Numerical Analysis – Lab 11

1 Euler's Method

Convergence of Euler's method. We have the following implementation.

```
function [t, y] = eulerstep(f,a,b,n,ya)
% Euler explicit time-stepping for
% y'=f(t,y), y(a) = ya
t = zeros(n+1,1); t(1) = a;
h = (b-a)/n; d = numel(ya);
y = zeros(n+1,d); y(1,:) = ya;
for i = 1:n
    y(i+1,:) = y(i,:) + h*f(t(i),y(i,:));
    t(i+1) = t(i) + h;
end
```

We solve the following IVP

$$y' = ty + t^3, \quad 0 \le t \le 1, \quad y(0) = 1$$

using Euler's method with number of steps $n = 10, 20, 40, 80, 160$ and $n = 10^4$. We use the solution from $n = 10^4$ as the exact solution to calculate the errors from smaller n at $t = 1$ and plot them against number of steps n.

```
f = @(t,y) t*y+t^3;
[t,y] = eulerstep(f,0,1,1e4,1);
%semilogy(t,y,'linewidth',2); hold on;
nrange = 2.^(0:4)*10;
e = zeros(numel(nrange),1); k= 1;
for n = nrange
    [t,yh] = eulerstep(f,0,1,n,1);
    e(k) = abs(y(end)-yh(end));
    k = k + 1;
    %semilogy(t,yh,'.-','linewidth',2);
end
%hold off;
loglog(nrange,e,'*-','LineWidth',2);
hold on;
loglog(nrange,1./nrange,'LineWidth',2);
legend('error at t=1','1/n','fontsize',18);
xlabel('n'); set(gca,'FontSize',16); grid on
shg
```

We can clearly see the error is $O(h)$ for the step size $h = 1/n$.