WU TIANWEI

Question 4:

Solution:

(a): $x(k+1) = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix} x(k) + \begin{bmatrix} 3 \\ 4 \end{bmatrix} u(k)$

$y(k) = [5 \quad 6]\, x(k)$

$\Rightarrow A = \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}, B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}, C = [5 \quad 6], d = 0$

$\Rightarrow |\lambda I - A| = \lambda^2 - 3\lambda + 2 - 2 = \lambda^2 - 3\lambda$

$\Rightarrow a_0 = 0,\ a_1 = -3$

$\Rightarrow W_o = \begin{bmatrix} 5 & 6 \\ 11 & 22 \end{bmatrix}, A_o = \begin{bmatrix} 0 & 0 \\ 1 & 3 \end{bmatrix}, C_o = [0 \quad 1], d_o = d = 0$

$\Rightarrow \tilde{W}_o = \begin{bmatrix} 0 & 1 \\ 1 & 3 \end{bmatrix}$; $W_o^{-1} = \frac{1}{110-66} \begin{bmatrix} 22 & -6 \\ -11 & 5 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & -\frac{3}{22} \\ -\frac{1}{4} & \frac{5}{44} \end{bmatrix}$

$\Rightarrow Q = W_o^{-1} \tilde{W}_o = \begin{bmatrix} -\frac{3}{22} & \frac{1}{11} \\ \frac{5}{44} & \frac{1}{11} \end{bmatrix}$

$\Rightarrow Q^{-1} = \frac{1}{\frac{1}{11}(-\frac{3}{22} - \frac{5}{44})} \begin{bmatrix} \frac{1}{11} & -\frac{1}{11} \\ -\frac{5}{44} & -\frac{3}{22} \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ 5 & 6 \end{bmatrix}$

$\Rightarrow B_o = Q^{-1} B = \begin{bmatrix} -4 & 4 \\ 5 & 6 \end{bmatrix} \begin{bmatrix} 3 \\ 4 \end{bmatrix} = \begin{bmatrix} 4 \\ 39 \end{bmatrix}$

$\Rightarrow$ The state-space representation in the OCF form is

$w(k) = \begin{bmatrix} -4 & 4 \\ 5 & 6 \end{bmatrix} x(k)$

$w(k+1) = \begin{bmatrix} 0 & 0 \\ 1 & 3 \end{bmatrix} w(k) + \begin{bmatrix} 4 \\ 39 \end{bmatrix} u(k)$

$y(k) = [0 \quad 1]\, w(k)$