

**Economics 403A: Homework 3**  
**Fall 2017, UCLA**  
**Instructor: Dr. Rojas**

**Due Date: Nov 2, 2017**

All problems require detailed worked out solutions to receive full credit. All problems are worth the same.

1. An article in The Engineer ("Redesign for Suspect Wiring," June 1990) reported the results of an investigation into wiring errors on commercial transport aircraft that may produce faulty information to the flight crew. Such a wiring error may have been responsible for the crash of a British Midland Airways aircraft in January 1989 by causing the pilot to shut down the wrong engine. Of 1600 randomly selected aircraft, eight were found to have wiring errors that could display incorrect information to the flight crew.
  - (a) Find a 99% confidence interval on the proportion of aircraft that have such wiring errors.
  - (b) Suppose we use the information in this example to provide a preliminary estimate of  $p$ . How large a sample would be required to produce an estimate of  $p$  that we are 99% confident differs from the true value by at most 0.008?
  - (c) Suppose we did not have a preliminary estimate of  $p$ . How large a sample would be required if we wanted to be at least 99% confident that the sample proportion differs from the true proportion by at most 0.008 regardless of the true value of  $p$ ?
  - (d) Comment on the usefulness of preliminary information in computing the needed sample size.
2. The proportion of residents in Phoenix favoring the building of toll roads to complete the freeway system is believed to be  $p = 0.3$ . If a random sample of 10 residents shows that 1 or fewer favor this proposal, we will conclude that  $p < 0.3$ .
  - (a) Find the probability of type I error if the true proportion is  $p = 0.3$ .
  - (b) Find the probability of committing a type II error with this procedure if  $p = 0.2$ .
  - (c) What is the power of this procedure if the true proportion is  $p = 0.2$ ?
3. Suppose that  $X_1$  and  $X_2$  have bivariate normal distribution for which  $E[X_1|X_2] = 3.7 - 0.15X_2$ ,  $E[X_2|X_1] = 0.4 - 0.6X_1$ , and  $var[X_2|X_1] = 3.64$ . Find the mean and the variance of  $X_1$ , the mean and the variance of  $X_2$ , and the correlation of  $X_1$  and  $X_2$ .
4. Suppose that a random sample of four observations is drawn from the Poisson distribution with mean  $\lambda$ , and let  $\bar{X}$  denote the sample mean. Show that

$$P\left(\bar{X} < \frac{1}{2}\right) = (4\lambda + 1)e^{-4\lambda}$$

5. Assume that  $X_1, \dots, X_9$  are i.i.d. having Bernoulli distribution with parameter  $p$ . Suppose that we wish to test the hypotheses

$$H_0 : p = 0.4$$

$$H_1 : p \neq 0.4$$

Let  $Y = \sum_{i=1}^9 X_i$ .

- (a) Find  $c_1$  and  $c_2$  such that  $P(Y \leq c_1 | p = 0.4) + P(Y \geq c_2 | p = 0.4)$  is as close as possible to 0.1 without being larger than 0.1.
  - (b) Let  $\delta$  be the test that rejects  $H_0$  if either  $Y \leq c_1$  or  $Y \geq c_2$ . What is the size of the test  $\delta_c$ ?
  - (c) Draw a graph of the power function  $\delta_c$ .
6. A 1992 article in the Journal of the *American Medical Association* (“A Critical Appraisal of 98.6 Degrees F, the Upper Limit of the Normal Body Temperature, and Other Legacies of Carl Reinhold August Wunderlich”) reported body temperature, gender, and heart rate for a number of subjects. The body temperatures for 25 female subjects follow: 97.8, 97.2, 97.4, 97.6, 97.8, 97.9, 98.0, 98.0, 98.0, 98.1, 98.2, 98.3, 98.3, 98.4, 98.4, 98.4, 98.5, 98.6, 98.6, 98.7, 98.8, 98.8, 98.9, 98.9, and 99.0.
- (a) Test the hypothesis  $H_0 : \mu = 98.6$  versus  $H_1 : \mu \neq 98.6$ , using  $\alpha = 0.05$ . Find the  $P$ -value.
  - (b) Check the assumption that female body temperature is normally distributed.
  - (c) Compute the power of the test if the true mean female body temperature is as low as 98.0.
  - (d) What sample size would be required to detect a true mean female body temperature as low as 98.2 if we wanted the power of the test to be at least 0.9?
  - (e) Explain how the question in part (a) could be answered by constructing a two-sided confidence interval on the mean female body temperature.
7. US Airways conducted a number of studies that indicated a substantial savings could be obtained by encouraging Dividend Miles frequent flyer customers to redeem miles and schedule award flights online (*US Airways Attache*, February 2003). One study collected data on the amount of time required to redeem miles and schedule an award flight over the telephone. A sample showing the time in minutes required for each of 150 award flights scheduled by telephone is contained in the data set `Flights.xlsx`. Use R to help answer the following questions.
- (a) What is the sample mean number of minutes required to schedule an award flight by telephone?
  - (b) What is the 95% confidence interval for the population mean time to schedule an award flight by telephone?

- (c) Assume a telephone ticket agent works 7.5 hours per day. How many award flights can one ticket agent be expected to handle a day?
  - (d) Discuss why this information supported US Airways plans to use an online system to reduce costs.
8. Home values tend to increase over time under normal conditions, but the recession of 2008 and 2009 has reportedly caused the sales price of existing homes to fall nationwide (*Businessweek*, March 9, 2009). You would like to see if the data support this conclusion. The file `HomePrices.xlsx` contains data on 30 existing home sales in 2006 and 40 existing home sales in 2009.
- (a) Provide a point estimate of the difference between the population mean prices for the two years.
  - (b) Develop a 99% confidence interval estimate of the difference between the resale prices of houses in 2006 and 2009.
  - (c) Would you feel justified in concluding that resale prices of existing homes have declined from 2006 to 2009? Why or why not?
9. Data were collected on the top 1000 financial advisers by Barron's (*Barron's*, February 9, 2009). Merrill Lynch had 239 people on the list and Morgan Stanley had 121 people on the list. A sample of 16 of the Merrill Lynch advisers and 10 of the Morgan Stanley advisers showed that the advisers managed many very large accounts with a large variance in the total amount of funds managed. The standard deviation of the amount managed by the Merrill Lynch advisers was  $s_1 = \$587$  million. The standard deviation of the amount managed by the Morgan Stanley advisers was  $s_2 = \$489$  million. Conduct a hypothesis test at  $\alpha = 0.10$  to determine if there is a significant difference in the population variances for the amounts managed by the two companies. What is your conclusion about the variability in the amount of funds managed by advisers from the two firms?
10. Suppose a sample of size 1 is taken from the pdf  $f_Y(y) = (1/\lambda)e^{-y/\lambda}$ ,  $y > 0$ , for the purpose of testing

$$H_0 : \lambda = 1$$

$$H_1 : \lambda > 1$$

The null hypothesis will be rejected if  $y \geq 3.20$ .

- (a) Calculate the probability of committing a Type I error.
- (b) Calculate the probability of committing a Type II error when  $\lambda = 4/3$ .
- (c) Draw a diagram that shows the  $\alpha$  and  $\beta$  calculated in parts (a) and (b).