WU TIANWEI

Question 2:

Solution:

If $x_3(k) = u(k)$, we have

$$0.1x_3(k+1) + 0.2x_3(k) = 0.3e(k)$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0.4 & 0.5 \\ 0.6 & 0.7 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0.8 \\ 0.9 \end{bmatrix} x_3(k)$$

$$\Rightarrow x_1(k+1) = 0.4x_1(k) + 0.5x_2(k) + 0.8x_3(k)$$

$$x_2(k+1) = 0.6x_1(k) + 0.7x_2(k) + 0.9x_3(k)$$

$$x_3(k+1) = -2x_3(k) + 3e(k)$$

$$y(k) = \begin{bmatrix} 1 & 2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$\Rightarrow y(k) = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 0.4 & 0.5 & 0.8 \\ 0.6 & 0.7 & 0.9 \\ 0 & 0 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix}, \quad d = 0$$

$$\Rightarrow x(k+1) = \begin{bmatrix} 0.4 & 0.5 & 0.8 \\ 0.6 & 0.7 & 0.9 \\ 0 & 0 & -2 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix} e(k); \quad x(k) = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

$$y(k) = \begin{bmatrix} 1 & 2 & 0 \end{bmatrix} x(k)$$