

$$(iii): x(k+1) = A_d x(k) + B_d u(k)$$

$\Rightarrow$  the characteristic equation is

$$\det(\lambda I - A_d) = 0$$

$$\Rightarrow \begin{vmatrix} \lambda - \frac{4}{3}e^{-\frac{1}{2}} + \frac{1}{3}e^{-2} & -\frac{1}{3}e^{-\frac{1}{2}} + \frac{1}{3}e^{-2} \\ \frac{4}{3}e^{-\frac{1}{2}} - \frac{4}{3}e^{-2} & \lambda + \frac{1}{3}e^{-\frac{1}{2}} - \frac{4}{3}e^{-2} \end{vmatrix} = 0$$

$$\Rightarrow \lambda^2 - (e^{-\frac{1}{2}} + e^{-2})\lambda + e^{-\frac{5}{2}} = 0$$

$$\Rightarrow (\lambda - e^{-\frac{1}{2}})(\lambda - e^{-2}) = 0$$

$$\Rightarrow \lambda_1 = e^{-\frac{1}{2}}, \lambda_2 = e^{-2}$$

So, the eigenvalues of  $A_d$  are  $\lambda_1 = e^{-\frac{1}{2}}$  and  $\lambda_2 = e^{-2}$