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Tuning of Multivariable PI Controllers by BLT Method for TITO Systems

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Abstract

The design of decentralized PI controller based on BLT tuning method is extended to design centralized PI controllers for TITO systems. The multivariable PI control structure proposed by Tanttu and Lieslehto (1991) is considered. The single detuning parameter (F) is selected in order to get the Biggest Log Modulus as 4dB. Decentralized PI controllers are also designed using the IMC to get the Biggest Log Modulus as 4dB. The centralized PI controllers give improved main responses and decreased interactions. This improvement is shown particularly for TITO systems having the relative gain element, $\lambda_{ij} < 1$. The robustness of these controllers is evaluated by the Inverse Maximum Singular Value versus Frequency plot for both the input and output multiplicative uncertainties. Simulation results are given for two TITO examples. The present controllers give better performances over that of the original Tanttu and Lieslehto method. The present decentralized PI controllers give improved performances over the decentralized PI controllers proposed by Xiong & Cai (2006).

KEYWORDS: Biggest Log Modulus, Centralized Controller, Decentralized Controller, IMC, TITO systems

INTRODUCTION

Multi input and multi output (MIMO) processes are more difficult to control when compared to the single-input single output (SISO) systems due to the existence of interactions between input and output variables. The most commonly used interaction measure is the Relative Gain Array (RGA) developed by Bristol (1966). The RGA method considers only the steady-state gains of the system and it gives a method to select a suitable pairing in the case of a decentralized control structure. The input-output pairing which has the relative gain element close to 1 should be considered. The input-output pairing that gives a negative RGA element should be avoided. Improved methods of pairing input and output variables considering the transient behaviour are given by McAvoy et al. (2003), Xiong et al. (2005), He et al. (2009)) and Monshizadh-Naini et al. (2009). The decentralized controllers work well if the interactions among the loops are only mild.

The centralized controllers are desirable for systems with significant interactions (Rajapandiyan & Chidambaram, 2012b; Vijay Kumar et al. 2012). In the literature, few methods are available to design a centralized control system. Reviews on the design of centralized PI controllers for a stable MIMO system are given by Tanttu and Lieslehto (1991), Maciejowski (1989), Wang et al. (2012) and Katebi (2012). Katebi (2012) has reviewed the simple methods of tuning centralized PI controllers for stable systems. These methods include Davison's method (Davison 1975) requiring only the steady state gain matrix (SSGM) and the Tanttu and Lieslehto method (Tanttu & Lieslehto 1991) requiring the transfer function matrix. More rigorous methods of designing multivariable

PI controllers are reviewed by Wang and Nie (2012). Shen et al. (2010) obtained a full controller matrix by the independent design approach. The controllers can be designed independently based on the gain and phase margin (GPM) method, RGA-based method (Vijay Kumar et al. 2012), RNGA-based method (Vijay Kumar et al. 2012), ERGA-based method (Xiong et al. 2007) and the analytical design method (Chen et al. 2013). In these methods, a suitable decoupler is designed first and for the resulting decoupled systems, suitable decentralised PI controllers are designed. The overall control scheme will result in centralized PI controllers with lead-lag filters. Vijay Kumar et al. (2012) have shown that the centralized control scheme works better for the systems having relative gain element less than 1. **Table *1*** shows the reported transfer function matrix for two input two output stable processes with the relative gain element less than 1.

Tanttu & Lieslehto (1991) have developed a method to tune the centralised PI controllers. First, a PI controller ($k_{c,ij}$) for each of the scalar transfer functions ($g_{P,ij}$) of the process is designed based on the IMC method. The same value of the tuning parameter (α) is used for all the controllers. Luyben (1986) has proposed a simple and practical design method called the biggest log modulus tuning (BLT) method, which is probably the most popular detuning technique (applied to multi loop PI control). Monica et al. (1988) have extended the BLT method for a multi-loop PID control systems. The methods use semi empirical functions and begin an iterative sequence with PI/PID controllers designed by Ziegler and Nichols' (ZN) settings based on the continuous cycling technique. These settings are then detuned by the factor F (typical value of F varies from 1 to 5 depending on the interactions). For interacting processes with, $\lambda_{ij}>1$, the detuning basically involves decreasing the control gains and increasing the integral times and decreasing derivative times.

Since the ZN settings give a large overshoot, the IMC method can be recommended to design the PI controllers. The BLT tuning method was originally proposed for the design of decentralised PI controllers. The simple structure for the centralized control system given by Tanttu and Lieslehto (1991) can be considered. In the centralized controllers proposed by Tanttu and Lieslehto (1991), only the controller gain alone is changed by tuning the value of α. Whereas in the present method, both the gains and the integral times of the controllers are tuned through the detuning factor F. The main objective of the present work is to extend the BLT tuning method meant for the decentralized control system (based on IMC rather than ZN) to design the centralized control systems based on the structure of Tanttu and Lieslehto method. The performance improvement of the centralized PI controllers over that of the decentralized PI controllers is to be evaluated along with the robustness of the control scheme.

MULTI VARIABLE SYSTEM

Consider a two-input two-output (TITO) multivariable system with time delay, $G_P(s)$ is process transfer function matrix and $G_c(s)$ is decentralized controller or centralized controller matrix. Generalized TITO transfer function matrix of the plant is given by:

$$G_P(s) = \begin{bmatrix} g_{P,11}(s) & g_{P,12}(s) \\ g_{P,21}(s) & g_{P,22}(s) \end{bmatrix} \tag{1}$$

Let each element of the process transfer function matrix be represented by a first-order plus dead time (FOPDT) stable model, i.e.,

$$g_{p,ij} = \frac{k_{p,ij}}{\tau_{ij} s + 1} e^{-\theta_{ij} s} \qquad (2)$$

The structure of the decentralized control system is given by:

$$G_C(s) = \begin{bmatrix} g_{C,1}(s) & 0 \\ 0 & g_{C,2}(s) \end{bmatrix} \qquad (3)$$

The structure of centralized control system is given by:

$$G_C(s) = \begin{bmatrix} g_{C,11}(s) & g_{C,12}(s) \\ g_{C,21}(s) & g_{C,22}(s) \end{bmatrix} \qquad (4)$$

The controller output and plant output are given by, $u_i=G_c e_i$ and $y_i=G_p u_i$, where $u_i (i=1,2)$ and $e_i=r_i-y_i$ $(i=1,2)$ are inputs to the plant and error signals to the controllers respectively.

CONTROL SYSTEM DESIGN

The TITO system is defined by

$$Y(s) = G_P(s) U(s) \qquad (5)$$

The present problem considers each transfer function $G_{P,ij}(s)$ as a first order plus time delay (FOPTD) system. For SISO systems, Rivera et al. (1986) and Morari & Zafiriou (1989) have discussed a tuning method based on an internal model control (IMC) principle. The first step in the proposed method is to calculate the IMC settings for each individual loop. The proportional gain (K_C), and the integral time (τ_I) for each diagonal transfer function of the process $G_{ii}(s)$ are calculated by the classical SISO method [Equation 0 and Equation 0].

PI controllers are designed using the IMC method as

$$\mathrm{K}_{\mathrm{C,ij\text{-}IMC}} = \frac{1}{k_{P,ij}}\left[\frac{\tau_{ij}}{\tau_{c,ij}+\theta_{ij}}\right] \tag{6}$$

$$\tau_{I,ij-IMC} = \tau_{ij} \tag{7}$$

$K_{c,ij}$ and $\tau_{I,ij}$ are the controller gain, closed loop integral time constant respectively, where $\tau_{c,ij}$=ατ_{ij},. For the Tanttu and Lieslehto method, α is the tuning parameter. In the proposed BLT method, it is assumed that α =1. Then the centralized PI settings can be designed by using the Equations 0 and 0. Hence the matrix K_c, for the centralised control system is given by (Tanttu & Lieslehto ,1991):

$$K_{C,ij-TL} = \begin{bmatrix} \dfrac{1}{K_{C,11-IMC}} & \dfrac{1}{K_{C,12-IMC}} \\ \dfrac{1}{K_{C,21-IMC}} & \dfrac{1}{K_{C,22-IMC}} \end{bmatrix}^{-1} \tag{8}$$

and

$$K_{I,ij-TL} = \begin{bmatrix} \dfrac{1}{K_{I,11-IMC}} & \dfrac{1}{K_{I,12-IMC}} \\ \dfrac{1}{K_{I,21-IMC}} & \dfrac{1}{K_{I,22-IMC}} \end{bmatrix}^{-1} \tag{9}$$

$$\tau_{I,ij-TL} = \left[\frac{K_{C,ij-TL}}{K_{I,ij-TL}}\right]_{2\times2} \tag{10}$$

The elements of the centralized controllers are assumed to be in the form of Equations 0 and 0, where K_C and τ_I, are matrices of size $n\times n$. $K_C=[k_{C,ij}]$ and $\tau_I=[\tau_{I,ij}]$, $(i,j=1,2)$. Here, $K_{C,ij}$ is the controller gain and $\tau_{I,ij}$ is the integral time for the loop ij.

BLT BASED DECENTRALIZED CONTROLLER DESIGN (LUYBEN & LUYBEN, 1997) USING IMC :

1. Calculate the IMC settings for each diagonal loop by assuming α=1 using equations (6) and (7).

2. A detuning factor F is assumed. For λ_{ij} <1, the value of F can be considered as 0.7. Other wise, a value of F=1.5 is recommended. The gains of diagonal feedback controllers $K_{c,ii}$ are calculated by dividing the IMC gains $K_{C,ii\text{-}IMC}$ by the factor F.

$$K_{C,ii} = \frac{K_{C,ii-IMC}}{F} \tag{11}$$

where $K_{C,ii\text{-}IMC}$ is Given in Equation 0. Then the integral time constants $\tau_{I,\mathrm{ii}}$ for the diagonal controllers are calculated by multiplying the IMC integral time constant $\tau_{I,ii\text{-}IMC}$ by the same factor *F*:

$$\tau_{I,ii} = \tau_{I,ii-IMC} \times F \tag{12}$$

where $\tau_{I,ii\text{-}IMC}$ is given in Equation 0.

3. Calculate the diagonal matrices of G_c and the corresponding multivariable Nyquist plot of the scalar function:

$$W(i\omega) = -1 + \det[I + G(i\omega)G_c(i\omega)] \tag{13}$$

We define a multivariable closed loop log modulus L_{cm}.

$$L_{cm}(i\omega) = 20\log_{10}\left|\frac{W}{1+W}\right| \tag{14}$$

4. The peak in the plot of L_{cm} over the entire frequency range is the biggest log modulus L_{cm}^{max}.

The F factor is varied (with an increment of 0.05) until L_{cm}^{max} is equal to $2N$, where N is the order of the system. For a $2 X 2$ system, a +4 dB value of L_{cm}^{max} is used.

PROPOSED BLT BASED CENTRALIZED CONTROLLER DESIGN

1. Calculate the individual PI controller settings by using Equations 0) & (7) with α=1.

2. A value for the detuning factor F is assumed. The gains K_{cij} and the integral time constants $\tau_{I,ij}$ of all the controllers are calculated as:

$$K_{C,ij} = \frac{K_{C,ij-IMC}}{F} \tag{15}$$

$$\tau_{C,ij} = \tau_{I,ij-IMC} \times F \tag{16}$$

where $K_{C,ii\text{-}IMC}$ and $\tau_{I,ij\text{-}IMC}$ are given by Equations (6) and (7)..

3. Calculate the multivariable controller settings by Equations (8) and (9) and the multivariable Nyquist plot of the function:

$$W(i\omega) = -1 + \det[I + G(i\omega)G_c(i\omega)] \tag{17}$$

We define a multivariable closed loop log modulus L_{cm}.

$$L_{cm}(i\omega) = 20\log_{10}\left|\frac{W}{1+W}\right| \tag{18}$$

The peak in the plot of L_{cm} over the entire frequency range is the biggest log modulus L_{cm}^{max}.

The F factor is varied (with an increment of 0.05) until L_{cm}^{max} is equal to $2N$, where N is the order of the system. For a $2 X 2$ system, a +4 dB value of L_{cm}^{max} is used.

It is to be noted that in the present method, α=1 and both the gains and the integral times of the controllers are tuned by F whereas in the centralised controllers proposed by Tanttu and Lieslehto (1991), only the controller's proportional gain alone is tuned by α. The relation between L_{cm} and the IMC tuning parameter (α) is given in the Appendix.

Performance Measure

To analyse the performance of the control systems, the following performance indices are used:

$$ISE = \int_0^\infty e(t)^2\,dt \tag{19}$$

$$IAE = \int_0^\infty \left|e(t)\right| dt \tag{20}$$

where, *e(s)=r(s)-u(s)*.

$$TV = \sum_{i=1}^{\infty} \left|u_i - u_{i-1}\right| \tag{21}$$

Total Variation (TV) is also a performance criterion for closed-loop response, but in contrast to the ISE and IAE in u, the objective for TV is to measure the total variation in the controller output signal, u. To evaluate the manipulated input usage we compute the total variation (TV) of the controller output u, which is sum of all its moves up and down. This provides a good measure of the smoothness of the control (Klan and Gorez, 2003).

Robust Stability Analysis

While the process model contains uncertainties in its parameters, the stability robustness of multi-loop control system becomes more important. Therefore, the maximum singular value uncertainty models are considered to demonstrate the stability robustness of the proposed control system. First, for a process multiplicative input uncertainty, $G(s)\left[I+\Delta_I(s)\right]$, the closed loop system is stable if (Maciejowski, 1989):

$$\left\|\Delta_I(j\omega)\right\| < \frac{1}{\overline{\sigma}}\left\{\left[I+G_C(j\omega)G(j\omega)\right]^{-1}G_C(j\omega)G(j\omega)\right\} \tag{22}$$

where, $\overline{\sigma}$ is maximum singular value of the closed loop system. For the process output uncertainty, $\left[I+\Delta_O(s)\right]G(s)$, the stability robustness of the closed-loop system can be obtained by (Maciejowski 1989). For the multiplicative output uncertainty, the closed-loop system is stable if

$$\left\|\Delta_O(j\omega)\right\| < \frac{1}{\overline{\sigma}}\left\{\left[I+G(j\omega)G_C(j\omega)\right]^{-1}G(j\omega)G_C(j\omega)\right\} \tag{23}$$

Where $\Delta_I(s)$ and $\Delta_O(s)$ are stable. The frequency plots obtained for the right-hand side part of Equation 0 and 0 indicate the stability bounds of the closed-loop system. The area under the curve represents the stability of the system. More area under the curve indicates the high stability of system. By using this plot, it is easy to compare the stability of the controllers. The control system which gives the maximum area under the curve is the most stable one.

SIMULATION STUDIES

Two *2 × 2* systems have been used to test the adequacy of this proposed method. In the following sections, we will present the results on a simulated ISP control system (Chien et al. 1999) and Cai-control problem (Xiong & Cai 2006) which have λ_{ij} <1.

***Example 1*:** The ISP Reactor transfer function matrix (Chien et al. 1999) is given as:

$$G(s)=\begin{bmatrix}\dfrac{22.89e^{-0.2s}}{4.572s+1} & \dfrac{-11.64e^{-0.4s}}{1.807s+1}\\ \dfrac{4.687e^{-0.2s}}{2.174s+1} & \dfrac{5.80e^{-0.4s}}{1.801s+1}\end{bmatrix} \tag{24}$$

The Relative Gain Array (Λ) can be calculated as

$$\wedge=\begin{bmatrix}0.7087 & 0.2913\\ 0.2913 & 0.7087\end{bmatrix} \tag{25}$$

The tuning procedure explained in the above Section is followed and the resulted BLT based centralized and decentralized PI tuning parameters are given in **Table *II*.** Since the relative gain element is less than 1, the detuning factor (F) is found to be less than 1.

The closed-loop system frequency responses for the decentralized and the centralized control systems are shown in Figure 1. Figure 2 shows the closed-loop servo responses and interactions for step change in yr_1, and the figure also shows the corresponding control actions in u1 and u2. Figure 2 also shows the closed-loop servo responses and interactions for step change in yr_2 and the control action. Though both the control systems are tuned for Biggest Log modulus as 4 dB, the frequency at which this occurs is different for these control schemes. Figure 3 shows the closed-loop regulatory responses for a step change given in v_1 and the corresponding control actions (u_1 and u_2). Figure 3 also shows the closed-loop regulatory responses for step change in v_2, and the corresponding control actions (u_1 and u_2) for step change in v_2. The Figures 2 and 3 show that the centralized PI controllers give improved performance when compared to that of the decentralized PI controllers. **Table *III*** shows that the sum of the ISE, IAE and TV values for the main responses and the interactions are lesser for the centralised control scheme.

As stated earlier, the frequency plot obtained for the right hand side part of Equation 0 and Equation 0 indicates the stability bounds of the closed loop system. Figure 4 shows the stability bounds for the Example 1. In this Figure, the region below the curve represents the stability region and above the curve represents instability region. From Figure 4, we can see that both the control schemes show almost same stability region. As stated earlier, the performance of the centralized control system is better than that of the decentralised control system.

It is desirable to compare the performance of the proposed method with that of the Tanttu and Lieslehto (TL) method for the centralized controller scheme. As stated earlier, in the TL method, the individual proportional gain of the controllers are tuned using the same value of tuning parameter α, whereas the integral times are kept as constants. The tuning parameter is varied from 0.5 to 1.5 in step of 0.5 and it is found that good performance is obtained if α=0.5 is used. The stability bounds are found to be same as that of the proposed centralised controllers (similar to Figure 4). The ISE performance indices are tabulated in ***Table V*** for the proposed centralized and TL centralized control systems. The present method gives improved performances (better responses and lesser interactions). The performance of the method is evaluated and the results are given in Table 3. The present tuning method gives the better performances than that of the Tanttu and Lieslehto method (based on tuning only the controllers gain and keeping the integral times as constants).

Selecting the closed loop time constant equal to its individual open loop time constant (i.e., α=1) is a reasonable choice; because a compromise is to be made between the improved responses and the increased interaction with decreased α. The proposed controllers' settings with α=1 show reduced interactions and improved responses. To check that the robustness of the decentralized and centralized controllers is not different, the robustness analysis is carried out as discussed. Since there is no tuning parameter selected other than F in the BLT method, robustness analysis to tune the value of α may not be required. For this example, with α=1, the detuning factor F by BLT method is obtained as F=0.573 and by using α=1.5, the value of F obtained by the BLT method is 0.518. The centralised and the decentralised PI controller settings are calculated and the performances are evaluated and compared. The performances are shown in Fig 5. The improved performances and reduced interactions similar to α=1.0 (similar to Fig 2) are obtained.

***Example 2*:** Consider the example reported by Xiong & Cai (2006):

$$G_P(s) = \begin{bmatrix} \dfrac{2.5e^{-5s}}{15s+1} & \dfrac{5e^{-3s}}{4s+1} \\ \dfrac{1e^{-4s}}{5s+1} & \dfrac{-4e^{-6s}}{20s+1} \end{bmatrix} \tag{26}$$

Notice that in this case the interactions are such that $\lambda_{ij} < 1$. The Relative Gain Array (Λ) can be calculated as

$$\wedge = \begin{bmatrix} 0.6667 & 0.3333 \\ 0.3333 & 0.6667 \end{bmatrix} \tag{27}$$

The parameters of the resulted BLT based centralized and decentralized PI controllers are tabulated in **Table *II*.** The closed-loop system frequency responses for the decentralized and the centralized control systems are shown in Figure 6. Though both the control systems are tuned for Biggest Log modulus as 4 dB, the frequency at which this occurs is different for these control schemes.

Figure 6 shows the closed-loop servo responses and interactions for a step change in yr_1 and the manipulated variable versus time behaviour (in u_1 and u_2). Figure 7 also shows the closed-loop servo responses and interactions for step a change in yr_2 and the manipulated variable versus time behaviour. Figure 7 shows the closed-loop regulatory responses for a step change given in v_1 and the manipulated variable versus time behaviour. Figure 8 also shows the closed-loop regulatory responses for a step change in v_2 and the corresponding control actions in u_1 and u_2. Both the servo and regulatory performances are better for the centralized control scheme when compared with the decentralized control scheme. **Table IV** shows that the sum of the ISE, IAE and TV values for the main responses and the interactions are lesser for the centralised control scheme. As stated earlier, the frequency plot obtained for the right hand side part of Equation 0 and Equation 0 indicates the stability bounds of the closed loop system. Figure 8 shows stability bounds for the Example Problem 2. In this Figure, the region below the curve represents the stability region and above the curve represents instability region. From Figure 9, we can see that both the control schemes show almost same stability region. As stated earlier, the performance of the centralized control system is better than that of the decentralised control system.

As stated earlier, it is desirable to compare the performance of the proposed method with that of the Tanttu and Lieslehto (TL) method for the centralized controller scheme. The tuning parameter is varied from 0.5 to 1.5 in step of 0.5 and it is found that a good performance is obtained if α=1.0 is used. The ISE performance indices are tabulated in ***Table V*** for the proposed centralized and TL centralized control systems. The present method gives improved performances (better responses and lesser interactions).

***Comparison Of The BLT Method With That Of* Xiong And Cai**

For this example, the performance of the proposed decentralized PI controllers is compared with that of the decentralized PI controllers reported by Xiong and Cai (2006). The decentralized PI controller parameters are given as ($kc_1=0.0233$, $kc_2=0.1094$) and ($\tau_{I1}=4$, $\tau_{I2}=5$). The ISE, IAE and TV values of the proposed BLT based decentralized controller and the reported decentralized controller are given in the **Table IV**. The servo responses, interactions and control actions are shown in Figure 7 , and the regulatory responses and control action are shown in Figure 8. The proposed method gives improved responses with lesser interactions.

CONCLUSIONS

The centralized PI controllers, based on the control structure given by Tanttu and Lieslehto, are designed by selecting the detuning parameter (F) so as to get the Biggest Log Modulus as 4dB. The simulation applications are given for two TITO examples (with the relative gain element λ_{ij}<1). The proposed centralized PI controllers give improved responses and decreased interactions when compared to that of the decentralised PI controllers. The centralised control scheme shows an improvement in the main responses about 76% and reduces the interaction by 26%. Similar improvement is also obtained for the regulatory problems. Though both the centralized and decentralized PI controllers are designed to get the Biggest Log Modulus as 4dB, the centralised scheme gives better performances since the biggest log modulus occurs at a different frequency value. The proposed centralized PI controllers based on IMC-BLT method with the tuning parameter F gives better performances when compared to that of the original Tanttu and Lieslehto method with the tuning parameter α.

APPENDIX : RELATION BETWEEN THE LOG MODULUS AND IMC TUNING PARAMETER

We consider a PI controller:

$$G_c(s) = k_c\left(1+\frac{1}{\tau_I s}\right) = k_c + \frac{k_I}{s} \tag{A1}$$

PI controller settings by Tanttu and Lielehto method (TL-Controller) are given by:

$$k_c = \begin{bmatrix} \frac{1}{k_{c,11}} & \frac{1}{k_{c,12}} \\ \frac{1}{k_{c,21}} & \frac{1}{k_{c,22}} \end{bmatrix}^{-1} \text{ and } k_I = \begin{bmatrix} \frac{1}{k_{I,11}} & \frac{1}{k_{I,12}} \\ \frac{1}{k_{I,21}} & \frac{1}{k_{I,22}} \end{bmatrix}^{-1} \tag{A2}$$

Individual IMC PI controller settings are given by:

$$k_{c,ij} = \frac{\tau_{ij}}{k_{p,ij}(\alpha\tau_{ij}+\theta_{ij})}; \qquad k_{I,ij} = \frac{k_{c,ij}}{\tau_{ij}} = \frac{1}{k_{p,ij(\alpha\tau_{ij}+\theta_{ij})}} \tag{A3}$$

Substituting Eq (A3) in Eq (A2) and after simplification we get:

$$k_c = \begin{bmatrix} \dfrac{k_{p,22}(\tau_{11}\tau_{12}\tau_{21})(\theta_{22}+\alpha\tau_{22})}{\sigma_1} & \dfrac{-k_{p,11}(\tau_{11}\tau_{21}\tau_{22})(\theta_{12}+\alpha\tau_{12})}{\sigma_1} \\ \dfrac{-k_{p,21}(\tau_{11}\tau_{12}\tau_{22})(\theta_{21}+\alpha\tau_{21})}{\sigma_1} & \dfrac{k_{p,11}(\tau_{12}\tau_{21}\tau_{22})(\theta_{11}+\alpha\tau_{11})}{\sigma_1} \end{bmatrix} \tag{A4}$$

$$k_I = \begin{bmatrix} \dfrac{k_{p,22}(\theta_{22}+\alpha\tau_{22})}{\sigma_2} & \dfrac{-k_{p,11}(\theta_{12}+\alpha\tau_{12})}{\sigma_2} \\ \dfrac{-k_{p,21}(\theta_{21}+\alpha\tau_{21})}{\sigma_2} & \dfrac{k_{p,11}(\theta_{11}+\alpha\tau_{11})}{\sigma_2} \end{bmatrix} \tag{A5}$$

where

$$\sigma_1 = \begin{bmatrix} k_{p,11}k_{p,22}(\tau_{12}\tau_{21})\theta_{11}\theta_{22} \\ -k_{p,12}k_{p,21}(\tau_{11}\tau_{22})\theta_{12}\theta_{21} \end{bmatrix} + \alpha \begin{bmatrix} k_{p,11}k_{p,22}(\tau_{12}\tau_{21})(\tau_{11}\theta_{22}+\tau_{22}\theta_{11}) \\ -k_{p,12}k_{p,21}(\tau_{11}\tau_{22})(\tau_{12}\theta_{21}+\tau_{21}\theta_{12}) \end{bmatrix}$$
$$+\alpha^2\left[\tau_{11}\tau_{12}\tau_{21}\tau_{22}(k_{p,11}k_{p,22}-k_{p,12}k_{p,21})\right]$$

$$\sigma_2 = \begin{bmatrix} k_{p,11}k_{p,22}\theta_{11}\theta_{22} \\ -k_{p,12}k_{p,21}\theta_{12}\theta_{21} \end{bmatrix} + \alpha \begin{bmatrix} k_{p,11}k_{p,22}(\tau_{11}\theta_{22}+\tau_{22}\theta_{11}) \\ -k_{p,12}k_{p,21}(\tau_{12}\theta_{21}+\tau_{21}\theta_{12}) \end{bmatrix} + \alpha^2 \begin{bmatrix} k_{p,11}k_{p,22}\tau_{11}\tau_{22} \\ -k_{p,12}k_{p,21}\tau_{12}\tau_{21} \end{bmatrix}$$

Case 1: Ideal Controller

$$G_c(i\omega) = \frac{1}{\lambda s}G_p^{-1} = \frac{1}{\lambda(i\omega)}G_p^{-1}(i\omega) \tag{A8}$$

$$W(i\omega) = -1+\det[I+\frac{I}{\lambda(i\omega)}] \tag{A9}$$

Case 2: Proportional Controller

$$G_c(s) = \frac{k_c}{F} \tag{A10}$$

$$W(i\omega) = -1+\det[I+G_p(i\omega)\frac{k_c}{F}] \tag{A11}$$

Case 3: Proportional Integral Controller

$$G_c(i\omega) = \frac{k_c}{F}+\frac{k_I}{F^2(i\omega)} \tag{A12}$$

$$W(i\omega) = -1 + \det[I + G_p(i\omega)\left(\frac{k_c}{F} + \frac{k_I}{F^2(i\omega)}\right)] \quad \text{(A13)}$$

Closed loop log modules equation

$$L_{cm}(i\omega) = 20\log_{10}\left|\frac{W}{1+W}\right| \quad \text{(A14)}$$

Substituting Eqs (A9),(A11) &(A13) in Eq (A14), we get the relationship between closed loop log modulus and detuning factor *F*, and the IMC tuning parameter *α*. Here k_c, k_I given by equations (A4) & (A5) are function of *α*. For a assumed value of α (say 1 to 2), the suitable value of F will be given by the BLT tuning method to get L_{cm} as 2N.

NOMENCLATURE

BLT	Biggest Log Modulus
e_i	Error signal to the controller
F	Detuning factor
$G_{C,II}$	Decentralized controller Transfer Function Matrix
$G_{C,IJ}$	Centralized controller Transfer Function Matrix
G_P	Process Transfer Function Matrix
ISE	Integral of the squared error
IMSV	Inverse Maximum Singular Value
IAE	Integral Absolute Error
k_p	Process steady state gain
$K_{C,IMC}$	Internal Model Control IMC gain
$K_{C,IJ\text{-}TL}$	Tanttu & Lieslehto controller Gain
$K_{C,ij}$	Centralized Controller Gain

$K_{C,ii}$	Decentralized Controller Gain
L_c^{max}	Maximum Closed loop Log Modulus
MIMO	Multi-input Multi-output
MU	Multiplicative Uncertainty
PI	Proportional integral
RGA	Relative Gain Array
RNGA	Relative normalized gain array
s	Laplace variable
SISO	Single Input Single Output
SSGM	Steady state gain matrix
TV	Total Variation
t	Time
τ_{ij}	Process time constant
TITO	Two input − two output
$\tau_{I,ij\text{-}TL}$	Tanttu & Lieslehto controller's Integral time constant
$\tau_{I,ij}$	Centralized Controller integral Time Constant
$\tau_{I,IMC}$	IMC Time Constant
$\tau_{I,i\text{-}IMC}$	Decentralized Controller Time Constant
u_i	Manipulated variable
v_i	Load variable
y_i	System output
y_{ri}	Reference inputs,
Λ	Relative gain array

Λ_{ij} Relative gain array element

θ_{ij} Process time delay

Δ_o Output uncertainty

Δ_I Input uncertainty

$\bar{\sigma}$ Maximum singular value

Subscripts

i, j: loop representation

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Table I. List of TITO Transfer functions with each element of $\lambda_{ij} < 1$

S No	Process	Transfer Function	RGA
1.	Reactive Ion Etcher(Rashap et al. 1995);(Maji et al. 2006)	$\begin{bmatrix} \frac{16e^{-0.5s}}{5.88s+1} & \frac{0.355}{0.8s+1} \\ \frac{-2.047e^{-0.5s}}{4.76s+1} & \frac{0.024}{0.202s+1} \end{bmatrix}$	$\begin{bmatrix} 0.3457 & 0.6543 \\ 0.6543 & 0.3457 \end{bmatrix}$
2.	Assumed TFM by (Lin et al. 2009); (Rajapandiyan & Chidambaram 2012a)	$\begin{bmatrix} \frac{7e^{-5s}}{10s+1} & \frac{4}{10s+1} \\ \frac{4}{10s+1} & \frac{-6e^{-10s}}{20s+1} \end{bmatrix}$	$\begin{bmatrix} 0.7241 & 0.2759 \\ 0.2759 & 0.7241 \end{bmatrix}$
3.	Assumed TFM by (Mc Avoy et al. 2003)(He et al. 2009)	$\begin{bmatrix} \frac{5e^{-40s}}{100s+1} & \frac{e^{-4s}}{10s+1} \\ \frac{-5e^{-40s}}{10s+1} & \frac{5e^{-40s}}{100s+1} \end{bmatrix}$	$\begin{bmatrix} 0.8333 & 0.1667 \\ 0.1667 & 0.8333 \end{bmatrix}$
4.	Assumed TFM by (Grosdidier & Morari 1986)	$\begin{bmatrix} \frac{5}{4s+1} & \frac{2.5e^{-5s}}{4s+1} \\ \frac{-4e^{-6s}}{20s+1} & \frac{1}{3s+1} \end{bmatrix}$	$\begin{bmatrix} 0.3333 & 0.6667 \\ 0.6667 & 0.3333 \end{bmatrix}$
5.	Assumed TFM by (Rajapandiyan & Chidambaram 2012a) (He et al. 2009)	$\begin{bmatrix} \frac{1}{10s+1} & \frac{0.6}{10s+1} \\ \frac{-0.6}{s+1} & \frac{1}{10s+1} \end{bmatrix}$	$\begin{bmatrix} 0.7353 & 0.2647 \\ 0.2647 & 0.7353 \end{bmatrix}$

Table II. Controller parameters for all three case studies

		BLT-Decentralized Controller (B-D)	BLT Centralized controller (B-C)
Example 1	k_c	$\begin{bmatrix} 0.0577 & 0 \\ 0 & 0.1944 \end{bmatrix}$ (F=0.7260)	$\begin{bmatrix} 0.0511 & 0.1026 \\ -0.0370 & 0.1720 \end{bmatrix}$ (F=0.5733)
	τ_I	$\begin{bmatrix} 3.3192 & 0 \\ 0 & 1.3075 \end{bmatrix}$	$\begin{bmatrix} 2.2015 & 2.1952 \\ 1.8242 & 0.8711 \end{bmatrix}$
Example 2	k_c	$\begin{bmatrix} 0.3074 & 0 \\ 0 & -0.1970 \end{bmatrix}$ (F=0.9760)	$\begin{bmatrix} 0.2281 & 0.3838 \\ 0.0789 & -0.1463 \end{bmatrix}$ (F=0.6891)
	τ_I	$\begin{bmatrix} 14.6396 & 0 \\ 0 & 19.5195 \end{bmatrix}$	$\begin{bmatrix} 5.7438 & 28.7892 \\ 23.0071 & 7.6704 \end{bmatrix}$

Table III. Performance comparisons for Example 1.

Step change in	Error	ISE			IAE			TV		
	Error in	y1	y2	∑(y1+y2)	y1	y2	∑(y1+y2)	u1	u2	∑(u1+u2)
yr_1	B-D	1.303	0.01835	1.3214	2.017	0.2986	2.3156	0.0366	0.057	0.0936
	B-C	1.066	0.00047	1.0665	1.926	0.04539	1.9714	0.0297	0.0332	0.0629
yr_2	B-D	3.893	0.9868	4.8798	4.275	1.781	6.056	0.1614	0.2535	0.4149
	B-C	1.036	0.788	1.824	2.441	1.288	3.729	0.0597	0.248	0.3077
v1	B-D	367.2	3.886	371.086	57.54	4.98	62.52	1.0846	1.0009	2.0855
	B-C	183.8	3.51	187.310	37.34	3.681	41.021	1.1089	0.5543	1.6632
v2	B-D	30.35	14.61	44.960	13.77	8.048	21.818	0.6858	1.464	2.1498
	B-C	27.36	6.798	34.158	9.377	4.673	14.05	0.0016	1.3167	1.3183

(B-D: BLT Decentralized, B-C: BLT Centralized Controllers)

Table IV. Performance comparisons for Example 2.

Step change in	Error in	ISE			IAE			TV		
		y1	y2	∑(y1+y2)	y1	y2	∑(y1+y2)	u1	u2	∑(u1+u2)
yr_1	R-D	14.39	1.966	16.356	23.79	10.58	34.37	0.3509	0.1078	0.4587
	B-D	10.23	1.466	11.696	14.29	6.604	20.894	0.3779	0.1394	0.5173
	B-C	7.173	0.4524	7.6254	12.17	4.349	16.51	0.2519	0.0611	0.313
yr_2	R-D	13.28	16.42	29.7	26.89	24.52	51.41	0.5443	0.1598	0.7041
	B-D	12.5	11.76	24.26	17.32	16.52	33.84	0.7481	0.2206	0.9687
	B-C	2.571	8.831	11.004	8.723	14.21	22.933	0.4064	0.1239	0.5303
v1	R-D	93.54	13.48	107.02	70.7	30.54	101.24	1.4312	0.3586	1.7898
	B-D	60.94	7.21	68.15	47.61	18.22	65.83	1.3307	0.4359	1.7666
	B-C	25.11	4.94	30.05	29.8	13.55	43.35	1.2278	0.179	1.4068
v2	R-D	358	298.	657.5	155	126	282.5	3.44	1.25	4.6988

		.9	6		.8	.7		51	37	
	B-D	201 .8	185. 4	387.2	94. 24	98. 84	193.08	3.86 96	1.20 61	5.0757
	B-C	122 .1	86.5 2	208.62	67. 52	59. 25	126.77	2.65 5	1.14 61	3.8011

(R-D: Reported Decentralized, B-D: BLT Decentralized, B-C: BLT Centralized Controllers)

Table V. ISE Performance comparisons for BLT centralized controller vs TL centralized Controller.

		Example 1			Example 2		
Step change in	Error	ISE			ISE		
	Error in	y1	y2	∑(y1+y2)	y1	y2	∑(y1+y2)
yr1	T-C	1.183	0.0003	1.1833	8.671	0.3638	9.0348
	B-C	0.618	0.001972	0.6209	7.173	0.4524	7.6254
yr2	T-C	0.5118	0.8401	1.3519	2.084	22.72	24.804
	B-C	0.1071	0.5568	0.6639	2.571	8.831	11.004
v1	T-C	261.3	5.471	266.771	40.92	27.22	68.14
	B-C	56.04	1.617	57.657	25.11	4.94	30.05
v2	T-C	43.43	10.78	54.21	154	150.4	304.4
	B-C	19.69	4.898	24.588	122.1	86.52	208.62

(T-C: TL Centralized, B-C: BLT Centralized Controllers)

Figure 1. Closed loop log modulus curves of the BLT based Decentralized and Centralized control systems for Example 1 (solid: centralized; dash: decentralized)

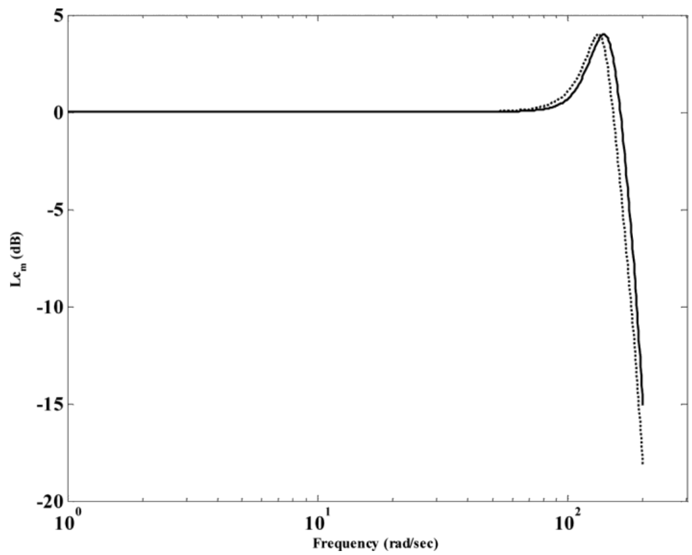

Figure 2. Servo responses (left), control action (right) for the Example 1 (solid: centralized; dash: decentralized)

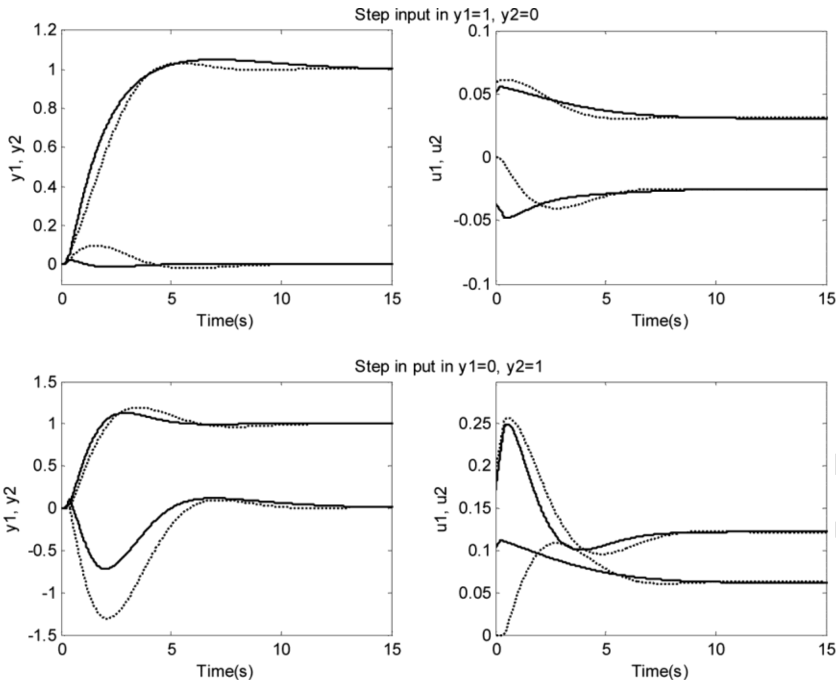

Figure 3. Regulatory responses (left), control action (right) for the Example 1(solid: centralized; dash: decentralized)

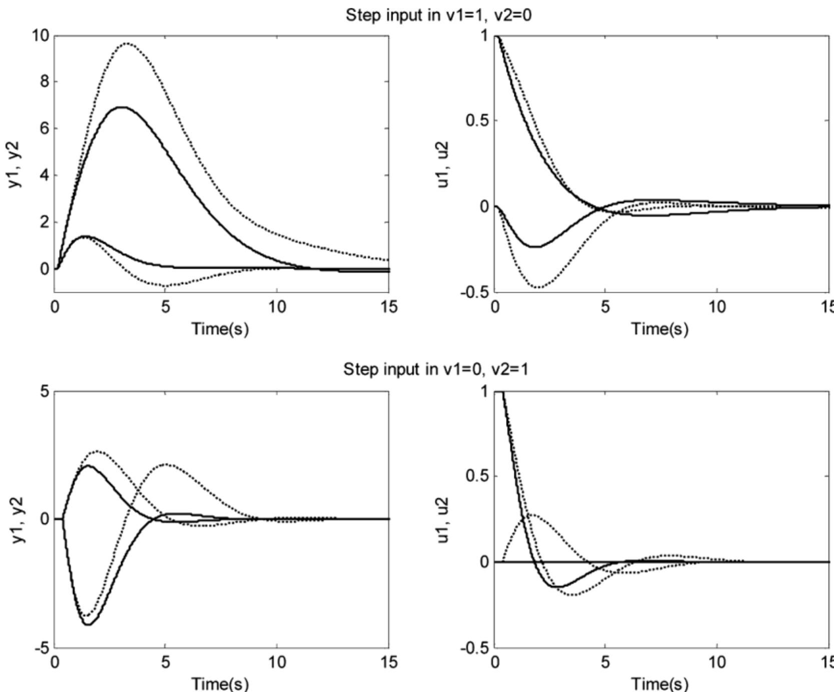

Figure 4. Stability regions of input and output uncertainties for Example 1. (solid: centralized; dash: decentralized)

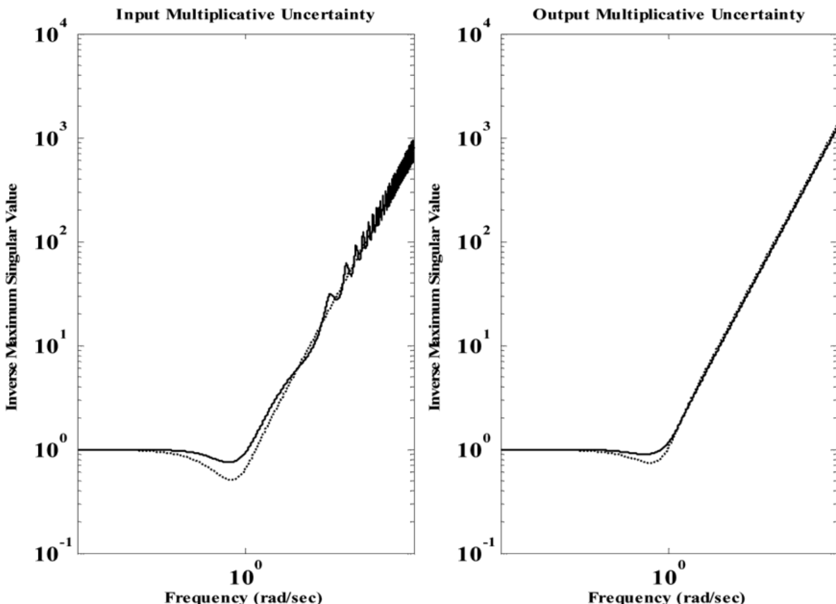

Figure 5: Servo responses (left), control action (right) for the Example 1 (solid: centralized; dash: decentralized) [α=1.5]

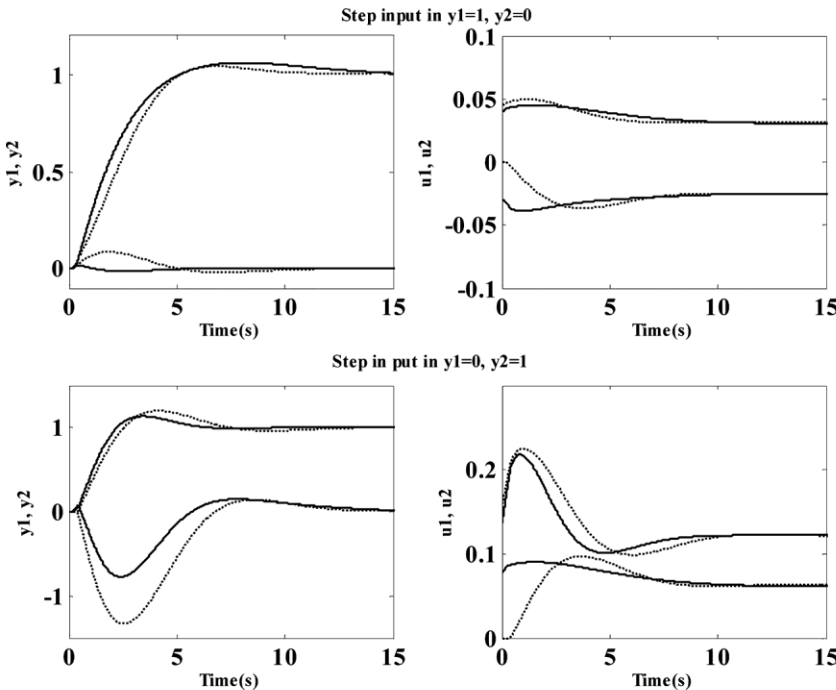

Figure 6: Closed loop log modulus curves of the BLT based Decentralized and Centralized control systems for Example 2(solid: centralized; dash: decentralized; solid-dash: reported decentralized)

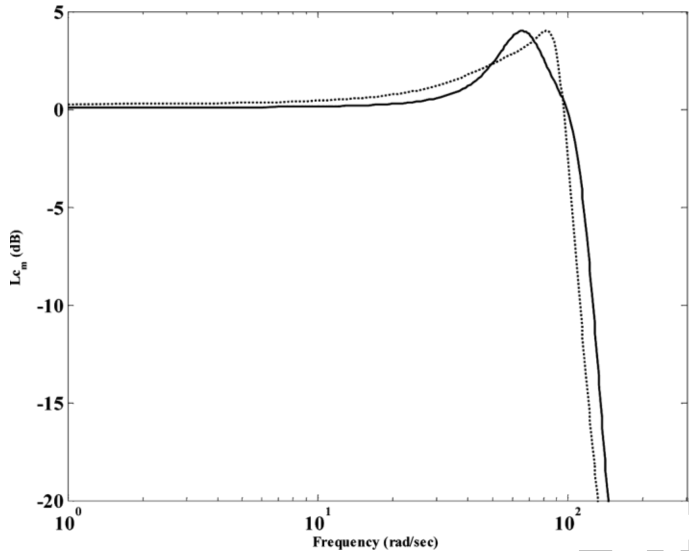

Figure 7: Servo responses (left), control action (right) for the Example 2(solid: centralized; dash: decentralized; solid-dash: reported decentralized)

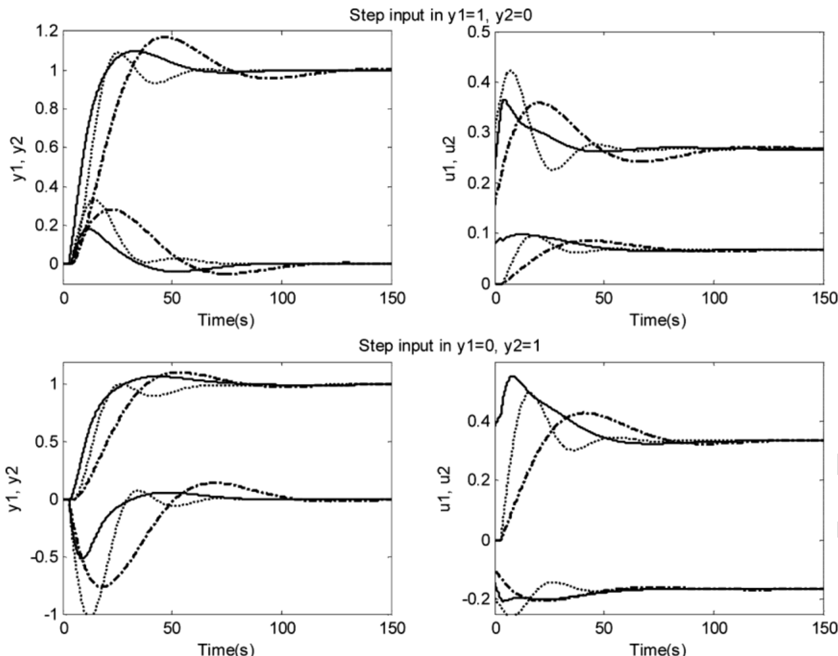

Figure 8: Regulatory responses (left), control action (right) for the Example 2 (solid: centralized; dash: decentralized; solid-dash: reported decentralized)

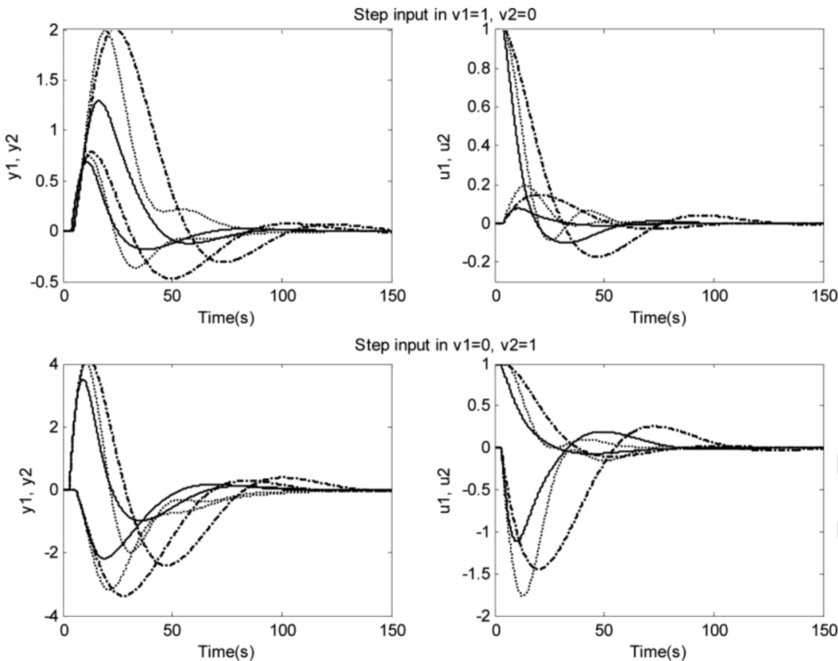

Figure 9. Stability regions of input and output uncertainties for Example 2. (solid: centralized; dash: decentralized)