



P265 1.

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1. 有A, B, C三个厂生产的电池, 各随机抽取5只电池, 经试验得其寿命(h)如下:

A	40	42	48	45	38
B	26	28	34	32	30
C	39	50	40	50	43

试在显著性水平 0.05 下检验电池的
平均寿命有无显著性差异。

若是求 $\mu_A - \mu_B$, $\mu_A - \mu_C$ 和 $\mu_B - \mu_C$
的置信水平为 95% 的置信区间。

解: 以 μ_A, μ_B, μ_C 表示 A, B, C 厂电池平均寿命
在 $\alpha = 0.05$ 下假设

$$H_0: \mu_A = \mu_B = \mu_C$$

$$H_1: \mu_A, \mu_B, \mu_C \text{ 不相等}$$

将 A, B, C 三厂抽出的样本依次记为 $x_{11}, x_{21}, \dots, x_{n1};$

$x_{12}, x_{22}, \dots, x_{n2}; x_{13}, x_{23}, \dots, x_{n3}$. 且 $S=3$.

$$n_1 = n_2 = n_3 = 5, n = n_1 + n_2 + n_3 = 15,$$

$$T_1 = \sum_{i=1}^5 x_{i1} = 213, T_2 = \sum_{i=1}^5 x_{i2} = 150,$$

$$T_3 = \sum_{i=1}^5 x_{i3} = 222, T_{..} = \sum_{j=1}^3 \sum_{i=1}^5 x_{ij} = 585.$$

$$S_T = \sum_{j=1}^3 \sum_{i=1}^{n_j} x_{ij}^2 - T_{..}^2/n = 23647 - 22815 = 832,$$

$$S_A = \sum_{j=1}^3 T_j^2/n_j - T_{..}^2/n = 23430.6 - 22815 = 615.6,$$

$$S_E = S_T - S_A = 216.4,$$

S_T, S_A, S_E 的自由度分别为 $n-1=14, S-1=2, n-S=12$.

方差分析表如下所示:

方差来源	平方和	自由度	均方	$F_{\alpha}(d=0.05)$
因素A	615.6	2	$\bar{S}_A = 307.8$	$\bar{S}_A / \bar{S}_E = 17.07$
误差E	216.4	12	$\bar{S}_E = 18.03$	
总和T	832	14		

由于 $F_{0.05}(2, 14) = 3.81, F_{\alpha} = 17.07 > 3.81$
所以在显著性水平 0.05 下拒绝 H_0 .

现在来求 $\mu_A - \mu_B, \mu_A - \mu_C$ 和 $\mu_B - \mu_C$ 置信
水平为 $1 - \alpha = 0.95$ 的置信区间。

$$t_{\alpha/2}(n-S) = t_{0.025}(15-3) = 2.1788,$$

$$t_{0.025}(12) \times \sqrt{\bar{S}_E \left(\frac{1}{n_i} + \frac{1}{n_k} \right)} = 5.85 \quad (i, k = 1, 2, 3),$$

$$\hat{\mu}_A = \bar{x}_1 = T_1/5 = 42.6,$$

$$\hat{\mu}_B = \bar{x}_2 = T_2/5 = 30,$$

$$\hat{\mu}_C = \bar{x}_3 = T_3/5 = 44.4,$$

可以分别求出对应置信水平为 95%
的置信区间为:

$$(\hat{\mu}_A - \hat{\mu}_B \pm 5.85) = (6.75, 18.45),$$

$$(\hat{\mu}_A - \hat{\mu}_C \pm 5.85) = (-7.65, 4.05),$$

$$(\hat{\mu}_B - \hat{\mu}_C \pm 5.85) = (-20.25, -8.55).$$