for which x(t+T) = x(t) for all T.
- The smallest value of T for which the above condition is satisfied is called the *fundamental period* of x(t). The reciprocal of the same is known as the *fundamental frequency*.
- If for a signal x(t) there exists no value of T to satisfy x(t+T) = x(t) then it is known as aperiodic.

- A discrete time signal is said to be periodic if it satisfies the condition x(n+N) = x(n) for all N where N is a positive integer.
- The smallest value of N for which the above is satisfied is known as the *fundamental period* of x(n).

4. Energy and Power signals

4. Energy And Power Signals.

In electrical slm. signal may represents voltage or current.



Consider voltage v(t) developed across the resistor R producing current I(t). The instantaneous power dissipated in resistor R is defined by

$$p(t) = \frac{v^2(t)}{R} \longrightarrow ①$$

equivalently $p(t) = R \cdot I^2(t) \longrightarrow ②$

In both the cases p(t) is proportional to square of the amplitude of the signal. further. for a resistor $R = 1\Omega$ equations ① & ② takes the same mathematical form hence the instananeous power signal can be expressed as

$$p(t) = x^2(t) \longrightarrow ③$$

On the basis of this, we can define total energy of continuous time signal as.

$$E = \int_{-\infty}^{\infty} x^2(t) \cdot dt \quad \text{or} \quad E = \lim_{T \to \infty} \int_{-T/2}^{T/2} x^2(t)\, dt.$$

Average power as. $p = \lim_{T \to \infty} \frac{1}{2T} \int_{-T/2}^{T/2} [x(t)]^2 \cdot dt$

The square root of the average power P is called the root mean square (RMS) value of the periodic signal x(t).

In case of discrete time signal $x(n)$, the total energy is

$$E = \sum_{n=-\infty}^{\infty} [x(n)]^2$$

and average power is

$$P = \lim_{N \to \infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x(n)|^2$$

$x(n)$ is periodic with fundamental period N. Average power is,

$$P = \frac{1}{N} \sum_{n=0}^{N-1} |x(n)|^2$$

lll'y for C.T.S $P = \frac{1}{T} \int_0^T |x(t)|^2 \, dt$

Note:

* A signal is referred to as energy signal if and only if the total energy of the signal satisfies the condition $0 < E < \infty$.
* A signal is referred to as power signal if and only if the average power of the signal satisfies the condition $0 < P < \infty$
* The Energy and power classification of signals are mutually exclusive. In particular, an energy signal has zero time average power, where as power signa has infinite energy.

5. Deterministic and Random signals.

- A deterministic signal is one about which there is no uncertainty about its value at any time. These signals are described uniquely by a mathematical expression, graph or well defined rule.

- This means that if we know the value of the signal at t = t1, we can precisely predict the value at any other time t2.
- A random signal is one about which there is uncertainty about its value before its actual occurrence. These signals cannot be described by an equation of graph. These signals are defined by their statistical properties eg. range, average values, probability distribution etc.

Elementary Signals or Functions

Elementary signals occur frequently in nature and are the basic building blocks that construct more complex signals.

Some of the important elementary continuous and discrete signals are

1. Exponential signal
2. Sinusoidal signal
3. Exponentially damped sinusoidal signal
4. Unit step function
5. Unit impulse function
6. Unit ramp function

1. Exponential signals

A continuous time real exponential signal is given by

$$\boxed{x(t) = B \cdot e^{at}}$$

where B and a are real parameters.

- The parameter 'B' is the amplitude of the exponential signal measured at t=0

- The behavior of the signal depends on the parameter 'a' and can be of two types:
    1. Growing exponential

       If 'a' is positive then the signal x(t) is called a growing exponential signal.
    2. Decaying exponential

       If 'a' is negative then the signal x(t) is called a decaying exponential signal.

2. Sinusoidal signals

The continuous time version of sinusoidal signals in its most basic form may be written as

x(t) = Acos(wt + phi) or x(t) = Asin(wt + phi)

A → amplitude

w → angular frequency in radians/sec

phi → phase angle in radians

- A sinusoidal signal is an example of periodic signal with T = 2pi/2

Proof:
$$
\begin{aligned}
x(t+T) &= A\cos[\omega(t+T)+\phi] \\
&= A\cos[\omega t+\omega T+\phi] \\
&= A\cos\left[\omega t+\omega\left[\tfrac{2\pi}{\omega}\right]+\phi\right] \\
&= A\cos[\omega t+2\pi+\phi] \\
&= A\cos[\omega t+\phi]
\end{aligned}
$$
$$x(t+T) = x(t)$$

<u>Discrete time version of sinusoidal signal:</u>

The period of a periodic discrete time signal is measures in samples. Thus x(n) is said to be periodic with a period of 'N' samples.

i.e $x(n) = A\cos[\Omega n + \phi]$

for periodic, $x(n+N) = x(n)$

$\therefore x(n) = A\cos[\Omega(n+N) + \phi]$

Only if $\Omega N = 2\pi m$ radians

i.e $N = \frac{2\pi m}{\Omega}$ samples where m, N are integers.

3. Exponentially damped sinusoidal signal

This signal is obtained as a result of multiplication of a sinusoidal signal by a real-valued decaying exponential signal.

i.e $\boxed{x(t) = Ae^{-\alpha t}\sin(\omega t + \phi)}$, $\alpha > 0$

where,

$x(t)$ = Exponential Damped sinusoidal signal

$e^{-\alpha t}$ = real valued decaying exponential signal.

$A\sin(\omega t + \phi)$ = Sinusoidal signal.

4. Unit step function

The continuous time version of the unit step function is defined by

$$u(t) = \begin{cases} 1 & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$

The discrete time version of the unit step function is defined by

$$u(n) = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

A step function is said to exhibit a discontinuity at t=0 as the value of u(t) changes instantaneously from 0 to 1 at t=0.

5. Impulse and delta function

Unit impulse can be regarded as a rectangular pulse with a width that has become infinitesimal and height that has become infinitely large, with the overall area remaining as unity.

- Hence unit impulse signal is a signal with 0 amplitude everywhere except t=0. The amplitude is infinite such that the area under the curve = 1.
- Mathematically, the continuous time version of the unit impulse function is given by

  S(t) = 0, for t ≠ 0

  = i, for t = 0
- The discrete time version of the unit impulse signal is given by

  S[n] = 1, n=0

  = 0, n≠0

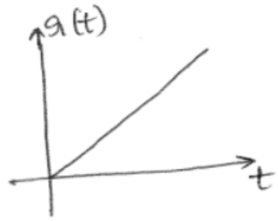
6. Ramp signal

The continuous time version of the ramp signal or function is,

$$r(t) = \begin{cases} t, & t \geq 0 \\ 0, & t < 0 \end{cases} \quad \& \quad r(t) = t \cdot u(t)$$

The impulse function S(t) is the derivative of the step function u(t) wrt time.
The integral of the step function u(t) is the ramp function of the unit slope.

The physical example is a constant current flowing through a capacitor leads to a ramp voltage across capacitor

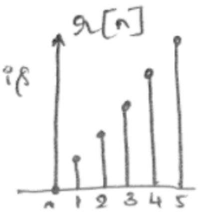


$$V_c(t) = \frac{1}{C}\int i \, d\tau$$

$$= \frac{I}{C}\int d\tau = \frac{I}{C}\cdot t$$

$$\boxed{V_c(t) \propto t}$$

The Discrete time version of the ramp function is defined by $r[n] = \begin{cases} n, & n>0 \\ 0, & n<0 \end{cases}$ & $r[n] = n\cdot u[n]$



7. Complex exponential signal

- The complex exponential signal is defined by

$$x(t) = A\, e^{j\omega_0 t}$$

where

A → amplitude

w0 → angular frequency

- A complex exponential signal can be resolved into real and imaginary parts ie:

$$x(t) = A \cdot e^{j\omega_0 t}$$

$$= A\left[\cos\omega_0 t + j\sin\omega_0 t\right]$$

Exponential sinusoidal signal is defined as

$$x(t) = A\, e^{at} \sin\omega_0 t$$

The exponential Discrete time signal is defined by

$$e(n) = a^n \cdot x(n)$$

8. Signum signal

Continuous time signum signal is defined as

$$\mathrm{Sgn}(t) = \begin{cases} 1 & ; t > 0 \\ 0 & ; t = 0 \\ -1 & ; t < 0 \end{cases}$$

Discrete time signum signal is defined as

$$\mathrm{Sgn}(n) = \begin{cases} 1 & ; n > 0 \\ 0 & ; n = 0 \\ -1 & ; n < 0 \end{cases}$$

9. Rectangular function

- The rectangular function is defined as

$$rect(t) = \begin{cases} 1 & ; |t| < 0.5 \\ 0 & ; |t| > 0.5 \end{cases}$$

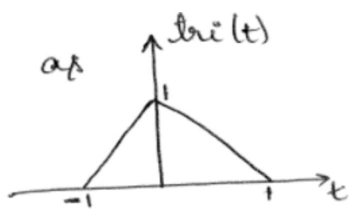
Discrete rectangular function is.

$$rect_{N_0}(n) = 1 \quad : \quad |n| \leq N_0$$
$$= 0 \quad\quad |n| > N_0$$

10. Unit triangular function

The triangular function is defined as

$$tri(t) = 1 - |t| \quad : |t| \leq 1$$
$$= 0 \quad\quad ; |t| > 1$$



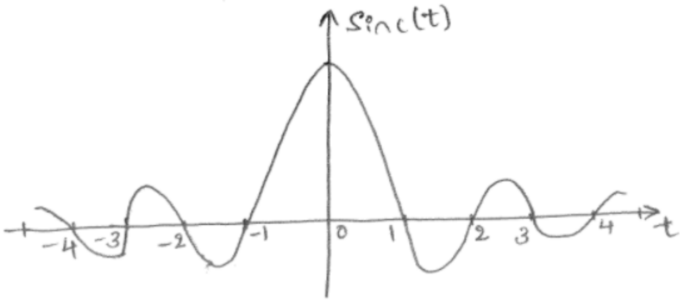
The discrete time triangular function is defined as.

$$tri(n) = 1 - \frac{|n|}{N_0} \quad ; \quad |n| \leq N_0$$
$$\quad\quad : \quad |n| > N_0$$

11. Sinc function

Sinc function

The Sinc function is defined as $\text{Sinc}(t) = \frac{\sin(\pi t)}{\pi t}$



At t=0:

$$\text{Sinc}(t)\Big|_{t=0} = 1$$

Operations on Signals

One dimensional signals can be defined using two variables : dependent and independent.

Dependent variable corresponds to the amplitude or value of the signal whereas the independent variable is the continuous or discrete time signal, 't' or 'n' respectively.

Operations on the dependent variable

1. Amplitude scaling

- If x(t) is a continuous time signal then the signal obtained from amplitude scaling is given by

  y = cx(t) where c is the scaling factor.

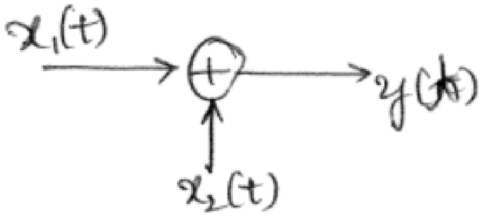
- A physical device that performs amplitude scaling is an amplifier.

2. Addition

- If x1(t) and x2(t) are two C.T.S. then the signal obtained from addition of the two is given by

  y = x1(t) + x2(t)

Addition is represented as



- A physical device that performs addition is the volume mixer.

3. Multiplication

- If x1(t) and x2(t) are two C.T.S. then y(t) resulting from multiplication of the two is given by

  y = x1(t).x2(t)

- A physical device that performs multiplication is the modulator in which the audio frequency signal is multiplied by a high frequency sinusoidal called carrier.

4. Differentiation

- Let x(t) denote a C.T.S. The derivative of x(t) wrt t is given by

$$\boxed{y(t) = \frac{d\,x(t)}{dt}}$$

- A physical device that performs differentiation is the inductor.

5. Integration

- Let x(t) denote a C.T.S. Then the integral of x(t) wrt time is given by

$$y(t) = \frac{1}{C}\int_{-\infty}^{t} x(t)\cdot d\tau$$

- A physical device that performs integration is capacitor.

Operations on the independent variable

1. Time scaling

If x(t) is a C.T.S. then a signal y(t) is obtained by scaling the independent variable 't' by a factor 'a' as:

y(t) = x(at)

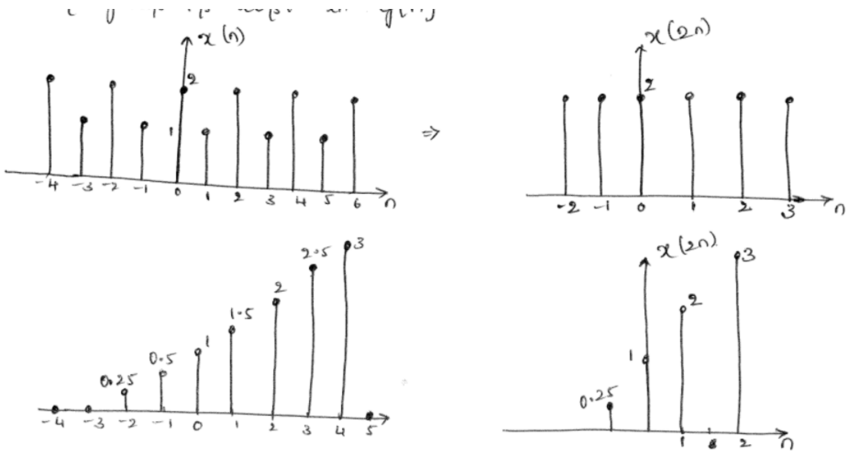
- If a>1, then the resultant signal y(t) is a compressed version of x(t)
- If a<1, then the resultant signal is an expanded version of x(t)

A physical device that performs time scaling is the tape recording, which can play audio at fast-forwarded or slowed-down speeds.

Similarly in case of D.T.S, y[n] = x[kn]

where k>0 and can only take integer values.

- If k>1 then some values of the D.T.S. are lost in y[n]



2. Time shifting

Let x(t) be a C.T.S. then the time shifted version of x(t) is given by

y(t) = x(t-t0)

where t_0 = time shift

- If $t_0 > 0$, then $x(t-t_0)$ represents the delayed version of x(t). It is shifted to the right relative to the time axis.
- If $t_0 < 0$, then $x(t-t_0)$ represents the advanced version of x(t). It is shifted to the left relative to time axis.

In case of DTS,

$y[n] = x[n-n_0]$

where n_0 is an integer value of time shift

3. Reflection or time reversal

If x(t) is a CTS then y(t) is the signal obtained by replacing time with -t as shown

$y(t) = x(-t)$

In case of DTS,

$y[n] = x[-n]$

Precedence rule for time shifting and scaling

Let y(t) denote a CTS that is derived from another CTS x(t) through a combination of time shifting and time scaling.

$y(t) = x(at - b)$

This relation between y(t) and x(t) satisfies the conditions:

→ $y(0) = x(-b)$

→ $x(0) = y(b/a)$

which provides valuable info on y(t) from x(t)

To obtain y(t) from x(t), time shifting and time scaling operations must be performed in the correct order. The proper order is based on the fact that the scaling operation always replaces t by at whereas the time shifting operation replaces t by t-b. Hence the time shifting is performed on x(t) resulting in v(t) = x(t-b)

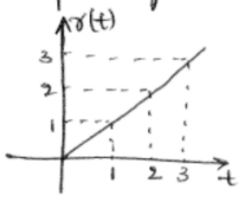
The next operation performed on v(t) replaces t by at, resulting in the desired output

$y(t) = v(at)$

$\Rightarrow y(t) = v(at - b)$

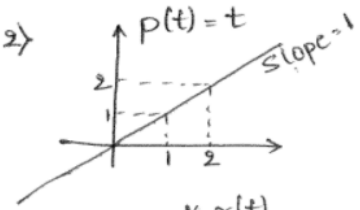
Note:

1) Ramp signal with unit slope



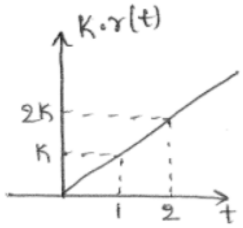
slope $m = \frac{y}{x} = \frac{1}{1} = 1$

2)



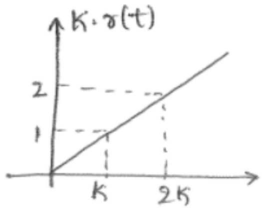
p(t) is a function passing through t=0 existing from $-\infty$ to ∞.

3)

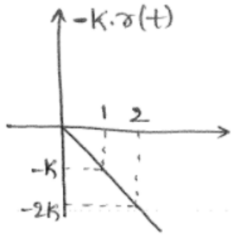


Signal K·r(t) is a ramp of slope K starting at t=0 where K>1

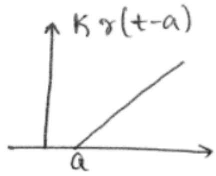
4)



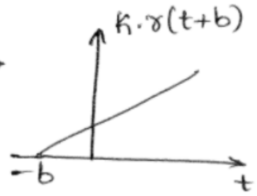
Here, signal K·r(t) is a ramp of slope 1/K starting at t=0 where K<1.

5> 

Signal $-K.r(t)$ is a ramp of slope $-K$

6> 

$K.r(t-a)$ is a delayed ramp shifted by 'a' units and K is a slope and signal starts at $t=a$ and extending in the region $t=(a, \infty)$

7> 

$K.r(t+b)$ is a delayed ramp by $-b$ units and K is a slope and the signal starts at $t=-b$ and extending in the region $t=(-b, \infty)$

8> $K.r(b-t)$ is a ramp of slope K. starting at $t=b$ and extending in the region $t=[-\infty, b]$ is a left sided signal.

9> The differentiation of the ramp signal is a unit step function. $\frac{dr(t)}{dt} = u(t)$.

10>. Differentiation of step function gives an impulse $\frac{du(t)}{dt} = \delta(t)$. By repeated integration we can get original signal.

11> The ramp signal can be used as the building block point to point addition of two ramps of opposite slopes produces step function.

12>. Step x Ramp = ramp   Step + (-ve ramp) = -ve ram
Step + (+ve ramp) = +ve ramp

13>. consider a signal as the difference of two step function

$x(t) = u(t-a) - u(t-b) \quad b > a$

$$\therefore y(t) = \{x_1(t)\, x_1(t-1) + |x_2(t)|\} - \cos(x_4(t))$$

$$= \{x_1(t)\, x_1(t-1) + |x_2(t)|\} - \cos(y_3(t))$$

$$y(t) = \{x_1(t)\, x_1(t-1) + |x_2(t)|\} - \cos(1 + 2x_3(t))$$

$$x_1(t) = x_2(t) = x_3(t) = x(t)$$

$$\therefore \boxed{H: \; y(t) = x(t)\, x(t-1) + |x(t)| - \cos(1 + 2x(t))}$$

Properties of systems

The properties of a system describe the characteristics of the operator 'H' representing the system. The different properties of a system are:

1. Memory
2. Stability
3. Causality
4. Linearity
5. Time Invariance
6. Invertibility

1. Memory

A system is said to possess memory if its output signal depends on past and/or future values of the input signal. It is then also referred to as a dynamic system.

eg : Sequential logic circuits, flip-flops etc.

A ~~syste~~ physical element, inductor has a memory since the current $i(t)$ flowing through it is related to the past values of the applied vlg.

present

-3 -2 -1 0 1 2 3

1) past future

2) past value present value future value

current $i(t)$ and vlg $v(t)$ is given by

$$i(t) = \frac{1}{L}\int_{-\infty}^{\infty} v(t)\,dt$$

where L is the inductance of the inductor. The current through a inductor at time t depends on all past values

the vlg $v(t)$. the memory of the inductor extend upto ∞

A system is said to be memoryless or static if its output signal depends only on the present value of the input signal.

eg: Resistor.

2. Stability

A system is said to be bounded input, bounded output [BIBO] stable if and only if every bounded input results in bounded output. The output of such a system does not diverge if the input does not diverge.

ex: if $y(t) = t \cdot x(t)$

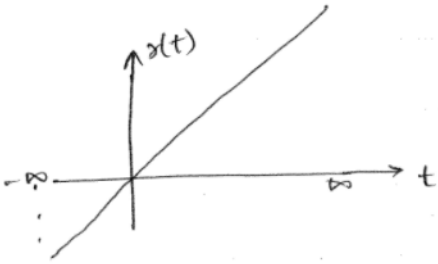
$x(t) \rightarrow \boxed{s/m} \rightarrow y(t) = t \cdot x(t)$

let $x(t) = u(t)$

$u(t) \rightarrow \boxed{s/m} \rightarrow y(t) = t \cdot u(t)$

$u(t)$ ↓ bounded i/p

$y(t) = r(t)$ ↓ not bounded



∴ s/m is unstable

3. Causality

A system is said to be causal if the output of the system is independent of future values of input or if the o/p signal is dependent only on the present and past values of i/p signal.

A system is said to be non-causal if the output of the system depends on the future values of the input signal.

An *anti-causal* system is exactly the opposite of a causal system where its output values depend only on the future values of the input signal.

4. Linearity

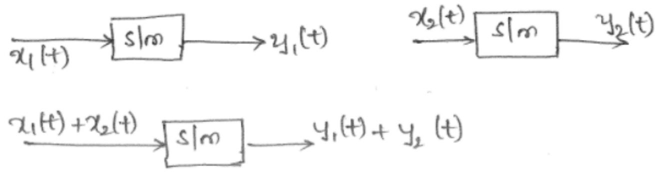
A system is said to be linear in terms of the system input and the system output if it satisfies the following properties :

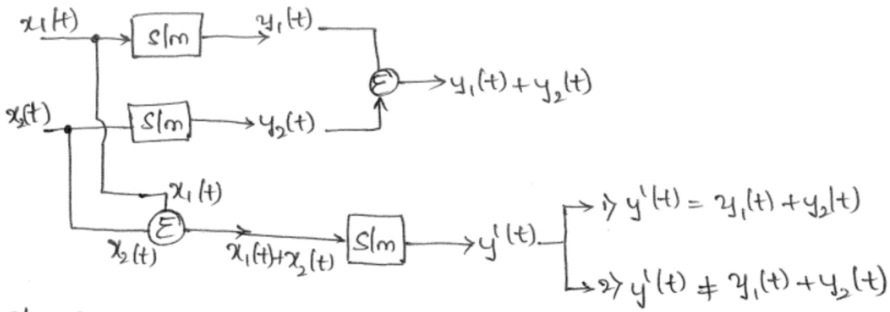
→ Law of superposition

→ Law of Homogenity

- Law of superposition is also known as law of additivity
- Law of homogenity is also known as law of multiplication or scalar multiplication.

Law of Superposition [LOA]



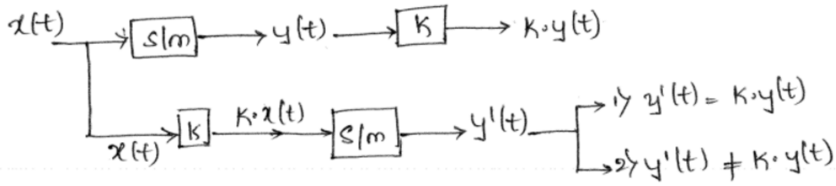


S/m is same in all cases.

if $y'(t) = y_1(t)+y_2(t) \Rightarrow$ s/m is following law of addivity

$y'(t) \neq y_1(t)+y_2(t) \Rightarrow$ s/m is not following Law of addivity.

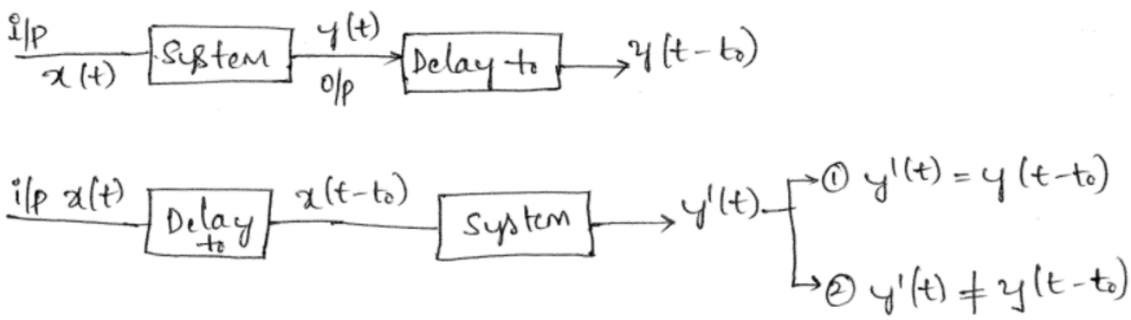
Law of Homogeneity [LOH]



if $y'(t) = K \cdot y(t) \Rightarrow$ s/m is following Law of Homogenity

$y'(t) \neq K \cdot y(t) \Rightarrow$ s/m is not following Law of Homogenity

5. Time Invariance



1> if $y'(t) = y(t-t_0) \Rightarrow$ Time invariant s/m

2> if $y'(t) \neq y(t-t_0) \Rightarrow$ Time variant s/m.

A system is said to be time invariant if a time delay or advance of the input leads to an identical time shift in the output signal.

6. Invertibility

A system is said to be invertible if the input signal can be recovered from the system output. A system is thus said to be invertible if distinct inputs lead to distinct outputs (one-to-one mapping)