$$
\begin{aligned}
\alpha_t|\alpha_{t-1} &\sim \mathcal{N}(\alpha_{t-1}, \sigma^2 I) \\
\hat{\alpha_t}|\alpha_t &\sim \mathcal{N}(\alpha_t, \hat{T_t}^2 I) \\
m_t &= (\frac{\hat{T_t}^2}{V_{t-1}+\sigma^2+\hat{T_t}^2})m_{t-1} + (1-\frac{\hat{T_t^2}}{V_{t-1}+\sigma^2+\hat{T_t^2}})\hat{\alpha}_t \\
V_t &= (\frac{\hat{T_t^2}}{V_{t-1}+\sigma^2+\hat{T_t^2}})(V_{t-1}+\sigma^2) \\
\widetilde{m}_{t-1} &= (\frac{\sigma^2}{V_{t-1}+\sigma^2})m_{t-1} + (1-\frac{\sigma^2}{V_{t-1}+\sigma^2})\widetilde{m}_t \\
\widetilde{V}_{t-1} &= V_{t-1} + (\frac{V_{t-1}}{V_{t-1}+\sigma^2})^2(\widetilde{V}_t - (V_{t-1}+\sigma^2)) \\
L(\alpha_t) &\geq -\frac{1}{2\sigma^2}\|\tilde{m}_t - \tilde{m_{t-1}}\|^2 - \frac{1}{\sigma^2}Tr(\tilde{T}_t) + \frac{1}{2\sigma^2}(Tr(\tilde{V_0} - Tr(\tilde{V_T})) \\
&+ \sum_t(\sum_n \mathbb{E}[s_{t,n}]\sum_m y_{t,n,m})\alpha_{t,0} + \sum_t(\sum_n \mathbb{E}[s_{t,n}]\sum_m(1-y_{t,n,m}))\alpha_{t,1} \\
&- \sum_{t,n}\mathbb{E}[s_{t,n}]\ln\hat{\xi}_t - \sum_{t,n}\mathbb{E}[s_{t,n}]\frac{\mathbb{E}[\exp\alpha_{t,0}+\exp\alpha_{t,1}]}{\hat{\xi}_t} + const \\
&= -\frac{1}{2\sigma^2}\|\tilde{m}_t - \tilde{m_{t-1}}\|^2 - \frac{1}{\sigma^2}Tr(\tilde{T}_t) + \frac{1}{2\sigma^2}(Tr(\tilde{V_0} - Tr(\tilde{V_T})) \\
&- \sum_{t,n}\mathbb{E}[s_{t,n}]\ln\hat{\xi}_t - \sum_{t,n}\mathbb{E}[s_{t,n}]\frac{\mathbb{E}[\exp\alpha_{t,0}+\exp\alpha_{t,1}]}{\hat{\xi}_t} + const \\
&(\sum_t(\sum_n \mathbb{E}[s_{t,n}]\sum_m y_{t,n,m})\alpha_{t,0} = \sum_t(\sum_n \mathbb{E}[s_{t,n}]\sum_m(1-y_{t,n,m}))\alpha_{t,1} = 0) \\
\frac{\partial L_(\alpha_t)}{\partial\hat{\xi}_t} &= -\frac{\sum_{t,n}\mathbb{E}[s_{t,n}]}{\hat{\xi}_t} + \frac{\sum_{t,n}\mathbb{E}[s_{s,t}]\mathbb{E}[\exp\alpha_{t,0}+\exp\alpha_{t,1}]}{\hat{\xi}_t^2} \\
&= 0 \\
\hat{\xi}_t &= \mathbb{E}[\exp\alpha_{t,0}+\exp\alpha_{t,1}] \\
&= \sum_{i=0,1}\exp(\tilde{m}_{t,i} + \tilde{V}_{t,i}/2) \\
L(\alpha_t) &\geq -\frac{1}{2\sigma^2}\|\tilde{m}_t - \tilde{m_{t-1}}\|^2 - \frac{1}{\sigma^2}Tr(\tilde{T}_t) + \frac{1}{2\sigma^2}(Tr(\tilde{V_0} - Tr(\tilde{V_T})) \\
&- \sum_{t,n}\mathbb{E}[s_{t,n}]\ln\hat{\xi}_t - \sum_{t,n}\mathbb{E}[s_{t,n}] + const
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial L_(\alpha_t)}{\partial \hat{\alpha}_t} &= -\frac{1}{\sigma^2}(\widetilde{m}_t - \widetilde{m}_{t-1})(\frac{\partial \widetilde{m}_t}{\partial \hat{\alpha}_t} - \frac{\partial \widetilde{m}_{t-1}}{\partial \hat{\alpha}_t}) \\
&= 0 \\
\frac{\partial m_{t'}}{\partial \hat{\alpha}_t} &= (\frac{\hat{T}_t^2}{V_{t-1} + \sigma^2 + \hat{T}_t^2})\frac{\partial m_{t'-1}}{\partial \hat{\alpha}_t} \\
&\quad +(1 - \frac{\hat{T}_t^2}{V_{t-1} + \sigma^2 + \hat{T}_t^2})\delta_(t', t)
\end{aligned}
$$

with the initial condition

$$
\frac{\partial m_0}{\partial \hat{\alpha}_t} = 0
$$

$$
\frac{\partial \widetilde{m}_{t-1}}{\partial \hat{\alpha}_t} = (\frac{\sigma^2}{V_{t-1} + \sigma^2})\frac{\partial m_{t-1}}{\partial \hat{\alpha}_t} + (1 - \frac{\sigma^2}{V_{t-1} + \sigma^2})\frac{\partial \widetilde{m}_t}{\partial \hat{\alpha}_t}
$$

with the initial condition

$$
\frac{\partial \widetilde{m}_t}{\partial \hat{\alpha}_t} = \frac{\partial m_t}{\partial \hat{\alpha}_t}
$$