Econ 403A: Homework #1

Minxuan(Shaun) Wang 705060599

Question #1:

In this question, there are two contradiction to the paradox.

i. If the barber does not shave himself, he will be a person belongs to the set: “people who don’t shave themselves”, such that he should shave himself.

ii. If the barber shaves himself, he will be a person belongs to the set: “people shave themselves” such that the barber should not shave himself.

In mathematical term, $S = \{x|x \notin S\}$,.

If s doesn’t belongs to S, it satisfies $x \notin S$, $\Rightarrow s \in S$;

If s belongs to S, it doesn’t satisfy $x \notin S$, $\Rightarrow s \notin S$

Question #2:

Based on the Bayes formula in conditional probability problem, we can get

$$\mathbb{P}(G|T) = \frac{\mathbb{P}(T|G)\mathbb{P}(G)}{\mathbb{P}(T)}$$

To get the result of $\mathbb{P}(G|T) = \mathbb{P}(T|G)$, it must satisfy the condition $\mathbb{P}(G) = \mathbb{P}(T)$.

Question #3:

In this case it equals to the question “How many nonnegative integer solutions are there of the equation $a + b + c + d = 100$?” and it has a constraint of $1 \le a \le 10$, $0 \le b$, $2 \le c$, $20 \le d \le 30$. We can reassign some new variables to help us handle this problem.

Let $x = a - 1$, so that $0 \le x \le 9$; let $y = c - 2$, so that $y \ge 0$; let $z = d - 20$, so that $0 \le z \le 10$.

First of all, we only consider the case each variable is greater than zero. Now we changed the problem to “How many nonnegative integer solutions are there of the equation $x + b + y + z = 100 - 23 = 77$”. Then we get there are $C(77 + 4 - 1, 4 - 1) = C(80, 3)$ solutions.

Then, we consider the case carriable $x \le 9$ and $z \le 10$, to get this solution, we can count how many solutions are there such that $x \ge 10$ and $z \ge 11$. Similarly, we set two new variable we call them $x_1 = x - 10$ and $z_1 = z - 11$, so that $x_1 \ge 0$, $z_1 \ge 0$. We find the solutions of the tow equations following:

$x_1 + b + y + z = 77 - 10 = 67$, and there are $C(67 + 4 - 1, 4 - 1) = C(70, 3)$ solutions;

$x + b + y + z_1 = 77 - 11 = 66$, and there are $C(66 + 4 - 1, 4 - 1) = C(69, 3)$ solutions.

Finally, these two conditions have a coincident part:

$x_1 + b + y + z_1 = 77 - 10 - 11 = 56$, and there are $C(56 + 4 - 1, 4 - 1) = C(59, 3)$ solutions.

So the answer is there are $C(80, 3) - C(70, 3) - C(69, 3) + C(56, 3)$ solutions of the equation $a + b + c + d = 100$.

Question #4:

For these problems we'd better think from the opposite side.

i. Let the probability that a person get at least 1 six when 6 dice are rolled is $P(A)$, so $P(A')$ is the probability that he gets no six, $P(A') = \left(\frac{5}{6}\right)^6 \Rightarrow P(A) = 1 - \left(\frac{5}{6}\right)^6$.

ii. Let the probability that a person get at least 1 six when 12 dices are rolled is $P(B)$, so $P(B')$ is the probability that he gets no six and only 1 six, $P(B') = \left(\frac{5}{6}\right)^{12} + C(12,1) \times \frac{1}{6} \times \left(\frac{5}{6}\right)^{11}$, $\Rightarrow$ $P(B) = 1 - \left(\frac{5}{6}\right)^{12} - C(12,1) \times \frac{1}{6} \times \left(\frac{5}{6}\right)^{11}$.

iii. Let the probability that a person get at least 1 six when 18 dices are rolled is $P(C)$, so $P(C')$ is the probability that he gets no six, only 1 six and 2 six, $P(C') = \left(\frac{5}{6}\right)^{18} + C(18,1) \times \frac{1}{6} \times \left(\frac{5}{6}\right)^{17} + C(18,2) \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^{16} \Rightarrow P(C) = 1 - \left(\frac{5}{6}\right)^{18} - C(18,1) \times \frac{1}{6} \times \left(\frac{5}{6}\right)^{17} - C(18,2) \times \left(\frac{1}{6}\right)^2 \times \left(\frac{5}{6}\right)^{16}$.

Question #5:

I think there is no definite answer to this question, the only solution is they both choose the landmarks of New York to see if they can meet each other, such as Time Square and Train Station...

Here I read two references:

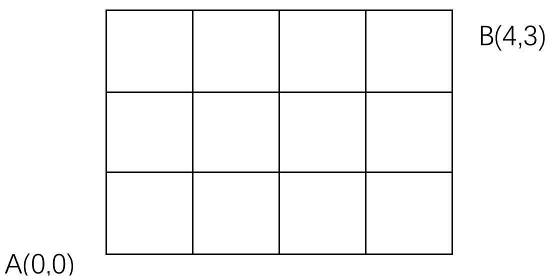
Quo vadis? (Mosteller, 1965);

Symmetric Rendezvous Search (Richard Weber Talk to the Adams Society, 21 February, 2007 and to LSE, 8 March, 2007) Statistical Laboratory, Centre for Mathematical Sciences, University of Cambridge.

In the second one they discussed a detailed explanation of this problem.

Question #6:

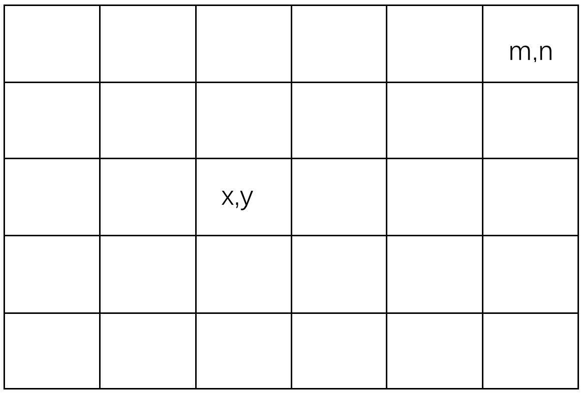
(a).



To get B from A, there are 7 roads to go through in total. And we should choose 4 lateral or 3 longitudinal roads from this set. So there are $C(7,4) = C(7,3)$ shortest routes to get point B.

(b). Similarly, when there are $m + n$ roads to go through in total, we have $C(m + n, m) = C(m + n, n)$ routes to choose.

(c).

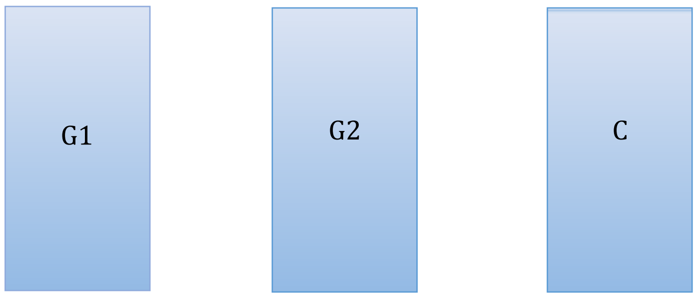


First we get the point (x, y), we have $C(x+y, x) = C(x+y, y)$ routes, then from (x, y) to (m, n), we have $C(m-x+n-y, m-x) = C(m-x+n-y, n-y)$ routes. So totally we have $C(x+y, x) \times C(m-x+n-y, m-x)$ choices.

Question #7:

It is to my advantage to switch my choice.

Suppose that 2 doors witch have goats behind are G1 and G2; the door has a car behind is C.



i. I pick G1 and it becomes the door No. 1, the probability of this event is $1/3$, then the host opens the door G2.

If I change (to door C), I'll get a car. The probability is 1;

If I don't change, I will not get a car. The probability is 0.

So in this situation, the probability of the event "I switch my choice and get a car" is $1/3$, the probability of the event "I don't switch my choice and get a car" is 0.

ii. I pick G2 and it becomes the door No. 1, the probability of this event is $1/3$, then the host opens the door G1.

If I change (to door C), I'll get a car. The probability (of getting a car) is 1;

If I don't change, I will not get a car. The probability (of getting a car) is 0.

So in this situation, the probability of the event "I switch my choice and get a car" is $1/3$, the probability of the event "I don't switch my choice and get a car" is 0.

iii. I pick C and it becomes the door No. 1, the probability of this event is $1/3$, then the host opens the door G2 or G1.

If I change (to door G2 or G1), I will not get a car. The probability (of getting a car) is 0;

If I don't change, I will get a car. The probability (of getting a car) is 1.

So in this situation, the probability of the event "I switch my choice and get a car" is 0; the probability of the event "I don't switch my choice and get a car" is $1/3$.

So ,the probability of the event “I switch my choice and get a car” is $1/3 + 1/3 = 2/3$; the probability of the event “I don’t switch my choice and get a car” is $1/3$. So I’d better change my choice.

Question #8:

I picked out the data from 10/1/2014 to 10/5/2017, because there are no S&P 500 data at weekends and the Effective Federal Funds rate always remains the same, there are many missing values. When collecting the Effective Federal Funds rate I only pick out the dates that this data changes, for the S&P 500 data, I choose the adjusted close value as the daily value and use the open value take place the missing value at weekends. Here is the result I get:

		Interest Rates	
S&P 500		increase	decrease
	increase	28	24
	decrease	59	34

We get $\mathrm{P}\,(\mathrm{SP}\ 500\ \downarrow |\, \mathrm{I}\ \uparrow) = \frac{59}{145} \approx 41\%$.

PS: I have seen the answer you posted on the course website, it was totally different. I handled these data in Excel and counted the number by myself... I have no idea how to solve this problem in R ☹)

By increasing the federal funds rate, the federal government attempts to reduce the supply of money available for purchasing or doing things, by making money more expensive to obtain. it does not directly affect the stock market itself but it becomes more expensive for banks to borrow money from the federal government, it increases the cost of financial institutions, so they will shift this part of burden onto the customers, who are in the stock market. So the customers will have less discretionary money. In other words, when the banks make borrowing more expensive, companies might not borrow as much and will pay higher rates of interest on their loans. The investors see this condition they will reduce their investment on this company and the stock price will decrease.

But why there is a bias between the hypothesis and reality? I think the first reason is that the impact of interest rate on stock market is too indirect, and there are many other factors that may have an impact on stock market; the second reason is the market's response is delayed, it usually takes a long time for any increase or decrease in interest rates to be felt in a widespread economic way.

Reference: *How Do Interest Rates Affect the Stock Market? (Investopedia, February 15, 2017)*

Question #9:

i. We should think from the opposite side, what if there is no one has the same birthday?

First we choose one day from 365 days, then get rid of this date and pick another one... for 25 times. Finally we get the probability that none of them has the same birthday $\mathrm{P}(A')$:

$$\mathrm{P}(A') = \frac{C(365,1) \times C(364,1) \times \ldots \times C(341,1)}{365^{25}}$$

So, the probability that two or more of them have the same birthday $\mathrm{P}(\mathrm{A})$:

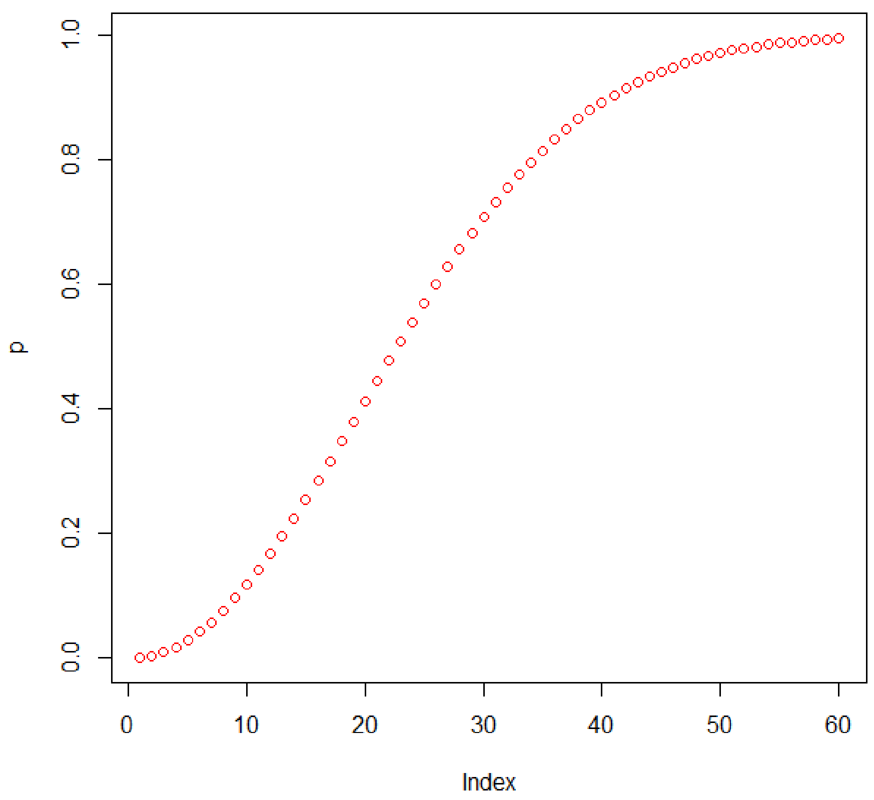
$$\mathrm{P}(\mathrm{A}) = 1 - \mathrm{P}(A') = 1 - \frac{C(365,1) \times C(364,1) \times \ldots \times C(341,1)}{365^{25}}$$

ii. R code:

```
(a) > p <- prod(365:341)/365^25
    > q <- 1-p
    > q
  [1] 0.5686997
```

```
(b) > p <- 60
    > for (n in 1:60) {
    + q <- 1 - (0:(n - 1))/365
    + p[n] <- 1 - prod(q) }
    > plot(p, col="red")
```

Then we get the graph (I print it in red color):




```
iii. > p <- 60
  > for (n in 1:60) {
  + q <- 1 - (0:(n - 1))/365
  + p[n] <- 1 - prod(q) }
  > plot(p, col="red")
  > print(p)

[1] 0.000000000 0.002739726 0.008204166 0.016355912 0.027135574 0.040462484
```

```
[7] 0.056235703 0.074335292 0.094623834 0.116948178 0.141141378 0.167024789
[13] 0.194410275 0.223102512 0.252901320 0.283604005 0.315007665 0.346911418
[19] 0.379118526 0.411438384 0.443688335 0.475695308 0.507297234 0.538344258
[25] 0.568699704 0.598240820 0.626859282 0.654461472 0.680968537 0.706316243
[31] 0.730454634 0.753347528 0.774971854 0.795316865 0.814383239 0.832182106
[37] 0.848734008 0.864067821 0.878219664 0.891231810 0.903151611 0.914030472
[43] 0.923922856 0.932885369 0.940975899 0.948252843 0.954774403 0.960597973
[49] 0.965779609 0.970373580 0.974431993 0.978004509 0.981138113 0.983876963
[55] 0.986262289 0.988332355 0.990122459 0.991664979 0.992989448 0.994122661
```

I print p out to see each value of them, we can see that when $n = 23$, $p \approx 0.5$, so the minimum number of people in a room such that the probability of a match is greater than or equal to 50% is 23.

Question #10:

R code:

```
> n <- 25;    m <- 100000;    x <- numeric(m)
> for (i in 1:m)    {
+ b <- sample(1:365, n, repl=T)
+ x[i] <- n - length(unique(b))    }
> cut <- (0:(max(x) + 1)) - 0.5
> hist(x, breaks=cut, freq=F, col=8)
```