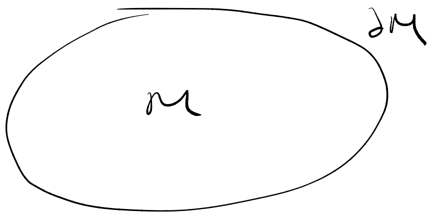
Bulk - boundary correspondence :



QFT in ∂M (CFT, TQFT, ...)

$\xrightarrow[\text{determine}]{\text{many to one}}$ top. phase in M

BQFT carry anomaly of bulk is top. non-trivial.

Such BQFT can't be regularized on a lattice

anomaly : 't Hooft (related to symmetry) (usually gauge)

couple to gravity (background) and check diffeomorphism

gravitational (related to top.)

gauge - gravitational mixed

invertible / non-invertible anomaly

(no / have (GSD, anyon (or extended object))

⋮

top. order { category : anyon
TQFT

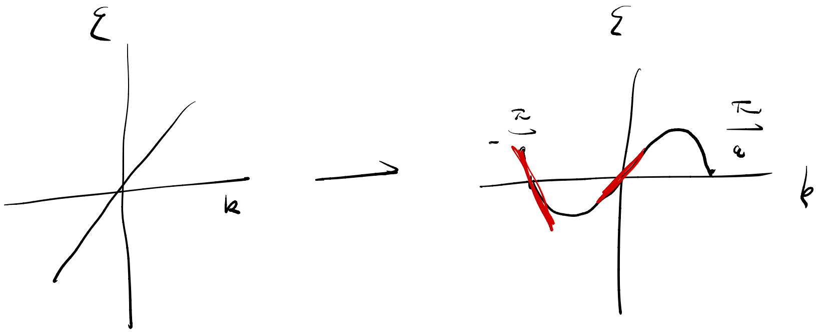
anomalous TQFT $\longrightarrow$ boundary of some T.O.

① IQH: $\sigma_{xy} = N_R - N_L$ ABJ anomaly   Nielsen-Ninomiya

1D chiral fermion $H = \int dx\,(-iv)\,\psi^\dagger \partial_x \psi$

$\xrightarrow{\text{lattice regularization}}$ $H = \sum_n \frac{-iv}{2}\left[\psi_n^\dagger \psi_{n+1} - \psi_{n+1}^\dagger \psi_n\right]$

$\rightarrow \mathcal{E} = \frac{v}{a} \sin ka$

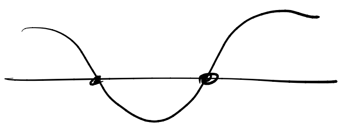


Fermion doubling!

$N_L = N_R$

(no ABJ anomaly)

Fermi surface:



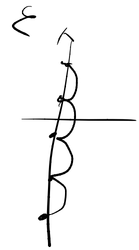
Coupling to A_μ $\Rightarrow$ $\partial_\mu j^\mu_{U(1)} = -\frac{1}{4\pi}\varepsilon^{\mu\nu}F_{\mu\nu}$

$\times F$

Spectrum flow:

$$A_x = -\frac{2\pi}{L}\frac{t}{T} \Rightarrow \varepsilon = vk - A_x = vk + \frac{2\pi}{L}\frac{t}{T}$$

$\uparrow$ insert 2π flux

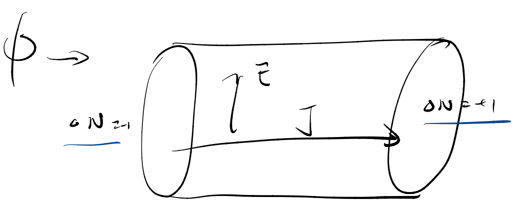


$\Rightarrow$ total charge $\Delta N = 1$ $\left(\partial_\mu j^\mu_{U(1)} \neq 0\right)$

particle not conserved

This is equivalent to Laughlin's argument:

(charge pump · spectrum flow)



boundary is anomalous

can't live on its own, must be boundary of some bulk.

no anomaly in whole

(anomaly cancellation)

$\longrightarrow$ Actually gravitational anomaly

② $H = H_L + H_R \longrightarrow \underset{\substack{\uparrow \\ N_L + N_R}}{U(1)_{em}} , \underset{\substack{\uparrow \\ N_L - N_R}}{U(1)_{axial}}$ symmetry

(chiral)

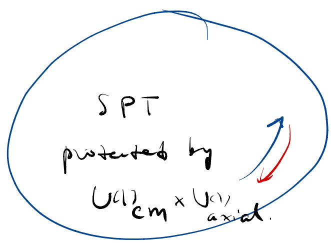
Axial anomaly

$$\partial_\mu j_L \propto \underset{\substack{\uparrow}}{*F} \qquad \partial_\mu j_R \propto - *F \quad \Longrightarrow \quad \partial_\mu j^\mu_{em} = 0$$

$$\partial_\mu j^\mu_{axial} = \frac{1}{2\pi} *F$$

turning on background EM field $(U(1)_{em})$ (gauge response)

anomalous boundary $\Longrightarrow$



't Hooft anomaly

global symmetry $\xrightarrow[\text{turning on gauge field}]{\text{gauging}}$ inconsistency

(including discrete, continuous)

SPT / $\boxed{\text{SET}}$ on M 't Hooft anomalies ∂M.

more studied

· bulk T.O $\longleftrightarrow$ boundary anomalous QFT

In this talk:

2D TQFT with symmetry + 3D bulk
(SET)

Example:

abelian FQH:

symmetry properties

$$\mathcal{L} = \frac{m}{4\pi} \varepsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda + \frac{\boxed{1}}{2\pi} \varepsilon^{\mu\nu\lambda} A_\mu \partial_\nu a_\lambda$$

anyon spectrum
- statistic
- charge
- ⋮

TQFT
(Chern-Simons)

External coupling (probing)
(gauge response)

$\longrightarrow$ anyon fractional charge

(different type of T.O. (SET) depend on parameter of symmetry properties)

Enrich.

Example T.O. $\xrightarrow{U(1)}$ 4 types

Rmk. categorical description (of anyon: quantum dimension, fusion ring, braiding, ...) $\equiv$ TQFT description (defined by top. inv. = observables, no need for $\mathcal{L}$)

Local symmetry action

local degeneracy

$$U(g)|a,\mu\rangle = \sum_{\nu} (U_g^a)_{\nu\mu} |a,\nu\rangle$$

anyon state

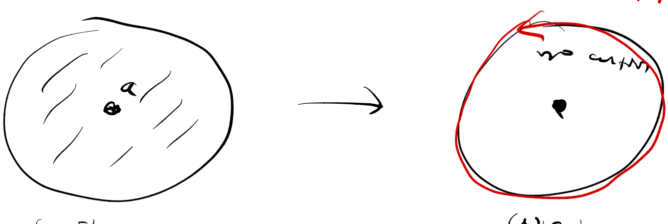
projective rep of G

$$U_g^a U_h^a = e^{i\alpha_a(g,h)} U_{gh}^a$$

Physical meaning $\alpha_a(g,h) = \theta_{a,\omega(g,h)}$ ← mutual statistics

anyon 1 anyon 2

mutual statistics



← boundary of local action

$U_{gh}^{-1} U_g U_h$

$\omega(g,h)$ = string operator

$\left\{\begin{array}{l} \text{2-cocycle for } \omega = \text{fusion of anyons} \\ \text{coboundary transformation} = \text{anyon phase redefinition} \end{array}\right.$

PVIR of G = anyon fractional charge.

= symmetry properties of TQFT (SET)

These TQFT (SET) carry anomaly

(being boundary to some SPT bulk)

for example

anomaly indicator

(SET)		(SPT)
$\sum$ boundary anyon properties	=	bulk T.O. physical property