Mathematics of Discrete Sine Transform (DCT)[1]

DST_I:

$$y(k) = \sqrt{\frac{2}{N+1}} \sum_{n=0}^{N-1} x(n) sin(\frac{\pi(n+1)(k+1)}{N+1}) \tag{1}$$

DST_II:

$$y(k) = \sqrt{\frac{2-\delta_{kN}}{N}} \sum_{n=0}^{N-1} x(n) sin(\frac{\pi(n+\frac{1}{2})(k+1)}{N}) \tag{2}$$

DST_III:

$$y(k) = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} x(n) \frac{1}{\sqrt{1+\delta_{nN}}} sin(\frac{\pi(n+1)(k+\frac{1}{2})}{N}) \tag{3}$$

DST_IV:

$$y(k) = \sqrt{\frac{2}{N}} \sum_{n=0}^{N-1} x(n) sin(\frac{\pi(n+\frac{1}{2})(k+\frac{1}{2})}{N}) \tag{4}$$

References:
http://onlinelibrary.wiley.com/doi/10.1002/cta.447/pdf

The general equation for a 2D DST

References:
https://uk.mathworks.com/matlabcentral/fileexchange/49875-2d-discrete-sine-transform-theory

[1] Implemented Mathematics
The series are indexed from n = 0 and k = 0 to N-1.
All variants of the DCT are unitary (or, equivalently, orthogonal): To find their inverses, switch k and n in each definition. In particular, DST-1 and DST-4 are their own inverses, and DST-2 and DST-3 are inverses of each other.

2D discrete sine transform:

We assume that the length of the domain is equal in X and Y directions. Moreover, we assume that number of nodes are equal in both directions. The discrete sine transform can be written as

$$a_{mn} = \left(\frac{2}{l}\right)^2 \int_0^l \int_0^l u(x,y) \sin\left(\frac{\pi m x}{l}\right) \sin\left(\frac{\pi n y}{l}\right) dx dy$$

$$a_{mn} = \left(\frac{2}{l}\right)^2 \int_0^l \left[\sum_{i=1}^{N-1} u\big((i-1)\Delta x, y\big) \sin\left(\frac{\pi m (i-1)\Delta x}{l}\right) + u(i\Delta x, y) \sin\left(\frac{\pi m i \Delta x}{l}\right)\right] \frac{\Delta x}{2} \sin\left(\frac{\pi n y}{l}\right) dy$$

$$a_{mn} = \left(\frac{2}{N-1}\right)^2 \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} \left[\begin{array}{l} U(i,j) \sin\left(\frac{\pi m (i-1)}{N-1}\right) \sin\left(\frac{\pi n (j-1)}{N-1}\right) + U(i,j+1) \sin\left(\frac{\pi m (i-1)}{N-1}\right) \sin\left(\frac{\pi n j}{N-1}\right) \\ + U(i+1,j) \sin\left(\frac{\pi m i}{N-1}\right) \sin\left(\frac{\pi n (j-1)}{N-1}\right) + U(i+1,j+1) \sin\left(\frac{\pi m i}{N-1}\right) \sin\left(\frac{\pi n j}{N-1}\right) \end{array}\right]$$

2D inverse sine transform:

$$U(i,j) = \sum_{m=1}^{N-1} \sum_{n=1}^{N-1} a_{mn} \sin\left(\frac{\pi m (i-1)}{N-1}\right) \sin\left(\frac{\pi n (j-1)}{N-1}\right)$$