WU TIAN WEI

(b): From (a), we have

$$A=\begin{bmatrix}1 & 2\\ 1 & 2\end{bmatrix},\ B=\begin{bmatrix}3\\ 4\end{bmatrix},\ C=\begin{bmatrix}5 & 6\end{bmatrix},\ d=0$$

$$a_0=0,\ a_1=-3$$

$$\Rightarrow W_c=\begin{bmatrix}3 & 11\\ 4 & 11\end{bmatrix},\ A_c=\begin{bmatrix}0 & 1\\ 0 & 3\end{bmatrix},\ B_c=\begin{bmatrix}0\\ 1\end{bmatrix},\ d_c=d=0$$

$$\Rightarrow \tilde{W}_c=\begin{bmatrix}0 & 1\\ 1 & 3\end{bmatrix}\Rightarrow \tilde{W}_c^{-1}=-1\times\begin{bmatrix}3 & -1\\ -1 & 0\end{bmatrix}=\begin{bmatrix}-3 & 1\\ 1 & 0\end{bmatrix}$$

$$\Rightarrow P=W_c\tilde{W}_c^{-1}=\begin{bmatrix}3 & 11\\ 4 & 11\end{bmatrix}\begin{bmatrix}-3 & 1\\ 1 & 0\end{bmatrix}=\begin{bmatrix}2 & 3\\ -1 & 4\end{bmatrix}$$

$$\Rightarrow C_c=CP=\begin{bmatrix}5 & 6\end{bmatrix}\begin{bmatrix}2 & 3\\ -1 & 4\end{bmatrix}=\begin{bmatrix}4 & 39\end{bmatrix}$$

$\Rightarrow$ The state-space representation in the CCF form is

$$w(k)=P^{-1}x(k)=\begin{bmatrix}\frac{4}{11} & -\frac{3}{11}\\ \frac{1}{11} & \frac{2}{11}\end{bmatrix}x(k)$$

$$w(k+1)=\begin{bmatrix}0 & 1\\ 0 & 3\end{bmatrix}w(k)+\begin{bmatrix}0\\ 1\end{bmatrix}u(k)$$

$$y(k)=\begin{bmatrix}4 & 39\end{bmatrix}w(k)$$