

Discrete Mathematics

2023-24 Semester 1 Assignment 02

Due Time: 5:30pm, 14th December 2023

There are SEVEN questions in this assignment. Answer ALL the questions.

1. (15 points) Drones of eight different colors (red, orange, yellow, green, blue, indigo, violet and black) are chosen for a performance. There are twenty drones for each color.
 - (a) How many drones do we need to choose to guarantee that seven drones are of the same color? (Steps required.)
 - (b) How many drones do we need to choose to guarantee that we have 5 red drones, 4 green drones and 3 blue drones? (Steps required.)
2. (10 points) $A = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $B = \{u, v, w, x, y, z\}$. How many onto functions can be formed from A to B ? (Steps required.)
3. (15 points) A pair of fair dice is rolled. Let $X((i, j))$ be the random variable, where i and j are the numbers that appear on the first die and the second die, respectively. $X((i, j))$ is defined as follows:

$$X((i, j)) = \begin{cases} 2i + j, & \text{if } i < j; \\ i^2, & \text{if } i = j; \\ 3i + j, & \text{if } j < i. \end{cases}$$

- (a) What is the distribution of the random variable X ? (Steps required.)
 - (b) What is the expected value X ? Correct the answer to 4 decimal places. (Steps required.)
 - (c) What is the variance of X ? Correct the answer to 4 decimal places. (Steps required.)
4. (15 points) Prove by Mathematical Induction, or disprove, that $4^p + 7^p \leq 11^p$ for all natural numbers $p \geq 1$.
5. (15 points) Prove by Mathematical Induction, or disprove, that any natural number a can be expressed as $\sum_{j=0}^{n_a} c_{a,j} 2^j$, where n_a is a non-negative integer and $c_{a,j} \in \{0, 1\}$.

6. (15 points) (a) $60 \in S$.
 If $x \in S$, then $x - 15 \in S$.
 Based on the above recursive definition, describe S in at most two sentences.
- (b) $\lambda \in T$.
 If $y \in T$, then “00” $y \in T$ and y “11” $\in T$.
 Write down any five non-empty strings in T .
 Use at most two sentences to describe the elements of T .
7. (15 points) Consider the poset $(\{4, 5, 8, 10, 20, 24, 25, 30, 32, 40\}, |)$.
- Write down all the maximal element(s). (Write down 'Nil' if there isn't any maximal element.)
 - Write down all the minimal element(s). (Write down 'Nil' if there isn't any minimal element.)
 - Write down all the upper bound(s) of $\{8, 10\}$. (Write down 'Nil' if there isn't any upper bound.)
 - Write down all the lower bound(s) of $\{20, 40\}$. (Write down 'Nil' if there isn't any lower bound.)

Reminders:

- No mark will be given to late assignments.
- This is an individual assignment. You are encouraged to discuss with your classmates. But, please ensure that you will use your own words to answer the questions.
- Do NOT post/broadcast your answers on any shared platform. Zero mark will be given to those students who violate this regulation.
- Collusion and plagiarism are serious offences and may result in disciplinary action. A mark of zero will be given for the piece of coursework and; in addition, the final grade of the course may be affected (for example, it may be lowered from D to F). Please carefully read the section about Collusion and Plagiarism in your Student Handbook.
- Assignment (hard copy) should be put into the assignment box before the deadline AND soft copy should be submitted via the SOUL platform before the deadline.