Simplex Example

- iteration-by-iteration standard-form example

Example

$$A = \begin{bmatrix} -2 & 1 & 1 & 0 & 0 \\ -1 & 2 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad b = \begin{bmatrix} 2 \\ 7 \\ 3 \end{bmatrix}, \quad c = \begin{bmatrix} -1 \\ -2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

iteration 0: $\mathcal{B} = \{ 3, 4, 5 \}, \quad \mathcal{N} = \{ 1, 2 \}$

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- $B = I = B^{-1}$

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- ratio test: $q = \underset{q \in \{ 1, \dots, m \} | d_{\beta_q} < 0}{\arg\min} -\frac{x_{\beta_q}}{d_{\beta_q}} \quad \longrightarrow \quad q = 2,\ \beta_q = 4$ **exits** basis

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- $z_N \geq 0 \quad \longrightarrow \quad$ basis is **optimal**