Homework 1

Consider a **convex optimization** problem

$$\begin{aligned} \min \quad & f(x) \\ \text{s.t.} \quad & g(x) \leq 0 \\ & h(x) = 0 \end{aligned}$$

Its PHR Augmented Lagrangian is defined as

$$\mathcal{L}_\rho(x, \lambda, \mu) := f(x) + \frac{\rho}{2}\left\{\left\|h(x) + \frac{\lambda}{\rho}\right\|^2 + \left\|\max\left[g(x) + \frac{\mu}{\rho}, 0\right]\right\|^2\right\} - \frac{1}{2\rho}\left\{\|\lambda\|^2 + \|\mu\|^2\right\}$$

where $\rho > 0, \mu \succeq 0.$

Prove that **the PHR Lagrangian is always convex with respect to x**.

Homework 2

Provided a low-dimensional strictly convex QP solver that only solves:

$$\min_{x\in\mathbb{R}^n} \frac{1}{2}x^{\mathrm{T}}M_{\mathcal{Q}}x + c_{\mathcal{Q}}^{\mathrm{T}}x, \text{ s.t. } A_{\mathcal{Q}}x \le b_{\mathcal{Q}}$$

where $M_{\mathcal{Q}} \succ 0$.

Please design a scheme to approximately solve the case where $M_{\mathcal{Q}} \succeq 0$ but $M_{\mathcal{Q}} \nsucc 0$.

You can **combine the low-dimensional strictly convex QP solver with a "proximal" term**.

A C++ version of the solver is provided as below:

https://github.com/ZJU-FAST-Lab/SDQP

A test example for a positive semi-definite case:

$$M_{\mathcal{Q}} = \begin{pmatrix} 2 & -1 & 0 \\ 0 & 1 & -1 \\ 2 & -2 & 1 \end{pmatrix} \quad c_{\mathcal{Q}} = \begin{pmatrix} 1 \\ 3 \\ -2 \end{pmatrix} \quad A_{\mathcal{Q}} = \begin{pmatrix} 0 & -1 & -2 \\ -1 & 1 & -3 \\ 1 & -2 & 0 \\ -1 & -2 & -1 \\ 3 & 5 & 1 \end{pmatrix} \quad b_{\mathcal{Q}} = \begin{pmatrix} -1 \\ 2 \\ 7 \\ 2 \\ -1 \end{pmatrix} \quad x^* = \begin{pmatrix} -\frac{38}{3} \\ \frac{25}{6} \\ \frac{109}{6} \end{pmatrix}$$

Homework 3

Please solve the SOCP below via Conic ALM.

$$
\begin{aligned}
\min_{a,b,c,d,e,f,g\in\mathbb{R}} \quad & a+2b+3c+4d+5e+6f+7g \\
\text{s.t.} \quad & \|(7a+1, 6b+3, 5c+5, 4d+7, 3e+9, 2f+11, g+13)\| \le a+1.
\end{aligned}
$$

The approximate solution is:

$$
(-0.127286, -0.506097, -1.01317, -1.77744, -3.06097, -5.66462, -13.7682)
$$