Numerical Optimization in Robotics

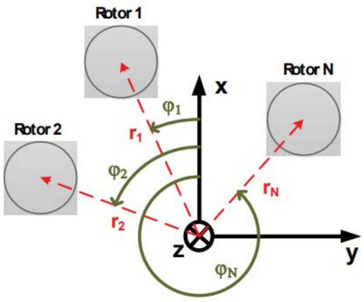
Homework_3 Instruction

1. Problem description:

● KKT condition

You have to prove the question of the course in your homework report.

● Control Allocation Problem

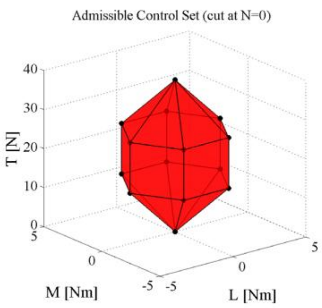


Control allocation as constrained optimization

$$\min_{\nu_1,\dots,\nu_N} \left\| \begin{bmatrix} \bar{f}_{\text{total}} \\ \bar{\tau}_{\text{roll}} \\ \bar{\tau}_{\text{pitch}} \\ \bar{\tau}_{\text{yaw}} \end{bmatrix} - \begin{bmatrix} t_1 & \cdots & t_N \\ l_1 & \cdots & l_N \\ m_1 & \cdots & m_N \\ n_1 & \cdots & n_N \end{bmatrix} \begin{bmatrix} \bar{\omega}_1^2 + \nu_1 \\ \vdots \\ \bar{\omega}_N^2 + \nu_N \end{bmatrix} \right\|_W^2 + \rho \left\| \begin{bmatrix} \nu_1 \\ \vdots \\ \nu_N \end{bmatrix} \right\|^2$$

$$\text{s.t.} \quad \begin{bmatrix} \omega_{\min}^2 - \bar{\omega}_1^2 \\ \vdots \\ \omega_{\min}^2 - \bar{\omega}_N^2 \end{bmatrix} \le \begin{bmatrix} \nu_1 \\ \vdots \\ \nu_N \end{bmatrix} \le \begin{bmatrix} \omega_{\max}^2 - \bar{\omega}_1^2 \\ \vdots \\ \omega_{\max}^2 - \bar{\omega}_N^2 \end{bmatrix}$$

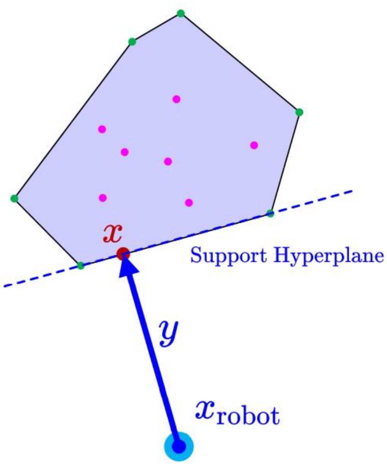
$$A \begin{bmatrix} t_1 & \cdots & t_N \\ l_1 & \cdots & l_N \\ m_1 & \cdots & m_N \\ n_1 & \cdots & n_N \end{bmatrix} \begin{bmatrix} \bar{\omega}_1^2 + \nu_1 \\ \vdots \\ \bar{\omega}_N^2 + \nu_N \end{bmatrix} \le b$$



This is a strictly convex low-dim QP as long as N<10, whose exact solution can be obtain in linear time.

● Collision Distance Computation

Collision vector from a robot to a polytope obstacle: ***V-rep*** cases



Any collision vector is the normal of its support hyperplane.

Due to the convexity, the following two problems are equivalent

Minimize length of the V-polytope's collision vector

Maximize length of the hyperplane's normal vector

Obviously, the separating halfspace is $\{x \in \mathbb{R}^d \mid y^{\mathrm{T}}(x - x_{\text{robot}}) \le y^{\mathrm{T}} y\}$

$$\max_{y \in \mathbb{R}^d} y^{\mathrm{T}} y, \quad \text{s.t.} \ (v_i - x_{\text{robot}})^{\mathrm{T}} y \ge y^{\mathrm{T}} y, \ \forall i \in \{1, \dots, m\}$$

Notice that if we use $z = y/(y^{\mathrm{T}} y)$ or equivalently $y = z/(z^{\mathrm{T}} z)$

$$\min_{z \in \mathbb{R}^d} z^{\mathrm{T}} z, \quad \text{s.t.} \ (v_i - x_{\text{robot}})^{\mathrm{T}} z \ge 1, \ \forall i \in \{1, \dots, m\}$$

Infeasibility implies collision occurs, else $x = y + x_{\text{robot}} = z/(z^{\mathrm{T}} z) + x_{\text{robot}}$

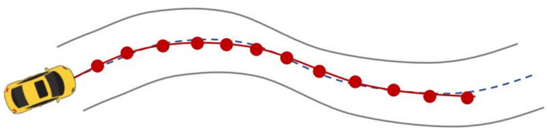
This is a low-dim QP again !

Complete the code for Lec_3_homework_part_2, make sure it could work.

Optional: you can implement the low-dim QP to solve the CAP or CDC problem.

- ## Nonlinear Model Predict Control

Example: Control longitudinal acceleration and steering angle of the vehicle simultaneously for autonomous driving of tracking a reference trajectory



Time-discretized state and control input:

$$s_{k+1} = F(s_k, u_k) := f(s_k, u_k)\tau + s_k$$

where

$$s_k = \begin{bmatrix} x_k \\ y_k \\ \phi_k \\ v_k \end{bmatrix}, \; u_k = \begin{bmatrix} a_k \\ \delta_k \end{bmatrix}, \; \tau \text{ is a constant duration}$$

Our objective function is to minimize tracking error and energy:

$$J(s_1, \ldots, s_N, u_0, \ldots, u_N) := \sum_{k=1}^{N} \left[(x_k - x_k^{\text{ref}})^2 + (y_k - y_k^{\text{ref}})^2 + w_v(a_k - a_{k-1})^2 + w_\delta(\delta_k - \delta_{k-1})^2 \right]$$

Physical limits requires

$$\begin{aligned} &\forall\, k \in \{0, \ldots, N\} \\ &a_{\min} \le a_k \le a_{\max} \\ &\delta_{\min} \le \delta_k \le \delta_{\max} \\ &v_{\min} \le v_k \le v_{\max} \end{aligned} \qquad G(s_k, u_k) \le 0$$

You have to implement PHR-ALM to solve the MPC problem given above.

2. Problem analysis

Model Predict Control: MPC is an iterative process of optimizing the predictions of robot states in the future limited horizon while manipulating inputs for a given horizon. The forecasting is achieved using the process model. Thus, a dynamic model is essential while implementing MPC. These process models are generally nonlinear, but for short periods of time, there are methods such as tailor expansion to linearize these models.

For more:

https://www.bilibili.com/video/BV1cL411n7KV?spm_id_from=333.999.0.0
https://www.bilibili.com/video/BV16U4y1c7EG?spm_id_from=333.999.0.0&vd_source=a1c2e0e10d18e90004496c6ea8093833

3. Assignment requirements

- ✓ Your homework ***should*** be a ***zip*** including your code, an documentation and an instruction.
- ✓ You can complete this chapter assignment in different programming languages, but I suggest you apply ***MATLAB*** or ***C++*** for this homework.
- ✓ You ***must*** given an instruction named ***'readme'*** to tell the reader how to run your code and check your answer.
- ✓ You ***have to*** give an report for this assignment which includes (1) the ***workflow and result*** of your homework; (2) your ***analysis*** of the homework; (3) any ***question or suggestion*** of the course and the homework.
- ✓ You can add your notes of this course to your homework.

4. Scoring Criteria

Unqualified: The results are incorrect, or the assignment is not written in the required format.

Qualified: Finish one of the first or second task completely.

Good: Finish both the first and the second tasks completely or complete the MPC project correctly.

OutStanding: Finish all the task completely!