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P223.29

设X为取值1,2,3的随机变量,分布列如下.

$P(X=1)=\frac{1}{2}$, $P(X=2)=\frac{1}{4}$, $P(X=3)=\frac{1}{4}$.

求X的矩母函数并且得到前三个矩 $E[X]$, $E[X^2]$, $E[X^3]$.

解: 矩母函数 $M(s)=E[e^{sx}]=\frac{1}{2}e^{s}+\frac{1}{4}e^{2s}+\frac{1}{4}e^{3s}$

$$E(X)=\frac{dM(s)}{ds}\Big|_{s=0}=\left(\frac{1}{2}e^{s}+\frac{1}{2}e^{2s}+\frac{3}{4}e^{3s}\right)\Big|_{s=0}=\frac{1}{2}+\frac{1}{2}+\frac{3}{4}=\frac{7}{4}.$$

$$E(X^2)=\frac{d^2M(s)}{ds^2}\Big|_{s=0}=\left(\frac{1}{2}e^{s}+e^{2s}+\frac{9}{4}e^{3s}\right)\Big|_{s=0}=\frac{1}{2}+1+\frac{9}{4}=\frac{15}{4}.$$

$$E(X^3)=\frac{d^3M(s)}{ds^3}\Big|_{s=0}=\left(\frac{1}{2}e^{s}+2e^{2s}+\frac{27}{4}e^{3s}\right)\Big|_{s=0}=\frac{1}{2}+2+\frac{27}{4}=\frac{37}{4}.$$