WU TIANWEI

Question 5:

Solution:

(a): According to the system given, we have

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_2(t) = -\frac{2}{125}x_1(t) + \frac{1}{625}u(t)$$

$$y_1(t) = x_1(t)$$

$$\Rightarrow \dot{x}(t) = \begin{bmatrix} 0 & 1 \\ -\frac{2}{125} & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \frac{1}{625} \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

(b): $A = \begin{bmatrix} 0 & 1 \\ -\frac{2}{125} & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \frac{1}{625} \end{bmatrix}$, $C = [1\ 0]$, $d = 0$, $T = 0.5\,\text{sec}$

$$\Rightarrow \phi(t) = \mathcal{L}^{-1}\{[sI - A^{-1}]\} = \mathcal{L}^{-1}\left\{\begin{bmatrix} \frac{125s}{125s^2+2} & \frac{125}{125s^2+2} \\ -\frac{2}{125s^2+2} & \frac{125s}{125s^2+2} \end{bmatrix}\right\}$$

$$= \begin{bmatrix} \cos\frac{\sqrt{10}}{25}t & \frac{5}{2}\sqrt{10}\sin\frac{\sqrt{10}}{25}t \\ -\frac{\sqrt{10}}{25}\sin\frac{\sqrt{10}}{25}t & \cos\frac{\sqrt{10}}{25}t \end{bmatrix}$$

$$\Rightarrow \phi(T) = \phi(0.5) = \begin{bmatrix} 0.998 & 0.4997 \\ -0.008 & 0.998 \end{bmatrix} = A_d$$

$$\theta(T) = \left[\int_0^{0.5}\phi(\tau)d\tau\right]\begin{bmatrix} 0 \\ \frac{1}{625} \end{bmatrix} = \begin{bmatrix} 1.999\times10^{-4} \\ 7.995\times10^{-4} \end{bmatrix} = B_d$$

$\Rightarrow$ The ZOH equivalent of the continuous-time system is

$$x(k+1) = \begin{bmatrix} 0.998 & 0.4997 \\ -0.008 & 0.998 \end{bmatrix} x(k) + \begin{bmatrix} 1.999\times10^{-4} \\ 7.995\times10^{-4} \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$