SPT phases in $\boxed{\text{free}}$ fermion systems are classified by K theory

but interaction is usually unavoidable.

Example:

- 1D Haldane chain: (interacting bosonic)

  meanfield (large S, semiclassical) → spin wave

  but Haldane find it's not the case.

  to study the H_{Haldane}, we can deform to AKLT model (no phase transition, preserving symmetry)

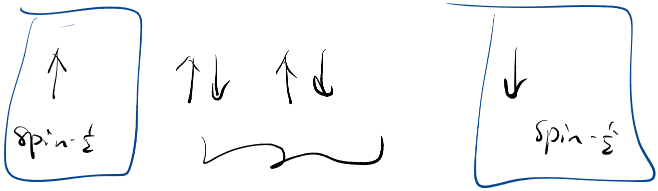
AKLT: $$H = \sum_i P_2(S_i \cdot S_{i+1}) = \sum_i \Big[\tfrac{1}{2} S_i \cdot S_{i+1} + \tfrac{1}{6}(S_i \cdot S_{i+1})^2 + \dots\Big]$$

spin-2 ↓ (at $S_i \cdot S_{i+1}$); Haldane ↓ (at $\frac{1}{2} S_i \cdot S_{i+1}$); same phase ↓ (at $\frac{1}{6}(S_i \cdot S_{i+1})^2$)

↑ (at P_2) project out spin 2 (only spin 0, 1).

Spin-1: $\underbrace{\uparrow\ \uparrow}_{\boxed{\text{relevant D.o.F}}}$ (spin $\frac{1}{2}$) project out spin-0 → spin-1 effective theory

$\Rightarrow$ G.S.



bulk: decoupled spin-$\frac{1}{2}$

preserving spin-rotation symmetry

Further:

1) spin-0 + perturbation (not changing phase)

$\Rightarrow$

Fixed point $\Psi_{G.S.}$

(renormalization = adiabatic deforming H (perturbing)

fixed pt: Long-range limit, only topology matters)

here: projection is ruled out

spin-$\frac{1}{2}$ is free D.o.F on boundary

Key observation:

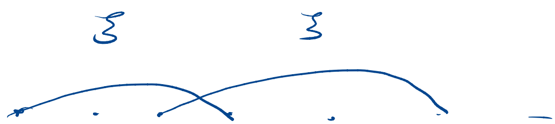
edge state form $\boxed{\text{PUR}}$ of G

$H^2(G, U(1))$ nontrivial elements

SPT phase is gapped G.S. with unique ground state and the correlation is finite length (short-range order)

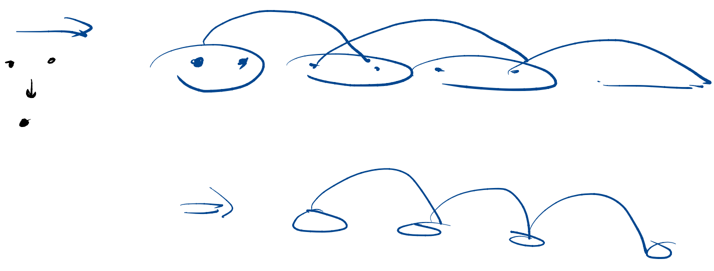
RG fixed point ψ:

out of correlation length: tensor product states



renormalization

粗粒化

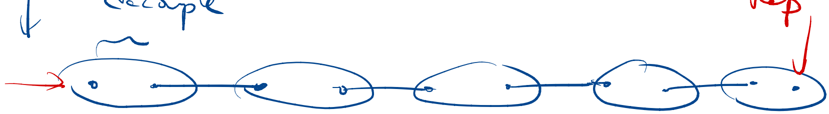


U_s

decouple

ω^{-1}

projective rep ω



symmetric maximally entangled state.

any 1+1d gapped phase!

(including fermionic ???)

$\Longrightarrow$ classification of 1D bosonic SPT phase

$\longrightarrow H^2(G, U(1))$ 2-cocycle ω

(classifying through boundary state in G.S. (wave functional))

Example. in higher dim

1. Levin - Gu model

磁矩 $\xrightarrow{\text{condensation}}$ disordered phase (paramagnetic phase)

(Symmetry breaking order vanishes)

Two disordered phase

$$\begin{cases} \text{trivial} & |\Psi_{G.S.}\rangle = \sum (+1)^{\#d.w.} \,|\text{domain walls}\rangle \\ \text{nontrivial} & |\Psi_{G.S.}\rangle = \sum (-1)^{\#d.w.} \,|D.W.\rangle \end{cases}$$

domain wall

Like $\boxed{(\text{dual}!)}$ $\begin{cases} \text{T.C.} \\ \text{Double semion.} \end{cases}$ closed string

G.S $\Longrightarrow$ exactly soluble H

dual! (lattice)

T.O. $\Longleftrightarrow$ SPT

D.O.F　　　　D.O.F

duality: D.o.F $\to \uparrow$ $\mathbb{Z}_2$ gauge D.o.F in T.C.

$\Rightarrow \downarrow$

no $\mathbb{Z}_2$ anomaly $\Longleftrightarrow$ $U \to \infty$ in T.C.

(terms breaking $\mathbb{Z}_2$)

external magnetic field $\longleftrightarrow$ string tension.

Double semion can be described using Chern-Simons

similar for T.C.

Anomaly detecting : detecting nontrivial phase

global G ⟶ couple to gauge-G field

→ flux, charge, anyon ...

Example :

(flux ↔ charge duality with double semion)

nontrivial Ising phase ($\mathbb{Z}_2$), gauging → flux carry semion statistics

⟹ degenerate edge state

(Heisenberg algebra have no 1D rep !)

It can be shown the boundary is anomalous, though bulk is trivial (similar with T.I.T.S.C)

SPT phases are classified by

(group action)

↓

equivalence class of generalized symmetric

LU transform with 1-dim support space

(short range entangled)

branching: remove symmetry from orientation of graph itself (only topological)

↓

more general wave functions

no more constraint from graph orientation symmetry

Result:

Interacting bosonic $\longrightarrow$ $\mathcal{H}^{\text{ted}}(G, U(1))$

$\Updownarrow$

(closed manifold) <u>unique</u> ground state Ψ_{GS} (fixed-point)

and exactly solvable lattice models (H)

<u>group super-cohomology</u>

(physically, putting extra fermion/Majorana-chain .... on the lattice).

$\longrightarrow$ differential geometry

(supersymmetric QM)