

Mathematics of Discrete Cosine Transform (DCT)¹

DCT_I:

$$y(k) = \sqrt{\frac{2}{N}} \sum_{n=1}^N x(n) \frac{1}{\sqrt{1 + \delta_{n1} + \delta_{nN}}} \frac{1}{\sqrt{1 + \delta_{k1} + \delta_{kN}}} \cos\left(\frac{\pi(n-1)(k-1)}{N-1}\right) \quad (1)$$

DCT_II:

$$y(k) = \sqrt{\frac{2}{N}} \sum_{n=1}^N x(n) \frac{1}{\sqrt{1 + \delta_{k1}}} \cos\left(\frac{\pi(2n-1)(k-1)}{2N}\right) \quad (2)$$

DCT_III:

$$y(k) = \sqrt{\frac{2}{N}} \sum_{n=1}^N x(n) \frac{1}{\sqrt{1 + \delta_{n1}}} \cos\left(\frac{\pi(n-1)(2k-1)}{2N}\right) \quad (3)$$

DCT_IV:

$$y(k) = \sqrt{\frac{2}{N}} \sum_{n=1}^N x(n) \cos\left(\frac{\pi(2n-1)(2k-1)}{4N}\right) \quad (4)$$

References:

<https://uk.mathworks.com/help/signal/ref/dct.html>

The general equation for a 2D DCT

$$F(u, v) = \sqrt{\frac{2}{N}} \sqrt{\frac{2}{M}} \sum_{i=0}^{N-1} \sum_{j=0}^{M-1} \Lambda(i) \Lambda(j) \cos\left(\frac{\pi(2i+1)(u)}{2N}\right) \cos\left(\frac{\pi(2j+1)(v)}{2M}\right) f(i, j) \quad (5)$$

¹Implemented Mathematics

The series are indexed from n = 1 and k = 1 instead of the usual n = 0 and k = 0, because MATLAB® vectors run from 1 to N instead of from 0 to N - 1.

All variants of the DCT are unitary (or, equivalently, orthogonal): To find their inverses, switch k and n in each definition. In particular, DCT-1 and DCT-4 are their own inverses, and DCT-2 and DCT-3 are inverses of each other.

and the corresponding inverse 2D DCT transform is $F(u, v)^{-1}$ where

$$\Lambda(\kappa) := \begin{cases} \frac{1}{\sqrt{2}} & \text{for } \kappa = 0 \\ 1 & \text{otherwise} \end{cases}$$

References:

<https://users.cs.cf.ac.uk/Dave.Marshall/Multimedia/node231.html>