Modeling and Forecasting Exchange Rate

Based on USD/EUR exchange rate*

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Abstract

Modeling and forecasting exchange rate has long been a area of study in the field of economics. Since the January 1st, 1999, the creation of euro by European Central Bank, euro and dollar have been the two major world reserve currencies. A 2014 study by the Federal Reserve Bank showed an average daily volume of USD/EUR in the market of $140 billion (Investopedia). The importance of USD/EUR exchange rate links to people's everyday life, no matter you are a multinational corporation or an individual traveller. From prior theories and research papers, we understand that a currency's value is determined by the purchasing power parity theory and interest rate parity theory. However, observing the historical data, we see a correlation between the macroeconomic variables and the actual exchange rate but the correlation (R squared) is quite low.

This report attempts to model and forecast USD/EUR exchange rate using ARIMA model and ARCH Model. We use daily data of US Dollar's exchange rate against the Euro for the period of September 6th, 1999, about nine month after the creation of the euro, to February 28th, 2018. The data is collected from Quandl, where Quandl collects the USD/EUR reference exchange rates from European Central Bank. We first check the normality and stationarity of the USD/EUR exchange rate using both Jarque-Bera statistics, Dickey-Fuller test and visual inspection. After confirming the non-stationary, the series are transformed to natural logarithm. It can also facilitate the study of the percentage change in the exchange rate by giving it economic implications. In the model estimation stage, we use auto-regression and moving average process

*UCLA Econ 403B Final Report, winter 2017 to fit the series to an ARIMA model. Then, check the residuals to determine the stability of our model (the fitting performance) and conduct a short-term forecasting. Finally, capture the volatilities of the series with an ARCH Model.

Figure 1: USD/EUR exchange rate from 1999-2018

1 Introduction to Topic and Data

Since the January 1st, 1999, the European Union introduced its new currency, a currency widely used in all 27 member states, the euro, euro and dollar have been the two major world reserve currencies. A 2014 study by the Federal Reserve Bank showed an average daily volume of eurodollar in the market of $140 billion (Investopedia). The huge market of USD/EUR exchange rate market sparks interests of many economists and finance professions. Many theories have been proposed to explain the movement behavior of the exchange rate but many have failed to provide a comprehensive and accurate picture of the exchange rate movement. One common method used to forecast exchange rates involves gathering factors that we believe affect the movement of a certain currency and creating a model that relates these factors to the exchange rate[1]. The factors used in econometric models are normally based on economic theory, but any variable can be added if it is believed to significantly influence the exchange rate. Like concluding the factors that are most influential are: the interest rate differential between the U.S. and EU (INT), the difference in GDP growth rates (GDP), and income growth rate (IGR) differences between the two countries. The econometric model he comes up with is shown as:

$$USD/EUR_{1year} = z + a(INT) + b(GDP) + c(IGR)$$

In theory, a currency's value should converge to its real long-run equilibrium if the observed period is long enough. The most prominent theories, the purchasing power parity, and the interest rate parity theories have some macroeconomics

[1] https://www.investopedia.com/articles/forex/11/4-ways-to-forecast-exchange-rates.asp

explanatory power for the exchange rate series, however, these two models in practice are undermined due to the low coefficient of determination (R squared).

Since traditional Auto-Regressive Integrated Moving Average (ARIMA) model assume that variance of residuals is constant, avoiding the heteroscedasticity, we use ARCH (Autoregressive Conditional Heteroskedasticity) to capture the volatility of the USD/EUR exchange rate. To evaluate the accuracy of the forecast models, five different criteria have been used; consist of: Mean Error(ME), Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean Percentage Error(MPE), and Mean Absolute Percentage Error(MAPE).

This paper is prepared as follows: In Section 4-6, we present the movement of USD/EUR exchange rate from September 6th, 1999, about nine month after the creation of the euro, to February 28th, 2018 and we build a model with the process of ARIMA-ARCH model. Lastly, We test performance of the model. In each section we focused on the empirical results followed by conclusions which is discussed in Section 7.

2 Methodology

In this section, we present the models identification, estimation, and model checking to model the future movement of USD/EUR and the volatility of this time series. We first establish a stationarity of the time series by converting the non-stationary USD/EUR exchange rate time series to a covariance stationary time series, the natural logarithm form of the exchange rate. Secondly, we test various models to find out the most fitted one. Thirdly, the model checking is performed by diagnostics of randomness of the residuals. The residuals are required to be uncorrelated and normally distributed (white noise). Finally, one can perform forecasting with the chosen model over future finite time space.

3 Data Acquirement

We get our data directly from Quandl.[2]

4 Model Fitting

The three main steps of this model, such as Identification, Estimation, and Model checking are elaborated as follows.

Firstly, the stationarity of the time series is established. Next, the conditional mean model for the given data is identified. There are three rules to identify

[2] https://www.quandl.com/data/CURRFX/USDEUR-Currency-Exchange-Rates-USD-vs-EUR

ARIMA model: If ACF (autocorrelation graph) cut off after lag n, PACF (partial autocorrelation graph) dies down: ARIMA(0,d,n) $\rightarrow$ identify MA(q). If ACF dies down, PACF cut off after lag n: ARIMA(n,d,0) $\rightarrow$ identify AR(p). If ACF and PACF die down: mixed ARIMA model, need differencing. Secondly, the model parameters are estimated by utilizing the maximum likelihood method. Thirdly, the model checking is performed by diagnostics of randomness of the residuals. The residuals are required to be uncorrelated and normally distributed. Finally, one can perform forecasting with the chosen model over future finite time space.

4.1 Model Identification

a) Jarque-Bera Statistics

Jarque-Bera statistics is used to test the nonnormality of the EUR/USD exchange rate.

```
Jarque Bera Test

data:  rate
X-squared = 913.97, df = 2, p-value < 2.2e-16
```

According to the Jarque-Bera statistics, the USD/EUR rate is non-normal at the confidence interval of 99%, since probability is 2.2×10^{-16} which is less than 0.01. So, it is required to transform the USD/EUR exchange rate series into the return series.

Generally, the movements of the foreign exchange rates are usually non-stationary as well as quite random and not suitable for the time series analysis. In financial time series, it is often that the series is transformed by logging and then the differencing is performed. This is because financial time series is usually exposed to exponential growth, and thus log transformation can smooth out (linearize) the series and differencing will help stabilize the variance of the time series.

b) Transformation of the USD/EUR Exchange Rate Series

The concept of time series second order weakly stationary (covariance stationary)[3]:
Means: $\mu_{Y_1} = \mu_{Y_2} = ... = \mu_{Y_T} = \mu$
Variances: $\sigma^2_{Y_1} = \sigma^2_{Y_2} = ... = \sigma^2_{Y_T}$
Time Independent Covariances: $\rho_{Y_T, Y_{T-l}} = \rho_{|k|}$

Hence we should check the stationarity of the time series, if it is not stationary, taking the log of y_T and then take the first difference of this transformed series is required.

[3]Econ403B Lec1 Prof.Rojas

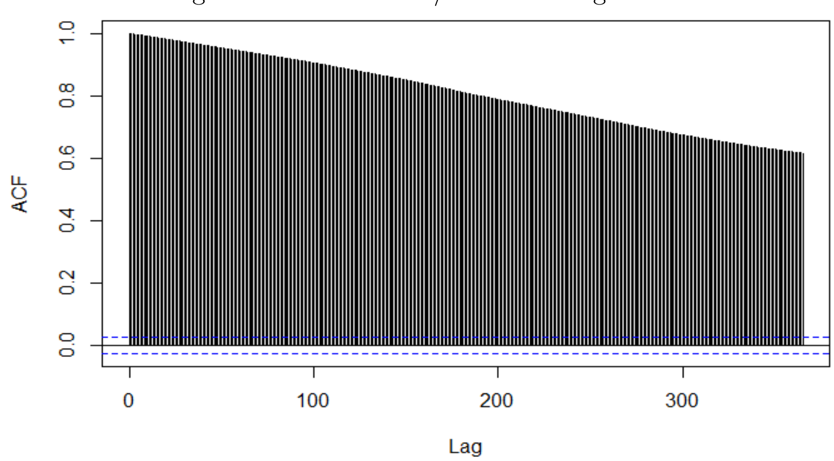
First, We will take the natural logarithm of the exchange rate and conduct Augmented Dickey-Fuller Test together with visual inspection from Auto-Correlation Function, all the following steps are based on the logged rate data as *lograte*.

```
        Augmented Dickey-Fuller Test

data:  lograte
Dickey-Fuller = -1.5676, Lag order = 17, p-value = 0.7615
alternative hypothesis: stationary
```

Due to the high p-value, we can't reject the null hypothesis: non-stationary, so our logged exchange rate time series is non-stationary. Also from visual inspection, we notice that the ACF tails off (Figure 2):

Figure 2: ACF of USD/EUR exchange rate

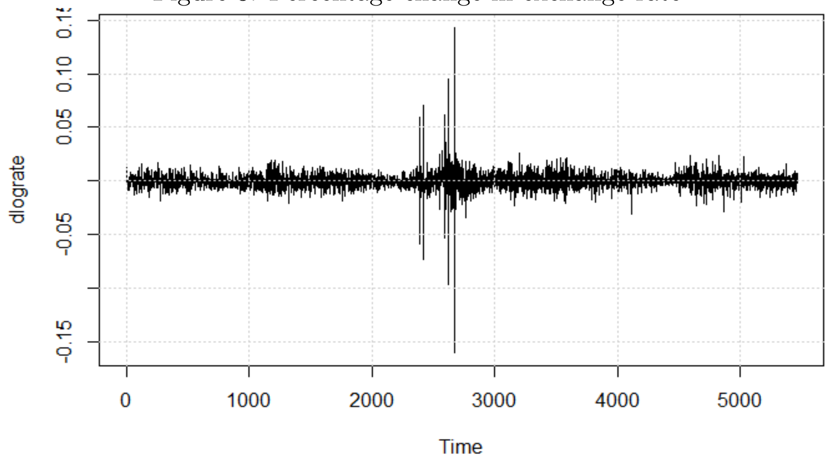


Based on the results above, our logged exchange rate time series is non-stationary, we have to take difference before future analysis.
In order to convert non-stationary series to stationary, differencing method can be used in which the series is lagged 1 step and subtracted from previous series: $Y_t = Y_{t-1} + e_t \rightarrow \mathrm{e}_t = Y_t - Y_{t-1}$ now we get the percentage change in the exchange rate as

$$dlograte = ln(rate_t) - ln(rate_{t-1}) = ln\left(\frac{rate_t}{rate_{t-1}}\right)$$

The differenced log rate, I.e. the return series, has economic meaning now.

Figure 3: Percentage change in exchange rate

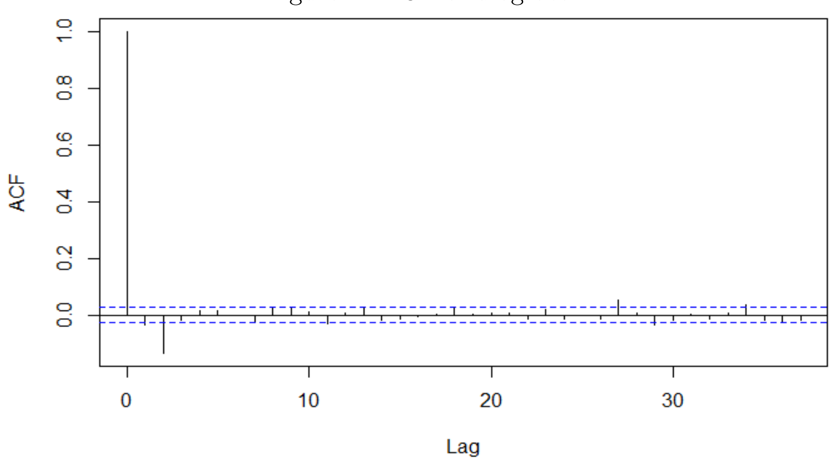


If we check the ADF test and the ACF plot again, the series after taking the first difference is stationary now (for ADF test the p-value is less than 0.05, which means we can reject the null hypothesis, for ACF plot it cuts off, figure 4).

```
     Augmented Dickey-Fuller Test

data:  dlograte
Dickey-Fuller = -17.082, Lag order = 17, p-value = 0.01
alternative hypothesis: stationary
```

Figure 4: ACF of dlograte

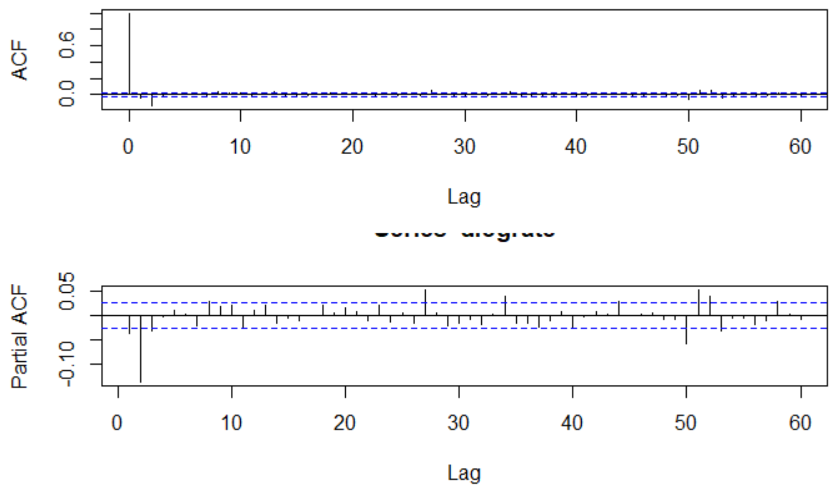


4.2 Model Estimation

The general ARIMA model introduced by Box and Jenkins includes Autoregressive as well as moving average parameters. Specifically, the two types of parameter in the model are; the auto regressive parameter (p), and the moving average parameter (q). The models are summarized as ARIMA(p,d,q). The general form of the ARIMA (p,d,q) process is of the form, in which θ is the parameter of MA process and ϕ is the parameter of AR process:

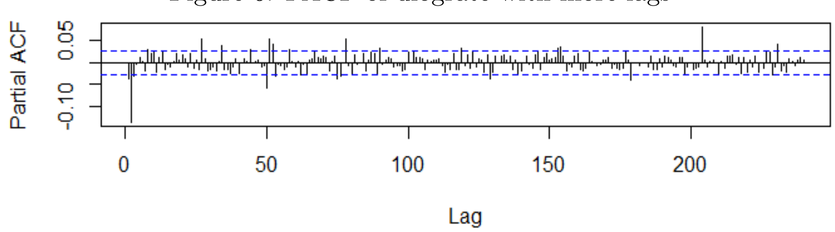
$$\hat{y_t} = \mu + \phi_1 \times y_{t-1} + ... + \phi_p \times y_{t-p} + \theta_1 \times e_{t-1} + ... + \theta q \times e_{t-q}$$

Figure 5: ACF and PACF of dlograte



From the ACF and PACF plot of "dlograte", We notice ACF cuts off at lag.2, so our q = 2. Note that PACF has a significant spike at lag.2, we can consider p = 2 (Figure 5). However, when we expend the lag limit, more spikes are coming up (Figure 6), so basically we will try both of them: p = 2 and p = 0, e.g. ARIMA(2,1,2) and ARIMA(0,1,2) and compare their AIC and BIC value.

Figure 6: PACF of dlograte with more lags



From Table 1, we know ARIMA(0,1,2) has both smaller AIC and BIC values.

Table 1: AIC and BIC comparison

	AIC	BIC
ARIMA(0,1,2)	-38943.85	-38924.04
ARIMA(2,1,2)	-38943.59	-38910.56

We can also use "auto.arima" function, let R help us:

```
    Series: lograte
ARIMA(0,1,2)

Coefficients:
          ma1      ma2
      -0.0465  -0.1321
s.e.   0.0134   0.0131

sigma^2 estimated as 4.685e-05:  log likelihood=19474.93
AIC=-38943.85   AICc=-38943.85   BIC=-38924.04
```

Hence, we choose ARIMA(0,1,2) model as follows:

$$Y_t = -0.0465\epsilon_{t-1} - 0.1321\epsilon_{t-2} + \epsilon_t$$

Where,
Y_t = the dependent variable, i.e. our log rate data at time t
ϵ_t = the residual term
$\epsilon_{t-1}, \epsilon_{t-2}$ and ϵ_{t-p} = previous values of the residual

4.3 Model Checking

a) Checking randomness of residuals

The procedure includes observing residual vs fitted values plot and its ACF PACF diagram, and check Ljung-Box result. If ACF PACF of the model residuals show no significant lags, the selected model is appropriate.

Residuals VS Fitted plot

For each fitted (predicted) values, the residuals are centered around 0, which means out ARIMA model fits the original data very well (Figure 7).

ACF and PACF plot of Residuals

In general, after fitting a time series model, the residuals should be white noise. So they should have no autocorrelation. If the residuals have significant autocorrelation and not just one spike at high lag order in the ACF/PACF plots,

Figure 7: Residuals vs Fitted plot

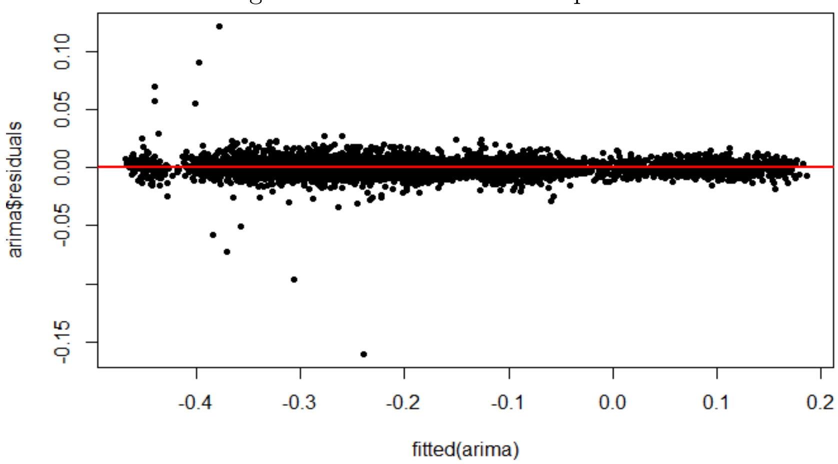
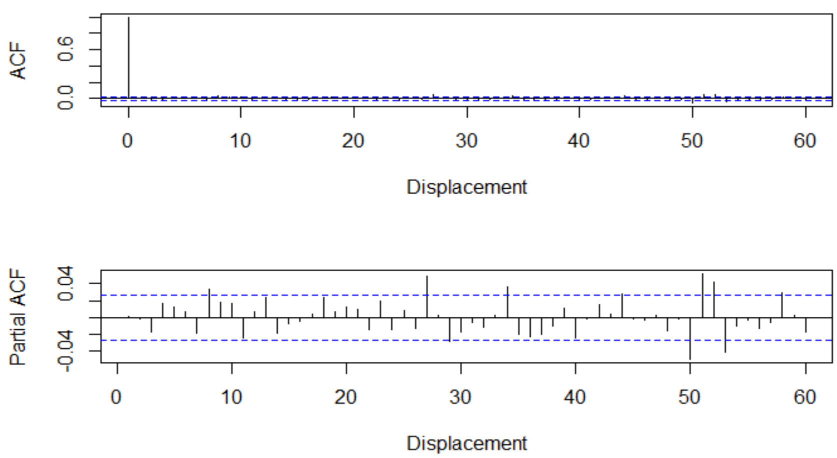


Figure 8: ACF and PACF plot of Residuals



this maybe an indicate that the model is wrong.

From the plots (Figure 8), the residuals' Autocorrelations and Partial Autocorrelations do not have significant lag (in the dotted line), which means they have no significant autocorrelation with itself's previous value or error terms, indicating ARIMA(0,1,2) is a good model to represent the series.

In addition, Ljung-Box test also provides a different way to double check the model. Basically, Ljung-Box is a test of autocorrelation in which it verifies whether the autocorrelations of a time series are different from 0. In other words, if the result rejects the hypothesis, this means the data is independent and uncorrelated; otherwise, there still remains serial correlation in the series and the model needs modification.

```
        Box-Ljung test

data:  arima$res
X-squared = 0.020263, df = 1, p-value = 0.8868
```

The output from R show that p-values are all greater than 0.05, so we cannot reject the hypothesis that the autocorrelation is different from 0. Therefore, the selected model is an appropriate one of USD/EUR exchange rate.

Check the persistence of Recursive Residuals and the Stability of Parameters

Recursive residuals are standardized one-step-ahead prediction errors. Under the usual assumptions for the linear regression model they are (asymptotically) normal and i.i.d.. If model is correctly specified, recursive residuals have mean zero. From the result (Figure 9), the recursive residuals are centered around 0, our model is perfect.

Figure 9: Recursive Residuals

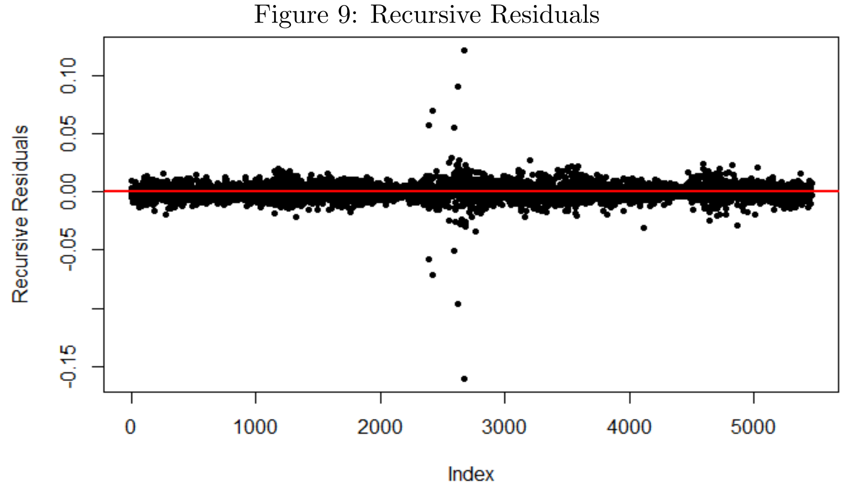
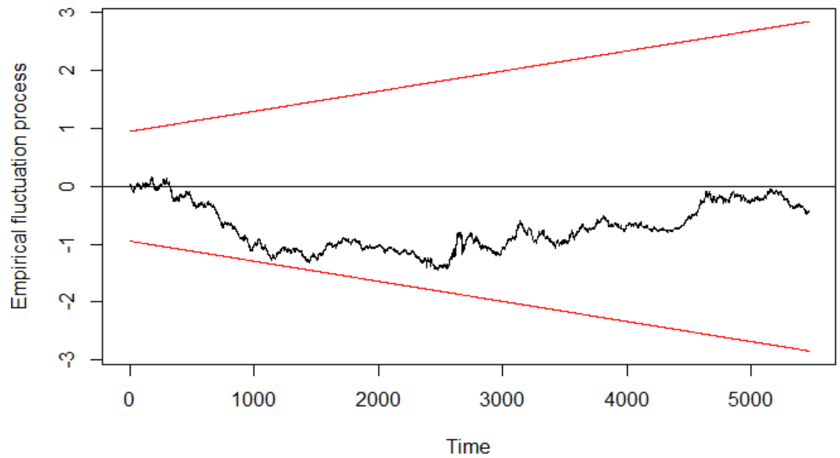


Figure 10: Recursive CUSUM Test

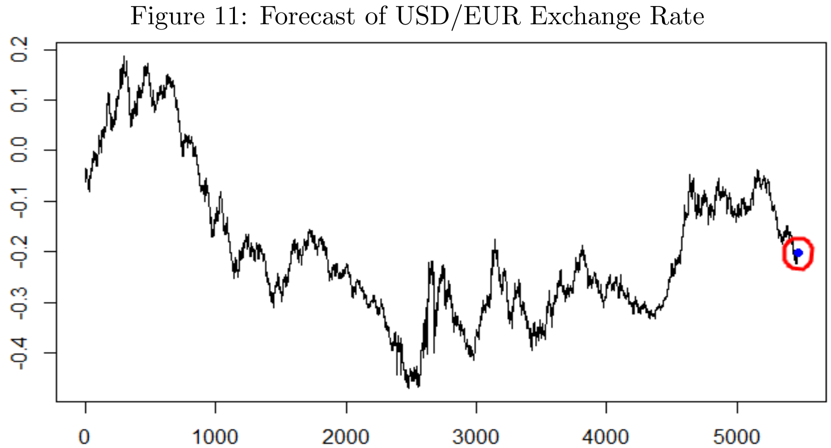


The function efp (in R) returns a one-dimensional empirical process of sums of residuals, if there is a single structural change point, the recursive CUSUM path will depart from its mean 0 at this point. Overall, the sums of residuals don't exceed the boundary line (Figure 10, the boundary line is 5% significant level critical line), the parameters' stability is pretty good.

5 Forecasting

5.1 Forecast 2-steps ahead[4]

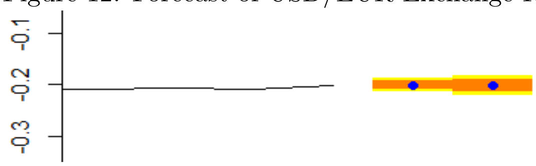
Figure 11: Forecast of USD/EUR Exchange Rate



[4]To get higher accuracy, we only use 2-days ahead.

To see the detail:

Figure 12: Forecast of USD/EUR Exchange Rate



5.2 Test Predictive Performance - accuracy

It is important to evaluate forecast accuracy using genuine forecasts. That is, it is invalid to look at how well a model fits the historical data; the accuracy of forecasts can only be determined by considering how well a model performs on new data that were not used when estimating the model. When choosing models, it is common to use a portion of the available data for testing (test set), and use the rest of the data for estimating (training set) the model. Then the testing data can be used to measure how well the model is likely to forecast on new data.

In this part, we split our log exchange rate data into a training set (before 2018-02-26) and a test set (2018-02-27 and 2018-02-28). To check the accuracy of our forecasting method, we will estimate the parameters using the training data, and forecast the next 2 observations. These forecasts can then be compared to the test data (actual data). Here is the table of predicted value versus the actual value: From R command "accuracy()", we can get all the common

Table 2: Predicted vs Actual lograte comparison

	Actual	Forecast	Error	Percent.error
2018-02-27	-0.2082796	-0.2057904	0.0024892	0.01195124
2018-02-28	-0.2013331	-0.2062818	0.0049487	0.02457966

statistics used to test the predictive performance. For example, one of the most common used is mean absolute percentage error (MAPE), it is a measure of prediction accuracy of a forecasting method in statistics, for example in our ARIMA model estimation.

$$MAPE = \frac{100}{n} \sum_{t=1}^{n} \left| \frac{A_t - F_t}{A_t} \right|$$

Following is what we get from R:

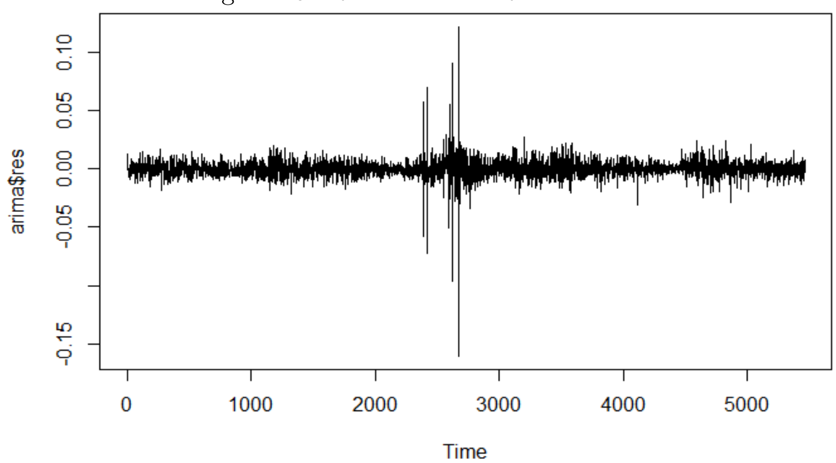
```
         ME        RMSE         MAE       MPE     MAPE ACF1 Theil's U
-0.001229785 0.003916986 0.003718926 0.5947267 1.804278 -0.5  10.07018
```

We can see at longer horizon forecasts, all the testing statistics are mainly concentrat in very small values, close to 0. Our model is good.

6 ARCH Model

Although ACF&PACF of residuals have no significant lags, the time series plot of residuals shows some cluster of volatility (Figure 13). It is important to remember that ARIMA is a method to linearly model the data and the forecast width remains constant because the model does not reflect recent changes or incorporate new information. In other words, it provides best linear forecast for the series, and thus plays little role in forecasting model nonlinearly. In order to model volatility, ARCH (or even GARCH) method comes into play.

Figure 13: Residuals of ARIMA model



Firstly, check if residual plot displays any cluster of volatility. Next, observe the squared residual plot. Check if the e_t^2 term has serial correlation, where $e_t = r_t - \mu_t$. If there are clusters of volatility, ARCH should be used to model the volatility of the series to reflect more recent changes and fluctuations in the series. Finally, ACF&PACF of squared residuals will help confirm if the residuals $\{e_t\}$(noise term) are not independent and can be predicted. As mentioned earlier, a strict white noise cannot be predicted either linearly or nonlinearly while general white noise might not be predicted linearly yet done so nonlinearly. If the residuals are strict white noise, they are independent with zero mean, normally distributed, and ACF&PACF of squared residuals displays no significant lags[5].

Hence, after we get the ARIMA model, there are three things to do:

[5]https://talksonmarkets.files.wordpress.com/2012/09/time-series-analysis-with-arima-e28093-arch013.pdf

First, use the residuals of the mean equation to test for ARCH effects. Ljung-Box Test for ϵ_t^2, or Lagrange Multiplier Test. Specify a volatility model if ARCH effects are statistically significant and perform a joint estimation of the mean and volatility equations. Check the fitted model carefully and refine it if necessary.

6.1 Test the Squared Residuals

There are two possible methods to test if the squared residuals in our ARIMA model follow a random walk.

a) Ljung-Box Test

```
	Box-Ljung test

data:  (arima$residuals)^2
X-squared = 53.359, df = 1, p-value = 2.779e-13
```

The p-value is less than 0.01, so we can reject the null hypothesis: the residuals square is white noise. I.e. There is no correlation in the residual sequence, there exists ARCH effect.

b) ArchTest

This function is in the package "FinTS", this package has been removed from the R CRAN library, but I time-traveled to the past and brought it back! Magic :)

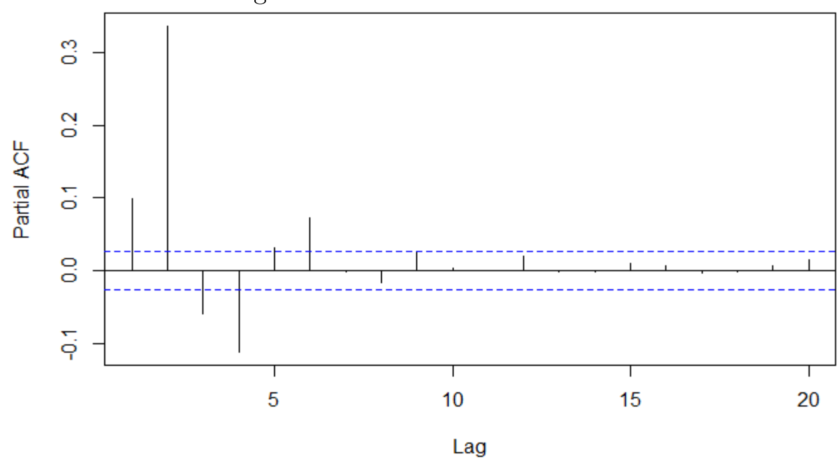
```
	ARCH LM-test; Null hypothesis: no ARCH effects

data:  arima$residuals
Chi-squared = 776.1, df = 12, p-value < 2.2e-16
```

The null hypothesis of Arch test is: The ARMA model doesn't have ARCH effect. Because the p-value is less than 0.01, so we can reject the null hypothesis. I.e. there exists ARCH effect.

After we determine the ARCH effect, we can determine the order of the ARCH model. To determine the order, we use Partial Autocorrelation of squared series. There are 4 significant spikes in PACF plot of the residual square sequance. It's an ARCH(4) model.

Figure 14: PACF of the Residuals



6.2 Fit ARCH Model

Let's say $\epsilon_t = \sigma_t a_t$ is return residual obtained after modelling a mean process or in other terms, $r_t = \mu_t + \sigma_t^2 a_t$. The random variable a_t is white noise. The variance of residual series σ_t^2 is modelled as;

$$\sigma_t^2 = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \cdots + \alpha_q \epsilon_{t-q}^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2$$

where $\alpha_0 > 0$, $\alpha_i \geq 0$ and $i > 0$.

And here is the summary we got from R:

```
    Call:
garch(x = arima$residuals, order = c(0, 4), trace = F)

Model:
GARCH(0,4)

Residuals:
      Min        1Q    Median        3Q       Max
-25.10840  -0.48254  -0.01523   0.45356   5.48880

Coefficient(s):
    Estimate  Std. Error  t value Pr(>|t|)
a0 2.456e-05   4.550e-07    53.97   <2e-16 ***
a1 9.903e-02   8.278e-03    11.96   <2e-16 ***
a2 1.637e-01   1.240e-02    13.20   <2e-16 ***
a3 1.174e-01   3.322e-03    35.34   <2e-16 ***
```

```
a4 1.337e-01   8.163e-03    16.38   <2e-16 ***
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Diagnostic Tests:
Jarque Bera Test

data:  Residuals
X-squared = 1450200, df = 2, p-value < 2.2e-16


Box-Ljung test

data:  Squared.Residuals
X-squared = 0.049877, df = 1, p-value = 0.8233
```

The p-value for all parameters are less than 0.05, indicating that they are statistically significant. The p-value of Box-Ljung test is way greater than 0.05, we can't reject the null hypothesis: the autocorrelation of residuals is differ from 0, i.e. it is white noise. The model thus adequately represents the residuals.

So the full ARCH(4) model is:

$$\sigma_t^2 = 2.456\times10^{-5}\epsilon_{t-1}^2+9.903\times10^{-2}\epsilon_{t-1}^2+1.637\times10^{-1}\epsilon_{t-2}^2+1.174\times10^{-1}\epsilon_{t-3}^2+1.337\times10^{-2}\epsilon_{t-4}^2$$

7 Weakness and Future Work

There are some parts that we think can be improved:

a) Our data plot has some problems in the x-axis because they are not dates. That is because our data has some missing values like they didn't collect the exchange rate data during weekends and holidays. Hence when plotting the time series, we can't use the usual way to set the frequency as 365 days. We came up with two possible solutions: First we can try to use weekly frequency to avoid the missing values in weekends, second we can try to use some methods like linear regression to impute the data.

b) In ARCH model part, the order from PACF is not clear, when we try different orders it may influence the parameters' significance. Besides, we didn't use the combined model ARIMA-ARCH to re-predict the exchange rate. We can definitely do that to see if the predictive performance could get better when adding ARCH effect. What's more, ARCH model has its weakness itself: The model assumes that positive and negative shocks have the same effects on volatility because it depends on the square of the previous shocks. In practice, it is well known that price of a financial asset responds differently to positive and negative shocks. The ARCH model is rather restrictive. The constraint becomes complicated for higher order ARCH models. In practice, it limits the ability of ARCH models with Gaussian innovations to capture excess kurtosis. The ARCH model does not provide any new insight for understanding the source of variations of a financial time series. It merely provides a mechanical way to describe the behavior of the conditional variance. It gives no indication about what causes such behavior to occur. ARCH models are likely to over-predict the volatility because they respond slowly to large isolated shocks to the return series. These chould be the reasons why we need GARCH model.

c) The most important thing is, we have tried a lot of different exchange rate data and ALL of them failed. After taking log and the first difference to make it stationary, the series became white noise, which can not be fitted by an ARIMA model because we can not determine the p and q order, it's ARIMA(0,1,0) model. We discussed a lot about this and read tons of papers that using ARIMA/GARCH model to model exchange rate but they did not include a lot details. We DON'T trust them. Before we saw the paper from Prof. Tornell we didn't realize how complex this topic is, maybe for such financial data like exchange rate and stock prices ARIMA model is not a good choice at all. Basically we think why our model works is more like a coincidence (only for this USD/EUR exchange rate data, even reverse it as EUR/USD will not work!), hence we could find another better way to analyze exchange rate time series.