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template

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WUST

# So Like Coding? You Baldy

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## 0 Header

### 0.1 pbds

#### 0.1.1 head

```
1 #include <bits/extc++.h>
2 #pragma comment(linker, "/STACK:102400000,102400000")
3 using namespace __gnu_pbds; // tree, gp_hash_table, trie
4 using namespace __gnu_cxx; // rope
5 tree<TYPE, null_type, less<>, rb_tree_tag, tree_order_statistics_node_update> tr;
6 // 可并堆
7 #include <ext/pb_ds/priority_queue.hpp>
8 using namespace __gnu_pbds;
9 __gnu_pbds::priority_queue<int, greater<int>, pairing_heap_tag> q[maxn];
10 //q[i].join(q[j]) 将j堆并入i
11
```

### 0.2 FastIO

#### 0.2.1 FastScanner

```
1 // 适用于正负整数
2 template <class T>
3 inline bool scan(T &ret){
4     char c;
5     int sgn;
6     if (c = getchar(), c == EOF) return 0; //EOF
7     while (c != '-' && (c < '0' || c > '9')) c = getchar();
8     sgn = (c == '-') ? -1 : 1;
9     ret = (c == '-') ? 0 : (c - '0');
10    while (c = getchar(), c >= '0' && c <= '9') ret = ret * 10 + (c - '0');
11    ret *= sgn;
12    return 1;
13 }
14
15 template <class T>
16 inline void out(T x) {
17     if (x > 9) out(x / 10);
18     putchar(x % 10 + '0');
19 }
20
21 inline int read() {
22     int x = 0;
23     char ch = getchar();
24     while (ch > '9' || ch < '0') ch = getchar();
25     while (ch >= '0' && ch <= '9') {
26         x = x * 10 + ch - '0';
27         ch = getchar();
28     }
29     return x;
30 }
```

#### 0.2.2 FastPowAndAdd

```
1 // 精确快速乘
2 ll qpmul(ll a, ll b) {
3     a %= mod; b %= mod;
```

```
4     ll res = 0;
5     while (b > 0) {
6         if (b & 1) {
7             res = (res + a);
8             if (res >= mod) res -= mod;
9         }
10        a = (a + a);
11        if (a >= mod) a -= mod;
12        b >>= 1;
13    }
14    return res;
15 }
16
17 // 0(1)快速乘
18 ll mul2(ll x,ll y,ll p) {
19     ll res=(x*y-(ll)((long double)x/p*y+1.0e-8)*p);
20     return res<0?res+p:res;
21 }
22
23 //int128
24 ll ans = ((__int128) a * b) % p;
25
26 // 10进制快速幂, 直接读入%s,c 预处理字符串len
27 char c[1000005], len;
28 ll qp(ll a) {
29     len--;
30     a %= mod;
31     ll s = a;
32     ll res = 1;
33     while (len >= 0) {
34         ll cur = s;
35         for (int i = 1; i <= c[len] - '0'; ++i) {
36             res = res * s % mod;
37         }
38         for (int i = 1; i < 10; ++i) {
39             cur = cur * s % mod;
40         }
41         s = cur;
42         len--;
43     }
44     return res;
45 }
```

### 0.2.3 PythonInput

```
1 // python一行读入
2 a,b = map(int, input().split())
3
4 a = []
5 for i in input().split():
6     a.append(int(i))
```

### 0.2.4 SpecialInput

```
1 // 代替gets
2 scanf("%[^\n]%"c", ss)
3
4 void out2(int x, int flag = 1) {
```



```
5     if (x == 0) {
6         if (flag) putchar('0');
7         return;
8     }
9     out2(x >> 1, 0);
10    putchar('0' + x % 2);
11 }
```

### 0.3 header

```
1  // Editor -> Live Templates
2  // add template group acm
3  // add template main
4  // C++ Declaration
5
6  #include <bits/stdc++.h>
7  #define ll long long
8  using namespace std;
9
10 template <class T>
11 inline bool scan(T &ret){
12     char c;
13     int sgn;
14     if (c = getchar(), c == EOF) return 0; //EOF
15     while (c != '-' && (c < '0' || c > '9')) c = getchar();
16     sgn = (c == '-') ? -1 : 1;
17     ret = (c == '-') ? 0 : (c - '0');
18     while (c = getchar(), c >= '0' && c <= '9') ret = ret * 10 + (c - '0');
19     ret *= sgn;
20     return 1;
21 }
22
23 const ll mod = 1e9+7;
24 const int maxn = $MAXN$;
25
26 ll qp(ll x, ll n) {
27     ll res = 1; x %= mod;
28     while (n > 0) {
29         if (n & 1) res = res * x % mod;
30         x = x * x % mod;
31         n >>= 1;
32     }
33     return res;
34 }
35
36 int main(int argc, char* argv[]) {
37     $CONTENT$
38     return 0;
39 }
40
41 // C++ Expression debug
42 freopen("data.in", "r", stdin);
43 freopen("data.out", "w", stdout);
44 clock_t ST = clock();
45 cerr << "time: " << ((clock()-ST)*1000.0 / CLOCKS_PER_SEC) << "ms" << endl;
46
47
48 // C++ Expression tkase
```

```
49 int T;
50 scanf("%d", &T);
51 for (int kase = 1; kase <= T; ++kase) { $CONTENT$;
52 }
```

### 0.3.1 comp

```
1 // 1 create directory comp
2 // 2 create directory comp/test
3 // 3 create duipai.cpp
4 // add text
5
6 #include<bits/stdc++.h>
7 using namespace std;
8 int main(){
9     int i;
10    for (i=1;;i++){
11        printf("The result of No. %d Case is: ",i);
12        system("python3 rand.py");
13        system("./std < test/data.in > test/std.out");
14        system("./my < test/data.in > test/my.out");
15        if (system("diff test/std.out test/my.out")){
16            printf("Wrong Answer\n");
17            return 0;
18        }
19        else printf("Accepted\n");
20    }
21    return 0;
22 }
23
24
25 // 4 create duipai.sh
26 #!/bin/bash
27 g++ std.cpp -o std
28 g++ my.cpp -o my
29 python3 rand.py
30 ./a.out
31
32 // 5 create rand.py
33 # coding=utf-8
34 from random import randint, choice, shuffle
35 # with open("../cmake-build-debug/data.in", "w") as f:
36 with open("test/data.in", "w") as f:
37     n = randint(1, 10)
38     m = randint(1, 10)
39     f.write(f"{n} {m}")
40
41 // 6 terminal: g++ duipai.cpp
42 // 7 terminal: sudo chmod 777 duipai.sh
43 // 8 add my.cpp and std.cpp
44 // 9 ./duipai.sh
```

# 1 Math

## 1.1 素数

### 1.1.1 Eratosthenes 筛法

```
1 bool vis[(int)1e6+5];
2 int prim[(int)1e5], tot; // tot = 78499
3 void init() {
4     for (int i = 2; i < 1e6+5; ++i) {
5         if (vis[i]) continue;
6         prim[++tot] = i;
7         for (int j = i + i; j < 1e6+5; j += i) vis[j] = 1;
8     }
9 }
```

### 1.1.2 Euler 筛

```
1 const int maxn = 1e6 + 10;
2 int prime[maxn], v[maxn], n, cnt; //每个合数只会被它的最小质因子p筛一次
3
4 void Euler_Sieve()
5 {
6     for(int i = 2; i <= n; i++)
7     {
8         if(!v[i]) v[i] = i, prime[++cnt] = i;
9         for(int j = 1; j <= cnt && i * prime[j] <= n; j++)
10         {
11             v[i * prime[j]] = prime[j];
12             if(i % prime[j] == 0) break;
13         }
14     }
15 }
```

### 1.1.3 MillerRabin 素性测试

```
1 typedef long long ll;
2
3 bool check(ll a, ll n)
4 {
5     if(n == 2 || a >= n) return true;
6     if(n == 1 || !(n & 1)) return false;
7     ll d = n - 1;
8     while(!(d & 1)) d >>= 1;
9     ll t = qp(a, d, n);
10    while(d != n - 1 && t != 1 && t != n - 1)
11    {
12        t = mul(t, t, n);
13        d <<= 1;
14    }
15    return t == n - 1 || d & 1;
16 }
17
18 bool Miller_Rabin(ll n)
19 {
20     static vector<ll> t = {2, 325, 9375, 28178, 450775, 9780504, 1795265022};
21     if (n <= 1) return false;
22     for (ll k: t) if (!check(k, n)) return false;
```

```

23     return true;
24 }

```

#### 1.1.4 PollardRho 快速因数分解

```

1  mt19937 mt(time(0));
2  ll pollard_rho(ll n, ll c) {
3      ll x = uniform_int_distribution<ll>(1, n - 1)(mt), y = x;
4      auto f = [&](ll v) { ll t = mul(v, v, n) + c; return t < n ? t : t - n; };
5      while (1) {
6          x = f(x); y = f(f(y));
7          if (x == y) return n;
8          ll d = __gcd(abs(x - y), n);
9          if (d != 1) return d;
10     }
11 }
12
13 ll fac[100], fcnt;
14 void get_fac(ll n, ll cc = 19260817) {
15     if (n == 4) { fac[fcnt++] = 2; fac[fcnt++] = 2; return; }
16     if (Miller_Rabin(n)) { fac[fcnt++] = n; return; }
17     ll p = n;
18     while (p == n) p = pollard_rho(n, --cc);
19     get_fac(p); get_fac(n / p);
20 }

```

#### 1.1.5 求素因子

```

1  vector<pair<ll, int>> getFactors(ll x)
2  {
3      vector<pair<ll, int>> fact;
4      for (int i = 0; prime[i] <= x / prime[i]; i++)
5      {
6          if (x % prime[i] == 0)
7          {
8              fact.emplace_back(prime[i], 0);
9              while (x % prime[i] == 0) fact.back().second++, x /= prime[i];
10         }
11     }
12     if (x != 1) fact.emplace_back(x, 1);
13     return fact;
14 }

```

## 1.2 约数

### 1.2.1 EulerPhi

```

1  //计算欧拉phi函数, phi(n)且与n互素的正整数个数
2
3  int Euler(int n){
4      int rea=n;
5      for(int i=2; i*i<=n; i++)
6          if(n%i==0)//第一次找到的必为素因子
7          {
8              rea=rea-rea/i;
9              do
10                 n/=i;//把该素因子全部约掉

```

```
11         while(n%i==0);
12     }
13     if(n>1)
14         rea=rea-rea/n;
15     return rea;
16 } //单点欧拉 O(sqrt(n))
17
18 bool boo[50000];
19 int p[20000];
20 void prim(){
21     memset(boo,0,sizeof(boo));
22     boo[0]=boo[1]=1;
23     int k=0;
24     for(int i=2; i<50000; i++)
25     {
26         if(!boo[i])
27             p[k++]=i;
28         for(int j=0; j<k&& i*p[j]<50000; j++)
29         {
30             boo[i*p[j]]=1;
31             if(!(i%p[j]))
32                 break;
33         }
34     }
35 } //筛选法打表
36 int phi(int n)
37 {
38     int rea=n;
39     for(int i=0; p[i]*p[i]<=n; i++) //对于一些不是素数的可不遍历
40         if(n%p[i]==0)
41         {
42             rea=rea-rea/n;
43             do
44                 n/=p[i];
45             while(n%p[i]==0);
46         }
47     if(n>1)
48         rea=rea-rea/n;
49     return rea;
50 } //素数+欧拉
51
52 int euler[maxn];
53 void init() {
54     int i, j;
55     for(i=1; i<maxn; i++)
56         euler[i]=i;
57     for(i=2; i<maxn; i+=2)
58         euler[i]/=2;
59     for(i=3; i<maxn; i+=2)
60         if(euler[i]==i) {
61             for(j=i; j<=maxn; j+=i)
62                 euler[j]=euler[j]/i*(i-1);
63         }
64 } //递推欧拉表
```

### 1.2.2 Sieve

1 //用类似筛法的方法计算 $\phi(1), \phi(2), \dots, \phi(n)$

```
2 int phi[maxn];
3
4 void phi_table(int n)
5 {
6     for (int i = 2; i <= n; i++) phi[i] = 0;
7     phi[1] = 1;
8     for (int i = 2; i <= n; i++) if (!phi[i])
9         for (int j = i; j <= n; j += i)
10             {
11                 if (!phi[j]) phi[j] = j;
12                 phi[j] = phi[j] / i * (i - 1);
13             }
14 }
```

### 1.2.3 gcd

```
1 ll gcd(ll a, ll b) {while(b^=a^=b%=b);return a;}
2
3 void exgcd(ll a, ll b, ll& x, ll& y, ll& c) {
4     if(!b) {y = 0; x = 1; c = a; return;}
5     exgcd(b, a % b, y, x); y -= a / b * x;
6 }
```

### 1.2.4 解乘法逆元

```
1 void exgcd(ll a, ll b, ll c, ll d, ll &x, ll &y) {
2     ll z = (a + b - 1) / b;
3     if (z <= c / d) {
4         x = z;
5         y = 1;
6         return;
7     }
8     a -= (z - 1) * b; c -= (z - 1) * d;
9     exgcd(d, c, b, a, y, x);
10    x += (z - 1) * y;
11 }
12
13 int main(int argc, char* argv[]) {
14     int T;
15     scanf("%d", &T);
16     ll p, x;
17     for (int kase = 1; kase <= T; ++kase) {
18         scanf("%lld%lld", &p, &x);
19         ll b, y;
20         exgcd(p, x, p, x - 1, b, y);
21         printf("%lld/%lld\n", b * x - p * y, b);
22     }
23     return 0;
24 }
```

## 1.3 同余

### 1.3.1 扩展欧几里得算法

```
1 void exgcd(int a, int b, int &x, int &y)
2 {
3     if(b == 0) { x = 1; y = 0; return; }
```

```
4     exgcd(b, a % b, x, y);
5     int t = x; x = y, y = t - a / b * y;
6 }
```

### 1.3.2 中国剩余定理

```
1  typedef long long ll;
2
3  void exgcd(ll a, ll b, ll &x, ll &y)
4  {
5      if(b == 0) { x = 1; y = 0; return; }
6      exgcd(b, a % b, x, y);
7      ll t = x; x = y, y = t - a / b * y;
8  }
9
10 ll crt(ll *a, ll *m, int n)
11 {
12     ll M = 1, ans = 0;
13     for(int i = 1; i <= n; i++) M *= m[i];
14     for(int i = 1; i <= n; i++)
15     {
16         ll x = 0, y = 0;
17         ll Mi = M / m[i];
18         exgcd(Mi, m[i], x, y);
19         ans = (ans + Mi % M * x % M * a[i] % M + M) % M;
20     }
21     if(ans < 0) ans += M;
22     return ans;
23 }
```

### 1.3.3 扩展中国剩余定理

```
1  typedef long long ll;
2
3  const int N = 1e5 + 10;
4
5  int n;
6  ll a[N], r[N];
7
8  ll exgcd(ll a, ll b, ll& x, ll& y)
9  {
10     if(b == 0) { x = 1, y = 0; return a; }
11     ll ret = exgcd(b, a % b, y, x); y -= a / b * x;
12     return ret;
13 }
14
15 ll excrt()
16 {
17     ll M = a[1], R = r[1], x, y, d;
18     for(int i = 2; i <= n; i++)
19     {
20         d = exgcd(M, a[i], x, y);
21         if((R - r[i]) % d) return -1;
22         x = (R - r[i]) / d * x % a[i];
23         R -= M * x;
24         M = M / d * a[i];
25         R %= M;
26     }
```

```
27     return (R % M + M) % M;
28 }
```

#### 1.3.4 BSGS

```
1  int qp(int a, int n, int mod)
2  {
3      long long ans = 1, base = a;
4      while(n)
5      {
6          if(n & 1) (ans *= base) %= mod;
7          (base *= base) %= mod;
8          n >>= 1;
9      }
10     return ans;
11 }
12
13 int BSGS(int a, int b, int p)
14 {
15     map<int, int> hash;
16     b %= p;
17     int t = (int)sqrt(p) + 1;
18     for(int j = 0; j < t; j++)
19     {
20         int val = 1ll * b * qp(a, j, p) % p;
21         hash[val] = j;
22     }
23     a = qp(a, t, p);
24     if(a == 0) return b == 0 ? 1 : -1;
25     for(int i = 0; i <= t; i++)
26     {
27         int val = qp(a, i, p);
28         int j = hash.find(val) == hash.end() ? -1 : hash[val];
29         if(j >= 0 && i * t - j >= 0) return i * t - j;
30     }
31     return -1;
32 }
```

#### 1.3.5 exBSGS

```
1  unordered_map<int, int> Hash;
2
3  int exBSGS(int a, int b, int p)
4  {
5      a %= p, b %= p;
6      if(b == 1) return 0;
7      if(!b && !a) return 1;
8      if(!a) return -1;
9      if(!b)
10     {
11         int ret = 0, d;
12         while((d = __gcd(a, p)) != 1)
13         {
14             ++ret, p /= d;
15             if(p == 1) return ret;
16         }
17         return -1;
18     }
```



```

19     int ret = 0, A = a, B = b, P = p, C = 1, d;
20     while((d = __gcd(A, P)) != 1)
21     {
22         if(B % d) return -1;
23         P /= d, B /= d;
24         C = 111 * C * (A / d) % P;
25         ++ ret;
26         if(C == B) return ret;
27     }
28     Hash.clear();
29     int f = 1, t = sqrt(P) + 1;
30     for(int i = 0; i < t; i++)
31     {
32         Hash[111 * f * B % P] = i;
33         f = 111 * f * A % P;
34     }
35     int tf = f;
36     f = 111 * f * C % P;
37     for(int i = 1; i <= t; i++)
38     {
39         if(Hash.find(f) != Hash.end()) return ret + i * t - Hash[f];
40         f = 111 * f * tf % P;
41     }
42     return -1;
43 }

```

### 1.3.6 逆元

```

1  /*
2  1. 费马小定理
3  条件: mod为素数
4  */
5  ll inv(ll x){return qp(x,mod-2);}
6
7  /*
8  2. 扩展欧几里得
9  条件: gcd(a,mod)==1
10  如果gcd(a,mod)!=1 返回-1
11  */
12  ll inv(ll a,ll p)
13  {
14      ll g,x,y;
15      g=exgcd(a,p,x,y);
16      return g==1?(x+p)%p:-1;
17  }
18
19  /*
20  3. 公式
21  a/b%mod=c
22  ->a%(b*mod)/b=c
23  */
24
25  /*
26  4. 逆元打表
27  p是模
28  p要求是奇素数
29  */
30  ll inv[MAX];

```

```

31 void getinv(int n,ll p)
32 {
33     ll i;
34     inv[1]=1;
35     for(i=2;i<=n;i++) inv[i]=(p-p/i)*inv[p%i]%p;
36 }
37
38 // log逆元
39 ll dlog(ll g, ll b, ll p) {
40     ll m = sqrt(p - 1);
41     map<ll, ll> powers;
42     for (long j = 0; j < m; j++) powers[qp(g, j, p)] = j;
43     long gm = qp(g, -m + 2 * (p - 1), p);
44     for (int i = 0; i < m; i++) {
45         if (powers[b]) return i * m + powers[b];
46         b = b * gm % p;
47     }
48     return -1;
49 }

```

### 1.3.7 模素数二次同余方程

```

1 // 要求模为素数, 输入n, mod, 返回  $x^2 \bmod = n$ , 可解任意一次二元方程
2
3 bool Legendre(ll a,ll p) {
4     return qp(a,p-1>>1,p)==1;
5 }
6
7 ll modsqr(ll a,ll p) {
8     ll x;
9     ll i,k,b;
10    if(p==2) x=a%p;
11    else if(p%4==3) x=qp(a,p+1>>2,p);
12    else {
13        for(b=1;Legendre(b,p);++b);
14        i=p-1>>1;
15        k=0;
16        do
17        {
18            i>>=1;
19            k>>=1;
20            if(!((1LL*qp(a,i,p)*qp(b,k,p)+1)%p)) k+=p-1>>1;
21        }while(!(i&1));
22        x=1ll*qp(a,i+1>>1,p)*qp(b,k>>1,p)%p;
23    }
24    return min(x, p - x);
25 //     if(p-x<x) x=p-x;
26 //     if(x==p-x) printf("%d\n",x);
27 //     else printf("%d %d\n",x,p-x);
28 }

```

## 1.4 矩阵与线性方程组

### 1.4.1 矩阵快速幂

```

1 const int mod = 1e9 + 7;
2 typedef long long ll;
3

```

```

4  int cur;
5  struct Matrix {ll a[105][105]; };
6
7  Matrix mul(Matrix a, Matrix b)
8  {
9      Matrix res;
10     memset(res.a, 0, sizeof res.a);
11     for(int i = 0; i < cur; i++)
12         for(int j = 0; j < cur; j++)
13             for(int k = 0; k < cur; k++)
14                 (res.a[i][j] += a.a[i][k] * b.a[k][j] % mod) %= mod;
15     return res;
16 }
17
18 Matrix pow(Matrix a, ll n)
19 {
20     Matrix ans, base = a;
21     for(int i = 0; i < cur; i++) ans.a[i][i] = 1;
22     while(n)
23     {
24         if(n & 1) ans = mul(ans, base);
25         base = mul(base, base);
26         n >>= 1;
27     }
28     return ans;
29 }

```

#### 1.4.2 高斯消元

```

1  const int N = 20 + 10;
2
3  int n;
4  double b[N], c[N][N];
5  //c: 系数矩阵, b: 常数; 二者一起构成增广矩阵
6
7  void Gaussian_Elimination()
8  {
9      for(int i = 1; i <= n; i++)
10     {
11         //找到x[i]的系数不为0的一个方程
12         for(int j = i; j <= n; j++) if(fabs(c[j][i]) > 1e-8)
13         {
14             for(int k = 1; k <= n; k++) swap(c[i][k], c[j][k]);
15             swap(b[i], b[j]);
16         }
17         //消去其他方程的x[i]的系数
18         for(int j = 1; j <= n; j++)
19         {
20             if(i == j) continue;
21             double rate = c[j][i] / c[i][i];
22             for(int k = i; k <= n; k++) c[j][k] -= c[i][k] * rate;
23             b[j] -= b[i] * rate;
24         }
25     }
26 }

```

#### 1.4.3 线性基

```
1  typedef long long ll;
2
3  struct LinearBasis
4  {
5      ll d[64], tot;
6
7      void ins(ll x)  //插入线性基
8      {
9          for(int i = 63; i >= 0; i --)
10             {
11                 if((x >> i) & 1)
12                     {
13                         if(!d[i]) return void(d[i] = x);
14                         x ^= d[i];
15                     }
16             }
17     }
18
19     ll max_xor()  //在一个序列中取若干个数，使其异或和最大
20     {
21         ll ans = 0;
22         for(int i = 63; i >= 0; i --)
23             if((ans ^ d[i]) > ans) ans ^= d[i];
24         return ans;
25     }
26
27     void init()
28     {
29         for(int i = 0; i < 64; i ++){ if(d[i])
30             for(int j = 0; j < i; j ++){
31                 if(d[i] & (1ll << j)) d[i] ^= d[j];
32             }
33         }
34
35         ll k_th(ll k)  //取任意个元素进行异或的第k小个数
36         {
37             //考虑能异或出0的情况，tot表示线性基中的元素个数
38             k -= (n != tot);
39             if(k > (1ll << tot)) return -1;
40             ll ans = 0;
41             for(int i = 0; i < tot; i ++){ if(k & (1ll << i)) ans ^= d[i];
42             }
43         }
44     };
```

## 1.5 组合数学

### 1.5.1 Lucas

```
1  const int maxn = 1e6 + 10;
2
3  ll fac[maxn], inv[maxn], facinv[maxn];
4
5  void init()
6  {
7      fac[0] = inv[0] = facinv[0] = 1;
8      fac[1] = inv[1] = facinv[1] = 1;
9      for(int i = 2; i < maxn; i++)
10     {
```

```
11     fac[i] = fac[i - 1] * i % mod;
12     inv[i] = mod - mod / i * inv[mod % i] % mod;
13     facinv[i] = facinv[i - 1] * inv[i] % mod;
14 }
15 }
16
17 ll C(int n, int k)
18 {
19     if(k > n || k < 0) return 0;
20     return fac[n] * facinv[k] % mod * facinv[n - k] % mod;
21 }
22
23 ll lucas(ll n, ll m)
24 {
25     ll res = 1;
26     while(n && m)
27     {
28         res = res * C(n % mod, m % mod) % mod;
29         n /= mod;
30         m /= mod;
31     }
32     return res;
33 }
```

### 1.5.2 exLucas

```
1  typedef long long ll;
2
3  ll p, n, m;
4
5  ll exgcd(ll a, ll b, ll &x, ll &y)
6  {
7      if(!b) { x = 1; y = 0; return a; }
8      ll res = exgcd(b, a % b, x, y), t;
9      t = x, x = y, y = t - a / b * y;
10     return res;
11 }
12
13 ll qp(ll a, ll n, ll mod)
14 {
15     ll ans = 1, base = a;
16     for(; n; n >>= 1, (base *= base) %= mod) if(n & 1) (ans *= base) %= mod;
17     return ans;
18 }
19
20 ll fac(ll n, ll a, ll b)
21 {
22     if(!n) return 1;
23     ll res = 1;
24     for(ll i = 2; i <= b; i++)
25         if(i % a) (res *= i) %= b;
26     res = qp(res, n / b, b);
27     for(ll i = 2; i <= n % b; i++)
28         if(i % a) (res *= i) %= b;
29     return res * fac(n / a, a, b) % b;
30 }
31
32 ll inv(ll n, ll mod)
```

```

33 {
34     ll x, y;
35     exgcd(n, mod, x, y);
36     return (x += mod) > mod ? x - mod : x;
37 }
38
39 ll CRT(ll b, ll mod) { return b * inv(p / mod, mod) % p * (p / mod) % p; }
40
41 ll C(ll n, ll m, ll a, ll b)
42 {
43     ll up = fac(n, a, b), d1 = fac(m, a, b), d2 = fac(n - m, a, b);
44     ll k = 0;
45     for(ll i = n; i; i /= a) k += i / a;
46     for(ll i = m; i; i /= a) k -= i / a;
47     for(ll i = n - m; i; i /= a) k -= i / a;
48     return up * inv(d1, b) % b * inv(d2, b) % b * qp(a, k, b) % b;
49 }
50
51 ll exlucas(ll n, ll m)
52 {
53     ll res = 0, tmp = p, b;
54     int lim = sqrt(p) + 5;
55     for(int i = 2; i <= lim; i++) if(tmp % i == 0)
56     {
57         b = 1;
58         while(tmp % i == 0) tmp /= i, b *= i;
59         (res += CRT(C(n, m, i, b), b)) %= p;
60     }
61     if(tmp > 1) (res += CRT(C(n, m, tmp, tmp), tmp)) %= p;
62     return res;
63 }

```

### 1.5.3 递推组合数

$0 \leq m \leq n \leq 1000$

```

1 const int maxn = 1010;
2 ll C[maxn][maxn];
3 void init() {
4     C[0][0] = 1;
5     for (int i = 1; i < maxn; i++)
6     {
7         C[i][0] = 1;
8         for (int j = 1; j <= i; j++) C[i][j] = (C[i - 1][j - 1] + C[i - 1][j]) % mod;
9     }
10 }

```

$0 \leq m \leq n \leq 10^5$ , 模  $p$  为素数

```

1 const int maxn = 100010;
2 ll f[maxn];
3 ll inv[maxn]; // 阶乘的逆元
4 void CalFact() {
5     f[0] = 1;
6     for (int i = 1; i < maxn; i++) f[i] = (f[i - 1] * i) % p;
7     inv[maxn - 1] = qp(f[maxn - 1], p - 2);
8     for (int i = maxn - 2; ~i; i--) inv[i] = inv[i + 1] * (i + 1) % p;
9 }
10 ll C(int n, int m) { return f[n] * inv[m] % p * inv[n - m] % p; }

```

### 1.5.4 小模数组合数

$p$  小  $n, m$  大

```
1
2  const int NICO = 100000+10;
3  const int MOD = 99991;
4  ll f[NICO];
5
6  ll Lucas(ll a, ll k)
7  {
8      ll res = 1;
9      while(a && k)
10     {
11         ll a1 = a % MOD;
12         ll b1 = k % MOD;
13         if(a1 < b1) return 0;
14         res = res*f[a1]*qp(f[b1]*f[a1-b1]%MOD, MOD-2)%MOD;
15         a /= MOD;
16         k /= MOD;
17     }
18     return res;
19 }
20
21 void init()
22 {
23     f[0] = 1;
24     for(int i=1; i<=MOD; i++)
25     {
26         f[i] = f[i-1]*i%MOD;
27     }
28 }
29
30 int main()
31 {
32     init();
33     cout << Lucas(5,2) << endl;
34 }
```

### 1.5.5 大模数组合数

$n, m$  小  $p$  大

```
1  map<int, ll> m;
2
3  const int MOD = 1e9+7;
4  void fun(int n, int k) {
5      for (int i = 2; i <= sqrt(n * 1.0); i++) {
6          while (n % i == 0) {
7              n /= i;
8              m[i] += k;
9          }
10     }
11     if (n > 1) {
12         m[n] += k;
13     }
14 }
15
```

```
16 ll C(ll a, ll b) {
17     if (a < b || a < 0 || b < 0)
18         return 0;
19     m.clear();
20     ll ret = 1;
21     b = min(a - b, b);
22     for (int i = 0; i < b; i++) {
23         fun(a - i, 1);
24     }
25     for (int i = b; i >= 1; i--) {
26         fun(i, -1);
27     }
28     for (__typeof(m.begin()) it = m.begin(); it != m.end(); it++) {
29         if ((*it).second != 0) {
30             ret *= qp((*it).first, (*it).second);
31             ret %= MOD;
32         }
33     }
34     return ret;
35 }
36
37 int main(int argc, char *argv[])
38 {
39     ll a, b;
40     while (scanf("%lld%lld", &a, &b) != EOF) {
41         printf("%lld\n", C(a, b));
42     }
43     return 0;
44 }
```

## 1.6 卷积

### 1.6.1 FFT

```
1  const int maxn = 1e7 + 10;
2  const double Pi = acos(-1.0);
3
4  struct complex
5  {
6      double x, y;
7      complex (double xx = 0, double yy = 0) { x = xx, y = yy; }
8  }a[maxn], b[maxn];
9
10 complex operator + (complex a, complex b) { return complex(a.x + b.x, a.y + b.y); }
11 complex operator - (complex a, complex b) { return complex(a.x - b.x, a.y - b.y); }
12 complex operator * (complex a, complex b) { return complex(a.x * b.x - a.y * b.y, a.x * b.y + a.y *
    b.x); }
13
14 int n, m;
15 int l, r[maxn];
16 int limit;
17
18 void FFT(complex *A, int type)
19 {
20     for(int i = 0; i < limit; i++)
21         if(i < r[i]) swap(A[i], A[r[i]]);
22     for(int mid = 1; mid < limit; mid <= 1)
23     {
24         complex Wn(cos(Pi / mid), type * sin(Pi / mid));
```



```
25     for(int R = mid << 1, j = 0; j < limit; j += R)
26     {
27         complex w(1, 0);
28         for(int k = 0; k < mid; k ++, w = w * Wn)
29         {
30             complex x = A[j + k], y = w * A[j + mid + k];
31             A[j + k] = x + y;
32             A[j + mid + k] = x - y;
33         }
34     }
35 }
36 }
37
38 void mul()
39 {
40     l = 0, limit = 1;
41     while(limit <= n + m) limit <=< 1, l ++;
42     for(int i = 0; i < limit; i ++ )
43         r[i] = (r[i >> 1] >> 1) | ( (i & 1) << (l - 1));
44     FFT(a, 1);
45     FFT(b, 1);
46     for(int i = 0; i <= limit; i ++ ) a[i] = a[i] * b[i];
47     FFT(a, -1);
48     for(int i = 0; i <= n + m; i ++ )
49         printf("%d ", (int)(a[i].x / limit + 0.5));
50 }
```

### 1.6.2 NTT

```
1  const int maxn = 2097152;
2  const int mod = 998244353;
3  const int root = 3;
4  // 998244353 -> 3, 1e9+7 -> 5,
5
6  template<long long mod, long long root>
7  struct NTT {
8      vector<long long> omega;
9
10     NTT() {
11         omega.resize(maxn + 1);
12         long long x = fpow(root, (mod - 1) / maxn);
13         omega[0] = 1ll;
14         for (int i = 1; i <= maxn; ++i)
15             omega[i] = omega[i - 1] * x % mod;
16     }
17
18     long long fpow(long long a, long long n) {
19         (n += mod - 1) %= mod - 1;
20         long long r = 1;
21         for (; n >>= 1; n >>= 1) {
22             if (n & 1) (r *= a) %= mod;
23             (a *= a) %= mod;
24         }
25         return r;
26     }
27
28     void bitrev(vector<long long> &v, int n) {
29         int z = __builtin_ctz(n) - 1;
```

```

30     for (int i = 0; i < n; ++i) {
31         int x = 0;
32         for (int j = 0; j <= z; ++j) x ^= (i >> j & 1) << (z - j);
33         if (x > i) swap(v[x], v[i]);
34     }
35 }
36
37 void ntt(vector<long long> &v, int n) {
38     bitrev(v, n);
39     for (int s = 2; s <= n; s <= 1) {
40         int z = s >> 1;
41         for (int i = 0; i < n; i += s) {
42             for (int k = 0; k < z; ++k) {
43                 long long x = v[i + k + z] * omega[maxn / s * k] % mod;
44                 v[i + k + z] = (v[i + k] + mod - x) % mod;
45                 (v[i + k] += x) %= mod;
46             }
47         }
48     }
49 }
50
51 void intt(vector<long long> &v, int n) {
52     ntt(v, n);
53     for (int i = 1; i < n / 2; ++i) swap(v[i], v[n - i]);
54     long long inv = fpow(n, -1);
55     for (int i = 0; i < n; ++i) (v[i] *= inv) %= mod;
56 }
57
58 vector<long long> operator()(vector<long long> a, vector<long long> b) {
59     int sz = 1;
60     while (sz < a.size() + b.size() - 1) sz <= 1;
61     while (a.size() < sz) a.push_back(0);
62     while (b.size() < sz) b.push_back(0);
63     ntt(a, sz), ntt(b, sz);
64     vector<long long> c(sz);
65     for (int i = 0; i < sz; ++i) c[i] = a[i] * b[i] % mod;
66     intt(c, sz);
67     while (c.size() && c.back() == 0) c.pop_back();
68     return c;
69 }
70
71 vector<long long> operator()(vector<long long> a, int n) {
72     int sz = 1;
73     while (sz < n * a.size()) sz <= 1;
74     while (a.size() < sz) a.push_back(0);
75     ntt(a, sz);
76     for (int i = 0; i < sz; ++i) a[i] = fpow(a[i], n);
77     intt(a, sz);
78     while (a.size() && a.back() == 0) a.pop_back();
79     return a;
80 }
81 };
82
83 NTT<mod, root> conv;

```

### 1.6.3 原根

```
1 #include<bits/stdc++.h>
```

```

2  #define ll long long
3  #define IL inline
4  #define RG register
5  using namespace std;
6
7  ll prm[1000],tot,N,root;
8
9  ll Power(ll bs,ll js,ll MOD){
10     ll S = 1,T = bs;
11     while(js){
12         if(js&1)S = S*T%MOD;
13         T = T*T%MOD;
14         js >>= 1;
15     } return S;
16 }
17
18 IL ll GetRoot(RG ll n){
19     RG ll tmp = n - 1 , tot = 0;
20     for(RG ll i = 2; i <= sqrt(tmp); i ++){
21         if(tmp%i==0){
22             prm[++tot] = i;
23             while(tmp%i==0)tmp /= i;
24         }
25     }
26     if(tmp != 1)prm[++tot] = tmp;           //质因数分解
27     for(RG ll g = 2; g <= n-1; g ++){
28         bool flag = 1;
29         for(RG int i = 1; i <= tot; i ++){   //检测是否符合条件
30             if(Power(g,(n-1)/prm[i],n) == 1)
31                 { flag = 0; break; }
32         }
33         if(flag)return g;
34     }return 0;                             //无解
35 }
36
37 int main(){
38     cin >> N;
39     root = GetRoot(N);
40     cout<<root<<endl;
41     return 0;
42 }

```

#### 1.6.4 FWT

```

1  //C_k=\sum_{i \oplus j=k} A_i B_j$
2  //FWT 完后需要先模一遍
3  template<typename T>
4  void fwt(ll a[], int n, T f) {
5      for (int d = 1; d < n; d *= 2)
6          for (int i = 0, t = d * 2; i < n; i += t)
7              for(int j = 0; j < d; j ++)
8                  f(a[i + j], a[i + j + d]);
9  }
10
11 void AND(ll& a, ll& b) { a += b; }
12 void OR(ll& a, ll& b) { b += a; }
13 void XOR (ll& a, ll& b) {
14     ll x = a, y = b;

```

```
15     a = (x + y) % mod;
16     b = (x - y + mod) % mod;
17 }
18 void rAND(ll& a, ll& b) { a -= b; }
19 void rOR(ll& a, ll& b) { b -= a; }
20 void rXOR(ll& a, ll& b) {
21     static ll INV2 = (mod + 1) / 2;
22     ll x = a, y = b;
23     a = (x + y) * INV2 % mod;
24     b = (x - y + mod) * INV2 % mod;
25 }
26
27 //FWT 子集卷积
28 a[popcount(x)][x] = A[x]
29 b[popcount(x)][x] = B[x]
30 fwt(a[i]) fwt(b[i])
31 c[i + j][x] += a[i][x] * b[j][x]
32 rfw(c[i])
33 ans[x] = c[popcount(x)][x]
```

## 1.7 多项式

### 1.7.1 拉格朗日插值

```
1  typedef long long ll;
2
3  const int mod = 998244353;
4  const int maxn = 1e5 + 10;
5
6  int x[maxn], y[maxn];
7
8  int qp(int a, int n)
9  {
10     ll ans = 1, base = a;
11     for(; n; (base *= base) %= mod, n >>= 1) if(n & 1) (ans *= base) %= mod;
12     return ans;
13 }
14
15 int lagrange(int n, int *x, int *y, int xi)
16 {
17     int ans = 0;
18     for(int i = 0; i <= n; i++)
19     {
20         int s1 = 1, s2 = 1;
21         for(int j = 0; j <= n; j++) if(i != j)
22         {
23             s1 = 1ll * s1 * (xi - x[j]) % mod;
24             s2 = 1ll * s2 * (x[i] - x[j]) % mod;
25         }
26         ans = (1ll * ans + 1ll * y[i] * s1 % mod * qp(s2, mod - 2) % mod) % mod;
27     }
28     return (ans + mod) % mod;
29 }
```

### 1.7.2 拉格朗日插值 (连续取值)

```
1  const int mod = 'edit';
2  const int maxn = 'edit';
```

```

3
4 int x[maxn], y[maxn];
5 int s1[maxn], s2[maxn], ifac[maxn];
6
7 //如果x的取值是连续一段, 可以做到O(n)求解
8 int lagrange(int n, int *x, int *y, int xi)
9 {
10     int ans = 0;
11     s1[0] = (xi - x[0]) % mod, s2[n + 1] = 1;
12     for(int i = 1; i <= n; i++) s1[i] = 1ll * s1[i - 1] * (xi - x[i]) % mod;
13     for(int i = n; i >= 0; i--) s2[i] = 1ll * s2[i + 1] * (xi - x[i]) % mod;
14     ifac[0] = ifac[1] = 1;
15     for(int i = 2; i <= n; i++) ifac[i] = -1ll * mod / i * ifac[mod % i] % mod;
16     for(int i = 2; i <= n; i++) ifac[i] = 1ll * ifac[i] * ifac[i - 1] % mod;
17     for(int i = 0; i <= n; i++)
18         (ans += 1ll * y[i] * (i == 0 ? 1 : s1[i - 1]) % mod * s2[i + 1] % mod * ifac[i] % mod * (((
19             n - i) & 1) ? -1 : 1) * ifac[n - i] % mod) %= mod;
20     return (ans + mod) % mod;
21 }

```

## 1.8 Others

1. 约数定理: 若  $n = \prod_{i=1}^k p_i^{a_i}$ , 则
  - (a) 约数个数  $f(n) = \prod_{i=1}^k (a_i + 1)$
  - (b) 约数和  $g(n) = \prod_{i=1}^k (\sum_{j=0}^{a_i} p_i^j)$
2. 小于  $n$  且互素的数之和为  $n\varphi(n)/2$
3. 若  $\gcd(n, i) = 1$ , 则  $\gcd(n, n - i) = 1 (1 \leq i \leq n)$
4. 错排公式:  $D(n) = (n - 1)(D(n - 2) + D(n - 1)) = \sum_{i=2}^n \frac{(-1)^k n!}{k!} = \lfloor \frac{n!}{e} + 0.5 \rfloor$
5. 威尔逊定理:  $p \text{ is prime} \Rightarrow (p - 1)! \equiv -1 \pmod{p}$
6. 欧拉定理:  $\gcd(a, n) = 1 \Rightarrow a^{\varphi(n)} \equiv 1 \pmod{n}$
7. 欧拉定理推广:  $\gcd(n, p) = 1 \Rightarrow a^n \equiv a^{n \% \varphi(p)} \pmod{p}$
8. 模的幂公式:  $a^n \pmod{m} = \begin{cases} a^n \pmod{m} & n < \varphi(m) \\ a^{n \% \varphi(m) + \varphi(m)} \pmod{m} & n \geq \varphi(m) \end{cases}$
9. 素数定理: 对于不大于  $n$  的素数个数  $\pi(n)$ ,  $\lim_{n \rightarrow \infty} \pi(n) = \frac{n}{\ln n}$
10. 位数公式: 正整数  $x$  的位数  $N = \log_{10}(n) + 1$
11. 斯特灵公式  $n! \approx \sqrt{2\pi n} (\frac{n}{e})^n$
12. 设  $a > 1, m, n > 0$ , 则  $\gcd(a^m - 1, a^n - 1) = a^{\gcd(m, n)} - 1$
13. 设  $a > b, \gcd(a, b) = 1$ , 则  $\gcd(a^m - b^m, a^n - b^n) = a^{\gcd(m, n)} - b^{\gcd(m, n)}$
14. 若  $\gcd(m, n) = 1$ , 则:
  - (a) 最大不能组合的数为  $m * n - m - n$
  - (b) 不能组合数个数  $N = \frac{(m-1)(n-1)}{2}$
15.  $(n + 1) \text{lcm}(C_n^0, C_n^1, \dots, C_n^{n-1}, C_n^n) = \text{lcm}(1, 2, \dots, n + 1)$
16. 若  $p$  为素数, 则  $(x + y + \dots + w)^p \equiv x^p + y^p + \dots + w^p \pmod{p}$

17. 卡特兰数: 1, 1, 2, 5, 14, 42, 132, 429, 1430, 4862, 16796, 58786, 208012

$$h(0) = h(1) = 1, h(n) = \frac{(4n-2)h(n-1)}{n+1} = \frac{C_{2n}^n}{n+1} = C_{2n}^n - C_{2n}^{n-1}$$

18. 伯努利数:  $B_n = -\frac{1}{n+1} \sum_{i=0}^{n-1} C_{n+1}^i B_i$

$$\sum_{i=1}^n i^k = \frac{1}{k+1} \sum_{i=1}^{k+1} C_{k+1}^i B_{k+1-i} (n+1)^i$$

19. 二项式反演:

$$f_n = \sum_{i=0}^n (-1)^i \binom{n}{i} g_i \Leftrightarrow g_n = \sum_{i=0}^n (-1)^i \binom{n}{i} f_i$$

$$f_n = \sum_{i=0}^n \binom{n}{i} g_i \Leftrightarrow g_n = \sum_{i=0}^n (-1)^{n-i} \binom{n}{i} f_i$$

20. 莫比乌斯反演:

(a) 令  $f(d) = \sum_{i=1}^n \sum_{j=1}^m [gcd(i, j) = d]$

(b)  $F(n) = \sum_{n|d} f(d) = \lfloor \frac{N}{n} \rfloor \lfloor \frac{M}{n} \rfloor$

(c) 有  $f(n) = \sum_{n|d} \mu(\lfloor \frac{d}{n} \rfloor) F(d)$

(d)  $\phi(n) = \sum_{d|n} d * \mu(n/d)$

21. 2 的 n 次方, 在 pow 时可以精确输出最大  $2^{1023}$ , pow(2,1023)

22. FFT 常用素数

$r \cdot 2^k + 1$	$r$	$k$	$g$
3	1	1	2
5	1	2	2
17	1	4	3
97	3	5	5
193	3	6	5
257	1	8	3
7681	15	9	17
12289	3	12	11
40961	5	13	3
65537	1	16	3
786433	3	18	10
5767169	11	19	3
7340033	7	20	3
23068673	11	21	3
104857601	25	22	3
167772161	5	25	3
469762049	7	26	3
998244353	119	23	3
1004535809	479	21	3
2013265921	15	27	31
2281701377	17	27	3
3221225473	3	30	5
75161927681	35	31	3
77309411329	9	33	7
206158430209	3	36	22
2061584302081	15	37	7
2748779069441	5	39	3
6597069766657	3	41	5
39582418599937	9	42	5
79164837199873	9	43	5
263882790666241	15	44	7
1231453023109121	35	45	3
1337006139375617	19	46	3
3799912185593857	27	47	5
4222124650659841	15	48	19
7881299347898369	7	50	6
31525197391593473	7	52	3
180143985094819841	5	55	6
1945555039024054273	27	56	5
4179340454199820289	29	57	3

### 1.8.1 BM

```

1 //Berlekamp-Massey
2 typedef vector<int> VI;
3 namespace linear_seq
4 {
5 #define rep(i,a,n) for (int i=a;i<n;i++)
6 #define SZ(x) ((int)(x).size())
7 #define pb(x) push_back(x)
8     const ll mod=1e9+7;
9     ll powmod(ll a,ll b){ll res=1;a%=mod; assert(b>=0); for(;;b>>=1){if(b&1)res=res*a%mod;a=a*a%mod;}return res;}
10     const int N=10010;
11     ll res[N],base[N],_c[N],_md[N];
12     vector<int> Md;
13     void mul(ll *a,ll *b,int k)
14     {

```

```

15     rep(i,0,k+k) _c[i]=0;
16     rep(i,0,k) if (a[i]) rep(j,0,k) _c[i+j]=(_c[i+j]+a[i]*b[j])%mod;
17     for (int i=k+k-1;i>=k;i--) if (_c[i])
18         rep(j,0,SZ(Md)) _c[i-k+Md[j]]=(_c[i-k+Md[j]]-_c[i]*_md[Md[j]])%mod;
19     rep(i,0,k) a[i]=_c[i];
20 }
21 int solve(ll n,VI a,VI b){
22     ll ans=0,pnt=0;
23     int k=SZ(a);
24     assert(SZ(a)==SZ(b));
25     rep(i,0,k) _md[k-1-i]=-a[i];_md[k]=1;
26     Md.clear();
27     rep(i,0,k) if (_md[i]!=0) Md.push_back(i);
28     rep(i,0,k) res[i]=base[i]=0;
29     res[0]=1;
30     while ((ll)<pnt)<=n) pnt++;
31     for (int p=pnt;p>=0;p--) {
32         mul(res,res,k);
33         if ((n>p)&1) {
34             for (int i=k-1;i>=0;i--) res[i+1]=res[i];res[0]=0;
35             rep(j,0,SZ(Md)) res[Md[j]]=(res[Md[j]]-res[k]*_md[Md[j]])%mod;
36         }
37     }
38     rep(i,0,k) ans=(ans+res[i]*b[i])%mod;
39     if (ans<0) ans+=mod;
40     return ans;
41 }
42 VI BM(VI s){
43     VI C(1,1),B(1,1);
44     int L=0,m=1,b=1;
45     rep(n,0,SZ(s)){
46         ll d=0;
47         rep(i,0,L+1) d=(d+(ll)C[i]*s[n-i])%mod;
48         if(d==0) ++m;
49         else if(2*L<=n){
50             VI T=C;
51             ll c=mod-d*powmod(b,mod-2)%mod;
52             while (SZ(C)<SZ(B)+m) C.pb(0);
53             rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
54             L=n+1-L; B=T; b=d; m=1;
55         } else {
56             ll c=mod-d*powmod(b,mod-2)%mod;
57             while (SZ(C)<SZ(B)+m) C.pb(0);
58             rep(i,0,SZ(B)) C[i+m]=(C[i+m]+c*B[i])%mod;
59             ++m;
60         }
61     }
62     return C;
63 }
64 int gao(VI a,ll n)
65 {
66     VI c=BM(a);
67     c.erase(c.begin());
68     rep(i,0,SZ(c)) c[i]=(mod-c[i])%mod;
69     return solve(n,c,VI(a.begin(),a.begin()+SZ(c)));
70 }
71 };//linear_seq::gao(VI{},n-1)

```



### 1.8.2 exBM

```
1 // given first m items init[0..m-1] and coefficients trans[0..m-1] or
2 // given first 2 *m items init[0..2m-1], it will compute trans[0..m-1]
3 // for you. trans[0..m] should be given as that
4 //      init[m] = sum_{i=0}^{m-1} init[i] * trans[i]
5 struct LinearRecurrence
6 {
7     using int64 = long long;
8     using vec = std::vector<int64>;
9
10    static void extend(vec& a, size_t d, int64 value = 0)
11    {
12        if (d <= a.size()) return;
13        a.resize(d, value);
14    }
15    static vec BerlekampMassey(const vec& s, int64 mod)
16    {
17        std::function<int64(int64)> inverse = [&](int64 a) {
18            return a == 1 ? 1 : (int64)(mod - mod / a) * inverse(mod % a) % mod;
19        };
20        vec A = {1}, B = {1};
21        int64 b = s[0];
22        for (size_t i = 1, m = 1; i < s.size(); ++i, m++)
23        {
24            int64 d = 0;
25            for (size_t j = 0; j < A.size(); ++j)
26            {
27                d += A[j] * s[i - j] % mod;
28            }
29            if (!(d % mod)) continue;
30            if (2 * (A.size() - 1) <= i)
31            {
32                auto temp = A;
33                extend(A, B.size() + m);
34                int64 coef = d * inverse(b) % mod;
35                for (size_t j = 0; j < B.size(); ++j)
36                {
37                    A[j + m] -= coef * B[j] % mod;
38                    if (A[j + m] < 0) A[j + m] += mod;
39                }
40                B = temp, b = d, m = 0;
41            }
42            else
43            {
44                extend(A, B.size() + m);
45                int64 coef = d * inverse(b) % mod;
46                for (size_t j = 0; j < B.size(); ++j)
47                {
48                    A[j + m] -= coef * B[j] % mod;
49                    if (A[j + m] < 0) A[j + m] += mod;
50                }
51            }
52        }
53        return A;
54    }
55    static void exgcd(int64 a, int64 b, int64& g, int64& x, int64& y)
56    {
57        if (!b)
```

```

58         x = 1, y = 0, g = a;
59     else
60     {
61         exgcd(b, a % b, g, y, x);
62         y -= x * (a / b);
63     }
64 }
65 static int64 crt(const vec& c, const vec& m)
66 {
67     int n = c.size();
68     int64 M = 1, ans = 0;
69     for (int i = 0; i < n; ++i) M *= m[i];
70     for (int i = 0; i < n; ++i)
71     {
72         int64 x, y, g, tm = M / m[i];
73         exgcd(tm, m[i], g, x, y);
74         ans = (ans + tm * x * c[i] % M) % M;
75     }
76     return (ans + M) % M;
77 }
78 static vec ReedsSloane(const vec& s, int64 mod)
79 {
80     auto inverse = [](int64 a, int64 m) {
81         int64 d, x, y;
82         exgcd(a, m, d, x, y);
83         return d == 1 ? (x % m + m) % m : -1;
84     };
85     auto L = [](const vec& a, const vec& b) {
86         int da = (a.size() > 1 || (a.size() == 1 && a[0])) ? a.size() - 1 : -1000;
87         int db = (b.size() > 1 || (b.size() == 1 && b[0])) ? b.size() - 1 : -1000;
88         return std::max(da, db + 1);
89     };
90     auto prime_power = [&](const vec& s, int64 mod, int64 p, int64 e) {
91         // linear feedback shift register mod p^e, p is prime
92         std::vector<vec> a(e), b(e), an(e), bn(e), ao(e), bo(e);
93         vec t(e), u(e), r(e), to(e, 1), uo(e), pw(e + 1);
94         ;
95         pw[0] = 1;
96         for (int i = pw[0] = 1; i <= e; ++i) pw[i] = pw[i - 1] * p;
97         for (int64 i = 0; i < e; ++i)
98         {
99             a[i] = {pw[i]}, an[i] = {pw[i]};
100            b[i] = {0}, bn[i] = {s[0] * pw[i] % mod};
101            t[i] = s[0] * pw[i] % mod;
102            if (t[i] == 0)
103            {
104                t[i] = 1, u[i] = e;
105            }
106            else
107            {
108                for (u[i] = 0; t[i] % p == 0; t[i] /= p, ++u[i])
109                ;
110            }
111        }
112        for (size_t k = 1; k < s.size(); ++k)
113        {
114            for (int g = 0; g < e; ++g)
115            {
116                if (L(an[g], bn[g]) > L(a[g], b[g]))

```

```

117         {
118             ao[g] = a[e - 1 - u[g]];
119             bo[g] = b[e - 1 - u[g]];
120             to[g] = t[e - 1 - u[g]];
121             uo[g] = u[e - 1 - u[g]];
122             r[g] = k - 1;
123         }
124     }
125     a = an, b = bn;
126     for (int o = 0; o < e; ++o)
127     {
128         int64 d = 0;
129         for (size_t i = 0; i < a[o].size() && i <= k; ++i)
130         {
131             d = (d + a[o][i] * s[k - i]) % mod;
132         }
133         if (d == 0)
134         {
135             t[o] = 1, u[o] = e;
136         }
137         else
138         {
139             for (u[o] = 0, t[o] = d; t[o] % p == 0; t[o] /= p, ++u[o])
140                 ;
141             int g = e - 1 - u[o];
142             if (L(a[g], b[g]) == 0)
143             {
144                 extend(bn[o], k + 1);
145                 bn[o][k] = (bn[o][k] + d) % mod;
146             }
147             else
148             {
149                 int64 coef = t[o] * inverse(to[g], mod) % mod * pw[u[o] - uo[g]] % mod;
150                 int m = k - r[g];
151                 extend(an[o], ao[g].size() + m);
152                 extend(bn[o], bo[g].size() + m);
153                 for (size_t i = 0; i < ao[g].size(); ++i)
154                 {
155                     an[o][i + m] -= coef * ao[g][i] % mod;
156                     if (an[o][i + m] < 0) an[o][i + m] += mod;
157                 }
158                 while (an[o].size() && an[o].back() == 0) an[o].pop_back();
159                 for (size_t i = 0; i < bo[g].size(); ++i)
160                 {
161                     bn[o][i + m] -= coef * bo[g][i] % mod;
162                     if (bn[o][i + m] < 0) bn[o][i + m] -= mod;
163                 }
164                 while (bn[o].size() && bn[o].back() == 0) bn[o].pop_back();
165             }
166         }
167     }
168 }
169 return std::make_pair(an[0], bn[0]);
170 };
171
172 std::vector<std::tuple<int64, int64, int>> fac;
173 for (int64 i = 2; i * i <= mod; ++i)
174 {
175     if (mod % i == 0)

```

```

176         {
177             int64 cnt = 0, pw = 1;
178             while (mod % i == 0) mod /= i, ++cnt, pw *= i;
179             fac.emplace_back(pw, i, cnt);
180         }
181     }
182     if (mod > 1) fac.emplace_back(mod, mod, 1);
183     std::vector<vec> as;
184     size_t n = 0;
185     for (auto&& x : fac)
186     {
187         int64 mod, p, e;
188         vec a, b;
189         std::tie(mod, p, e) = x;
190         auto ss = s;
191         for (auto&& x : ss) x %= mod;
192         std::tie(a, b) = prime_power(ss, mod, p, e);
193         as.emplace_back(a);
194         n = std::max(n, a.size());
195     }
196     vec a(n), c(as.size()), m(as.size());
197     for (size_t i = 0; i < n; ++i)
198     {
199         for (size_t j = 0; j < as.size(); ++j)
200         {
201             m[j] = std::get<0>(fac[j]);
202             c[j] = i < as[j].size() ? as[j][i] : 0;
203         }
204         a[i] = crt(c, m);
205     }
206     return a;
207 }
208
209 LinearRecurrence(const vec& s, const vec& c, int64 mod) : init(s), trans(c), mod(mod), m(s.size()
    ()) {}
210 LinearRecurrence(const vec& s, int64 mod, bool is_prime = true) : mod(mod)
211 {
212     vec A;
213     if (is_prime)
214         A = BerlekampMassey(s, mod);
215     else
216         A = ReedsSloane(s, mod);
217     if (A.empty()) A = {0};
218     m = A.size() - 1;
219     trans.resize(m);
220     for (int i = 0; i < m; ++i)
221     {
222         trans[i] = (mod - A[i + 1]) % mod;
223     }
224     std::reverse(trans.begin(), trans.end());
225     init = {s.begin(), s.begin() + m};
226 }
227 int64 calc(int64 n)
228 {
229     if (mod == 1) return 0;
230     if (n < m) return init[n];
231     vec v(m), u(m << 1);
232     int msk = !!n;
233     for (int64 m = n; m > 1; m >>= 1) msk <<= 1;

```

```

234     v[0] = 1 % mod;
235     for (int x = 0; msk; msk >>= 1, x <= 1)
236     {
237         std::fill_n(u.begin(), m * 2, 0);
238         x |= !(n & msk);
239         if (x < m)
240             u[x] = 1 % mod;
241         else
242             { // can be optimized by fft/ntt
243                 for (int i = 0; i < m; ++i)
244                 {
245                     for (int j = 0, t = i + (x & 1); j < m; ++j, ++t)
246                     {
247                         u[t] = (u[t] + v[i] * v[j]) % mod;
248                     }
249                 }
250                 for (int i = m * 2 - 1; i >= m; --i)
251                 {
252                     for (int j = 0, t = i - m; j < m; ++j, ++t)
253                     {
254                         u[t] = (u[t] + trans[j] * u[i]) % mod;
255                     }
256                 }
257             }
258         v = {u.begin(), u.begin() + m};
259     }
260     int64 ret = 0;
261     for (int i = 0; i < m; ++i)
262     {
263         ret = (ret + v[i] * init[i]) % mod;
264     }
265     return ret;
266 }
267
268 vec init, trans;
269 int64 mod;
270 int m;
271 };

```

### 1.8.3 杜教筛

```

1  #include <bits/stdc++.h>
2  #include <tr1/unordered_map>
3
4  using namespace std;
5  typedef long long ll;
6
7  const int N = 5e6;
8
9  bool vis[N + 1];
10 int mu[N + 1], sumu[N + 1], prim[N + 1], cnt;
11
12 tr1::unordered_map<int, int> Smu;
13
14 void get_mu(int n)
15 {
16     mu[1] = 1;
17     for(int i = 2; i <= n; i++)

```

```

18     {
19         if(!vis[i]) { prim[++ cnt] = i; mu[i] = -1; }
20         for(int j = 1; j <= cnt && prim[j] * i <= n; j ++)
21             {
22                 vis[prim[j] * i] = 1;
23                 if(i % prim[j] == 0) break;
24                 else mu[i * prim[j]] = -mu[i];
25             }
26     }
27     for(int i = 1; i <= n; i ++) sumu[i] = sumu[i - 1] + mu[i];
28 }
29
30 int phi[N + 1]; ll sumphi[N + 1];
31 tr1::unordered_map<ll, ll> Sphi;
32
33 void get(int n)
34 {
35     phi[1] = mu[1] = 1;
36     for(int i = 2; i <= n; i ++)
37     {
38         if(!vis[i])
39         {
40             prim[++ cnt] = i;
41             mu[i] = -1; phi[i] = i - 1;
42         }
43         for(int j = 1; j <= cnt && prim[j] * i <= n; j ++)
44         {
45             vis[i * prim[j]] = 1;
46             if(i % prim[j] == 0)
47             {
48                 phi[i * prim[j]] = phi[i] * prim[j];
49                 break;
50             }
51             else mu[i * prim[j]] = -mu[i], phi[i * prim[j]] = phi[i] * (prim[j] - 1);
52         }
53     }
54 }
55 for(int i = 1; i <= n; i ++) sumu[i] = sumu[i - 1] + mu[i], sumphi[i] = sumphi[i - 1] + phi[i];
56 }
57
58 ll getSum_mu(int x)
59 {
60     if(x <= N) return sumu[x];
61     if(Smu[x]) return Smu[x];
62     int ans = 1;
63     for(int l = 2, r; l >= 0 && l <= x && r < 2147483647; l = r + 1)
64     {
65         r = x / (x / l);
66         ans -= (r - l + 1) * getSum_mu(x / l);
67     }
68     return Smu[x] = ans;
69 }
70
71 ll getSum_phi(ll x)
72 {
73     if(x <= N) return sumphi[x];
74     if(Sphi[x]) return Sphi[x];
75     ll ans = x * (x + 1) / 2;
76     for(ll l = 2, r; l <= x; l = r + 1)

```

```
77     {
78         r = x / (x / 1);
79         ans -= (r - 1 + 1) * getSum_phi(x / 1);
80     }
81     return Sphi[x] = ans;
82 }
```

#### 1.8.4 欧拉降幂

```
1  const int maxn = 1e7+50;
2
3  int prim[maxn], vis[maxn];
4  int tot, phi[maxn];
5  struct node {
6      ll res;
7      bool v;
8  };
9
10 node qpow(ll A, ll B, ll C) {
11     ll re = 1;
12     bool flag = true;
13     while (B) {
14         if (B & 1) {
15             if ((re * A) >= C) flag = 0;
16             re = re % C;
17         }
18         B = B >> 1;
19         if (B) {
20             if (A >= C) flag = 0;
21             A %= C;
22             if ((A * A) >= C) flag = 0;
23             A %= C;
24         }
25     }
26     return node{re, flag};
27 }
28
29 void init(int n) {
30     phi[1] = 1;
31     for (int i = 2; i <= n; i++) {
32         if (!vis[i]) {
33             prim[++tot] = i;
34             phi[i] = i - 1;
35         }
36         for (int j = 1; j <= tot && prim[j] * i <= n; j++) {
37             vis[i * prim[j]] = 1;
38             if (i % prim[j] == 0) {
39                 phi[i * prim[j]] = phi[i] * prim[j];
40                 break;
41             } else phi[i * prim[j]] = phi[i] * (prim[j] - 1);
42         }
43     }
44 }
45 }
46
47
48 inline ll Euler(ll x) {
49     if (x <= maxn) return phi[x];
```

```
50     return 0;
51 }
52
53 node f(ll a, ll k, ll p) {
54     if (p == 1) return node{0, 0};
55     if (k == 0) return node{a % p, a < p};
56     ll ep = Euler(p);
57     node tmp = f(a, k - 1, ep);
58     if (__gcd(a, p) == 1) return qpow(a, tmp.res, p);
59     if (!tmp.v) {
60         tmp.res += ep;
61     }
62     return qpow(a, tmp.res, p);
63 }
64
65 int main() {
66     ll a, k, p;
67     init(1e7+2);
68     int T;
69     scanf("%d", &T);
70     for (int kase = 1; kase <= T; ++kase) {
71         // k次a次方模p的值
72         scanf("%lld%lld%lld", &a, &k, &p);
73         if (k == 0) printf("%lld\n", 1 % p);
74         else printf("%lld\n", f(a, k - 1, p).res);
75     }
76     return 0;
77 }
```



## 2 Graph Theory

### 2.1 路径

#### 2.1.1 Dijkstra

```
1  const int maxn = 1e5 + 10;
2  const int inf = 0x3f3f3f3f;
3
4  int head[maxn], dis[maxn], cnt, n;
5
6  struct Edge { int nex,to,w; }edge[20*maxn];
7
8  void add(int u,int v,int w)
9  {
10     edge[++cnt].nex=head[u];
11     edge[cnt].w=w;
12     edge[cnt].to=v;
13     head[u]=cnt;
14 }
15
16 void dijkstra(int s)
17 {
18     priority_queue<pair<int, int>, vector<pair<int, int> >, greater<pair<int, int> > > que;
19     memset(dis, 0x3f, sizeof dis);
20     que.push({0, s}); dis[s] = 0;
21     while(!que.empty())
22     {
23         auto f = que.top(); que.pop();
24         int u = f.second, d = f.first;
25         if(d != dis[u]) continue;
26         for(int i = head[u]; ~i; i = edge[i].nex)
27         {
28             int v = edge[i].to, w = edge[i].w;
29             if(dis[u] + w < dis[v])
30             {
31                 dis[v] = dis[u] + w;
32                 que.push({dis[v], v});
33             }
34         }
35     }
36 }
```

#### 2.1.2 Euler Path

```
1  int S[N << 1], top;
2  Edge edges[N << 1];
3  set<int> G[N];
4
5  void DFS(int u) {
6      S[top++] = u;
7      for (int eid: G[u])
8      {
9          int v = edges[eid].get_other(u);
10         G[u].erase(eid);
11         G[v].erase(eid);
12         DFS(v);
13         return;
14     }
```

```
15 }
16
17 void fleury(int start)
18 {
19     int u = start;
20     top = 0; path.clear();
21     S[top++] = u;
22     while (top)
23     {
24         u = S[--top];
25         if (!G[u].empty())
26             DFS(u);
27         else path.push_back(u);
28     }
29 }
```

### 2.1.3 K shortest Path(Astar)

```
1  const int inf = 0x3f3f3f3f;
2  const int maxn = 1000 + 10;
3  const int maxm = 100000 + 10;
4
5  int n, k, cnt, head[maxn], revhead[maxn], dis[maxn];
6  bool vis[maxn];
7
8  struct node { int v, w, nex; } edge[maxm], revedge[maxm];
9
10 void init()
11 {
12     cnt = 0;
13     memset(head, 0xff, sizeof head);
14     memset(revhead, 0xff, sizeof revhead);
15 }
16
17 void add(int u, int v, int w)
18 {
19     edge[cnt].v = v, revedge[cnt].v = u;
20     edge[cnt].w = revedge[cnt].w = w;
21     edge[cnt].nex = head[u];
22     revedge[cnt].nex = revhead[v];
23     head[u] = revhead[v] = cnt;
24     cnt++;
25 }
26
27 void spfa(int src)    //建立反向图，求图中所有点到终点的最短路径
28 {
29     for (int i = 1; i <= n; i++) dis[i] = inf;
30     memset(vis, false, sizeof vis);
31     vis[src] = 0;
32     queue<int> que;
33     que.push(src);
34     dis[src] = 0;
35     while (!que.empty())
36     {
37         int u = que.front();
38         que.pop();
39         vis[u] = false;
40         for (int i = revhead[u]; ~i; i = revedge[i].nex)
```

```

41     {
42         int v = revedge[i].v, w = revedge[i].w;
43         if (dis[v] > dis[u] + w)
44         {
45             dis[v] = dis[u] + w;
46             if (!vis[v])
47             {
48                 que.push(v);
49                 vis[v] = true;
50             }
51         }
52     }
53 }
54 }
55
56 struct A
57 {
58     int f, g, h;    //f(n),g(n),h(n)函数
59     int id;        //当前点的编号
60     bool operator<(const A a) const
61     {
62         //定义比较函数
63         if (a.f == f) return a.g < g;
64         return a.f < f;
65     };
66
67     int Astar(int src, int des)
68     {
69         int cnt = 0;
70         priority_queue<A> Q;
71         if (src == des) k++;    //如果起点即为终点
72         if (dis[src] == inf) return -1;    //如果起点不能到达终点
73         A st, now, tmp;
74         st.id = src, st.g = 0, st.f = st.g + dis[src];    //定义起始节点
75         Q.push(st);
76         while (!Q.empty())
77         {
78             now = Q.top();
79             Q.pop();
80             if (now.id == des)    //如果当前节点为终点
81             {
82                 cnt++;
83                 if (cnt == k) return now.g;    //找到第k短路
84             }
85             for (int i = head[now.id]; ~i; i = edge[i].nex)
86             {
87                 tmp.id = edge[i].v;
88                 tmp.g = now.g + edge[i].w;    //到该点的实际花费
89                 tmp.f = tmp.g + dis[tmp.id];    //到最终状态的估计花费
90                 Q.push(tmp);
91             }
92         }
93         return -1;    //路径总数小于k
94     }
95
96     int main()
97     {
98         int m, s, t, u, v, w;
99         while (scanf("%d%d", &n, &m) != EOF)

```

```

100     {
101         init();
102         while (m--)
103         {
104             scanf("%d%d%d", &u, &v, &w);
105             add(u, v, w);
106         }
107         scanf("%d%d%d", &s, &t, &k);
108         spfa(t);    //求所有点到终点的最短路
109         printf("%d\n", Astar(s, t));
110     }
111     return 0;
112 }

```

#### 2.1.4 K shortest Path(可持久化可并堆)

```

1  #include <bits/stdc++.h>
2  #include<ext/pb_ds/priority_queue.hpp>
3
4  using namespace std;
5
6  const int N = '';
7  const int M = '';
8  const int logM = 20;
9  const int inf = 0x3f3f3f3f;
10
11 int n, m, k, S, T;
12
13 struct Edge{ int nex, to, w; };
14
15 struct Graph
16 {
17     int head[N], cnt;
18     Edge edge[M];
19     void init(int n) { for(int i = 0; i <= n; i++) head[i] = 0; cnt = 0; }
20     void addedge(int u, int v, int val) { edge[++ cnt].nex = head[u], edge[cnt].to = v, edge[cnt].w
        = val, head[u] = cnt; }
21 }g, rg;
22
23 int dis[N];
24
25 void dijkstra()
26 {
27     priority_queue<pair<int, int>, vector<pair<int, int> >, greater<pair<int, int> > > que;
28     memset(dis, inf, sizeof dis);
29     que.push({0, T}); dis[T] = 0;
30     const int *head = rg.head; const Edge *edge = rg.edge;
31     while(!que.empty())
32     {
33         auto f = que.top(); que.pop();
34         int u = f.second, d = f.first;
35         if(d != dis[u]) continue;
36         for(int i = head[u]; i; i = edge[i].nex)
37         {
38             int v = edge[i].to, w = edge[i].w;
39             if(dis[u] + w < dis[v]) { dis[v] = dis[u] + w; que.push({dis[v], v}); }
40         }
41     }

```

```
42 }
43
44 bool tree_edge[M], vis[N];
45 int fa[N], st[N], top;
46
47 void dfs(int u)
48 {
49     vis[u] = true;
50     st[++ top] = u;
51     for(int i = rg.head[u]; i; i = rg.edge[i].nex)
52     {
53         int v = rg.edge[i].to;
54         if(!vis[v] && dis[v] == dis[u] + rg.edge[i].w)
55         {
56             fa[v] = u;
57             tree_edge[i] = true;
58             dfs(v);
59         }
60     }
61 }
62
63 namespace LT
64 {
65     int son[M * logM][2];
66     int ht[M * logM], val[M * logM], id[M * logM];
67     int tot;
68
69     int newnode(int _val, int _id, int _dis = 0)
70     {
71         int now = ++ tot;
72         val[now] = _val, id[now] = _id;
73         ht[now] = _dis, son[now][0] = son[now][1] = 0;
74         return now;
75     }
76
77     int _copy(int ori)
78     {
79         int now = ++tot;
80         val[now] = val[ori], id[now] = id[ori];
81         ht[now] = ht[ori], son[now][0] = son[ori][0], son[now][1] = son[ori][1];
82         return now;
83     }
84
85     int merge(int a, int b)
86     {
87         if(!a || !b) return a | b;
88         if(val[a] > val[b]) swap(a, b);
89         int now = _copy(a);
90         son[now][1] = merge(son[now][1], b);
91         if(ht[son[now][0]] < ht[son[now][1]]) swap(son[now][0], son[now][1]);
92         ht[now] = ht[son[now][1]] + 1;
93         return now;
94     }
95
96     void insert(int &rt, int val, int id) { rt = merge(newnode(val, id), rt); }
97 }
98
99 int rt[M];
100
```

```

101 void build_heap()
102 {
103     for(int i = 1; i <= top; i++)
104     {
105         int u = st[i];
106         rt[u] = rt[fa[u]];
107         for(int i = g.head[u]; i; i = g.edge[i].nex)
108         {
109             int v = g.edge[i].to;
110             if(!tree_edge[i] && dis[v] != inf) LT::insert(rt[u], dis[v] - dis[u] + g.edge[i].w, v);
111         }
112     }
113 }
114
115 int solve(int k)
116 {
117     if(k == 1) return dis[S];
118     __gnu_pbds::priority_queue<pair<int, int>, greater<pair<int, int> > > que;
119     que.push({dis[S] + LT::val[rt[S]], rt[S]});
120     while(!que.empty())
121     {
122         pair<int, int> f = que.top(); que.pop();
123         if(--k == 1) return f.first;
124         int v = f.first, u = f.second;
125         int lc = LT::son[u][0], rc = LT::son[u][1], o = LT::id[u];
126         if(rt[o]) que.push({v + LT::val[rt[o]], rt[o]});
127         if(lc) que.push({v + LT::val[lc] - LT::val[u], lc});
128         if(rc) que.push({v + LT::val[rc] - LT::val[u], rc});
129     }
130     return -1;
131 }
132
133 void init()
134 {
135     g.init(n), rg.init(n);
136     memset(rt, 0, sizeof rt);
137     memset(tree_edge, 0, sizeof tree_edge);
138     top = LT::tot = 0;
139 }
140
141 void getans()
142 {
143     //input S-T
144     init();
145     dijkstra();
146     dfs(T);
147     build_heap();
148     cout << solve(k);
149 }

```

## 2.2 生成树

### 2.2.1 Kruskal

```

1  const int maxn = 1e5 + 10;
2
3  int n, m, pre[maxn];
4  struct edge {int u, v, w; } es[maxn];
5  int Find(int x) { return x == pre[x] ? x : pre[x] = Find(pre[x]); }

```

```
6 bool cmp(const edge &x, const edge &y) { return x.cost < y.cost; }
7
8 int kruskal()
9 {
10     sort(es, es + m, cmp);
11     int res = 0;
12     for(int i = 0; i < m; i ++)
13     {
14         int fx = Find(es[i].u), fy = Find(es[i].v);
15         if(fx != fy) pre[fx] = fy, res += es[i].cost;
16     }
17     return res;
18 }
```

### 2.2.2 Prim

```
1 const int maxn = 1000 + 10;
2 const int inf = 0x3f3f3f3f;
3
4 int n, mp[maxn][maxn], cost[maxn];
5 bool vis[maxn];
6
7 int prim()
8 {
9     for(int i = 0; i < n; i ++) cost[i] = inf, vis[i] = false;
10    int res = 0; cost[0] = 0;
11    for(;;)
12    {
13        int v = -1;
14        for(int u = 0; u < n; u ++)
15            if(!vis[u] && (v == -1 || cost[u] < cost[v])) v = u;
16        if(v == -1) break;
17        res += cost[v];
18        vis[v] = true;
19        for(int u = 0; u < n; u ++) cost[u] = min(cost[u], mp[v][u]);
20    }
21    return res;
22 }
```

### 2.2.3 最小树形图

```
1 const int INF = 0x3f3f3f3f;
2 const int maxn = 10000;
3 const int maxm = 10000;
4
5 struct Edge{int u,v,cost; } edge[maxm];
6
7 int pre[maxn], id[maxn], vis[maxn], in[maxn];
8
9 int zhuliu(int root, int n, int m)
10 {
11     int res=0, u, v;
12     for(;;)
13     {
14         for(int i=0; i<n; i++) in[i] = INF;
15         for(int i=0; i<m; i++) if(edge[i].u != edge[i].v && edge[i].cost < in[edge[i].v])
16             {
17                 pre[edge[i].v] = edge[i].u;
```

```

18         in[edge[i].v] = edge[i].cost;
19     }
20     for(int i=0; i<n; i++) if(i != root && in[i] ==INF) return -1;
21     int tn=0;
22     memset(id, 0xff, sizeof id);
23     memset(vis, 0xff, sizeof vis);
24     in[root] = 0;
25     for(int i=0; i<n;i++)
26     {
27         res += in[i];
28         v = i;
29         while( vis[v] != i && id[v] == -1 && v!= root) vis[v] = i, v = pre[v];
30         if(v != root && id[v] == -1)
31         {
32             for(int u = pre[v]; u != v; u = pre[u]) id[u] = tn;
33             id[v] = tn++;
34         }
35     }
36     if(tn == 0) break;
37     for(int i=0; i<n; i++) if(id[i] == -1) id[i] = tn++;
38     for(int i=0; i<m; )
39     {
40         v = edge[i].v;
41         edge[i].u = id[edge[i].u];
42         edge[i].v = id[edge[i].v];
43         if(edge[i].u != edge[i].v) edge[i++].cost -= in[v];
44         else swap(edge[i], edge[--m]);
45     }
46     n = tn;
47     root = id[root];
48 }
49 return res;
50 }

```

## 2.2.4 Matrix Tree

```

1  const int N = 305;
2  const int mod = 1e9 + 7;
3
4  int n, m, a[N][N];
5
6  int Gauss(int n) {
7      int ans = 1;
8      for (int i = 1; i <= n; i++) {
9          for (int k = i + 1; k <= n; k++) {
10             while (a[k][i]) {
11                 int d = a[i][i] / a[k][i];
12                 for (int j = i; j <= n; j++) {
13                     a[i][j] = (a[i][j] - 1LL * d * a[k][j] % mod + mod) % mod;
14                 }
15                 std::swap(a[i], a[k]);
16                 ans = - ans;
17             }
18         }
19         ans = 1LL * ans * a[i][i] % mod;
20     }
21     return (ans % mod + mod) % mod;
22 }

```



```
23 int main() {
24     scanf("%d%d", &n, &m);
25     for (int i = 1; i <= m; i++) {
26         int u, v;
27         scanf("%d%d", &u, &v);
28         a[u][v]--, a[v][u]--;
29         a[u][u]++, a[v][v]++;
30     }
31     printf("%d\n", Gauss(n - 1));
32     return 0;
33 }
```

## 2.3 连通性

### 2.3.1 割点

```
1  const int maxn = 1e4 + 10;
2
3  vector<int> edge[maxn];
4  int n, dfn[maxn], low[maxn], cnt = 0;
5  bool vis[maxn], cut[maxn];
6
7  void Tarjan(int u, int fa)
8  {
9      dfn[u] = low[u] = ++cnt;
10     vis[u] = true;
11     int children = 0;
12     for (int i = 0; i < edge[u].size(); i++)
13     {
14         int v = edge[u][i];
15         if (v != fa && vis[v])
16             low[u] = min(low[u], dfn[v]);
17         else if (!vis[v])
18         {
19             Tarjan(v, u);
20             children++;
21             low[u] = min(low[u], low[v]);
22             if (fa == -1 && children > 1) //若u是根节点且子节点数大于1
23                 cut[u] = true;        //u是割点
24             else if (fa != -1 && low[v] >= dfn[u]) //若u不是根节点且v不能访问到u的父节点
25                 cut[u] = true;        //u是割点
26         }
27     }
28 }
```

### 2.3.2 桥

```
1  const int maxn = 1e4 + 10;
2
3  vector<int> edge[maxn];
4  int n, dfn[maxn], low[maxn], father[maxn], cnt = 0;
5  bool bridge[maxn][maxn];
6
7  void Tarjan(int u, int fa)
8  {
9      dfn[u] = low[u] = ++cnt;
10     for (int i = 0; i < edge[u].size(); i++)
11     {
```

```

12     int v = edge[u][i];
13     if (!dfn[v])    //未访问节点v
14     {
15         Tarjan(v, u);
16         low[u] = min(low[u], low[v]);
17         if (low[v] > dfn[u]) //节点v到达祖先必须经过(u,v)
18             bridge[u][v] = bridge[v][u] = true;    //(u,v)是桥
19     }
20     else if (fa != v)    //u的父节点不是v, (u,v)不存在重边
21         low[u] = min(low[u], dfn[v]);
22 }
23 }

```

### 2.3.3 强连通分量

```

1  const int maxn=1000+10;
2
3  vector<int> edge[maxn];
4
5  int dfn[maxn], low[maxn];
6  int stack[maxn], index, tot;
7  int belong[maxn], inde[maxn], outde[maxn], scc;
8  bool vis[maxn];
9
10 void add(int u, int v)
11 {
12     edge[u].push_back(v);
13     edge[v].push_back(u);
14 }
15
16 void Tarjan(int u)
17 {
18     dfn[u] = low[u] = ++tot;
19     stack[++index] = u;
20     vis[u] = true;
21     int v;
22     for(int i = 0; i < edge[u].size(); i++)
23     {
24         v=edge[u][i];
25         if(!dfn[v])
26         {
27             Tarjan(v);
28             low[u] = min(low[v], low[u]);
29         }
30         else if(vis[v]) low[u] = min(low[v], dfn[u]);
31     }
32     if(dfn[u] == low[u])
33     {
34         scc++;
35         do
36         {
37             v = stack[index--];
38             vis[v] = false;
39             belong[v] = scc;
40         }while(v != u);
41     }
42 }

```

## 2.3.4 点双联通分量

```

1  const int maxn = 10000 + 10;
2
3  struct Edge{ int u, v; };
4  vector<int> G[maxn], bcc[maxn];
5
6  int dfn[maxn], low[maxn], bccno[maxn], idx, bcc_cnt, bridge;
7  bool iscut[maxn];
8
9  stack<Edge> st;
10
11 void dfs(int u, int pre)
12 {
13     dfn[u] = low[u] = ++idx;
14     int child = 0;
15     for(auto v : G[u])
16     {
17         if(v == pre) continue;
18         if(!dfn[v])
19         {
20             child++;
21             st.push({u, v});
22             dfs(v, u);
23             low[u] = min(low[u], low[v]);
24             if(low[v] >= dfn[u])
25             {
26                 iscut[u] = true;
27                 bcc[++bcc_cnt].clear();
28                 Edge x;
29                 do
30                 {
31                     x = st.top(); st.pop();
32                     if(bccno[x.u] != bcc_cnt) { bcc[bcc_cnt].push_back(x.u); bccno[x.u] = bcc_cnt;
33
34                     if(bccno[x.v] != bcc_cnt) { bcc[bcc_cnt].push_back(x.v); bccno[x.v] = bcc_cnt;
35
36                     } while(x.u != u || x.v != v);
37                 }
38                 if(low[v] > dfn[u]) ++ bridge;
39             }
40             else if(dfn[v] < dfn[u])
41             {
42                 st.push({u, v});
43                 low[u] = min(low[u], dfn[v]);
44             }
45         }
46         if(pre < 0 && child == 1) iscut[u] = 0;
47     }
48 }
49
50 void find_bcc(int n)
51 {
52     memset(dfn, 0, sizeof dfn);
53     memset(iscut, 0, sizeof iscut);
54     memset(bccno, 0, sizeof bccno);
55     for(int i = 1; i <= bcc_cnt; i++) bcc[i].clear();
56     idx = bcc_cnt = bridge = 0;
57     for(int i = 0; i < n; i++) if(!dfn[i]) dfs(i, -1);
58 }

```

### 2.3.5 边双联通分量

```

1  const int maxn = 10000 + 10;
2
3  int low[maxn], dfn[maxn], head[maxn], cnt, idx;
4  int cutEdge[maxn << 2];
5  struct Edge { int nex, v; } edge[maxn << 2];
6
7  void add(int u, int v) { edge[cnt].nex = head[u], edge[cnt].v = v, head[u] = cnt++; }
8
9  void dfs(int u, int pre)
10 {
11     low[u] = dfn[u] = ++idx;
12     for(int i = head[u]; ~i; i = edge[i].nex)
13     {
14         int v = edge[i].v;
15         if(v == pre) continue;
16         if(!dfn[v])
17         {
18             dfs(v, u);
19             low[u] = min(low[u], low[v]);
20             if(low[v] > dfn[u]) cutEdge[i] = cutEdge[i ^ 1] = 1;
21         }
22         else if(dfn[v] < dfn[u]) low[u] = min(low[u], dfn[v]);
23     }
24 }
```

## 2.4 二分图匹配

1. 二分图中的最大匹配数 = 最小点覆盖数
2. 最小路径覆盖 = 最小路径覆盖 =  $|G|$  - 最大匹配数
3. 二分图最大独立集 = 顶点数 - 最小点覆盖
4. 二分图的最大团 = 补图的最大独立集

### 2.4.1 Hungary Algorithm

```

1  const int maxn = 150;
2
3  int n;
4  int edge[maxn][maxn];
5  int linker[maxn];
6  bool vis[maxn];
7
8  bool path(int u)
9  {
10     for (int v = 1; v <= n; v++)
11     {
12         if (edge[u][v] && !vis[v])
13         {
14             vis[v] = true;
15             if (linker[v] == -1 || path(linker[v]))
16             {
17                 linker[v] = u;
18                 return true;
19             }
20         }
21     }
```

```
22     return false;
23 }
24
25 int hungary()
26 {
27     int res = 0;
28     memset(linker, 0xff, sizeof(linker));
29     for (int i = 1; i <= n; i++)
30     {
31         memset(vis, false, sizeof(vis));
32         res += path(i);
33     }
34     return res;
35 }
```

### 2.4.2 Hopcroft-karp Algorithm

```
1  //复杂度 $O(n^{0.5} * m)$ , 注意这个板子的下标是从0开始的
2
3  const int MAXN = 3010; //左边节点数量、右边节点数量
4  const int MAXM = 3010 * 3010; //边的数量
5  const int INF = 0x3f3f3f3f;
6
7  struct Edge
8  {
9      int v;
10     int next;
11 } edge[MAXM];
12
13 int nx, ny;
14 int cnt;
15 int dis;
16
17 int first[MAXN];
18 int xlink[MAXN], ylink[MAXN];
19 /*xlink[i]表示左集合顶点所匹配的右集合顶点序号, ylink[i]表示右集合i顶点匹配到的左集合顶点序号。*/
20 int dx[MAXN], dy[MAXN];
21 /*dx[i]表示左集合i顶点的距离编号, dy[i]表示右集合i顶点的距离编号*/
22 int vis[MAXN]; //寻找增广路的标记数组
23
24 void init()
25 {
26     cnt = 0;
27     memset(first, -1, sizeof(first));
28     memset(xlink, -1, sizeof(xlink));
29     memset(ylink, -1, sizeof(ylink));
30 }
31
32 void read_graph(int u, int v)
33 {
34     edge[cnt].v = v;
35     edge[cnt].next = first[u], first[u] = cnt++;
36 }
37
38 int bfs()
39 {
40     queue<int> q;
41     dis = INF;
```

```
42     memset(dx, -1, sizeof(dx));
43     memset(dy, -1, sizeof(dy));
44     for (int i = 0; i < nx; i++)
45     {
46         if (xlink[i] == -1)
47         {
48             q.push(i);
49             dx[i] = 0;
50         }
51     }
52     while (!q.empty())
53     {
54         int u = q.front();
55         q.pop();
56         if (dx[u] > dis) break;
57         for (int e = first[u]; e != -1; e = edge[e].next)
58         {
59             int v = edge[e].v;
60             if (dy[v] == -1)
61             {
62                 dy[v] = dx[u] + 1;
63                 if (ylink[v] == -1) dis = dy[v];
64                 else
65                 {
66                     dx[ylink[v]] = dy[v] + 1;
67                     q.push(ylink[v]);
68                 }
69             }
70         }
71     }
72     return dis != INF;
73 }
74
75 int find(int u)
76 {
77     for (int e = first[u]; e != -1; e = edge[e].next)
78     {
79         int v = edge[e].v;
80         if (!vis[v] && dy[v] == dx[u] + 1)
81         {
82             vis[v] = 1;
83             if (ylink[v] != -1 && dy[v] == dis) continue;
84             if (ylink[v] == -1 || find(ylink[v]))
85             {
86                 xlink[u] = v, ylink[v] = u;
87                 return 1;
88             }
89         }
90     }
91     return 0;
92 }
93
94 int MaxMatch()
95 {
96     int ans = 0;
97     while (bfs())
98     {
99         memset(vis, 0, sizeof(vis));
100        for (int i = 0; i < nx; i++)
```

```

101         if (xlink[i] == -1)
102             ans += find(i);
103     }
104     return ans;
105 }

```

### 2.4.3 二分图多重匹配

```

1  const int maxn = 1e2 + 5; //左边最大点数
2  const int maxm = 1e2 + 5; //右边最大点数
3  int graph[maxn][maxm], vis[maxm]; //图G和增广路访问标记
4  int match[maxm][maxn]; //左边元素与右边元素第n次匹配
5  int nx, ny, m; //左边点数, 右边点数, 边数
6  int vol[maxm]; //右边点多重匹配可容纳值
7  int cnt[maxm]; //右边点已匹配值
8
9  bool find_path(int u) //找增广路
10 {
11     for (int i = 0; i < ny; i++) //注意, 这里节点是从0开始编号, 题目有时是从1开始编号
12     {
13         if (graph[u][i] && !vis[i]) //不在增广路
14         {
15             vis[i] = 1; //放进增广路
16             if (cnt[i] < vol[i]) //如果当前已匹配数量小于可容纳量, 则直接匹配
17             {
18                 match[i][cnt[i]++] = u;
19                 return true;
20             }
21             for (int j = 0; j < cnt[i]; j++)
22             {
23                 if (find_path(match[i][j])) //如果先前已匹配右边的点能另外找到增广路, 则此点仍可匹配
24                 {
25                     match[i][j] = u;
26                     return true;
27                 }
28             }
29         }
30     }
31     return false;
32 }
33
34 int max_match() //计算多重匹配的最大匹配数
35 {
36     int res = 0;
37     memset(match, -1, sizeof(match));
38     memset(cnt, 0, sizeof(cnt));
39     for (int i = 0; i < nx; i++)
40     {
41         memset(vis, 0, sizeof(vis));
42         if (find_path(i)) res++;
43     }
44     return res;
45 }
46
47 bool all_match() //判断左边的点是否都与右边的点匹配了
48 {
49     memset(cnt, 0, sizeof(cnt));
50     for (int i = 0; i < nx; i++)

```

```

51     {
52         memset(vis, 0, sizeof(vis));
53         if (!find_path(i)) return false;
54     }
55     return true;
56 }

```

#### 2.4.4 二分图最大权匹配 (KM 算法)

```

1  const int maxn=1000+10;
2  const int inf=0x3f3f3f3f;
3
4  int n;
5  int lx[maxn],ly[maxn],edge[maxn][maxn];
6  int match[maxn],delta;
7  bool vx[maxn],vy[maxn];
8
9  bool dfs(int x) //DFS增广, 寻找相等子图的完备匹配
10 {
11     vx[x]=true;
12     for(int y=1;y<=n;y++)
13     {
14         if(!vy[y])
15         {
16             int tmp=lx[x]+ly[y]-edge[x][y];
17             if(!tmp) //edge(x,y)为可行边
18             {
19                 vy[y]=true;
20                 if(!match[y]||dfs(match[y]))
21                 {
22                     match[y]=x;
23                     return true;
24                 }
25             }
26             else delta=min(delta,tmp);
27         }
28     }
29     return false;
30 }
31
32 void KM()
33 {
34     for(int i=1;i<=n;i++) //初始化可行顶标的值
35     {
36         lx[i]=-inf;
37         ly[i]=0;
38         for(int j=1;j<=n;j++)
39             lx[i]=max(lx[i],edge[i][j]);
40     }
41     memset(match,0,sizeof(match));
42     for(int x=1;x<=n;x++)
43     {
44         for(;;)
45         {
46             delta=inf;
47             memset(vx,0,sizeof(vx));
48             memset(vy,0,sizeof(vy));
49             if(dfs(x)) break;

```



```
50         for(int i=1;i<=n;i++)    //修改顶标
51         {
52             if(vx[i]) lx[i]-=delta;
53             if(vy[i]) ly[i]+=delta;
54         }
55     }
56 }
57 }
```

#### 2.4.5 一般图匹配带花树

```
1  //一般图匹配，带花树算法
2  const int maxn = 1000 + 10;
3
4  vector<int> edge[maxn];
5  queue<int> que;
6
7  int n, pre[maxn], type[maxn], link[maxn], nex[maxn], vis[maxn];
8
9  void add(int u, int v)
10 {
11     edge[u].push_back(v);
12     edge[v].push_back(u);
13 }
14
15 int Find(int x)
16 {
17     return x == pre[x] ? x : pre[x] = Find(pre[x]);
18 }
19
20 void combine(int x, int lca)    //如果找到奇环，对当前点x和找到的
21 {
22     while (x != lca)
23     {
24         int u = link[x], v = nex[u];
25         if (Find(v) != lca) nex[v] = u;
26         if (type[u] == 1) type[u] = 2, que.push(u);
27         pre[Find(x)] = Find(u);
28         pre[Find(u)] = Find(v);
29         x = v;
30     }
31 }
32
33 void contrack(int x, int y)
34 {
35     int lca = x;
36     memset(vis, 0, sizeof(vis));
37     for (int i = x; i; i = nex[link[i]])
38     {
39         i = Find(i);
40         vis[i] = 1;
41     }
42     for (int i = y; i; i = nex[link[i]])
43     {
44         i = Find(i);
45         if (vis[i])
46         {
47             lca = i;
```

```
48         break;
49     }
50 }
51 if (lca != Find(x)) nex[x] = y;
52 if (lca != Find(y)) nex[y] = x;
53 combine(x, lca);
54 combine(y, lca);
55 }
56
57 void bfs(int s)
58 {
59     memset(type, 0, sizeof(type));
60     memset(nex, 0, sizeof(nex));
61     for (int i = 1; i <= n; i++) pre[i] = i;
62     while (!que.empty()) que.pop();
63     que.push(s);
64     type[s] = 2;
65     while (!que.empty())
66     {
67         int x = que.front();
68         que.pop();
69         for (int i = 0; i < edge[x].size(); i++)
70         {
71             int y = edge[x][i];
72             if (Find(x) == Find(y) || link[x] == y || type[y] == 1) continue;
73             if (type[y] == 2) contrack(x, y);
74             else if (link[y])
75             {
76                 nex[y] = x;
77                 type[y] = 1;
78                 type[link[y]] = 2;
79                 que.push(link[y]);
80             } else
81             {
82                 nex[y] = x;
83                 int pos = y, u = nex[pos], v = link[u];
84                 while (pos)
85                 {
86                     link[pos] = u;
87                     link[u] = pos;
88                     pos = v;
89                     u = nex[pos];
90                     v = link[u];
91                 }
92                 return;
93             }
94         }
95     }
96 }
97
98 int maxmatch()
99 {
100     for (int i = 1; i <= n; i++) if (!link[i]) bfs(i);
101     int ans = 0;
102     for (int i = 1; i <= n; i++) if (link[i]) ans++;
103     return ans / 2;
104 }
105
106 void init()
```

```

107 {
108     for (int i = 1; i <= n; i++) edge[i].clear();
109     memset(link, 0, sizeof(link));
110 }

```

## 2.5 网络流

### 2.5.1 Dinic

```

1  const int MAX_V = 1000 + 10;
2  const int INF = 0x3f3f3f3f;
3
4  //用于表示边的结构体 (终点, 流量, 反向边)
5  struct edge{int to, cap, rev;};
6
7  vector<edge> G[MAX_V]; //图的邻接表表示
8  int level[MAX_V]; //顶点到源点的距离标号
9  int iter[MAX_V]; //当前弧
10
11 void add(int from, int to, int cap)
12 {
13     G[from].push_back((edge){to, cap, (int)G[to].size()});
14     G[to].push_back((edge){from, 0, (int)G[from].size() - 1});
15 }
16
17 //计算从源点出发的距离标号
18 void bfs(int s)
19 {
20     memset(level, -1, sizeof(level));
21     queue<int> que;
22     level[s] = 0;
23     que.push(s);
24     while(!que.empty())
25     {
26         int v = que.front(); que.pop();
27         for(int i = 0; i < G[v].size(); i++)
28         {
29             edge &e = G[v][i];
30             if(e.cap > 0 && level[e.to] < 0)
31             {
32                 level[e.to] = level[v] + 1;
33                 que.push(e.to);
34             }
35         }
36     }
37 }
38
39 //通过DFS寻找增广路
40 int dfs(int v, int t, int f)
41 {
42     if(v == t) return f;
43     for(int &i = iter[v]; i < G[v].size(); i++)
44     {
45         edge &e = G[v][i];
46         if(e.cap > 0 && level[v] < level[e.to])
47         {
48             int d = dfs(e.to, t, min(f, e.cap));
49             if(d > 0)
50             {

```

```

51         e.cap -= d;
52         G[e.to][e.rev].cap += d;
53         return d;
54     }
55 }
56 }
57 return 0;
58 }
59
60 //求解从s到t的最大流
61 int max_flow(int s, int t)
62 {
63     int flow = 0;
64     for(;;)
65     {
66         bfs(s);
67         if(level[t] < 0) return flow;
68         memset(iter, 0, sizeof(iter));
69         int f;
70         while((f = dfs(s,t,INF)) > 0) flow += f;
71     }
72 }

```

### 2.5.2 ISAP

```

1  struct Edge {
2      int from, to, cap, flow;
3      Edge(int u, int v, int c, int f) : from(u), to(v), cap(c), flow(f) {}
4  };
5
6  bool operator<(const Edge& a, const Edge& b) {
7      return a.from < b.from || (a.from == b.from && a.to < b.to);
8  }
9
10 struct ISAP {
11     int n, m, s, t;
12     vector<Edge> edges;
13     vector<int> G[maxn];
14     bool vis[maxn];
15     int d[maxn];
16     int cur[maxn];
17     int p[maxn];
18     int num[maxn];
19
20     void AddEdge(int from, int to, int cap) {
21         edges.push_back(Edge(from, to, cap, 0));
22         edges.push_back(Edge(to, from, 0, 0));
23         m = edges.size();
24         G[from].push_back(m - 2);
25         G[to].push_back(m - 1);
26     }
27
28     bool BFS() {
29         memset(vis, 0, sizeof(vis));
30         queue<int> Q;
31         Q.push(t);
32         vis[t] = 1;
33         d[t] = 0;

```

```
34     while (!Q.empty()) {
35         int x = Q.front();
36         Q.pop();
37         for (int i = 0; i < G[x].size(); i++) {
38             Edge& e = edges[G[x][i] ^ 1];
39             if (!vis[e.from] && e.cap > e.flow) {
40                 vis[e.from] = 1;
41                 d[e.from] = d[x] + 1;
42                 Q.push(e.from);
43             }
44         }
45     }
46     return vis[s];
47 }
48
49 void init(int n) {
50     this->n = n;
51     for (int i = 0; i < n; i++) G[i].clear();
52     edges.clear();
53 }
54
55 int Augment() {
56     int x = t, a = INF;
57     while (x != s) {
58         Edge& e = edges[p[x]];
59         a = min(a, e.cap - e.flow);
60         x = edges[p[x]].from;
61     }
62     x = t;
63     while (x != s) {
64         edges[p[x]].flow += a;
65         edges[p[x] ^ 1].flow -= a;
66         x = edges[p[x]].from;
67     }
68     return a;
69 }
70
71 int Maxflow(int s, int t) {
72     this->s = s;
73     this->t = t;
74     int flow = 0;
75     BFS();
76     memset(num, 0, sizeof(num));
77     for (int i = 0; i < n; i++) num[d[i]]++;
78     int x = s;
79     memset(cur, 0, sizeof(cur));
80     while (d[s] < n) {
81         if (x == t) {
82             flow += Augment();
83             x = s;
84         }
85         int ok = 0;
86         for (int i = cur[x]; i < G[x].size(); i++) {
87             Edge& e = edges[G[x][i]];
88             if (e.cap > e.flow && d[x] == d[e.to] + 1) {
89                 ok = 1;
90                 p[e.to] = G[x][i];
91                 cur[x] = i;
92                 x = e.to;
```

```
93         break;
94     }
95 }
96 if (!ok) {
97     int m = n - 1;
98     for (int i = 0; i < G[x].size(); i++) {
99         Edge& e = edges[G[x][i]];
100         if (e.cap > e.flow) m = min(m, d[e.to]);
101     }
102     if (--num[d[x]] == 0) break;
103     num[d[x] = m + 1]++;
104     cur[x] = 0;
105     if (x != s) x = edges[p[x]].from;
106 }
107 }
108 return flow;
109 }
110 };
```

### 2.5.3 MCMF

```
1  const int maxn = 10000 + 10;
2  const int inf = 0x3f3f3f3f;
3
4  struct Edge { int from, to, cap, flow, cost; };
5
6  struct MCMF
7  {
8      int n, m;
9      vector<Edge> edges;
10     vector<int> G[maxn];
11     bool inq[maxn];
12     int dis[maxn], path[maxn], a[maxn];
13
14     void init(int n)
15     {
16         this->n = n;
17         for(int i = 0; i <= n; i++)
18             G[i].clear();
19         edges.clear();
20     }
21
22     void addEdge(int from, int to, int cap, int cost)
23     {
24         edges.push_back(Edge{from, to, cap, 0, cost});
25         edges.push_back(Edge{to, from, 0, 0, -cost});
26         m = edges.size();
27         G[from].push_back(m - 2);
28         G[to].push_back(m - 1);
29     }
30
31     bool Bellman_Ford(int s, int t, int& flow, int& cost)
32     {
33         for(int i = 0; i <= n; i++) dis[i] = inf;
34         memset(inq, 0, sizeof inq);
35         dis[s] = 0, inq[s] = true, path[s] = 0, a[s] = inf;
36         queue<int> Q;
37         Q.push(s);
```

```

38     while(!Q.empty())
39     {
40         int u = Q.front(); Q.pop();
41         inq[u] = false;
42         for(int i = 0; i < G[u].size(); i++)
43         {
44             Edge& e = edges[G[u][i]];
45             if(e.cap > e.flow && dis[e.to] > dis[u] + e.cost)
46             {
47                 dis[e.to] = dis[u] + e.cost;
48                 path[e.to] = G[u][i];
49                 a[e.to] = min(a[u], e.cap - e.flow);
50                 if(!inq[e.to])
51                 {
52                     Q.push(e.to);
53                     inq[e.to] = true;
54                 }
55             }
56         }
57     }
58     if(dis[t] == inf) return false;    //求最小费用最大流
59     //if(1ll * dis[t] * a[t] > 0) return false; 求可行流最小费用, 因此当费用增量大于0时不继续增加
流量
60     flow += a[t];
61     cost += dis[t] * a[t];
62     for(int u = t; u != s; u = edges[path[u]].from)
63     {
64         edges[path[u]].flow += a[t];
65         edges[path[u] ^ 1].flow -= a[t];
66     }
67     return true;
68 }
69
70 int mincostMaxFlow(int s, int t)
71 {
72     int flow = 0, cost = 0;
73     while(Bellman_Ford(s, t, flow, cost));
74     return cost;
75 }
76 };

```

## 2.5.4 Trick

### 建模技巧

**二分图带权最大独立集。**给出一个二分图，每个结点上有一个正权值。要求选出一些点，使得这些点之间没有边相连，且权值和最大。

**解：**在二分图的基础上添加源点  $S$  和汇点  $T$ ，然后从  $S$  向所有  $X$  集合中的点连一条边，所有  $Y$  集合中的点向  $T$  连一条边，容量均为该点的权值。 $X$  结点与  $Y$  结点之间的边的容量均为无穷大。这样，对于图中的任意一个割，将割中的边对应的结点删掉就是一个符合要求的解，权和为所有权减去割的容量。因此，只需要求出最小割，就能求出最大权和。

**公平分配问题。**把  $m$  个任务分配给  $n$  个处理器。其中每个任务有两个候选处理器，可以任选一个分配。要求所有处理器中，任务数最多的那个处理器所分配的任务数尽量少。不同任务的候选处理器集  $\{p_1, p_2\}$  保证不同。

**解：**本题有一个比较明显的二分图模型，即  $X$  结点是任务， $Y$  结点是处理器。二分答案  $x$ ，然后构图，首先从源点  $S$  出发向所有的任务结点引一条边，容量等于 1，然后从每个任务结点出发引两条边，分别到达它所能分配到的两个处理器结点，容量为 1，最后从每个处理器结点出发引一条边到汇点  $T$ ，容量为  $x$ ，表示选择该处理器的任务不能超过  $x$ 。

这样网络中的每个单位流量都是从  $S$  流到一个任务结点，再到处理器结点，最后到汇点  $T$ 。只有当网络中的总流量等于  $m$  时才意味着所有任务都选择了一个处理器。这样，我们通过  $O(\log m)$  次最大流便算出了答案。

**区间  $k$  覆盖问题。**数轴上有一些带权值的左闭右开区间。选出权和尽量大的一些区间，使得任意一个数最多被  $k$  个区间覆盖。

**解：**本题可以用最小费用流解决，构图方法是把每个数作为一个结点，然后对于权值为  $w$  的区间  $[u, v)$  加边  $u \rightarrow v$ ，容量为 1，费用为  $-w$ 。再对所有相邻的点加边  $i \rightarrow i+1$ ，容量为  $k$ ，费用为 0。最后，求最左点到最右点的最小费用最大流即可，其中每个流量对应一组互不相交的区间。如果数值范围太大，可以先进行离散化。

**最大闭合子图。**给定带权图  $G$ （权值可正可负），求一个权和最大的点集，使得起点在该点集中的任意弧，终点也在该点集中。

**解：**新增附加源  $s$  和附加汇  $t$ ，从  $s$  向所有正权点引一条边，容量为权值；从所有负权点向汇点引一条边，容量为权值的相反数。求出最小割以后， $S - \{s\}$  就是最大闭合子图。

**最大密度子图。**给出一个无向图，找一个点集，使得这些点之间的边数除以点数的值（称为子图的密度）最大。

**解：**如果两个端点都选了，就必然要选边，这就是一种推导。如果把每个点和每条边都看成新图中的结点，可以把问题转化为最大闭合子图。

**无源汇有上下界可行流：**附加源  $S$  和汇  $T$ ；对于边  $(u, v, min, max)$ ，记  $d[u]- = min, d[v]+ = max$ ，并添加弧  $(u, v, max - min)$ ；对于流量不平衡的点  $u$ ，设多余流量为  $W$ ，如果  $W > 0$ ，添加弧  $S \rightarrow u : W$ ，否则若  $W < 0$ ，添加弧  $u \rightarrow T : -W$ ，求改造后的网络  $S - T$  最大流即可，当且仅当所有附加弧满载时原图有可行流。

**有源汇有上下界可行流：**建  $t \rightarrow s$ ，容量为  $inf$ ，然后和无源汇相同。

**有源汇有上下界最大/最小流：**与上面相同，跑完可行流  $S \rightarrow T$  后去掉边  $t \rightarrow s$ ，最大流为加  $s \rightarrow t$ ，最小流为  $G[s][t].cap - maxflow(t, s)$ 。

## 2.6 Others

### 2.6.1 拓扑排序

```

1  const int maxn = 1e5 + 10;
2
3  vector<int> edge[maxn];
4  int indegree[maxn];
5
6  void add(int u, int v)
7  {
8      edge[u].push_back(v);
9      indegree[v]++;
10 }
11
12 void Toposort(int n)
13 {
14     queue<int> que;
15     for (int i = 1; i <= n; i++)
16         if (!indegree[i]) que.push(i);    //将图中没有前驱，即入度为0的点加入队列
17     while (!que.empty())
18     {
19         int u = que.front();
20         que.pop();
21         indegree[u] = -1;    //从图中删去此顶点
22         for (int i = 0; i < edge[u].size(); i++)
23         {
24             int v = edge[u][i];
25             indegree[v]--;    //删去图中以u为尾的弧

```



```

26         if (!indegree[v]) que.push(v);    //将新增的当前入度为0的点压入队列中
27     }
28 }
29 }

```

## 2.6.2 2-SAT

```

1  /*2-SAT连边含义: 选A必选B
2     点 $x_i$ 表示选,  $x_i'$ 表示不选
3     1. 必选 $x_i$ , 等价于 $x_i=1$ :  $x_i \rightarrow x_i$ 
4     2. 必不选 $x_i$ , 等价于 $x_i=0$ ,  $x_i \rightarrow x_i'$ 
5     3.  $x_i$ 与 $x_j$ 中至少选择一个, 等价于 $x_i \vee x_j=1$ , 连边 $x_i \rightarrow x_j, x_j' \rightarrow x_i$ 
6     4.  $x_i$ 与 $x_j$ 不都选, 等价于 $x_i \wedge x_j=0$ , 连边 $x_i \rightarrow x_j', x_j \rightarrow x_i'$ 
7     5.  $x_i$ 与 $x_j$ 情况相同, 等价于 $x_i \oplus x_j=0$ , 连边 $x_i \rightarrow x_j, x_i' \rightarrow x_j', x_j \rightarrow x_i, x_j' \rightarrow x_i'$ 
8     6.  $x_i$ 与 $x_j$ 情况相反, 等价于 $x_i \oplus x_j=1$ , 连边 $x_i \rightarrow x_j', x_i' \rightarrow x_j, x_j \rightarrow x_i', x_j' \rightarrow x_i$ 
9  */
10
11  const int maxn = 2e6 + 10;
12
13  int n, m, a, va, b, vb;
14  int low[maxn], dfn[maxn], color[maxn], cnt, scc_cnt;
15  bool instack[maxn];
16
17  vector<int> g[maxn];
18  stack<int> st;
19
20  void Tarjan(int u)
21  {
22      low[u] = dfn[u] = ++cnt;
23      st.push(u);
24      instack[u] = true;
25      for(const auto &v : g[u])
26      {
27          if(!dfn[v]) Tarjan(v), low[u] = min(low[u], low[v]);
28          else if(instack[v]) low[u] = min(low[u], dfn[v]);
29      }
30      if(low[u] == dfn[u])
31      {
32          ++scc_cnt;
33          do {
34              color[u] = scc_cnt;
35              u = st.top(); st.pop();
36              instack[u] = false;
37          } while(low[u] != dfn[u]);
38      }
39  }
40
41  inline void add(int a, int b) { g[a].push_back(b); }
42
43  inline void AND(int a, int b, int c)
44  {
45      if(c == 1) add(a, a + n), add(b, b + n);
46      else add(a + n, b), add(b + n, a);
47  }
48
49  inline void OR(int a, int b, int c)
50  {
51      if(c == 0) add(a + n, a), add(b + n, b);

```

```

52     else add(a, b + n), add(b, a + n);
53 }
54
55 inline void XOR(int a, int b, int c)
56 {
57     if(c == 0) add(a, b), add(a + n, b + n), add(b, a), add(b + n, a + n);
58     else add(a, b + n), add(a + n, b), add(b, a + n), add(b + n, a);
59 }
60
61 bool TWO_SAT()
62 {
63     input();
64     for(int i = 1; i <= (n << 1); i++) if(!dfn[i]) Tarjan(i);
65     for(int i = 1; i <= n; i++)
66         if(color[i] == color[i + n]) return false;
67     for(int i = 1; i <= n; i++)
68         printf("%d ", color[i] > color[i + n]);
69     return true;
70 }

```

### 2.6.3 差分约束系统

```

1  //以 $x_i - x_j$ 为约束条件，建图求最短路后得到的是最大解。所有的解都不大于且尽可能逼近 $dis[x0]$ 
2  //最短路对应最大解，最长路对应最小解
3
4  const int maxn = 1000 + 10;
5  const int inf = 0x3f3f3f3f;
6
7  struct Edge
8  {
9      int nex, to, w;
10 } edge[10 * maxn];
11
12 int head[maxn], cnt, dis[maxn], n;
13 bool vis[maxn];
14
15 void init()
16 {
17     cnt = 0;
18     memset(head, 0xff, sizeof head);
19 }
20
21 void add(int u, int v, int w)
22 {
23     edge[cnt].nex = head[u];
24     edge[cnt].to = v;
25     edge[cnt].w = w;
26     head[u] = ++cnt;
27 }
28
29 void spfa(int u)
30 {
31     int u, v, w;
32     for (int i = 1; i <= n; i++) dis[i] = inf, vis[i] = false;
33     dis[u] = 0;
34     queue<int> que;
35     que.push(u);
36     vis[u] = true;

```

```

37     while (!que.empty())
38     {
39         u = que.front();
40         que.pop();
41         vis[u] = false;
42         for (int i = head[u]; ~i; i = edge[i].nex)
43         {
44             v = edge[i].v, w = edge[i].w;
45             if (dis[u] + w < dis[v])
46             {
47                 dis[v] = dis[u] + w;
48                 if (!vis[v])
49                 {
50                     que.push(v);
51                     vis[v] = true;
52                 }
53             }
54         }
55     }
56 }

```

#### 2.6.4 支配树

```

1  const int N = 2e5 + 10;
2
3  int n, m;
4
5  struct G
6  {
7      vector<int> edge[N];
8      inline void add(int u, int v) { edge[u].push_back(v); }
9  }a, b, c, d;
10
11 int dfn[N], id[N], fa[N], cnt;
12
13 void dfs(int u)
14 {
15     dfn[u] = ++ cnt; id[cnt] = u;
16     int len = a.edge[u].size();
17     for(auto v : a.edge[u]) if(!dfn[v]) { fa[v] = u; dfs(v); }
18 }
19
20 int semi[N], idom[N], belong[N], val[N];
21
22 int find(int x)
23 {
24     if(x == belong[x]) return x;
25     int tmp = find(belong[x]);
26     if(dfn[semi[val[belong[x]]]] < dfn[semi[val[x]]]) val[x] = val[belong[x]];
27     return belong[x] = tmp;
28 }
29
30 void tarjan()
31 {
32     for(int i = cnt; i > 1; i --)
33     {
34         int u = id[i];
35         for(auto v : b.edge[u])

```

```
36     {
37         if(!dfn[v]) continue;
38         find(v);
39         if(dfn[semi[val[v]]] < dfn[semi[u]]) semi[u] = semi[val[v]];
40     }
41     c.add(semi[u], u);
42     belong[u] = fa[u];
43     u = fa[u];
44     for(auto v : c.edge[u])
45     {
46         find(v);
47         if(semi[val[v]] == u) idom[v] = u;
48         else idom[v] = val[v];
49     }
50 }
51 for(int i = 2; i <= cnt; i++)
52 {
53     int u = id[i];
54     if(idom[u] != semi[u]) idom[u] = idom[idom[u]];
55 }
56 }
57
58 int ans[N];
59
60 void dfs_ans(int u)
61 {
62     ans[u] = 1;
63     for(auto v : d.edge[u]) dfs_ans(v), ans[u] += ans[v];
64 }
65
66 void solve()
67 {
68     int u, v;
69     scanf("%d%d", &n, &m);
70     while(m--)
71     {
72         scanf("%d%d", &u, &v);
73         a.add(u, v);
74         b.add(v, u);
75     }
76     for(int i = 1; i <= n; i++) semi[i] = belong[i] = val[i] = i;
77     dfs(1);
78     tarjan();
79     for(int i = 2; i <= n; i++) d.add(idom[i], i);
80     dfs_ans(1);
81     for(int i = 1; i <= n; i++) printf("%d ", ans[i]);
82 }
```

## 3 DataStructure

### 3.1 SegmentTreeDS

#### 3.1.1 SegmentTree

```
1  const int maxn = 2e5+5;
2  // 序列
3  int a[maxn];
4
5  struct SegmentTree {
6  #define TYPE int
7  #define USELAZY 0
8      TYPE val[maxn << 2];
9      int sz;
10 //    check this type
11     vector<int> lazy;
12
13     inline TYPE comb(const TYPE& a, const TYPE& b) {
14         TYPE res;
15         res = a + b;
16         return res;
17     }
18
19     int le, re, k;
20
21     inline void build(int rt, int l, int r) {
22         if (USELAZY) lazy[rt] = 0;
23         if (l == r) {
24             val[rt] = a[l];
25             return;
26         }
27         int mid = l + r >> 1;
28         build(rt << 1, l, mid);
29         build(rt << 1 | 1, mid + 1, r);
30         pushup(rt);
31     }
32     inline void build() {build(1, 1, sz);}
33
34     inline void init(int sz_) {
35         sz = sz_;
36         lazy.resize(sz_ << 2);
37         build();
38     }
39     inline void pushup(int rt) {val[rt] = comb(val[rt << 1], val[rt << 1 | 1]);}
40     inline void deal(int rt, int kt) {
41 //        todo:
42         val[rt] = comb(val[rt], kt);
43     }
44     inline void pushdown(int rt, int len) {
45         if (lazy[rt]) {
46             // check the lazy change
47             lazy[rt << 1] += lazy[rt];
48             lazy[rt << 1 | 1] += lazy[rt];
49             deal(rt << 1, lazy[rt]);
50             deal(rt << 1 | 1, lazy[rt]);
51             lazy[rt] = 0;
52         }
53     }
```

```

54
55     inline void update(int rt, int l, int r) {
56         if (le <= l && r <= re) {
57             deal(rt, k);
58             return;
59         }
60         if (USELAZY) pushdown(rt, r - l + 1);
61         int mid = l + r >> 1;
62         if (le <= mid) update(rt << 1, l, mid);
63         if (re > mid) update(rt << 1 | 1, mid + 1, r);
64         pushup(rt);
65     }
66
67     inline TYPE query(int rt, int l, int r) {
68         if (le <= l && r <= re) {
69             return val[rt];
70         }
71         if (USELAZY) pushdown(rt, r - l + 1);
72         // check the zero type
73         TYPE res;
74         int mid = l + r >> 1;
75         if (le <= mid) res = comb(res, query(rt << 1, l, mid));
76         if (re > mid) res = comb(res, query(rt << 1 | 1, mid + 1, r));
77         return res;
78     }
79
80     // check return type
81     inline int query(int l, int r) {
82         le = l, re = r;
83         return query(1, 1, sz);
84     }
85     inline void modify(int l, int r, int kt) {
86         le = l, re = r, k = kt;
87         update(1, 1, sz);
88     }
89
90     // inline void pt(int rt, int l, int r) {
91     //     if (l == r) {
92     //         printf("%d ", val[l]);
93     //         return;
94     //     }
95     //     pushdown(rt, r - l + 1);
96     //     int mid = l + r >> 1;
97     //     if (le <= mid) pt(rt << 1, l, mid);
98     //     if (re > mid) pt(rt << 1 | 1, mid + 1, r);
99     // }
100
101 #undef TYPE
102 };

```

### 3.1.2 离散化区间

```

1 // 原题1e5个区间有2e5个端点，离散化出来4e5个区间
2 // 然后线段树需要4e5*4=16e5的大小
3 // 注意三个数组要开离散化数量的四倍，如果不需要sz可以不用这个数组。
4 int val[maxn << 4];
5 int lpos[maxn << 2], rpos[maxn << 2], tot, sz[maxn << 2];
6 vector<int> xpos;

```

```

7  sort(xpos.begin(), xpos.end());
8  xpos.erase(unique(xpos.begin(), xpos.end()), xpos.end());
9  tot = 1;
10 lpos[1] = rpos[1] = xpos[0];
11 sz[1] = 1;
12 for (int i = 1; i < xpos.size(); ++i) {
13     if (xpos[i] - xpos[i - 1] != 1) {
14         lpos[++tot] = xpos[i - 1] + 1;
15         rpos[tot] = xpos[i] - 1;
16         sz[tot] = rpos[tot] - lpos[tot] + 1;
17     }
18     ++tot;
19     lpos[tot] = rpos[tot] = xpos[i];
20     sz[tot] = 1;
21 }
22 le = lower_bound(lpos + 1, lpos + 1 + tot, p[i].x) - lpos;
23 re = upper_bound(rpos + 1, rpos + 1 + tot, p[i].y) - rpos - 1;

```

### 3.1.3 动态区间最大子段和

```

1  namespace ST {
2      struct node{
3          ll ans,ls,rs,sum;
4      }xx[maxn << 2];
5      inline void pushdown(int x){
6          xx[x].sum=xx[x<<1].sum+xx[x<<1|1].sum;
7          xx[x].ls=max(xx[x<<1].ls,xx[x<<1].sum+xx[x<<1|1].ls);
8          xx[x].rs=max(xx[x<<1|1].rs,xx[x<<1|1].sum+xx[x<<1].rs);
9          xx[x].ans=max(xx[x<<1].ans,max(xx[x<<1|1].ans,xx[x<<1].rs+xx[x<<1|1].ls));
10         return;
11     }
12     inline void build(int k,int l,int r){
13         if(l==r){
14             xx[k].ls=xx[k].rs=xx[k].ans=xx[k].sum=0;
15             return;
16         }
17         int mid=l+r>>1;
18         build(k<<1,l,mid),build(k<<1|1,mid+1,r);
19         pushdown(k);
20         return;
21     }
22     inline void change(int k,int l,int r,int x,int y,int w){ // 1, 1, n
23         if(x<=l&&r<=y){
24             xx[k].ls += w;
25             xx[k].rs += w;
26             xx[k].ans += w;
27             xx[k].sum += w;
28         } // xx[k].ls=xx[k].rs=xx[k].ans=xx[k].sum=w;
29         return;
30     }
31     int mid=l+r>>1;
32     if(x<=mid) change(k<<1,l,mid,x,y,w);
33     if(mid<y) change(k<<1|1,mid+1,r,x,y,w);
34     pushdown(k);
35     return;
36 }
37 inline node query(int k,int l,int r,int x,int y){
38     if(x<=l&&r<=y) {

```

```

39         return xx[k];
40     }
41     int mid=l+r>>1;
42     if(x<=mid&&!(mid<y)) return query(k<<1,l,mid,x,y);
43     else if(!(x<=mid)&&mid<y) return query(k<<1|1,mid+1,r,x,y);
44     else{
45         node st,t1=query(k<<1,l,mid,x,y),t2=query(k<<1|1,mid+1,r,x,y);
46         st.sum=t1.sum+t2.sum;
47         st.ls=max(t1.ls,t1.sum+t2.ls);
48         st.rs=max(t2.rs,t2.sum+t1.rs);
49         st.ans=max(t1.ans,max(t2.ans,t1.rs+t2.ls));
50         return st;
51     }
52 }
53 }

```

### 3.1.4 动态开点权值线段树

```

1  int root[100005];
2  int ls[1800000], rs[1800000], sum[1800000];
3  int sz = 0;
4
5  void insert(int &k, int l, int r, int val){
6      if (!k) k = ++sz;
7      if (l == r) {
8          sum[k] = 1;
9          return;
10     }
11     int mid = (l + r) >> 1;
12     if (val <= mid) insert(ls[k], l, mid, val);
13     else insert(rs[k], mid + 1, r, val);
14     sum[k] = sum[ls[k]] + sum[rs[k]];
15 }
16
17 int query(int k, int l, int r, int rank) {
18     if (l == r) return l;
19     int mid = (l + r) >> 1;
20     if (sum[ls[k]] >= rank) return query(ls[k], l, mid, rank);
21     else return query(rs[k], mid + 1, r, rank - sum[ls[k]]);
22 }
23 int merge(int x, int y)
24 {
25     if (!x) return y;
26     if (!y) return x;
27     ls[x] = merge(ls[x], ls[y]);
28     rs[x] = merge(rs[x], rs[y]);
29     sum[x] = sum[ls[x]] + sum[rs[x]];
30     return x;
31 }
32 insert(root[i], 1, n, a[i]);
33 query(root[p], 1, n, x);

```

### 3.1.5 扫描线

```

1  // 范用型扫描线, del储存上界+1, add储存下界, 先del后add即可
2  struct node {
3      int lpos, rpos, linepos;
4      bool operator < (const node& oth) const {

```



```

5         return linepos < oth.linepos;
6     }
7 };
8 vector<node> add, del;
9 int delpos = 0;
10 int res = 0;
11 for (int addpos = 0; addpos < add.size(); ++addpos) {
12     while (delpos < del.size() && del[delpos].linepos <= add[addpos].linepos) {
13         up(del[delpos].lpos, del[delpos].rpos, -1);
14         delpos ++;
15     }
16     up(add[addpos].lpos, add[addpos].rpos, 1);
17     res = max(res, val[1]);
18 }
19
20 // 求面积并
21 #define maxn 222
22 #define tmp (st<<1)
23 #define mid ((l+r)>>1)
24 #define lson l,mid,tmp
25 #define rson mid+1,r,tmp|1
26 using namespace std;
27 int cnt[maxn<<2];
28 double sum[maxn<<2];
29 double x[maxn];
30 struct Seg{
31     double h,l,r;
32     int s;
33     Seg(){}
34     Seg(double a,double b,double c,int d):l(a),r(b),h(c),s(d){}
35     bool operator<(const Seg &cmp)const{
36         return h<cmp.h;
37     }
38 }ss[maxn];
39 void push_up(int st,int l,int r){
40     if(cnt[st])sum[st]=x[r+1]-x[l];
41     else if(l==r)sum[st]=0;
42     else sum[st]=sum[tmp]+sum[tmp|1];
43 }
44 void update(int L,int R,int c,int l,int r,int st){
45     if(L<=l&&r<=R){
46         cnt[st]+=c;
47         push_up(st,l,r);
48         return ;
49     }
50     if(L<=mid)update(L,R,c,lson);
51     if(R>mid)update(L,R,c,rson);
52     push_up(st,l,r);
53 }
54 int main(){
55     int n,tot=1,m;
56     while(scanf("%d",&n)&&n){
57         double a,b,c,d;
58         m=0;
59         while(n--){
60             scanf("%lf%lf%lf%lf",&a,&b,&c,&d);
61             x[m]=a;
62             ss[m++]=Seg(a,c,b,1);
63             x[m]=c;

```

```

64         ss[m++]=Seg(a,c,d,-1);
65     }
66     sort(x,x+m);
67     sort(ss,ss+m);
68     double ans=0;
69     for(int i=0;i<m;++i){
70         int l=lower_bound(x,x+m,ss[i].l)-x;
71         int r=lower_bound(x,x+m,ss[i].r)-x-1;
72         update(l,r,ss[i].s,0,m-1,1);
73         ans+=sum[1]*(ss[i+1].h-ss[i].h);
74     }
75     printf("Test case #%dnTotal explored area: %.2lfnn",tot++,ans);
76 }
77 return 0;
78 }
79
80 // 面积交
81 #include<bits/stdc++.h>
82 #define maxn 100005
83 #define lson l,mid,rt<<1
84 #define rson mid+1,r,rt<<1|1
85 #define pb push_back
86 using namespace std;
87
88 double tree[maxn<<2],tree2[maxn<<2];
89 int lazy[maxn<<2];
90 vector<double>ve;
91
92 struct seg{
93     double l,r,h;
94     int flag;
95     seg(){}
96     seg(double _l,double _r,double _h,int _flag){l=_l,r=_r,h=_h,flag=_flag;}
97     bool operator<(const seg &b)const{return h<b.h;}
98 }s[maxn];
99
100 void push_up(int l,int r,int rt){
101     if(lazy[rt]) tree[rt]=ve[r]-ve[l-1];
102     else if(l==r) tree[rt]=0;
103     else tree[rt]=tree[rt<<1]+tree[rt<<1|1];
104 }
105
106 void push_up2(int l,int r,int rt){
107     if(lazy[rt]>1) tree2[rt]=ve[r]-ve[l-1];
108     else if(l==r) tree2[rt]=0;
109     else if(lazy[rt]==1)tree2[rt]=tree[rt<<1]+tree[rt<<1|1];
110     else tree2[rt]=tree2[rt<<1]+tree2[rt<<1|1];
111 }
112
113 void build(int l,int r,int rt){
114     tree[rt]=0,lazy[rt]=0;
115     if(l==r) return;
116     int mid=l+r>>1;
117     build(lson);
118     build(rson);
119 }
120
121 void add(int L,int R,int v,int l,int r,int rt){
122     if(L<=l&&R>=r){

```

```

123         lazy[rt]+=v;
124         push_up(1,r,rt);
125         push_up2(1,r,rt);
126         return;
127     }
128     int mid=l+r>>1;
129     if(L<=mid) add(L,R,v,lson);
130     if(R>mid) add(L,R,v,rson);
131     push_up(1,r,rt);
132     push_up2(1,r,rt);
133 }
134
135 int getid(double x){ return lower_bound(ve.begin(),ve.end(),x)-ve.begin()+1;}
136
137 int main(){
138     int n;
139     int Case=1;
140     int T;
141     scanf("%d",&T);
142     while(T--){
143         scanf("%d",&n);
144         ve.clear();
145         int tot=0;
146         double x1,y1,x2,y2;
147         for(int i=1;i<=n;i++){
148             scanf("%lf %lf %lf %lf",&x1,&y1,&x2,&y2);
149             ve.pb(x1),ve.pb(x2);
150             s[++tot]=seg(x1,x2,y1,1);
151             s[++tot]=seg(x1,x2,y2,-1);
152         }
153         sort(ve.begin(),ve.end());
154         ve.erase(unique(ve.begin(),ve.end()),ve.end());
155         sort(s+1,s+tot+1);
156         int N=ve.size();
157         build(1,N,1);
158         double ans=0;
159         for(int i=1;i<tot;i++){
160             int L=getid(s[i].l);
161             int R=getid(s[i].r)-1;
162             add(L,R,s[i].flag,1,N,1);
163             ans+=tree2[1]*(s[i+1].h-s[i].h);
164         }
165         printf("%.2f\n",ans);
166     }
167 }
168
169 // 求周长并
170 #include<bits/stdc++.h>
171 #define maxn 100005
172 #define lson l,mid,rt<<1
173 #define rson mid+1,r,rt<<1|1
174 #define pb push_back
175 using namespace std;
176
177 int tree[maxn<<2];
178 int lazy[maxn<<2];
179 vector<int>ve[2];
180 int k;
181

```

```
182 struct seg{
183     int l,r,h;
184     int flag;
185     seg(){ }
186     seg(int _l,int _r,int _h,int _flag){l=_l,r=_r,h=_h,flag=_flag;}
187     bool operator< (const seg &b) const {return h<b.h;}
188 }s[maxn];
189
190 void push_up(int l,int r,int rt){
191     if(lazy[rt]) tree[rt]=ve[k][r]-ve[k][l-1];
192     else if(l==r) tree[rt]=0;
193     else tree[rt]=tree[rt<<1]+tree[rt<<1|1];
194 }
195
196 void build(int l,int r,int rt){
197     tree[rt]=0,lazy[rt]=0;
198     if(l==r) return;
199     int mid=l+r>>1;
200     build(lson);
201     build(rson);
202 }
203
204 void add(int L,int R,int v,int l,int r,int rt){
205     if(L<=l&&R>=r){
206         lazy[rt]+=v;
207         push_up(l,r,rt);
208         return;
209     }
210     int mid=l+r>>1;
211     if(L<=mid) add(L,R,v,lson);
212     if(R>mid) add(L,R,v,rson);
213     push_up(l,r,rt);
214 }
215
216 int getid(int x){return lower_bound(ve[k].begin(),ve[k].end(),x)-ve[k].begin()+1;}
217
218 int main(){
219     int n;
220     while(~scanf("%d",&n)){
221         ve[0].clear();
222         ve[1].clear();
223         int x1,y1,x2,y2;
224         for(int i=1;i<=n;i++){
225             scanf("%d %d %d %d",&x1,&y1,&x2,&y2);
226             ve[0].pb(x1),ve[0].pb(x2);
227             ve[1].pb(y1),ve[1].pb(y2);
228             s[i]=seg(x1,x2,y1,1);
229             s[i+n]=seg(x1,x2,y2,-1);
230             s[i+n+n]=seg(y1,y2,x1,1);
231             s[i+n+n+n]=seg(y1,y2,x2,-1);
232         }
233         int ans=0;
234         int pos=1;
235         for(k=0;k<2;k++){
236             sort(ve[k].begin(),ve[k].end());
237             ve[k].erase(unique(ve[k].begin(),ve[k].end()),ve[k].end());
238             sort(s+pos,s+pos+n*n);
239             int N=ve[k].size();
240             build(1,N,1);
```

```

241         int pre=0;
242         for(int i=pos;i<pos+n+n;i++){
243             int L=getid(s[i].l);
244             int R=getid(s[i].r)-1;
245             add(L,R,s[i].flag,1,N,1);
246             ans+=abs(tree[1]-pre);
247             pre=tree[1];
248         }
249         pos+=n+n;
250     }
251     printf("%d\n",ans);
252 }
253 }

```

## 3.2 HLD

### 3.2.1 HLD

```

1  #include <bits/stdc++.h>
2  #define ll long long
3  using namespace std;
4  /*
5   node 计算点权, path 下放后计算边权, edge 根据边的编号计算边权
6   work 中没有build需手动写
7   sz[] 数组, 以x为根的子树节点个数
8   top[] 数组, 当前节点的所在链的顶端节点
9   son[] 数组, 重儿子
10  deep[] 数组, 当前节点的深度
11  fa[] 数组, 当前节点的父亲
12  idx[] 数组, 树中每个节点剖分后的新编号
13  rnk[] 数组, idx的逆, 表示线段上中当前位置表示哪个节点
14  */
15
16  const int maxn = 1e5+5;
17
18  int sz[maxn], top[maxn], son[maxn], deep[maxn], fa[maxn], idx[maxn], rnk[maxn];
19  int tot;
20  int n, le, re;
21  ll k;
22
23  struct HLD {
24  #define type int
25
26      struct edge {
27          int a, b;
28          type v;
29
30          edge(int _a, int _b, type _v = 0) : a(_a), b(_b), v(_v) {}
31      };
32
33      struct node {
34          int to;
35          type w;
36
37          node() {}
38
39          node(int _to, type _w) : to(_to), w(_w) {}
40      };
41

```

```
42     vector<int> mp[maxn];
43     vector<edge> e;
44
45     void init(int _n) {
46         n = _n;
47         for (int i = 0; i <= n; i++) mp[i].clear();
48         e.clear();
49         e.push_back(edge(0, 0));
50     }
51
52     void add_edge(int a, int b, type v = 0) {
53 //         e.push_back(edge(a,b,v));
54         mp[a].push_back(b);
55         mp[b].push_back(a);
56     }
57
58     void dfs1(int x, int pre, int h) {
59         int i, to;
60         deep[x] = h;
61         fa[x] = pre;
62         sz[x] = 1;
63         for (i = 0; i < (int) (mp[x].size()); i++) {
64             to = mp[x][i];
65             if (to == pre) continue;
66             dfs1(to, x, h + 1);
67             sz[x] += sz[to];
68             if (son[x] == -1 || sz[to] > sz[son[x]]) son[x] = to;
69         }
70     }
71
72     void dfs2(int x, int tp) {
73         int i, to;
74         top[x] = tp;
75         idx[x] = ++tot;
76         rnk[idx[x]] = x;
77         if (son[x] == -1) return;
78         dfs2(son[x], tp);
79         for (i = 0; i < (int) (mp[x].size()); i++) {
80             to = mp[x][i];
81             if (to != son[x] && to != fa[x]) dfs2(to, to);
82         }
83     }
84
85     void work(int _rt = 1) {
86         memset(son, -1, sizeof son);
87         tot = 0;
88         dfs1(_rt, 0, 0);
89         dfs2(_rt, _rt);
90     }
91
92     int LCA(int x, int y) {
93         while (top[x] != top[y]) {
94             if (deep[top[x]] < deep[top[y]]) swap(x, y);
95             x = fa[top[x]];
96         }
97         if (deep[x] > deep[y]) swap(x, y);
98         return x;
99     }
100
```

```
101 void modify_node(int x, int y, type val) {
102     while (top[x] != top[y]) {
103         if (deep[top[x]] < deep[top[y]]) swap(x, y);
104         le = idx[top[x]], re = idx[x];
105         k = val;
106         update(1, 1, n);
107         x = fa[top[x]];
108     }
109     if (deep[x] > deep[y]) swap(x, y);
110     le = idx[x], re = idx[y];
111     k = val;
112     update(1, 1, n);
113 }
114
115 type query_node(int x, int y) {
116     type res = 0;
117     while (top[x] != top[y]) {
118         if (deep[top[x]] < deep[top[y]]) swap(x, y);
119         le = idx[top[x]], re = idx[x];
120         res += query(1, 1, n);
121         x = fa[top[x]];
122     }
123     if (deep[x] > deep[y]) swap(x, y);
124     le = idx[x], re = idx[y];
125     res += query(1, 1, n);
126     return res;
127 }
128
129 //path
130 // void init_path()
131 // {
132 //     v[idx[rt]]=0;
133 //     for(int i=1;i<n;i++)
134 //     {
135 //         if(deep[e[i].a]<deep[e[i].b]) swap(e[i].a,e[i].b);
136 //         a[idx[e[i].a]]=e[i].v;
137 //     }
138 //     build(n);
139 // }
140 void modify_edge(int id, type val) {
141     if (deep[e[id].a] > deep[e[id].b]) {
142         le = idx[e[id].a], re = idx[e[id].a];
143         k = val;
144         update(1, 1, n);
145     } else {
146         le = idx[e[id].b], re = idx[e[id].b];
147         k = val;
148         update(1, 1, n);
149     }
150 }
151
152 void modify_path(int x, int y, type val) {
153     while (top[x] != top[y]) {
154         if (deep[top[x]] < deep[top[y]]) swap(x, y);
155         le = idx[top[x]], re = idx[x];
156         k = val;
157         update(1, 1, n);
158         x = fa[top[x]];
159     }
```

```
160         if (deep[x] > deep[y]) swap(x, y);
161         if (x != y) {
162             le = idx[x] + 1, re = idx[y];
163             k = val;
164             update(1, 1, n);
165         }
166     }
167
168     type query_path(int x, int y) {
169         type res = 0;
170         while (top[x] != top[y]) {
171             if (deep[top[x]] < deep[top[y]]) swap(x, y);
172             le = idx[top[x]], re = idx[x];
173             res += query(1, 1, n);
174             x = fa[top[x]];
175         }
176         if (deep[x] > deep[y]) swap(x, y);
177         if (x != y) {
178             le = idx[x] + 1, re = idx[y];
179             res += query(1, 1, n);
180         }
181         return res;
182     }
183
184 #undef type
185 } hld;
```

### 3.3 RMQ

#### 3.3.1 RMQ

```
1  int A[maxn];
2  int maxx[maxn][22];
3  void RMQ(int n) {
4      for (int i = 1; i <= n; i++)
5          maxx[i][0] = A[i];
6      for (int j = 1; (1 << j) <= n; j++) {
7          for (int i = 1; i + (1 << j) - 1 <= n; i++) {
8              maxx[i][j] = max(maxx[i][j - 1], maxx[i + (1 << (j - 1))][j - 1]);
9          }
10     }
11 }
12 int query(int l, int r) {
13     int k = 0;
14     while ((1 << (k + 1)) <= r - l + 1) k++;
15     return max(maxx[l][k], maxx[r - (1 << k) + 1][k]);
16 }
17
18 template <typename T, class F = function<T(const T&, const T&)>>
19 class SparseTable {
20 public:
21     int n;
22     vector<vector<T>> mat;
23     F func;
24
25     SparseTable(const vector<T>& a, const F& f) : func(f) {
26         n = static_cast<int>(a.size());
27         int max_log = 32 - __builtin_clz(n);
28         mat.resize(max_log);
```



```

29     mat[0] = a;
30     for (int j = 1; j < max_log; j++) {
31         mat[j].resize(n - (1 << j) + 1);
32         for (int i = 0; i <= n - (1 << j); i++) {
33             mat[j][i] = func(mat[j - 1][i], mat[j - 1][i + (1 << (j - 1))]);
34         }
35     }
36 }
37
38 T get(int from, int to) const {
39     assert(0 <= from && from <= to && to <= n - 1);
40     int lg = 32 - __builtin_clz(to - from + 1) - 1;
41     return func(mat[lg][from], mat[lg][to - (1 << lg) + 1]);
42 }
43 };
44 //静态区间最大值
45 SparseTable<int> solve(v, [&](int i, int j) {return max(i, j);});

```

### 3.3.2 RMQbyIndex

```

1 //下标RMQ
2 int v[MAX],maxx[MAX][22],minn[MAX][22];
3 int pmax(int a,int b){return v[a]>v[b]?a:b;}
4 int pmin(int a,int b){return v[a]<v[b]?a:b;}
5 void RMQ(int n) {
6     int i,j;
7     for(i=1;i<=n;i++) {
8         maxx[i][0]=minn[i][0]=i;
9     }
10    for(j=1;1<<(j-1)<=n;j++) {
11        for(i=1;i+(1<<(j-1))-1<=n;i++) {
12            int t=1<<(j-1);
13            maxx[i][j]=pmax(maxx[i][j-1],maxx[i+t][j-1]);
14            minn[i][j]=pmin(minn[i][j-1],minn[i+t][j-1]);
15        }
16    }
17 }
18 int query(int l,int r) {
19     int j=(int)(log10(r-l+1)/log10(2))+1;
20     int i=r-(1<<(j-1))+1;
21     return pmax(maxx[i][j-1],maxx[i][j-1]);
22 // return pmin(minn[i][j-1],minn[i][j-1]);
23 }

```

### 3.3.3 RMQinNM

```

1 //二维RMQ
2 int v[302][302];
3 int maxx[302][302][9][9],minn[302][302][9][9];
4 void RMQ(int n,int m)
5 {
6     int i,j,ii,jj;
7     for(i=1;i<=n;i++)
8     {
9         for(j=1;j<=m;j++)
10        {
11            maxx[i][j][0][0]=minn[i][j][0][0]=v[i][j];
12        }
13    }
14 }

```

```

13     }
14     for(ii=0;(1<<ii)<=n;ii++)
15     {
16         for(jj=0;(1<<jj)<=m;jj++)
17         {
18             if(ii+jj)
19             {
20                 for(i=1;i+(1<<ii)-1<=n;i++)
21                 {
22                     for(j=1;j+(1<<jj)-1<=m;j++)
23                     {
24                         if(ii)
25                         {
26                             minn[i][j][ii][jj]=min(minn[i][j][ii-1][jj],minn[i+(1<<(ii-1))][j][ii
-1][jj]);
27                             maxx[i][j][ii][jj]=max(maxx[i][j][ii-1][jj],maxx[i+(1<<(ii-1))][j][ii
-1][jj]);
28                         }
29                         else
30                         {
31                             minn[i][j][ii][jj]=min(minn[i][j][ii][jj-1],minn[i][j+(1<<(jj-1))][ii][
jj-1]);
32                             maxx[i][j][ii][jj]=max(maxx[i][j][ii][jj-1],maxx[i][j+(1<<(jj-1))][ii][
jj-1]);
33                         }
34                     }
35                 }
36             }
37         }
38     }
39 }
40 int query(int x1,int y1,int x2,int y2)
41 {
42     int k1=0;
43     while((1<<(k1+1))<=x2-x1+1) k1++;
44     int k2=0;
45     while((1<<(k2+1))<=y2-y1+1) k2++;
46     x2=x2-(1<<k1)+1;
47     y2=y2-(1<<k2)+1;
48     return max(max(maxx[x1][y1][k1][k2],maxx[x1][y2][k1][k2]),max(maxx[x2][y1][k1][k2],maxx[x2][y2
][k1][k2]));
49 // return min(min(minn[x1][y1][k1][k2],minn[x1][y2][k1][k2]),min(minn[x2][y1][k1][k2],minn[x2][y2
][k1][k2]));
50 }

```

## 3.4 MO

### 3.4.1 MO

```

1 // const int maxn = 50005;
2
3 struct MO {
4     int l, r, id;
5 }q[maxn];
6
7 int n, m, col[maxn], block, belong[maxn];
8 int vis[maxn * 10];
9 ll res[maxn], ans;
10 bool cmp(const MO& a, const MO& b) { return belong[a.l] == belong[b.l] ? a.r < b.r : a.l < b.l; }

```

```
11 void add(int x) {
12     vis[x] ++;
13     ans += 1ll * x * (vis[x] * vis[x] - (vis[x] - 1) * (vis[x] - 1));
14 }
15
16 void del(int x) {
17     vis[x] --;
18     ans -= 1ll * x * ((vis[x] + 1) * (vis[x] + 1) - vis[x] * vis[x]);
19 }
20
21 int main() {
22     scanf("%d%d", &n, &m);
23     block = sqrt(n);
24     for (int i = 1; i <= n; ++i) {
25         scanf("%d", &col[i]);
26         belong[i] = i / block + 1;
27     }
28     for (int i = 1; i <= m; ++i) {
29         scanf("%d%d", &q[i].l, &q[i].r);
30         q[i].id = i;
31     }
32     sort(q + 1, q + 1 + m, cmp);
33     int l = 1, r = 0;
34     for (int i = 1; i <= m; ++i) {
35         while(r < q[i].r) add(col[++r]);
36         while(r > q[i].r) del(col[r--]);
37         while(l < q[i].l) del(col[l++]);
38         while(l > q[i].l) add(col[--l]);
39         res[q[i].id] = ans;
40     }
41     for (int i = 1; i <= m; ++i) printf("%lld\n", res[i]);
42     return 0;
43 }
```

### 3.4.2 MObyModify

```
1  #include <bits/stdc++.h>
2  #define ll long long
3  using namespace std;
4  const int maxn = 50005;
5
6  struct MO {
7      int l, r, id, oppre;
8  }q[maxn];
9
10 int n, m, col[maxn], block, belong[maxn], colpre[maxn];
11 int changepos[maxn], changepre[maxn], changenow[maxn];
12 int vis[maxn * 20];
13 int ans;
14 int res[maxn];
15 bool cmp(const MO& a, const MO& b) {
16     if (belong[a.l] != belong[b.l]) return a.l < b.l;
17     if (belong[a.r] != belong[b.r]) return a.r < b.r;
18     return a.oppre < b.oppre;
19 }
20 void add(int x) {}
21
22 void del(int x) {}
```

```
23
24 void unmodify(int pos, int now) {
25     if (q[pos].l <= changepos[now] && changepos[now] <= q[pos].r) {
26         del(changenow[now]);
27         add(changepre[now]);
28     }
29     col[changepos[now]] = changepre[now];
30 }
31
32 void modify(int pos, int now) {
33     if (q[pos].l <= changepos[now] && changepos[now] <= q[pos].r) {
34         del(changepre[now]);
35         add(changenow[now]);
36     }
37     col[changepos[now]] = changenow[now];
38 }
39
40 int main() {
41     scanf("%d%d", &n, &m);
42     block = pow(n, 0.66666);
43     for (int i = 1; i <= n; ++i) {
44         scanf("%d", &col[i]);
45         colpre[i] = col[i];
46         belong[i] = i / block + 1;
47     }
48     char s[2];
49     int t = 0, t2 = 0;
50     for (int i = 1; i <= m; ++i) {
51         scanf("%s", s);
52         if (s[0] == 'Q') {
53             ++t;
54             scanf("%d%d", &q[t].l, &q[t].r);
55             q[t].oppre = t2;
56             q[t].id = t;
57         } else {
58             ++t2;
59             scanf("%d%d", &changepos[t2], &changenow[t2]);
60             changepre[t2] = colpre[changepos[t2]];
61             colpre[changepos[t2]] = changenow[t2];
62         }
63     }
64     sort(q + 1, q + 1 + t, cmp);
65     int l = 1, r = 0, now = 0;
66     for (int i = 1; i <= t; ++i) {
67         while(r < q[i].r) add(col[++r]);
68         while(r > q[i].r) del(col[r--]);
69         while(l < q[i].l) del(col[l++]);
70         while(l > q[i].l) add(col[--l]);
71         while (now < q[i].oppre) modify(i, ++now);
72         while (now > q[i].oppre) unmodify(i, now--);
73         res[q[i].id] = ans;
74     }
75     for (int i = 1; i <= t; ++i) printf("%d\n", res[i]);
76     return 0;
77 }
```

### 3.4.3 分块

```
1 // 非预处理数组版
```

```
2 inline int belong(int x) { return (x - 1) / block + 1; }
3 inline int lpos(int x) { return 1 + (x - 1) * block; }
4 inline int rpos(int x) { return min(n, x * block); }
5 int sz = (n - 1) / block + 1;
6
7 // 预处理版, maxn大于1e6已经不可能处理了
8 const int maxb = 1005;
9 int n, m;
10 int belong[maxn], lpos[maxb], rpos[maxb];
11 int val[maxn], lazy[maxb];
12 int block;
13
14 scanf("%d", &n);
15 block = sqrt(n);
16 for (int i = 1; i <= n; ++i) {
17     scanf("%d", &val[i]);
18     belong[i] = (i - 1) / block + 1;
19 }
20 int sz = (n - 1) / block + 1;
21 for (int i = 1; i <= sz; ++i) {
22     lpos[i] = 1 + (i - 1) * block;
23     rpos[i] = i * block;
24 }
25 rpos[sz] = n;
```

#### 3.4.4 弹飞绵羊

```
1 int n, m;
2 int belong[maxn], lpos[maxn], rpos[maxn];
3 int val[maxn], nxt[maxn], k[maxn], lst[maxn];
4 int block;
5
6 void update(int pos) {
7     int llim = lpos[belong[pos]], rlim = rpos[belong[pos]];
8     for (int i = pos; i >= llim; --i) {
9         if (val[i] + i > rlim) {
10             k[i] = 1;
11             nxt[i] = val[i] + i;
12             if (val[i] + i > n) lst[i] = i;
13             else lst[i] = lst[nxt[i]];
14         } else {
15             k[i] = 1 + k[val[i] + i];
16             nxt[i] = nxt[val[i] + i];
17             lst[i] = lst[val[i] + i];
18         }
19     }
20 }
21
22 void init() {
23     for (int i = n; i >= 1; --i) {
24         int rlim = rpos[belong[i]];
25         if (val[i] + i > rlim) {
26             k[i] = 1;
27             nxt[i] = val[i] + i;
28             if (val[i] + i > n) lst[i] = i;
29             else lst[i] = lst[nxt[i]];
30         } else {
31             k[i] = 1 + k[val[i] + i];
```

```

32         nxt[i] = nxt[val[i] + i];
33         lst[i] = lst[val[i] + i];
34     }
35 }
36 }
37
38 int query(int pos) {
39     int res = 0;
40     while (pos <= n) {
41         res += k[pos];
42         if (nxt[pos] > n) printf("%d ", lst[pos]);
43         pos = nxt[pos];
44     }
45     return res;
46 }
47
48 int main(int argc, char* argv[]) {
49     scanf("%d%d", &n, &m);
50     block = sqrt(n) * 1.6 + 1;
51     for (int i = 1; i <= n; ++i) {
52         scanf("%d", &val[i]);
53         belong[i] = (i - 1) / block + 1;
54     }
55     int sz = (n - 1) / block + 1;
56     for (int i = 1; i <= sz; ++i) {
57         lpos[i] = 1 + (i - 1) * block;
58         rpos[i] = i * block;
59     }
60     rpos[sz] = n;
61     init();
62     while (m--) {
63         int op;
64         scanf("%d", &op);
65         if (op == 1) {
66             int pos;
67             scanf("%d", &pos);
68             printf("%d\n", query(pos));
69         } else {
70             int pos, kl;
71             scanf("%d%d", &pos, &kl);
72             val[pos] = kl;
73             update(pos);
74         }
75     }
76     return 0;
77 }

```

### 3.4.5 树莫队

```

1  // rnk保存欧拉序
2  int sz[maxn], top[maxn], son[maxn], deep[maxn], fa[maxn], idx[maxn], ed[maxn], rnk[maxn*2];
3  int tot, n, m;
4  vector<int> edge[maxn];
5  int val[maxn];
6  vector<int> xpos;
7
8  inline void dfs1(int u, int pre, int h) {
9      deep[u] = h;

```

```

10     fa[u] = pre;
11     sz[u] = 1;
12     for (auto to : edge[u]) {
13         if (to == pre) continue;
14         dfs1(to, u, h + 1);
15         sz[u] += sz[to];
16         if (son[u] == 0 || sz[to] > sz[son[u]]) son[u] = to;
17     }
18 }
19
20 inline void dfs2(int u, int tp) {
21     top[u] = tp;
22     idx[u] = ++tot, rnk[tot] = u;
23     if (son[u] == 0) {
24         ed[u] = ++tot, rnk[tot] = u;
25         return;
26     }
27     dfs2(son[u], tp);
28     for (auto to : edge[u]) {
29         if (to != son[u] && to != fa[u]) dfs2(to, to);
30     }
31     ed[u] = ++tot, rnk[tot] = u;
32 }
33
34 inline int LCA(int x, int y) {
35     while (top[x] != top[y]) {
36         if (deep[top[x]] < deep[top[y]]) swap(x, y);
37         x = fa[top[x]];
38     }
39     if (deep[x] > deep[y]) swap(x, y);
40     return x;
41 }
42
43 int belong[maxn*2], block;
44 int res[maxn], ans;
45 // 每个点是否访问 (欧拉序去重)
46 int vis[maxn];
47 // 标记数组
48 int pre[maxn];
49 struct MO {
50     int l, r, id, lca;
51     bool operator < (const MO& oth) const {
52         return belong[l] == belong[oth.l] ? r < oth.r : belong[l] < belong[oth.l];
53     }
54 }q[maxn];
55
56 inline void add(int x) {
57     pre[x] ++;
58     if (pre[x] == 1) ans ++;
59 }
60
61 inline void del(int x) {
62     pre[x] --;
63     if (pre[x] == 0) ans --;
64 }
65
66 inline void deal(int x) {
67     vis[x] ? del(val[x]) : add(val[x]);
68     vis[x] = !vis[x];

```

```

69 }
70
71 int main(int argc, char* argv[]) {
72     scanf("%d%d", &n, &m);
73     block = sqrt(n);
74     xpos.resize(n + 1);
75     for (int i = 1; i <= n; ++i) {
76         scanf("%d", &val[i]);
77         xpos[i] = val[i];
78     }
79     sort(xpos.begin(), xpos.end());
80     xpos.erase(unique(xpos.begin(), xpos.end()), xpos.end());
81     for (int i = 1; i <= n; ++i) val[i] = lower_bound(xpos.begin(), xpos.end(), val[i]) - xpos.
        begin();
82     // 欧拉序长度为n两倍所以分块要分两倍大小
83     for (int i = 1; i <= n * 2; ++i) {
84         belong[i] = (i - 1) / block + 1;
85     }
86     for (int i = 1, u, v; i < n; ++i) {
87         scanf("%d%d", &u, &v);
88         edge[u].push_back(v);
89         edge[v].push_back(u);
90     }
91     // 树剖预处理lca
92     dfs1(1, 0, 0);
93     dfs2(1, 1);
94     for (int i = 1, x, y; i <= m; ++i) {
95         scanf("%d%d", &x, &y);
96         if (idx[x] > idx[y]) swap(x, y);
97         int _lca = LCA(x, y);
98         q[i].id = i;
99         if (_lca == x) q[i].l = idx[x], q[i].r = idx[y], q[i].lca = 0;
100        else q[i].l = ed[x], q[i].r = idx[y], q[i].lca = _lca;
101        // cerr << q[i].l << " " << q[i].r << " " << q[i].id << " " << q[i].lca << endl;
102    }
103    sort(q + 1, q + 1 + m);
104    int l = 1, r = 0;
105    for (int i = 1; i <= m; ++i) {
106        while(r < q[i].r) deal(rnk[++r]);
107        while(r > q[i].r) deal(rnk[r--]);
108        while(l < q[i].l) deal(rnk[l++]);
109        while(l > q[i].l) deal(rnk[--l]);
110        if (q[i].lca) deal(q[i].lca);
111        res[q[i].id] = ans;
112        if (q[i].lca) deal(q[i].lca);
113    }
114    for (int i = 1; i <= m; ++i) {
115        printf("%d\n", res[i]);
116    }
117    return 0;
118 }

```

## 3.5 VirtualTree

### 3.5.1 VirtualTree

```

1 const int pow2 = 19;
2 const int maxn = 1 << pow2;
3 vector<int> adj0[maxn], adj1[maxn];

```



```

4  int st[maxn << 1][pow2 + 1], dep[maxn], euler[maxn], euler_clock;
5  // fa0 是原树的父节点
6  // fa1 是虚树的父节点
7  // len 是虚树每个节点的权重, 每个节点代表原树的几个节点, 也是虚树到它父节点的链的长度
8  int stk[maxn], fa0[maxn], fa1[maxn], len[maxn];
9  ll val[maxn];
10
11 void link0(int u, int v) { adj0[u].emplace_back(v); adj0[v].emplace_back(u); }
12 void link1(int u, int v) { adj1[u].emplace_back(v); adj1[v].emplace_back(u); }
13 void dfs0(int u, int p) {
14     fa0[u] = p;
15     dep[u] = dep[p] + 1;
16     st[++euler_clock][0] = u;
17     euler[u] = euler_clock;
18     for (const auto& v : adj0[u]) if (v != p) {
19         dfs0(v, u);
20         st[++euler_clock][0] = u;
21     }
22 }
23 inline bool cmp(int u, int v) {return dep[u] < dep[v];}
24 inline int upper(int u, int v) {return cmp(u, v) ? u : v;}
25 void lca_init() {
26     for (int i = 0; i != 31 - __builtin_clz(euler_clock); ++i)
27         for (int j = 1; j + (1 << (i + 1)) <= euler_clock; ++j)
28             st[j][i + 1] = upper(st[j][i], st[j + (1 << i)][i]);
29 }
30 inline int lca(int u, int v) {
31     if (u == v) return u;
32     u = euler[u];
33     v = euler[v];
34     if (u > v) swap(u, v);
35     int temp = 31 - __builtin_clz(++v - u);
36     return upper(st[u][temp], st[v - (1 << temp)][temp]);
37 }
38 void build(vector<int>& key) {
39     sort(key.begin(), key.end(), [&] (int u, int v) { return euler[u] < euler[v]; });
40     key.resize(unique(key.begin(), key.end()) - key.begin());
41     int top = 0;
42     for (const auto& u : key) {
43         if (!top) {
44             stk[++top] = u;
45             continue;
46         }
47         int p = lca(u, stk[top]);
48         while (euler[p] < euler[stk[top]]) {
49             if (euler[p] >= euler[stk[top - 1]]) {
50                 link1(p, stk[top]);
51                 if (stk[--top] != p) stk[++top] = p;
52                 break;
53             }
54             link1(stk[top - 1], stk[top]);
55             --top;
56         }
57         stk[++top] = u;
58     }
59     while (top > 1) {
60         link1(stk[top - 1], stk[top]);
61         --top;
62     }

```

```

63 }
64
65 void dfs1(int u, int p) {
66     fa1[u] = p;
67     val[u] = 0;
68     len[u] = dep[u] - dep[p];
69     for (const auto& v : adj1[u]) if (v != p) dfs1(v, u);
70 }
71
72 int main() {
73     // 多组清空操作
74     for (int i = 1; i <= n; ++i) {
75         adj0[i].clear();
76         adj1[i].clear();
77     }
78     euler_clock = 0;
79
80     // 读入原树 link0 加边
81     // 读入处理关键节点存入vector key, 包含1和链的端点和他们的lca的父节点 (lca如果为1就不加)。
82     dfs0(1, 0);
83     lca_init();
84
85     vector<int> key(1, 1);
86     for (auto& q : query) {
87         cin >> q.u >> q.v;
88         key.emplace_back(q.u);
89         key.emplace_back(q.v);
90         int p = lca(q.u, q.v);
91         if (p != 1) key.emplace_back(fa0[p]);
92     }
93
94     build(key);
95     dfs1(1, 0);
96     return 0;
97 }

```

## 3.6 PersistentDS

### 3.6.1 主席树区间 k 大

```

1  // const int maxn = 100005;
2  int n, m;
3  int a[maxn];
4  int root[maxn];
5  int cnt = 0;
6  vector<int> b;
7  struct node {
8      int l, r, val;
9  }p[maxn * 40];
10
11 void update(int l, int r, int pre, int &now, int pos) {
12     now = ++cnt;
13     p[now] = p[pre];
14     p[now].val++;
15     if (l == r) {
16         return;
17     }
18     int mid = l + r >> 1;
19     if (pos <= mid) update(l, mid, p[pre].l, p[now].l, pos);

```

```

20     else update(mid + 1, r, p[pre].r, p[now].r, pos);
21 }
22
23 int query(int l, int r, int x, int y, int k) {
24     if (l == r) return b[l - 1];
25     int mid = l + r >> 1;
26     int temp = p[p[y].l].val - p[p[x].l].val;
27     if (k <= temp) return query(l, mid, p[x].l, p[y].l, k);
28     return query(mid + 1, r, p[x].r, p[y].r, k - temp);
29 }
30
31 int main(int argc, char *argv[])
32 {
33     while (scanf("%d%d", &n, &m) != EOF) {
34         b.clear();
35         cnt = 0;
36         for (int i = 1; i <= n; ++i) scanf("%d", &a[i]), b.push_back(a[i]);
37         sort(b.begin(), b.end());
38         b.erase(unique(b.begin(), b.end()), b.end());
39         for (int i = 1; i <= n; ++i) {
40             update(1, b.size(), root[i - 1], root[i], lower_bound(b.begin(), b.end(), a[i]) - b.
begin() + 1);
41         }
42         int L, R, k;
43         while (m--) {
44             scanf("%d%d%d", &L, &R, &k);
45             printf("%d\n", query(1, b.size(), root[L - 1], root[R], k));
46         }
47     }
48     return 0;
49 }

```

### 3.6.2 可持久化数组

```

1  /*1、操作将u, v合并 2、操作回退 */
2  const int maxn = 2e5+5;
3  int n, m, sz;
4  int root[maxn], ls[maxn*40], rs[maxn*40], v[maxn*40], deep[maxn*40];
5  int has[maxn];
6
7  void build(int &k, int l, int r) {
8      if (!k) k = ++sz;
9      if (l == r) {
10         v[k] = l;
11         return;
12     }
13     int mid = (l + r) >> 1;
14     build(ls[k], l, mid);
15     build(rs[k], mid + 1, r);
16 }
17
18 void modify(int l, int r, int x, int &y, int pos, int val) {
19     y = ++sz;
20     if (l == r) {
21         v[y] = val;
22         deep[y] = deep[x];
23         return;
24     }

```

```
25     ls[y] = ls[x];
26     rs[y] = rs[x];
27     int mid = (l + r) >> 1;
28     if (pos <= mid)
29         modify(l, mid, ls[x], ls[y], pos, val);
30     else modify(mid + 1, r, rs[x], rs[y], pos, val);
31 }
32
33 int query(int k, int l, int r, int pos) {
34     if (l == r) return k;
35     int mid = (l + r) >> 1;
36     if (pos <= mid) return query(ls[k], l, mid, pos);
37     else return query(rs[k], mid + 1, r, pos);
38 }
39
40 void add(int k, int l, int r, int pos) {
41     if (l == r) {
42         deep[k]++;
43         return;
44     }
45     int mid = (l + r) >> 1;
46     if (pos <= mid) add(ls[k], l, mid, pos);
47     else add(rs[k], mid + 1, r, pos);
48 }
49
50 int find(int k, int x) {
51     int p = query(k, 1, n, x);
52     if (x == v[p]) return p;
53     return find(k, v[p]);
54 }
55
56 int main() {
57     int T = read();
58     while (T--) {
59         sz = 0;
60         memset(root, 0, sizeof root);
61         memset(ls, 0, sizeof ls);
62         memset(rs, 0, sizeof rs);
63         n = read();
64         has[0] = n;
65         m = read();
66         build(root[0], 1, n);
67         int f, k, a, b;
68         for (int i = 1; i <= m; i++) {
69             f = read();
70             if (f == 1) {
71                 root[i] = root[i - 1];
72                 has[i] = has[i - 1];
73                 a = read();
74                 b = read();
75                 int p = find(root[i], a), q = find(root[i], b);
76                 if (v[p] == v[q]) continue;
77                 has[i]--;
78                 if (deep[p] > deep[q]) swap(p, q);
79                 modify(1, n, root[i - 1], root[i], v[p], v[q]);
80                 if (deep[p] == deep[q]) add(root[i], 1, n, v[q]);
81             } else if (f == 2) {
82                 k = read();
83                 root[i] = root[k];
```

```
84         has[i] = has[k];
85     }
86     printf("%d\n", has[i]);
87 }
88 }
89 return 0;
90 }
```

## 3.7 Tree

### 3.7.1 LCA

```
1  // const int maxn = 1e5 + 10;
2
3  // 普通倍增lca
4  int n, dep[maxn], fa[maxn][30];
5  vector<int> edge[maxn];
6
7  void dfs(int u, int pre) {
8      dep[u] = dep[pre] + 1, fa[u][0] = pre;
9      for(int i = 1; (1 << i) <= n; i++)
10         fa[u][i] = fa[fa[u][i - 1]][i - 1];
11     for(auto v : edge[u]) if(v != pre) dfs(v, u);
12 }
13
14 int LCA(int u, int v) {
15     if(dep[u] < dep[v]) swap(u, v);
16     int d = dep[u] - dep[v];
17     for(int i = 0; (1 << i) <= d; i++)
18         if((1 << i) & d) u = fa[u][i];
19     if(u == v) return u;
20     for(int i = 20; i >= 0; i--)
21         if(fa[u][i] != fa[v][i])
22             u = fa[u][i], v = fa[v][i];
23     return fa[u][0];
24 }
25
26
27 // 欧拉序lca
28 // pow2 = 19
29 // maxn = 1 << pow2
30 int st[maxn << 1][pow2 + 1], dep[maxn], euler[maxn], euler_clock, fa[maxn];
31 void dfs(int u, int p) {
32     fa[u] = p;
33     dep[u] = dep[p] + 1;
34     st[++euler_clock][0] = u;
35     euler[u] = euler_clock;
36     for (const auto& v : adj0[u]) if (v != p) {
37         dfs(v, u);
38         st[++euler_clock][0] = u;
39     }
40 }
41 void lca_init() {
42     for (int i = 0; i != 31 - __builtin_clz(euler_clock); ++i)
43         for (int j = 1; j + (1 << (i + 1)) <= euler_clock; ++j)
44             st[j][i + 1] = upper(st[j][i], st[j + (1 << i)][i]);
45 }
46 inline int lca(int u, int v) {
47     if (u == v) return u;
```

```

48     u = euler[u];
49     v = euler[v];
50     if (u > v) swap(u, v);
51     int temp = 31 - __builtin_clz(++v - u);
52     return upper(st[u][temp], st[v - (1 << temp)][temp]);
53 }
54
55 // dfs(1, 0);
56 // lca_init();
57
58 // 另有树剖lca详见hld模板

```

### 3.7.2 前向星

```

1 // 清零 head 和 tot
2 const int maxm = 4e5+5;
3 int ver[maxm], Next[maxm], head[maxn], edge[maxm];
4 void addEdge(int u, int v, int w){
5     ver[++tot]=v;
6     Next[tot]=head[u];
7     head[u]=tot;
8     edge[tot]=w;
9 }
10
11 for(int i = head[u]; i; i=Next[i])

```

### 3.7.3 点分治

```

1 int n, k;
2
3 // 清零 head 和 tot
4 const int maxm = 2e4+5;
5 int ver[maxm], Next[maxm], head[maxn], edge[maxm];
6 int tot;
7 void addEdge(int u, int v, int w){
8     ver[++tot]=v;
9     Next[tot]=head[u];
10    head[u]=tot;
11    edge[tot]=w;
12 }
13
14 int sz[maxn], vis[maxn];
15 int rt, mxsz, has;
16
17 void getrt(int u, int pre) {
18     sz[u] = 1;
19     int mxnow = 0;
20     for (int i = head[u]; i; i = Next[i]) {
21         int v = ver[i];
22         if (v == pre || vis[v]) continue;
23         getrt(v, u);
24         sz[u] += sz[v];
25         mxnow = max(mxnow, sz[v]);
26     }
27     mxnow = max(mxnow, has - sz[u]);
28     if (mxnow < mxsz) {
29         mxsz = mxnow, rt = u;
30     }

```

```
31 }
32
33 int dl[maxn], r;
34 int val[maxn];
35
36 void getdis(int u, int pre) {
37     dl[r++] = val[u];
38     for (int i = head[u]; i; i = Next[i]) {
39         int v = ver[i];
40         if (v == pre || vis[v]) continue;
41         val[v] = val[u] + edge[i];
42         getdis(v, u);
43     }
44 }
45
46 ll cal(int u, int pre) {
47     r = 0;
48     val[u] = pre;
49     getdis(u, 0);
50     ll sum = 0;
51     sort(dl, dl + r);
52     r --;
53     int l = 0;
54     while (l < r) {
55         if (dl[l] + dl[r] > k) r --;
56         else sum += r - l, l ++;
57     }
58     return sum;
59 }
60
61 ll res = 0;
62 void dfs(int u) {
63     res += cal(u, 0);
64     vis[u] = 1;
65     for (int i = head[u]; i; i = Next[i]) {
66         int v = ver[i];
67         if (vis[v]) continue;
68         res -= cal(v, edge[i]);
69         has = sz[v];
70         mxsz = 0x3f3f3f3f;
71         getrt(v, 0);
72         dfs(v);
73     }
74 }
75
76 int main(int argc, char* argv[]) {
77     while (scanf("%d%d", &n, &k) != EOF && (n || k)) {
78         tot = 0; memset(head, 0, sizeof head);
79         memset(vis, 0, sizeof vis);
80         res = 0;
81         for (int i = 1, u, v, w; i < n; ++i) {
82             scanf("%d%d%d", &u, &v, &w);
83             addEdge(u, v, w);
84             addEdge(v, u, w);
85         }
86         mxsz = 0x3f3f3f3f;
87         has = n;
88         getrt(1, 0);
89         dfs(1);
```

```

90     printf("%lld\n", res);
91 }
92 return 0;
93 }

```

### 3.8 Others

#### 3.8.1 BITinNM

```

1  struct Fenwick_Tree {
2  #define type int
3      type bit[maxn][maxn];
4      int n, m;
5      void init(int _n, int _m) {
6          n = _n;
7          m = _m;
8          mem(bit, 0);
9      }
10     int lowbit(int x) { return x & (-x); }
11     void update(int x, int y, type v) {
12         int i, j;
13         for (i = x; i <= n; i += lowbit(i)) {
14             for (j = y; j <= m; j += lowbit(j)) {
15                 bit[i][j] += v;
16             }
17         }
18     }
19     type get(int x, int y) {
20         type i, j, res = 0;
21         for (i = x; i > 0; i -= lowbit(i)) {
22             for (j = y; j > 0; j -= lowbit(j)) {
23                 res += bit[i][j];
24             }
25         }
26         return res;
27     }
28     type query(int x1, int x2, int y1, int y2) {
29         x1--;
30         y1--;
31         return get(x2, y2) - get(x1, y2) - get(x2, y1) + get(x1, y1);
32     }
33 #undef type
34 } tr;
35
36 // 二维区间前缀和写法 (非树状数组)
37 inline void range_add(int xa, int ya, int xb, int yb) { add(xa, ya, 1), add(xa, yb + 1, -1), add(xb
    + 1, ya, -1), add(xb + 1, yb + 1, 1); }
38 inline ll range_ask(int xa, int ya, int xb, int yb){ return ask(xb, yb) - ask(xb, ya - 1) - ask(xa
    - 1, yb) + ask(xa - 1, ya - 1); }
39 inline void build() {
40     for (int i = 1; i < n + 5; ++i) {
41         for (int j = 1; j < m + 5; ++j) {
42             if (st[i][j] > 1) st[i][j] = 1;
43             st[i][j] += st[i - 1][j] + st[i][j - 1] - st[i - 1][j - 1];
44         }
45     }
46 }
47
48 // 二维树状数组区间加与求和

```



```

49 ll t1[maxn][maxn], t2[maxn][maxn], t3[maxn][maxn], t4[maxn][maxn];
50 void add(ll x, ll y, ll z){
51     for(int X = x; X <= n; X += X & -X)
52         for(int Y = y; Y <= m; Y += Y & -Y){
53             t1[X][Y] += z;
54             t2[X][Y] += z * x;
55             t3[X][Y] += z * y;
56             t4[X][Y] += z * x * y;
57         }
58 }
59 ll ask(ll x, ll y){
60     ll res = 0;
61     for(int i = x; i; i -= i & -i)
62         for(int j = y; j; j -= j & -j)
63             res += (x + 1) * (y + 1) * t1[i][j]
64                 - (y + 1) * t2[i][j]
65                 - (x + 1) * t3[i][j]
66                 + t4[i][j];
67     return res;
68 }
69
70 // 区间加, 询问单点: 直接维护前缀差分数组, 求单点=普通求前缀和

```

### 3.8.2 静态区间 k 大划分树

```

1 // const int maxn = 100010;
2 int tree[20][maxn];
3 // 读入sorted并排序, 赋值给tree的第0层
4 int sorted[maxn];
5 int toleft[20][maxn];
6 // 保存左子树的和
7 // ll sum[20][maxn];
8
9 // 1, n, 0
10 void build(int l, int r, int dep) {
11     if (l == r) return;
12     // sum[dep][0] = 0;
13     toleft[dep][0] = 0;
14     int mid = l + r >> 1;
15     int same = mid - l + 1;
16     for (int i = l; i <= r; ++i) {
17         if (tree[dep][i] < sorted[mid]) same--;
18     }
19     int lpos = l, rpos = mid + 1;
20     for (int i = l; i <= r; ++i) {
21         // sum[dep][i] = sum[dep][i - 1];
22         if (tree[dep][i] < sorted[mid]) {
23             // sum[dep][i] += tree[dep][i];
24             tree[dep + 1][lpos++] = tree[dep][i];
25         }
26         else if (tree[dep][i] == sorted[mid] && same > 0) {
27             // sum[dep][i] += tree[dep][i];
28             tree[dep + 1][lpos++] = tree[dep][i];
29             same--;
30         } else tree[dep + 1][rpos++] = tree[dep][i];
31         toleft[dep][i] = toleft[dep][l - 1] + lpos - 1;
32     }
33     build(l, mid, dep + 1);

```

```
34     build(mid + 1, r, dep + 1);
35 }
36
37 // (1~k-1)的数的和, 注意每次查询前初始化
38 // ll res = 0;
39
40 // L = 1, R = n, dep = 0, l, r是查询区间
41 int query(int L, int R, int l, int r, int dep, int k) {
42     if (l == r) return tree[dep][l];
43     int mid = (L + R) >> 1;
44     int cnt = toleft[dep][r] - toleft[dep][l - 1];
45     if (cnt >= k) {
46         int newl = L + toleft[dep][l - 1] - toleft[dep][L - 1];
47         int newr = newl + cnt - 1;
48         return query(L, mid, newl, newr, dep + 1, k);
49     } else {
50         int newr = r + toleft[dep][R] - toleft[dep][r];
51         int newl = newr - (r - l - cnt);
52         // res += sum[dep][r] - sum[dep][l - 1];
53         return query(mid + 1, R, newl, newr, dep + 1, k - cnt);
54     }
55 }
56
57
58 scan(n), scan(m);
59 for (int i = 1; i <= n; ++i) {
60     scan(sorted[i]);
61     tree[0][i] = sorted[i];
62 }
63 sort(sorted + 1, sorted + 1 + n);
64 build(1, n, 0);
65 int l, r, k;
66 while (m--) {
67     scan(l), scan(r), scan(k);
68     printf("%d\n", query(1, n, l, r, 0, k));
69 }
```

## 4 Geometry

### 4.1 Class

#### 4.1.1 geo

```

1  #define mp make_pair
2  #define fi first
3  #define se second
4  #define pb push_back
5  typedef double db;
6  const db eps=1e-6;
7  const db pi=acos(-1);
8  int sign(db k){
9      if (k>eps) return 1; else if (k<-eps) return -1; return 0;
10 }
11 int cmp(db k1,db k2){return sign(k1-k2);}
12 int inmid(db k1,db k2,db k3){return sign(k1-k3)*sign(k2-k3)<=0;}// k3 在 [k1,k2] 内
13 struct point{
14     db x,y;
15     point operator + (const point &k1) const{return (point){k1.x+x,k1.y+y};}
16     point operator - (const point &k1) const{return (point){x-k1.x,y-k1.y};}
17     point operator * (db k1) const{return (point){x*k1,y*k1};}
18     point operator / (db k1) const{return (point){x/k1,y/k1};}
19     int operator == (const point &k1) const{return cmp(x,k1.x)==0&&cmp(y,k1.y)==0;}
20     // 逆时针旋转
21     point turn(db k1){return (point){x*cos(k1)-y*sin(k1),x*sin(k1)+y*cos(k1)};}
22     point turn90(){return (point){-y,x};}
23     bool operator < (const point k1) const{
24         int a=cmp(x,k1.x);
25         if (a==1) return 1; else if (a==1) return 0; else return cmp(y,k1.y)==-1;
26     }
27     db abs(){return sqrt(x*x+y*y);}
28     db abs2(){return x*x+y*y;}
29     db dis(point k1){return ((*this)-k1).abs();}
30     point unit(){db w=abs(); return (point){x/w,y/w};}
31     void scan(){double k1,k2; scanf("%lf%lf",&k1,&k2); x=k1; y=k2;}
32     void print(){printf("%.11lf %.11lf\n",x,y);}
33     db getw(){return atan2(y,x);}
34     point getdel(){if (sign(x)==-1||(sign(x)==0&&sign(y)==-1)) return (*this)*(-1); else return (*this);}
35     int getP() const{return sign(y)==1||(sign(y)==0&&sign(x)==-1);}
36 };
37 int inmid(point k1,point k2,point k3){return inmid(k1.x,k2.x,k3.x)&&inmid(k1.y,k2.y,k3.y);}
38 db cross(point k1,point k2){return k1.x*k2.y-k1.y*k2.x;}
39 db dot(point k1,point k2){return k1.x*k2.x+k1.y*k2.y;}
40 db rad(point k1,point k2){return atan2(cross(k1,k2),dot(k1,k2));}
41 // -pi -> pi
42 int compareangle (point k1,point k2){
43     return k1.getP()<k2.getP()||(k1.getP()==k2.getP()&&sign(cross(k1,k2))>0);
44 }
45 point proj(point k1,point k2,point q){ // q 到直线 k1,k2 的投影
46     point k=k2-k1; return k1+k*(dot(q-k1,k)/k.abs2());
47 }
48 point reflect(point k1,point k2,point q){return proj(k1,k2,q)*2-q;}
49 int clockwise(point k1,point k2,point k3){// k1 k2 k3 逆时针 1 顺时针 -1 否则 0
50     return sign(cross(k2-k1,k3-k1));
51 }
52 int checkLL(point k1,point k2,point k3,point k4){// 求直线 (L) 线段 (S)k1,k2 和 k3,k4 的交点

```

```

53     return cmp(cross(k3-k1,k4-k1),cross(k3-k2,k4-k2))!=0;
54 }
55 point getLL(point k1,point k2,point k3,point k4){
56     db w1=cross(k1-k3,k4-k3),w2=cross(k4-k3,k2-k3); return (k1*w2+k2*w1)/(w1+w2);
57 }
58 int intersect(db l1,db r1,db l2,db r2){
59     if (l1>r1) swap(l1,r1); if (l2>r2) swap(l2,r2); return cmp(r1,l2)!=-1&&cmp(r2,l1)!=-1;
60 }
61 int checkSS(point k1,point k2,point k3,point k4){
62     return intersect(k1.x,k2.x,k3.x,k4.x)&&intersect(k1.y,k2.y,k3.y,k4.y)&&
63     sign(cross(k3-k1,k4-k1))*sign(cross(k3-k2,k4-k2))<=0&&
64     sign(cross(k1-k3,k2-k3))*sign(cross(k1-k4,k2-k4))<=0;
65 }
66 db disSP(point k1,point k2,point q){
67     point k3=proj(k1,k2,q);
68     if (inmid(k1,k2,k3)) return q.dis(k3); else return min(q.dis(k1),q.dis(k2));
69 }
70 db disSS(point k1,point k2,point k3,point k4){
71     if (checkSS(k1,k2,k3,k4)) return 0;
72     else return min(min(disSP(k1,k2,k3),disSP(k1,k2,k4)),min(disSP(k3,k4,k1),disSP(k3,k4,k2)));
73 }
74 int onS(point k1,point k2,point q){return inmid(k1,k2,q)&&sign(cross(k1-q,k2-k1))==0;}
75 struct circle{
76     point o; db r;
77     void scan(){o.scan(); scanf("%lf",&r);}
78     int inside(point k){return cmp(r,o.dis(k));}
79 };
80 struct line{
81     // p[0]->p[1]
82     point p[2];
83     line(point k1,point k2){p[0]=k1; p[1]=k2;}
84     point& operator [] (int k){return p[k];}
85     int include(point k){return sign(cross(p[1]-p[0],k-p[0]))>0;}
86     point dir(){return p[1]-p[0];}
87     line push(){ // 向外 ( 左手边 ) 平移 eps
88         const db eps = 1e-6;
89         point delta=(p[1]-p[0]).turn90().unit()*eps;
90         return {p[0]-delta,p[1]-delta};
91     }
92 };
93 point getLL(line k1,line k2){return getLL(k1[0],k1[1],k2[0],k2[1]);}
94 int parallel(line k1,line k2){return sign(cross(k1.dir(),k2.dir()))==0;}
95 int sameDir(line k1,line k2){return parallel(k1,k2)&&sign(dot(k1.dir(),k2.dir()))==1;}
96 int operator < (line k1,line k2){
97     if (sameDir(k1,k2)) return k2.include(k1[0]);
98     return compareangle(k1.dir(),k2.dir());
99 }
100 int checkpos(line k1,line k2,line k3){return k3.include(getLL(k1,k2));}
101 vector<line> getHL(vector<line> &L){ // 求半平面交 , 半平面是逆时针方向 , 输出按照逆时针
102     sort(L.begin(),L.end()); deque<line> q;
103     for (int i=0;i<(int)L.size();i++){
104         if (i&&sameDir(L[i],L[i-1])) continue;
105         while (q.size()>1&&!checkpos(q[q.size()-2],q[q.size()-1],L[i])) q.pop_back();
106         while (q.size()>1&&!checkpos(q[1],q[0],L[i])) q.pop_front();
107         q.push_back(L[i]);
108     }
109     while (q.size()>2&&!checkpos(q[q.size()-2],q[q.size()-1],q[0])) q.pop_back();
110     while (q.size()>2&&!checkpos(q[1],q[0],q[q.size()-1])) q.pop_front();
111     vector<line>ans; for (int i=0;i<q.size();i++) ans.push_back(q[i]);

```

```

112     return ans;
113 }
114 db closepoint(vector<point>&A,int l,int r){ // 最近点对 , 先要按照 x 坐标排序
115     if (r-l<=5){
116         db ans=1e20;
117         for (int i=l;i<=r;i++) for (int j=i+1;j<=r;j++) ans=min(ans,A[i].dis(A[j]));
118         return ans;
119     }
120     int mid=l+r>>1; db ans=min(closepoint(A,l,mid),closepoint(A,mid+1,r));
121     vector<point>B; for (int i=l;i<=r;i++) if (abs(A[i].x-A[mid].x)<=ans) B.push_back(A[i]);
122     sort(B.begin(),B.end(),[](point k1,point k2){return k1.y<k2.y;});
123     for (int i=0;i<B.size();i++) for (int j=i+1;j<B.size();j++) ans=min(ans,B[i].dis(B[j]));
124     return ans;
125 }
126 int checkposCC(circle k1,circle k2){// 返回两个圆的公切线数量
127     if (cmp(k1.r,k2.r)==-1) swap(k1,k2);
128     db dis=k1.o.dis(k2.o); int w1=cmp(dis,k1.r+k2.r),w2=cmp(dis,k1.r-k2.r);
129     if (w1>0) return 4; else if (w1==0) return 3; else if (w2>0) return 2;
130     else if (w2==0) return 1; else return 0;
131 }
132 vector<point> getCL(circle k1,point k2,point k3){ // 沿着 k2->k3 方向给出 , 相切给出两个
133     point k=proj(k2,k3,k1.o); db d=k1.r*k1.r-(k-k1.o).abs2();
134     if (sign(d)==-1) return {};
135     point del=(k3-k2).unit()*sqrt(max((db)0.0,d)); return {k-del,k+del};
136 }
137 vector<point> getCC(circle k1,circle k2){// 沿圆 k1 逆时针给出 , 相切给出两个
138     int pd=checkposCC(k1,k2); if (pd==0||pd==4) return {};
139     db a=(k2.o-k1.o).abs2(),cosA=(k1.r*k1.r+a-k2.r*k2.r)/(2*k1.r*sqrt(max(a,(db)0.0)));
140     db b=k1.r*cosA,c=sqrt(max((db)0.0,k1.r*k1.r-b*b));
141     point k=(k2.o-k1.o).unit(),m=k1.o+k*b,del=k.turn90()*c;
142     return {m-del,m+del};
143 }
144 vector<point> TangentCP(circle k1,point k2){// 沿圆 k1 逆时针给出
145     db a=(k2-k1.o).abs(),b=k1.r*k1.r/a,c=sqrt(max((db)0.0,k1.r*k1.r-b*b));
146     point k=(k2-k1.o).unit(),m=k1.o+k*b,del=k.turn90()*c;
147     return {m-del,m+del};
148 }
149 vector<line> TangentoutCC(circle k1,circle k2){
150     int pd=checkposCC(k1,k2); if (pd==0) return {};
151     if (pd==1){point k=getCC(k1,k2)[0]; return {(line){k,k}};}
152     if (cmp(k1.r,k2.r)==0){
153         point del=(k2.o-k1.o).unit().turn90().getdel();
154         return {(line){k1.o-del*k1.r,k2.o-del*k2.r},{k1.o+del*k1.r,k2.o+del*k2.r}};
155     } else {
156         point p=(k2.o*k1.r-k1.o*k2.r)/(k1.r-k2.r);
157         vector<point>A=TangentCP(k1,p),B=TangentCP(k2,p);
158         vector<line>ans; for (int i=0;i<A.size();i++) ans.push_back((line){A[i],B[i]});
159         return ans;
160     }
161 }
162 vector<line> TangentinCC(circle k1,circle k2){
163     int pd=checkposCC(k1,k2); if (pd<=2) return {};
164     if (pd==3){point k=getCC(k1,k2)[0]; return {(line){k,k}};}
165     point p=(k2.o*k1.r+k1.o*k2.r)/(k1.r+k2.r);
166     vector<point>A=TangentCP(k1,p),B=TangentCP(k2,p);
167     vector<line>ans; for (int i=0;i<A.size();i++) ans.push_back((line){A[i],B[i]});
168     return ans;
169 }

```

```

170 vector<line> TangentCC(circle k1,circle k2){
171     int flag=0; if (k1.r<k2.r) swap(k1,k2),flag=1;
172     vector<line>A=TangentoutCC(k1,k2),B=TangentinCC(k1,k2);
173     for (line k:B) A.push_back(k);
174     if (flag) for (line &k:A) swap(k[0],k[1]);
175     return A;
176 }
177 db getarea(circle k1,point k2,point k3){
178     // 圆 k1 与三角形 k2 k3 k1.o 的有向面积交
179     point k=k1.o; k1.o=k1.o-k; k2=k2-k; k3=k3-k;
180     int pd1=k1.inside(k2),pd2=k1.inside(k3);
181     vector<point>A=getCL(k1,k2,k3);
182     if (pd1>=0){
183         if (pd2>=0) return cross(k2,k3)/2;
184         return k1.r*k1.r*rad(A[1],k3)/2+cross(k2,A[1])/2;
185     } else if (pd2>=0){
186         return k1.r*k1.r*rad(k2,A[0])/2+cross(A[0],k3)/2;
187     } else {
188         int pd=cmp(k1.r,disSP(k2,k3,k1.o));
189         if (pd<=0) return k1.r*k1.r*rad(k2,k3)/2;
190         return cross(A[0],A[1])/2+k1.r*k1.r*(rad(k2,A[0])+rad(A[1],k3))/2;
191     }
192 }
193 circle getcircle(point k1,point k2,point k3){
194     db a1=k2.x-k1.x,b1=k2.y-k1.y,c1=(a1*a1+b1*b1)/2;
195     db a2=k3.x-k1.x,b2=k3.y-k1.y,c2=(a2*a2+b2*b2)/2;
196     db d=a1*b2-a2*b1;
197     point o=(point){k1.x+(c1*b2-c2*b1)/d,k1.y+(a1*c2-a2*c1)/d};
198     return (circle){o,k1.dis(o)};
199 }
200 circle getScircle(vector<point> A){
201     random_shuffle(A.begin(),A.end());
202     circle ans=(circle){A[0],0};
203     for (int i=1;i<A.size();i++)
204         if (ans.inside(A[i])==-1){
205             ans=(circle){A[i],0};
206             for (int j=0;j<i;j++)
207                 if (ans.inside(A[j])==-1){
208                     ans.o=(A[i]+A[j])/2; ans.r=ans.o.dis(A[i]);
209                     for (int k=0;k<j;k++)
210                         if (ans.inside(A[k])==-1)
211                             ans=getcircle(A[i],A[j],A[k]);
212                 }
213         }
214     return ans;
215 }
216 db area(vector<point> A){ // 多边形用 vector<point> 表示 , 逆时针
217     db ans=0;
218     for (int i=0;i<A.size();i++) ans+=cross(A[i],A[(i+1)%A.size()]);
219     return ans/2;
220 }
221 int checkconvex(vector<point>A){
222     int n=A.size(); A.push_back(A[0]); A.push_back(A[1]);
223     for (int i=0;i<n;i++) if (sign(cross(A[i+1]-A[i],A[i+2]-A[i]))== -1) return 0;
224     return 1;
225 }
226 int contain(vector<point>A,point q){ // 2 内部 1 边界 0 外部
227     int pd=0; A.push_back(A[0]);
228     for (int i=1;i<A.size();i++){

```

```

229     point u=A[i-1],v=A[i];
230     if (onS(u,v,q)) return 1; if (cmp(u.y,v.y)>0) swap(u,v);
231     if (cmp(u.y,q.y)>=0|cmp(v.y,q.y)<0) continue;
232     if (sign(cross(u-v,q-v))<0) pd*=1;
233 }
234 return pd<<1;
235 }
236 vector<point> ConvexHull(vector<point>A,int flag=1){ // flag=0 不严格 flag=1 严格
237     int n=A.size(); vector<point>ans(n*2);
238     sort(A.begin(),A.end()); int now=-1;
239     for (int i=0;i<A.size();i++){
240         while (now>0&&sign(cross(ans[now]-ans[now-1],A[i]-ans[now-1]))<flag) now--;
241         ans[++now]=A[i];
242     } int pre=now;
243     for (int i=n-2;i>=0;i--){
244         while (now>pre&&sign(cross(ans[now]-ans[now-1],A[i]-ans[now-1]))<flag) now--;
245         ans[++now]=A[i];
246     } ans.resize(now); return ans;
247 }
248 db convexDiameter(vector<point>A){
249     int now=0,n=A.size(); db ans=0;
250     for (int i=0;i<A.size();i++){
251         now=max(now,i);
252         while (1){
253             db k1=A[i].dis(A[now%n]),k2=A[i].dis(A[(now+1)%n]);
254             ans=max(ans,max(k1,k2)); if (k2>k1) now++; else break;
255         }
256     }
257     return ans;
258 }
259 vector<point> convexcut(vector<point>A,point k1,point k2){
260     // 保留 k1,k2,p 逆时针的所有点
261     int n=A.size(); A.push_back(A[0]); vector<point>ans;
262     for (int i=0;i<n;i++){
263         int w1=clockwise(k1,k2,A[i]),w2=clockwise(k1,k2,A[i+1]);
264         if (w1>=0) ans.push_back(A[i]);
265         if (w1*w2<0) ans.push_back(getLL(k1,k2,A[i],A[i+1]));
266     }
267     return ans;
268 }
269 int checkPoS(vector<point>A,point k1,point k2){
270     // 多边形 A 和直线 ( 线段 )k1->k2 严格相交 , 注释部分为线段
271     struct ins{
272         point m,u,v;
273         int operator < (const ins& k) const {return m<k.m;}
274     }; vector<ins>B;
275     //if (contain(A,k1)==2||contain(A,k2)==2) return 1;
276     vector<point>poly=A; A.push_back(A[0]);
277     for (int i=1;i<A.size();i++) if (checkLL(A[i-1],A[i],k1,k2)){
278         point m=getLL(A[i-1],A[i],k1,k2);
279         if (inmid(A[i-1],A[i],m)/*&&inmid(k1,k2,m)*/) B.push_back((ins){m,A[i-1],A[i]});
280     }
281     if (B.size()==0) return 0; sort(B.begin(),B.end());
282     int now=1; while (now<B.size()&&B[now].m==B[0].m) now++;
283     if (now==B.size()) return 0;
284     int flag=contain(poly,(B[0].m+B[now].m)/2);
285     if (flag==2) return 1;
286     point d=B[now].m-B[0].m;
287     for (int i=now;i<B.size();i++){

```

```

288         if (!B[i].m==B[i-1].m)&&flag==2) return 1;
289         int tag=sign(cross(B[i].v-B[i].u,B[i].m+d-B[i].u));
290         if (B[i].m==B[i].u||B[i].m==B[i].v) flag+=tag; else flag+=tag*2;
291     }
292     //return 0;
293     return flag==2;
294 }
295 int checkinp(point r,point l,point m){
296     if (compareangle(l,r)){return compareangle(l,m)&&compareangle(m,r);}
297     return compareangle(l,m)||compareangle(m,r);
298 }
299 int checkPosFast(vector<point>A,point k1,point k2){ // 快速检查线段是否和多边形严格相交
300     if (contain(A,k1)==2||contain(A,k2)==2) return 1; if (k1==k2) return 0;
301     A.push_back(A[0]); A.push_back(A[1]);
302     for (int i=1;i+1<A.size();i++){
303         if (checkLL(A[i-1],A[i],k1,k2)){
304             point now=getLL(A[i-1],A[i],k1,k2);
305             if (inmid(A[i-1],A[i],now)==0||inmid(k1,k2,now)==0) continue;
306             if (now==A[i]){
307                 if (A[i]==k2) continue;
308                 point pre=A[i-1],ne=A[i+1];
309                 if (checkinp(pre-now,ne-now,k2-now)) return 1;
310             } else if (now==k1){
311                 if (k1==A[i-1]||k1==A[i]) continue;
312                 if (checkinp(A[i-1]-k1,A[i]-k1,k2-k1)) return 1;
313             } else if (now==k2||now==A[i-1]) continue;
314             else return 1;
315         }
316     }
317     return 0;
318 }
319 // 拆分凸包成上下凸壳 凸包尽量都随机旋转一个角度来避免出现相同横坐标
320 // 尽量特判只有一个点的情况 凸包逆时针
321 void getUDP(vector<point>A,vector<point>&U,vector<point>&D){
322     db l=1e100,r=-1e100;
323     for (int i=0;i<A.size();i++) l=min(l,A[i].x),r=max(r,A[i].x);
324     int wherel,wherer;
325     for (int i=0;i<A.size();i++) if (cmp(A[i].x,l)==0) wherel=i;
326     for (int i=A.size();i;i--) if (cmp(A[i-1].x,r)==0) wherer=i-1;
327     U.clear(); D.clear(); int now=wherel;
328     while (1){D.push_back(A[now]); if (now==wherer) break; now++; if (now>=A.size()) now=0;}
329     now=wherel;
330     while (1){U.push_back(A[now]); if (now==wherer) break; now--; if (now<0) now=A.size()-1;}
331 }
332 // 需要保证凸包点数大于等于 3,2 内部 ,1 边界 ,0 外部
333 int containCoP(const vector<point>&U,const vector<point>&D,point k){
334     db lx=U[0].x,rx=U[U.size()-1].x;
335     if (k==U[0]||k==U[U.size()-1]) return 1;
336     if (cmp(k.x,lx)==-1||cmp(k.x,rx)==1) return 0;
337     int where1=lower_bound(U.begin(),U.end(),(point){k.x,-1e100})-U.begin();
338     int where2=lower_bound(D.begin(),D.end(),(point){k.x,-1e100})-D.begin();
339     int w1=clockwise(U[where1-1],U[where1],k),w2=clockwise(D[where2-1],D[where2],k);
340     if (w1==1||w2==1) return 0; else if (w1==0||w2==0) return 1; return 2;
341 }
342 // d 是方向 , 输出上方切点和下方切点
343 pair<point,point> getTangentCow(const vector<point> &U,const vector<point> &D,point d){
344     if (sign(d.x)<0||((sign(d.x)==0&&sign(d.y)<0)) d=d*(-1);
345     point whereU,whereD;
346     if (sign(d.x)==0) return mp(U[0],U[U.size()-1]);
347     int l=0,r=U.size()-1,ans=0;

```



```

347     while (l<r){int mid=l+r>>1; if (sign(cross(U[mid+1]-U[mid],d))<=0) l=mid+1,ans=mid+1; else r=
mid;}
348     whereU=U[ans]; l=0,r=D.size()-1,ans=0;
349     while (l<r){int mid=l+r>>1; if (sign(cross(D[mid+1]-D[mid],d))>=0) l=mid+1,ans=mid+1; else r=
mid;}
350     whereD=D[ans]; return mp(whereU,whereD);
351 }
352 // 先检查 contain, 逆时针给出
353 pair<point,point> getTangentCoP(const vector<point>&U,const vector<point>&D,point k){
354     db lx=U[0].x,rx=U[U.size()-1].x;
355     if (k.x<lx){
356         int l=0,r=U.size()-1,ans=U.size()-1;
357         while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid+1])==1) l=mid+1; else ans=mid,r=
mid;}
358         point w1=U[ans]; l=0,r=D.size()-1,ans=D.size()-1;
359         while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid+1])==-1) l=mid+1; else ans=mid,r=
mid;}
360         point w2=D[ans]; return mp(w1,w2);
361     } else if (k.x>rx){
362         int l=1,r=U.size(),ans=0;
363         while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid-1])==-1) r=mid; else ans=mid,l=mid
+1;}
364         point w1=U[ans]; l=1,r=D.size(),ans=0;
365         while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid-1])==-1) r=mid; else ans=mid,l=mid
+1;}
366         point w2=D[ans]; return mp(w2,w1);
367     } else {
368         int where1=lower_bound(U.begin(),U.end(),(point){k.x,-1e100})-U.begin();
369         int where2=lower_bound(D.begin(),D.end(),(point){k.x,-1e100})-D.begin();
370         if ((k.x==lx&&k.y>U[0].y)|| (where1&&clockwise(U[where1-1],U[where1],k)==1)){
371             int l=1,r=where1+1,ans=0;
372             while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid-1])==-1) ans=mid,l=mid+1; else
r=mid;}
373             point w1=U[ans]; l=where1,r=U.size()-1,ans=U.size()-1;
374             while (l<r){int mid=l+r>>1; if (clockwise(k,U[mid],U[mid+1])==-1) l=mid+1; else ans=mid,
r=mid;}
375             point w2=U[ans]; return mp(w2,w1);
376         } else {
377             int l=1,r=where2+1,ans=0;
378             while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid-1])==-1) ans=mid,l=mid+1; else
r=mid;}
379             point w1=D[ans]; l=where2,r=D.size()-1,ans=D.size()-1;
380             while (l<r){int mid=l+r>>1; if (clockwise(k,D[mid],D[mid+1])==-1) l=mid+1; else ans=mid
,r=mid;}
381             point w2=D[ans]; return mp(w1,w2);
382         }
383     }
384 }
385 struct P3{
386     db x,y,z;
387     P3 operator + (P3 k1){return (P3){x+k1.x,y+k1.y,z+k1.z};}
388     P3 operator - (P3 k1){return (P3){x-k1.x,y-k1.y,z-k1.z};}
389     P3 operator * (db k1){return (P3){x*k1,y*k1,z*k1};}
390     P3 operator / (db k1){return (P3){x/k1,y/k1,z/k1};}
391     db abs2(){return x*x+y*y+z*z;}
392     db abs(){return sqrt(x*x+y*y+z*z);}
393     P3 unit(){return (*this)/abs();}
394     int operator < (const P3 k1) const{
395         if (cmp(x,k1.x)!=0) return x<k1.x;

```

```

396     if (cmp(y,k1.y)!=0) return y<k1.y;
397     return cmp(z,k1.z)==-1;
398 }
399 int operator == (const P3 k1){
400     return cmp(x,k1.x)==0&&cmp(y,k1.y)==0&&cmp(z,k1.z)==0;
401 }
402 void scan(){
403     double k1,k2,k3; scanf("%lf%lf%lf",&k1,&k2,&k3);
404     x=k1; y=k2; z=k3;
405 }
406 };
407 P3 cross(P3 k1,P3 k2){return (P3){k1.y*k2.z-k1.z*k2.y,k1.z*k2.x-k1.x*k2.z,k1.x*k2.y-k1.y*k2.x};}
408 db dot(P3 k1,P3 k2){return k1.x*k2.x+k1.y*k2.y+k1.z*k2.z;}
409 //p=(3,4,5),l=(13,19,21),theta=85 ans=(2.83,4.62,1.77)
410 P3 turn3D(db k1,P3 l,P3 p){
411     l=l.unit(); P3 ans; db c=cos(k1),s=sin(k1);
412     ans.x=p.x*(l.x*l.x*(1-c)+c)+p.y*(l.x*l.y*(1-c)-l.z*s)+p.z*(l.x*l.z*(1-c)+l.y*s);
413     ans.y=p.x*(l.x*l.y*(1-c)+l.z*s)+p.y*(l.y*l.y*(1-c)+c)+p.z*(l.y*l.z*(1-c)-l.x*s);
414     ans.z=p.x*(l.x*l.z*(1-c)-l.y*s)+p.y*(l.y*l.z*(1-c)+l.x*s)+p.z*(l.x*l.x*(1-c)+c);
415     return ans;
416 }
417 typedef vector<P3> VP;
418 typedef vector<VP> VVP;
419 db Acos(db x){return acos(max(-(db)1,min(x,(db)1)));}
420 // 球面距离 , 圆心原点 , 半径 1
421 db Odist(P3 a,P3 b){db r=Acos(dot(a,b)); return r;}
422 db r; P3 rnd;
423 vector<db> solve(db a,db b,db c){
424     db r=sqrt(a*a+b*b),th=atan2(b,a);
425     if (cmp(c,-r)==-1) return {0};
426     else if (cmp(r,c)<=0) return {1};
427     else {
428         db tr=pi-Acos(c/r); return {th+pi-tr,th+pi+tr};
429     }
430 }
431 vector<db> jiao(P3 a,P3 b){
432     // dot(rd+x*cos(t)+y*sin(t),b) >= cos(r)
433     if (cmp(Odist(a,b),2*r)>0) return {0};
434     P3 rd=a*cos(r),z=a.unit(),y=cross(z,rnd).unit(),x=cross(y,z).unit();
435     vector<db> ret = solve(-(dot(x,b)*sin(r)),-(dot(y,b)*sin(r)),-(cos(r)-dot(rd,b)));
436     return ret;
437 }
438 db norm(db x,db l=0,db r=2*pi){ // change x into [l,r)
439     while (cmp(x,l)==-1) x+=(r-l); while (cmp(x,r)>=0) x-=(r-l);
440     return x;
441 }
442 db disLP(P3 k1,P3 k2,P3 q){
443     return (cross(k2-k1,q-k1)).abs()/(k2-k1).abs();
444 }
445 db disLL(P3 k1,P3 k2,P3 k3,P3 k4){
446     P3 dir=cross(k2-k1,k4-k3); if (sign(dir.abs())==0) return disLP(k1,k2,k3);
447     return fabs(dot(dir.unit(),k1-k2));
448 }
449 VP getFL(P3 p,P3 dir,P3 k1,P3 k2){
450     db a=dot(k2-p,dir),b=dot(k1-p,dir),d=a-b;
451     if (sign(fabs(d))==0) return {};
452     return {(k1*a-k2*b)/d};
453 }
454 VP getFF(P3 p1,P3 dir1,P3 p2,P3 dir2){// 返回一条线

```

```

455     P3 e=cross(dir1,dir2),v=cross(dir1,e);
456     db d=dot(dir2,v); if (sign(abs(d))==0) return {};
457     P3 q=p1+v*dot(dir2,p2-p1)/d; return {q,q+e};
458 }
459 // 3D Convex Hull Template
460 db getV(P3 k1,P3 k2,P3 k3,P3 k4){ // get the Volume
461     return dot(cross(k2-k1,k3-k1),k4-k1);
462 }
463 db rand_db(){return 1.0*rand()/RAND_MAX;}
464 VP convexHull2D(VP A,P3 dir){
465     P3 x={(db)rand(),(db)rand(),(db)rand()}; x=x.unit();
466     x=cross(x,dir).unit(); P3 y=cross(x,dir).unit();
467     P3 vec=dir.unit()*dot(A[0],dir);
468     vector<point>B;
469     for (int i=0;i<A.size();i++) B.push_back((point){dot(A[i],x),dot(A[i],y)});
470     B=ConvexHull(B); A.clear();
471     for (int i=0;i<B.size();i++) A.push_back(x*B[i].x+y*B[i].y+vec);
472     return A;
473 }
474 namespace CH3{
475     VVP ret; set<pair<int,int> >e;
476     int n; VP p,q;
477     void wrap(int a,int b){
478         if (e.find({a,b})==e.end()){
479             int c=-1;
480             for (int i=0;i<n;i++) if (i!=a&&i!=b){
481                 if (c==-1||sign(getV(q[c],q[a],q[b],q[i]))>0) c=i;
482             }
483             if (c!=-1){
484                 ret.push_back({p[a],p[b],p[c]});
485                 e.insert({a,b}); e.insert({b,c}); e.insert({c,a});
486                 wrap(c,b); wrap(a,c);
487             }
488         }
489     }
490     VVP ConvexHull3D(VP _p){
491         p=q=_p; n=p.size();
492         ret.clear(); e.clear();
493         for (auto &i:q) i=i+(P3){rand_db()*1e-4,rand_db()*1e-4,rand_db()*1e-4};
494         for (int i=1;i<n;i++) if (q[i].x<q[0].x) swap(p[0],p[i]),swap(q[0],q[i]);
495         for (int i=2;i<n;i++) if ((q[i].x-q[0].x)*(q[1].y-q[0].y)>(q[i].y-q[0].y)*(q[1].x-q[0].x))
496             swap(q[1],q[i]),swap(p[1],p[i]);
497         wrap(0,1);
498         return ret;
499     }
500     VVP reduceCH(VVP A){
501         VVP ret; map<P3,VP> M;
502         for (VP nowF:A){
503             P3 dir=cross(nowF[1]-nowF[0],nowF[2]-nowF[0]).unit();
504             for (P3 k1:nowF) M[dir].pb(k1);
505         }
506         for (pair<P3,VP> nowF:M) ret.pb(convexHull2D(nowF.se,nowF.fi));
507         return ret;
508     }
509     // 把一个面变成 ( 点 , 法向量 ) 的形式
510     pair<P3,P3> getF(VP F){
511         return mp(F[0],cross(F[1]-F[0],F[2]-F[0]).unit());
512     }

```

```

513 // 3D Cut 保留 dot(dir,x-p)>=0 的部分
514 VVP ConvexCut3D(VVP A,P3 p,P3 dir){
515     VVP ret; VP sec;
516     for (VP nowF: A){
517         int n=nowF.size(); VP ans; int dif=0;
518         for (int i=0;i<n;i++){
519             int d1=sign(dot(dir,nowF[i]-p));
520             int d2=sign(dot(dir,nowF[(i+1)%n]-p));
521             if (d1>=0) ans.pb(nowF[i]);
522             if (d1*d2<0){
523                 P3 q=getFL(p,dir,nowF[i],nowF[(i+1)%n])[0];
524                 ans.push_back(q); sec.push_back(q);
525             }
526             if (d1==0) sec.push_back(nowF[i]); else dif=1;
527             dif+=(sign(dot(dir,cross(nowF[(i+1)%n]-nowF[i],nowF[(i+1)%n]-nowF[i])))==-1);
528         }
529         if (ans.size()>0&&dif) ret.push_back(ans);
530     }
531     if (sec.size()>0) ret.push_back(convexHull2D(sec,dir));
532     return ret;
533 }
534 db vol(VVP A){
535     if (A.size()==0) return 0; P3 p=A[0][0]; db ans=0;
536     for (VP nowF:A)
537         for (int i=2;i<nowF.size();i++)
538             ans+=abs(getV(p,nowF[0],nowF[i-1],nowF[i]));
539     return ans/6;
540 }
541 VVP init(db INF) {
542     VVP pss(6,VP(4));
543     pss[0][0] = pss[1][0] = pss[2][0] = {-INF, -INF, -INF};
544     pss[0][3] = pss[1][1] = pss[5][2] = {-INF, -INF, INF};
545     pss[0][1] = pss[2][3] = pss[4][2] = {-INF, INF, -INF};
546     pss[0][2] = pss[5][3] = pss[4][1] = {-INF, INF, INF};
547     pss[1][3] = pss[2][1] = pss[3][2] = {INF, -INF, -INF};
548     pss[1][2] = pss[5][1] = pss[3][3] = {INF, -INF, INF};
549     pss[2][2] = pss[4][3] = pss[3][1] = {INF, INF, -INF};
550     pss[5][0] = pss[4][0] = pss[3][0] = {INF, INF, INF};
551     return pss;
552 }

```

#### 4.1.2 3D 凸包

```

1  #include<stdio.h>
2  #include<algorithm>
3  #include<string.h>
4  #include<math.h>
5  #include<stdlib.h>
6  using namespace std;
7  const int MAXN=1050;
8  const double eps=1e-8;
9
10 struct Point
11 {
12     double x,y,z;
13     Point() {}
14     Point(double xx,double yy,double zz):x(xx),y(yy),z(zz) {}
15     //两向量之差

```

```

16 Point operator -(const Point p1)
17 {
18     return Point(x-p1.x,y-p1.y,z-p1.z);
19 }
20 //两向量之和
21 Point operator +(const Point p1)
22 {
23     return Point(x+p1.x,y+p1.y,z+p1.z);
24 }
25 //叉乘
26 Point operator *(const Point p)
27 {
28     return Point(y*p.z-z*p.y,z*p.x-x*p.z,x*p.y-y*p.x);
29 }
30 Point operator *(double d)
31 {
32     return Point(x*d,y*d,z*d);
33 }
34 Point operator / (double d)
35 {
36     return Point(x/d,y/d,z/d);
37 }
38 //点乘
39 double operator ^(Point p)
40 {
41     return (x*p.x+y*p.y+z*p.z);
42 }
43 };
44 struct CH3D
45 {
46     struct face
47     {
48         //表示凸包一个面上的三个点的编号
49         int a,b,c;
50         //表示该面是否属于最终凸包上的面
51         bool ok;
52     };
53     //初始顶点数
54     int n;
55     //初始顶点
56     Point P[MAXN];
57     //凸包表面的三角形数
58     int num;
59     //凸包表面的三角形
60     face F[8*MAXN];
61     //凸包表面的三角形
62     int g[MAXN][MAXN];
63     //向量长度
64     double vlen(Point a)
65     {
66         return sqrt(a.x*a.x+a.y*a.y+a.z*a.z);
67     }
68     //叉乘
69     Point cross(const Point &a,const Point &b,const Point &c)
70     {
71         return Point((b.y-a.y)*(c.z-a.z)-(b.z-a.z)*(c.y-a.y),
72                     (b.z-a.z)*(c.x-a.x)-(b.x-a.x)*(c.z-a.z),
73                     (b.x-a.x)*(c.y-a.y)-(b.y-a.y)*(c.x-a.x)
74                     );

```

```

75     }
76     //三角形面积*2
77     double area(Point a,Point b,Point c)
78     {
79         return vlen((b-a)*(c-a));
80     }
81     //四面体有向体积*6
82     double volume(Point a,Point b,Point c,Point d)
83     {
84         return (b-a)*(c-a)^(d-a);
85     }
86     //正：点在面向向
87     double dblcmp(Point &p,face &f)
88     {
89         Point m=P[f.b]-P[f.a];
90         Point n=P[f.c]-P[f.a];
91         Point t=p-P[f.a];
92         return (m*n)^t;
93     }
94     void deal(int p,int a,int b)
95     {
96         int f=g[a][b];//搜索与该边相邻的另一个平面
97         face add;
98         if(F[f].ok)
99         {
100             if(dblcmp(P[p],F[f])>eps)
101                 dfs(p,f);
102             else
103             {
104                 add.a=b;
105                 add.b=a;
106                 add.c=p;//这里注意顺序，要成右手系
107                 add.ok=true;
108                 g[p][b]=g[a][p]=g[b][a]=num;
109                 F[num++]=add;
110             }
111         }
112     }
113     void dfs(int p,int now)//递归搜索所有应该从凸包内删除的面
114     {
115         F[now].ok=0;
116         deal(p,F[now].b,F[now].a);
117         deal(p,F[now].c,F[now].b);
118         deal(p,F[now].a,F[now].c);
119     }
120     bool same(int s,int t)
121     {
122         Point &a=P[F[s].a];
123         Point &b=P[F[s].b];
124         Point &c=P[F[s].c];
125         return fabs(volume(a,b,c,P[F[t].a]))<eps &&
126             fabs(volume(a,b,c,P[F[t].b]))<eps &&
127             fabs(volume(a,b,c,P[F[t].c]))<eps;
128     }
129     //构建三维凸包
130     void create()
131     {
132         int i,j,tmp;
133         face add;

```

```

134     num=0;
135     if(n<4)return;
136     //*****
137     //此段是为了保证前四个点不共面
138     bool flag=true;
139     for(i=1; i<n; i++)
140         if(vlen(P[0]-P[i])>eps)
141         {
142             swap(P[1],P[i]);
143             flag=false;
144             break;
145         }
146     if(flag)return;
147     flag=true;
148     //使前三个点不共线
149     for(i=2; i<n; i++)
150         if(vlen((P[0]-P[1])*(P[1]-P[i]))>eps)
151         {
152             swap(P[2],P[i]);
153             flag=false;
154             break;
155         }
156     if(flag)return;
157     flag=true;
158     //使前四个点不共面
159     for(int i=3; i<n; i++)
160         if(fabs((P[0]-P[1])*(P[1]-P[2])^(P[0]-P[i]))>eps)
161         {
162             swap(P[3],P[i]);
163             flag=false;
164             break;
165         }
166     if(flag)return;
167     //*****
168     for(i=0; i<4; i++)
169     {
170         add.a=(i+1)%4;
171         add.b=(i+2)%4;
172         add.c=(i+3)%4;
173         add.ok=true;
174         if(dblcmp(P[i],add)>0)swap(add.b,add.c);
175         g[add.a][add.b]=g[add.b][add.c]=g[add.c][add.a]=num;
176         F[num++]=add;
177     }
178     for(i=4; i<n; i++)
179         for(j=0; j<num; j++)
180             if(F[j].ok&&dblcmp(P[i],F[j])>eps)
181             {
182                 dfs(i,j);
183                 break;
184             }
185     tmp=num;
186     for(i=num=0; i<tmp; i++)
187         if(F[i].ok)
188             F[num++]=F[i];
189 }
190 //表面积
191 double area()

```

```

193     {
194         double res=0;
195         if(n==3)
196         {
197             Point p=cross(P[0],P[1],P[2]);
198             res=vlen(p)/2.0;
199             return res;
200         }
201         for(int i=0; i<num; i++)
202             res+=area(P[F[i].a],P[F[i].b],P[F[i].c]);
203         return res/2.0;
204     }
205     //体积
206     double volume()
207     {
208         double res=0;
209         Point tmp(0,0,0);
210         for(int i=0; i<num; i++)
211             res+=volume(tmp,P[F[i].a],P[F[i].b],P[F[i].c]);
212         return fabs(res/6.0);
213     }
214     //表面三角形个数
215     int triangle()
216     {
217         return num;
218     }
219     //表面多边形个数
220     int polygon()
221     {
222         int i,j,res,flag;
223         for(i=res=0; i<num; i++)
224         {
225             flag=1;
226             for(j=0; j<i; j++)
227                 if(same(i,j))
228                 {
229                     flag=0;
230                     break;
231                 }
232             res+=flag;
233         }
234         return res;
235     }
236     //三维凸包重心
237     Point barycenter()
238     {
239         Point ans(0,0,0),o(0,0,0);
240         double all=0;
241         for(int i=0; i<num; i++)
242         {
243             double vol=volume(o,P[F[i].a],P[F[i].b],P[F[i].c]);
244             ans=ans+(o+P[F[i].a]+P[F[i].b]+P[F[i].c])/4.0*vol;
245             all+=vol;
246         }
247         ans=ans/all;
248         return ans;
249     }
250     //点到面的距离
251     double ptoface(Point p,int i)

```



```
252     {
253         return fabs(volume(P[F[i].a],P[F[i].b],P[F[i].c],p)/vlen((P[F[i].b]-P[F[i].a])*(P[F[i].c]-P
[F[i].a]))));
254     }
255 };
256 CH3D hull;
257 int main()
258 {
259     while(~scanf("%d",&hull.n))
260     {
261         for(int i=0; i<hull.n; i++)
262             scanf("%lf%lf%lf",&hull.P[i].x,&hull.P[i].y,&hull.P[i].z);
263         hull.create();
264         printf("%d\n",hull.polygon());
265     }
266     return 0;
267 }
```

## 5 String

### 5.1 KMP

#### 5.1.1 KMP

```

1 // nxt[0]表示失配到完全不匹配
2 int nxt[maxm];
3
4 void getNext(char *s, int len) {
5     int i = 0, j = -1;
6     nxt[i] = j;
7     while (i < len) {
8         if (j == -1 || s[i] == s[j]) nxt[++i] = ++j;
9         else j = nxt[j];
10    }
11 }
12
13 // a为原串, b为模式串, 下标从0开始, 找第一个出现模式串的位置 (起点为1), 找不到返回-1
14 int KMP(char *a, char *b, int n, int m) {
15     getNext(b, m);
16     int i = 0, j = 0;
17     while (i < n && j < m) {
18         if (j == -1 || a[i] == b[j]) ++i, ++j;
19         else j = nxt[j];
20    }
21    return j == m ? i - m + 1 : -1;
22 }

```

#### 5.1.2 exKMP

```

1 const int maxn = 1e5 + 10;
2 int nex[maxn], extend[maxn];
3
4 //预处理计算Next数组
5 void getNext(char *str)
6 {
7     int i = 0, j, po, len = strlen(str);
8     nex[0] = len; //初始化nex[0]
9     while (str[i] == str[i + 1] && i + 1 < len) i++; //计算nex[1]
10    nex[1] = i;
11    po = 1; //初始化po的位置
12    for (int i = 2; i < len; i++)
13    {
14        if (nex[i - po] + i < nex[po] + po) //第一种情况, 可以直接得到nex[i]的值
15            nex[i] = nex[i - po];
16        else //第二种情况, 要继续匹配才能得到nex[i]的值
17        {
18            j = nex[po] + po - i;
19            if (j < 0) j = 0; //如果i>po+nex[po],则要从头开始匹配
20            while (i + j < len && str[j] == str[j + i]) j++;
21            nex[i] = j;
22            po = i; //更新po的位置
23        }
24    }
25 }
26
27 void EXKMP(char *s1, char *s2)
28 {

```

```
29     int i = 0, j, po, len = strlen(s1), l2 = strlen(s2);
30     getNext(s2);
31     while (s1[i] == s2[i] && i < l2 && i < len) i++;
32     extend[0] = i;
33     po = 0;
34     for (int i = 1; i < len; i++)
35     {
36         if (nex[i - po] + i < extend[po] + po)
37             extend[i] = nex[i - po];
38         else
39         {
40             j = extend[po] + po - i;
41             if (j < 0) j = 0;
42             while (i + j < len && j < l2 && s1[j + i] == s2[j]) j++;
43             extend[i] = j;
44             po = i;
45         }
46     }
47 }
```

## 5.2 Trie

### 5.2.1 Trie

```
1  const int maxn = 2e6 + 10;
2
3  int trie[maxn][30], tot;
4  bool flag[maxn];
5
6  void insert_ch(char *str)
7  {
8      int len = strlen(str);
9      int root = 0;
10     for (int i = 0; i < len; i++)
11     {
12         int id = str[i] - 'a';
13         if (!trie[root][id]) trie[root][id] = ++tot;
14         root = trie[root][id];
15     }
16     flag[root] = true;
17 }
18
19 bool find_ch(char *str)
20 {
21     int len = strlen(str);
22     int root = 0;
23     for (int i = 0; i < len; i++)
24     {
25         int id = str[i] - 'a';
26         if (!trie[root][id]) return false;
27         root = trie[root][id];
28     }
29     return true;
30 }
```

### 5.2.2 Persistence Trie

```
1  const int maxn = 1e5 + 10;
```

```

2
3 int a[maxn], rt[maxn], n;
4
5 struct Trie
6 {
7     int tot;
8     int child[maxn * 32][2], sum[maxn * 32];
9     int insert(int x, int val)
10    {
11        int tmp, y;
12        tmp = y = ++tot;
13        for(int i = 30; i >= 0; --i)
14        {
15            child[y][0] = child[x][0];
16            child[y][1] = child[x][1];
17            sum[y] = sum[x] + 1;
18            int t = val >> i & 1;
19            x = child[x][t];
20            child[y][t] = ++tot;
21            y = child[y][t];
22        }
23        sum[y] = sum[x] + 1;
24        return tmp;
25    }
26    int query(int l, int r, int val)
27    {
28        int tmp = 0;
29        for(int i = 30; i >= 0; --i)
30        {
31            int t = val >> i & 1;
32            if(sum[child[r][t^1]] - sum[child[l][t^1]]) tmp += (1<<i), r = child[r][t^1], l = child
[l][t^1];
33            else r = child[r][t], l = child[l][t];
34        }
35        return tmp;
36    }
37 }trie;

```

### 5.2.3 01Trie

```

1 struct Trie {
2     int tree[maxn*20][2], tot;
3     int flag[maxn*20];
4
5     void insert_ch(int x) {
6         int root = 0;
7         flag[0]++;
8         for (int i = 30; i >= 0; --i) {
9             int id = (x >> i) & 1;
10            if (!tree[root][id]) {
11                tree[root][id] = ++tot;
12                tree[tree[root][id]][0] = tree[tree[root][id]][1] = 0;
13                flag[tree[root][id]] = flag[tree[tree[root][id]][0]] = flag[tree[tree[root][id]]
]] [1]] = 0;
14            }
15            root = tree[root][id];
16            flag[root]++;
17        }

```

```
18     }
19
20     void del(int x) {
21         int root = 0;
22         flag[0]--;
23         for (int i = 30; i >= 0; --i) {
24             int id = (x >> i) & 1;
25             assert(tree[root][id]);
26             if (flag[tree[root][id]] == 1) {
27                 flag[tree[root][id]] = 0;
28                 tree[root][id] = 0;
29                 return;
30             }
31             root = tree[root][id];
32             flag[root]--;
33         }
34     }
35
36     int find_ch(int x, int flag = 0) { // flag 0 最小异或值, 1 最大异或值
37         int root = 0;
38         int res = 0;
39         for (int i = 30; i >= 0; --i) {
40             int id = ((x >> i) & 1);
41             if (flag) id = !id;
42             if (tree[root][id]) {
43                 root = tree[root][id];
44                 res = res << 1 | id;
45             } else {
46                 root = tree[root][!id];
47                 res = res << 1 | (!id);
48             }
49         }
50         return res;
51     }
52
53     void init() {
54         tree[0][0] = tree[0][1] = 0;
55         tot = 0;
56     }
57 };
```

## 5.3 Manachar

### 5.3.1 Manacher

```
1  const int maxn = 1e5 + 10;
2
3  char s[maxn];
4
5  char tmp[maxn << 1];
6  int Len[maxn << 1];
7
8  int init(char *str)
9  {
10     int len = strlen(str);
11     tmp[0] = '@';
12     for (int i = 1; i <= 2 * len; i += 2)
13     {
14         tmp[i] = '#';
```

```
15     tmp[i + 1] = str[i / 2];
16 }
17 tmp[2 * len + 1] = '#';
18 tmp[2 * len + 2] = '$';
19 tmp[2 * len + 3] = 0;
20 return 2 * len + 1;
21 }
22
23 int manacher(char *str)
24 {
25     int mx = 0, ans = 0, pos = 0;
26     int len = init(str);
27     for (int i = 1; i <= len; i++)
28     {
29         if (mx > i) Len[i] = min(mx - i, Len[2 * pos - i]);
30         else Len[i] = 1;
31         while (tmp[i - Len[i]] == tmp[i + Len[i]]) Len[i]++;
32         if (Len[i] + i > mx) mx = Len[i] + i, pos = i;
33     }
34 }
```

## 5.4 Aho-Corasick Automation

### 5.4.1 AC Automation

```
1 class AC_automation
2 {
3 public:
4     int trie[maxn][26], cnt;
5     int tag[maxn];
6     int fail[maxn], num[maxn], res[maxn], in[maxn], Map[maxn];
7
8     void init()
9     {
10         memset(trie, 0, sizeof trie);
11         memset(tag, 0, sizeof tag);
12         memset(fail, 0, sizeof fail);
13         cnt = 0;
14     }
15
16     void insert(char *str, int id)
17     {
18         int root = 0;
19         for (int i = 0; str[i]; i++)
20         {
21             int id = str[i] - 'a';
22             if (!trie[root][id]) trie[root][id] = ++cnt;
23             root = trie[root][id];
24         }
25         if (!tag[root]) tag[root] = id;
26         Map[id] = tag[root];
27     }
28
29     void build()
30     {
31         queue<int> que;
32         for (int i = 0; i < 26; i++) if (trie[0][i]) que.push(trie[0][i]);
33         while (!que.empty())
34         {
```

```

35     int k = que.front();
36     que.pop();
37     for (int i = 0; i < 26; i++)
38     {
39         if (trie[k][i])
40         {
41             fail[trie[k][i]] = trie[fail[k]][i];
42             que.push(trie[k][i]);
43             in[fail[trie[k][i]]] ++;
44         } else trie[k][i] = trie[fail[k]][i];
45     }
46 }
47 }
48
49 void toposort()
50 {
51     queue<int> que;
52     for(int i = 1; i <= cnt; i ++) if(in[i] == 0) que.push(i);
53     while(!que.empty())
54     {
55         int u = que.front(); que.pop();
56         res[tag[u]] = num[u];
57         int v = fail[u]; in[v] --;
58         num[v] += num[u];
59         if(in[v] == 0) que.push(v);
60     }
61 }
62
63 void query(char *str, int n)
64 {
65     int u = 0, len = strlen(s);
66     for(int i = 0; i < len; i ++)
67         u = trie[u][str[i] - 'a'], num[u] ++;
68     toposort();
69     for(int i = 1; i <= n; i ++) printf("%d\n", res[Map[i]]);
70 }
71 } AC;

```

## 5.5 Suffix Array

### 5.5.1 Suffix Array

```

1 char s[maxn];
2 int sa[maxn], t[maxn], t2[maxn], c[maxn], n;
3
4 //build_sa(n + 1, 130), sa, height下标从1开始, rk下标从0开始
5 void build_sa(int n, int m)
6 {
7     int *x = t, *y = t2;
8     for(int i = 0; i < m; i++) c[i] = 0;
9     for(int i = 0; i < n; i++) c[x[i]] = s[i]++;
10    for(int i = 1; i < m; i++) c[i] += c[i - 1];
11    for(int i = n - 1; i >= 0; i--) sa[--c[x[i]]] = i;
12    for(int k = 1; k <= n; k <= 1)
13    {
14        int p = 0;
15        for(int i = n - k; i < n; i++) y[p++] = i;
16        for(int i = 0; i < n; i++) if(sa[i] >= k) y[p++] = sa[i] - k;
17        for(int i = 0; i < m; i++) c[i] = 0;

```

```

18     for(int i = 0; i < n; i++) c[x[y[i]]]++;
19     for(int i = 0; i < m; i++) c[i] += c[i - 1];
20     for(int i = n - 1; i >= 0; i--) sa[--c[x[y[i]]]] = y[i];
21     swap(x, y);
22     p = 1; x[sa[0]] = 0;
23     for(int i = 1; i < n; i++)
24         x[sa[i]] = y[sa[i - 1]] == y[sa[i]] && y[sa[i - 1] + k] == y[sa[i] + k] ? p - 1 : p++;
25     if(p >= n) break;
26     m = p;
27 }
28 }
29
30 int rk[maxn], height[maxn];
31
32 void getHeight()
33 {
34     for(int i = 1; i <= n; i++) rk[sa[i]] = i;
35     for(int i = 0, k = 0; i < n; i++)
36     {
37         if(k) k--;
38         int j = sa[rk[i] - 1];
39         while(s[i + k] == s[j + k]) k++;
40         height[rk[i]] = k;
41     }
42 }
43
44 int dp[maxn][20];
45
46 void RMQ()
47 {
48     for(int i = 1; i <= n; i++) dp[i][0] = height[i];
49     for(int j = 1; (1 << j) < maxn; j++)
50         for(int i = 1; i + (1 << j) - 1 <= n; i++)
51             dp[i][j] = min(dp[i][j - 1], dp[i + (1 << (j - 1))][j - 1]);
52 }
53
54 int query(int l, int r)
55 {
56     int k = 0;
57     while((1 << (k + 1)) <= r - l + 1) k++;
58     return min(dp[l][k], dp[r - (1 << k) + 1][k]);
59 }
60
61 int lcp(int x, int y)
62 {
63     x = rk[x], y = rk[y];
64     if(x > y) swap(x, y);
65     return query(x + 1, y);
66 }

```

## 5.6 PalindromicTree

### 5.6.1 PalindromicTree

```

1  const int maxn = 2e6+6;
2  const int N = 26;
3  const int mod = 51123987;
4
5  struct Palindromic_Tree {

```



```

6  //    vector<pair<int, int> > next[maxn];
7  int next[maxn][N]; //next指针, next指针和字典树类似, 指向的串为当前串两端加上同一个字符构成
8  int fail[maxn]{}; //fail指针, 失配后跳转到fail指针指向的节点
9  int cnt[maxn]{}; //表示节点i表示的本质不同的串的个数 (建树时求出的不是完全的, 最后count()函数跑一遍
    以后才是正确的)
10 int num[maxn]{}; //表示以节点i表示的最长回文串的最右端点为回文串结尾的回文串个数
11 int len[maxn]{}; //len[i]表示节点i表示的回文串的长度 (一个节点表示一个回文串)
12 int S[maxn]{}; //存放添加的字符
13 int last{}; //指向新添加一个字母后所形成的最长回文串表示的节点。
14 int n{}; //表示添加的字符个数。
15 int p{}; //表示添加的节点个数。
16 //0向前加, 1向后加字符
17 //int last[2];
18 //int lpos, rpos;
19
20 int newnode(int l) { //新建节点
21 //    next[p].clear();
22     for (int i = 0; i < N; ++i) next[p][i] = 0;
23     cnt[p] = 0;
24     num[p] = 0;
25     len[p] = 1;
26     return p++;
27 }
28
29 void init() { //初始化
30     n = last = p = 0;
31     newnode(0);
32     newnode(-1);
33     S[n] = -1; //开头放一个字符集中没有的字符, 减少特判
34     fail[0] = 1;
35     // lpos 为字符串最大长度
36     // last[0] = last[1] = 0;
37     // lpos = 100000, rpos = lpos - 1;
38     // S[lpos - 1] = S[rpos + 1] = -1;
39 }
40
41 int get_fail(int x) { //和KMP一样, 失配后找一个尽量最长的
42     // op 0 向前, 1 向后
43     // if (op == 0) while (S[lpos + len[x] + 1] != S[lpos]) x = fail[x];
44     // else while (S[rpos - len[x] - 1] != S[rpos]) x = fail[x];
45     while (S[n - len[x] - 1] != S[n]) x = fail[x];
46     return x;
47 }
48
49 //    int find(int u, int c) {
50 //        vector<pair<int, int> > & x = next[u];
51 //        int sz = x.size();
52 //        for(int i = 0; i < sz; ++i) {
53 //            if(x[i].first == c) return x[i].second;
54 //        }
55 //        return 0;
56 //    }
57
58 int add(int c) {
59     // 注意清空左右字符
60     // if (op == 0) S[--lpos] = c, S[lpos - 1] = -1;
61     // else S[++rpos] = c, S[rpos + 1] = -1;
62     S[++n] = c;
63     int cur = get_fail(last); //通过上一个回文串找这个回文串的匹配位置

```

```

64 //     int x = find(cur, c);
65 //     if (!x) {
66 //         if (!next[cur][c]) { // 如果这个回文串没有出现过, 说明出现了一个新的本质不同的回文串
67 //             int now = newnode(len[cur] + 2); // 新建节点
68 //             x = now;
69 //             fail[now] = find(get_fail(fail[cur]), c);
70 //             next[cur].emplace_back(make_pair(c, now));
71 //             fail[now] = next[get_fail(fail[cur])][c]; // 和AC自动机一样建立fail指针, 以便失配后跳转
72 //             next[cur][c] = now;
73 //             num[now] = num[fail[now]] + 1;
74 //         }
75 //         last = x;
76 //         // 修改最终长度
77 //         if (len[last[op]] == rpos - lpos + 1) last[op ^ 1] = last[op];
78 //         last = next[cur][c];
79 //         cnt[last]++;
80 //         return num[last];
81 //     }
82
83 void count() {
84     for (int i = p - 1; i >= 0; --i) cnt[fail[i]] += cnt[i];
85     // 父亲累加儿子的cnt, 因为如果fail[v]=u, 则u一定是v的子回文串!
86 }
87 } solve;
88
89 char s[maxn];
90
91 // 求相交回文串数量
92 ll a[maxn], b[maxn];
93 int main() {
94     solve.init();
95     int n;
96     scanf("%d", &n);
97     scanf("%s", s);
98     for (int i = 0; i < n; ++i) a[i] = solve.add(s[i] - 'a');
99     solve.init();
100    for (int i = n - 1; i >= 0; --i) b[i] = (b[i + 1] + solve.add(s[i] - 'a')) % mod;
101    ll res = (b[0] * (b[0] - 1) / 2) % mod;
102    for (int i = 0; i < n; ++i) res = ((res - (a[i] * b[i + 1]) + mod) % mod) % mod;
103    printf("%lld\n", res);
104    return 0;
105 }

```

## 5.7 Hash

### 5.7.1 hash

```

1 // hash常用素数
2 // 61, 83, 113, 151, 211
3 // 91815541, 38734667, 68861641
4 // 917120411, 687840301, 386910137, 515880193
5 // 1222827239, 1610612741
6
7 typedef unsigned long long ull;
8 struct mhash {
9     // 自然溢出无模数 805306457
10    ull base[maxn];
11    ull hash_index[maxn];
12    ull seed; // 31, 131

```

```

13 void inithash(ull seedt = 31) {
14     base[0] = 1;
15     seed = seedt;
16     for (int i = 1; i < maxn; ++i) base[i] = base[i - 1] * seed;
17 }
18 void H(char *p, int n) { // from 1 to n
19     hash_index[0] = 0;
20     for (int i = 1; i <= n; ++i) hash_index[i] = hash_index[i - 1] * seed + p[i] - 'a';
21 }
22 ull gethash(int s, int e) {
23     return hash_index[e] - hash_index[s - 1] * base[e - s + 1];
24 }
25 };
26
27 // 26个素数, 解决加法hash
28 int prime[] = {34183,13513,152993,13591,19687,350869,111187,766091,769297,
29               633469,752273,298651,617191,880421,136067,
30               1408397,726899,458921,2133701,2599847,2730947,4696343,10267237,
31               18941059,34078909,69208409};

```

### 5.7.2 doubleHash

```

1 namespace Hash{
2
3     template<class __A,class __B>
4     class Hash{
5     private:
6         static const int size=2000000;
7         __B *hash; __A *0; int sz;
8     public:
9         Hash(int hash_size=size){ sz=hash_size;
10             hash=(__B *)malloc(sizeof(__B)*sz);
11             0=(__A *)malloc(sizeof(__A)*sz);
12             memset(0,0xff,sizeof(__A)*sz);
13         }~Hash(){free(0);free(hash);}
14         __B &operator [] (const __A &_0){
15             int loc=_0%sz;
16             while(~0[loc]&&0[loc]!=_0){
17                 ++loc;
18                 if(loc>sz)loc=0;
19             }if(!~0[loc])0[loc]=_0;
20             return hash[loc];
21         }
22         void clear(){memset(0,0xff,sizeof(__A)*sz);}
23     };
24
25     struct StringDoubleHashResult{
26         int32_t *H1,*H2,c_len,len;
27         StringDoubleHashResult(int32_t sz=0){
28             len=sz; c_len=0; //cur_len;
29             if(len<=0){
30                 H1=H2=0;
31                 return;
32             }
33             H1=(int32_t *)malloc(sizeof(int32_t)*sz);
34             H2=(int32_t *)malloc(sizeof(int32_t)*sz);
35         }
36         ~StringDoubleHashResult(){}

```

```

37     void clear(){free(H1);free(H2);len=0;H1=H2=0;}
38     void resize(int new_len){
39         int32_t *T1=(int32_t *)malloc(sizeof(int32_t)*new_len);
40         int32_t *T2=(int32_t *)malloc(sizeof(int32_t)*new_len);
41         for(int i=0;i<c_len;++i)T1[i]=H1[i],T2[i]=H2[i];
42         free(H1);free(H2); H1=T1; H2=T2; len=new_len;
43     }
44     void erase(int ers_len){//erase suffix
45         c_len-=ers_len;
46         if(c_len<0)c_len=0;
47     }
48     //erase prefix not better than recalc
49 };
50
51 namespace hash_random{
52     const int mod_tot=5;
53     const int mod[]={1000000009,1000000007,998244353,917120411,515880193};
54 };
55
56 class StringDoubleHash{
57 private:
58     static const int enable_random=1;
59     int32_t sz,HA1,HA2;
60     long long B,C;
61     int32_t *H1,*H2;
62 public:
63     StringDoubleHash(int32_t SZ=2e6+5,int32_t ha1=-1,int32_t ha2=-1,int32_t b=-1,int32_t c=-1){
64         sz=SZ;
65         if(enable_random){
66             std::mt19937 rnd(time(0)+19990630);
67             int z1= rnd() % hash_random::mod_tot;
68             int z2= (z1 +rnd()%(hash_random::mod_tot - 1) + 1) % hash_random::mod_tot;
69             if(ha1<0)ha1=hash_random::mod[z1];
70             if(ha2<0)ha2=hash_random::mod[z2];
71             if(b<0)b=rnd()%114514+23333;
72             if(c<0)c=rnd()%1919810+23333;
73         } else {
74             if(ha1<0)ha1=1e9+7;
75             if(ha2<0)ha2=1e9+9;
76             if(b<0)b=114514;
77             if(c<0)c=1919810;
78         }
79         HA1=ha1; HA2=ha2; B=b; C=c;
80         //cerr<<HA1<<" "<<HA2<<" "<<B<<" "<<C<<endl;
81         H1=(int32_t *)malloc(sizeof(int32_t)*sz);
82         H2=(int32_t *)malloc(sizeof(int32_t)*sz);
83         init_hash_val();
84     }
85     ~StringDoubleHash(){free(H1);free(H2);}
86     void init_hash_val(){
87         H1[0]=H2[0]=1;
88         for(int32_t i=1;i<sz;++i){
89             H1[i]=(H1[i-1]*B)%HA1;
90             H2[i]=(H2[i-1]*B)%HA2;
91         }
92     }
93     template <class _Tp>
94     StringDoubleHashResult culc_hash(const _Tp &s,int32_t len,int32_t tot_len=-1){
95         if(tot_len<0)tot_len=len;

```

```

96     StringDoubleHashResult R(tot_len);
97     if(len<=0)return R;
98     R.H1[0]=(s[0]+C)%HA1;
99     R.H2[0]=(s[0]+C)%HA2;
100    for(int32_t i=1;i<len;++i){
101        R.H1[i]=(R.H1[i-1]*B+s[i]+C)%HA1;
102        R.H2[i]=(R.H2[i-1]*B+s[i]+C)%HA2;
103    }
104    R.c_len=len;
105    return R;
106 }
107 // s is the char* first, len is the append length
108 template <class _Tp>
109 void append(StringDoubleHashResult &R,const _Tp &s,int32_t len){
110     if(len<=0)return;
111     int t_len=R.len;
112     while(R.c_len+len>t_len)t_len<=1;
113     if(t_len>R.len)R.resize(t_len);
114     for(int32_t i=R.c_len;i<R.c_len+len;++i){
115         if(i==0){
116             R.H1[i]=(s[i-R.c_len]+C)%HA1;
117             R.H2[i]=(s[i-R.c_len]+C)%HA2;
118         } else {
119             R.H1[i]=(R.H1[i-1]*B+s[i-R.c_len]+C)%HA1;
120             R.H2[i]=(R.H2[i-1]*B+s[i-R.c_len]+C)%HA2;
121         }
122     }
123     R.c_len+=len;
124 }
125 void append(StringDoubleHashResult &R, char s){
126     int t_len=R.len;
127     while(R.c_len+1>t_len)t_len<=1;
128     if(t_len>R.len)R.resize(t_len);
129     for(int32_t i=R.c_len;i<R.c_len+1;++i){
130         if(i==0){
131             R.H1[i]=(s+C)%HA1;
132             R.H2[i]=(s+C)%HA2;
133         } else {
134             R.H1[i]=(R.H1[i-1]*B+s+C)%HA1;
135             R.H2[i]=(R.H2[i-1]*B+s+C)%HA2;
136         }
137     }
138     R.c_len+=1;
139 }
140 //return hash [l,r)
141 ll gethash(const StringDoubleHashResult &R, int32_t l,int32_t r){
142     if(l>r||l<0||r-->R.c_len)return -1;//fail
143     ll v1=l>0?R.H1[l-1]*(long long)H1[r-l+1]%HA1:0;
144     ll v2=l>0?R.H2[l-1]*(long long)H2[r-l+1]%HA2:0;
145     v1=R.H1[r]-v1; v2=R.H2[r]-v2;
146     if(v1<0)v1+=HA1; if(v2<0)v2+=HA2;
147     return v1<<32|v2;
148 }
149 //merge two hashes as one(s1+s2), but need s2's length
150 ll merge_hash(const long long &hs1,const long long &hs2,int lenr){
151     int32_t m1=hs1>>32,m2=hs1&0xffffffffLL;
152     int32_t m3=hs2>>32,m4=hs2&0xffffffffLL;
153     m1=m1*(long long)H1[lenr]%HA1+m3;
154     if(m1>=HA1)m1-=HA1;

```

```

155         m2=m2*(long long)H2[lenr]%HA2+m4;
156         if(m2>=HA2)m2-=HA2;
157         return (long long)m1<<32|m2;
158     }
159 };
160 };

```

### 5.7.3 二维 hash

```

1  #define ull unsigned long long
2  const int maxn = 1005;
3  ull hs[maxn][maxn];
4  char a[maxn][maxn];
5  int n, m;
6  ull base1 = 131, base2 = 13331;
7  ull pwb1[maxn] = {1}, pwb2[maxn] = {1};
8
9  void init() {
10     for (int i = 1; i < maxn; ++i) {
11         pwb1[i] = pwb1[i - 1] * base1;
12         pwb2[i] = pwb2[i - 1] * base2;
13     }
14 }
15
16 void Hash() {
17     for(int i=1;i<=n;i++)
18         for(int j=1;j<=m;j++)
19             hs[i][j]=hs[i][j-1]*base1+a[i][j] - 'a';
20     for(int i=1;i<=n;i++)
21         for(int j=1;j<=m;j++)
22             hs[i][j]+=hs[i-1][j]*base2;
23 }
24
25 // 右下角(i,j), 行列长度n,m
26 ull getHs(int i, int j, int lenn, int lenm) {
27     return hs[i][j] - hs[i - lenn][j] * pwb2[lenn] -
28           hs[i][j - lenm] * pwb1[lenm] +
29           hs[i - lenn][j - lenm] * pwb2[lenn] * pwb1[lenm];
30 }

```

### 5.7.4 树 hash 同构

```

1  // n=1e5的话base开2e6+9, 可以输出看到top不比n小即可
2  const int base = 2e6+9;
3  // vis大小要开到素数大小, turn表示当前树的编号, p是预处理数组
4  int vis[base + 1], top, turn, p[base + 1];
5  // 程序开头调用一次
6  void init() {
7      top = 0;
8      for (int i = 2; i <= base; ++i) {
9          if (!vis[i]) {
10             p[++top] = i;
11         }
12         for (int j = 1; j <= top && i * p[j] <= base; ++j) {
13             vis[i * p[j]] = 1;
14             if (i % p[j] == 0) break;
15         }
16     }

```

```

17     assert(top >= maxn);
18 }
19
20 vector<int> edge[maxn];
21 // h[x]表示x这棵子树的hash值, g[x]表示以x为根的hash值
22 int h[maxn], g[maxn], sz[maxn];
23
24 struct TreeHash {
25     int n;
26     // 如果树比较多, 在类内部开edge可能会炸内存, 可以改到外面做前向星
27     // 除了hs是答案其他都可以改到外部, 只有edge需要清零
28     // vector<int> edge[maxn];
29     // int h[maxn], g[maxn], sz[maxn];
30     vector<int> hs;
31
32     void init(int n_ = 0) {
33         n = n_;
34         hs.clear();
35     }
36
37     void dfs1(int u, int pre) {
38         sz[u] = 1;
39         h[u] = 1;
40         for (auto v : edge[u]) {
41             if (v == pre) continue;
42             dfs1(v, u);
43             h[u] = (h[u] + 111 * h[v] * p[sz[v]] % mod) % mod;
44             sz[u] += sz[v];
45         }
46     }
47
48     void dfs2(int u, int pre, int V, int needres = 1) {
49         g[u] = (h[u] + 111 * V * p[n - sz[u]] % mod) % mod;
50         if (needres) hs.push_back(g[u]);
51         for (auto v : edge[u]) {
52             if (v == pre) continue;
53             dfs2(v, u, (g[u] - 111 * h[v] * p[sz[v]] % mod + mod) % mod);
54         }
55     }
56
57     void work(int needres = 1) {
58         // 无根树选一个不存在的点当pre即可, 当多棵无根树判重时需要sort
59         dfs1(1, 0);
60         dfs2(1, 0, 0, needres);
61         sort(hs.begin(), hs.end());
62     }
63 };
64
65 // 获取删掉某叶子节点后以与该叶子节点相邻点开头的hash值
66 // int res = (hs[edge[i][0]] - 2 + mod) % mod;

```

## 5.8 Suffix Automation

### 5.8.1 SAM

```

1 const int maxn = 2e4 + 10;
2
3 struct SuffixAutomation
4 {

```

```

5     int last, cnt;
6     int ch[maxn << 1][26], fa[maxn << 1], len[maxn << 1], pos[maxn << 1];
7     int sz[maxn << 1], a[maxn << 1], c[maxn << 1];
8
9     void init()
10    {
11        last = cnt = 1;
12        memset(ch[1], 0, sizeof ch[1]);
13        fa[1] = len[1] = 0;
14    }
15
16    int inline newnode(int idx)
17    {
18        ++cnt;
19        memset(ch[cnt], 0, sizeof ch[cnt]);
20        fa[cnt] = len[cnt] = 0;
21        pos[cnt] = idx;
22        return cnt;
23    }
24
25    void ins(int c)
26    {
27        int p = last, np = newnode(pos[last] + 1);
28        last = np, len[np] = len[p] + 1;
29        for(; p && !ch[p][c]; p = fa[p]) ch[p][c] = np;
30        if(!p) fa[np] = 1;
31        else
32        {
33            int q = ch[p][c];
34            if(len[p] + 1 == len[q]) fa[np] = q;
35            else
36            {
37                int nq = newnode(pos[p] + 1);
38                len[nq] = len[p] + 1;
39                memcpy(ch[nq], ch[q], sizeof ch[q]);
40                fa[nq] = fa[q], fa[q] = fa[np] = nq;
41                for(; ch[p][c] == q; p = fa[p]) ch[p][c] = nq;
42            }
43        }
44        sz[np] = 1;
45    }
46
47    int solve(int n)
48    {
49        /*求两个串的LCS:
50         对一个字符串建立SAM, 记录一个当前匹配的长度Len和当前节点v, 枚举另一个字符串的每个字符;
51         如果p有字符v的转移边出边, 则使Len加一, 并使p转移到出边指向的节点上;
52         否则不断向父节点上跳, 直到当前节点有字符p的转移出边, 或者跳到根节点;
53        */
54        int p = 1, ans = 0, now_len = 0;
55        for(int i = 0; s2[i]; i++)
56        {
57            if(ch[p][s2[i] - 'a']) p = ch[p][s2[i] - 'a'], now_len++;
58            else
59            {
60                for(;p && !ch[p][s2[i] - 'a']; p = fa[p]);
61                if(p == 0) now_len = 0, p = 1;
62                else now_len = len[p] + 1, p = ch[p][s2[i] - 'a'];
63            }

```



```
64         ans = max(now_len, ans);
65     }
66 }
67
68 void Toposort()
69 {
70     long long ans = 0;
71     for(int i = 1; i <= cnt; i++) c[len[i]] ++;
72     for(int i = 1; i <= cnt; i++) c[i] += c[i - 1];
73     for(int i = 1; i <= cnt; i++) a[c[len[i]] --] = i;
74     for(int i = cnt; i; i--) sz[fa[a[i]]] += sz[a[i]];
75 }
76 }sam;
```

## 5.9 Others

### 5.9.1 最小表示法

```
1  // 0起始
2  int Gao(char a[], int len) {
3      int i = 0, j = 1, k = 0;
4      while (i < len && j < len && k < len) {
5          int cmp = a[(j + k) % len] - a[(i + k) % len];
6          if (cmp == 0) k++;
7          else {
8              if (cmp > 0) j += k + 1;
9              else i += k + 1;
10             if (i == j) j++;
11             k = 0;
12         }
13     }
14     return min(i, j);
15 }
```

## 6 dp

### 6.1 BitDP

#### 6.1.1 数位 dp 计和

```

1  #include <bits/stdc++.h>
2  #define ll long long
3  using namespace std;
4  const int mod = 998244353;
5  pair<ll, ll> dp[20][1<<10];
6  bool vis[20][1<<10];
7  int k;
8  int t[20];
9  ll base[20];
10
11 pair<ll, ll> dfs(int pos, int state, bool limit, bool lead) {
12     if (pos == -1) return __builtin_popcount(state) <= k ? make_pair(1, 0) : make_pair(0, 0);
13     if (!limit && !lead && vis[pos][state]) return dp[pos][state];
14     int up = limit ? t[pos] : 9;
15     pair<ll, ll> res = {0, 0};
16     for (int i = 0; i <= up; ++i) {
17         int n_s = state;
18         if (lead && i == 0) n_s = 0;
19         else n_s = state | (1 << i);
20         auto tmp = dfs(pos - 1, n_s, limit && i == t[pos], lead && i == 0);
21         ll pre = 1ll * i * base[pos] % mod;
22         (res.first += tmp.first) %= mod;
23         (res.second += tmp.second + pre * tmp.first) %= mod;
24     }
25     if (!limit && !lead) dp[pos][state] = res, vis[pos][state] = 1;
26     return res;
27 }
28
29 ll solve(ll x) {
30     int pos = 0;
31     do {
32         t[pos++] = x % 10;
33     } while (x /= 10);
34     return dfs(pos - 1, 0, true, true).second;
35 }
36
37 int main(int argc, char *argv[])
38 {
39     base[0] = 1;
40     for (int i = 1; i < 20; ++i) base[i] = base[i - 1] * 10;
41     ll l, r;
42     scanf("%lld%lld%d", &l, &r, &k);
43     printf("%lld\n", (solve(r) - solve(l - 1) + mod) % mod);
44     return 0;
45 }

```

#### 6.1.2 两个数数位 dp

```

1  // 二进制数位dp, 求a $\\in$ $1\sim x$ 和 b $\\in$ $1\sim y$, 满足 $a \& b > c$ || $a \wedge b < c$的对数
2  ll dp[maxn][2][2][2][2];
3  int a[maxn], b[maxn], c[maxn];
4
5

```

```

6 void cal(int *xt, ll x) {
7     int has = 0;
8     while (x) {
9         xt[has++] = x % 2;
10        x /= 2;
11    }
12 }
13
14 ll dfs(int pos, int o1, int o2, int lim1, int lim2) {
15     if (pos < 0) return 1;
16     ll &t = dp[pos][o1][o2][lim1][lim2];
17     if (t != -1) return t;
18     int up1 = o1 ? a[pos] : 1;
19     int up2 = o2 ? b[pos] : 1;
20     ll res = 0;
21     for (int i = 0; i <= up1; ++i) {
22         for (int j = 0; j <= up2; ++j) {
23             int t1 = i & j;
24             int t2 = i ^ j;
25             if (lim1 && t1 > c[pos]) continue;
26             if (lim2 && t2 < c[pos]) continue;
27             res += dfs(pos - 1, o1 && i == up1, o2 && j == up2, lim1 && t1 == c[pos], lim2 && t2 ==
                c[pos]);
28         }
29     }
30     return t = res;
31 }
32
33 ll solve(ll x, ll y, ll z) {
34     memset(dp, -1ll, sizeof dp);
35     for (int i = 0; i < 33; ++i) a[i] = b[i] = c[i] = 0;
36     cal(a, x);
37     cal(b, y);
38     cal(c, z);
39     return dfs(32, 1, 1, 1, 1);
40 }
41
42 int main(int argc, char *argv[]) {
43     int T;
44     scanf("%d", &T);
45     ll x, y, z;
46     for (int kase = 1; kase <= T; ++kase) {
47         scanf("%lld%lld%lld", &x, &y, &z);
48         ll res = solve(x, y, z);
49         res -= max(0ll, y - z + 1);
50         res -= max(0ll, x - z + 1);
51         printf("%lld\n", x * y - res);
52     }
53     return 0;
54 }

```

## 6.2 Subsequence

### 6.2.1 MaxSum

```

1 // 传入序列a和长度n, 返回最大子序列和
2 int MaxSeqSum(int a[], int n)
3 {
4     int rt = 0, cur = 0;

```

```
5     for (int i = 0; i < n; i++)
6         cur += a[i], rt = max(cur, rt), cur = max(0, cur);
7     return rt;
8 }
```

### 6.2.2 LIS

```
1 // 简单写法(下标从0开始,只返回长度)
2 int dp[N];
3 int LIS(int a[], int n)
4 {
5     memset(dp, 0x3f, sizeof(dp));
6     for (int i = 0; i < n; i++) *lower_bound(dp, dp + n, a[i]) = a[i];
7     return lower_bound(dp, dp + n, INF) - dp;
8 }
9
10 // 小常数nlogn求序列用树状数组维护dp即可
11 // dp[i] = max(dp[j]) + 1 (j < i && a[j] < a[i])
```

### 6.2.3 LongestCommonIncrease

```
1 // 序列下标从1开始
2 int LCIS(int a[], int b[], int n, int m)
3 {
4     memset(dp, 0, sizeof(dp));
5     for (int i = 1; i <= n; i++)
6     {
7         int ma = 0;
8         for (int j = 1; j <= m; j++)
9         {
10             dp[i][j] = dp[i - 1][j];
11             if (a[i] > b[j]) ma = max(ma, dp[i - 1][j]);
12             if (a[i] == b[j]) dp[i][j] = ma + 1;
13         }
14     }
15     return *max_element(dp[n] + 1, dp[n] + 1 + m);
16 }
```

### 6.2.4 LCS

```
1 #include <stdio.h>
2 #include <string.h>
3
4 #define M 30005
5 #define SIZE 128
6 #define WORDMAX 3200
7 #define BIT 32
8
9 char s1[M], s2[M];
10 int nword;
11 unsigned int str[SIZE][WORDMAX];
12 unsigned int tmp1[WORDMAX], tmp2[WORDMAX];
13
14 void pre(int len)
15 {
16     int i, j;
17     memset(str, 0, sizeof(str));
```

```
18     for(i = 0; i < len; i ++)  
19         str[s1[i]][i / BIT] |= 1 << (i % BIT);  
20 }  
21  
22 void cal(unsigned int *a, unsigned int *b, char ch)  
23 {  
24     int i, bottom = 1, top;  
25     unsigned int x, y;  
26     for(i = 0; i < nword; i ++)  
27     {  
28         y = a[i];  
29         x = y | str[ch][i];  
30         top = (y >> (BIT - 1)) & 1;  
31         y = (y << 1) | bottom;  
32         if(x < y) top = 1;  
33         b[i] = x & ((x - y) ^ x);  
34         bottom = top;  
35     }  
36 }  
37  
38 int bitcnt(unsigned int *a)  
39 {  
40     int i, j, res = 0, t;  
41     unsigned int b[5] = {0x55555555, 0x33333333, 0x0f0f0f0f, 0x00ff00ff, 0x0000ffff}, x;  
42     for(i = 0; i < nword; i ++)  
43     {  
44         x = a[i];  
45         t = 1;  
46         for(j = 0; j < 5; j ++, t <= 1)  
47             x = (x & b[j]) + ((x >> t) & b[j]);  
48         res += x;  
49     }  
50     return res;  
51 }  
52  
53 void process()  
54 {  
55     int i, j, len1, len2;  
56     unsigned int *a, *b, *t;  
57     len1 = strlen(s1);  
58     len2 = strlen(s2);  
59     nword = (len1 + BIT - 1) / BIT;  
60     pre(len1);  
61     memset(tmp1, 0, sizeof(tmp1));  
62     a = &tmp1[0];  
63     b = &tmp2[0];  
64     for(i = 0; i < len2; i ++)  
65     {  
66         cal(a, b, s2[i]);  
67         t = a; a = b; b = t;  
68     }  
69     printf("%d\n", bitcnt(a));  
70 }  
71  
72 int main()  
73 {  
74     while(scanf("%s%s", s1, s2) != EOF)  
75         process();  
76     return 0;
```

77 }

### 6.3 Others

**问题** 设  $f(i) = \min(y[k] - s[i] \times x[k]), k \in [1, i-1]$ , 现在要求出所有  $f(i), i \in [1, n]$   
考虑两个决策  $j$  和  $k$ , 如果  $j$  比  $k$  优, 则

$$y[j] - s[i] \times x[j] < y[k] - s[i] \times x[k]$$

化简得:

$$\frac{y_j - y_k}{x_j - x_k} < s_i$$

不等式左边是个斜率, 我们把它设为  $\text{slope}(j, k)$

我们可以维护一个单调递增的队列, 为什么呢?

因为如果  $\text{slope}(q[i-1], q[i]) > \text{slope}(q[i], q[i+1])$ , 那么当前者成立时, 后者必定成立。即  $q[i]$  决策优于  $q[i-1]$  决策时,  $q[i+1]$  必然优于  $q[i]$ , 因此  $q[i]$  就没有存在的必要了。所以我们要维护递增的队列。

那么每次的决策点  $i$ , 都要满足

$$\begin{cases} \text{slope}(q[i-1], q[i]) < s[i] \\ \text{slope}(q[i], q[i+1]) \geq s[i] \end{cases}$$

一般情况去二分这个  $i$  即可。

如果  $s[i]$  是单调不降的, 那么对于决策  $j$  和  $k (j < k)$  来说, 如果决策  $k$  优于决策  $j$ , 那么对于  $i \in [k+1, n]$ , 都存在决策  $k$  优于决策  $j$ , 因此决策  $j$  就可以舍弃了。这样的话我们可以用单调队列进行优化, 可以少个  $\log$ 。

#### 单调队列滑动窗口最大值

```
1 // k为滑动窗口的大小, 数列下标从1开始, d为序列长度+1
2 deque<int> q;
3 for (int i = 0, j = 0; i + k <= d; i++)
4 {
5     while (j < i + k)
6     {
7         while (!q.empty() && a[q.back()] < a[j]) q.pop_back();
8         q.push_back(j++);
9     }
10    while (q.front() < i) q.pop_front();
11    // a[q.front()]为当前滑动窗口的最大值
12 }
```

#### 6.3.1 矩阵快速幂

```
1 struct Matrix {
2     int sz;
3     // int n, m;
4     ll a[maxn][maxn];
5     Matrix(int sz_ = 0):sz(sz_) {
6         memset(a, 0, sizeof a);
7     }
8     void pr() {
9         printf("*\n");
10        for(int i = 0; i < sz; ++i) {
11            for (int j = 0; j < sz; ++j) {
12                printf("%lld ", a[i][j]);
13            }
14            printf("\n");
15        }
16    }
17    void tr() {
```

```
18         for (int i = 0; i < sz; ++i) {
19             for (int j = i + 1; j < sz; ++j) {
20                 swap(a[i][j], a[j][i]);
21             }
22         }
23     }
24 }res, t1;
25
26 void init() {
27     ;
28 }
29
30 Matrix mul(Matrix a, Matrix b)
31 {
32     Matrix res(a.sz);
33     // if (a.m != b.n) return res;
34     for(int i = 0; i < res.sz; i++) // a.n
35         for(int j = 0; j < res.sz; j++) // b.m
36             for(int k = 0; k < res.sz; k++) // a.m, b.n
37                 (res.a[i][j] +=a.a[i][k] * b.a[k][j] % mod) %= mod;
38     return res;
39 }
40
41 Matrix pow(ll n)
42 {
43     init();
44     //for(int i = 0; i < cur; i++) res.a[i][i] = 1;
45     while(n > 0) {
46         if(n & 1) res = mul(res, t1);
47         t1 = mul(t1, t1);
48         n >>= 1;
49     }
50     return res;
51 }
```

## 7 Others

### 7.1 mint 类

```

1  const int mod = 998244353;
2
3  struct mint {
4      int n;
5      mint(int n_ = 0) : n(n_) {}
6  };
7
8  mint operator+(mint a, mint b) { return (a.n += b.n) >= mod ? a.n - mod : a.n; }
9  mint operator-(mint a, mint b) { return (a.n -= b.n) < 0 ? a.n + mod : a.n; }
10 mint operator*(mint a, mint b) { return 1LL * a.n * b.n % mod; }
11 mint &operator+=(mint &a, mint b) { return a = a + b; }
12 mint &operator-=(mint &a, mint b) { return a = a - b; }
13 mint &operator*=(mint &a, mint b) { return a = a * b; }
14 ostream &operator<<(ostream &o, mint a) { return o << a.n; }

```

### 7.2 不重叠区间贪心

```

1  #include <bits/stdc++.h>
2  #define ll long long
3  using namespace std;
4
5  const int maxn = 5e5+5;
6  pair<int, int> a[maxn];
7  int main() {
8      int n;
9      cin >> n;
10     for (int i = 1; i <= n; ++i) {
11         cin >> a[i].second >> a[i].first;
12     }
13     sort(a + 1, a + 1 + n);
14     int res = 1;
15     int tmp = a[1].first;
16     // printf("%d %d\n", a[1].second, a[1].first);
17     for (int i = 2; i <= n; ++i) {
18         if (a[i].second > tmp) {
19             res++;
20             // printf("%d %d\n", a[i].second, a[i].first);
21             tmp = a[i].first;
22         }
23     }
24     printf("%d\n", res);
25     return 0;
26 }

```

### 7.3 BigInt 类

```

1  const double PI = acos(-1.0);
2  struct Complex{
3      double x,y;
4      Complex(double _x = 0.0,double _y = 0.0){
5          x = _x;
6          y = _y;
7      }

```



```
8     Complex operator-(const Complex &b)const{
9         return Complex(x - b.x,y - b.y);
10    }
11    Complex operator+(const Complex &b)const{
12        return Complex(x + b.x,y + b.y);
13    }
14    Complex operator*(const Complex &b)const{
15        return Complex(x*b.x - y*b.y,x*b.y + y*b.x);
16    }
17 };
18 void change(Complex y[],int len){
19     int i,j,k;
20     for(int i = 1,j = len/2;i<len-1;i++){
21         if(i < j)    swap(y[i],y[j]);
22         k = len/2;
23         while(j >= k){
24             j = j - k;
25             k = k/2;
26         }
27         if(j < k)    j+=k;
28     }
29 }
30 void fft(Complex y[],int len,int on){
31     change(y,len);
32     for(int h = 2;h <= len;h<=1){
33         Complex wn(cos(on*2*PI/h),sin(on*2*PI/h));
34         for(int j = 0;j < len;j += h){
35             Complex w(1,0);
36             for(int k = j;k < j + h/2;k++){
37                 Complex u = y[k];
38                 Complex t = w*y[k + h/2];
39                 y[k] = u + t;
40                 y[k + h/2] = u - t;
41                 w = w*wn;
42             }
43         }
44     }
45     if(on == -1){
46         for(int i = 0;i < len;i++){
47             y[i].x /= len;
48         }
49     }
50 }
51 class BigInt
52 {
53     #define Value(x, nega) ((nega) ? -(x) : (x))
54     #define At(vec, index) ((index) < vec.size() ? vec[(index)] : 0)
55     static int absComp(const BigInt &lhs, const BigInt &rhs)
56     {
57         if (lhs.size() != rhs.size())
58             return lhs.size() < rhs.size() ? -1 : 1;
59         for (int i = lhs.size() - 1; i >= 0; --i)
60             if (lhs[i] != rhs[i])
61                 return lhs[i] < rhs[i] ? -1 : 1;
62         return 0;
63     }
64     using Long = long long;
65     const static int Exp = 9;
66     const static Long Mod = 1000000000;
```

```
67     mutable std::vector<Long> val;
68     mutable bool nega = false;
69     void trim() const
70     {
71         while (val.size() && val.back() == 0)
72             val.pop_back();
73         if (val.empty())
74             nega = false;
75     }
76     int size() const { return val.size(); }
77     Long &operator[](int index) const { return val[index]; }
78     Long &back() const { return val.back(); }
79     BigInt(int size, bool nega) : val(size), nega(nega) {}
80     BigInt(const std::vector<Long> &val, bool nega) : val(val), nega(nega) {}
81
82 public:
83     friend std::ostream &operator<<(std::ostream &os, const BigInt &n)
84     {
85         if (n.size())
86         {
87             if (n.nega)
88                 putchar('-');
89             for (int i = n.size() - 1; i >= 0; --i)
90             {
91                 if (i == n.size() - 1)
92                     printf("%lld", n[i]);
93                 else
94                     printf("%0*lld", n.Exp, n[i]);
95             }
96         }
97         else
98             putchar('0');
99         return os;
100     }
101     friend BigInt operator+(const BigInt &lhs, const BigInt &rhs)
102     {
103         BigInt ret(lhs);
104         return ret += rhs;
105     }
106     friend BigInt operator-(const BigInt &lhs, const BigInt &rhs)
107     {
108         BigInt ret(lhs);
109         return ret -= rhs;
110     }
111     BigInt(Long x = 0)
112     {
113         if (x < 0)
114             x = -x, nega = true;
115         while (x >= Mod)
116             val.push_back(x % Mod), x /= Mod;
117         if (x)
118             val.push_back(x);
119     }
120     BigInt(const char *s)
121     {
122         int bound = 0, pos;
123         if (s[0] == '-')
124             nega = true, bound = 1;
125         Long cur = 0, pow = 1;
```

```

126     for (pos = strlen(s) - 1; pos >= Exp + bound - 1; pos -= Exp, val.push_back(cur), cur = 0,
127         pow = 1)
128         for (int i = pos; i > pos - Exp; --i)
129             cur += (s[i] - '0') * pow, pow *= 10;
130     for (cur = 0, pow = 1; pos >= bound; --pos)
131         cur += (s[pos] - '0') * pow, pow *= 10;
132     if (cur)
133         val.push_back(cur);
134 }
135 BigInt &operator=(const char *s){
136     BigInt n(s);
137     *this = n;
138     return n;
139 }
140 BigInt &operator=(const Long x){
141     BigInt n(x);
142     *this = n;
143     return n;
144 }
145 friend std::istream &operator>>(std::istream &is, BigInt &n){
146     string s;
147     is >> s;
148     n=(char*)s.data();
149     return is;
150 }
151 BigInt &operator+=(const BigInt &rhs)
152 {
153     const int cap = std::max(size(), rhs.size()) + 1;
154     val.resize(cap);
155     int carry = 0;
156     for (int i = 0; i < cap - 1; ++i)
157     {
158         val[i] = Value(val[i], nega) + Value(At(rhs, i), rhs.nega) + carry, carry = 0;
159         if (val[i] >= Mod)
160             val[i] -= Mod, carry = 1;
161         else if (val[i] < 0)
162             val[i] += Mod, carry = -1;
163     }
164     if ((val.back() = carry) == -1) //assert(val.back() == 1 or 0 or -1)
165     {
166         nega = true, val.pop_back();
167         bool tailZero = true;
168         for (int i = 0; i < cap - 1; ++i)
169         {
170             if (tailZero && val[i])
171                 val[i] = Mod - val[i], tailZero = false;
172             else
173                 val[i] = Mod - 1 - val[i];
174         }
175     }
176     trim();
177     return *this;
178 }
179 friend BigInt operator-(const BigInt &rhs)
180 {
181     BigInt ret(rhs);
182     ret.nega ^= 1;
183     return ret;
184 }

```

```

184 BigInt &operator--(const BigInt &rhs)
185 {
186     rhs.nega ^= 1;
187     *this += rhs;
188     rhs.nega ^= 1;
189     return *this;
190 }
191 friend BigInt operator*(const BigInt &lhs, const BigInt &rhs)
192 {
193     int len=1;
194     BigInt ll=lhs,rr=rhs;
195     ll.nega = lhs.nega ^ rhs.nega;
196     while(len<2*lhs.size()||len<2*rhs.size())len<=1;
197     ll.val.resize(len),rr.val.resize(len);
198     Complex x1[len],x2[len];
199     for(int i=0;i<len;i++){
200         Complex nx(ll[i],0.0),ny(rr[i],0.0);
201         x1[i]=nx;
202         x2[i]=ny;
203     }
204     fft(x1,len,1);
205     fft(x2,len,1);
206     for(int i = 0 ; i < len; i++)
207         x1[i] = x1[i] * x2[i];
208     fft( x1 , len , -1 );
209     for(int i = 0 ; i < len; i++)
210         ll[i] = int( x1[i].x + 0.5 );
211     for(int i = 0 ; i < len; i++){
212         ll[i+1]+=ll[i]/Mod;
213         ll[i]%=Mod;
214     }
215     ll.trim();
216     return ll;
217 }
218 friend BigInt operator*(const BigInt &lhs, const Long &x){
219     BigInt ret=lhs;
220     bool negat = ( x < 0 );
221     Long xx = (negat) ? -x : x;
222     ret.nega ^= negat;
223     ret.val.push_back(0);
224     ret.val.push_back(0);
225     for(int i = 0; i < ret.size(); i++)
226         ret[i]*=xx;
227     for(int i = 0; i < ret.size(); i++){
228         ret[i+1]+=ret[i]/Mod;
229         ret[i] %= Mod;
230     }
231     ret.trim();
232     return ret;
233 }
234 BigInt &operator*=(const BigInt &rhs) { return *this = *this * rhs; }
235 BigInt &operator*=(const Long &x) { return *this = *this * x; }
236 friend BigInt operator/(const BigInt &lhs, const BigInt &rhs)
237 {
238     static std::vector<BigInt> powTwo{BigInt(1)};
239     static std::vector<BigInt> estimate;
240     estimate.clear();
241     if (absComp(lhs, rhs) < 0)
242         return BigInt();

```

```

243     BigInt cur = rhs;
244     int cmp;
245     while ((cmp = absComp(cur, lhs)) <= 0)
246     {
247         estimate.push_back(cur), cur += cur;
248         if (estimate.size() >= powTwo.size())
249             powTwo.push_back(powTwo.back() + powTwo.back());
250     }
251     if (cmp == 0)
252         return BigInt(powTwo.back().val, lhs.nega ^ rhs.nega);
253     BigInt ret = powTwo[estimate.size() - 1];
254     cur = estimate[estimate.size() - 1];
255     for (int i = estimate.size() - 1; i >= 0 && cmp != 0; --i)
256         if ((cmp = absComp(cur + estimate[i], lhs)) <= 0)
257             cur += estimate[i], ret += powTwo[i];
258     ret.nega = lhs.nega ^ rhs.nega;
259     return ret;
260 }
261 friend BigInt operator/(const BigInt &num, const Long &x){
262     bool negat = ( x < 0 );
263     Long xx = (negat) ? -x : x;
264     BigInt ret;
265     Long k = 0;
266     ret.val.resize( num.size() );
267     ret.nega = (num.nega ^ negat);
268     for(int i = num.size() - 1 ; i >= 0; i--){
269         ret[i] = ( k * Mod + num[i]) / xx;
270         k = ( k * Mod + num[i]) % xx;
271     }
272     ret.trim();
273     return ret;
274 }
275 bool operator==(const BigInt &rhs) const
276 {
277     return nega == rhs.nega && val == rhs.val;
278 }
279 bool operator!=(const BigInt &rhs) const { return nega != rhs.nega || val != rhs.val; }
280 bool operator>=(const BigInt &rhs) const { return !(*this < rhs); }
281 bool operator>(const BigInt &rhs) const { return !(*this <= rhs); }
282 bool operator<=(const BigInt &rhs) const
283 {
284     if (nega && !rhs.nega)
285         return true;
286     if (!nega && rhs.nega)
287         return false;
288     int cmp = absComp(*this, rhs);
289     return nega ? cmp >= 0 : cmp <= 0;
290 }
291 bool operator<(const BigInt &rhs) const
292 {
293     if (nega && !rhs.nega)
294         return true;
295     if (!nega && rhs.nega)
296         return false;
297     return (absComp(*this, rhs) < 0) ^ nega;
298 }
299 void swap(const BigInt &rhs) const
300 {
301     std::swap(val, rhs.val);

```

```
302         std::swap(nega, rhs.nega);
303     }
304 };
305 BigInt ba,bb;
306 int main(){
307     cin>>ba>>bb;
308     cout << ba + bb << '\n';//和
309     cout << ba - bb << '\n';//差
310     cout << ba * bb << '\n';//积
311     BigInt d;
312     cout << (d = ba / bb) << '\n';//商
313     cout << ba - d * bb << '\n';//余
314     return 0;
315 }
```

## 7.4 date

```
1  string dayOfWeek[] = {"Mo", "Tu", "We", "Th", "Fr", "Sa", "Su"};
2  // converts Gregorian date to integer (Julian day number)
3  int DateToInt (int m, int d, int y){
4      return
5          1461 * (y + 4800 + (m - 14) / 12) / 4 +
6          367 * (m - 2 - (m - 14) / 12 * 12) / 12 -
7          3 * ((y + 4900 + (m - 14) / 12) / 100) / 4 +
8          d - 32075;
9  }
10
11 // converts integer (Julian day number) to Gregorian date: month/day/year
12 void IntToDate (int jd, int &m, int &d, int &y){
13     int x, n, i, j;
14     x = jd + 68569;
15     n = 4 * x / 146097;
16     x -= (146097 * n + 3) / 4;
17     i = (4000 * (x + 1)) / 1461001;
18     x -= 1461 * i / 4 - 31;
19     j = 80 * x / 2447;
20     d = x - 2447 * j / 80;
21     x = j / 11;
22     m = j + 2 - 12 * x;
23     y = 100 * (n - 49) + i + x;
24 }
25 // converts integer (Julian day number) to day of week
26 string IntToDay (int jd){
27     return dayOfWeek[jd % 7];
28 }
```

## 7.5 Frac 类

```
1  struct Frac {
2      ll a, b;
3      void getJian() {
4          ll gcd = abs(__gcd(a, b));
5          a /= gcd;
6          b /= gcd;
7          if (b < 0) {
8              a = -a;
9              b = -b;
10         }
```

```
10     }
11 }
12 Frac(1l a_ = 1, 1l b_ = 1) {
13     a = a_;
14     b = b_;
15     getJian();
16 }
17 Frac add(const Frac& oth) {
18     1l bt = b * oth.b;
19     1l at = a * oth.b + oth.a * b;
20     return Frac(at, bt);
21 }
22 Frac multi(const Frac& oth) {
23     a *= oth.a;
24     b *= oth.b;
25     getJian();
26     return *this;
27 }
28 bool operator < (const Frac& oth) const {
29     return a * oth.b < b * oth.a;
30 }
31 bool operator == (const Frac& oth) const {
32     return a * oth.b == b * oth.a;
33 }
34 bool operator <= (const Frac& oth) const {
35     return a * oth.b <= b * oth.a;
36 }
37 };
```

## 7.6 模拟退火 (最小圆覆盖)

```
1  const int maxn = 1e5 + 10;
2  const double eps = 1e-8;
3  const double delta = 0.98;
4  const double inf = 1e18;
5
6  struct Point { double x, y; } p[maxn];
7
8  double dis(Point A, Point B) { return sqrt((A.x - B.x) * (A.x - B.x) + (A.y - B.y) * (A.y - B.y));
9  };
10
11 double Simulate_Annea(int n)
12 {
13     Point S;
14     S.x = S.y = 0;
15     double t = 1000;
16     double res = inf;
17     while(t > eps)
18     {
19         int k = 0;
20         for(int i = 0; i < n; i++) if(dis(S, p[i]) > dis(S, p[k])) k = i;
21         double d = dis(S, p[k]);
22         res = min(res, d);
23         S.x += (p[k].x - S.x) / d * t;
24         S.y += (p[k].y - S.y) / d * t;
25         t *= delta;
26     }
27     return res;
28 }
```

```
27 }
28
29 int main()
30 {
31     int n;
32     scanf("%d", &n);
33     for(int i = 0; i < n; i++) scanf("%lf%lf", &p[i].x, &p[i].y);
34     printf("%.3f\n", Simulate_Annea(n));
35     return 0;
36 }
```

## 7.7 string 类

```
1  const int maxn = 1005;
2  struct String{
3      int nex[maxn];
4      char x[maxn];
5      int len;
6      int getLength() {
7          return len;
8      }
9      void getNext() {
10         int n = len, i = 0, j = -1;
11         nex[0] = -1;
12         while (i < n) {
13             if (j == -1 || x[i] == x[j]) nex[++i] = ++j;
14             else j = -1;
15         }
16     }
17     void input() {
18         scanf("%s", x);
19         len = strlen(x);
20     }
21     void inputAndCal() {
22         scanf("%s", x);
23         len = strlen(x);
24         getNext();
25     }
26     void show() {
27         printf("%s\n", x);
28     }
29     bool operator < (const String&oth) const {
30         return strcmp(x, oth.x) < 0;
31     }
32     char operator [] (const int a) const {
33         return x[a];
34     }
35     bool substring(String b) {//b is the substring of a
36         int m = len, n = b.getLength();
37         int i = 0, j = 0;
38         while (i < m && j < n) {
39             if (j == -1 || x[i] == b[j]) ++i, ++j;
40             else j = b.nex[j];
41             if (j == n) return true;
42         }
43         return false;
44     }
45 };
```



## 7.8 前缀异或和

```

1 ll xor_sum(ll n) {
2     ll t=n&3;
3     if (t&1) return t/2ull^1;
4     return t/2ull^n;
5 }

```

## 7.9 约瑟夫环第 k 个

```

1 ll kth(ll n,ll m,ll k) {
2     if(m==1) return k;
3     ll res=(m-1)%(n-k+1);
4     for(ll i=n-k+2,stp=0;i<=n;i+=stp,res+=stp*m) {
5         if(res+m>=i) {
6             res=(res+m)%i;
7             i++;
8             stp=0;
9         }
10        else {
11            stp=(i-res-2)/(m-1);
12            if(i+stp>n)
13                {
14                    res+=(n-(i-1))*m;
15                    break;
16                }
17        }
18    }
19    return res+1;
20 }

```

## 7.10 二分

```

1 // a为二分数组, x为需要查找的数, 返回最左端和最右端
2 pair<int, int> F(vector<int> a, int x) {
3     int l = 0, r = a.size() - 1;
4     int lres = -1;
5     while (l <= r) {
6         int mid = l + r >> 1;
7         int tt = a[mid];
8         if (tt >= x) {
9             r = mid - 1;
10        } else if (tt < x) {
11            l = mid + 1;
12        }
13    }
14    if (l >= a.size() || a[l] != x) return make_pair(-1, -1);
15    lres = l;
16    l = 0, r = a.size() - 1;
17    while (l <= r) {
18        int mid = l + r >> 1;
19        int tt = a[mid];
20        if (tt > x) {
21            r = mid - 1;
22        } else if (tt <= x) {
23            l = mid + 1;
24        }
25    }
26 }

```

```
25     }
26     return make_pair(lres, r);
27 }
```

## 7.11 猛男 IO 挂

```
1  const int LEN = 100000;
2  struct fastio {
3      int it, len;
4      char s[LEN + 5];
5      fastio() {
6          it = len = 0;
7      }
8      char get() {
9          if (it < len) return s[it++];
10         it = 0, len = fread(s, 1, LEN, stdin);
11         return len ? s[it++] : EOF;
12     }
13     bool notend() {
14         char c;
15         for (c = get(); c == ' ' || c == '\n'; c = get());
16         if (it) it--;
17         return c != EOF;
18     }
19     void put(char c) {
20         if (it == LEN) fwrite(s, 1, LEN, stdout), it = 0;
21         s[it++] = c;
22     }
23     void flush() {
24         fwrite(s, 1, it, stdout);
25     }
26 } buff, bufo;
27 inline int getint() {
28     char c;
29     int res = 0, sig = 1;
30     for (c = buff.get(); c < '0' || c > '9'; c = buff.get()) if (c == '-') sig = -1;
31     for (; c >= '0' && c <= '9'; c = buff.get()) res = res * 10 + (c - '0');
32     return sig * res;
33 }
34 inline ll getll() {
35     char c;
36     ll res = 0, sig = 1;
37     for (c = buff.get(); c < '0' || c > '9'; c = buff.get()) if (c == '-') sig = -1;
38     for (; c >= '0' && c <= '9'; c = buff.get()) res = res * 10 + (c - '0');
39     return sig * res;
40 }
41 inline void putint(int x, char suf) {
42     if (!x) bufo.put('0');
43     else {
44         if (x < 0) bufo.put('-'), x = -x;
45         int k = 0;
46         char s[15];
47         while (x) {
48             s[++k] = x % 10 + '0';
49             x /= 10;
50         }
51         for (; k; k--) bufo.put(s[k]);
52     }
```

```

53     bufo.put(suf);
54 }
55 inline void putll(ll x, char suf) {
56     if (!x) bufo.put('0');
57     else {
58         if (x < 0) bufo.put('-'), x = -x;
59         int k = 0;
60         char s[25];
61         while (x) {
62             s[++k] = x % 10 + '0';
63             x /= 10;
64         }
65         for (; k; k--) bufo.put(s[k]);
66     }
67     bufo.put(suf);
68 }
69 inline char get_char() {
70     char c;
71     for (c = buff.get(); c == ' ' || c == '\n'; c = buff.get());
72     return c;
73 }

```

## 7.12 贪心结论

- 1 //  $n$ 个区间，挪到使得某个点被所有区间覆盖需要的最少步数时，选择的点是所有区间端点的中位数（ $mid-mid+1$ 答案都是一样的）

## 7.13 builtin

```

1 __builtin_popcount(unsigned int n) // 1的个数
2 __builtin_parity(unsigned int n) // 奇数个1返回1，偶数个返回0
3 __builtin_ctz(unsigned int n) // 判断n的二进制末尾后面0的个数
4 __builtin_clz(unsigned int n) // 返回前导0的个数

```

## 7.14 $n$ 以内 $k$ 因子的个数

```

1 // 返回1~n中k因子的个数
2 ll dig(ll n, ll k) {
3     ll res = 0;
4     while (n > 0) {
5         res += n / k;
6         n /= k;
7     }
8     return res;
9 }

```