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Question 1:

(1): Solution:

$$G(z) = Z[G(s)] = Z\left[\frac{1-e^{-Ts}}{s} \frac{1}{s(s+10)}\right]$$

$$= (1-z^{-1}) Z\left[\frac{1}{s^2(s+10)}\right]$$

$$= \frac{1-z^{-1}}{100} Z\left[\frac{10}{s^2} - \frac{1}{s} + \frac{1}{s+10}\right]$$

$$= \frac{1-z^{-1}}{100}\left[\frac{10Tz^{-1}}{(1-z^{-1})^2} - \frac{1}{1-z^{-1}} + \frac{1}{1-e^{-10T}z^{-1}}\right]$$

The sampling period T is 0.1 second, so

$$G(z) = \frac{1-z^{-1}}{100}\left[\frac{z^{-1}}{(1-z^{-1})^2} - \frac{1}{1-z^{-1}} + \frac{1}{1-e^{-1}z^{-1}}\right]$$

$$= \frac{(1-2e^{-1})z^{-2} + e^{-1}z^{-1}}{100e^{-1}z^{-2} - 100(1+e^{-1})z^{-1} + 100}$$

$$= \frac{0.2642z^{-2} + 0.3679z^{-1}}{36.7879z^{-2} - 136.7879z^{-1} + 100}$$

Therefore, the pulse transfer function of the close-loop system is

$$\frac{C(z)}{R(z)} = \frac{KG(z)}{1+KG(z)} = \frac{K[(1-2e^{-1})z^{-2} + e^{-1}z^{-1}]}{[K(1-2e^{-1}) + 100e^{-1}]z^{-2} + [Ke^{-1} - 100(1+e^{-1})]z^{-1} + 100}$$

$$= \frac{0.2642Kz^{-2} + 0.3679Kz^{-1}}{(0.2642K + 36.7879)z^{-2} + (0.3679K - 136.7879)z^{-1} + 100}$$

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(2): Solution:

The characteristic equation is

$$P(z) = 100z^2 + [Ke^{-1} - 100(1+e^{-1})]z + K(1-2e^{-1}) + 100e^{-1}$$

So, we can apply Jury test to determine the range of K.

From the CE, we have

$a_0 = 100$, $a_1 = Ke^{-1} - 100(1+e^{-1})$, $a_2 = K(1-2e^{-1}) + 100e^{-1}$

If the close-loop system is stable, the following conditions should be satisfied:

$$\begin{cases} |a_2| < a_0 \\ P(z)|_{z=1} > 0 \\ P(z)|_{z=-1} > 0 \end{cases} \Rightarrow \begin{cases} |K(1-2e^{-1}) + 100e^{-1}| < 100 \\ 100 + Ke^{-1} - 100(1+e^{-1}) + K(1-2e^{-1}) + 100e^{-1} > 0 \\ K(1-2e^{-1}) + 100e^{-1} + 100 - Ke^{-1} + 100(1+e^{-1}) > 0 \end{cases}$$

$$\Rightarrow \begin{cases} -517.6634 < K < 239.2211 \\ K > 0 \\ K < 273.6795 \end{cases}$$

Therefore, the range of K for stability of the close-loop system is

$$0 < K < 239.2211$$

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(3): Solution:

When K=1, the transfer function of the open loop system is

$$G(z) = \frac{(1-2e^{-1})z^{-2} + e^{-1}z^{-1}}{100e^{-1}z^{-2} - 100(1+e^{-1})z^{-1} + 100}$$

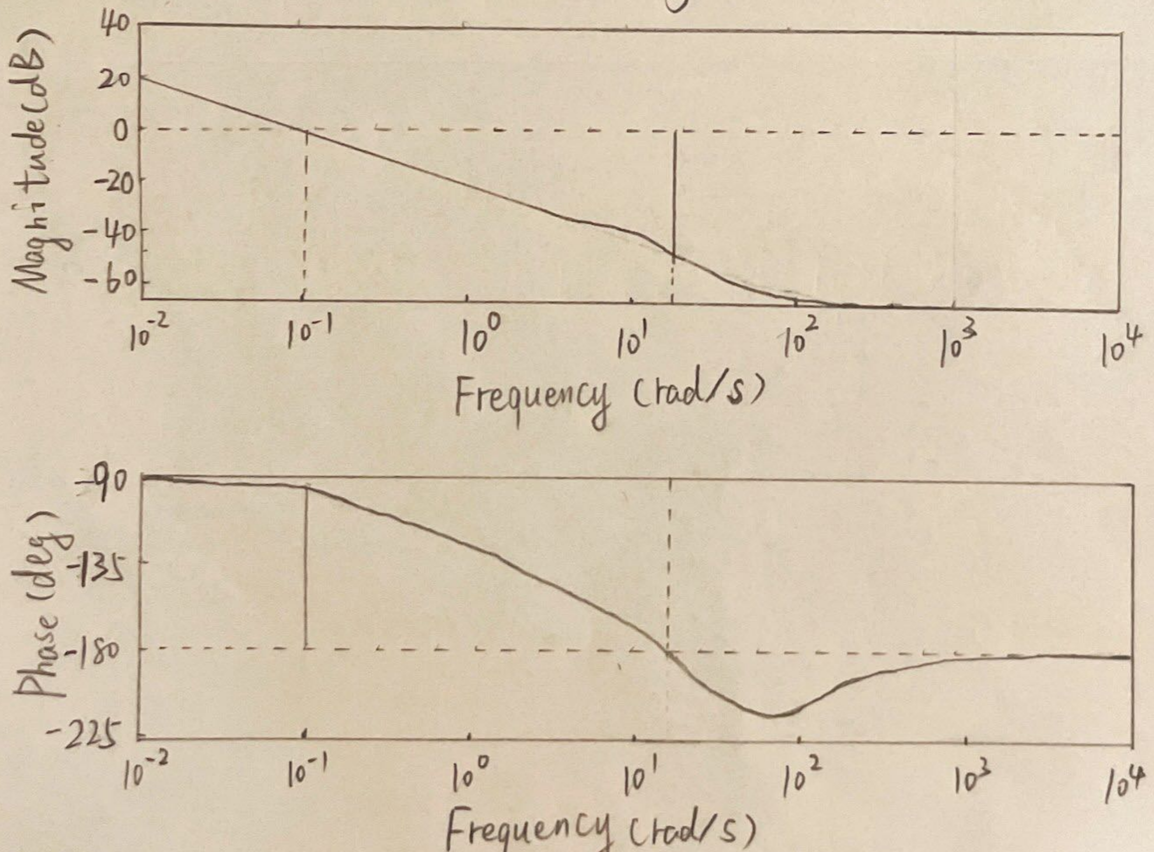
let $z = \left.\frac{1+\frac{wT}{2}}{1-\frac{wT}{2}}\right|_{T=0.1} = \frac{20+w}{20-w}$, we have

$$G(w) = \frac{(1-3e^{-1})w^2 - 40(1-2e^{-1})w + 400 - 400e^{-1}}{200(1+e^{-1})w^2 + (4000-4000e^{-1})w}$$

$$= \frac{-0.1036w^2 - 10.5696w + 252.8482}{273.5759w^2 + 2528.4822w}$$

Using Matlab, we can get the Bode Diagram.

Bode Diagram (K=1)



From Bode Diagram, ~~the~~ gain margin and phase margin can be determined.

Gain margin is 47.6 dB when $\omega = 15.6$ rad/s.

Phase margin is 89.1 deg when $\omega = 0.1$ rad/s.

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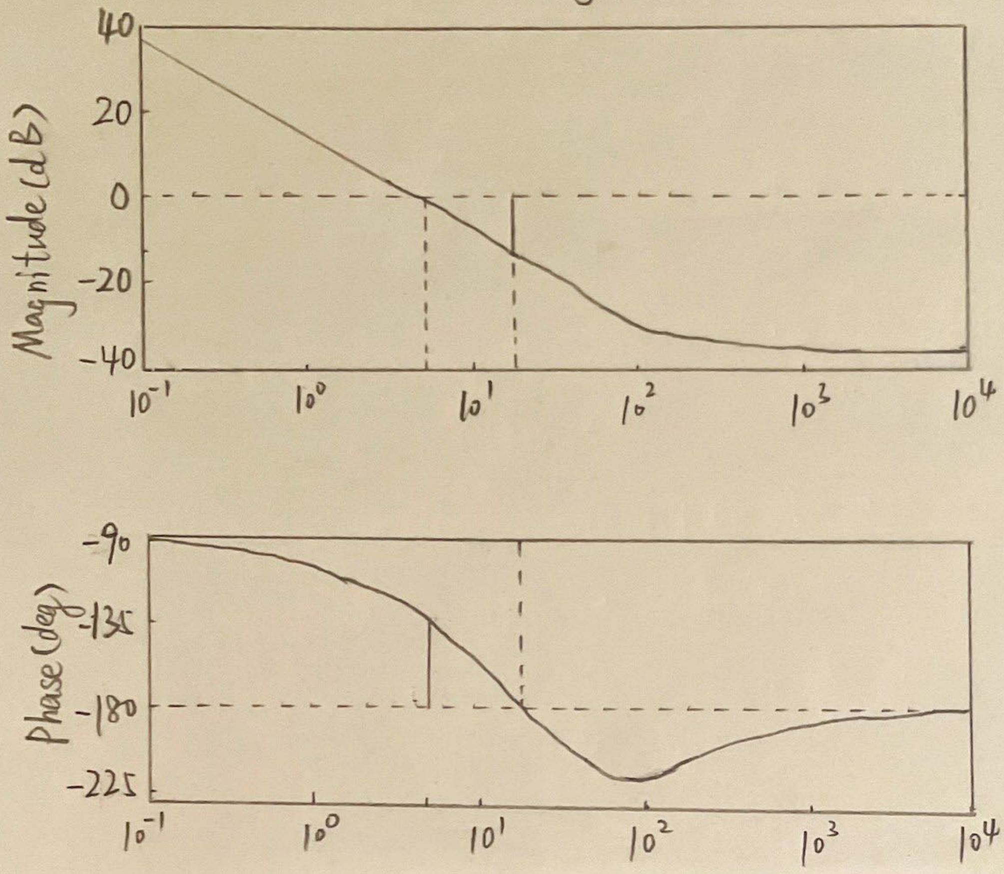
(4): Solution:

Bode Diagram (K=55)



From (3), we get the phase margin 89.1 deg when K=1. So, if we want to make phase margin decrease to 50 deg, K is need to increased.

When phase margin is 50 deg, the actual phase is $50° - 180° = -130°$.

According to the Bode Diagram, the corresponding ω is 5 rad/s and magnitude is -34.8 dB. Thus, we can get the equation

$$20 \log K = 34.8 \Rightarrow K = 55$$

From the Bode Diagram in (4), when K=55, the gain margin is 12.8 dB when ω is 15.6 rad/s