Homework 3

Minxuan Wang ID:705060599

November 1, 2017

1 The Engineer

(a)

We can know that the population obeys 0-1 distribution, let p_0 be the probability of aircraft that have such wiring errors. First of all we can get the value of p_0 from sample:

$$p_0 = \frac{8}{1600} = 0.005$$

The 99% confidence interval on p is:

$$\left(\left(\bar{X} - u_{\frac{\alpha}{2}}\sqrt{\bar{X}(1-\bar{X})/n}, \bar{X} + u_{\frac{\alpha}{2}}\sqrt{\bar{X}(1-\bar{X})/n}\right)\right)$$

Here, $n = 1600$, $\bar{X} = p_0 = 0.005$, $\alpha = 0.01$, $u_{\frac{0.01}{2}} = 2.576$, so the confidence interval is:

$$\left(0.005 - 2.576\sqrt{\frac{0.005(1-0.005)}{1600}}, 0.005 + 2.576\sqrt{\frac{0.005(1-0.005)}{1600}}\right)$$

i.e., $(0.000457631, 0.009542369)$.

(b)

Based on the current assumption, we can get the difference between true value and our estimate is:

$$2.576 \times \sqrt{\frac{0.005 \times (1-0.005)}{n}} = 0.008$$

$n = 515.8279 \approx 516$.

(c)

If we wanted to be at least 99% confident that the sample proportion differs from the true proportion by at most 0.008 regardless of the true value of p, we should set p to 0.5:

$$2.576 \times \sqrt{\frac{0.5 \times (1 - 0.5)}{n}} = 0.008$$

$n = 25921$.

(d)

Basically, if we have preliminary information-p_0, we can get a same result with a smaller sample size, we don't need to collect a large amount of samples to make sure the sample proportion differs from the true proportion by at most 0.008. In other words, given a fixed number of samples, we can get a more accurate result when we have preliminary information.

2 Phoenix Road

(a)

The null hypothesis H_0 is $p = 0.3$, the alternative hypothesis H_1 is $p < 0.3$. The type I error is we refuse H_0 when H_0 is right. I.e. $p = 0.3$ but there is no person or just 1 person favors this proposal in the random sample of 10 residents. Let the probability of type I error occurs is α and the number of residents favor this proposal is n:

$$\begin{aligned}
\alpha &= P(n = 0|p = 0.3) + P(n = 1|p = 0.3) \\
&= C_{10}^0 \times 0.3^0 \times 0.7^{10} + C_{10}^1 \times 0.3^1 \times 0.7^9 \\
&= 0.1493083
\end{aligned}$$

(b)

The type II error is we accept H_0 when H_0 is false. I.e. $p = 0.2$ but there are more than 1 person favor this proposal in the random sample of 10 residents. Let the probability of type I error occurs is β:

$$\begin{aligned}
\beta &= P(n > 1|p = 0.2) \\
&= 1 - \left(C_{10}^0 \times 0.2^0 \times 0.8^{10} + C_{10}^1 \times 0.2^1 \times 0.8^9\right) \\
&= 0.6241904
\end{aligned}$$

(c)

The power of this procedure when the true proportion is $p = 0.2$ is:

$$1 - \beta = 1 - 0.6241904 = 0.3758096$$

3 Bivariate Normal Distribution

Similar to the previous question in Homework 2, we suppose $X_1 \sim N(\mu_1, \sigma^2)$, $X_2 \sim N(\mu_2, \tau^2)$, the correlation of X_1 and X_2 is ρ, we can easily prove that:

$$\begin{aligned}
f_{X_1|X_2}(x_1|x_2) &= \frac{f(x_1, x_2)}{f_{X_1}(x_2)} \\
&= \frac{\frac{1}{2\pi\sigma\tau\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left[\frac{(x_1-\mu_1)^2}{\sigma^2} - 2\rho\frac{(x_1-\mu_1)(x_2-\mu_2)}{\sigma\tau} + \frac{(x_2-\mu_2)^2}{\tau^2}\right]}}{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x_2-\mu_2)^2}{2\tau^2}}} \\
&= \frac{1}{2\sqrt{2\pi}\sigma\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left(\frac{x_1-\mu_1}{\sigma} - \rho\frac{x_2-\mu_2}{\tau}\right)^2} \\
&= \frac{1}{2\sqrt{2\pi}\sigma\sqrt{1-\rho^2}} e^{-\frac{1}{2\sigma^2(1-\rho^2)}\left[x_1-\mu_1-\frac{\sigma}{\tau}\rho(x_2-\mu_2)\right]^2}
\end{aligned}$$

So, when $X_2 = x_2$, $x_1 \sim N\left(\mu_1 + \frac{\sigma}{\tau}\rho(x_2 - \mu_2), \sigma^2(1-\rho^2)\right)$
$E(X_1|X_2 = x_2) = \mu_1 + \frac{\sigma}{\tau}\rho(x_2 - \mu_2)$
$E(X_1|X_2) = \mu_1 + \frac{\sigma}{\tau}\rho(X_2 - \mu_2)$, $D(X_1|X_2) = \sigma^2(1-\rho^2)$
Symmetrically, when $X_1 = x_1$, $x_2 \sim N\left(\mu_2 + \frac{\tau}{\sigma}\rho(x_1 - \mu_1), \tau^2(1-\rho^2)\right)$
$E(X_2|X_1 = x_1) = \mu_2 + \frac{\tau}{\sigma}\rho(x_1 - \mu_1)$
$E(X_2|X_1) = \mu_2 + \frac{\tau}{\sigma}\rho(X_1 - \mu_1)$, $D(X_2|X_1) = \tau^2(1-\rho^2)$
Now we get:

$$\begin{cases}
\mu_1 - \frac{\sigma}{\tau}\rho\mu_2 = 3.7 \\
\frac{\sigma}{\tau}\rho = -0.15 \\
\mu_2 - \frac{\tau}{\sigma}\rho\mu_1 = 0.4 \\
\frac{\tau}{\sigma}\rho = -0.6 \\
\tau^2(1-\rho^2) = 3.64
\end{cases}$$

$$\Downarrow$$

$$\begin{cases} \mu_1 = 4 \\ \mu_2 = -2 \\ \sigma = 1 \\ \tau = 2 \\ \rho = -0.3 \end{cases}$$

So the mean and the variance of X_1 is (4,1), the mean and the variance of X_2 is (-2,4), and the correlation of X_1 and X_2 is -0.3.

4 Poisson Distribution

The Population X fits a Poisson distribution with a mean of λ, the probability distribution of X is:

$$P(X = k) = \frac{\lambda^k}{k!} e^{-\lambda}$$

The important thing is that Poisson distribution is a discrete distribution, and the value of k must be non-negative integers, which means that when we pick up a random sample of four observations to let the mean of these four observations $\bar{X} < \frac{1}{2}$, the only two possible combinations are: all of the four observations are 0/only 1 of the four observations is 1 and others are 0. So:

$$\begin{aligned} P\left(\bar{X} < \frac{1}{2}\right) &= [P(X = 0)]^4 + C_4^1 \times P(X = 1) \times [P(X = 0)]^3 \\ &= \left(\frac{\lambda^0}{0!} e^{-\lambda}\right)^4 + C_4^1 \times \frac{\lambda^1}{1!} e^{-\lambda} \times \left(\frac{\lambda^0}{0!} e^{-\lambda}\right)^3 \\ &= e^{-4\lambda} + 4\lambda e^{-\lambda} \times e^{-3\lambda} \\ &= (4\lambda + 1) e^{-4\lambda} \end{aligned}$$

5 Bernoulli Distribution

(a)

A Bernoulli distribution obeys that:

$$f(x) = p^x (1-p)^{1-x} = \begin{cases} p & if\ x = 1 \\ 1-p & if\ x = 0 \\ 0 & otherwise \end{cases}$$

$Y = \sum_{i}^{9} X_i$, We can compute the following results:

$$P(Y = k | p = 0.4) = C_9^k \times 0.4^k \times 0.6^{9-k}$$

Y=k	$P(Y=k\|p=0.4)$
0	0.0100777
1	0.06046618
2	0.1612431
3	0.2508227
4	0.2508227
5	0.1672151
6	0.07431782
7	0.02123366
8	0.003538944
9	0.000262144

Because $P(Y \leq c_1|p=0.4) + P(Y \geq c_2|p=0.4) \to 1$, so $P(Y \leq c_1|p=0.4) \leq 1$ and $P(Y \geq c_2|p=0.4) \leq 1$. So we can know the possible value of c_1 is (0,1) and c_2 is (6,7,8,9). When $c_2 = 6$, $P(Y \geq c_2|p=0.4) = 0.09935257$, which is closest to 1. But meanwhile $P(Y \leq c_1|p=0.4)$ can't be greater than 0.00064743, so $c_1 < 0$. We got:

$$c_1 < 0, \quad c_2 = 6$$

(b)

The size α of the test δ means that we reject H_0 when H_0 is true. So it is the same as

$$P(Y \leq c_1|p=0.4) + P(Y \geq c_2|p=0.4)$$

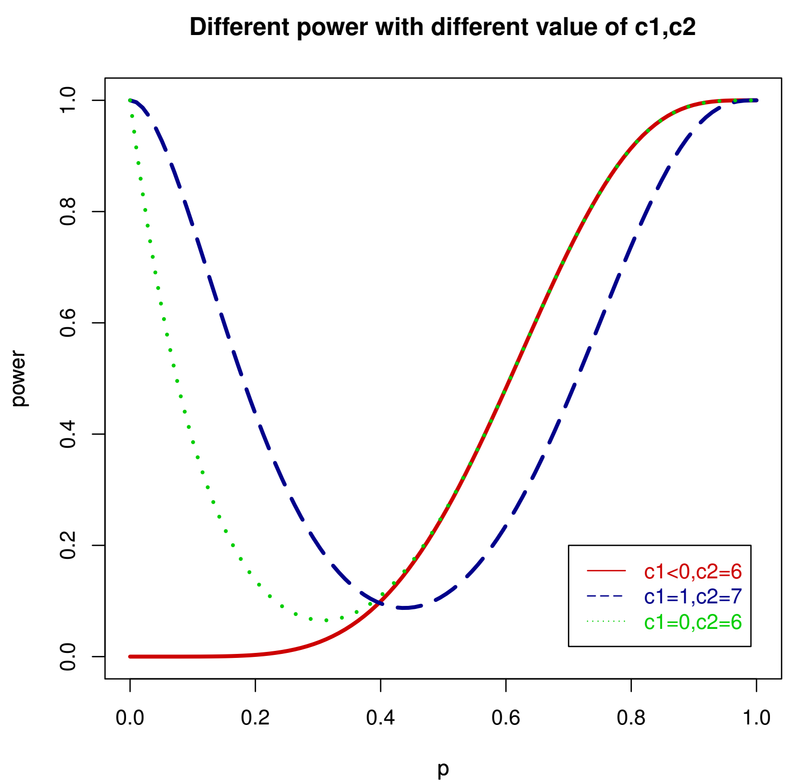
In (a) we got the result 0.09935257 when $c_1 < 0$ and $c_2 = 6$, so in this case the size of test δ_c is 0.09935257. When c_1, c_2 have different values this result would change.

(c)

$\beta = P(c_1 < Y < c_2|p \neq 0.4)$ i.e. we accept H_0 when it is false, and the power function is $1-\beta$. We can use R to plot three different power function graph with different c_1 and c_2:

```
p<-seq(0,1,length=100)
# c1<0, c2=6
# getting 5 or fewer successes leads to not rejecting H0.
beta1<-pbinom(5,9,p)
round(cbind(p,beta1),3)
power<-1-pbinom(5,9,p)
# c1=1, c2=7
# getting successes between 1 and 6 leads to not rejecting H0.
```

```
beta2<-pbinom(6,9,p)-pbinom(1,9,p)
round(cbind(p,beta2),3)
power2<-1-(pbinom(6,9,p)-pbinom(1,9,p))
# c1=0, c2=6
# getting successes between 0 and 5 leads to not rejecting H0.
beta3<-pbinom(5,9,p)-pbinom(0,9,p)
round(cbind(p,beta3),3)
power3<-1-(pbinom(5,9,p)-pbinom(0,9,p))
# plot the graph
plot(p,power,type='l',col='red3',lty=1,lwd=3)
lines(p,power2,type='l',col='blue4',lty=5,lwd=3)
lines(p,power3,type='l',col='green3',lty=3,lwd=3)
legend(0.7,0.2,c("c1<0,c2=6","c1=1,c2=7","c1=0,c2=6"),
col=c("red3","blue4","green3"),
text.col=c("red3","blue4","green3"),lty=c(1,5,3))
title("Different power with different value of c1,c2",
xlab="p",ylab="power")
```



6 American Medical Association

(a)

First of all we should compute the mean and standard deviation of sample:

$$\bar{X} = \frac{97.8 + 97.2 + ... + 98.9 + 99.0}{25} = 98.264$$

$S = 0.4820788 \approx 0.48$.

H_0: $\mu_0 = 98.6$

Then construct the pivot:

$$T = \frac{\bar{X} - \mu}{S/\sqrt{n}} \sim t(24)$$

Under the H_0 hypothesis:

$$T_0 = \frac{\bar{X} - \mu_0}{S/\sqrt{n}} \sim t(24)$$

So that

$$P\{|T_0| > t_{0.025}(24)\} = P\left\{\left|\frac{\bar{X} - \mu_0}{S/\sqrt{25}}\right| > t_{0.025}(24)\right\} = 0.05$$

where $t_{0.025}(24) = 2.064$.
The rejection region is:

$$C = \left\{(x_1, x_2, ..., x_n : \left|\frac{\bar{X} - 98.6}{s/\sqrt{25}}\right| > 2.064\right\}$$

The observed sample $(x_1, x_2, ..., x_n)$ satisfies:

$$|t_0| = \left|\frac{\bar{X} - 98.6}{s/\sqrt{25}}\right| = \left|\frac{98.264 - 98.6}{0.48/5}\right| = 3.5 > 2.064$$

The observed sample is under the rejection region, we reject the H_0: $\mu = 98.6$
Compute the p-value:

$$p = P\left\{|T_0| > |t_0|\right\} = P\left\{|T_0| > 3.5\right\} = 2P\{T_0 > 3.5\}$$

From the Upper critical values of Student's t distribution table,
We can see that when the freedom degree is 24,

$$0.0005 < P\{T_0 > 3.5\} < 0.001$$

so the p-value is between $(2 \times 0.0005, 2 \times 0.001) \Rightarrow (0.001, 0.002)$.

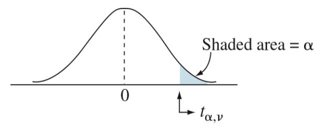


TABLE 2

Percentage points of Student's t distribution

df/α =	.40	.25	.10	.05	.025	.01	.005	.001	.0005
1	0.325	1.000	3.078	6.314	12.706	31.821	63.657	318.309	636.619
2	0.289	0.816	1.886	2.920	4.303	6.965	9.925	22.327	31.599
3	0.277	0.765	1.638	2.353	3.182	4.541	5.841	10.215	12.924
4	0.271	0.741	1.533	2.132	2.776	3.747	4.604	7.173	8.610
5	0.267	0.727	1.476	2.015	2.571	3.365	4.032	5.893	6.869
6	0.265	0.718	1.440	1.943	2.447	3.143	3.707	5.208	5.959
7	0.263	0.711	1.415	1.895	2.365	2.998	3.499	4.785	5.408
8	0.262	0.706	1.397	1.860	2.306	2.896	3.355	4.501	5.041
9	0.261	0.703	1.383	1.833	2.262	2.821	3.250	4.297	4.781
10	0.260	0.700	1.372	1.812	2.228	2.764	3.169	4.144	4.587
11	0.260	0.697	1.363	1.796	2.201	2.718	3.106	4.025	4.437
12	0.259	0.695	1.356	1.782	2.179	2.681	3.055	3.930	4.318
13	0.259	0.694	1.350	1.771	2.160	2.650	3.012	3.852	4.221
14	0.258	0.692	1.345	1.761	2.145	2.624	2.977	3.787	4.140
15	0.258	0.691	1.341	1.753	2.131	2.602	2.947	3.733	4.073
16	0.258	0.690	1.337	1.746	2.120	2.583	2.921	3.686	4.015
17	0.257	0.689	1.333	1.740	2.110	2.567	2.898	3.646	3.965
18	0.257	0.688	1.330	1.734	2.101	2.552	2.878	3.610	3.922
19	0.257	0.688	1.328	1.729	2.093	2.539	2.861	3.579	3.883
20	0.257	0.687	1.325	1.725	2.086	2.528	2.845	3.552	3.850
21	0.257	0.686	1.323	1.721	2.080	2.518	2.831	3.527	3.819
22	0.256	0.686	1.321	1.717	2.074	2.508	2.819	3.505	3.792
23	0.256	0.685	1.319	1.714	2.069	2.500	2.807	3.485	3.768
24	0.256	0.685	1.318	1.711	2.064	2.492	2.797	3.467	3.745
25	0.256	0.684	1.316	1.708	2.060	2.485	2.787	3.450	3.725

Using R:

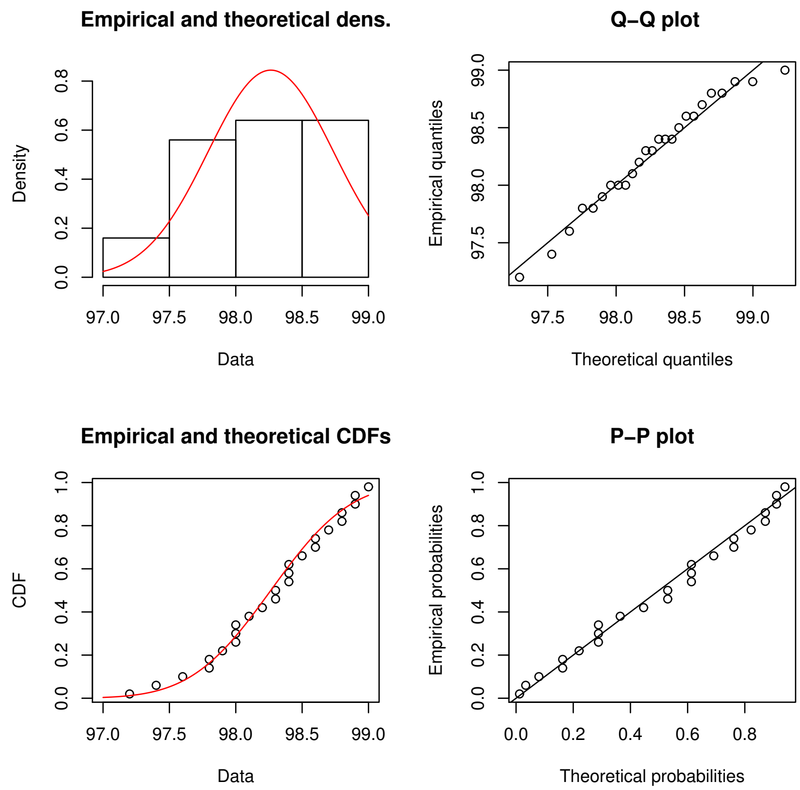
```
t.test(c,mu=98.6)

        One Sample t-test

data:  c
t = -3.4849, df = 24, p-value = 0.001912
alternative hypothesis: true mean is not equal to 98.6
95 percent confidence interval:
 98.06501 98.46299
sample estimates:
mean of x
   98.264
```

So, the p-value is 0.001912.

(b)

Using R:

```
fitc<-fitdist(c,"norm")
plot(fitc)
```



Easily we can see data frame c(the body temperatures for 25 female subjects) fits normal distribution well.

(c)

First we should compute the effect size d, then it can be solved by R(using "pwr" package).

$$d = \frac{|\mu_1 - \mu_2|}{s} = \frac{98.6 - 98}{0.48} = 1.25$$

```
install.packages('pwr')
library(pwr)
pwr.t.test(n=25,d=1.25,sig.level=0.05,type=c("one.sample"))
```

```
     One-sample t test power calculation

              n = 25
              d = 1.25
      sig.level = 0.05
          power = 0.9999716
    alternative = two.sided
```

The power is 0.9999716.

(d)

We can still use the same function, because we can get the last unknown number when we have the other three values(n, d, sig.level and power). This time the effect size d is

$$d = \frac{|\mu_1 - \mu_2|}{s} = \frac{98.6 - 98.2}{0.48} = 0.83333$$

```
pwr.t.test(d=0.83333,sig.level=0.05,power=0.9,type=c("one.sample"))
```

```
     One-sample t test power calculation

              n = 17.16719
              d = 0.83333
      sig.level = 0.05
          power = 0.9
    alternative = two.sided
```

We can easily find that n should at least be 18 to make sure the power reaches 0.9.

(e)

We can know that $n = 25$, $\sigma = 0.48$, $\alpha = 0.05$, checking the table we can know $u_{0.025} = 2.064$. So,

$$u_{\frac{\alpha}{2}} \times \frac{\sigma}{\sqrt{n}} = 2.064 \times \frac{0.48}{5} = 0.198$$

So the 95% confidence interval of μ is

$$(\bar{X} - 0.198, \bar{X} + 0.198) = (98.264 - 0.198, 98.264 + 0.198) = (98.066, 98.462)$$

$\mu_0 = 98.6$ doesn't belong to this range, so we reject H_0: $\mu_0 = 98.6$.

7 US Airways

(a)

```
setwd("D:/Econ 403A/Homework 3/Homework 3-20171027")
time<-read.csv("Flights.csv")
t<-sapply(time[,c(1)],as.numeric)
t
  [1] 10.1 12.9 13.0 15.3 14.4 15.6 10.4 15.5 16.0 10.7 11.5
 [12]  9.4 14.2 14.6  9.0 14.8  9.0  9.7 13.6 16.6  7.4 16.4
 [23] 15.6 15.0 10.7 13.1 11.7 11.2 17.0 15.7 14.6 15.5  5.5
 [34] 12.0 10.7 28.0 19.9 20.4 12.9  6.5 13.7 13.2 20.2  8.7
 [45] 10.1 18.9 12.6 22.3 17.2 13.5 11.7 18.1 16.3 12.9 16.9
 [56]  4.9 16.6 16.7 22.9 12.3 20.1 14.3 16.4 12.7 11.2 11.4
 [67] 13.6 11.9  5.2 15.4 19.3 16.3 12.8 17.6 13.3 16.0  5.3
 [78] 10.8 13.9 12.7 20.9 17.6 15.3 11.8 16.0 15.5 21.7 13.7
 [89] 11.2 12.6 14.5  9.7 12.4 13.3 16.2 13.8  9.0  8.7 13.9
[100] 21.9 22.2  9.2 17.0 11.9 14.3 10.8 17.7 15.9 13.5 11.9
[111] 13.1 16.0  5.2 14.6 15.4 10.7 13.9 10.9 12.8 13.3 15.9
[122] 10.8 15.6  4.8 17.3 16.1 14.7 14.5 15.8 20.8 14.2 15.1
[133] 14.6 15.0 15.1 18.2 11.7 18.6  9.1 16.4 10.9 14.8 14.3
[144] 15.7 16.8 16.1  7.8 12.6 14.5 16.2
mean(t)
[1] 14
```

The sample mean number of minutes required to schedule an award flight by telephone is 14.

(b)

```
t.test(t)

        One Sample t-test

data:  t
t = 44.681, df = 149, p-value < 2.2e-16
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 13.38085 14.61915
sample estimates:
mean of x
       14
```

So the 95% confidence interval for the population mean time to schedule an award flight by telephone is (13.38085,14.61915).

(c)

$$7.5 \times 60 \div 14 = 32.14286$$

So one ticket agent is expected to handle 32.14 award flights a day.

(d)

It can help the US Airways know exactly how many people they need to accomplish such kind of works, reduce the unnecessary waste of human resources(reduce costs).

8 US Airways

(a)

Because the MLE of the mean of the normal population is unbiased estimate, the MLE of μ is $\hat{\mu} = \bar{X}$. The proof process is as follows:
The density of population X is

$$f(x;\mu,\sigma^2) = \frac{1}{\sqrt{x\pi}\sigma} exp\left\{-\frac{(x-\mu)^2}{2\sigma}\right\}$$

. The likelihood function

$$\begin{aligned} L(\mu,\sigma^2) &= \prod_{i=1}^{n} f(x;\mu,\sigma) \\ &= (2\pi\sigma^2)^{-\frac{n}{2}} exp\left\{-\frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2\right\} \end{aligned}$$

So $\ln L(\mu,\sigma^2) = -\frac{n}{2} ln(2\pi\sigma^2) - \frac{1}{2\sigma^2}\sum_{i=1}^{n}(x_i-\mu)^2$

FOC:

$$\begin{cases} \frac{1}{\sigma^2}\sum_{i=1}^{n}(x_i-\mu) = 0 \\ -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\sum_{i=1}^{n}(x_i-\mu)^2 = 0 \end{cases}$$

$\Rightarrow \mu_0 = \frac{1}{n}\sum_{i=1}^{n} x_i = \bar{x}$, $\sigma = \frac{1}{n}\sum_{i=1}^{n}(x_i-\bar{x})^2$.
So the point estimate of the population mean price is the mean of samples.

$$\bar{X}_{06} = 225896.7, \bar{X}_{09} = 170992.5$$

$$\bar{X}_d = (\bar{X}_{06} - \bar{X}_{09}) = 54904.2$$

(b)

X_{06} and X_{09} are two independent normal populations with standard deviation $S_{06} = 55207.36$, $S_{09} = 44958.26$. And the joint sample deviation is

$$\begin{aligned} S^2 &= \frac{n_{06}-1}{n_{06}+n_{09}-2}S_{06}^2 + \frac{n_{09}-1}{n_{06}+n_{09}-2}S_{09}^2 \\ &= \frac{29}{68} \times 55207.36^2 + \frac{39}{68} \times 44958.26^2 \\ &= 2459063028 \\ S &= 49588.94 \end{aligned}$$

Now we can get the pivot

$$T = \frac{\bar{X}_{06} - \bar{X}_{09} - \mu}{S\sqrt{\frac{1}{n_{06}} + \frac{1}{n_{09}}}} \sim t_{0.01}(n_{06} + n_{09} - 2)$$

$$T_0 = \frac{54904.2}{11976.86} = 4.58419 \sim t_{0.01}(68)$$

So the asymptotic confidence interval of μ is:

$$(54904.2 - t_{0.005}(68) \times 11976.86,\ 54904.2 + t_{0.005}(68) \times 11976.86)$$

$$\Downarrow$$

$$(23165.52,\ 86642.88)$$

(c)

I think the resale prices of existing homes have declined from 2006 to 2009. Because the 99% confidence interval estimate of the difference between the resale prices of houses in 2006 and 2009 is (23165.52, 86642.88), in 99% confidence interval there is a huge gap between the prices of 2006 and 2009.

9 Financial Advisers

We use F-test to test the ratio of variances of two independent Normal populations. The null and alternative hypothesis are:

$$\begin{aligned} H_0 &: \ \sigma_1^2 = \sigma_2^2 \\ H_1 &: \ \sigma_1^2 \neq \sigma_2^2 \end{aligned}$$

From the sample:

$$F_0 = \frac{S_1^2}{S_2^2} = \frac{587^2}{489^2} = 1.440982$$

The number of observations in two samples are $n_1 = 16$, $n_2 = 10$, so the degree of freedom is 15 and 9. So we should find $F_{0.05}(15, 9)$ next, if $F_0 < F_{0.05}(15, 9)$, we can't reject null hypothesis.

F **检验临界值表**（α=0.05(b)）

自由度	自变量数目（m）									显著性水平：α=0.05
(df)	**11**	**12**	**13**	**14**	**15**	**16**	**17**	**18**	**19**	**20**
n-m-1										
1	242.983	243.906	244.690	245.364	245.950	246.464	246.918	247.323	247.686	248.013
2	19.405	19.413	19.419	19.424	19.429	19.433	19.437	19.440	19.443	19.446
3	8.763	8.745	8.729	8.715	8.703	8.692	8.683	8.675	8.667	8.660
4	5.936	5.912	5.891	5.873	5.858	5.844	5.832	5.821	5.811	5.803
5	4.704	4.678	4.655	4.636	4.619	4.604	4.590	4.579	4.568	4.558
6	4.027	4.000	3.976	3.956	3.938	3.922	3.908	3.896	3.884	3.874
7	3.603	3.575	3.550	3.529	3.511	3.494	3.480	3.467	3.455	3.445
8	3.313	3.284	3.259	3.237	3.218	3.202	3.187	3.173	3.161	3.150
9	3.102	3.073	3.048	3.025	3.006	2.989	2.974	2.960	2.948	2.936
10	2.943	2.913	2.887	2.865	2.845	2.828	2.812	2.798	2.785	2.774
11	2.818	2.788	2.761	2.739	2.719	2.701	2.685	2.671	2.658	2.646
12	2.717	2.687	2.660	2.637	2.617	2.599	2.583	2.568	2.555	2.544

From the table we get $F_{0.05}(15,9)$=3.006, so that $F_0 = 1.441 < F_{0.05}(15,9)$. It's not in the reject region, so that we can't reject $H_0: \ \sigma_1^2 = \sigma_2^2$. I.e. The variability in the amount of funds managed by advisers from the two firms can be regarded as the same.

10 Financial Advisers

(a)

The Type I error is we reject $H_0: \ \lambda = 1$ when H_0 is true. That is:

$$P(y \geq 3.2|\lambda = 1)$$

We can compute $P(y < 3.2|\lambda = 1)$ instead.

$$\begin{aligned} P(y < 3.2|\lambda = 1) &= \int_0^{3.2} e^{-y} dy \\ &= -e^{-y}|_0^{3.2} \\ &= 1 - e^{-3.2} \end{aligned}$$

So $P(y \geq 3.2|\lambda = 1) = 1 - (1 - e^{-3.2}) = e^{-3.2} \approx 0.04076$.

(b)

The Type II error is we accept $H_0: \ \lambda = 1$ when H_0 is false. That is:

$$P(y < 3.2|\lambda = \frac{4}{3})$$

$$\begin{aligned} P(y < 3.2|\lambda = \frac{4}{3}) &= \int_0^{3.2} \frac{3}{4} e^{-\frac{3}{4}y} dy \\ &= -e^{-\frac{3}{4}y}|_0^{3.2} \\ &= 1 - e^{-2.4} \\ &\approx 0.90928 \end{aligned}$$

(c)

To plot the marginal density curves, using R:

```
a<-function(x) exp(-x)
b<-function(x) 0.75*exp(-0.75*x)

plot(a,xlim=c(-3,3),type="l",col="red3",lwd=2)
plot(b,xlim=c(-3,3),type="l",col="blue3",lwd=2,add = TRUE)
```