$\nu = \frac{1}{3}$, Laughlin state:

$$\Psi_3 = \prod_{i<j} \underline{(z_i - z_j)^3}\, e^{-\frac{1}{4}\sum_i |z_i|^2}$$

↳ fractional charge / statistics

⟶ 90s T.O. Wen.

T.O. Def: Gapped quantum phases without Sym Brking and Long-range correlation, but can't be adiabatically connected to a trivial disorder phase without phase transi.

Symmetry breaking state is a trivial gapped state

no long-range order (correlation)

trivial: tensor product state

for fermion: atomic insulator

top. phase
- intrinsic top. phase (Long-range entanglement) most order!
- SPT (short-range entanglement)

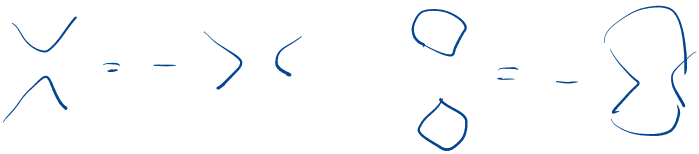
Example: toric code

$\downarrow$

1. double semion model: (Lattice projective H model)

rule for determine wave function

$\longrightarrow$ local deformation of $\mathbb{Z}_2$ string
(action of H, e.g)



$$\Longrightarrow \quad |\Psi\rangle = \sum_{\partial X = 0} (-1)^{n(X)} |X\rangle$$

excitation: semion.

$\mathbb{Z}_2$ gauge theory

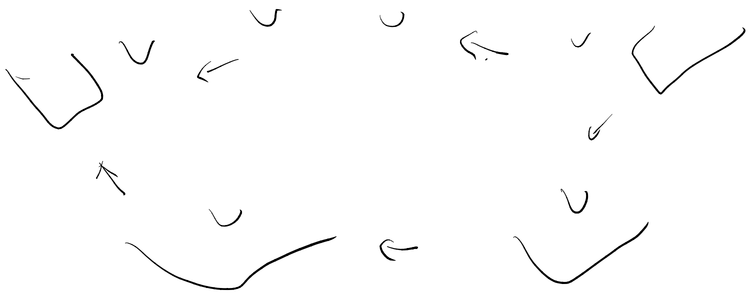
$H^2(\mathbb{Z}_2, U(1))$ $\begin{cases} \text{T.C} \\ \text{double semion.} \end{cases}$

Topological consistent condition:

for fixed-point wavefunction

(RG. long-range. effectively depend only on topology)

$\Longrightarrow$ F-move (local)



Classifying T.O. with G.S.-ψ

$$\left[\psi_1 \xleftrightarrow[\text{local unitary evolution}]{} \psi_2 \right]$$

local unitary evolution ← adiabatic evolution of H w/o closing gap

Long-range entanglement (in G.S.)

LU $\not\Longrightarrow$ tensor product state.

(short-range entangled).

⊗ extra states won't change the phase !

support space

the reduced matrix of a ψ in region A

may act on subspace of $\mathcal{H}_{tot}$

basis transformation.

$\longrightarrow$ generalized LU $\quad U_g = U_1 P_g U_2$

$$|\Psi\rangle = \sum_{\text{conf}} \Psi_{\substack{\text{fix} \\ \text{point}}}(\text{conf})\,|\text{conf}\rangle$$

conditions

Input data: phase, ...

It's hard to construct lattice model for chiral top. phase! (fermion doubling anomaly)

Ψ-$\boxed{\text{renormalization}}$ (long-range, fixed-point only topology matter, all triangles should be equivalent)

F-move: changing basis = retriangulation (dual lattices)

$\longrightarrow$ consistent condition: Pentagon eqn (solving possible F)

GLU (part of)

By applying all 4 types of wave-function renormalization

(GLU of G.S., Ψ_{fix}, topological)

one can deform $\boxed{\Psi_{G.S.}}$ from one trivalent graph to any other trivalent graph. the consistency conditions and unitary conditions gives eqn of these data, these solution define UTC

Fermionic lack this condition

$\Longrightarrow$ there're enough (with saturate condition) to classify all 2D $\boxed{\text{non-chiral}}$ Bosonic topological order (phases) (with $\Psi_{G.S.}$)

models $\longrightarrow$ Local H (projective : exactly soluable, gapped)