UTS IM FINAL EXAM

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Class: 31/10/2020

Student: _____

Part 1: Portfolio Theory and Portfolio Choices

Topic1: Expected return and variance of a portfolio（考点）

$$E(r_p) = \sum_{k=1}^{N} w_k E(r_k) \quad \text{and} \quad r_p = \sum_{k=1}^{N} w_k r_k$$

Note: weights must sum to 1

- **For two assets,**

$$E(r_{A,B}) = w_A E(r_A) + w_B E(r_B)$$

$$\sigma_{A,B}{}^2 = w_A^2\ \sigma_A^2 + w_B^2\sigma_B^2 + 2w_A w_B \sigma_{A,B}$$

$$\rho_{AB} = \frac{\mathrm{Cov}(r_A, r_B)}{\sigma_A \sigma_B} = \frac{\sigma_{AB}}{\sigma_A \sigma_B} \text{ so } \sigma_{AB} = \rho_{AB}\sigma_A\sigma_B$$

$$\boxed{\sigma_q^2 = w_A^2\sigma_A^2 + w_B{}^2\sigma_B^2 + 2w_A w_B \rho_{AB}\sigma_A\sigma_B}$$

Topic 2: Portfolios of One risky and one risk free asset portfolio（考点）

- $E(r_q) = (1 - w_p)r_f + w_p E(r_p)$
- Risk free asset has no risk

$$\sigma_{rf}^2 = 0 \text{ and } \sigma_{rfp} = 0$$

Therefore

$$\sigma_q^2 = w_p{}^2\sigma_p^2 \quad \text{and } \sigma_q = w_p\sigma_p$$

- **Weight on risky asset**

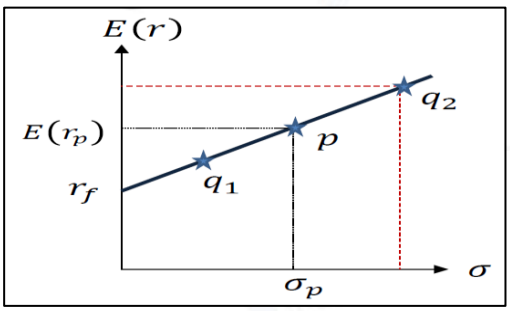
$$\boxed{\sigma_q = w_p\sigma_p \text{ so } w_p = \frac{\sigma_q}{\sigma_p}}$$

$$\boxed{E(r_q) = r_f + \frac{\sigma_q}{\sigma_p}[E(r_p) - r_f]} \qquad \boxed{\frac{E(r_q) - r_f}{\sigma_q} = \frac{E(r_p) - r_f}{\sigma_p}}$$

- ✓ These equations show the relationship between the expected return on the portfolio and its risk.（收益和风险的关系）
- ✓ Portfolios that are mixtures of the risk-free asset (lending/borrowing) and a risky asset will plot on this straight line in E(r) and σ space.
  - This is because the risk-free asset does not make any contribution to the variance of portfolio returns （因为组合风险与风险资产的风险，即 SD 是

线性关系，组合的期望收益与无风险资产和风险资产的收益为线性关系，所以组合在收益风险空间呈现的是一条直线）

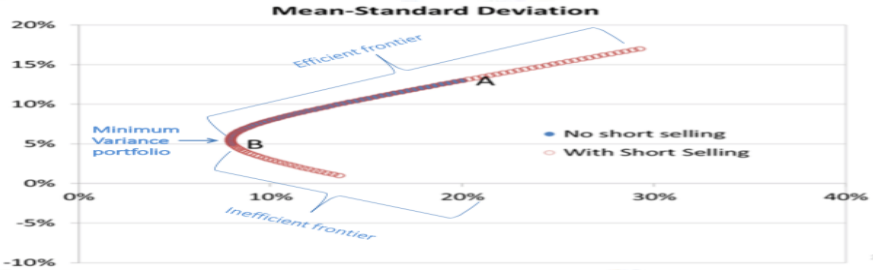
- When portfolio "q" is made up of the risk-free asset and the optimal risky asset, **the straight line is known as the capital allocation line. CAL（资本配置线）**CAL 表达的是当组合的两种 asset 权重不同时的组合的期望与风险的图像。



- "p" is the risky asset and the risk-free asset is cash
- In q_1 the weight on "p" is $0 < w_p < 1$. Portfolio q_1 invests in cash (lending at r_f) and "p".
- In q_2 the weight on "p" is $w_p > 1$. Portfolio q_2 borrows cash (margin loan at r_f) to invest more than 100% in "p".

Topic 3: Efficient and Inefficient frontiers

- The efficient frontier is the set of portfolios that have the highest return for a level of risk, or the lowest risk for a level of return （资产的有效前沿就是当同一水平风险时，收益最高；或者同一水平收益时，风险最低）-理性人的选择
- There is a unique portfolio that have the smallest variance. It is called the Minimum Variance Portfolio **(MVP)（最小方差组合）**
- All the portfolios you can construct can be divided by the MVP into efficient and inefficient portfolios.



Inefficient frontier 就是我们不选择的组合，因为相同的 risk 情况下，我们为何不选择上面 efficient frontier 的大 return 呢？

Topic 4: Utility Functions（考点）

- Utility function to represent risk/return preferences. (效用函数表示投资者的风险/收益偏好)

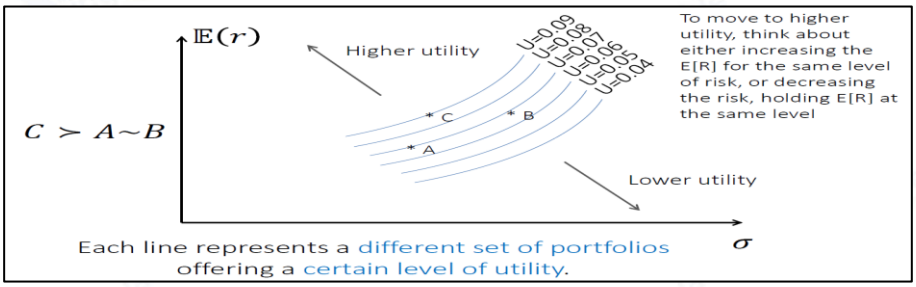
$$U_i(E(r),\sigma) = \underbrace{E(r)}_{reward} - \underbrace{\frac{1}{2}A_i\sigma^2}_{risk}$$

The higher the number the more 'utility' the choice has. U 数值越大则给投资者带来的

满足度越大，但是边际效用递减

- ✓ High risk averse investors will choose a less risky portfolio.
- ✓ Low risk averse investors will choose a riskier portfolio.
- ✓ Risk neutral investors will choose the portfolio with the highest return (A=0)

➢ **Indifference curve:** same expected utility on the same curve（同一条无差异曲线上的投资组合给投资者带来的效用是一样的）



Topic 5: Optimal Capital Allocation (最优资本配置)（考点）

For the risk-free asset: $U(r_f) = E(r_f) = r_f$

For the risky asset: $U(r_p) = \mathbb{E}(r_p) - \frac{1}{2}A_i\sigma_p{}^2$

➢ Find the optimal weight on risky asset portfolio

$$E(r_q) = (1 - w_p)r_f + w_p E(r_p)$$
$$U_i(q) = E(r_q) - \frac{1}{2}A_i\sigma_q^2$$

✓ Step 1:

$$U_i(q) = (1 - w_p)r_f + w_p E(r_p) - \frac{1}{2}A_i w_p{}^2\sigma_p^2$$

✓ Step 2: investors wants to maximize utility, so take the derivative of utility（求导）with respect to *wwpp*, set it equal to zero and then solve for w_p

$$w_p{}^* = \frac{E(r_p) - r_f}{A_i\sigma_p^2}$$

Practice:

Investor *A* and investor *B* are mean-variance optimisers possessing quadratic utility.
Consider the following investments:

Portfolio	Expected return, E(r)	Standard deviation, s
P1	4%	0%
P2	8%	5%
P3	20%	40%

a) Investor *A* has a risk aversion coefficient of 4. How would investor *A* rank the three portfolios above? Show all working.

b) Using the correlation matrix below determine the excepted return and variance for the equally weighted portfolio

Correlation	P1	P2	P3
P1	1	0	0
P2	0	1	0.5
P3	0	0.5	1

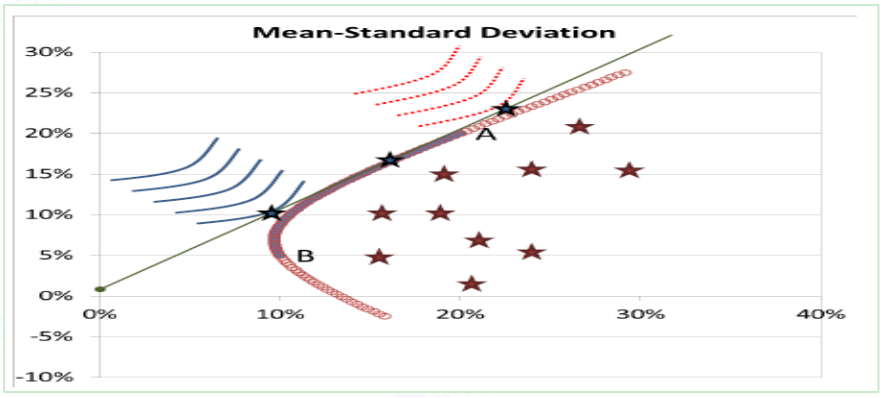
100

c) Suppose investor *A* (from part (a)) is allocating between cash and the evenly weighted portfolio above. What is her optimal allocation? Assume the risk-free asset has a return of 4% per annum.

d) Determine the reward to volatility ratio for the overall (optimal) portfolio.

Part 2: Portfolio Theory and CAPM

Topic1: Optimal Portfolio (考点)



- **Optimal risky asset portfolio (tangent portfolio)–the risky asset portfolio tangent to the capital allocation line (CAL).**(最优的风险资产组合，和资本配置线相切)

✓ The slope of the CAL**, Sharpe ratio**,$S_p = \frac{E[r_p]-r_f}{\sigma_p}$

✓ Optimal risky asset portfolio has highest Sharpe ratio= = slope of the CAL that is tangent to the efficient frontier.（最优的风险资产组合有最高的夏普比率）

- **Optimal combined portfolio –this is the portfolio where an indifference curve is tangent to the CAL.**

✓ All investors will choose the same optimal risky portfolio. However, their optimal combined portfolios may be different as they have different levels of risk aversion(A) (different indifference curve).（所有的投资者都选择相同的最优风险资产，但因为他们无差异曲线（效用函数）不同，和资本配置线相切的位置不同，即有对风险资产和无风险资产配置的权重不同，所以会有不同的最优组合）

Topic2: CAPM（考点加重点）

❖ *CML*（资本市场线）

$$E[r_p] = r_f + \sigma_p \frac{E[r_m]-r_f}{\sigma_m}$$

✓ Market portfolio that includes all available risky assets

✓ When Tangency Portfolio is Market portfolio, the capital allocation line (CAL) is now called the **capital market line (CML)**（当切点组合为市场组合时，资本配置线就是资本市场线-这用来推导 CPAM）

❖ *CAPM*

The **goal** of CAPM is to establish a fair rate of return for a Security given its systematic risk（CAPM 模型的目的是在给定某个证券的市场风险下，求出它的公允收益）

- **Assumptions:**

✓ Markets are frictionless（市场没有摩擦）

- ✓ All investors are mean-variance optimisers.（所有的投资者都是均值方差最优化）
- ✓ Investors can differ in their degree of risk aversion.（投资者的风险偏好可不同）
- ✓ There are no informational asymmetries（投资者可以没有信息不对称）

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f)$$

The CAPM equation says that any asset will return you the risk-free rate plus an adjustment for market risk. This adjustment is given by beta times the market risk premium.（CAPM 模型表示任意资产都有无风险收益以及承担市场风险的收益补偿）

- ➢ **Risk Decomposition**
  - The absolute risk (The total risk (variance) of the asset): σ_i^2 (绝对风险)
  - The relative risk: (相对风险) $\beta_i = \frac{\sigma_{i,m}}{\sigma_m^2}$

- ➢ **Beta:** （β 衡量系统性风险或者说市场风险）

Beta provides a measure of systematic risk (or market risk) for an asset. Its meanings:

- ✓ The return on assets that have a higher(lower) beta will be more(less) sensitive to movements in the market portfolio's returns.（β 越高对市场收益的变动越敏感）
- ✓ Assets with returns more(less) correlated with the market portfolio returns will have a higher(lower) beta.（当资产收益和市场收益相关性越大则 β 越高）
  - When the market rises, assets with higher(lower) betas will achieve higher(lower) expected returns.（当市场上扬，高 β 则会有更高的资产收益，反之）

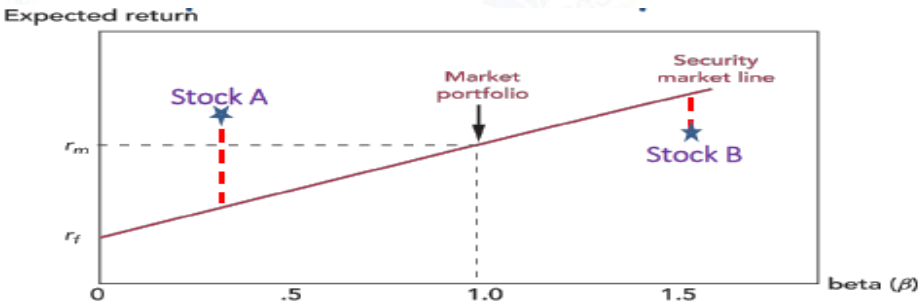
- ➢ Portfolio based on CAPM
- ✓ Return: $r_P = w_1 r_1 + w_2 r_2$
- ✓ Beta: $\boldsymbol{\beta_P = \frac{cov(r_P, r_M)}{\sigma_M^2} = w_1\beta_1 + w_2\beta_2}$ （组合的 β 是资产 β 的加权平均）

- **A testable consequence of the CAPM:(对 CAPM 检测，市场组合应该位于有效前沿上)**
  - ✓ The market portfolio should lie on the efficient frontier generated by all available assets

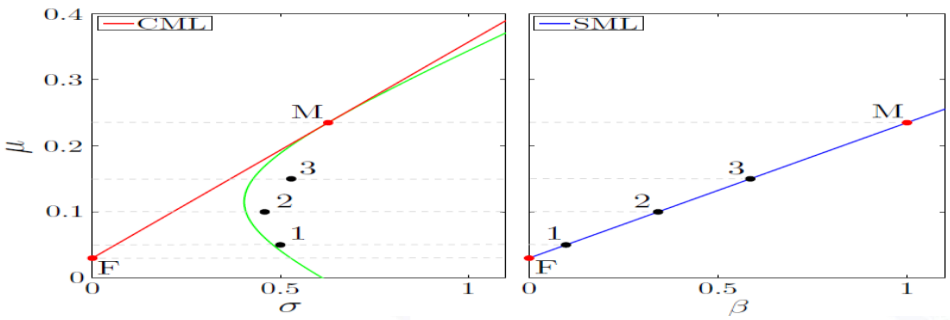
❖ **SML**

- The security market line (SML) is the line drawn in expected return is the line drawn in expected return –beta space using the CAPM equation.

$$E[r_i] = r_f + \beta_i\left(E[r_m] - r_f\right)$$



- ✓ Asset on the SML is priced correctly
- ✓ **Asset A above the SML is underpriced.** The actual price is lower than expected. Investors will buy Asset A.
- ✓ **Asset B below the SML is overpriced.** The actual price is higher than expected. Investor will sell Asset B
- The differences between CML and SML



- ✓ The CML describes the various combinations of risky market portfolio and the risk-free asset which yield the maximum expected return for any level of standard deviation.（资本市场线描述无风险资产和风险资产的各种组合，最大化期望收益）
- ✓ The SML provides the required rate of return necessary to compensate investors for risk of an investment as measured by beta.（证券市场线描述当承担市场风险时，对风险进行补偿所要求的收益）

- **According to the CAPM, in equilibrium all assets/portfolios lie on the SML**.(

  根据 CAPM，均衡状态下，所有的资产或者组合都应该位于 SML 上-SML 的 test)

Practice:

Consider the following information is provided for a stock market:

	$E(R_j)$	σ_j	ρ_{jM}
Asset 1	0.18	0.6	0.65
Asset 2	0.08	0.2	0.45
Asset 3	0.02	0.2	-0.45
Market Portfolio	0.1	.15	1

The risk-free interest rate is 5%.

a. Define and interpret the beta coefficient, b_j, in the context of the CAPM.

b. Calculate the beta coefficients from the above information and hence obtain the SML for this market.

c. Discuss how you would compare the "riskiness" of the assets in the context of the above information.

d. You are informed that a fourth asset is available. It has an average return of 17%, standard deviation of return, $\sigma_4 = 0.45$, and correlation of return with the market portfolio, $\rho_{4M} = 0.6$. What inferences, if any, would you draw from this information?

Part3: Factor Model and APT（考点加重点）

Topic1: SIM

- A single factor model assumes all asset returns depend on the systematic risk of one common factor. (When the common factor is the market index it is called the single index model (SIM).（单一指数模型是指当所有资产的收益都取决于市场指数）
- ✓ **Uses realized excess returns not realized returns.** （用的是实现的超额收益而不是实现的收益）

$$r_{it} - r_{ft} = \alpha_i + \beta_{im}(r_{mt} - r_{ft}) + \varepsilon_{it}$$

General Equation：

$$R_{it} = \alpha_i + \beta_{im} R_{mt} + \varepsilon_{it}$$

- ✓ The CAPM predicts that ai=0 （CAPM 预测 alpha 为 0，当 alpha 大于 0，则资产收益会高于由 CAPM 预测出来的收益，反之）
- ✓ αi is called "alpha". **A positive alpha (αi> 0)** would imply an asset's return is higher than predicted by CAPM. **A negative alpha (αi< 0)** implies the asset's return is lower.

➢ Variance and covariance (Risk) of SIM

$$\sigma_i^2 = \beta_{im}^2\sigma_m^2 + \sigma_{\varepsilon i}^2$$
$$Cov(R_i, R_j) = \sigma_{ij} = \beta_{im}\beta_{jm}\sigma_m^2$$
$$\beta_{im}=Cov(R_i, R_m)/\sigma_m^2=\sigma_{im}/\sigma_m^2$$

➢ **Risk Decomposition**

$$\sigma_{it}^2 = \beta_{im}^2\sigma_m^2 + \sigma_{\varepsilon i}^2$$

Total risk (σ_i^2)= systematic risk $(\beta_{im}^2\sigma_m^2)$ + asset-specific risk $(\sigma_{\varepsilon i}^2)$

R_i^2 = Systematic risk/total risk

$$R_i^2 = \frac{\beta_{im}^2\sigma_m^2}{\sigma_i^2} = \rho_{im}^2$$

where $\rho_{im} = Corr(r_{it}, r_{mt})$.

Practice:

Consider two stocks with the following data under the Single Index Model. Assume $R_m = 8\%, \sigma_m = 5\%, r_f = 3.5\%$.

Stock	Alpha	Beta	Firm Specific Variance	Weight in Portfolio
A	5%	1.2	20%	50%
B	2%	0.8	5%	50%

a. Write down the general equation for stock returns under the Single Index model.
b. Calculate the expected return of the portfolio described in the table.
c. Calculate the covariance between the returns on this portfolio and the returns on the market.
d. State and calculate the two components that make up the variance of the returns of stock A.
e. Determine the correlation between the returns of A and the returns of B. What key assumption is being made in the calculation?

Topic 2: Arbitrage pricing theory (APT) （套利定价理论）

- ✓ Assets are mispriced if they do not have a zero alpha.(当资产 alpha 不等于 0 则为错误定价)
- ✓ Arbitrage trading will exploit mispricing by buying an underpriced asset financed by short selling an overpriced asset.（套利交易是通过用卖掉高估证券的钱来买被低估的证券）
- ➢ Example of arbitrage trading

$$R_A = 3\% + 0.7R_m + \varepsilon_A$$
$$R_B = -2\% + 1.2R_m + \varepsilon_B$$

- ✓ Asset A has a positive alpha. **A's excess return is higher than predicted by the CAPM so it is under-price**d. It would be profitable to buy A, this would push up A's price and decrease A's return until it lay on the SML.
- ✓ Asset B has a negative alpha. **B's excess return is lower than predicted by the CAPM so it is overpriced**. It would be profitable to sell B, this would push down B's price and increase B's return until it lay on the SML.
- ✓ This mispricing will be exploited by buying A and selling B to create portfolio**,** **"*q*", ** that is free from systematic risk.（套利组合没有市场风险-只考虑市场风险因子）
- ✓ Portfolio "q" will have no systematic risk if is beta is zero. So the sum of the weighted asset's betas must equal zero.（没有市场风险则 beta 等于 0）

$$\beta_q = w_A\beta_A + w_B\beta_B = 0 \quad w_A + w_B = 1$$

$$w_A = \frac{-\beta_B}{\beta_A - \beta_B} \quad w_B = \frac{\beta_A}{\beta_A - \beta_B}$$

- ✓ **Excess return of "q"**

$$R_q = w_A\alpha_A + w_B\alpha_B$$

Practice:

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Suppose that the risk-free rate is 5% and a well-diversified portfolio, *A*, with beta of 1.3 has an alpha of 3% and another well-diversified portfolio, *B*, with beta of 0.8 has an alpha of 1%.

Does an arbitrage opportunity exist? If so, explain how to exploit this opportunity.

- As the portfolio alpha's are not zero, it ay be possible to set up a zero-beta portfolio has a certain rate of return that differs from the risk-free rate

- Form a new portfolio Q with the following proportion using:

$$w_A = \frac{-\beta_B}{\beta_A - \beta_B} \quad \& \quad w_B = \frac{-\beta_A}{\beta_B - \beta_A}$$

$$w_A = -1.6 \quad \& \quad w_B = 2.6$$

- These weights add up to 1.0 and result in a portfolio with beta = −1.6 × 1.3 + 2.6 × 0.8 = 0. The alpha of the portfolio is: −1.6 × 3% + 2.6 × 1% = −2.2%. This means that the riskless portfolio will earn a rate of return less than the risk-free rate. Hence to complete the arbitrage we need to short-sell the portfolio and invest at the risk-free rate.

Part 4: Market Efficiency

Topic1: Forms of market efficiency:

- ✓ **Weak form:** asset prices reflect all historical market trading information (i.e. past prices, returns and trading volume).
- ✓ **Semi-strong form:** asset prices reflect all publicly available information(i.e. earnings forecasts, quality of management, balance sheet composition as well as past prices, returns and trading volume)
- ✓ **Strong form:** asset prices reflect all publicly available information as well as information available only to company insiders.

Topic2: Behaviour Finance

Information Processing	Behavioural Bias
1. Forecasting error – Higher weight is placed on the most recent experiences.	1. Framing – Decisions are affected by how the choices are presented.
2. Overconfidence – People think they can do better than the average person.	2. Mental Accounting – Decisions are separated rather than being considered together.
3. Conservatism – People are slow to change their beliefs.	3. Regret Avoidance – People regret bad outcomes so only make conventional decisions.
4. Sample size neglect and Representativeness – People use findings from a small sample to represent a broad population.	4. Prospect Theory – Higher wealth is regarded as providing higher utility but at a diminishing rate. The utility depends on whether you start from a gain or loss, rather than levels of wealth.

总结

Correlation coefficient:

$$\rho_{AB} = \frac{Cov(r_A, r_B)}{\sigma_A \sigma_B}$$

Quadratic utility:

$$U_i(E[r_p], \sigma_p) = E[r_p] - \frac{1}{2} A_i \sigma_p^2$$

Optimal allocation:

$$w_p^* = \frac{E(r_p) - r_f}{A_i \sigma_p^2}$$

Portfolio:

- $r_p = w_A r_A + w_B r_B$
- $E(r_p) = w_A E(r_A) + w_B E(r_B)$
- $\sigma_p^2 = w_A^2 \sigma_A^2 + w_B^2 \sigma_B^2 + 2 w_A w_B \sigma_A \sigma_B \rho_{AB}$

CAPM:

$$E[r_i] = r_f + \beta_i (E[r_m] - r_f)$$

CML:

$$E[r_p] = r_f + \sigma_p \frac{E[r_m] - r_f}{\sigma_m}$$

APT:

$$E[\bar{r}_p] = r_f + \sum_{i=1}^{N} \beta_{p,k} (E[\bar{f}_k] - r_f)$$

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Single Index Model:

- $R_i = \alpha_i + \beta_i R_m + \varepsilon_i$
- $E[\epsilon_i] = 0;\ Cov(\varepsilon_i, R_m) = 0\ and\ Cov(\epsilon_i, \epsilon_j) = 0\ for\ i \neq j$
- $E(R_i) = \alpha_i + \beta_i E(R_m)$
- $\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_{\varepsilon,i}^2 \qquad Cov(R_i, R_j) = \beta_i \beta_j \sigma_m^2$

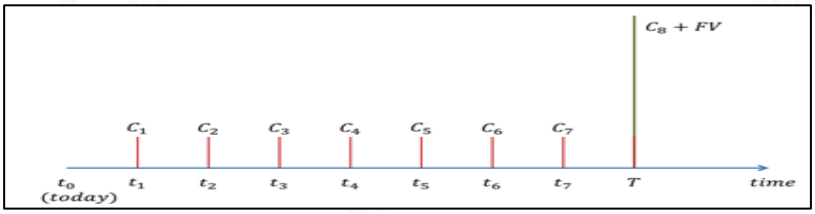
- $R - square = \frac{\beta_i^2 \sigma_m^2}{\sigma_i^2}$

Part 5: Bond Pricing and Term Structure of Yields

Topic 1: Bond

- **A bond** is a promise of a fixed (or formulaic) series of cash-flows at fixed times in the future (债券是在未来固定时间偿付固定现金流的承诺)
  - ✓ For the issuer it is a promise to pay cash flows
  - ✓ For the holder it is a right to receive cash flows
  - ✓ The interim payments are known as coupons.（利息）
  - ✓ The final payment usually involves a coupon and the bond's face value (known as its notional value or par value)（面值或者名义值）
- **A zero-coupon bond (ZCB)** is a bond that only pays a face value when it matures and pays no coupons.（零息债券在到期日偿付面值，期间不支付利息）
  - ✓ When issued these bonds sell at a discount to their face value and at maturity pay their face value.（折价发行，最后收面值来获益）
- **A consol bond** is perpetual and only pays coupons and no face value.（永续债权只付利息不偿还面值）

Topic 2: Bond Pricing （必考点加重点）



- **A ZCB only pays face value**. The price of a $1 ZCB (The discount factor) will be

$$P_{\$}(t) = \frac{\$1}{(1+y(t))^t} = (1+y(t))^{-t}$$

- **Coupon Paying Bond**

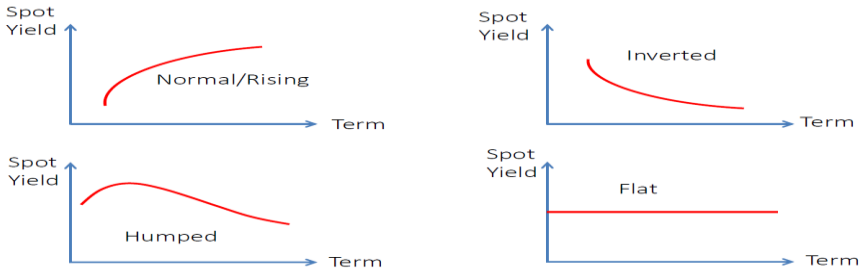
$$Price = \sum_{t=1}^{T} \frac{C}{(1+y(t))^t} + \frac{F}{(1+y(T))^T}$$

In terms of discount factors, $P_{\$}(t)$, the bond price becomes:

$$Price = \underbrace{\sum_{t=1}^{T} C \times P_{\$}(t)}_{\substack{discounted \\ coupons}} + \underbrace{F \times P_{\$}(T)}_{\substack{discounted \\ face\ value}}$$

Topic 3: Term Structure of Yields

- The relationship between yields on ZCBs and their maturities is known as the 'term structure of yields (interest rates)'.（利率的期限结构是零息债券的收益与到期日的关系，即每个到期日对应着零息债券的收益）
  - ✓ ZCB yields are also referred to as **spot yields (or spot rates).（即期收益率）**



$$y(t) = \left(\frac{F}{P_{\$}(t)}\right)^{\frac{1}{t}} - 1$$

Practice:

A two-year non-callable government bond with a face value of $1,000,000 pays semi-annual coupons at a rate of 4.6% p.a. Price the bond using the following ZCB (pure)

yield curve.

Government ZCB Prices Face Value = $1,000		
Maturity (months)	Price ($)	ZCB Yield (% p.a.)
6	988	?
12	950	?
18	920	?
24	860	?

Topic 4: Forward Yields（考点加重点）

- Forward Yields （远期利率）
  - ✓ 买 1 年 bond 相当于买完 1 年 bond 再买 1 年 bond

$$(1+y(1))(1+f_{1\,to\,2}) = (1+y(2))^2$$

- **通用公式: 买 k 年 bond 相当于买完 i 年 bond 再买（k – i）年 bond**

$$(1+y(i))^i (1+f_{i\,to\,k})^{k-i} = (1+y(k))^k$$

- Forward Rate from time i to k (T = k-i in years)

$$f_{i\,to\,k} = \left(\frac{(1+y(k))^k}{(1+y(i))^i}\right)^{1/T} - 1 = \left(\frac{P_i}{P_k}\right)^{1/T} - 1$$

Practice: Calculate both the six-month and 12-month forward yields out of 12 months using this data:

Maturity (months)	ZCB Price
6	$950
12	$890
18	$852
24	$830

Topic 5: Classification of Bonds（了解即可）

There are two broad bond classes：（两大类：政府债券和公司债券）

- **Government Bonds**
  - ✓ Most government bonds are fixed interest (pay fixed coupons semi-annually) or indexed for inflation
- **Corporate Bonds**

  The characteristics of corporate bonds can vary widely.

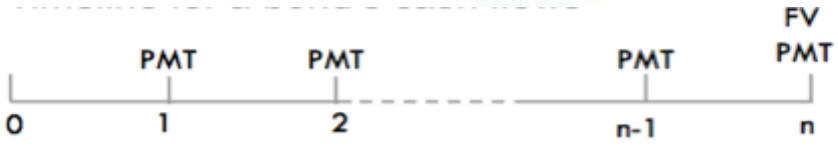
  - ✓ Asset-backed bonds（资产支持的债券）
  - ✓ Has default risk (difficult to price)（有违约风险）
  - ✓ **Convertible bonds（可转换债券）:** allow for conversion into a fixed number of shares
  - ✓ **Catastrophe bonds（巨灾债券）:** don't pay if a defined catastrophe occurs
  - ✓ **Callable bond （可赎回债券）**(right is held by the company) **and Puttable bonds（可申购债券）** (right is held by the investor)

Part 6: YTM and Yield Curve Theory

Topic 1: YTM（必考点加重点）

The internal rate of return (IRR) for a bond is known as its yield-to-maturity (YTM).（到期收益率是债券的内部收益）(The IRR is the interest rate that sets the present value of the bond's cash flows equal to the bond's price)(债券的内部收益率就是当债券的未来所有现金流现值等于债券价格时的利率)

- Use YTM for bond pricing



$$coupon\ C = \frac{r \times FV}{N}$$

r: coupon rate
N: 一年内给出的 coupon 次数

$$PV = P = \left(\frac{C}{1+i} + \frac{C}{(1+i)^2} + \cdots + \frac{C}{(1+i)^n}\right) + \frac{FV}{(1+i)^n}$$

公式：$PV = PMT\left(\frac{1-(1+i)^{-n}}{i}\right) + \frac{FV}{(1+r)^n}$

PV：present value / price

FV：Future value / Face value

i: market interest rate, market yield, YTM

$$Price = \frac{C}{r}\left(1 - \frac{1}{(1+r)^n}\right) + \frac{FV}{(1+r)^n} \quad \text{or}$$

$$Price = C\left(\frac{1-(1+r)^{-n}}{r}\right) + \frac{FV}{(1+r)^n}$$

Practice:

The bond has a maturity of 5.5 years* and a face value of $1m. The coupon is paid semi-annually and at a rate of 1.75% p.a.. The best bid expressed as a yield-to-maturity is 1.98% p.a.

- Relationship between YTM and Coupon Rate（YTM 和债券利息率的关系）
  - ✓ If YTM > coupon rate => PV < FV => **discount bond（折价债券）**
  - ✓ If YTM < coupon rate => PV > FV => **premium bond（溢价债券）**
  - ✓ If YTM = coupon rate => PV = FV ： **Par Bond (平价债券）**

Practice:

Consider a 5-year $100 bond paying annual coupons at a rate of 5%. Use a spreadsheet to calculate the prices for each of the following cases:

- A) The YTM is 5.1%: P = $99.57
  - YTM > coupon rate: DISCOUNT bond Price < Face Value
- B) The YTM is 5.0%: P = $100
  - YTM = coupon rate: PAR bond Price = Face Value
- C) the YTM is 4.9%: P = $100.43
  - YTM < coupon rate: PREMIUM bond Price > Face Value

Topic 2: Holding Period Return (HPR)

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For bonds paying semi-annual coupons:

$$YTM = 2 \times \left(\left[\frac{FV + FV_{coupons}}{P_{buy}} \right]^{1/n} - 1 \right)$$

$$FV_{coupons} = C \times \left[\frac{(1+r)^n - 1}{r} \right]$$

FV coupon 是指你在持有期已经收到的利息，小心 n 是指你收到的利息的次数而不是年份。每次收到利息都以 r 进行再投资。

Holding Period Return (HPR)

– Simple HPR:

$$Simple\ HPR = \frac{Coupon + (P_{sell} - P_{buy})}{P_{buy}}$$

– For bonds paying semi-annual coupons:

$$HPR = 2 \times \left(\left[\frac{P_{sell} + FV_{coupons}}{P_{buy}} \right]^{1/n} - 1 \right)$$

where P_{sell} = selling price.

这里的 P sell 是指站在你要卖出债券的那个时点来对债券进行估值。把那个时点到债券持有到期日之间产生的所有现金流进行折现求出。

Topic 3: Term Structure Theories（考点）

1. **The Expectation Theory （期望理论）**

➢ The forward yields are equivalent to the market's expectation of future short rates.（远期利率是未来即期利率的期望）

✓ Example: market (i.e., investors) have some expectation about ${}_1y_2$

– $(1+y_2)^2 = (1+y_1)^*(1+E({}_1y_2))$

➢ In reality: hold in short term (0-1 year), not longer term（实际只在短期成立）

➢ Implication for the yield curve shape

✓ Upward Sloping: Expectation of increasing short-term interest rates

✓ Downward Sloping: Expectation of decreasing short-term interest rates

✓ On average, the yield curve should be flat because the average rates will rise and fall equal amounts. （平均来说，收益曲线是平的因为平均上扬和下扬的曲线概率一致）

2. **The Liquidity Preference Theor**（流动性偏好理论）

✓ We know risks attract premiums on returns.

✓ Short-term bonds are highly liquid and have less price uncertainty than long term bonds.（短期债券高度流动，价格有更少的不确定性）

✓ Short-term bonds are less sensitive to changes in yields.（短期债券对利率的变动更不敏感）

- With short-term bonds, if you can't find a buyer, you can just hold them until maturity which will be in less than one year's time.

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- With long-term bonds you can be "locked in" for several years as not many investors want to trade long-term bonds.

➢ Under the Liquidity Preference Theory, the forward rate is the expected future short rate plus a liquidity premium （长期利率等于期望的未来即期利率加上流动性溢价-即因为持有长期债券带来的流动性风险进行补偿的额外收益）

$$f_{i\ to\ k} = E[r_{i\ to\ k}] + liquidity\ premium$$

✓ Consequently, on average term structure will be upward sloping (to allow for the premium on long term bonds). （平均来说，利率曲线是上扬的，因为长期债券有流动性风险补偿，所以收益率更高）

3. Market Segmentation Theory（市场分割理论）

➢ The Market Segmentation Theory states that different bond issuers have specific funding needs and therefore issue only at specific maturities.（每个债券发行者有特定的融资需求，所以只会发行特定到期日的债券）

➢ Investors also have specific needs and invest in specific maturities.（投资者同理）

e.g. Bank – prefer short term investment
Pension – Prefer long term investment

➢ Interest rates are determined separately in each segment of the market depending on the demand and supply conditions in that segment（利率是在不同子市场单独决定，由那个市场的供给和需求决定）

➢ Implications for shape of term structure:（利率曲线结构可以是任意形状的）

✓ This could generate any shape, in particular humps and troughs.

✓ The short-term bond yield would reflect banks expectations of future interest rates and long-term bond yield would reflect the expectations of pension funds.

4. Preferred Habitat Theory （投资者偏好理论-是市场分割理论的变形）

➢ This is a variation on the Market Segmentation Theory. It states that while investors have a preferred maturity segment, they will accept a less than ideal bond investment if compensated with higher yields.（当投资者有偏好的到期日，则会接受不那么理想的债券投资，只要有更高的收益补偿）

Practice 1:

a. What is the relationship between forward rates and the market's expectation of future short rates? Explain in the context of both the expectations and liquidity preference theories of the term structure of interest rates.

According to the expectations theory of the term structure of interest rates, the liquidity premium is zero so that the forward rate is equal to the market's expectation of the future short rate. This could result in either upward sloping or downward sloping term structures, and in general, the term structure should be flat since the same probability for upward sloping

and downward sloping in general.

The liquidity preference theory, on the other hand, specifies that the liquidity premium is positive so that the forward rate is greater than the market's expectation of the future short rate. This could result in an upward sloping term structure even if the market does not anticipate an increase in interest rates

b. Under the expectation hypothesis, if the yield curve is upward--sloping, the market must expect an expect an increase in short--term interest rates. True/false/uncertain? Why?

True. Under the expectations hypothesis, there are no risk built into bond prices. so the only reason for long--term yields to exceed short term yields is an expectation of higher short--term rates in the future.

c. Under the liquidity preference theory, if inflation is expected to be falling over the next few years, long--term interest rates will be higher than interest rates. True/false/uncertain? Why?

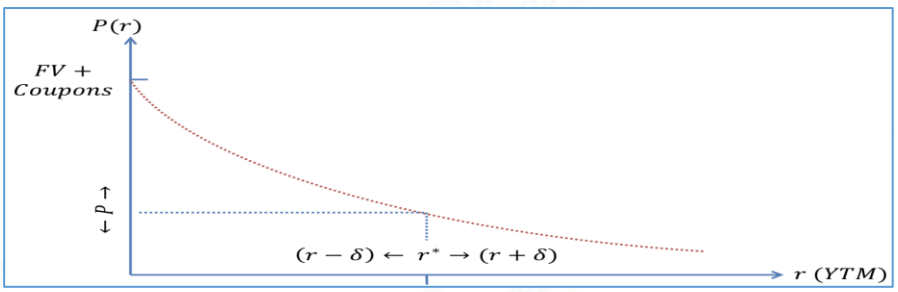
Uncertain. Expectations of lower inflation will usually lead to lower nominal interest rates. Nevertheless, if the liquidity premium is sufficiently great, long--term yields may exceed short term yields despite expectations of falling short rates.

Part 7: Interest Rate Risk and Duration (Bond Portfolio Management)

Topic 1: Interest Rate Risk（必考点加重点）

- **Interest rate risk:** interest rate change, bond price change. There is an Inverse relationship between bond price and yield.(利率风险，即当利率发生改变时，债券的价格发生改变。债券的价格和利率是相反的关系)

$$Price = \frac{C}{r}\left(1 - \frac{1}{(1+r)^n}\right) + \frac{FV}{(1+r)^n}$$

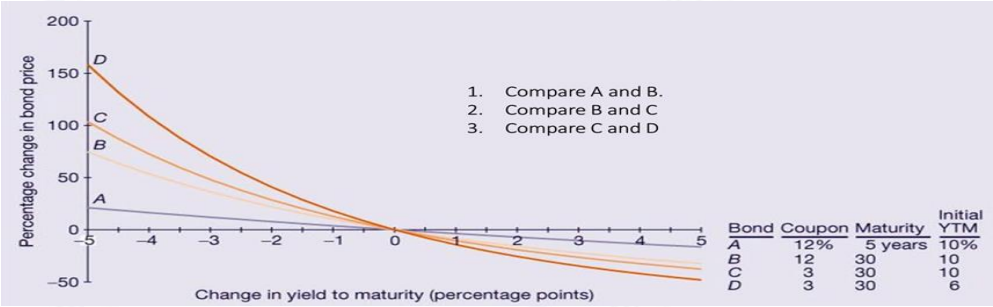


- **Yield Sensitivity(收益率敏感度，债券收益率对利率变化的敏感性)**

A measure of the average maturity of a bond's cash flows would provide the "effective

maturity" of the bond. Yield sensitivity is influenced by:（有效到期日，衡量债券现金流的平均到期日）

- ✓ **Maturity (main determinant):** Long term bonds are more sensitive to interest rate risk. （到期日是最主要影响因素，越长对利率风险越敏感）
- ✓ **Coupon Size:** Low-coupon bonds are more sensitive to interest rate risk.(利息率低的债券对利率风险更敏感)
- ✓ **YTM:** Lower YTM bonds are more sensitive to interest rate risk.（债券的到期收益率越低则对利率风险越敏感）



Bond	Coupon	Maturity	Initial YTM
A	12%	5 years	10%
B	12	30	10
C	3	30	10
D	3	30	6

Example: A and B, B 对利率风险更敏感因为到期日更长； B and C, C 对利率风险更敏感因为 C 的 coupon rate 更低；C and D, D 对利率风险更敏感因为 D 的 YTM 更低。

Topic 2: Macaulay's Duration（麦考利久期-衡量有效到期日）（必考点加重点）

Macaulay's duration provides a measure of "effective maturity"

- ➢ It is the weighted average of the times until each payment, with weights being proportional to the present value of the payment（麦考利久期是收到债券所有现金流的平均时间，权重为每个现金流的现值占总现值比例）
- ✓ A ZCB pays no coupons, so its risk can be simply measured by its maturity.（由于零息债券不付息，所以到期日就等于久期，因而风险可由到期日衡量）
- ✓ Macaulay's duration strips the cash-flows from a coupon-paying bond and treats each cash-flow as a ZCB. （麦考利久期把每个付息债券的每笔现金流拆开当成零息债券来计算）
- ✓ **Only ZCB have Duration = Maturity (D = n)**

$$D = \sum_{t=1}^{T} t \times w_t$$

$$\text{where} \quad w_t = \frac{1}{P}\frac{CF_t}{(1+r)^t}$$

- ➢ **The** factors that influence Duration（影响久期的因素）

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Factor	More or less sensitive to yield (YTM) changes?	Increase or decrease duration
Longer time to maturity	More	Increase
Larger coupon rate	Less	Decrease
Higher yield (YTM)	Less	Decrease
Semi-annual rather than annual coupons	Less	Decrease
Better credit rating	More	Increase

Practice:

Consider the following bonds and assume coupons are paid annually. Assume the YTM is 10% p.a. for all bonds.

Bond	Maturity (in years)	Coupon
A	10	10%
B	10	5%
C	12	5%
D	20	0%

a. Sort the bonds in terms of shortest duration to longest duration

b. In general, does a bond paying a semi-annual payment have a shorter or longer duration that a bond paying annual coupons?

Answers: (a) Bond A, B, C, D (b) Shorter duration.

- **Bond Portfolio Duration**（组合的久期为单个债券久期的加权平均）

The duration of a portfolio of bonds with equal yields (which is extremely rare) will be the weighted average of durations of individual bonds

$$D_{portfolio} = weight_{BOND1} \times D_{BOND1} + weight_{BOND2} \times D_{BOND2}$$

- **Modified Duration**（修正久期-麦考利久期除以（1 加 YTM））

$$D^* = \frac{D}{1+r}$$

- ✓ Modified duration increases as the coupon decreases.
- ✓ Modified duration decreases as maturity decreases.

（重要）:当利息减少时修正久期增加，当到期日缩短则修正久期下降。

去年考题：修正久期和麦考利久期区别：修正久期衡量对于 YTM 的微小变化引起的价格波动，能更精准描述债券价格对 YTM 变动的敏感度

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- **The use of Duration** （久期的用途，用来评估债券价格变化的 **amount**）
  - ✓ To approximate price changes with Macaulay duration:

$$\frac{\Delta P}{P} \approx -D\frac{\Delta r}{1+r} \text{ so } \Delta P \approx -PD\frac{\Delta r}{1+r}$$

  - ✓ To approximate price changes with modified duration:

$$\frac{\Delta P}{P} \approx -D^*\Delta r \text{ so } \Delta P \approx -PD^*\Delta r$$

- **Duration Shortcut** （计算麦考利久期的一个“捷径）

Calculating D for a 20-year bond is rather onerous – but we can derive the following (ugly) formula for D.

$$D = \frac{1+r}{r} - \frac{1+r+n(c-r)}{c[(1+r)^n-1]+r}$$

Lets use this shortcut to calculate the Macaulay's duration for the three-year $1,000 non-callable bond that is paying annual coupons at 6% p.a. and currently trading at a YTM of 7% p.a.

$$D = \frac{1.07}{0.07} - \frac{1+0.07+3\times(0.06-0.07)}{0.06[(1.07)^3-1]+0.07} = 2.83$$

Practice:

Bond A: 4 years to maturity, pays annual coupon of 8%, face value of $5000. The current price of Bond A is $5000. Bond B: A zero-coupon bond with 3 year to maturity.

(a) Calculate the durations of Bond A and Bond B.

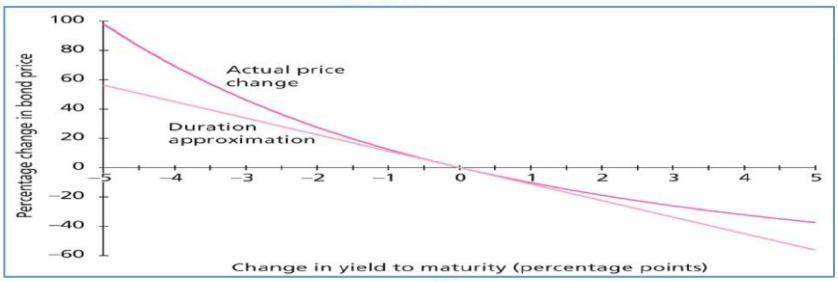
(b) Compute the duration of an equally-weighted portfolio of Bond A and Bond B.

(c) By how much do the price of bonds A and B change (in %) if interest rates rise by 0.5% (approximately)

Topic 3: Convexity（久期表示债券的到期收益率和价格变动的线性关系。凸性表示非线性关系）（考点，考试会给这个值，求价格变动的值）

Convexity measures the **curvature** of the bond's price--yield curve. （曲度）

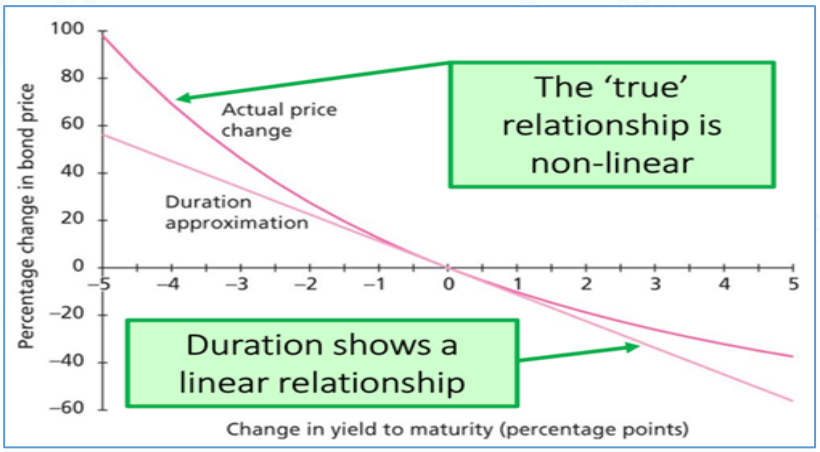
- Duration explains a linear relationship between price changes and YTM changes. However, bond prices are non-linear(convex) functions of YTM.
  - ✓ Duration is a good approximation for small moves in YTM, but will not be good for large moves.（久期是对 YTM 变动很小时候价格变动的一个很好的估测，但对大变动则不是很好的估测）



- Including convexity for the percentage change in the price (考试会直接给这个值):

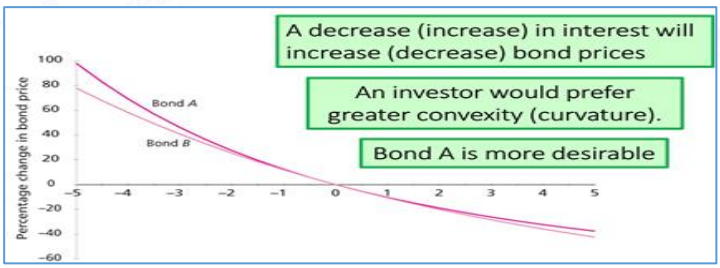
$$\frac{\Delta P}{P} = -D^{*}\Delta r + \frac{1}{2}Convexity(\Delta r)^{2}$$

- Convexity is also an approximation. Duration involves the 1stderivative and convexity the 2ndderivative.
- **Callable bonds are subject to positive and negative convexity**.（可赎回债券有正的和负的 convexity）
  - ✓ The call option is a source of negative convexity



Note: **Duration and convexity of callable bond < or = non-callable bond**

- Will an investor prefer greater or lesser convexity?
  - ✓ Greater convexity (curvature) as a decrease in YTM will increase price by more while an increase in YTM will decrease price by less.（更大的凸性会在 YTM 下降时候，债券价格上升更多；会在 YTM 上升时候，价格下降更少。所以投资者会偏好更多的 convexity）



Practice :

Consider a 5-year Government bond, with a $100,000 face value, a coupon rate of 10% p.a. and a convexity of 18.4775. Today's YTM is 12% and term structure is flat. Coupon frequency and compounding frequency are assumed to be annual.

a. What is the Macaulay duration of this bond? What is the modified duration of the bond?

b. Briefly, explain why modified duration is a better measure than maturity when calculating the bond's sensitivity to changes in interest rates.

Modified duration is a better measure than maturity when calculating the bond's sensitivity to changes in interest rates as it accounts for timing and size of cash flows not just final payment.

c. What is the exact price change in dollars if interest rates increase by 20 basis points (a parallel shift) . By 100 bp?

d. Use duration to calculate the approximate price change in dollars if interest rates increase by 20 basis points? By 100 bp?

e. Incorporate convexity to calculate the approximate price change in dollars if interest rates increase by 20 basis points? By 100 bp?

总结

Bond price:

$$P = \frac{C}{r}\left(1 - \frac{1}{(1+r)^n}\right) + \frac{FV}{(1+r)^n} \qquad P = \sum_{i=1}^{n} \frac{C}{(1+r)^i} + \frac{FV}{(1+r)^n}$$

Duration and Convexity:

- $D = \sum_{i=1}^{n} i \times w_i \ where \ w_i = \frac{1}{P} \times \frac{CF_i}{(1+r)^i}$ $\qquad D = \frac{1+r}{r} - \frac{1+r+n(c-r)}{c[(1+r)^n-1]+r}$
- $D^* = \frac{D}{1+r}$
- $\frac{\Delta P}{P} \approx -D^*\Delta r + \frac{1}{2}Convexity(\Delta r)^2$
- $\Delta P \approx -PD^*\Delta r$

Part 8: Equity Evaluation

Topic 1: Intrinsic Value and Market Price（了解即可）

- **Intrinsic Value (Vo):** present value of all future cash flows（内在价值是所有未来现金流的现值）
- ✓ For equity the cash flows are the future dividends and capital gain/loss.（对于股票来说，未来的现金流是所有的股息已经资本利得/损失）
- **Market Price:** What we see in the market trading, Po（市价是投资者在市场进行买卖股票的价格）
- ✓ If Po < Vo, the share may be undervalued（当市价小于内在价值时，则股票被低估，反之高估）
- ✓ If Po > Vo, the share may be overvalued.
- **Boundaries exist for equity valuation （了解即可）**
- ✓ The lower bound is liquidation value or 'scrap value' – Selling assets and repaying debt （清算价格。即当公司进行破产清算，有多少钱能还给债券人-公司价值最低的时候）
- ✓ Upper bound is replacement value – Cost of reproducing the same company（重置价值。用来创造一个相同公司的价钱。）

In the long run this should hold:
liquidation value < market price < replacement value

Topic 2: Dividend Discount Model（股利折现模型）（必考点）

Type I: Constant-growth DDM (Gordon Model)（股利稳定增长模型-高登模型）

- If we assume that dividends will grow at a constant rate g then we can rewrite our model in terms of g and the dividend that was most recently paid (D_0). （股利以一个不变的速度增长，第一期股利为最近 pay 的股利）

$$V_0 = \frac{D_0(1+g)}{1+k} + \frac{D_0(1+g)^2}{(1+k)^2} + \frac{D_0(1+g)^3}{(1+k)^3} + \cdots$$

$$V_0 = \sum_{i=1}^{\infty} D_0 \frac{(1+g)^i}{(1+k)^i} = D_0 \sum_{i=1}^{\infty} \left(\frac{1+g_1}{1+k}\right)^i$$

But this just a geometric sum, so **provided $k > g$** we can use the formula for an infinite geometric sum :（小心，k 一定要大于 g，否则等式无法收敛，得不出确定的值）。

$$V_0 = D_0 \frac{1+g}{k-g} = \frac{D_1}{k-g}$$

The formula $V_0 = \frac{D_1}{k-g}$ cannot be applied if $k \leq g$ because the formula was derived under the strict condition that $k > g$.

If $k \leq g$ then $V_0 = \sum_{i=1}^{\infty} D_0 \frac{(1+g)^i}{(1+k)^i}$ becomes unbounded (goes to infinity)

The constant growth DDM implies the value of a firm's share will higher when:

- The latest dividend, D_0, is larger.
- The market capitalization rate, k, is lower.
- The expected dividend growth rate, g, is higher.

$$V_0 = \boldsymbol{D_0} \frac{1+g}{k-g}$$

几个结论：当 Do 越大，则 Value 越高；当 k 越小，则 value 越高；当 g 越大，则 value 越高。

- Decomposing Growth

When a firm has positive earnings it must decide whether to pay them out (as dividends) or retain the earning and invest them in new or existing projects.

- Remember 'dividend payout rate' = (1-b) = D/EPS where b = 'plowback rate' and EPS = earnings per share
- There is a trade-off: (i) high dividends now and low growth, vs (ii) low dividends now and high growth

✓ 如果在 E 中拿出 b% 来当做再投资的比率，则我们的 Dividend = E*(1-b)；我们的 growth rate=ROE * b （ROE 在这里为新投资所带来的总收益，等于 Return on investment，考试会直接给）

$$V_0 = \frac{D_1}{k-g} = \frac{E_1(1-b)}{k - ROE \times b}$$

Practice:

MF Corp. has an ROE of 16% and a plowback ratio of 50%. If the coming year's earnings are expected to be $2 per share,

a.at what price will the stock sell? The market capitalization rate is 12%.

b. What price do you expect MF shares to sell for in 3 years?

➢ **Present Value of Growth Opportunities (PVGO)（公司增长机会的现值）考点**

✓ Now, if b = 0 (no retained earnings) then g = 0 and $D1 = E1$. So the 'no-growth value' of the firm, VNG, would be:（当 b 等于 0，则公司没有保留收益进行再投资，即挣的全部用来付股息，则公司没有增长 value）

$$V_{NG} = \frac{D_1}{k} = \frac{E_1}{k}$$

✓ **We can view a firm as being the sum of the no-growth value and the present value its growth opportunities （PVGO）:**（把公司的价值看成没有增长的价值加上未来增长机会的现值）

$$P_0 = V_{NG} + PVGO = \frac{E_1}{k} + PVGO$$

✓ PVGO can be positive or negative.（PVGO 可为正的也可以是负的）

- A firm can have value if has negative PVGO, as long as its no-growth value exceeds the negative PVGO.（只要公司不进行增长的值大于负的 PVGO，则 P 仍然可以大于 0，公司有 value）
- A firm can have value if its earnings are zero or negative as long as it has positive PVGO.（当收益为 0 或者负的，只要有正的 PVGO,公司仍然可以有 value）

Let's look at BHP again; according to Yahoo Finance the earnings-per-share (EPS) is $2.70.

Let's assume the market risk premium is 7% p.a.. The current market price is $P_0 = 37.17, \beta_{BHP} = 1.06, r_f = 2.6\%$. Then

$$k = 0.026 + 1.06 \times 0.07 = 0.1002$$

So we can establish that the PVGO is

$$PVGO = P_0 - \frac{E_1}{k} = 37.17 - \frac{2.70}{0.1002} = \$10.22$$

Type II: Multi-stage growth DDM（多阶段股利增长模型-前提：k 大于 g）考点

A firm's dividends may grow quickly (well above the capitalization rate) for a few years after a new venture has been launched. Later, once the venture has matured and competition starts to erode profits, the growth in dividends might decline to below the capitalization rate. Let S be the point at which growth changes from $g1$ to$g2$ with$g2 <k$.（公司可能在前几年因为新的市场投资股利高速增长，后来随着竞争加剧则股利增速变慢）

Theory （考核与股利稳定增长模型的对比）-去年考题

In what circumstances is it most important to use multistage dividend what circumstances is it most important to use multistage dividend discount models rather than rather than constant--growth models？

Answer: It is most important to use multi--stage dividend discount models when valuing companies with temporarily high growth rates. These companies tend to be companies in the early phases of their life cycles, when they have numerous opportunities for reinvestment, resulting in relatively rapid growth and relatively low dividends (or, in many cases, no dividends at all). As these firms mature, attractive investment opportunities are less numerous so that growth rates slow. （当对暂时有高收益的公司进行估值时尤其要用多阶段股利增长模型。这些公司一般都处于早期发展阶段，有各种新的投资机会，导致高速增长并且低股利。当市场成熟以后，新投资机会变少，则增速变慢）

$$V_0 = \underbrace{D_0 \sum_{i=1}^{S} \frac{(1+g_1)^i}{(1+k)^i}}_{\substack{present\ value \\ of\ divideds\ up \\ to\ time\ S}} + \underbrace{D_0 \frac{(1+g_1)^S}{(1+k)^S} \sum_{i=S+1}^{\infty} \frac{(1+g_2)^i}{(1+k)^i}}_{\substack{present\ value \\ of\ dividends\ from \\ time\ S\ onwards}}$$

Or （考核实操）

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$$V_0 = \sum_{i=1}^{S} \frac{D_i}{(1+k)^i} + \frac{V_S}{(1+k)^S}$$

where

$$V_S = \frac{D_S(1+g_2)}{k-g_2} \quad \text{and} \quad D_i = D_0(1+g_1)^i$$

Practice:

Deployment Specialists pay a current (annual) dividend of $1.00 and is expected to grow at 20% for 2 years and then at 4% thereafter. If the required return for Deployment Specialists is 8.5%, what is the intrinsic value of Deployment Specialists stock?

Topic 3: Limitations of DDM and Comparative Valuation Ratio

- Limitations of DDM（DDM 的缺陷）

- The biggest limitation with the DDM is when k is close to g the model becomes very sensitive to small changes in g.

$$V_0 = D_0 \frac{1+g}{k-g}$$

- Another limitation is that for a multi-stage DDM the analyst needs to define the exact length of different growth stages. It is very difficult to forecast when you move from one growth stage to another growth stage.

$$V_0 = \underbrace{D_0 \sum_{i=1}^{S} \frac{(1+g_1)^i}{(1+k)^i}}_{\substack{\text{present value} \\ \text{of dividends up} \\ \text{to time } S}} + \underbrace{D_0 \frac{(1+g_1)^S}{(1+k)^S} \sum_{i=S+1}^{\infty} \frac{(1+g_2)^i}{(1+k)^i}}_{\substack{\text{present value} \\ \text{of dividends from} \\ \text{time } S \text{ onwards}}}$$

- The DDM works better for firms with stable dividend policy.

DDM 的最大缺点是当 k 趋近于 g，则模型对 g 的小小变动都变得十分敏感。另一个缺点是在多阶段股利增长模型中，分析师要对不同股利增长的年限进行预测，这十分困难。因此，DDM 更适用于对有稳定股利政策的公司进行估值）

- **P/E Ratio Analysis (因为 DDM 有缺陷，所以用相对比值法来估值，克服 DDM 的缺陷)**

$$P_0 = \frac{E_1}{k} + PVGO$$

$$\frac{P_0}{E_1} = \frac{1}{k} + \frac{PVGO}{E_1}$$

Alternatively we can use the constant growth DDM:

$$P_0 = \frac{D_1}{k-g} = \frac{E_1 \times (1-b)}{k-g}$$

this can be rewritten as a P/E ratio:

$$\frac{P_0}{E_1} = \frac{1-b}{k-g}$$

小心：当用 E1，则为 leading P/E ratio；当用 Eo， 则为 trailing P/E ratio（revision 题目有区别，lecture 没有）

- **What type of firm would have higher P/E ratios? （要会理论判断）**
- ✓ Less riskier stocks will have higher P/E ratios. They will have lower betas which means they will have lower capitalization rates (k)（风险越低的股票有更高的 P/E ratio）
- ✓ Firms with higher growth rates (g) will have higher P/E ratios. g=ROE * b, so a firm's P/E will increase as its ROE increases.（当 g 越大时则 P/E ratio 越高）
- ✓ Start up firms often have high P/E ratios.（刚起步的公司会有更高的 P/E ratio，虽然风险高，但投资者对公司未来增长率比较看好，会付高价买）

Practice 1:

The market consensus is that Analog Electronic Corporation has an ROE of 9%, has a beta of 1.25, and plans to maintain indefinitely its traditional plowback ratio of 2/3.

This year's earnings were $3 per share. The annual dividend was just paid. The consensus estimate of the coming year's market return is 14%, and T- bills currently offer a 6% return.

a. Find the price at which Analog stock should sell.

b. Calculate the P/E ratio.

c. Calculate the present value of growth opportunities

Practice 2:

P/E will tend to be higher when the plowback rate is higher. True, false or uncertain?

Answer : depends on a comparison of the expected rate of return on reinvested earnings with the market capitalization rate. If the former (the project expected return) is higher than the latter, than the P/E will increase as the plowback ratio increases.

➢ Some other Ratios（了解即可）

- The *price-to-book* (aka market-to-book) ratio is the ratio of the price per share divided by the book value per share.
  - This is one of the FF3F characteristics (HML)

$$r_i - r_f = \alpha_i + \beta_{i,m} \times (r_m - r_f) + \beta_{i,HML} r_{HML} + \beta_{i,SMB} r_{SMB} + e_i$$

- The *price-to-cash-flow* ratio uses the flow of cash to avoiding the accounting manipulation of earnings numbers. Of course there are several ways to define cash-flow.
- The *price-to-sales ratio* (stock price to annual sales per share) is sometimes applied to start-up firms that have no earnings.
- The *cyclically adjusted price-to-equity (CAPE)* ratio is the price divided by the average of the last 10 years of earnings adjusted for inflation.

总结 （Q4）

Valuation:

- $V_0 = \frac{D_1}{k-g} = \frac{E_1(1-b)}{k-ROE \times b}$
- $P_0 = V_{NG} + PVGO$ $\qquad P_0 = \frac{E_1}{K} + PVGO$

Part 9: Investor and Investment Process

1. Managed Fund: Listed vs. Unlisted Investment vehicles

➢ **Listed:**

✓ Listed on a public stock exchange (eg.ASX): Examples: ETFs and A-REITs(在公开股票交易所挂牌交易)

✓ Traded through a broker (brokerage fees)（通过经纪人交易）

- ✓ Tax advantage compared to unlisted one (no exposed to capital gains)（相比非挂牌有税收优势，因为不交资本利得税）
- ✓ Closed-end fund (fixed number, buy/sell at the market)（封闭型基金：不允许新的投资者进来或出去，市场买卖有固定的数量）
- ✓ More liquid than unlisted investments（比非挂牌流动性更强）
- ➢ **Unlisted:**
- ✓ Not listed
- ✓ Traded through a licensed dealer (financial planner) (buy-sell spread that paid to investment manager)（通过有牌照的交易商进行交易）
- ✓ Open-end funds (allow new investors enter or investors exit the fund)(开放型基金，允许新的投资者不断进行进入或者退出)

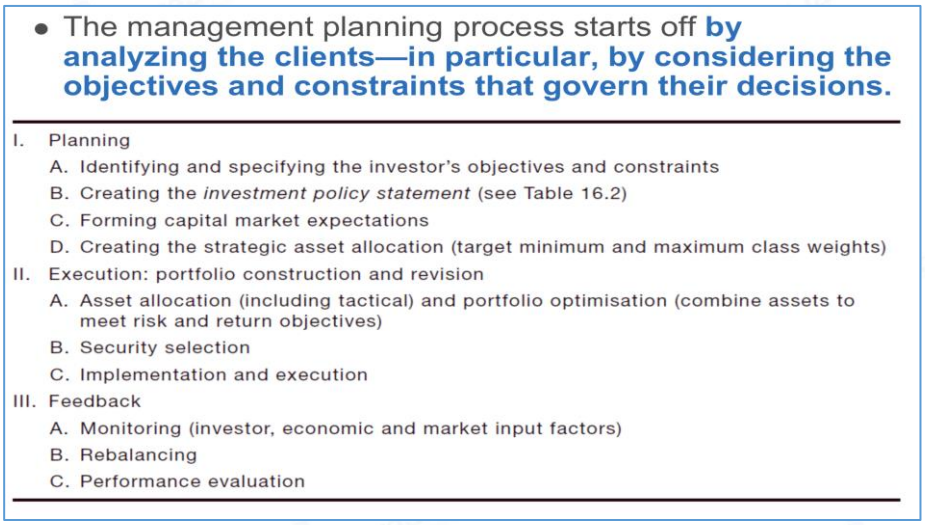
2. Single vs Multi-asset diversified fund

- ➢ Single: invest in one sector
- ➢ Multi: multi sectors (super fund)
- ✓ Stable fund: low risk, money-market instruments and bonds（稳定型基金，风险低，投资货币市场证券）
- ✓ Balanced fund: hold equity and fixed-income securities in relatively stable proportions（平衡型基金，持有稳定比例的股票和固定收益证券）
- ✓ Growth fund: high risk, higher return. Mainly hold equities（成长型基金，高收益高风险，一般投资于各种股票）

3. Passive and Active management

- ➢ **Passive**（消极管理，认为市场定价有效，跟踪市场指数的表现，经理不用自己额外分析研究，所以管理费用比较低）
- ✓ Believe markets are efficiently priced.
- ✓ Follow a market index
- ✓ Lower fee
- ➢ **Active**（主动管理，认为市场无效，通过研究错误定价获得超额收益。管理费高）
- ✓ Believe can profit from mispriced securities, buy low sell high
- ✓ Abnormal returns/Excess Returns/ Alpha
- ✓ Higher fee

4. Investment Management Process



- The management planning process starts off **by analyzing the clients—in particular, by considering the objectives and constraints that govern their decisions.**

I. Planning
  A. Identifying and specifying the investor's objectives and constraints
  B. Creating the *investment policy statement* (see Table 16.2)
  C. Forming capital market expectations
  D. Creating the strategic asset allocation (target minimum and maximum class weights)
II. Execution: portfolio construction and revision
  A. Asset allocation (including tactical) and portfolio optimisation (combine assets to meet risk and return objectives)
  B. Security selection
  C. Implementation and execution
III. Feedback
  A. Monitoring (investor, economic and market input factors)
  B. Rebalancing
  C. Performance evaluation

➢ **Responsible Investment (reflect investors' value)**-反映投资者个人价值观的投资

Focus sustainability (environmental social and governance (ESG) dimensions); today: climate change and environment.（从社会环境政府管制三个维度考虑）

✓ Positive screening (inclusionary screening): energy saving
✓ Negative screening (exclusionary screening): tobacoo, gambling, pollution

Part 10: Performance Evaluation（必考点）

Risk-adjusted returns need to be used in performance evaluation. (要用经过风险调整过的收益来评估基金经理的表现，因为高收益可能是因为承担了高风险，不能反映经理的真正管理能力)

Topic 1: Measurement of Portfolio Performance

1. Sharpe ratio（夏普比率，考虑全部风险，适合分析整个组合，值越大越好）

✓ Consider total risk, including systematic and idiosyncratic risk.
✓ Appropriate to evaluate entire portfolio (slope of CAL)
✓ The higher, the better.
✓ Related to CAL

$$S_i = \frac{E(r_i) - r_f}{\sigma_i}$$

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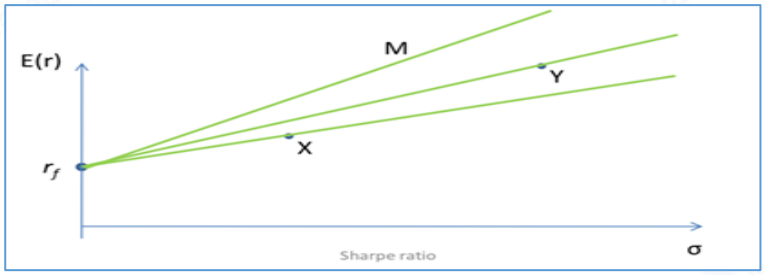


2. M^2（评估当组合承担和市场一样的风险时所获得的额外收益，评估整个组合，值越大越好）

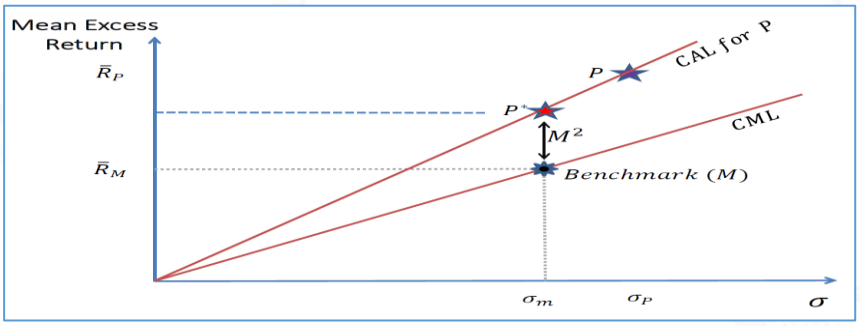
✓ Forming a hypothetical portfolio P'(which has same standard deviation as the market portfolio), reflect the additional return from portfolio P.

✓ Used to evaluate entire portfolio

✓ The higher, the better

✓ Related to CML

$$M^2 = (S_P - S_M)\sigma_M$$
$$M^2 = \left(\frac{\bar{R}_P}{\sigma_p} - \frac{\bar{R}_M}{\sigma_M}\right)\sigma_M$$
$$M^2 = \frac{\bar{R}_P \sigma_M}{\sigma_p} - \bar{R}_M$$



小心：图上纵坐标是 **excess return** 不是 **expected return**。

3. Treynor Measure (特雷诺指数：评估高度分散化的组合，只考虑系统性风险)

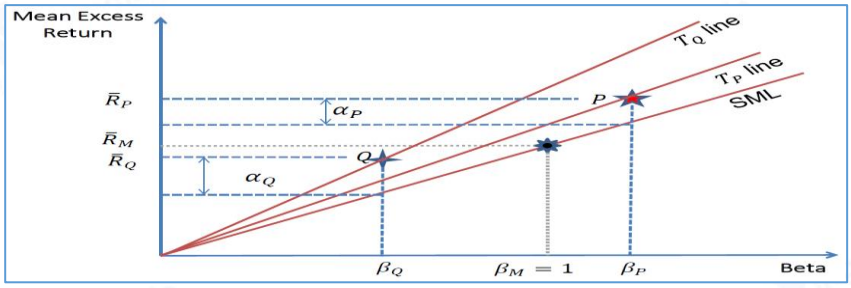
✓ Evaluate individual assets in a diversified portfolio

✓ Only consider systematic risk. That is beta. (Slope of SML)

✓ the higher the better.

✓ Related to SML

$$T_P = \frac{r_p - r_f}{\beta_p} = \frac{\bar{R}_P}{\beta_P}$$



4. **Jensen's alpha (NOT IN LECTURE, BUT IN FORMULA SHEET)（詹森指数，把市场指数当作 benchmark，把组合实际收益和由 CAPM 模型计算的收益对比 over 还是 under performance。适用于评估高度分散化的以及积极管理的组合）**

✓ Recall CAPM, Jensen's alpha measures over/under- performance relative to systematic risk benchmark

✓ Evaluate individual asset in an active portfolio

✓ The higher the better

✓ Related to SML

$$E(r_i) - r_f = \alpha_i + \beta_i \left[E(r_M) - r_f \right]$$

5. **Information Ratio（信息指数，衡量积极管理组合的表现，体现基金经理通过研究 misprice 获得超额收益的能力）**

✓ Evaluate individual asset in an active portfolio

✓ The information ratio is appropriate if you are looking for mispriced assets and trying to combine them with a passive diversified benchmark portfolio (like market portfolio)

✓ The higher the better

$$IR = \frac{\alpha_p}{\sigma_{\varepsilon p}}$$

✓ when an actively managed portfolio is added to a passive portfolio, new Sharpe ratio will be:

$$S_{New}{}^2 = S_M^2 - \left(\frac{\alpha_p}{\sigma_{\varepsilon p}}\right)^2$$

Summary:

Appropriate performance measures are as follows:

- ***Sharpe or M²:*** Should be used when the selected portfolio represents the entire investment fund.
- ***Treynor or Jensen***: Should be used when the selected portfolio represents one subportfolio of many.
- ***Information Ratio*** : Should be used when the selected portfolio represents the active portfolio to be optimally mixed with the passive portfolio. It measures abnormal return per unit of risk that in principle could be diversified away by holding a market index portfolio.

Practice:

Fund	Average Return%	Risk Free%	Standard Deviation%	β	$\sigma(e_p)$	R^2
A	8	2	15	0.67	?	?
M (Market)	9	2	21	1.00	?	?
Z	10	2	32	1.33	?	?

Fund	Sharpe Ratio	Treynor Ratio	Jensen's Alpha	IR	M^2
A	?	?	?	?	?
M	?	?	?	?	?
Z	?	?	?	?	?

Topic 3: Style Analysis（对基金经理或者投资者投资风格的分析）

Style analysis is used to determine the investment strategy adopted by an investor or managed fund.

Examples: Active/Passive investing; Growth/ Value investing; Small cap and Large Cap investing;

1. **Return-based style analysis（基于收益的风格分析）**

➢ This determines the investment style of a portfolio by comparing its return with the return on a market index.（把收益和相应的市场指数对比，通过多年的历史数据进行回归得出回归系数，类多因子模型）

✓ This comparison is undertaken over time (e.g1 year, 3 years, 5 years).

✓ This allows returns to be attributed to each index.

✓ The regression coefficient on each market index would give the implicit allocation to that 'style'. (multi-factor models)

2. **Holding-based style analysis （基于持有成分股的风格分析）**

- This determines the investment style of a portfolio by analysing the characteristics of each of the assets in the portfolio. The results are aggregated through a weighted average at a portfolio level to obtain the style of the whole portfolio.（在某个时点分析基金经理管理组合中的各个·成分股的特点来判断风格。更花时间）
- ✓ The characteristics can be the size (small, medium or large), the industry of the company issuing the asset and if they are value or growth assets.
- ✓ It is typically conducted at a single point in time.
- ✓ It is more time consuming to do than return-based style analysis.

Topic 4: Performance Attribution（组合的业绩归因；超重点）

- Performance attribution determines which decisions by the portfolio manager resulted in superior or inferior performance.（用来决定组合业绩来自于经理的哪种能力）
- ✓ **Asset Allocation:** return difference due to deviation from benchmark weights across various asset sectors（择时能力，即通过配置与 benchmark 不同的资产大类的比重）
- ✓ **Security Selection:** return difference due to selecting securities different from benchmark within each asset sectors（择股能力，即在不同资产大类内部高配被低估的股票，低配被高估的股票）

$$r_B = \sum_{i=1}^{n} w_{B,i} r_{B,i} \text{ and } r_P = \sum_{i=1}^{n} w_{P,i} r_{P,i}$$

- **Total impact = asset allocation impact + security selection impact.**

$$\text{Total impact} = r_P - r_B = \sum_{i=1}^{n} (w_{P,i} r_{P,i} - w_{B,i} r_{B,i})$$

Impact of asset allocation is given by $\sum_{i=1}^{n} (w_{P,i} - w_{B,i}) r_{B,i}$
- Impact of having different asset class weights from the benchmark.

Impact of security selection is given by $\sum_{i=1}^{n} w_{P,i} (r_{P,i} - r_{B,i})$
- Impact of having different asset class returns from the benchmark.

Practice:

Asset Class	FVF Weight	Benchmark Weight	FVF Return	Benchmark Return
Equity	25%	30%	-3.5%	-5.0%
Fixed Income	48%	50%	7.5%	8.2%
Property	22%	13%	9.2%	8.8%
Cash	5%	7%	3.5%	3.6%

a. What was the manager's return in this particular year? Did the manager over- or underperform?

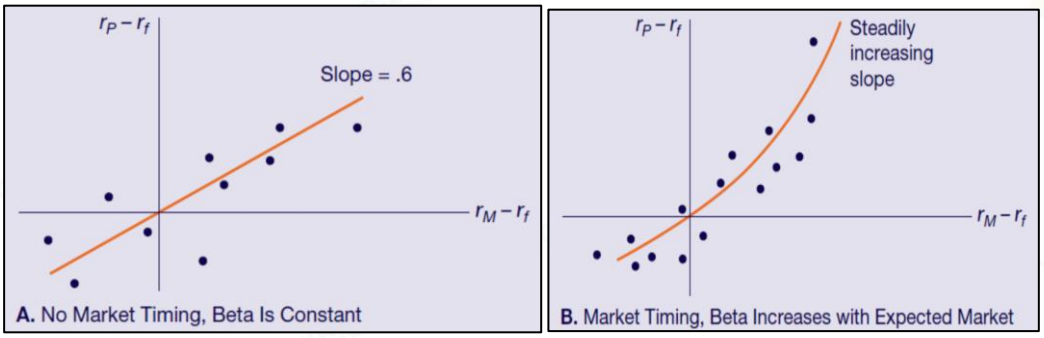
b. What was the contribution of asset allocation to relative performance?

c. What was the contribution of security selection to relative performance?

Topic 5: Market Timing and Factor Tilting

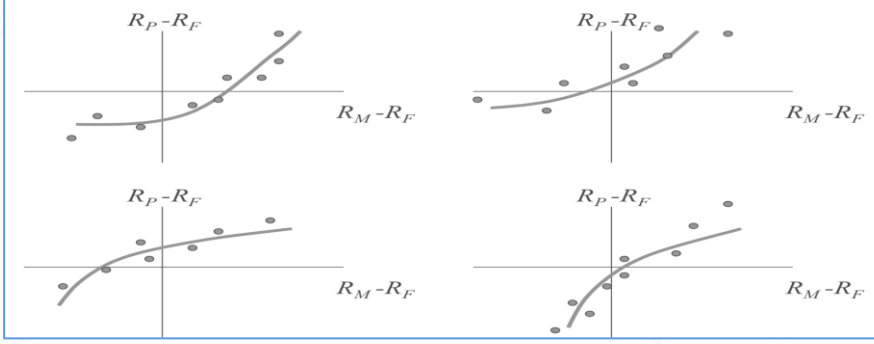
Two forms of Active Portfolio Management:（为主动管理的两种形式）

- **Market timing** –This focuses on macroeconomic factors and is an investment strategy that outweighs a particular asset class relative to the benchmark portfolio.
  Example: bull share markets, more weight to stocks（聚焦宏观因子）
- **Factor tilting:** Factor tilting –This focuses on microeconomic factors and is an investment strategy that outweighs a particular investment style relative to the benchmark portfolio.（聚焦微观因子）
  Example: More small cap and value stocks than benchmark portfolio



Practice:

Evaluate the market timing and security selection abilities of four managers whose performances are plotted in the accompanying diagrams.



总结：（Q5）

Performance measures:

- $S_P = \frac{\bar{R}_p}{\sigma_P}$
- $T_P = \frac{\bar{R}_p}{\beta_P}$
- $M^2 = \frac{\bar{R}_p \sigma_M}{\sigma_P} - \bar{R}_M$ $\qquad M^2 = (S_P - S_M)\sigma_M$
- $\alpha_P = \bar{R}_p - \beta_p \bar{R}_M$
- $IR = \frac{\alpha_P}{\sigma_{eP}}$

Attribution Analysis:

- $r_B = \sum_{i=1}^{n} w_{B,i} r_{B,i}$ $\qquad r_P = \sum_{i=1}^{n} w_{P,i} r_{P,i}$
- Impact of asset allocation is given by $\sum_{i=1}^{n} (w_{P,i} - w_{B,i}) r_{B,i}$
- Impact of security selection is given by $\sum_{i=1}^{n} w_{P,i} (r_{P,i} - r_{B,i})$