A simple MPC example using CVX to motivate the EE6225 class

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Given a plant $G(z) = \dfrac{b(z)}{a(z)}$

```
clear
bz = [0.1]; az=[1 -0.9];
ts=0.1; %this is a discrete-time model, so sampling time is arbitary
Gz=tf(bz,az,ts)
```

```
Gz =

    0.1
  -------
  z - 0.9

Sample time: 0.1 seconds
Discrete-time transfer function.
```

It is convenient to use its state space representation

```
Gz = ss(Gz);
Ap=Gz.A; Bp=Gz.B; Cp=Gz.C; Dp=Gz.D;
nx=size(Ap,1);
```

Now, let's design a MPC for the plant above.

1. MPC needs a model of plant
2. Tuning parameters: N1, N2, Nu and Lambda
3. Form the prediction equations Y = Phi x_k + G U

```
Model = ss(Ap,Bp,Cp,Dp,ts);
A=Model.A; B=Model.B; C=Model.C;

N1=1; N2=3; Nu=1; Lambda=0.01;

Phi = [C*A; C*A^2; C*A^3];
G = [C*B; C*A*B; C*A^2*B];
```

Now, do the closed loop simulation

```
tsim = 20;
SetPt = [0 ones(1,tsim/2) zeros(1,tsim/2)];

for k=1:tsim

    if (mod(k,10)==0), k, end

    % measure the plant state and the set-point
    wk = SetPt(k);
    if k > 1
        xk = x;
    else
```

```
        xk = zeros(nx,1);
    end
    yk = C*xk;

    % Compute the MPC control signal
    %
    % For unconstrained MPC, we have closed-form
    % solution as follows:
    %W = wk*ones(size(Phi,1),1);
    %U = (G'*G+Lambda*eye(Nu,Nu))\G'*(W-Phi*xk);

    % Alternatively, use CVX which can solve
    % MPC with constraints, and other more general
    % formulations
    %
    [U,J] = MPC(wk,xk,Phi,G,N1,N2,Nu,Lambda);

    % Apply control u(k) to the plant
    uk = U(1);
    x = Ap*xk + Bp*uk;

    % Save the plant y, u and set-point for plotting later
    y_plot(k) = yk;
    u_plot(k) = uk;
    w_plot(k) = wk;
    J_plot(k) = J;
end
```

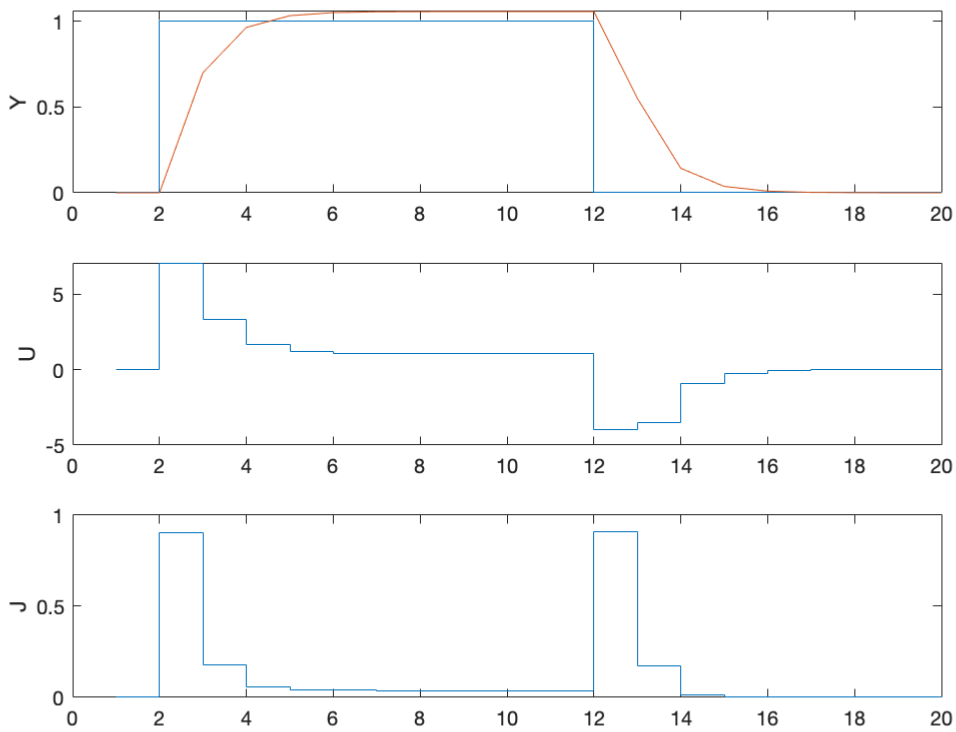
```
k = 10
k = 20
```

Let's look at the results, and notice the non-zero steady state error. But for w=0, there is no steady state error! Why?

```
subplot(311),stairs(w_plot),hold on,plot(y_plot)
subplot(312),stairs(u_plot)

subplot(3,1,1)
ylabel('Y')
subplot(3,1,2)
ylabel('U')

subplot(313),stairs(J_plot)
subplot(3,1,3)
ylabel('J')
```



What modification is needed to achieve zero steady state error?

Can you modify the code to

1. investigate the effect of MPC tuning parameters: N1, N2, Nu and Lambda
2. include constraints in the MPC design
3. include disturbance response (good for checking whether MPC or any controller has integral action or not)
4. investigate effects of modelling error (i.e., Plant and MPC Model not the same)

Feel free to add any other improvement to the code to impress your classmates ;-)

```
function [U,J] = MPC(wk,xk,Phi,G,N1,N2,Nu,Lambda)
    cvx_begin quiet
      variable U(Nu)
      Y = Phi*xk + G*U;
      W = wk*ones(size(Y,1),1);
      OBJ = (W-Y)'*(W-Y) + Lambda*U'*U;
      minimize(OBJ)

      % The following lines add constrains on U
      UMIN = -4*ones(size(U,1),1);
      UMAX = 7*ones(size(U,1),1);
      subject to
        UMIN <= U <= UMAX;
    cvx_end
    J = cvx_optval;
end
```