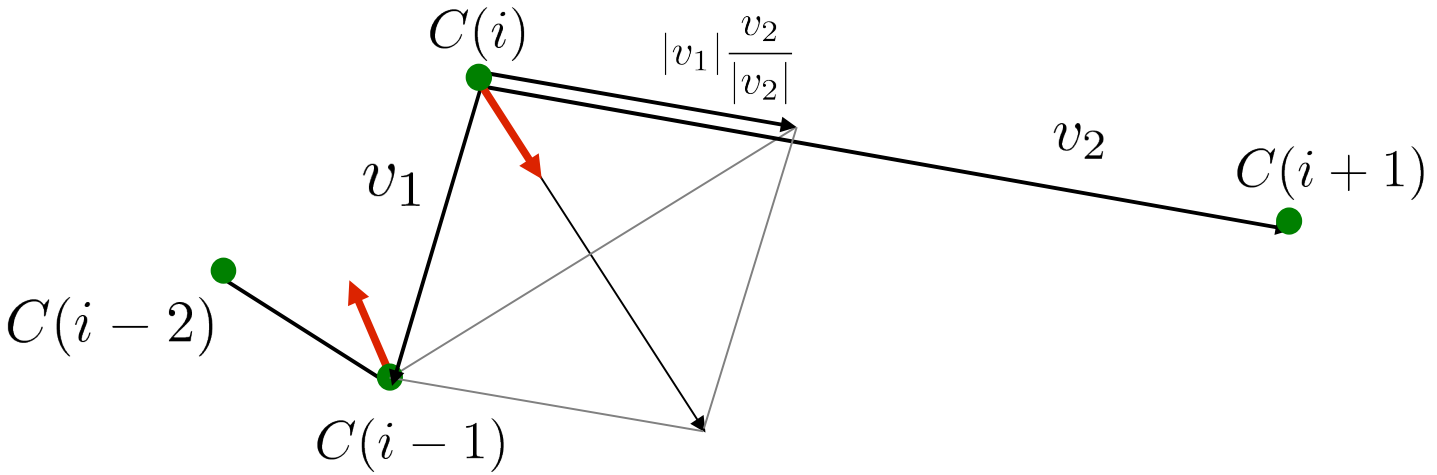
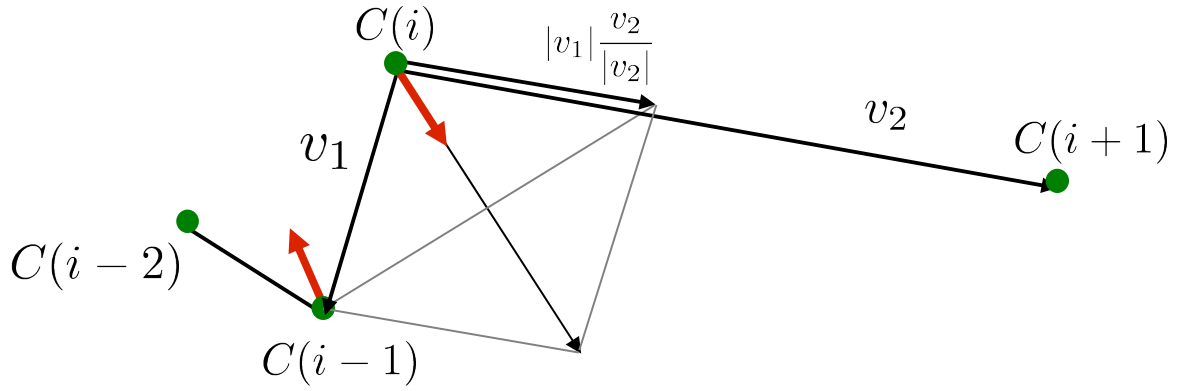
Smoothing a curve:
Discrete Curve Shortening

- Basic Idea: smooth curve by moving points proportional to curvature, using a rough discrete estimate of curvature and normal
  - No interpolation in the process
- This algorithm will be denoted *curve shortening* or CSM
- Given *C, the estimate at a sample i is:*



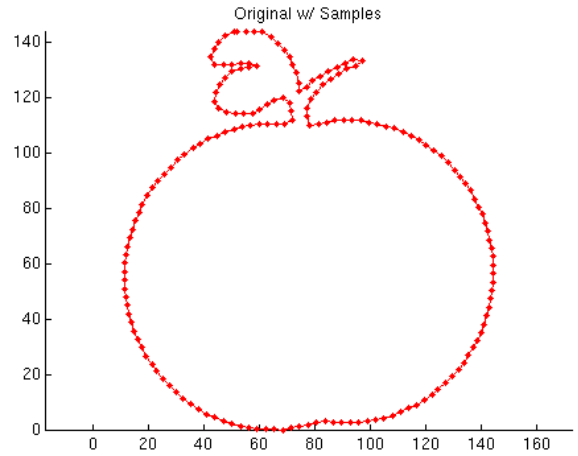
Red: final vector on which we move the points

Smoothing a curve: Discrete Curve Shortening

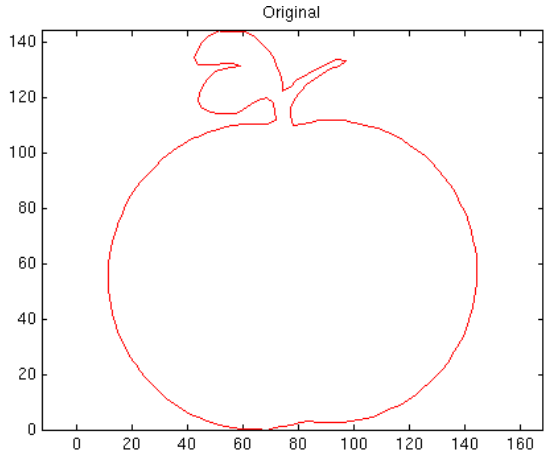


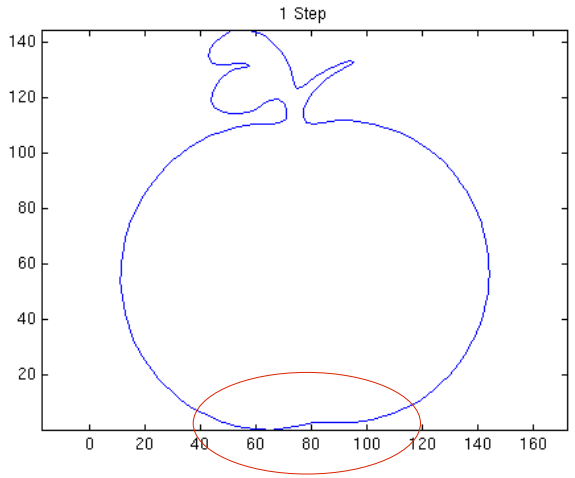
- Algorithm
  - Pick the smallest of v_1 and v_2
    - In this case, this is v_1
  - We move according to the vector $\vec{k} = \left(v_1 + |v_1|\frac{v_2}{|v_2|}\right)\frac{1}{4}$
    - Divide by 2 since other points are moving too.
  - Move point *i and store result in a new curve;*
  - Repeat, taking the new curve as input.

Discrete Curve Shortening: Results

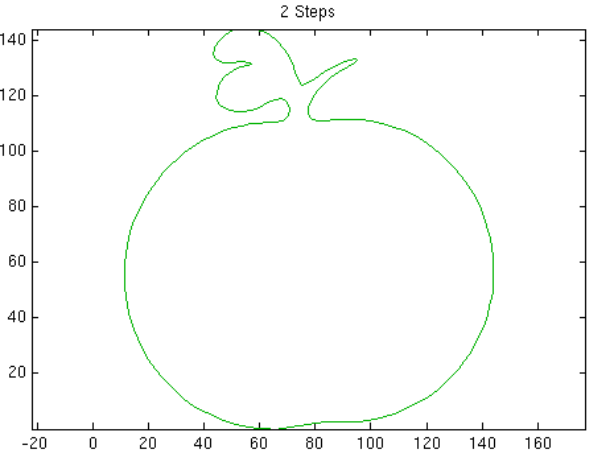


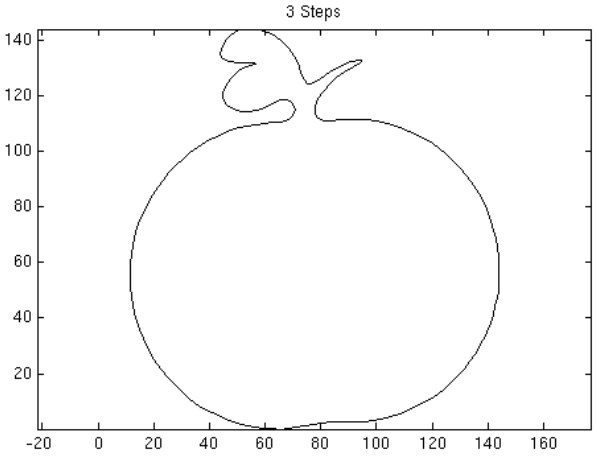
apple05

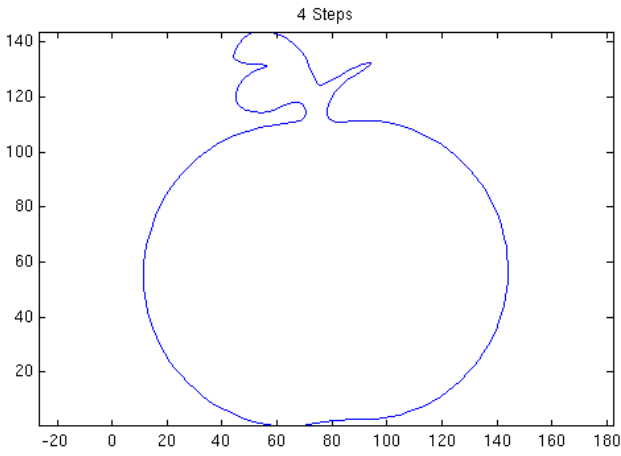




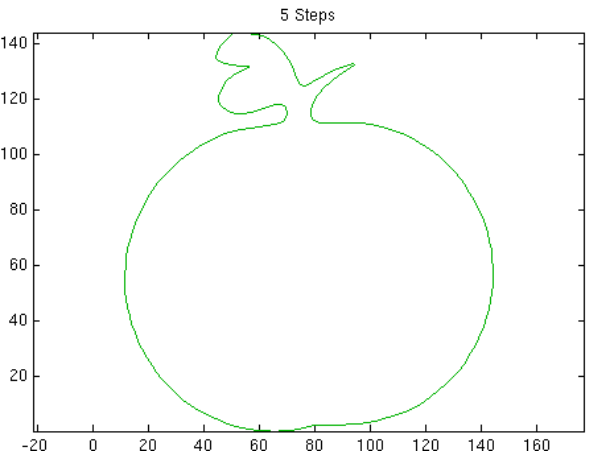
Bumps persist

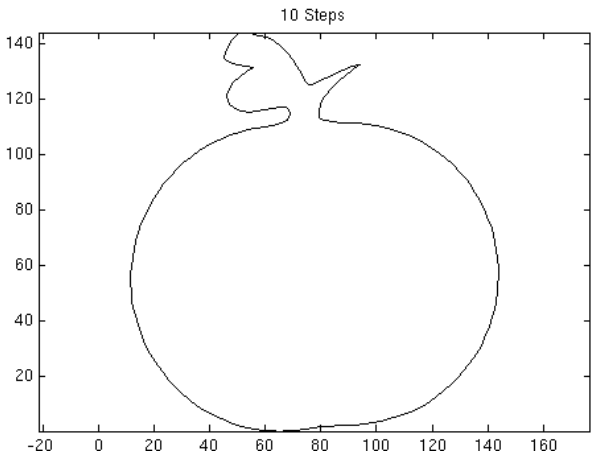


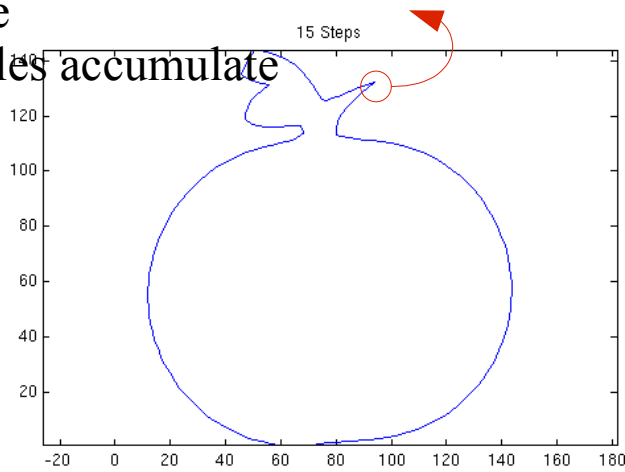


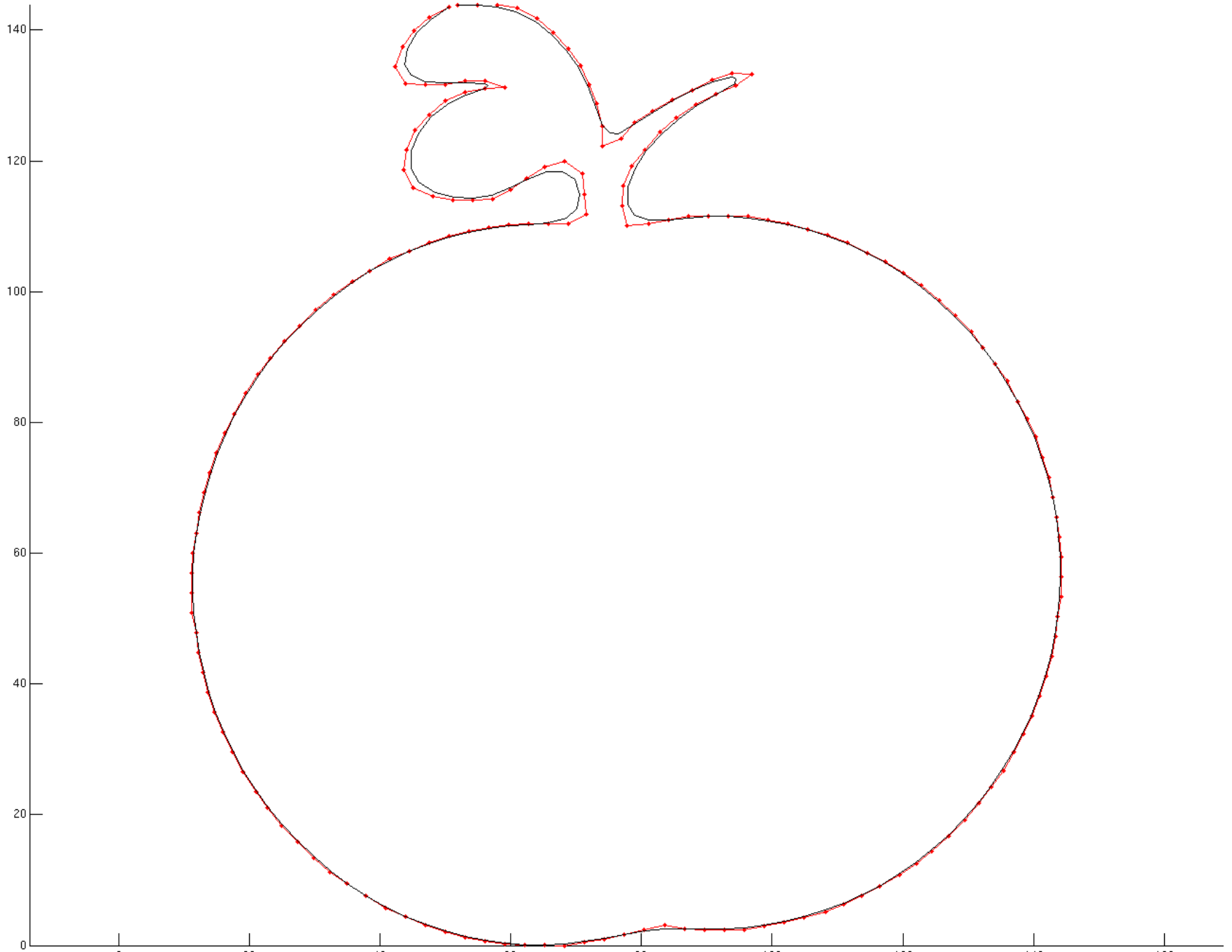


Sharp edges develop where samples accumulate

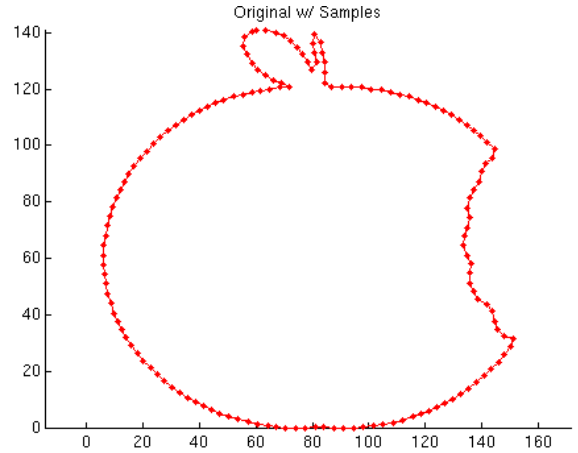




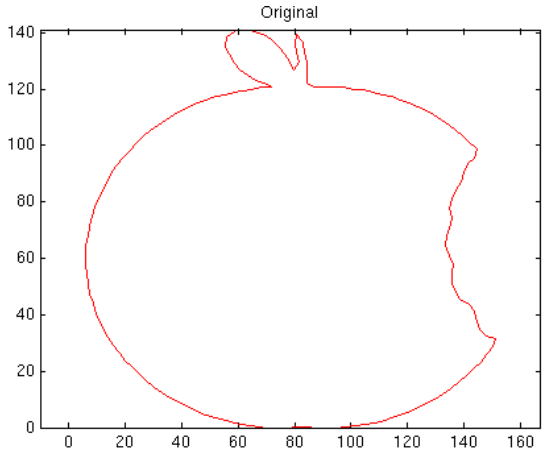


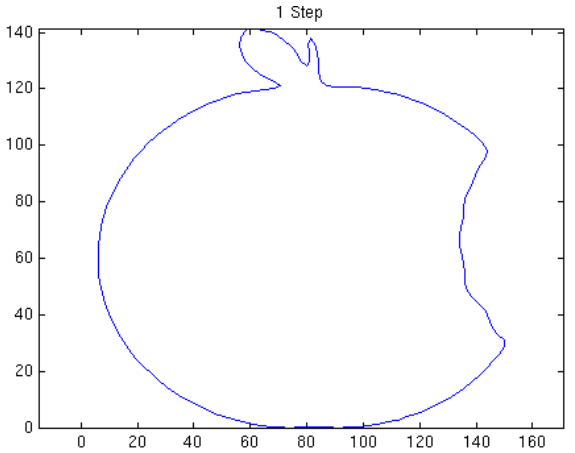


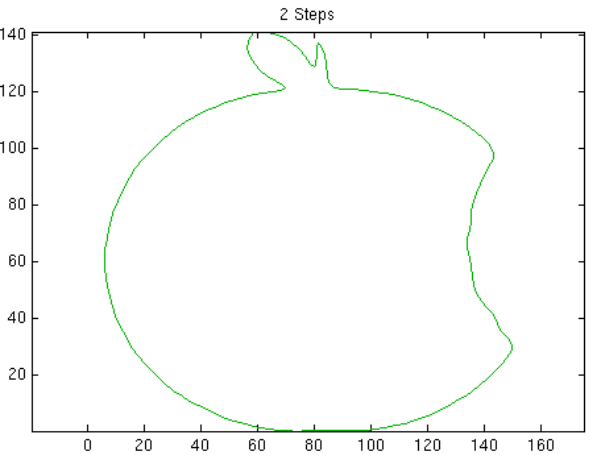
3 steps

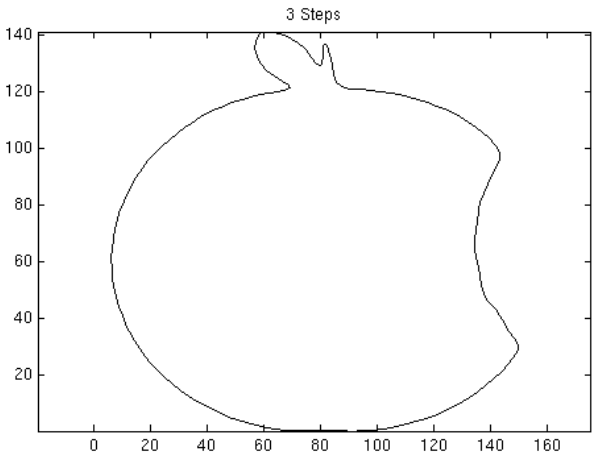

Original w/ Samples

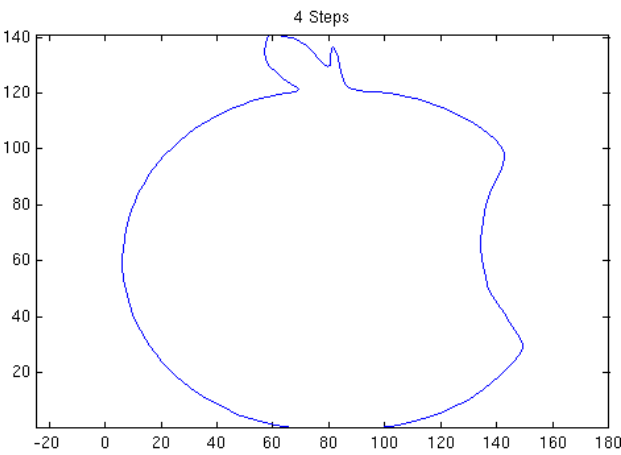
apple09

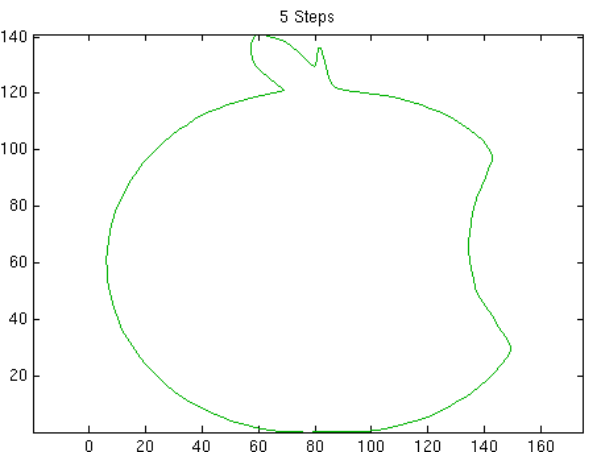

Original

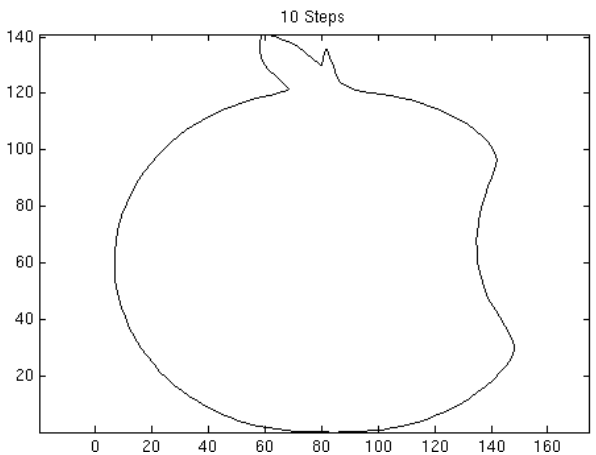

1 Step

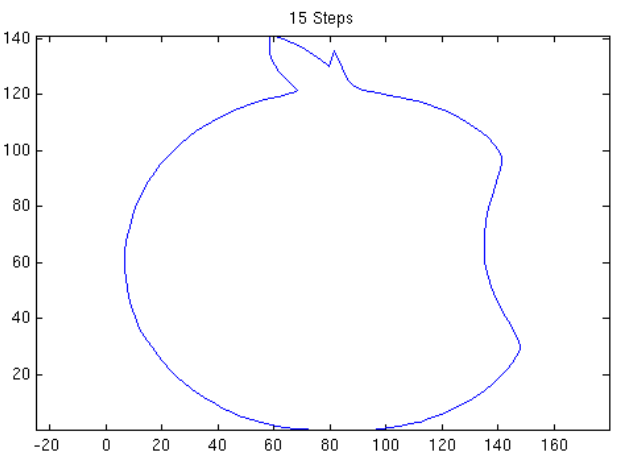

2 Steps

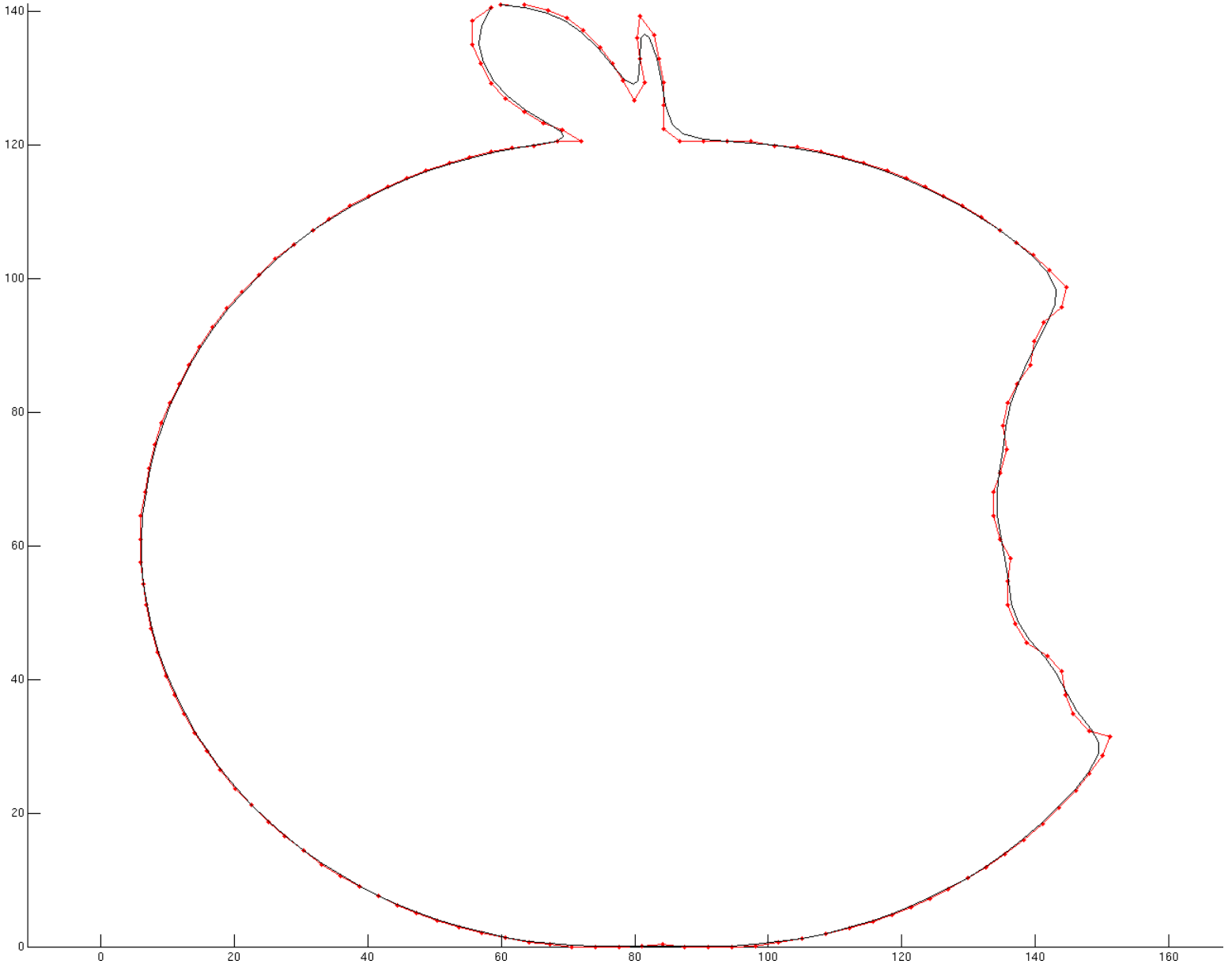

3 Steps


4 Steps

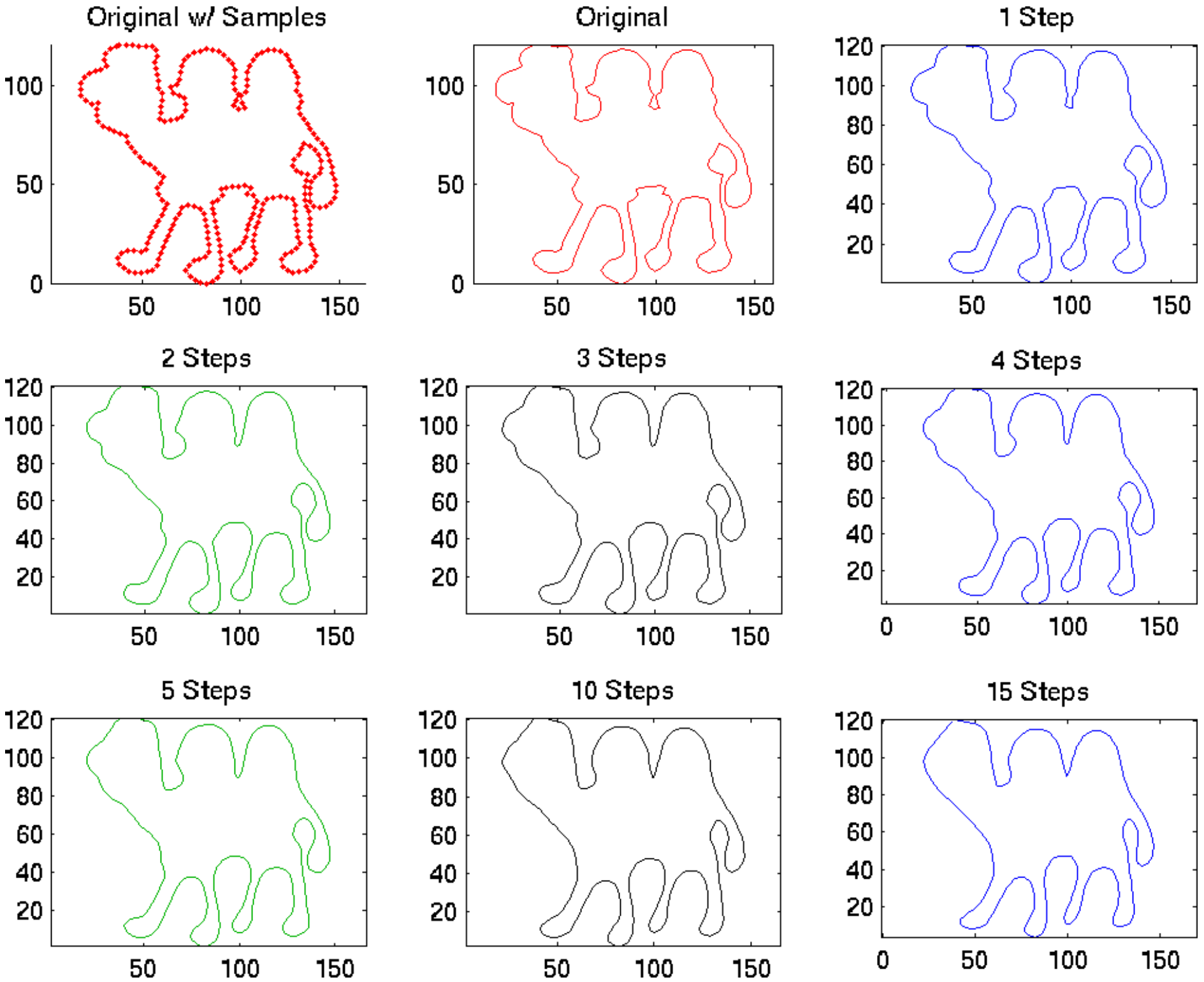

5 Steps

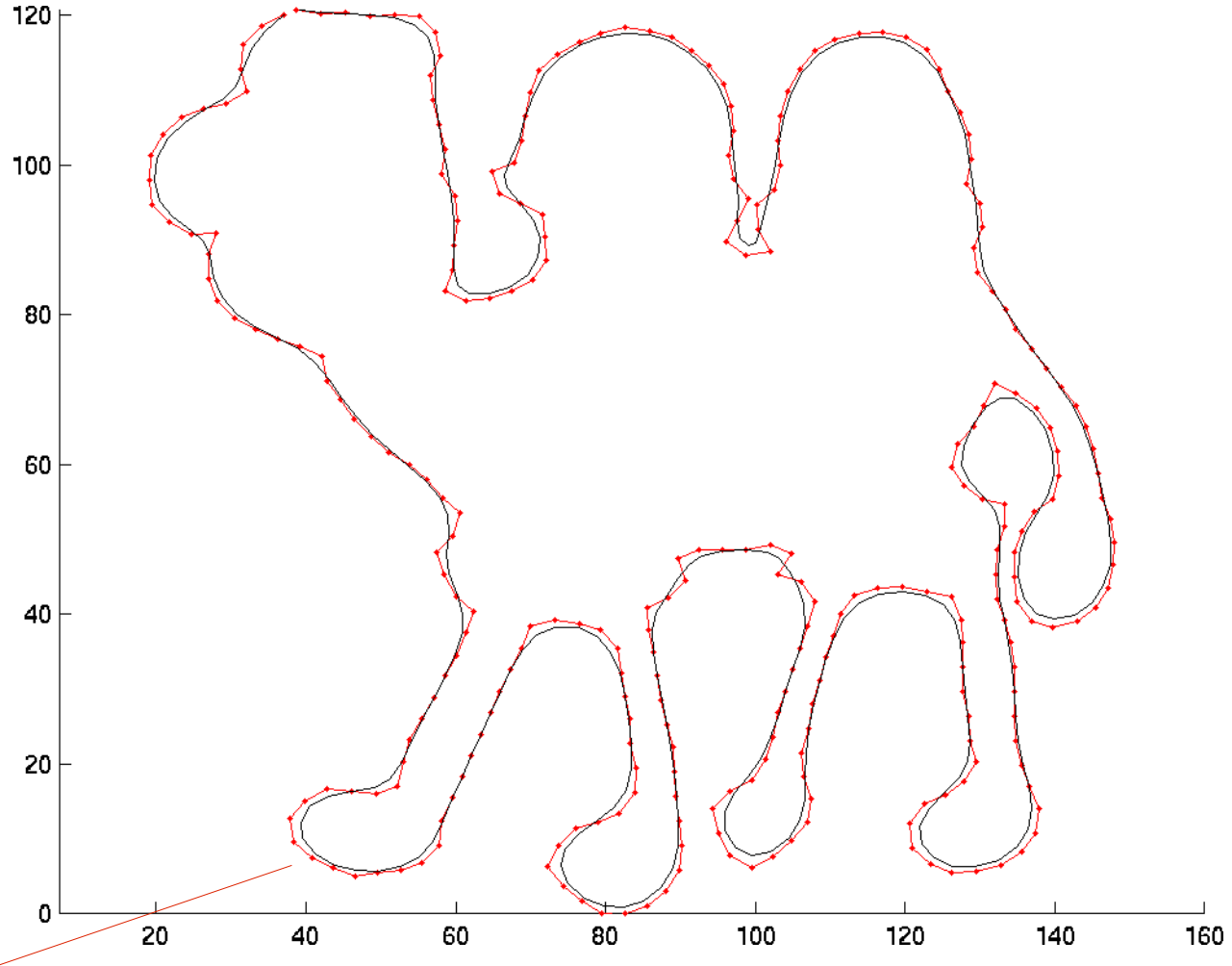

10 Steps


15 Steps



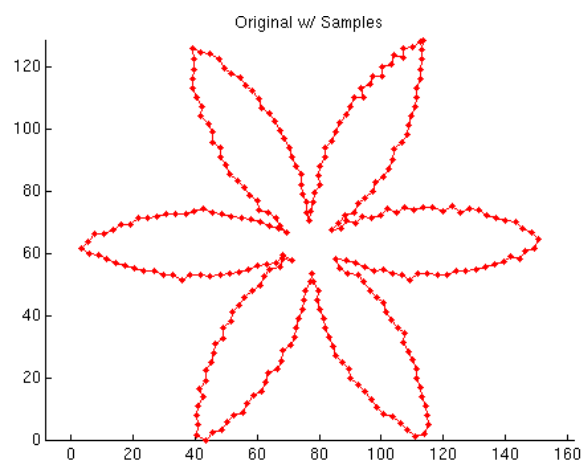
3 steps

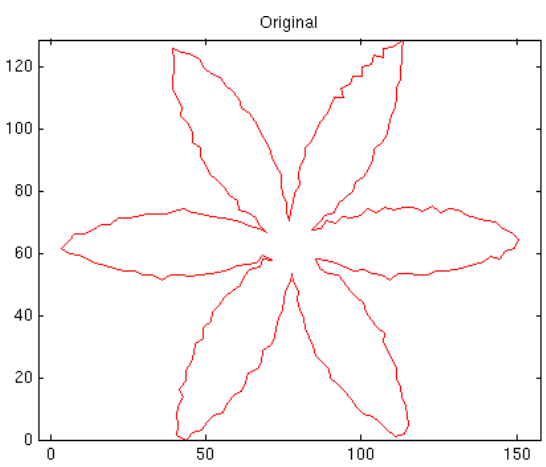

Original w/ Samples
Original
1 Step
2 Steps
3 Steps
4 Steps
5 Steps
10 Steps
15 Steps

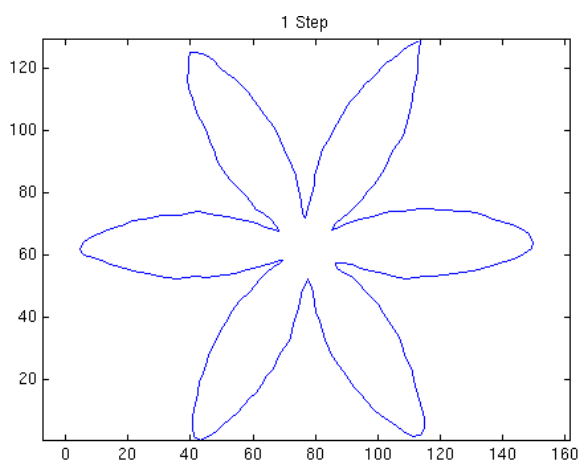


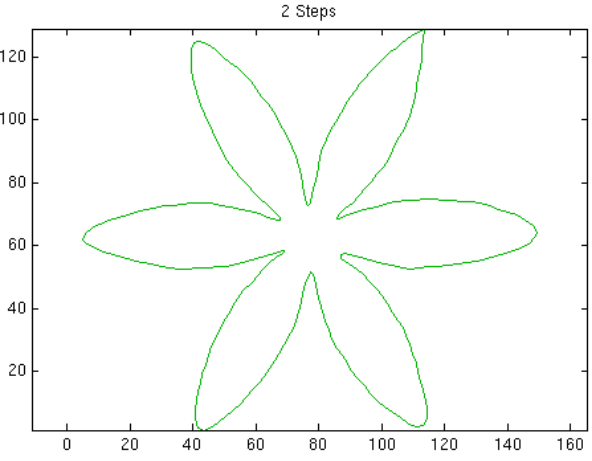
3 steps

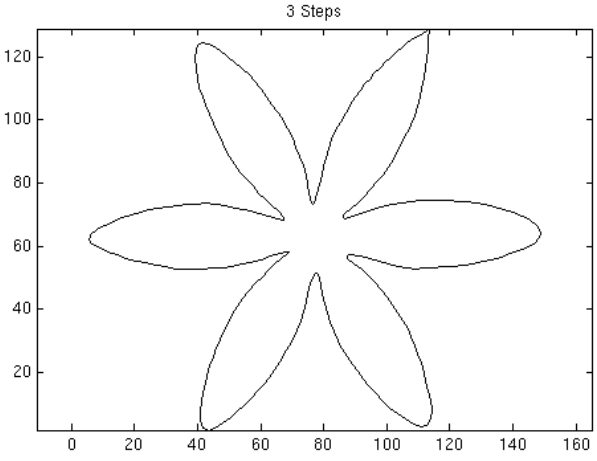
High-curvature features affected
even before removal of slight low-curvature bumps in the shape

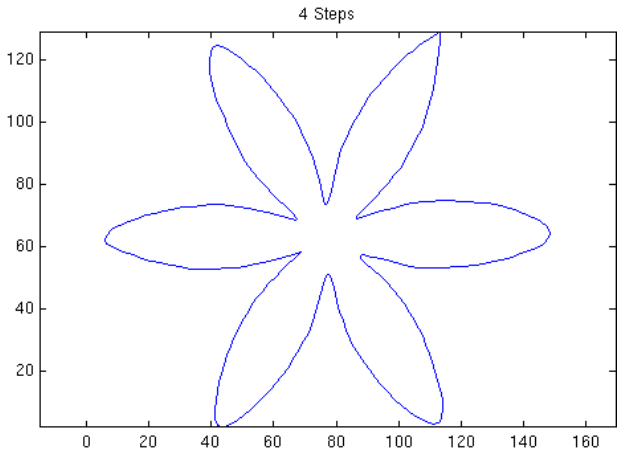

Original w/ Samples

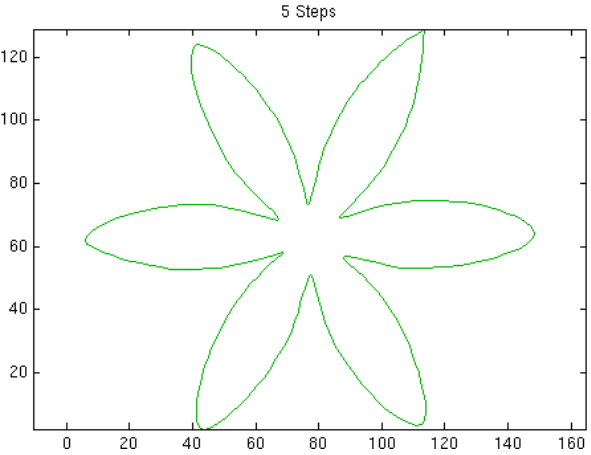

Original

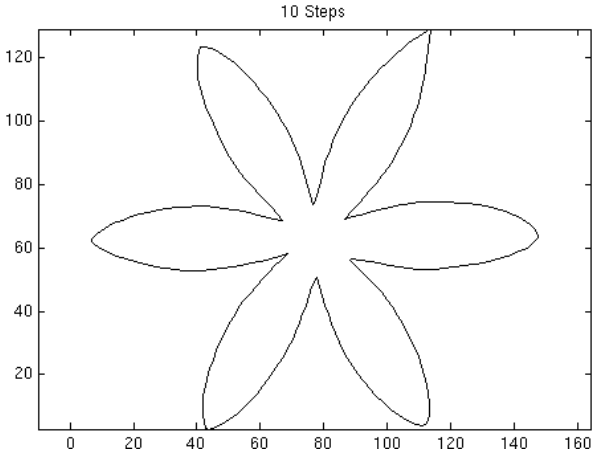

1 Step

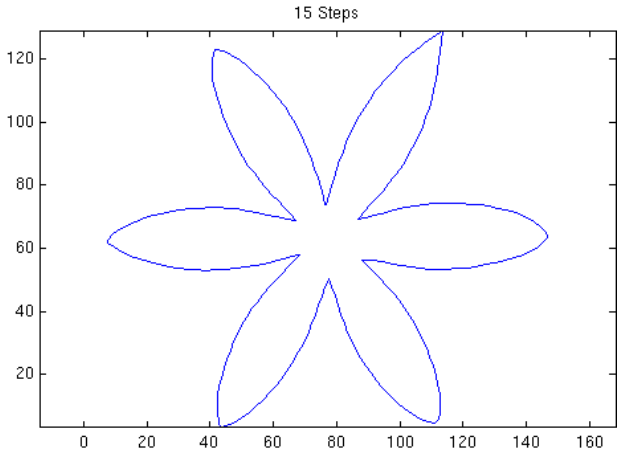

2 Steps

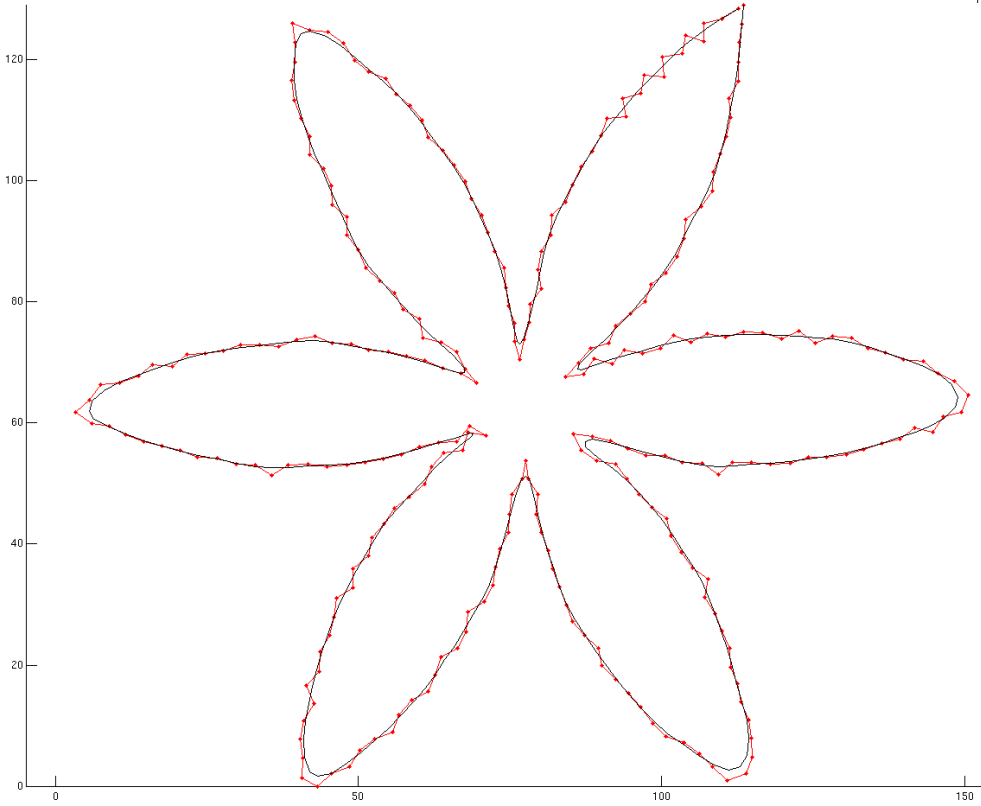

3 Steps


4 Steps

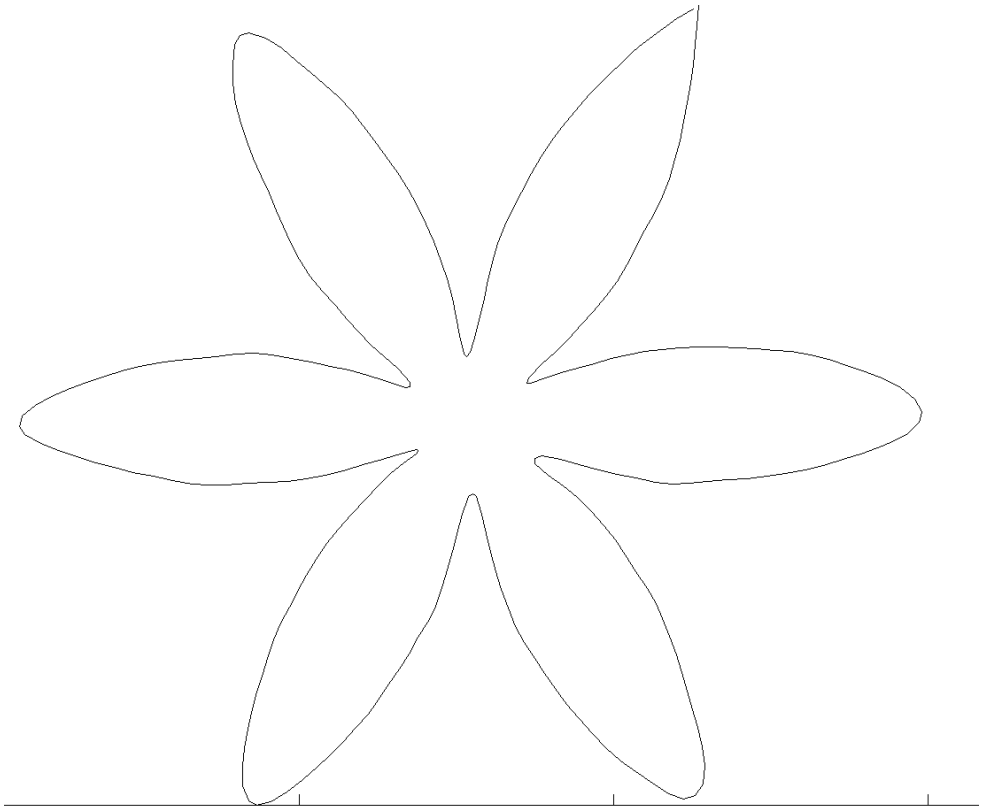

5 Steps


10 Steps

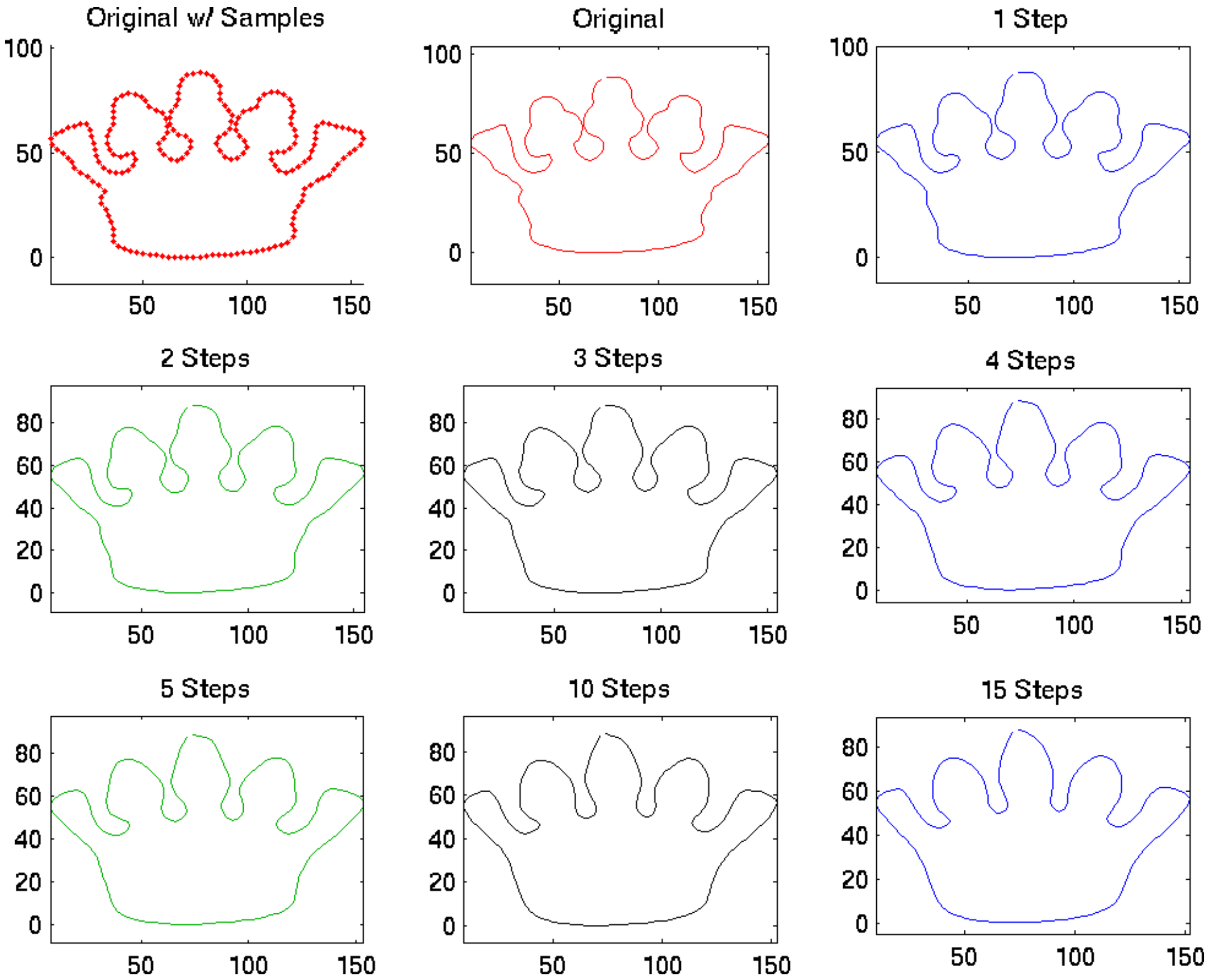

15 Steps

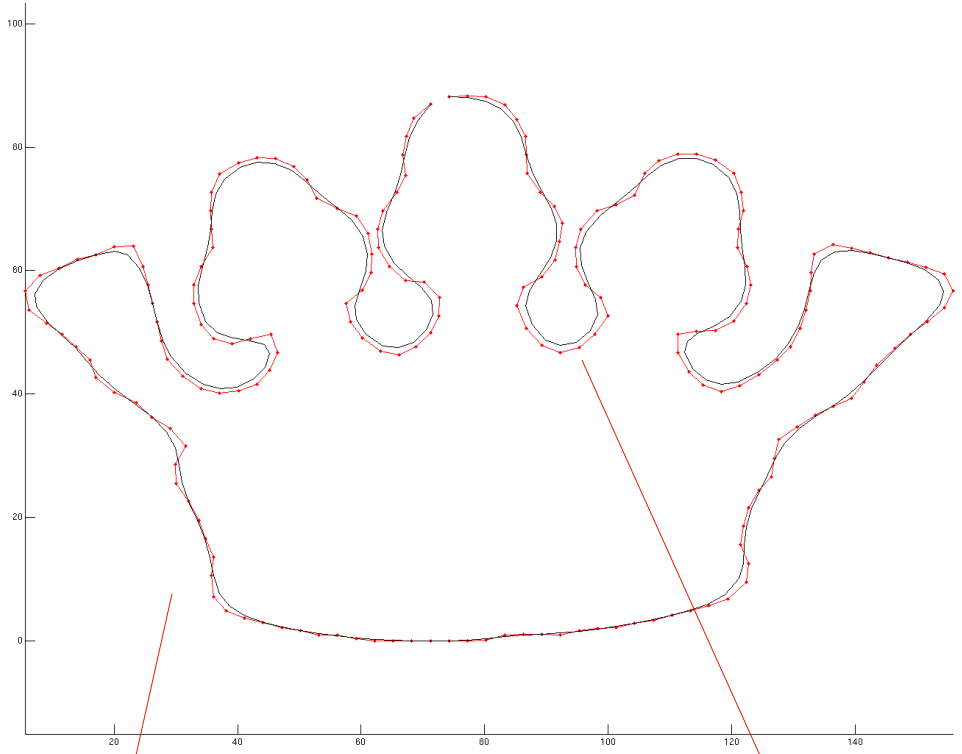


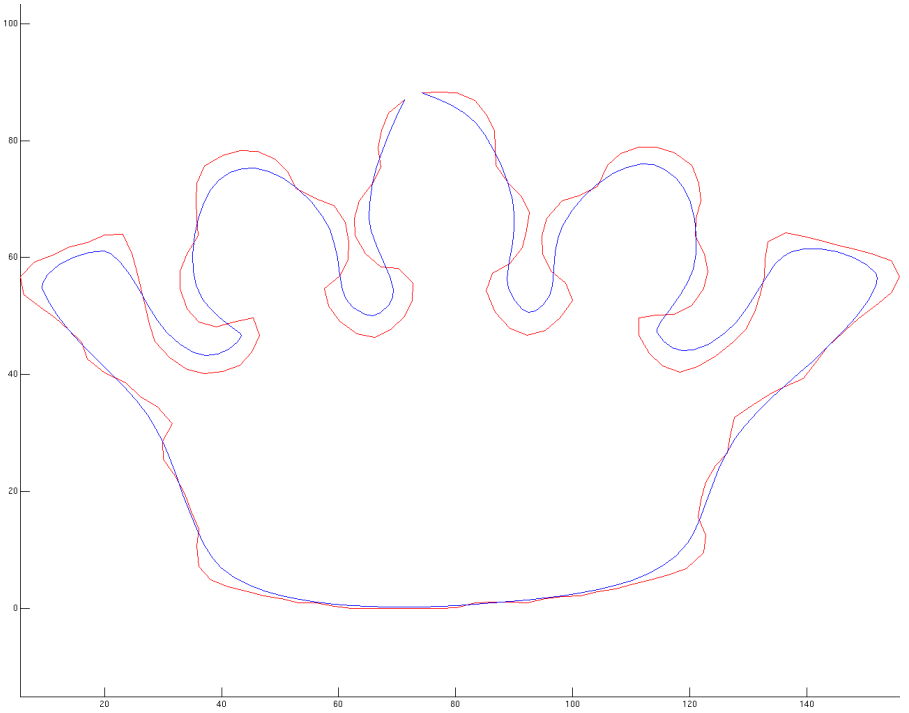
3 steps



3 steps (alone)
Not as smooth as we want,
and high-curvatures are already affected


Original w/ Samples
Original
1 Step
2 Steps
3 Steps
4 Steps
5 Steps
10 Steps
15 Steps



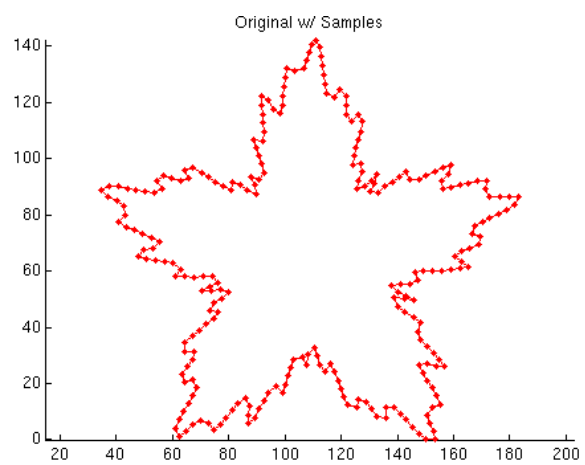


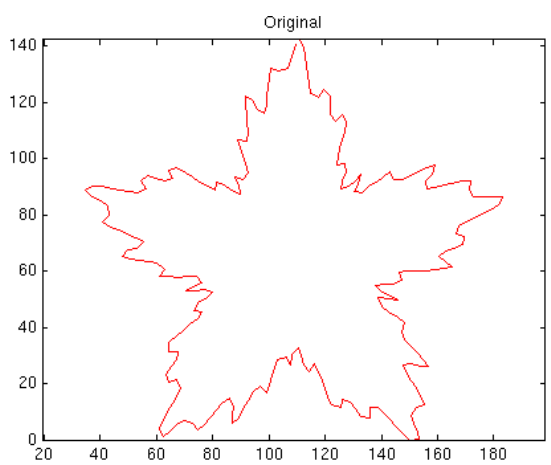
15 steps

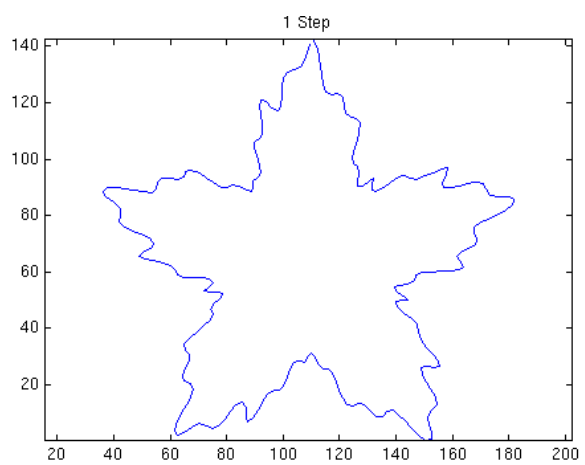
3 steps

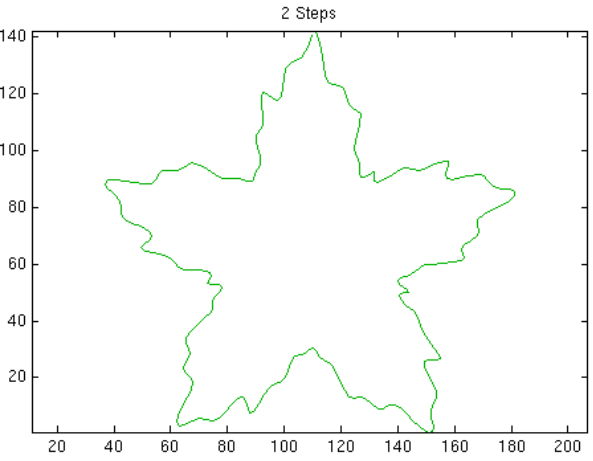
High curvatures get affected

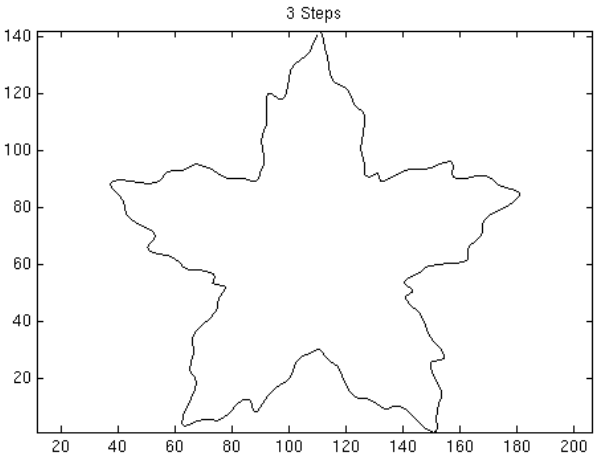
Bumps persist (see also previous slide)

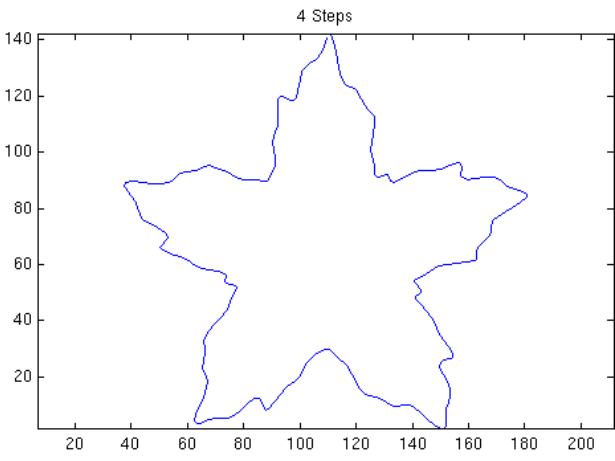

Original w/ Samples

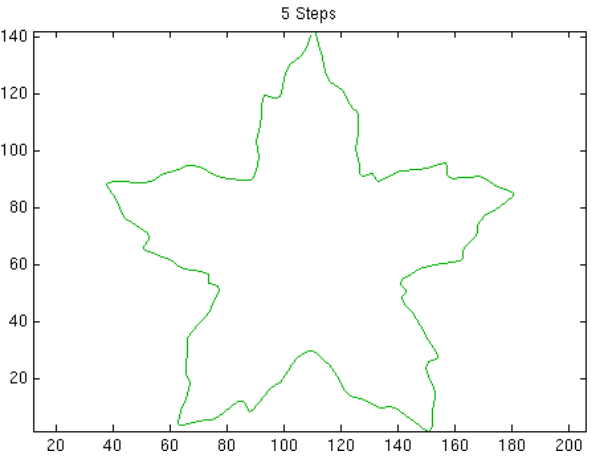

Original

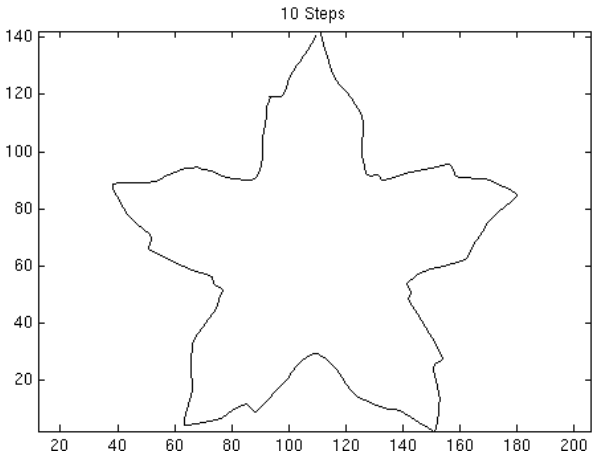

1 Step

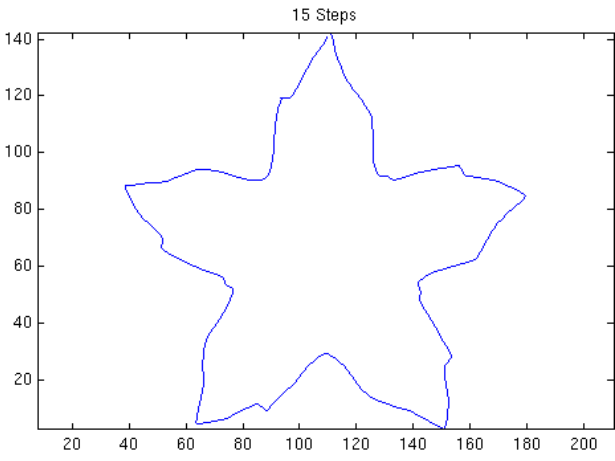

2 Steps

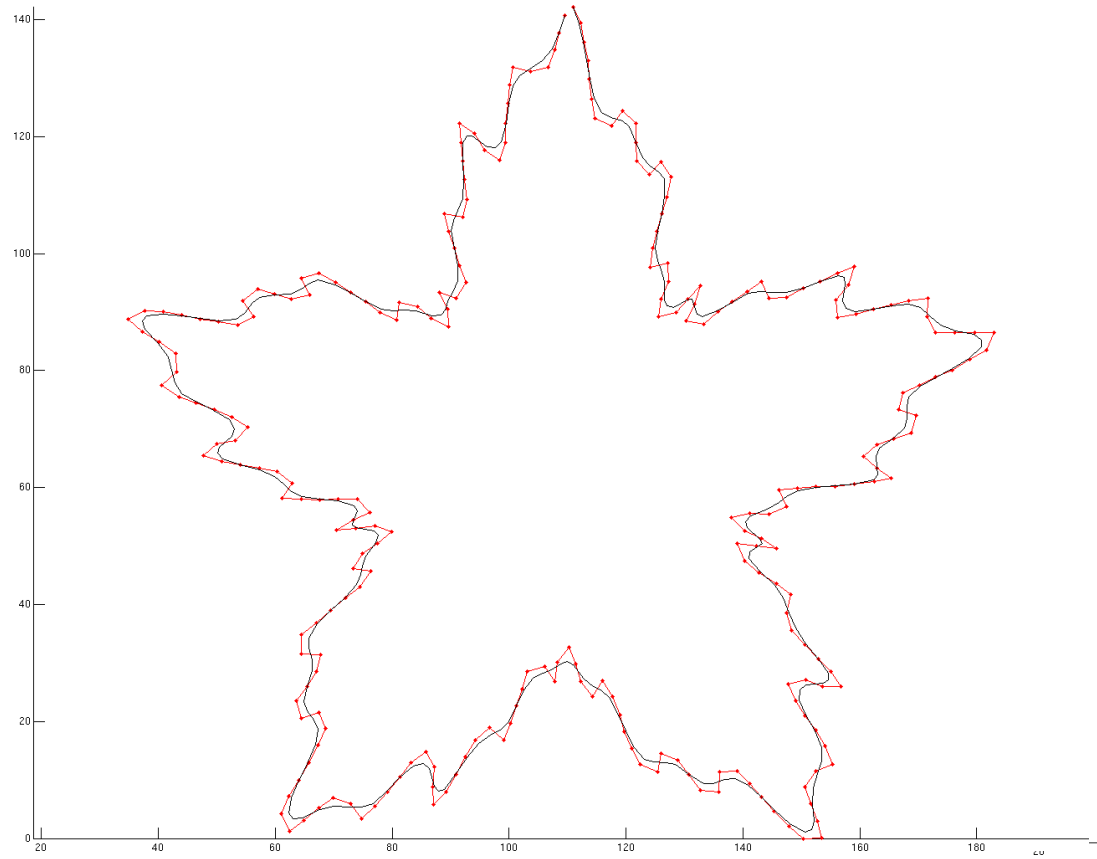

3 Steps


4 Steps

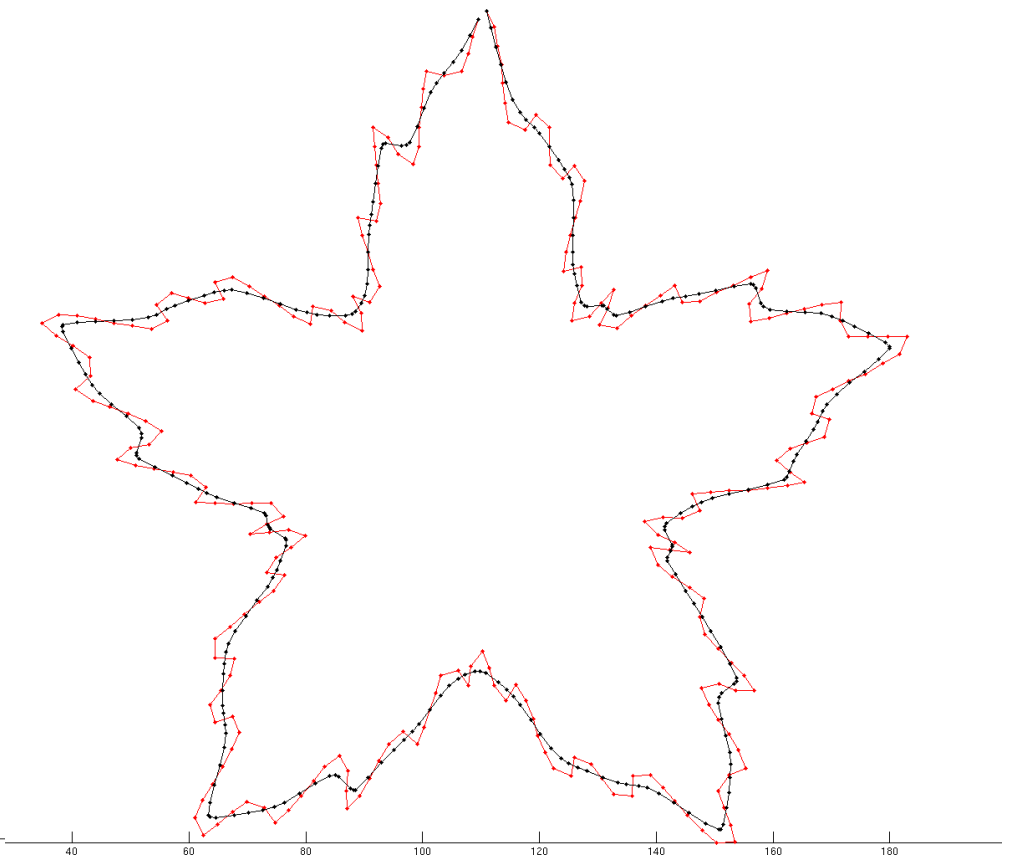

5 Steps


10 Steps

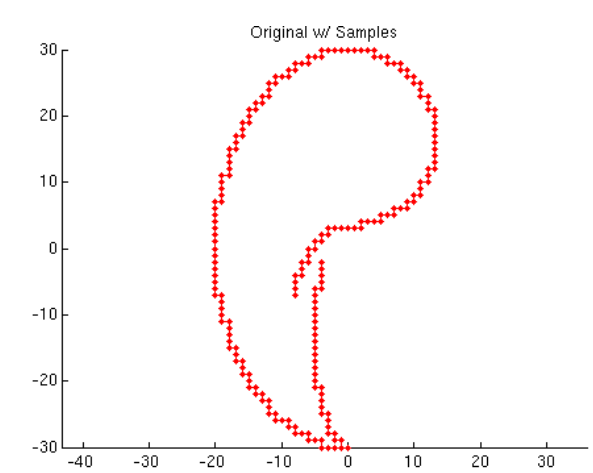

15 Steps

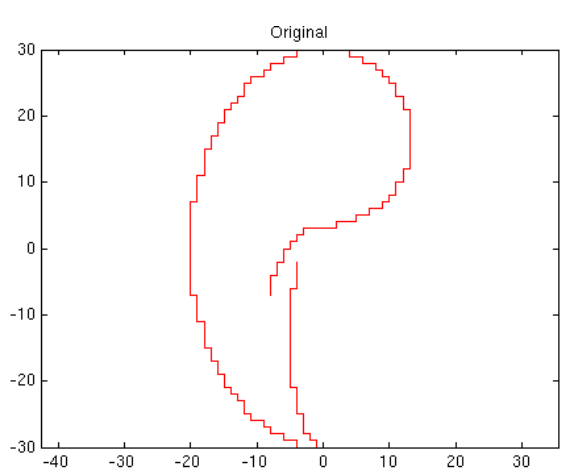


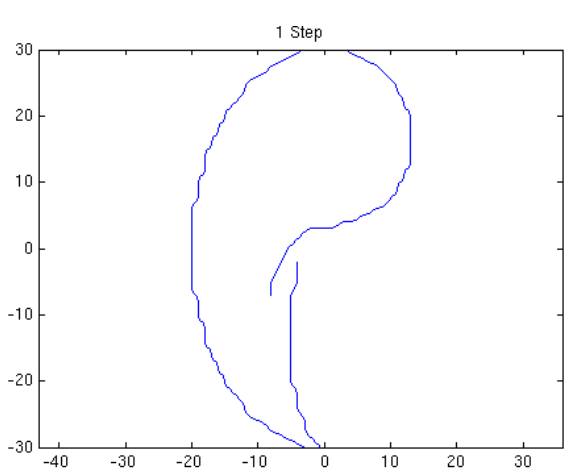
3 steps

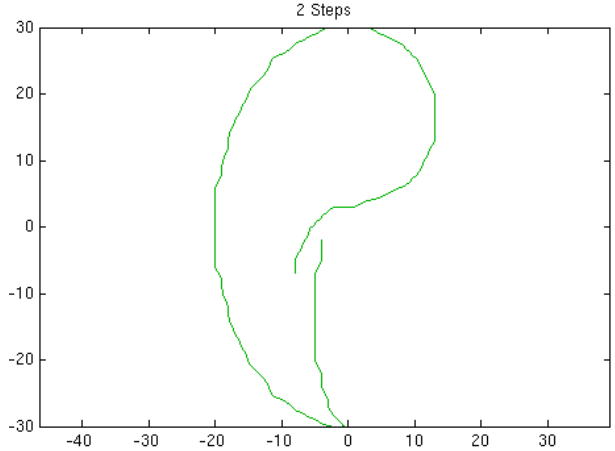


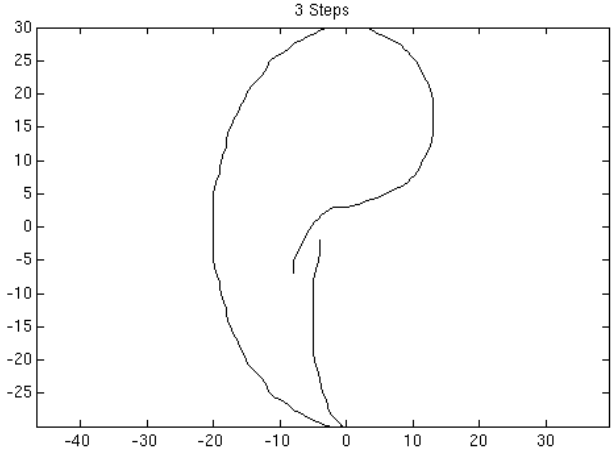
15 steps
Notice accumulation
of samples at sharp corners
This makes smoothing stop
at these regions.

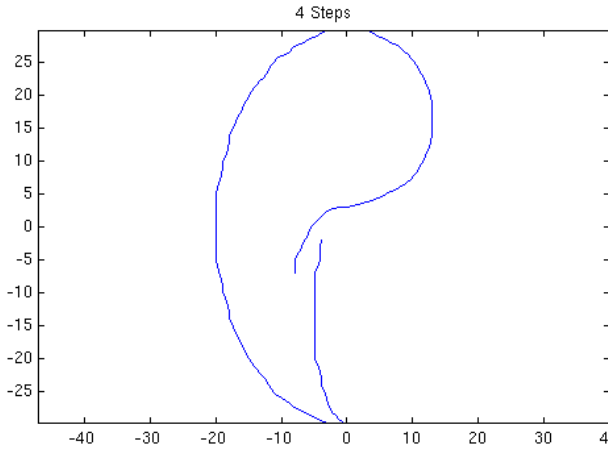

Original w/ Samples

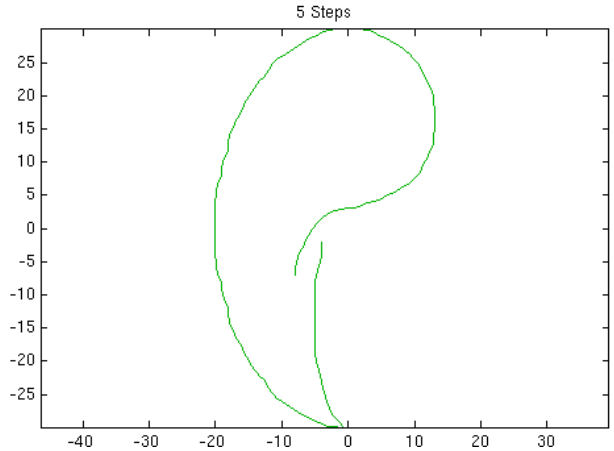

Original

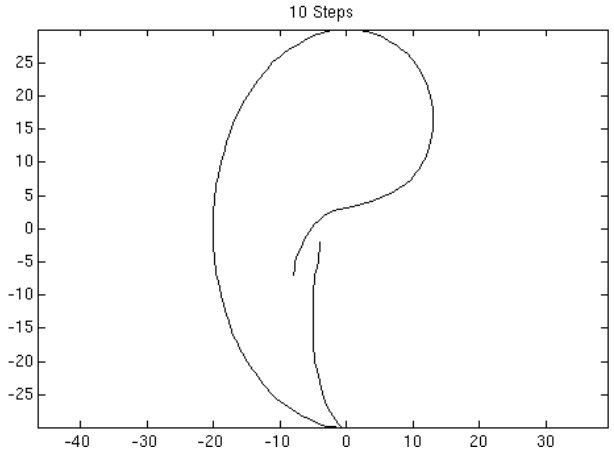

1 Step

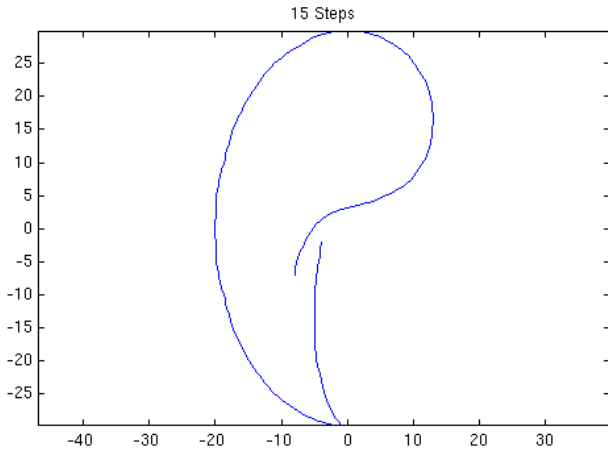

2 Steps

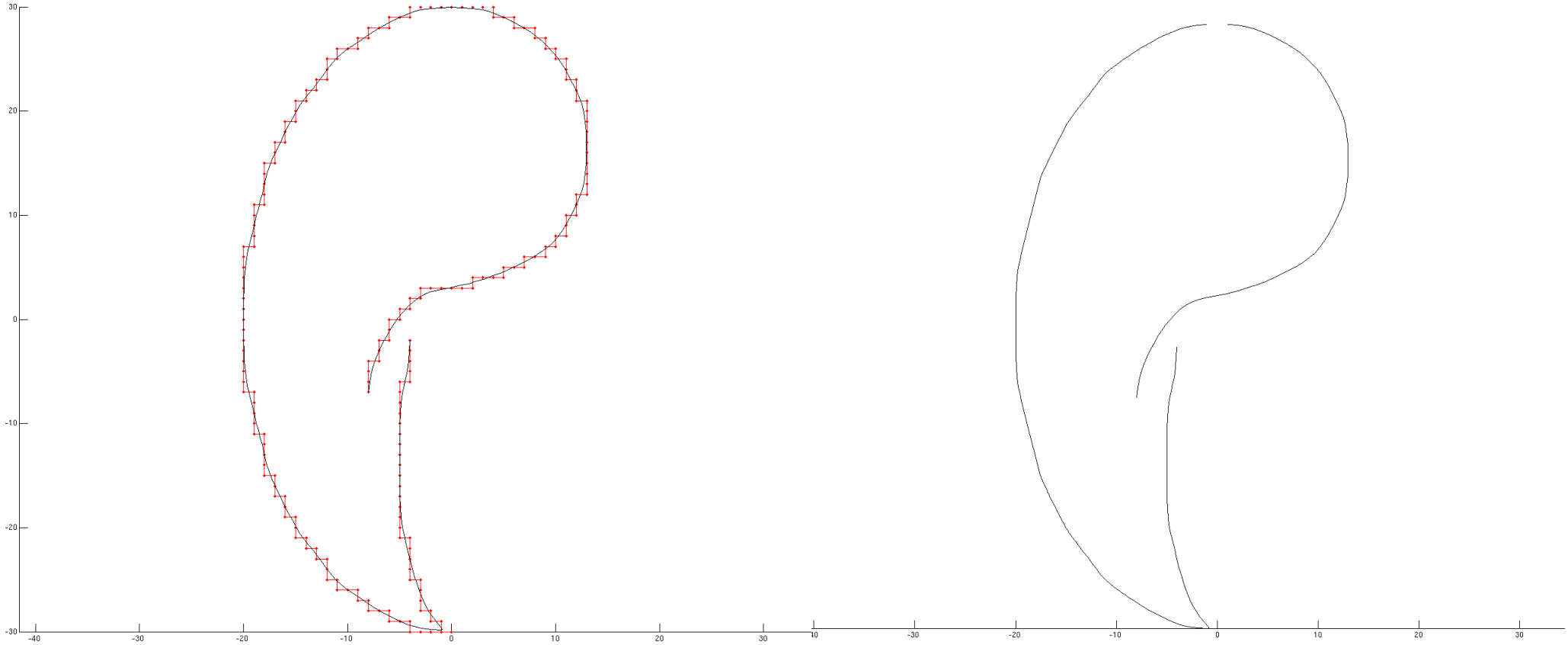

3 Steps


4 Steps

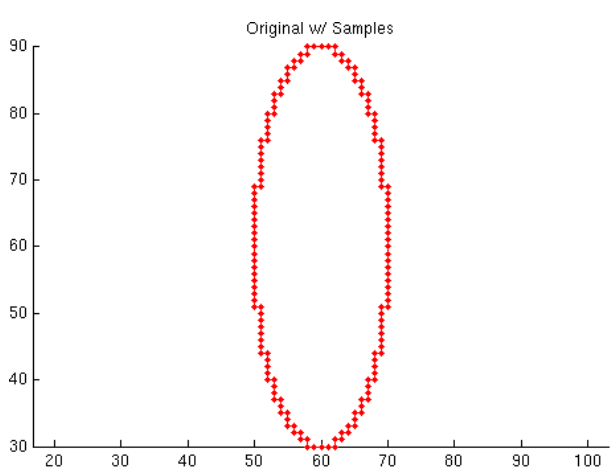

5 Steps

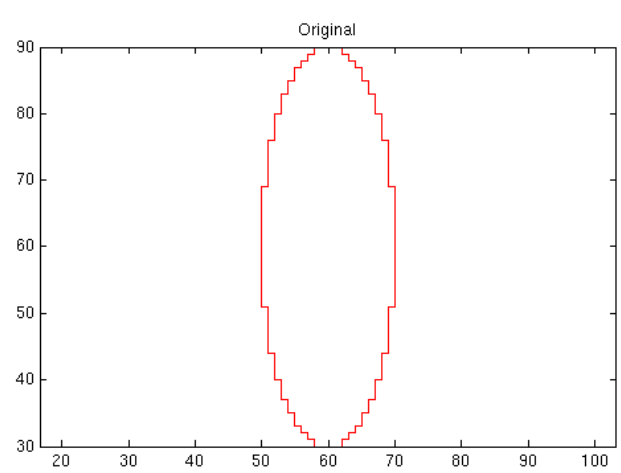

10 Steps

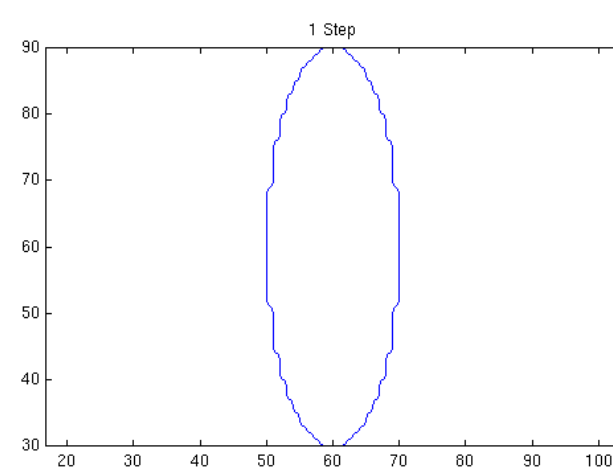

15 Steps

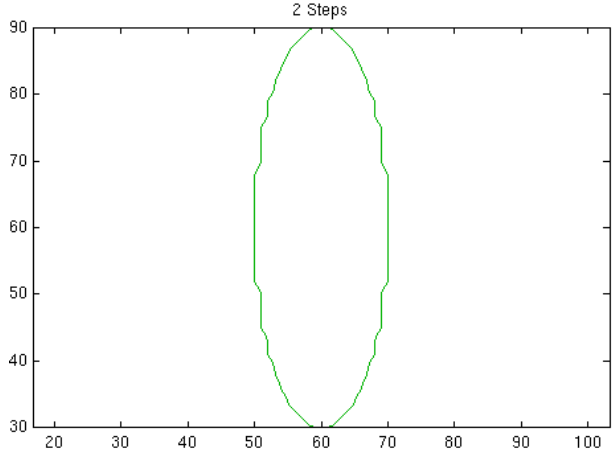


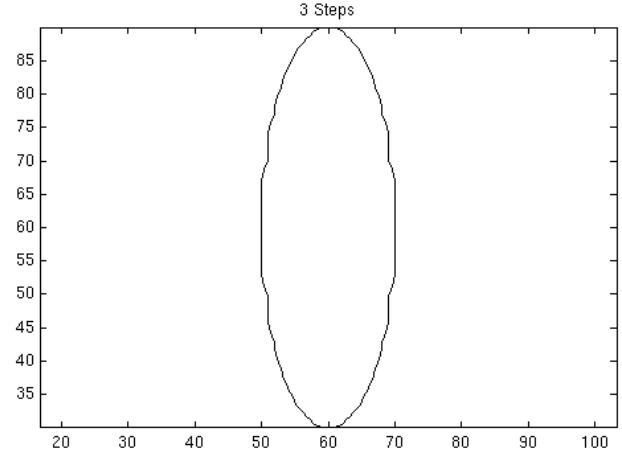
10 steps
Bumps not 100% removed!

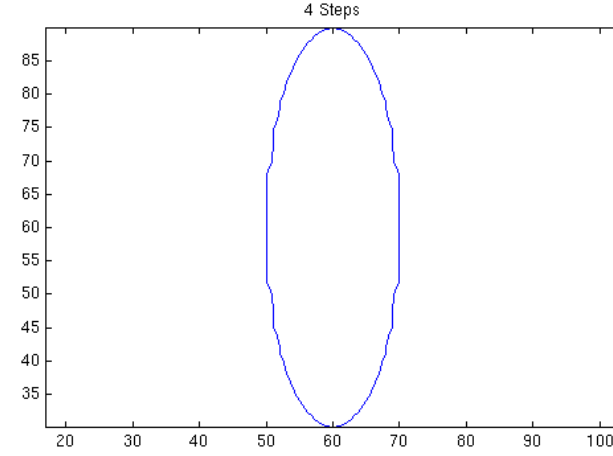

Original w/ Samples

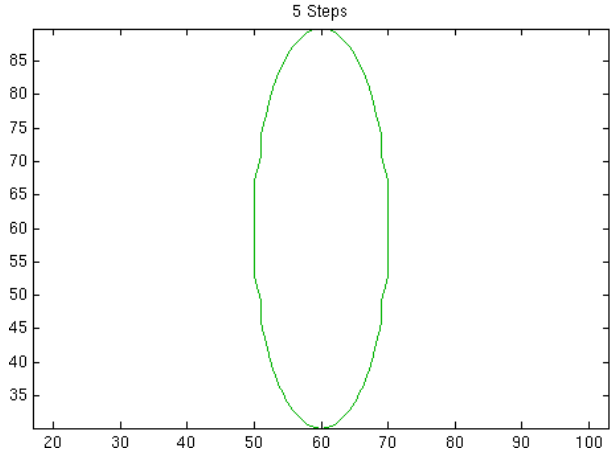

Original

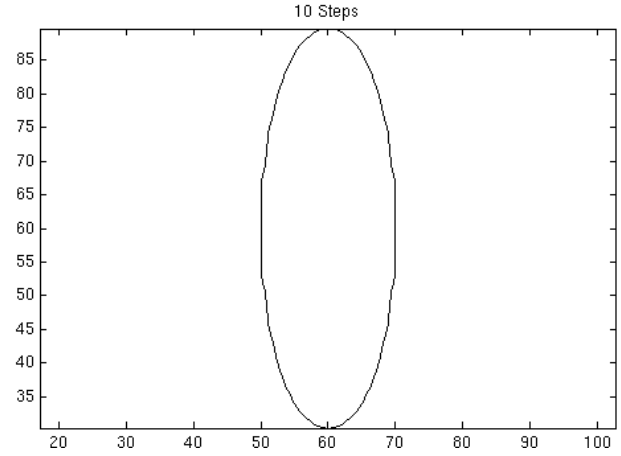

1 Step

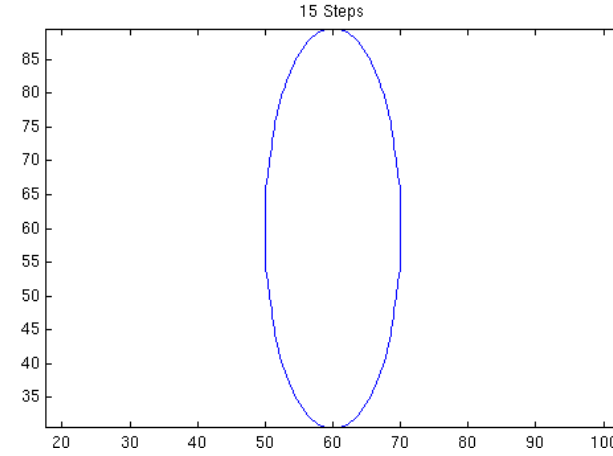

2 Steps

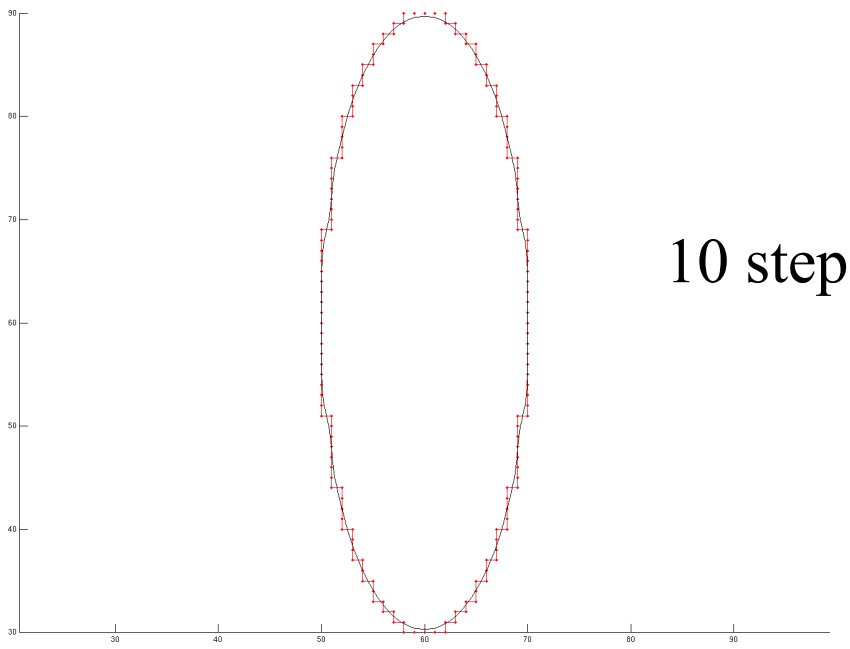

3 Steps


4 Steps

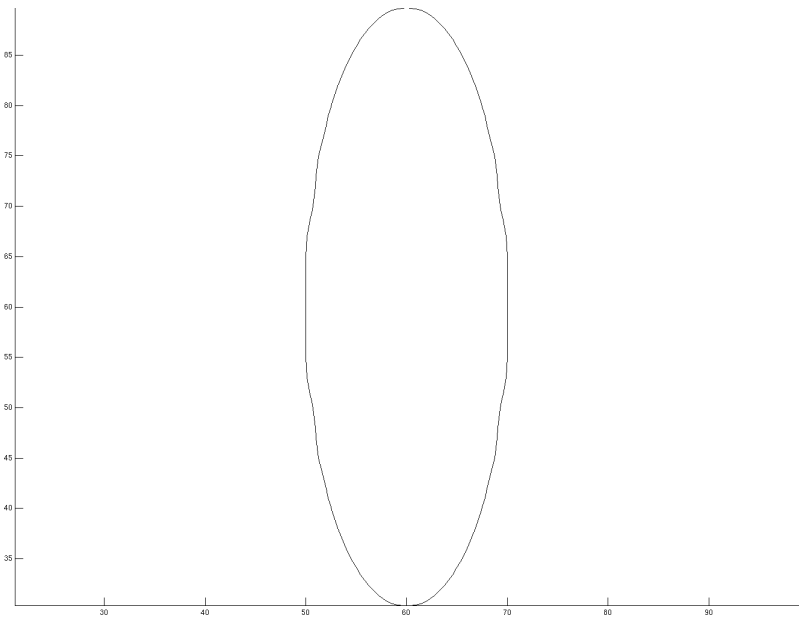

5 Steps

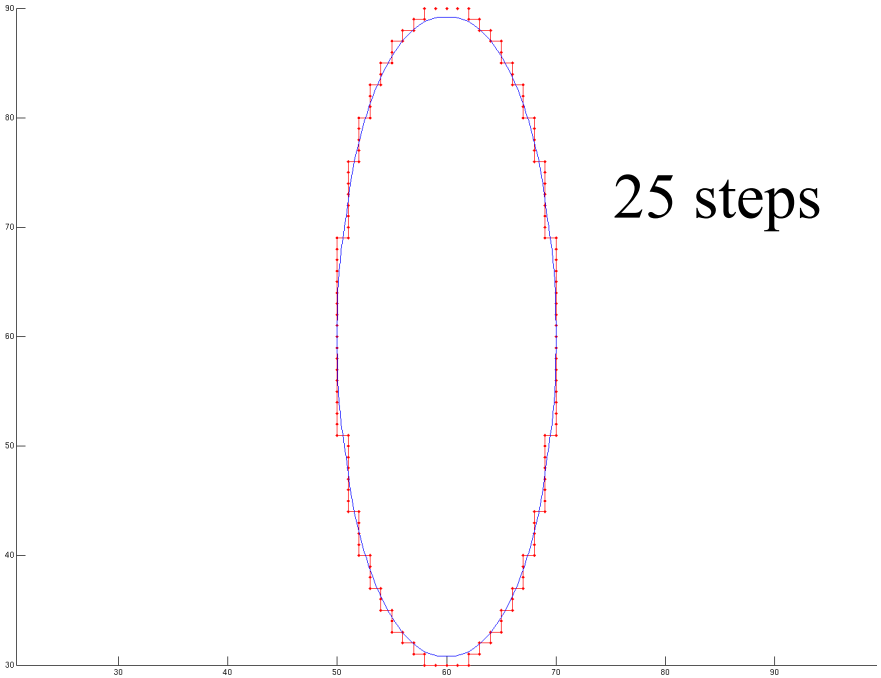

10 Steps


15 Steps



10 steps





25 steps

25 steps
Degrades peak
(locally 0 curvature)