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Parallel Self-Assembly under Uniform Control Inputs

Sheryl Manzoor and Aaron T. Becker

Abstract— **We present fundamental progress on parallel self-assembly using large swarms of micro- or nano-scale particles in complex environments, controlled not by individual navigation, but by a uniform global, external force with the same effect on each particle. Consider a 2D grid world, in which all obstacles and particles are unit squares, and for each actuation, robots move maximally until they collide with an obstacle or another robot. We present algorithms that, given an arbitrary 2D structures, design an obstacle layout. When actuated, this layout generates copies of the 2D structure. We analyze the spatial and time complexity of the factory layouts. We present hardware results on both a macro-scale, gravity-based system and a milli-scale, magnetically actuated system.**

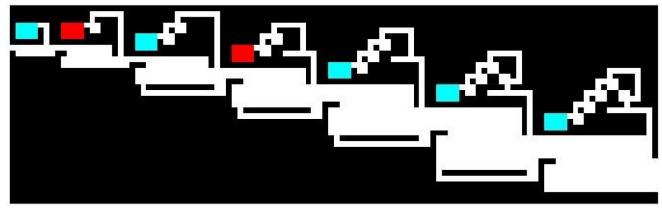

Fig. 1. A seven tile factory. Each particle is actuated simultaneously by the same global control field. The factory (black tiles) is designed so each clockwise control input assembles another component.

I. Introduction

One of the exciting new directions of robotics is the design and development of micro- and nanorobot systems, with the goal of letting a massive swarm of robots perform complex operations in a complicated environment. Due to scaling issues, individual control of the involved robots becomes physically impossible: while energy storage capacity drops with the third power of robot size, medium resistance decreases much slower. As a consequence, current micro- and nanorobot systems with many robots are steered and directed by an external force that acts as a common control signal [1]–[7]. These common control signals include global magnetic or electric fields, chemical gradients, and turning a light source on and off.

A. Selective Control with Global Inputs

Clearly, having only one global signal that uniformly affects all robots at once poses a strong restriction on the ability of the swarm to perform complex operations. This control symmetry can be broken using interactions between the robot swarm and obstacles in the environment. The key challenge is to establish if interactions with obstacles are sufficient to perform complex operations, ideally by analyzing the complexity of possible logical operations. In previous work [8]–[10], we were able to demonstrate how a subset of logical functions can be implemented; however, devising a fan-out gate (and thus the ability to replicate and copy information) appeared to be prohibitively challenging. In this paper, we resolve this crucial question by showing that only using unit-sized robots is insufficient for achieving computational universality. Remarkably, adding a limited number of domino-shaped objects *is sufficient* to let a common control signal, mobile particles, and unit-sized obstacles simulate a computer. While this does not imply that large-scale computational tasks should be run on these particle computers instead of current electronic devices, it establishes that future nano-scale systems are able to perform arbitrarily complex operations *as part of the physical system*, instead of having to go through external computational devices.

B. Model

This paper builds on the techniques for controlling many simple robots with uniform control inputs presented in [8]–[10], using the following rules:

1) A planar grid *workspace* W is filled with a number of unit-square robots (each occupying one cell of the grid) and some fixed unit-square blocks. Each unit square in the workspace is either *free*, which a robot may occupy or *obstacle* which a robot may not occupy. Each square in the grid can be referenced by its Cartesian coordinates $\boldsymbol{x} = (x, y)$.
2) All robots are commanded in unison: the valid commands are "Go Up" (u), "Go Right" (r), "Go Down" (d), or "Go Left" (l).
3) Robots all move in the commanded direction until they
   a) hit an obstacle
   b) hit a stationary robot.
   c) share an edge with a compatible robot

   If a robot shares an edge with a compatible robot the two robots bond and from then on move as a unit. A *move sequence* $\boldsymbol{m}$ consists of an ordered sequence of moves m_k, where each $m_k \in \{u, d, r, l\}$ A representative move sequence is $\langle u, r, d, l, d, r, u, \ldots \rangle$. We assume the area of W is finite and issue each command long enough for the robots to reach their maximum extent.

II. Related Work

Our efforts have similarities with *mechanical computers*, computers constructed from mechanical, not electrical com-

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ponents. For a fascinating nontechnical review, see [11]. These devices have a rich history, from the *Pascaline*, an adding machine invented in 1642 by a nineteen-year old Blaise Pascal; Herman Hollerith's punch-card tabulator in 1890; to the mechanical devices of IBM culminating in the 1940s. These devices used precision gears, pulleys, or electric motors to carry out calculations. Though our GRID-WORLD implementations are rather basic, we require none of these precision elements—merely unit-size obstacles and particles.

A. Sliding-Block Puzzles

Sliding-block puzzles use rectangular tiles that are constrained to move in a 2D workspace. The objective is to move one or more tiles to desired locations. They have a long history. Hearn [16] and Demaine [17] showed tiles can be arranged to create logic gates, and used this technique to prove PSPACE complexity for a variety of sliding-block puzzles. Hearn expressed the idea of building computers from the sliding blocks—many of the logic gates could be connected together, and the user could propagate a signal from one gate to the next by sliding intermediate tiles. This requires the user to know precisely which sequence of gates to enable/disable. In contrast to such a hands-on approach, with our architecture we can build circuits, store parameters in memory, and then actuate the entire system in parallel using a global control signal.

B. Other Related Work on Programmable Matter

Clearly there is a wide range of interesting scenarios for developing approaches to programmable matter. One such model is the *abstract Tile-Assembly Model* (aTAM) by Winfree [18]–[20], which has sparked a wide range of theoretical and practical research. In this model, unit-sized pixels ("tiles") interact and bond with the help of differently labeled edges, eventually composing complex assemblies. Even though the operations and final objectives in this model are quite different from our particle computation with global inputs (e.g., key features of the aTAM are that tiles can have a wide range of different edge types, and that they keep sticking together after bonding), there is a remarkable geometric parallelism to a key result of our present paper: While it is widely believed that at the most basic level of interaction (called *temperature 1*), computational universality *cannot be achieved* [21]–[23] in the aTAM with only unit-sized pixels, very recent work [24] shows that computational universality *can be achieved* as soon as even slightly bigger tiles are used. This resembles the results of our paper, which shows that unit-size particles are insufficient for universal computation, while employing bigger particles suffices

III. THEORY

Two design factories that build arbitrary 2D shaped polyominos, we first assign species to individual tiles of the polyomino, next discover a build path, then iteratively build factory components that add each tile to partially assembled polyomino.

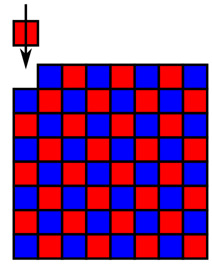

Fig. 2. Any polyomino can be constructed with two compatible robot species.

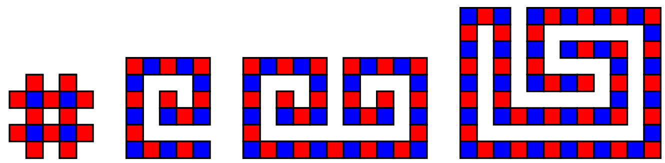

Fig. 3. Polyomino parts. Difficulty increases from left to right. The rightmost part cannot be built by additive construction.

A. Arbitrary 2D shapes require two particle species

A *polyomino* is a 2D geometric figure formed by joining one or more equal squares edge to edge. Polyominoes have four-point connectivity.

Lemma 1: Any polyomino can be constructed using just two species

Proof: Label a grid with an alternating pattern like a checkerboard. Any desired polyomino can be constructed on this checkerboard, and all joints are between dissimilar species. An example shape is shown in Fig. 2. ■

The sufficiency of two species to construct any shape gives many options for implementation. The two species could correspond to any gendered connection, including electric charge, ionic charge, magnetic polarity, or hook-and-loop type fasteners.

B. Complexity Handled in This Paper

Different 2D part geometries are more difficult to construct than others. Fig. 3 shows three parts of varying complexity. The part of the left is shaped as a '#' symbol. Though it has an interior hole, any of the 16 particles could serve as the seed particle, and the shape could be constructed around it. The second shape is a spiral, and must be constructed from the inside-out. If the outer spiral was completed first, there would be no path to add particles to finish the interior because added particles would have to slide past compatible particles. Increasing the number of species would not solve this problem, because there is a narrow passage through the spiral that forces incoming parts to slide past the edges of all the bonded particles.

The third shape on the right is two mirrored spirals that are connected. This part cannot be assembled by adding one particle at a time, because each spiral must be constructed from the inside-out. Instead, this part must be divided into sub-assemblies that are each constructed, and then combined.

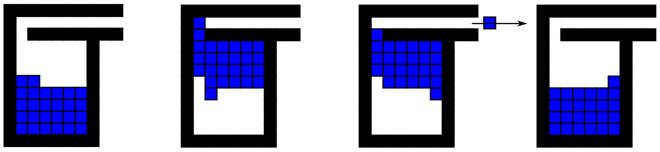

Fig. 4. This hopper is filled with similarly-labelled robots that will not combine. Every clockwise command sequence $\langle u, r, d, l\rangle$ releases one robot from the hopper. replace with new hopper design

A polyomino is said to be *column convex* if each column has no holes. Similarly, a polyomino is said to be row convex if each row has no holes. A polyomino is said to be *convex* if it is row and column convex.

Lemma 2: Any convex polyomino can be constructed by adding one particle at a time

Proof: Select any pixel as the *seed block*, or root node. Perform a breadth-first search starting at the seed block, labelling each block in the order they are expanded. Constructing the shape according to the ordering ensures that the polyomino is convex at every step of construction. ■

The proof of 2 assumes the existence of fixtures for assembly.

```
describe fixtures for adding one
particle at a time
```

Some non-convex polynominos cannot be constructed one particle at a time, as illustrated in Fig. 3. For instance, a polynomino consisting of a clockwise and a counterclockwise square spiral, joined at the ends with a gap of one unit between the spirals must be constructed by first assembling each spiral, and then combining the sub assemblies.

C. Discovering a Build Path

```
move algorithm here
```

D. Assembling Tiles

1) Hopper Construction: Two-part adhesives react when the components mix. Placing the components in separate containers prevents mixing. Similarly, storing many particles of a single species in separate containers allows controlled mixing.

We can design *part hoppers*, containers that store similarly labelled particles. These particles will not bond with each other. The hopper shown in Fig. 4 releases one particle every cycle.

E. Part Assembly Jigs

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Sheryl, add the algorithmic environment
for Build Factory
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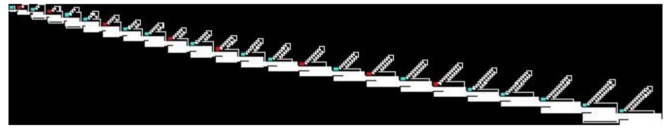

Fig. 5. A twenty four tile factory

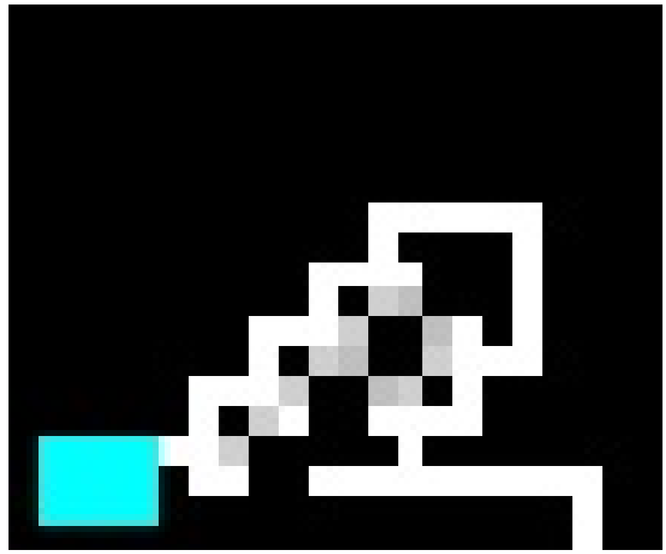

Fig. 6. Hopper with delays

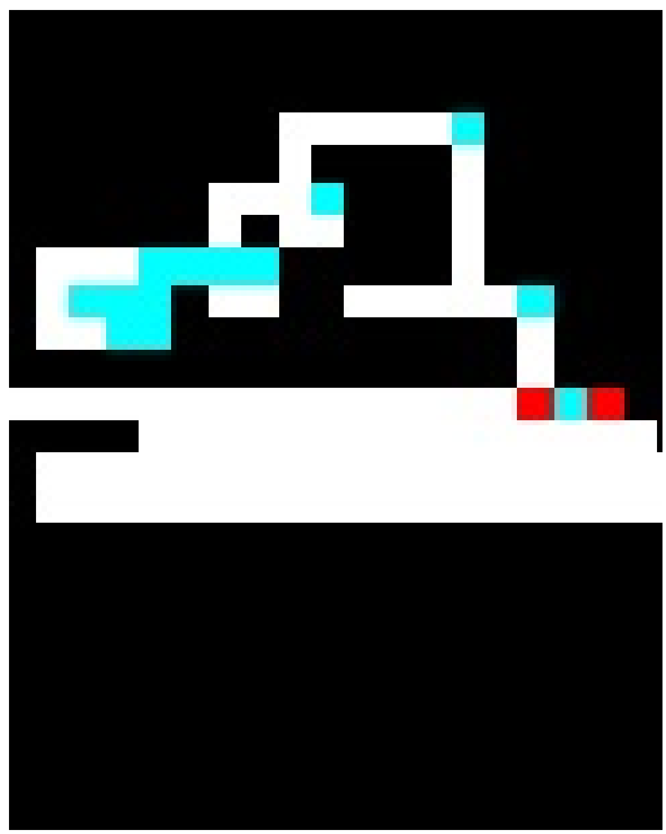

Fig. 7. Tile being attached to a three tile part by down move

Algorithm 1 BUILDFACTORY($\mathbf{P}, n$)

$\mathbf{P}$ is the x, y coordinates of a 4-connected polyomino. n is the number of parts desired. Returns a two dimensional array $\mathbf{F}$ containing the factory obstacles and filled hoppers.

$\{\mathbf{m}, \mathbf{C}, \mathbf{c}\} \leftarrow$ FINDBUILDPATH($\mathbf{P}$)
if $\varnothing = \mathbf{m}$ **then** return
end if
$\mathbf{F} \leftarrow \{\}$ ▷ the factory obstacle array
$\mathbf{b} \leftarrow \{\}$ ▷ the part being built
for $i \leftarrow 2, i \leq |\mathbf{m}|)$ **do**
  $\{\mathbf{A}, \mathbf{b}\} \leftarrow$FACTORYADDTILE($n, \mathbf{b}, \mathbf{m}_i, \mathbf{C}_i, \mathbf{c}_i$)
  $\mathbf{F} \leftarrow$CONCATFACTORIES($\mathbf{F}, \mathbf{A}$)
end for
return F

Algorithm 2 FindBuildPath algorithm

$\{foundPath$,sequence,dirs,partColoredArray$\}\leftarrow$FINDBUILDPATH($part_{xy}$)
for $m = 1 : partXY(:,1)$ **do**
$Start = partXY(m,:)$;
$\{Output$,Seq,Tmppart$\}\leftarrow$DEPTHFIRSTSEARCH($part_{xy}$,$Start$)
$partColored = labelColor(Tmppart(:,:,1))$; $partialAssembly = zeros(size(Tmppart(:,:,1),1), size(Tmppart(:,:,1),2))$; $partialAssembly(Output(1,1), Output(1,2)) = 1$;
$dirsFinal = size(partXY,1) - 1$;
$dirsFinal = char(dirsFinal)$;
$dirs2 = ['d';'l';'u';'r']$;
  for $i = 2 : size(Output,1)$ **do**
    for $j = 1 : 4$ **do**
      $\{move\} \leftarrow$CHECKPATHTILE($partialAssembly$, $Output(i,:)$,$dirs2(j,:)$,$partColored$)
      if $strcmp(move,'true')$ **then** $partialAssembly(Output(i,1), Output(i,2)) = 1$; $dirsFinal(i-1,:) = num2str(dirs2(j,:))$;
$break$;
      end if
    end for
    if $strcmp(move,'false')\&m = size(partXY,1)$ **then** $clearoutput, seq, tmppart, partColored, partialAssembly, dirsfinal$;
$break$;
    end if
    if $strcmp(move,'true')\&i == size(Output,1)$ **then** $foundPath = true$;
    end if
  end for
  if $foundPath == true$ **then** $sequence = Output$; $dirs = dirsFinal$; $partColoredArray = partColored$; $break$;
  end if
end for
[3]

Algorithm 3 The factoryAddTile algorithm

$\{partXYupdated$,factoryObstacleAdditionArray,align, hopperSize$\}\leftarrow$FACTORYADDTILE($partXY$, $tileXY$, dir, $tileColor$, $numCopies$, pos)
$\{hopper$,hopperSize$\}\leftarrow$HOPPER($tileColor$,$numCopies$,4,pos)
if $dir = d$ **then** $maxpartx = max(partXY(:,2))$;
  if $tileXY(1,2) <= maxpartx$ **then**
    $\{factoryObstacleAdditionArray$,align$\}\leftarrow$DOWNDIR($hopper$,$partXY$,$tileXY$)
  else
    $\{factoryObstacleAdditionArray$,align$\}\leftarrow$LEFTDIR($hopper$,$partXY$,$tileXY$)
  end if
end if
if $dir =' l'$ **then** $maxparty = max(partXY(:,1))$;
  if $tileXY(1,1) <= maxparty$ **then**
    $\{factoryObstacleAdditionArray$,align$\}\leftarrow$LEFTDIR($hopper$,$partXY$,$tileXY$)
  else
    $\{factoryObstacleAdditionArray$,align$\}\leftarrow$UPDIR($hopper$,$partXY$,$tileXY$)
  end if
end if
if $dir =' u'$ **then** $minpartx = min(partXY(:,2))$;
  if $tileXY(1,2) >= minpartx$ **then**
    $\{factoryObstacleAdditionArray$,align$\}\leftarrow$UPDIR($hopper$,$partXY$,$tileXY$)
  else
    $\{factoryObstacleAdditionArray$,align$\}\leftarrow$RIGHTDIR($hopper$,$partXY$,$tileXY$)
  end if
end if
if $dir =' r'$ **then** $minparty = min(partXY(:,1))$;
  if $tileXY(1,1) >= minparty$ **then**
    $\{factoryObstacleAdditionArray$,align$\}\leftarrow$RIGHTDIR($hopper$,$partXY$,$tileXY$)
  else
    $\{factoryObstacleAdditionArray$,align$\}\leftarrow$DOWNDIR($hopper$,$partXY$,$tileXY$)
  end if
end if
[2]

IV. ANALYSIS

A. Running Time

B. Space Required

C. Simulation Results

Algorithms ?? ?? were coded in MATLAB and are available at ??. This section examines

experiments that generate actual running time and space required for different parts.

FIG: xy plot that shows number of tiles on the x-axis, and running time on y-axis.

FIG: xy plot that shows number of tiles on the x-axis, and space required on y-axis. (two lines: vertical space and horizontal space and a line for the best and the worst case bounds from your previous section)

V. EXPERIMENT

A. Macro-scale, Gravity-Based Prototype

B. Milli-scale, Magnetic-Based Prototype

VI. CONCLUSION

In this paper we

This work, along with [8]–[10], introduces a new model for additive assembly. Interesting applications will aim at microfluidics work.

Future work could extend Algorithms ?? to three dimensions. Parts can be decomposed into subassemblies, which would enable more complex parts to be created and enable construction in logarithmic time.

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