§ Introduction

In strong correlated systems, H is very complex, the perturbative methods are all not valid

Effective field theory construction involve projective construction

§ Projective / parton construction

(split micro D.o.F into particles
finding relevant effective D.o.F)

e.g.
- heavy fermion
- high T_c
- FQHE
- ⋮

topological phases of matter with global symm
- SPT
- SET

Example: t-J model

Strong correlation

superexchange

$$H_{tJ} = -t \sum_{\langle ij \rangle} c_{i\sigma}^{+} c_{j\sigma} + J \sum_{\langle ij \rangle} \vec{S}_i \cdot \vec{S}_j$$

weak correlation

with $\sum_{s} c_{is}^{+} c_{js} \le 1$ (non-doubly occupation, this is $U \to \infty$ expansion (limit) of Hubbard model) → low energy region.

(no $\uparrow\downarrow$)

Slave-particle technique: (one type of projective construction)

(axillary boson)

physical electron → $c_i = h_i^{+} f_{i\sigma}$ ← spinon. spin-$\frac{1}{2}$. neutral. fermionic (statistic)

↑

holon

charge -1, spinless, bosonic

Low energy subspace:

$\sum_{\sigma} c_{i\sigma}^{+} c_{i\sigma} \le 1 \Longrightarrow$ (*) single-occupancy $\boxed{\text{constraint}}$

$h_i^{+} h_i + \sum_{\sigma} f_{i\sigma}^{+} f_{i\sigma} = 1$

↑ ↓

Lagrangian multiplier

redundant (emergent) U(1) gauge symmetry

"gluing" two auxilliary D.O.F

$$\begin{cases} h_i \to h_i e^{i\theta_i} \\ f_{i\sigma} \to f_{i\sigma} e^{i\theta_i} \end{cases} \to C_{i\sigma} \text{ gauge invariant (physical)}$$

local. depend on spacetime.

Producing gauge field in CMP: dynamical; Berry phase (not Dynamical)

$\Longrightarrow$ meanfield treatment of spinon & holon

D.O.F	spin	charge

$$S \to S + \underbrace{\int \sum \overset{\text{Langrangle multiplier}}{A_\mu} (\overbrace{J_\mu^h + J_\mu^f}^{\text{gauge invariant}} - 1)}_{\text{projection to effective space } (*)}$$

$\xrightarrow{\int DA_\mu}$ δ(constraint)

① A_μ is the redundant U(1) gauge field

② $g_{UV} = \infty$ on lattice. (no Maxwell term: $\frac{1}{g^2}(1-\cos F) \Rightarrow 0 \quad g \to \infty$)

(g hidden in A)

③ Physical G.S. :

$$|\Psi\rangle = \underset{\uparrow}{\underline{P_G}} \left| \begin{array}{l} \text{meanfield of spinons} \\ \text{and holons} \end{array} \right\rangle$$

projection

this is like quantization of gauge theory

(applying the physical constraint
project to $\mathcal{H}_{phy}$ (satisfying $\sum c_i^\dagger c_i \le 1$))

due to ②, we need to suppress gauge fluctuation

introduce (Chern-Simons) top. mass
(gapping gauge fluctuation)

Example : $\boxed{\text{Laughlin } \nu = \frac{1}{3} \text{ state of FQHE}}$

$c = \underbrace{f_1 f_2 f_3}_{}$ $\underbrace{J_\mu^c = J_\mu^1 = J_\mu^2 = J_\mu^3}_{}$

color singlet

3 fermions (spinless).

bounding 3 fermion to a physical fermion.

gauge structure: $\underline{SU(3)}$

but $SU(3)$ is accidental in meanfield theory

$$SU(3) \xrightarrow[\text{usually}]{} \begin{matrix}\text{maximal}\\ \text{torus}\end{matrix} \quad \underline{U(1)\times U(1)}$$

$$c = \underbrace{f_1 \underbrace{f_2}_{U(1)} f_3}_{}$$

$A_\mu^1 \quad U(1) \quad U(1) \quad A_\mu^2$

$$\begin{cases} f_1 \to f_1 e^{i\theta_1} \\ f_3 \to f_3 e^{-i\theta_2} \\ f_2 \to f_2 e^{-i\theta_1 + i\theta_2} \end{cases}$$

meanfield ansatz:

$$|\phi_{f_i}\rangle = |LLL\rangle$$

(the Landau level need not to be same for all i (breaking $SU(3)$))

$$\Longrightarrow \begin{cases} \mathcal{L}_\mu^{\text{parton}} = \dfrac{1}{4\pi} \displaystyle\sum_{I=1}^{3} a_\mu^I \partial_\nu a_\lambda^I \varepsilon^{\mu\nu\lambda} \\ J_\mu^I = \dfrac{1}{2\pi} \varepsilon^{\mu\nu\lambda} \partial_\nu a_\lambda^I \end{cases} \qquad \begin{cases} B_a \to J_0 \\ E_a \to \vec{J} \end{cases}$$

$$(\partial_\mu J^{I\mu} \equiv 0)$$

$\Longrightarrow$

$$\mathcal{L} = \underbrace{\frac{1}{4\pi}\sum_{I=1}^{3} a^I_\mu \partial_\nu a^I_\lambda \varepsilon^{\mu\nu\lambda}}_{\substack{\text{free part} \\ \text{(meanfield)}}} + \boxed{A^1_\mu}(J^1_\mu - J^2_\mu) + \boxed{A^2_\mu}(J^2_\mu - J^3_\mu) + \underbrace{\frac{1}{3} A^{em}_\mu (J^1_\mu + J^2_\mu + J^3_\mu)}_{\text{gauge (EM) response}}$$

Langrang multiplier, redundant gauge field

confining (each U(1))

two constraint ($U(1) \times U(1)$)

$\nu = \frac{1}{3}$ state

$\Longrightarrow$ integrate out A^1_μ, A^2_μ.

$(a^1_\mu = a^2_\mu = a^3_\mu = a_\mu)$

$\hookrightarrow$ projection $\Rightarrow$ only one relevant DOF

$$\boxed{\mathcal{L} = \frac{3}{4\pi} a_\mu \partial_\nu a_\lambda \varepsilon^{\mu\nu\lambda} + \frac{1}{2\pi} A^{em}_\mu \partial_\nu a_\lambda \varepsilon^{\mu\nu\lambda}}$$

Chern-Simons theory of $\nu = \frac{1}{3}$ Laughlin state.

<u>Rmk</u> In Strong — systems, "guessing" is very important

$\longrightarrow$ $(z_i - z_j)^3$

$\uparrow$ 3 partons confined to become a physical e

$\Longrightarrow$ gauge responses:

integrate out a_μ

$$\mathcal{L}[A_\mu^{em}] = \frac{1}{4\pi}\frac{1}{3} A_\mu^{em} \partial_\nu A_\lambda^{em} \varepsilon^{\mu\nu\lambda}$$

$$\Longrightarrow \sigma_{xy} = \frac{1}{2\pi}\frac{1}{3} \underset{\text{dimension}}{=} \boxed{\frac{1}{3}\frac{e^2}{h}}$$

$\Longrightarrow$ integrate out $a_\mu^1, a_\mu^2, a_\mu^3$

$\mathcal{L}^{eff}[A_\mu^i] \sim A^i \partial A^i$ topological mass generated by C-S terms (suppressing gauge fluctuation)

C-S >> Maxwell

meanfield method valid

Another example: see the slide

result:

① diagonal of K in effective C-S: all even

$\Longrightarrow$ bosonic

② $|\det K| = 1 \Longrightarrow$ GSD on torus is unique.

(no topological order)

③ # positive eigenvalue = # negative eigenvalues

$\Rightarrow$ non-chiral (central charge $\Rightarrow 0$)

chiral

BF term (3 e l d)

topological

trivial

nontrivial

$$\left.\begin{matrix}\text{trivial}\\ \\ \text{nontrivial}\end{matrix}\right\} \text{TQFT} \xrightarrow{\text{symmetry}} \left\{\begin{matrix}\text{SPT phase.}\\ \text{SET.}\end{matrix}\right.$$

For more detail, refer to the 2016 paper.