# **Parallel Self-Assembly under Uniform Control Inputs**

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Abstract—We present fundamental progress on parallel self-assembly using large swarms of micro-scale particles in complex environments, controlled not by individual navigation, but by a uniform global, external force with the same effect on each particle. Consider a 2D grid world, in which all obstacles and particles are unit squares, and for each actuation, robots move maximally until they collide with an obstacle or another robot. We present algorithms that, given an arbitrary 2D structure, designs an obstacle layout. When actuated, this layout generates copies of the input 2D structure. We analyze the spatial and time complexity of the factory layouts. We present hardware results on both a macro-scale, gravity-based system and a milliscale, magnetically actuated system.

#### I. INTRODUCTION

One of the exciting new directions of robotics is the design and development of micro- and nanorobot systems, with the goal of letting a massive swarm of robots perform complex operations in a complicated environment. Due to scaling issues, individual control of the involved robots becomes physically impossible: while energy storage capacity drops with the third power of robot size, medium resistance decreases much slower. As a consequence, current micro- and nanorobot systems with many robots are steered and directed by an external force that acts as a common control signal [1]–[7]. These common control signals include global magnetic or electric fields, chemical gradients, and turning a light source on and off.

#### A. Selective Control with Global Inputs

Clearly, having only one global signal that uniformly affects all robots at once poses a strong restriction on the ability of the swarm to perform complex operations. This control symmetry can be broken using interactions between the robot swarm and obstacles in the environment. This paper builds on the techniques for controlling many simple particles with uniform control inputs presented in [8]–[10], where we demonstrated how such a system could implement digital computation.

### B. Model

Assume the following rules:

1) A planar grid *workspace* W is filled with a number of unit-square robots (each occupying one cell of the grid)





Fig. 1. (a) A milli-scale magnetic based prototype. (b) A seven tile factory. Each particle is actuated simultaneously by the same global field. The factory is designed so each clockwise control input assembles another component.

and some fixed unit-square blocks. Each unit square in the workspace is either *free*, which a particle may occupy or *obstacle* which a robot may not occupy. Each square in the grid can be referenced by its Cartesian coordinates x = (x, y).

- 2) All particles are commanded in unison: the valid commands are "Go Up" (u), "Go Right" (r), "Go Down" (d), or "Go Left" (l).
- 3) Particles all move until they
  - a) hit an obstacle
  - b) hit a stationary particle.
  - c) share an edge with a compatible particle

If a particle shares an edge with a compatible robot the two robots bond and from then on move as a unit. A move sequence m consists of an ordered sequence of moves  $m_k$ , where each  $m_k \in \{u,d,r,l\}$  A representative move sequence is  $\langle u,r,d,l,d,r,u,\ldots \rangle$ . We assume the area of W is finite and issue each command long enough for the robots to reach their maximum extent.

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### II. RELATED WORK

Our efforts have similarities with *mechanical computers*, computers constructed from mechanical, not electrical components. For a fascinating nontechnical review, see [?]. These devices have a rich history, from the *Pascaline*, an

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adding machine invented in 1642 by a nineteen-year old Blaise Pascal; Herman Hollerith's punch-card tabulator in 1890; to the mechanical devices of IBM culminating in the 1940s. These devices used precision gears, pulleys, or electric motors to carry out calculations. Though our GRID-WORLD implementations are rather basic, we require none of these precision elements—merely unit-size obstacles and particles.

### A. Sliding-Block Puzzles

Sliding-block puzzles use rectangular tiles that are constrained to move in a 2D workspace. The objective is to move one or more tiles to desired locations. They have a long history. Hearn [?] and Demaine [?] showed tiles can be arranged to create logic gates, and used this technique to prove PSPACE complexity for a variety of sliding-block puzzles. Hearn expressed the idea of building computers from the sliding blocks—many of the logic gates could be connected together, and the user could propagate a signal from one gate to the next by sliding intermediate tiles. This requires the user to know precisely which sequence of gates to enable/disable. In contrast to such a hands-on approach, with our architecture we can build circuits, store parameters in memory, and then actuate the entire system in parallel using a global control signal.

### B. Other Related Work on Programmable Matter

Clearly there is a wide range of interesting scenarios for developing approaches to programmable matter. One such model is the abstract Tile-Assembly Model (aTAM) by Winfree [?], [?], [?], which has sparked a wide range of theoretical and practical research. In this model, unit-sized pixels ("tiles") interact and bond with the help of differently labeled edges, eventually composing complex assemblies. Even though the operations and final objectives in this model are quite different from our particle computation with global inputs (e.g., key features of the aTAM are that tiles can have a wide range of different edge types, and that they keep sticking together after bonding), there is a remarkable geometric parallelism to a key result of our present paper: While it is widely believed that at the most basic level of interaction (called temperature 1), computational universality cannot be achieved [?], [?], in the aTAM with only unitsized pixels, very recent work [?] shows that computational universality can be achieved as soon as even slightly bigger tiles are used. This resembles the results of our paper, which shows that unit-size particles are insufficient for universal computation, while employing bigger particles suffices

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### III. RELATED WORK

### A. Microscale Biomanufacturing

Naturally derived biomaterials as building blocks for functional materials and devices are increasingly desired because they are environmentally and biologically safer than purely synthetic materials. One such class of materials, polysaccharide based hydrogels, are intriguing because they can reversibly encapsulate a variety of smaller components. Many groups have termed these loaded-alginate particles as artificial cells, in that they mimic the basic structure of living cells (membrane, cytoplasm, organelles, etc.) [11] [1-3]. Construction with these micron-sized gels has numerous applications in industry, including cell manipulation, tissue engineering, and micro-particle assembly [4-8], but requires fundamental research in biology, medicine, and colloidal science. While there are several methods to efficiently fabricate these particulate systems, it is still challenging to construct larger composite materials out of these units [9]. Traditional methods of assembling larger macroscale systems are unemployable due to the change of dominant forces at small length scales. In particular, forces due to electromagnetic interactions dominate gravitational forces at the microscale resulting in strong adhesion and sudden shifts in the position of microparts under atmospheric conditions. To form constructs out of microgels, groups have traditionally turned to non-robotic microfluidic systems that utilize a variety of actuation methods, including mechanical, optical, dielectrophoretic, acoustophoretic, and thermophoretic [10-14]. While each of these methods has proven to be capable of manipulating biological cells, each method has significant drawbacks that limit their widespread application. For example, microscale mechanical, acoustophoretic, and thermophoretic manipulation methods use stimuli that can be potentially lethal to live cells [15]. Furthermore, most, if not all, of these techniques require expensive equipment and lack control schemes necessary to precisely manipulate large numbers of cells autonomously.

### B. Control Microrobotic Swarms Using Only Global Signals

Today one of the most exciting new frontiers in robotics is the development of micro- and nanorobotic systems, which hold the potential to revolutionize the fields of manufacturing and medicine. Chemist, biologist, and roboticist have shown the ability to produce very large populations  $(10^3-10^{14})$  of small scale  $(10^{-9}-10^{-6} \text{ m})$  robots using a diverse array of materials and techniques [16-18]. Untethered swarms of these tiny robots may be ideal for on-site construction of high-resolution macroscale materials and devices. While these new types of large-population, small-sized, robotic systems have many advantages over their larger-scale counterparts, they also present a set of unique challenges in terms of their control. Due to current limitations in fabrication, micro- and nanorobots have little-to-no onboard computation, along with limited computation and communication ability [18-20]. These limitations make controlling swarms of these robots individually impractical. Thus, these robotic systems are often controlled by a uniform global external signal (e.g. chemical gradients, electric and magnetic fields), which makes motion planning for large robotic populations in tortuous environments difficult. We recently demonstrated that obstacles present in the workspace can break the symmetry of approximately identical robotic swarms, enabling positional configuration of robots [21]. Given a large-enough free space, a single obstacle is sufficient for positional control over N particles. This method can be used to form complex assemblies out of large swarms of mobile microrobotic building blocks, using only a single global input signal.

#### C. Microrobot Based Microassembly

The ability to create microrobots, and control algorithms capable of autonomous manipulation and assembly of small scale components into functional materials is currently a major manufacturing challenge [1]. To address this challenge, teams of microrobotic systems must work together intelligently to coordinate manipulation tasks in novel environments. While several microrobots capable of performing simple manipulation and assembly tasks have been reported [2-7], few have shown the ability to pattern intricate designs or assemble complex multi-component parts. Recently, some groups have begun to develop cell-safe magnetically actuated microrobotic systems for cell patterning, yet their method is limited in that these systems are manually controlled, not automated, and suffer from low spatial resolution [22, 23]. In this paper, we seek to combine the use of microscale hybrid organic/inorganic actuators along with novel swarm control algorithms for mask free programmable patterning and micro-assembly. Specifically, this paper applies swarm control and particle logic computations to magnetically actuate artificial cells, so as to use them as micro-scale robotic swarms, to create complex, high resolution, 2D and 3D patterns and assemblies. ¿¿¿¿¿¿¿ origin/master

#### IV. THEORY

This section explains how to design factories that build arbitrary 2D shaped polyominos. We first assign species to individual tiles of the polyomino, second discover a build path, and finally build an assembly line of factory components that each add one tile to partially assembled polyomino and pass the polyomino to the next component.

### A. Arbitrary 2D shapes require two particle species

A *polyomino* is a 2D geometric figure formed by joining one or more equal squares edge to edge. Polyominoes have four-point connectivity.

Lemma 1: Any polyomino can be constructed using just two species

*Proof:* Label a grid with an alternating pattern like a checkerboard. Any desired polyomino can be constructed on this checkerboard, and all joints are between dissimilar species. An example shape is shown in Fig. 2.

The sufficiency of two species to construct any shape gives many options for implementation. The two species could correspond to any gendered connection, including electric charge, ionic charge, magnetic polarity, or hook-and-loop type fasteners.

### B. Complexity Handled in This Paper

Different 2D part geometries are more difficult to construct than others. Fig. 3 shows four parts of varying complexity. Label the first particle in the assembly process the seed particle. The part on the left is shaped as a '#' symbol.

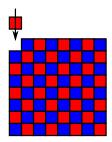


Fig. 2. Any polyomino can be constructed with two compatible robot species.

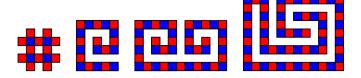


Fig. 3. Polyomino parts. Difficulty increases from left to right. The rightmost part cannot be built by additive construction.

Though it has an interior hole, any of the 16 particles could serve as the seed particle, and the shape could be constructed around it. The second shape is a spiral, and must be constructed from the inside-out. If the outer spiral was completed first, there would be no path to add particles to finish the interior because added particles would have to slide past compatible particles. Increasing the number of species would not solve this problem, because there is a narrow passage through the spiral that forces incoming parts to slide past the edges of all the bonded particles.

The third shape is the combination of a left-handed and a right-handed spiral. This part cannot be assembled by adding one particle at a time, because each spiral must be constructed from the inside-out. Instead, this part must be divided into sub-assemblies that are each constructed, and then combined. The fourth shape contains compound overhangs, and may be impossible to construct with additive manufacturing. The algorithms in this paper detect if the desired shape can be constructed one particle at a time. If so, a build order is provided, and a factory layout is designed.

### C. Discovering a Build Path

Given a polyomino, Alg. 1 determines if the polyomino can be built by adding one component at a time. The forward problem of determining a build order is difficult because there are O(n!) possible build orders, and many of them may be violate the constraints give in Section . Each new tile must have a straight-line path to its goal position in the polyomino that does not collide with any other particle, does not slide past an opposite species of tile, and terminates in a mating configuration with an opposite species tile. As in many robotics problem, the inverse problem is easier. The inverse problem of deconstruction simply checks (1) if any of the polyomino tiles can be removed along a straight-line path without colliding with any other particle or sliding past an opposite species of tile, and (2) that its removal does

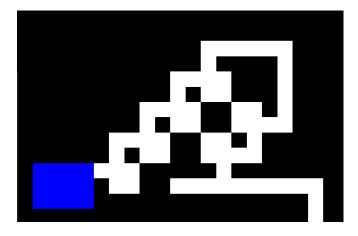


Fig. 4. Hopper with delays. The hopper is filled with similarly-labelled robots that will not combine. Every clockwise command sequence  $\langle u,r,d,l\rangle$  releases one robot from the hopper.

not fragment the remaining polyomino into more than one piece. This algorithm requires at most 1/2n(1+n) terations, becasue there are n particles to remove, and each iteration considers one less particle than the previous iteration.

### Algorithm 1 FINDBUILDPATH(P)

 $\mathbf{P}$  is the x,y coordinates of a 4-connected polyomino. Returns  $\mathbf{C}$ ,  $\mathbf{c}$  and  $\mathbf{m}$  where  $\mathbf{C}$  contains sequence of polyomino coordinates,  $\mathbf{c}$  is a vector of color labels and  $\mathbf{m}$  is a vector of directions for assembly.

```
1: \mathbf{C} \leftarrow \{\}, \mathbf{c} \leftarrow \{\}, \mathbf{m} \leftarrow \{\}

2: \mathbf{c} \leftarrow \mathsf{LABELCOLOR}(\mathbf{P})

3: \mathbf{for} \ m \leftarrow 1, m \leq |\mathbf{P}|) \ \mathbf{do}

4: \mathbf{C} \leftarrow \mathsf{DEPTHFIRSTSEARCH}(\mathbf{P}_m, \mathbf{P})

5: \mathbf{m} \leftarrow \mathsf{CHECKPATHTILE}(\mathbf{C}, \mathbf{c})

6: \mathbf{if} \ \{\} \neq \mathbf{m} \ \mathbf{then}

7: \mathbf{break}

8: \mathbf{end} \ \mathbf{if}

9: \mathbf{end} \ \mathbf{for}

10: \mathbf{return} \ \{\mathbf{C}, \mathbf{c}, \mathbf{m}\}
```

### D. Assembling Tiles

1) Hopper Construction: Two-part adhesives react when the components mix. Placing the components in separate containers prevents mixing. Similarly, storing many particles of a single species in separate containers allows controlled mixing.

We can design *part hoppers*, containers that store similarly labelled particles. These particles will not bond with each other. The hopper shown in Fig. 4 releases one particle every cycle.



Fig. 5. A twenty-four tile factory

### Algorithm 2 BUILDFACTORY( $\mathbf{P}, n$ )

 ${f P}$  is the x,y coordinates of a 4-connected polyomino. n is the number of parts desired. Returns a two dimensional array  ${f F}$  containing the factory obstacles and filled hoppers.

```
1: F ← {}

    b the factory obstacle array

        2: b ← {}

    being built
    being
        3: \{C, c, m\} \leftarrow FINDBUILDPATH(P)
        4: if \{\} \neq \mathbf{m} then
                                                                 for i \leftarrow 1, i \leq |\mathbf{m}|) do
        5:
                                                                                                   \{\mathbf{A}, \mathbf{b}\} \leftarrow \text{FACTORYADDTILE}(n, \mathbf{b}, \mathbf{m}_i, \mathbf{C}_i, \mathbf{c}_i, w)
        6:
                                                                                                 \mathbf{F} \leftarrow \text{ConcatFactories}(\mathbf{F}, \mathbf{A})
        7:
                                                                  end for
        8:
        9: end if
10: return F
```

### **Algorithm 3** FACTORYADDTILE $(n, \mathbf{b}, m, C, c, w)$

```
1: \{\mathbf{hopper}\} \leftarrow \mathsf{HOPPER}(c,n,w)

2: if m = d and (C_y \leq \max \mathbf{b}_y \text{ or } C_y > \min \mathbf{b}_x) then

3: \{\mathbf{A}, \mathbf{b}\} \leftarrow \mathsf{DOWNDIR}(\mathbf{hopper}, \mathbf{b}, \mathbf{C})

4: else if m = l and (C_x \leq \max \mathbf{b}_x \text{ or } C_y > \min \mathbf{b}_y) then

5: \{\mathbf{A}, \mathbf{b}\} \leftarrow \mathsf{LEFTDIR}(\mathbf{hopper}, \mathbf{b}, \mathbf{C})

6: else if m = u and (C_y \leq \min \mathbf{b}_y \text{ or } C_x > \min \mathbf{b}_x) then

7: \{\mathbf{A}, \mathbf{b}\} \leftarrow \mathsf{UPDIR}(\mathbf{hopper}, \mathbf{b}, \mathbf{C})

8: else if m = r and (C_y \leq \min \mathbf{b}_x \text{ or } C_y > \min \mathbf{b}_y) then

9: \{\mathbf{A}, \mathbf{b}\} \leftarrow \mathsf{RIGHTDIR}(\mathbf{hopper}, \mathbf{b}, \mathbf{C})

10: end if

11: return \{\mathbf{A}, \mathbf{b}\}
```

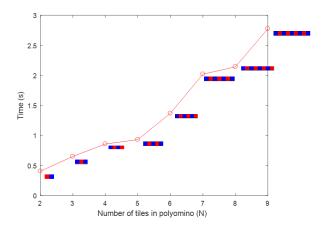


Fig. 6. Running time plotted against number of tiles (N). add max and min bounds, show results for column and row parts and arbitrary parts

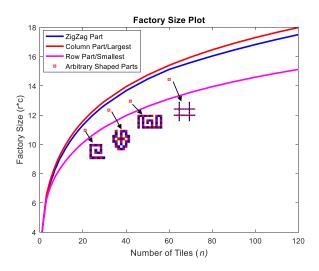


Fig. 7. Factory size (rows\*columns) plotted against number of tiles (n).

E. Part Assembly Jigs

#### V. ANALYSIS

- A. Running Time
- B. Space Required
- C. Simulation Results

Algorithms 1, 2, and 3 were coded in MATLAB and are available at [12].

This section examines experiments that generate actual running time and space required for different parts.

FIG: xy plot that shows number of tiles on the x-axis, and running time on y-axis.

FIG: xy plot that shows number of tiles on the x-axis, and space required on y-axis. (two lines: vertical space and horizontal space and a line for the best and the worst case bounds from your previous section)

#### VI. EXPERIMENT

- A. Macro-scale, Gravity-Based Prototype
- B. Milli-scale, Magnetic-Based Prototype

To demonstrate the algorithm, we developed a magnetic control stage and alginate micro-particles.

a) Experimental setup: This stage generates a magnetic drag force by moving a permanent magnet. The permanent magnet is able to move x, y direction as following two mail shafts. The permanent magnet has T and the dimension is  $cm^2$ . The main channel is made up PDMS and it was filled



Fig. 8. A large-scale demonstration of particle assembly using gravity as the external force and magnetic attraction between red and blue particles for assembly.

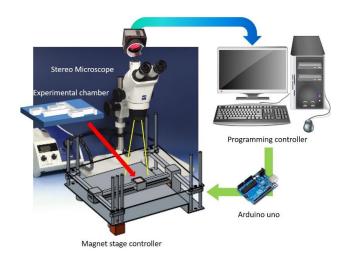


Fig. 9. Experiment

with motility buffer. The alginate microrobot was fabricated using . After the alginate microrobots were located at each chamber in the channel, the experimental channel was located on the center of the stage where a magnet was positioned initially. The stage controller was manipulated by a C++ programming through an Arduino UNO. The channel was observed by a stereo microscope and the installed camera captured all sequent images (fps). The scheme of system is shown in Fig. 9.

b) Experimental result: Using one of construction maps, it is available to demonstrate the map using multiple alginate microrobots. The initial scene is shown in Fig. 10a and the first assemble was manipulated moving the magnet in a clockwise direction as indicated in Fig. 10b. The alginate microrobots moved in the oriented direction until coming into contact with an object. The final completion of a square polyomino is shown in the lower right corner in Fig. 10c. In addition, other polyominoes were simultaneously being manufactured.

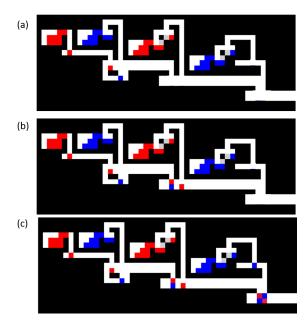


Fig. 10. Fig. Construction of a microrobotic polyomino from four alginate artificial cells. (a) Initial position of alginate microrobots at all chambers, (b) First assemble by two microrobots from two chambers, (c) Final result of construction.

### VII. CONCLUSION

This work introduces a new model for additive assembly that enables efficient parallel construction because it does not depend on individual control of each agent. Instead, the workspace is designed to direct particles. This enables a simple global control input to produce a complex output.

Interesting applications will aim at microfluidics work.

Future work could extend Algorithms 1–3 to three dimensions. To build a polyomino, our current algorithm requires time that grows linearly with the number of tiles in a polyomino. Parts can be decomposed into subassemblies, which would enable more complex parts to be created and enable construction in logarithmic time.

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## CONTENTS

I	Introduction		
	I-A I-B	Selective Control with Global Inputs . Model	1 1
II	Related	Work	1
	II-A	Sliding-Block Puzzles	2
	II-B	Other Related Work on Programmable	_
		Matter	2
Ш	Related	Work	2
	III-A	Microscale Biomanufacturing	2
	III-B	Control Microrobotic Swarms Using	
		Only Global Signals	2
	III-C	Microrobot Based Microassembly	3
IV	Theory		3
	IV-A	Arbitrary 2D shapes require two par-	
		ticle species	3
	IV-B	Complexity Handled in This Paper	3
	IV-C	Discovering a Build Path	3
	IV-D	Assembling Tiles	4
		IV-D.1 Hopper Construction	4
	IV-E	Part Assembly Jigs	5
V	Analysis		5
	V-A	Running Time	5
	V-B	Space Required	5
	V-C	Simulation Results	5
VI	Experiment		5
	VI-A	Macro-scale, Gravity-Based Prototype	5
	VI-B	Milli-scale, Magnetic-Based Prototype	5
VII	Conclus	sion	6
References			6