Mathematics of Discrete Sine Transform (DST)[1]

DST_I:

$$y(k) = \sqrt{\frac{2}{N+1}} \sum_{n=0}^{N-1} x(n) sin(\frac{\pi(n+1)(k+1)}{N+1}) \tag{1}$$

DST_II:

$$y(k) = \sqrt{\frac{2-\delta_{kN}}{N}} \sum_{n=0}^{N-1} x(n) sin(\frac{\pi(n+\frac{1}{2})(k+1)}{N}) \tag{2}$$

DST_III:

$$y(k) = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} x(n) \frac{1}{\sqrt{1+\delta_{nN}}} sin(\frac{\pi(n+1)(k+\frac{1}{2})}{N}) \tag{3}$$

DST_IV:

$$y(k) = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} x(n) sin(\frac{\pi(n+\frac{1}{2})(k+\frac{1}{2})}{N}) \tag{4}$$

References:
http://onlinelibrary.wiley.com/doi/10.1002/cta.447/pdf

The general equation for a 2D DST

References:
https://uk.mathworks.com/matlabcentral/fileexchange/49875-2d-discrete-sine-transform-theory

[1] Implemented Mathematics
The series are indexed from n = 0 and k = 0 to N-1.
All variants of the DCT are unitary (or, equivalently, orthogonal): To find their inverses, switch k and n in each definition. In particular, DST-1 and DST-4 are their own inverses, and DST-2 and DST-3 are inverses of each other.

2D discrete sine transform:

We assume that the length of the domain is equal in X and Y directions. Moreover, we assume that number of nodes are equal in both directions. The discrete sine transform can be written as

$$a_{mn} = \left(\frac{2}{l}\right)^2 \int_0^l \int_0^l u(x,y) \sin\left(\frac{\pi m x}{l}\right) \sin\left(\frac{\pi n y}{l}\right) dx dy$$

$$a_{mn} = \left(\frac{2}{l}\right)^2 \int_0^l \left[\sum_{i=1}^{N-1} u\big((i-1)\Delta x, y\big) \sin\left(\frac{\pi m (i-1)\Delta x}{l}\right) + u(i\Delta x, y) \sin\left(\frac{\pi m i \Delta x}{l}\right)\right] \frac{\Delta x}{2} \sin\left(\frac{\pi n y}{l}\right) dy$$

$$a_{mn} = \left(\frac{2}{N-1}\right)^2 \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} \left[\begin{array}{l} U(i,j) \sin\left(\frac{\pi m (i-1)}{N-1}\right) \sin\left(\frac{\pi n (j-1)}{N-1}\right) + U(i,j+1) \sin\left(\frac{\pi m (i-1)}{N-1}\right) \sin\left(\frac{\pi n j}{N-1}\right) \\ +U(i+1,j) \sin\left(\frac{\pi m i}{N-1}\right) \sin\left(\frac{\pi n (j-1)}{N-1}\right) + U(i+1,j+1) \sin\left(\frac{\pi m i}{N-1}\right) \sin\left(\frac{\pi n j}{N-1}\right) \end{array}\right]$$

2D inverse sine transform:

$$U(i,j) = \sum_{m=1}^{N-1} \sum_{n=1}^{N-1} a_{mn} \sin\left(\frac{\pi m (i-1)}{N-1}\right) \sin\left(\frac{\pi n (j-1)}{N-1}\right)$$

Mathematics of Discrete Cosine Transform (DCT)[2]

DCT_I:

$$y(k) = \sqrt{\frac{2}{N}} \sum_{n=1}^{N} x(n) \frac{1}{\sqrt{1+\delta_{n1}+\delta_{nN}}} \frac{1}{\sqrt{1+\delta_{k1}+\delta_{kN}}} cos(\frac{\pi(n-1)(k-1)}{N-1}) \tag{5}$$

DCT_II:

$$y(k) = \sqrt{\frac{2}{N}} \sum_{n=1}^{N} x(n) \frac{1}{\sqrt{1+\delta_{k1}}} cos(\frac{\pi(2n-1)(k-1)}{2N}) \tag{6}$$

DCT_III:

$$y(k) = \sqrt{\frac{2}{N}} \sum_{n=1}^{N} x(n) \frac{1}{\sqrt{1+\delta_{n1}}} cos(\frac{\pi(n-1)(2k-1)}{2N}) \tag{7}$$

DCT_IV:

$$y(k) = \sqrt{\frac{2}{N}} \sum_{n=1}^{N} x(n) cos(\frac{\pi(2n-1)(2k-1)}{4N}) \tag{8}$$

References:
https://uk.mathworks.com/help/signal/ref/dct.html

The general equation for a 2D DCT

$$F(u,v) = \sqrt{\frac{2}{N}} \sqrt{\frac{2}{M}} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \Lambda(i)\Lambda(j) cos(\frac{\pi(2i+1)(u)}{2N}) cos(\frac{\pi(2j+1)(v)}{2M}) f(i,j) \tag{9}$$

[2]Implemented Mathematics
The series are indexed from n = 1 and k = 1 instead of the usual n = 0 and k = 0, because MATLAB® vectors run from 1 to N instead of from 0 to N – 1.
All variants of the DCT are unitary (or, equivalently, orthogonal): To find their inverses, switch k and n in each definition. In particular, DCT-1 and DCT-4 are their own inverses, and DCT-2 and DCT-3 are inverses of each other.

and the corresponding inverse 2D DCT transform is $F(u,v)^{-1}$
where

$$\Lambda(\kappa) := \begin{cases} \frac{1}{\sqrt{2}} & for\ \kappa = 0 \\ 1 & otherwise \end{cases}$$

References:
https://users.cs.cf.ac.uk/Dave.Marshall/Multimedia/node231.html

The general equation for a 3D DCT

$$F(u,v,w) = \sqrt{\frac{2}{N}}\sqrt{\frac{2}{M}}\sqrt{\frac{2}{P}}\sum_{i=0}^{N-1}\sum_{j=0}^{M-1}\sum_{j=0}^{P-1}\Lambda(i)\Lambda(j)\Lambda(k)cos(\frac{\pi(2i+1)(u)}{2N})cos(\frac{\pi(2j+1)(v)}{2M})$$

$$cos(\frac{\pi(2k+1)(v)}{2P})f(i,j,k) \tag{10}$$

and the corresponding inverse 3D DCT transform is $F(u,v,w)^{-1}$
where

$$\Lambda(\xi) := \begin{cases} \frac{1}{\sqrt{2}} & for\ \xi = 0 \\ 1 & otherwise \end{cases}$$