1 Classical Mechanics

1.1 Basics: a harmonic oscillator

2 Time Integration

3 Optimization

3.1 Implicit Newmark

$$x_{n+1} = x_n + \frac{h}{2}(v_n + v_{n+1}) \tag{1a}$$

$$v_{n+1} = v_n = \frac{h}{2}(f_n + f_{n+1}) \tag{1b}$$

where x and v are positions and velocities, and $f_n \equiv f(x_n)$. Thus we have

$$x_{n+1} - x_n - hv_n = \frac{h^2}{4}(f_n + f_{n+1}) \tag{2}$$

Let $y = x_n + h_n v_n + \frac{h^2}{4} f_n$, $x_{n+1} = x$, note that $\nabla E = -f$, then (2) becomes

$$x - y = \frac{h^2}{4} f \Rightarrow x - y + \frac{h^2}{4} f \nabla E = 0 \tag{3}$$

Let $g(x) = \frac{1}{2}\|x - y|^2 + \frac{h^2}{4}E$, then solving g equals to solve x for

$$\min_x \quad g(x) \tag{4}$$

3.2 Optimization Problems

Problem formulation:

$$\min_x \quad g(x), \quad x \in R^n, \quad g \in R. \tag{5}$$

Optimization problems can be categorized into constrained or unconstrained problems, or convex or non-convex problems.

Theorem 1. *For a convex problem (where both objective and feasible set are convex), if the objective is* C^2*, then the Hessian* $H \succeq 0$*, and the local minimum is the global minimum.*

Definition 1. *A linear programming (LP) is a problem with linear objective and linear equality or inequality constraints. A quadratic programming (QP) is* **the same as LP** *except with a quadratic objective.*

Note 1. *Convex QP has polynomial time solver but non-convex QP is NP-hard.*

Example 1. *A non-convex QP:*

$$\min_x \quad \frac{1}{2}\|Ax\|^2, \tag{6a}$$

$$s.t. \quad \|x\|_2 = 1. \tag{6b}$$

Note 2. *Software package for solving non-convex problem:* IpOPT, KNITRO, *or NEOS-Guide*

3.2.1 Solving unconstrained problems

There are two ways to solve an unconstrained problem: descent method or trust-region method.

3.3 Numerical Linear Algebra

4 Elastic Materials and Finite Element Simulation

5 Something more...

6 Papers and books

1. Nonlinear Continuum Mechanics for Finite Element Analysis by J. Bonet
2. Convex optimization by S. Boyd
3. Numerical optimization by Nocedal