Problem Chosen	2024	Team Control Number
B	MCM/ICM Summary Sheet	2401859

Submarine localization and search strategy based on hydrodynamics and Monte Carlo simulation

Summary

As the production and deployment of submarines escalate, nowadays it's a pressing issue how to search for a submarine in case of communication loss or mechanical defect. In this paper, we comprehensively take multiple factors into consideration and set up models to search for missing submersibles.

For problem 1: We construct **a hydrodynamic-based model of the movement of an unpowered submarine,** including two situations in which it is at a neutral buoyancy point and at the bottom. To locate a possible area where the missing submersible might appear, we use **Gaussian random process** to simulate the current and adopt **Monte-Carlo simulation** figure out the possible area over time. We also analyze uncertainties brought by different factors.

For problem 2: We fix sonars, magnetic sondes and ROVs as device to take for rescue vessels. Based on **sector search method**, we give the formula to calculate time needed to check a suspicious object. By calculating t**he function of combining time and economic benefit**, we establish the mode to equip each rescue vessel with one sonar, one magnetic sonde and one ROV as a search unit.

For problem 3: In the first part we use **multilevel Monte-Carlo simulation** to determine the searching area. In the second part, we adopt **tree search algorithm** and **rewarding function** to find the search route. By iterating the process we ultimately locate the best search route along which it's most possible to find the missing submersible. With emulation of motion of the missing submersible, we can calculate the probability for the submersible to be found based on **Bayes theory**. We set the threshold value of the probability to be **95%**. We think the submersible could be successfully found if the probability is over 95% Via results from repeat of emulation and simulation, we can calculate the success rate. Considering success rate, total price of device and search time, We used **analytic hierarchy process (AHP)** to evaluate strategies, and figure out that putting in 7 vessels for searching is the best strategy.

For problem 4: We extrapolate our model from the Ionian Sea to the Caribbean Sea. Firstly with adjustments about parameters in our initial model, we give prediction of possible location of a missing submersible in the Caribbean Sea. Secondly we set up a model for search of multiple submersibles. In the new model, we still set **95%** as the threshold value for each submersible to be found. And we give the search route with help of **5** rescue vessels.

After set-up of our models, we analyze the sensitivity of our model. We change the value of specific parameters and find out that our model has good stability.

At the end of our paper, we conclude the strengths and weaknesses of our model. We discuss further about the adaptability and optimization of our model as well. Finally, we write a memo based on our model to the Greek government to ask for approval of our project.

Keywords: Fluid dynamics, Monte-Carlo simulation, Tree search algorithm

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1 Introduction

1.1 Problem Background

With the rapid development of marine technology, submarines have not only become a military focus, but also attract attention as a new tool for underwater sightseeing. Nevertheless, in recent years there are news reports about submarine wrecks from time to time. Before putting submersibles into tourism practice, security is supposed to come as the first priority for companies, which results in urgent need of safety procedures in case of emergence.

The movement of submersibles could be affected by numerous factors besides its own mechanical power. When accidents happen, loss of communication or mechanical defect contributes to the difficulty of position prediction as well. This paper comprehensively takes factors above into consideration and build models to help predict the location of missing submarines with advice about how to search for them.

1.2 Restatement of the Problem

Our work aims to help the company MCMS establish a set of rescue procedures if they lose connection with any submersible. The problems we need to solve come as follows:

• Problem 1: If a submersible loses connection, develop a model to predict its location over time with the information sent prior to the accident. What equipment would be needed? What might cause the uncertainty?

• Problem 2: What search equipment should the host ship and rescue vessels bring additionally to assist? We have to take costs, maintenance and availability into account.

• Problem 3: Develop a model for searching procedure with results from previous two problems. How can we minimize the searching time? How possible can we find the missing submersible? What might be the searching results?

• Problem 4: Besides the Ionian Sea, how can we apply our model to other sea areas such as the Caribbean Sea? How to adjust our model if there are multiple submersibles moving in the same area?

1.3 Our Approach

The whole problem requires us to both predict the location of some missing submersibles and come up with a searching strategy considering financial and time efficiency. Our work mainly includes the following:

• For problem 1: We analyze the motion of an unpowered submersible according to fluid dynamics and set up a physical model. To deal with uncertainty of current, we adopt Gaussian random process to simulate possible velocity of current. With simulated current and physical motion model, we give the prediction of the submersible's position over time.

• For problem 2: We search documents and determine three kinds of device needed for rescue vessels additionally. For each rescue vessel, we figure out the strategy about how to equip device based comprehensive consideration of time and financial efficiency.

• For problem 3: We firstly locate possible area of the missing submersible by Monte-Carlo simulation. Based on the possible area, we set up a model to predict the best search route utilizing tree search algorithm and rewarding function.

• For problem 4: We extrapolate our model to the area of the Caribbean Sea and give a result of position prediction based on certain initial information. Then we consider that there're multiple submersibles to be found. We make some adjustments to the model in problem 3 and give prediction of submersibles' positions with advice about the searching route.

2 Assumptions and Explanations

Considering that practical problems always contain complex factors, we firstly make some reasonable assumptions to simplify our model, which are followed by their corresponding explanations.

• Assumption 1: All submersibles are voyaging in the deep ocean over 200 meters.

△Explanation: Firstly, the company we work for aims to carry passengers to the deepest of ocean. Secondly, a submersible diving less than 200 meters might be able to operate rescue on their own.

• Assumption 2: We take the position of the last communication as the initial position where a submersible loses contact with the host ship. Before an incident occurs, the captain is able to send a message to the host ship

• Assumption 3: The submersible without power is probably positioned at some point of neutral buoyancy or on the seabed.

△Explanation: Over time, a submersible with mechanical defects is very likely to reach a balanced state in vertical direction.

• Assumption 4: We regard the structure of a submersible as an ellipsoid.

△Explanation: Ellipsoid structure is similar to the shape of submersibles and helps simplify the model due to high symmetry.

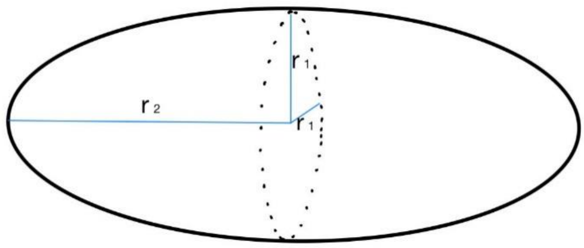

Figure 1: Structure of a submersible in our model

3 Notations

The key mathematical notations used in this paper are listed in Table 1.

Table 1: Notations used in this paper

Symbol	Description	Unit
η	Coefficient of Viscosity of Seawater	$Pa \cdot s$
ρ	Density of Seawater	kg/m^3
r_1	Length of semi-minor axis of submersibles	m
r_2	Length of semi-major axis of submersibles	m

m	Weight of a submersible	kg
$v_{current}$	Velocity of the current near the submersible	$m \cdot s^{-1}$
v_{sail}	Velocity of the submersible	$m \cdot s^{-1}$
a	Accelerated velocity of submersible	$m \cdot s^{-2}$
R_1	Search radius of a sonar	m
R_2	Search radius of a magnetic sonde	m
R_3	Search radius of a ROV	m
Σ	Covariance matrix of two-dimensional current velocity	

4 Model 1: Location Prediction Based on Monte-Carlo Simulation

4.1 Data Extraction

In order to comprehensively analyze the movement of submersibles underwater, we need the data about the speed and direction of currents, the temperature and density of seawater and the topography of the seabed.

For information about current, we visit the website of *Moored Current Meters* and extract relevant data[1]. For the temperature, salinity and density of seawater, we extract data from *World Ocean Database.* We also refer to a website which visualizes the motion of current globally[2]. Lastly, we access global topographic data with accuracy of 1 second of arc from *Generic Mapping Tools*[3].

After checking borders of the Ionian Sea from Wikipedia, we determine 36~39E, 17~21N as an approximate area of the Ionian Sea. The topographic figure goes as follows.

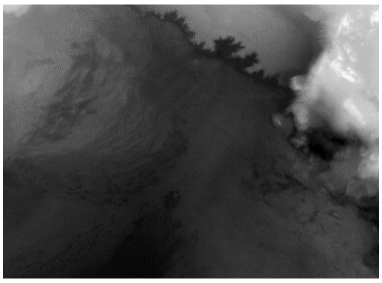

Figure 2: Topography of the Ionian Sea

4.2 Build Physical Model Based on Fluid Dynamics

For a rigid body underwater, we separate the force exerted on it to two kinds, one in vertical and the other in the horizontal plane. As assumed, the submersible stays stationary vertically. So we care more about analysis of the force in the horizontal plane, which is mainly brought by the current. To analyze different occasions, we respectively consider that a submersible stays at points of neutral buoyancy and on the seabed.

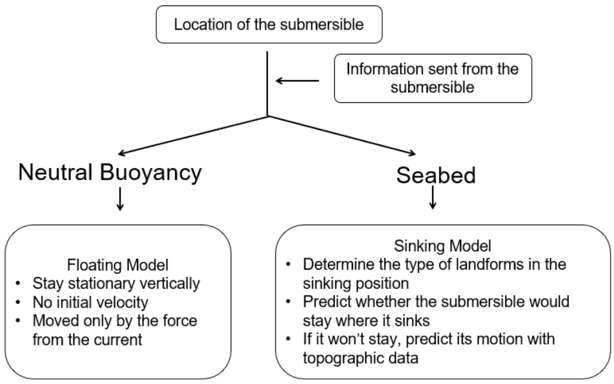

Figure 3: Mind map of analysis

4.2.2 A Submersible Stays at Some Point of Neutral Buoyancy

When a rigid body moves underwater, it is mainly affected by impact force and viscous force from seawater. We assume that impact force is caused by pressure difference. According to Bernoulli Equation, the formula to calculate impact force is as follows:

$$F_{impact} = \frac{1}{2}\rho S v^2 \tag{1}$$

Where ρ stands for the density of seawater, $\boldsymbol{v}$ equals to $\boldsymbol{v_{current}} - \boldsymbol{v_{sail}}$ as relative velocity and S for area of the projection of the submersible on the plane that is vertical to the direction of $\boldsymbol{v}$.

According to Newton's law of viscosity, viscous force is caused by velocity gradient of movement in fluid. The formula is as follows:

$$F_{viscous} = \eta S \frac{dv}{dx} \tag{2}$$

η stands for coefficient of viscosity, which can be determined by table look-up.

Because the velocity gradient is hard to compute and the submersible could be regarded as a whole, we replace the differential equation with difference equation to calculate

$$F_{viscous} = \eta S \frac{dv}{dx} \approx \eta S \frac{\Delta v}{\Delta x} \tag{3}$$

It's noteworthy that two kinds of force above are all caused by relative motion between the submersible and seawater. Therefore, the direction of force is parallel to the direction of relative velocity.

To find the numerical value of force, the first step is to calculate the area S. With accordance to our assumptions, the direction of current's velocity is parallel to the horizontal plane. In geometry the projection of a submersible is ellipse, of which the length of semi-minor axis is r_1. So the only thing left is to find the length of semi-major axis. Let's consider a plane parallel to the horizontal plane which passes through the center of the ellipsoid.

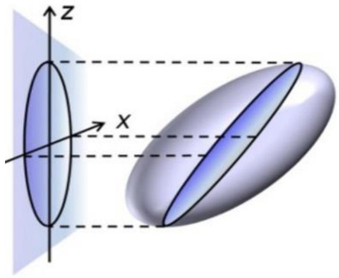

Figure 4: Projection of ellipsoid

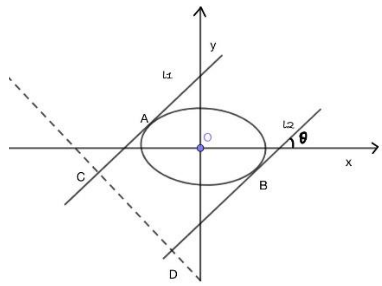

Figure 5: section plane in coordinate system

In the two-dimensional Cartesian coordinate system which Figure 5 shows, the analytical equation of the ellipsoid writes as follows:

$$\frac{x^2}{r_2^2}+\frac{y^2}{r_1^2}=1 \tag{4}$$

The intersection angel of the direction of velocity and the x-axis is θ, we can find two points of tangency A, B on the ellipse with speed lines. Doing some math gives the coordinate of them:

$$A\left(-\frac{r_2^2 tan\theta}{\sqrt{r_2^2\tan^2\theta+r_1^2}},\frac{r_1^2}{\sqrt{r_2^2\tan^2\theta+r_1^2}}\right), B\left(\frac{r_2^2 tan\theta}{\sqrt{r_2^2\tan^2\theta+r_1^2}},-\frac{r_1^2}{\sqrt{r_2^2\tan^2\theta+r_1^2}}\right) \tag{5}$$

With the coordinates come the analytical expressions of tangent lines l_1 and l_2. The distance between two tangent lines equals to the length of major axis of the projection ellipse.

The length of semi-major axis is $r_2'=\sqrt{r_2^2\sin^2\theta+r_1^2\cos^2\theta}$, consequently $S=\pi r_1\sqrt{r_2^2\sin^2\theta+r_1^2\cos^2\theta}$.

Above all, if a submersible stays at some point of neutral buoyancy, we think it has no speed vertically and only moves horizontally. Horizontal joint force it receives is combination of impact force and viscous force, which can be expressed as follows:

$$F_{joint}=\frac{1}{2}\rho S v^2+\eta S\frac{2v}{r_1+r_2{}'}=ma \tag{6}$$

v stands for relative velocity. m stands for mass of a submersible and a stands for accelerated velocity of the submersible. Once the accelerated velocity is determined, the motion curve of the submersible could be figured out.

4.2.3 Further Analysis

Though via mathematical calculation we get the precise expression to compute force, in practical the angle θ is hard to determine for it keeps changing with the rotation of submersibles. If there's no significant change of current velocity, the angle θ would eventually stay zero or keep $90°$ all the time, the latter of which is a very extreme occasion. To further simplify the procedure of computation, we could think the area of projection always stays the same as when θ is zero.

4.2.4 A Submersible Stays on the Seabed

Preparing work:

1) Divide the area of the Ionian Sea into grids with lines parallel to longitude or latitude.
2) In every grid, calculate the slope gradient of seabed via topographic data.
3) Identify landforms of every grid according to slope gradient. Classify them as sea-bottom plain, trench or upland.

If a submersible is positioned on the seabed before losing contact, with the location information sent from it we can locate the initial position in a small grid on the topographic map. The first step is to estimate the landform near the initial position. If it stays on the sea-bottom plain, we think it keeps still or moves very slowly over time. If not so, there's tendency that the submersible might move with current. If the terrain along the direction of current, the submersible would be stopped by the terrain. Otherwise, the submersible will move somewhere with lower terrain in the direction of the current.

4.3 Solving the Physical Model with Gaussian Random Process

While the information of current information may not be very accurate, we need to come up with certain method to simulate the motion of current. With simulated current and physical motion model, we can predict the position of the missing submersible over time. Our process goes as follows:

1) Select current velocity vectors over 600 meters deep from 14349114 data points within the area of latitude 30 to 60 degrees north. As the Ionian Sea is situated in the northern hemisphere, current velocity data from northern latitude is better for current simulation for they are both influenced by Coriolis force. Use all vectors selected to calculate the covariance matrix $\Sigma = \begin{bmatrix} 0.00831335 & 0.00186226 \\ 0.00186226 & 0.0109959 \end{bmatrix}$.
2) Extract information about temperature, density and salinity from all data points in the area of the Ionian Sea. Figures about the data are as follows

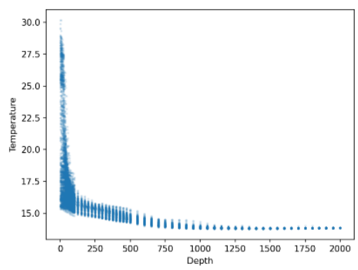

Figure 6: data about temperature

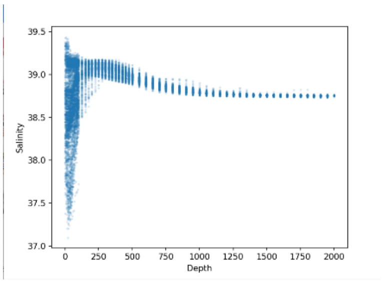

Figure 7: data about salinity

Because the sightseeing submersibles voyage in deep ocean, so we select temperature and salinity over 600m deep as the data set. Average temperature, density and salinity of the data set are 13.91℃, $1.06 \times 10^3 kg/m^3$ and 38.79%. We regard the average value as index of seawater of the Ionian Sea in the following simulation.

3) Because searching time is usually longer than hours, so we set a time step t_0 to be $60s$ with consideration of our computility.
4) From the beginning, we use the information sent from the missing submersible to calculate the accelerated velocity a. After a time step, $\boldsymbol{v_{sail}}$ equals to $\boldsymbol{a}t_0$. And the displacement is $\frac{1}{2}\boldsymbol{a}t_0^2$.
5) Then we adopt Gaussian random process to simulate the motion of current.

$$\boldsymbol{v_{simu}} = \alpha \boldsymbol{v_{current}} + \beta \boldsymbol{v_{ran}} \ , \qquad \alpha + \beta = 1 \tag{7}$$

$\boldsymbol{v_{current}}$ is a fixed velocity sent back from the submersible and $\boldsymbol{v_{simu}}$ is the simulated velocity. $\boldsymbol{v_{ran}}$ is random vector obeying two-dimensional normal distribution $N(\boldsymbol{0}, \Sigma)$. α and β are weighing coefficients. β increases as time passes.

6) With $\boldsymbol{v_{simu}}$ as the velocity of current, the accelerated velocity $\boldsymbol{a}$ could be computed. At the beginning of a new time step, the velocity of the submersible is $\boldsymbol{v_{begin}}$ and it's positioned on $\boldsymbol{x_0}$. At the end of the time step, the velocity becomes $\boldsymbol{v_{begin}} + \boldsymbol{a}t_0$ and its position changes to $\boldsymbol{x_0} + \boldsymbol{v_{begin}}t_0 + \frac{1}{2}\boldsymbol{a}t_0^2$. By doing such iteration in every time step, we obtain the motion curve of the submersible over time.

4.4 Uncertainty Analysis

4.4.1 What might contribute to uncertainty

The motion of a submersible underwater is influenced by multiple factors, and uncertainty in every factor contributes to the whole. We are going to analyze the uncertainty that different factors bring.

• Current: The accuracy of data about the current velocity is two degrees in longitude and latitude. It's much larger compared to the size of submersibles, making it impossible to figure out precise estimation about current velocity near the missing submersible. As a result, the current might take the submersible somewhere quite different from our prediction.

• Topography of seabed: The minimum distance between data points is 20~30m, which is larger than the length of submersibles. Meanwhile, the database of global topography was updated over two years ago. So landforms of seabed nowadays differ from that years age to some extent.

• Temperature and density of seawater: Change of density influences buoyancy, causing movement in vertical. Besides, the coefficient of viscosity changes with temperature, density and salinity. Uncertainty about these factors will affect the force imposed on the submersible and its velocity.

4.4.2 How to decrease uncertainty with information prior to an incident

In order to decrease uncertainty of our prediction, more accurate data is required. So we need a submersible able to observe and send information of its surroundings. Different types of information and equipment required are listed as follows.

Table 2: Information and Equipment Needed

Equipment	Message to send	Use of message
Communication equipment	All kinds of message, including amount of water in tank	Determine if the submersible will sink
Velocity meter	The velocity of currents nearby	Determine future possible movement
Density measure instrument	Measure the density of seawater	Determine the value of η and calculate force
Thermometer	Temperature of seawater	
Conductivity meter	Salinity of seawater	

5 Model 2: Equip Device Strategy

According to documents and researches about marine rescue[4], we list three kinds of equipment necessary for searching:

Table 3: Equipment for Searching

Equipment	Use	Search Radius
Sonar	Search for submersibles with reflection of sound wave	$\boldsymbol{R_1 = 3000m}$
Magnetic Sonde	Judge whether suspicious objects are made of metal	$\boldsymbol{R_2 = 300m}$
ROV	Determine whether suspicious objects are submersibles	$\boldsymbol{R_3 = 30m}$

Due to different functions of the equipment, we take one rescue vessel as a search unit. Every rescue vessel is provided with one sonar, $\boldsymbol{N_{mag}}$ magnetic sondes and $\boldsymbol{N_{rov}}$ ROVs. where the numbers of $\boldsymbol{N_{mag}}$ and $\boldsymbol{N_{rov}}$ are over 1.

To find the most suitable deployment, firstly we come up with a method for searching. It's assumed that rescue vessels primarily use sonars to search the ocean. If suspicious objects is detected by a sonar, then they let out magnetic sondes to search the area of sonar detection. After that, ROVs are sent out to search the area of magnetic sound detection. The following figure shows the whole process.

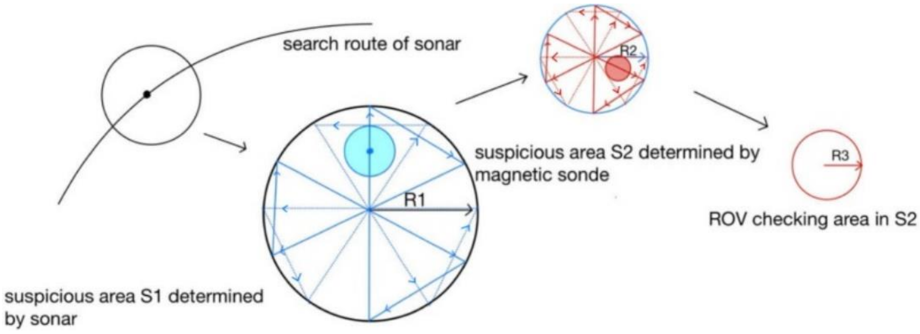

Figure 8: Searching Method

The searching method adopted by magnetic sondes and ROVs as showed above is sector searching method. For N_{mag} magnetic sondes searching in an area of sonar detection altogether, the total search time is as follows:

$$t_{mag} = \frac{18 \cdot R_1}{v_{mag} \cdot N_{mag}} \tag{8}$$

v_{mag} stands for the velocity of magnetic sondes. Time for N_{rov} ROVs to in the area of magnetic sonde detection calculates the same.

$$t_{rov} = \frac{18 \cdot R_2}{v_{rov} \cdot N_{rov}} \tag{9}$$

v_{rov} is the velocity of ROVs.

Table 4: Information about searching device

Device	Price	Labor cost	Maintenance cost	Velocity
Sonar	\$5000	\$10/h	-	-
Magnetic sonde	\$10000	\$1000/h	\$10/h	$1.54m/s$
ROV	\$100000	\$5000/h	\$100/h	$1.54m/s$

Besides search time, financial efficiency is another factor to consider. We think the total cost consists of labor cost and expenditure on purchase and maintenance. Labor cost is proportionate to search time. As for maintenance, we assume that the longer a detector stays underwater, the more we need to spend on its maintenance. The index we adopt to determine the quantity of device is defined as follows:

$$\begin{gathered} C_{mag} = N_{mag} \cdot p_{mag} + c_{mag} \cdot t_{mag} \cdot N_{mag} + t_{mag} \cdot c_{labor}^{1} \\ C_{rov} = N_{rov} \cdot p_{rov} + c_{rov} \cdot t_{rov} \cdot N_{rov} + t_{rov} \cdot c_{labor}^{2} \end{gathered} \tag{10}$$

In the expression above, p_{mag} and p_{rov} stand for price of one magnetic sonde or one ROV, c_{mag} and c_{rov} for cost of maintenance in unit time, c_{labor}^{1} and c_{labor}^{2} for labor cost respectively. To find the optimal number of each device, we regard C_{mag} and C_{rov} as dependent variables of N_{mag} and N_{rov}. We take the value of N_{mag} which enables C_{mag} to reach its minimum as the optimal value. The optimal value of $\boldsymbol{N_{mag}}$ equals to $\sqrt{\frac{18R_1 \cdot c_{labor}^1}{v_{mag} \cdot p_{mag}}}$ (if the optimal value isn't an integer, then we consider the positive integers adjacent to it). Similarly, the optimal value for $\boldsymbol{N_{rov}}$ is $\sqrt{\frac{18R_2 \cdot c_{labor}^2}{v_{rov} \cdot p_{rov}}}$. After computing with numerical values, we get the numerical value: $\boldsymbol{N_{mag} = 1, N_{rov} = 1}$. To conclude, we're going to equip each rescue vessel with one sonar, one magnetic sonde and one ROV as a search unit.

In our model for problem 2, every rescue unit is equipped with fixed device. Therefore, the total cost is proportionate to the number of rescue units. The specific number of rescue units is highly associated with searching area, so we're going to analyze that in concrete situations in our following models.

6 Model 3: Search Strategy for the Submersible

The solution to finding a missing submersible mainly involves two parts. The first is to locate a possible area. The next is to figure out a strategy to search the area as fast as possible. Our solution to this problem is listed as follows.

6.1 Determine Searching Area by Monte-Carlo Simulation

- Simulate current by Gaussian random process
  In this model, simulation of current velocity keeps up with the principal in model 1. The only difference is that real-time data about current velocity can be obtained via searching equipment. Once the velocity is obtained, we update $v_{current}$ with the latest message and reset the value of α and β. Then we iterate with new data according to the process in model 1.
- Determine possible searching area
  In this part we adopt Monte-Carlo simulation to find possible searching area. The process is as follows.
  At first, we suppose there're M balls, whose parameters are the same as the missing submersible. Then put all balls at the initial position of the missing submersible at the same time. For each ball, we predict where it might move to with simulated current over time. After a same period of time, every ball is brought to different positions in the simulation. We select the area where balls locate as the possible searching area.
- Further analysis of searching area
  Due to limited searching capability, it's necessary to evaluate where the missing submersible is more likely to appear.
  As we plan to reset our searching strategy at the start of each day, the first thing is to predict searching area in a new day. Possible area in day $n + 1$ is the location of M balls after motion simulation for one day based on their simulated positions in day n.
  Then let all balls weigh the same initially. After searching in the first day, the problem will be solved if the missing submersible is found. Otherwise, we reduce the weight of balls that locate in searched area. To be more detailed, if a ball is scanned by a sonar the weight of the ball decreases to $1 - p$. If a ball is scanned k times a day but they fail to

find the submersible, the weight of the ball reduces to $\prod_{i=1}^{k}(1-p_i)$. Such adjustment indicates that the missing submersible is less likely to follow the motion of such balls. Thus, we would pay more attention to positions of other balls in future searching.

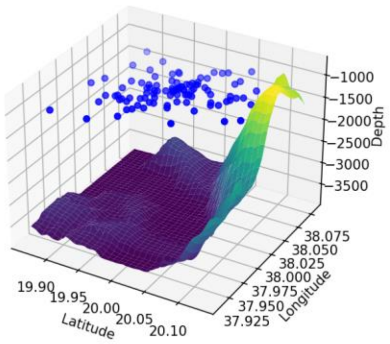

Figure 9: simulated positions of the submersible in day 1

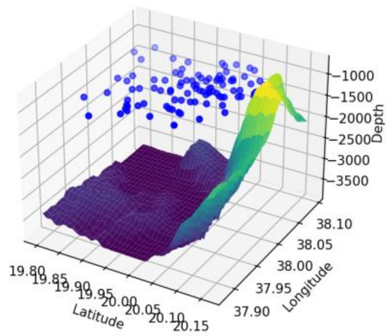

Figure 10: simulated positions of the submersible in day 2

6.2 Searching Strategy

To reduce complexity, we make a few hypotheses as follows. First, we only take the searching of sonars into account. The search radius of every search unit equals to the search radius of sonars. Second, all search units continuously work in a fixed period of time daily. Third, confirmation of the loss and organizing a rescue team bring some time delay. So we think the rescue work starts one day after a submersible is missing in the following model. Fourth, it's natural that the farther an object is to a sonar, the less probable it could be detected by the sonar. To be more precise in math, the probability p that an object could be detected is defined as follows:

$$p = \begin{cases} 1, & 0 \le d \le R_0 \\ e^{\frac{R_0-d}{750}}, & R_0 \le d \le R_1 \\ 0, & d > R_1 \end{cases}$$
$$P = \begin{cases} 1 - scaling, & 0 \le d \le R_0 \\ \left(e^{\frac{R_0-d}{750}}\right)^{1-scaling} - scaling, & R_0 < d \end{cases}, 0 < scaling < 1 \tag{11}$$

In this expression, d stands for distance between the object and the sonar. R_1 stands for the search radius of sonars. R_0 is less than R_1 and defined by characteristic of sonars. Limited by computility of our computers, only finite balls are put into Monte-Carlo simulation.

Bringing in the variable $scaling$ is an attempt to smoothen the simulation with limited computility, in which the number $scaling$ is non-probabilistic but defined by means of Machine Learning.

Under the settings we need to find the best search route before searching starts. Process of our work is as follows:

- Use clustering algorithm to classify balls

  Suppose there're N search units. We take Euclidean distance as index for classification. Then we adopt k-means in clustering algorithm to divide M balls into N subsets. Balls in a same subset is close to each other. For the i-th subset, we can calculate the weighed average position of the subset as its center.

$$\boldsymbol{x}^{\boldsymbol{i}}_{\boldsymbol{center}} = \frac{\sum_{j=1}^{n_i} w_{ij}\boldsymbol{x_{ij}}}{\sum_{j=1}^{n_i} w_{ij}} \tag{12}$$

  n_i stands for the number of balls in the i-th subset. w_{ij} stands for the weight for each ball and $\boldsymbol{x_{ij}}$ for position vector of each ball. We repeat the calculation for each subset and get N center position vectors. At the beginning of searching, we put N search units on these centers as starting positions for one day's searching.

- Compute the best search route—using tree search algorithm and rewarding function based on Monte-Carlo simulation

  With initial positions of search units confirmed, we let the units able to move in 12 directions. The angel between two most similar directions is 30 degree and a search unit can change its moving direction anytime.

  The rewarding system works as follows. For a ball positioned within the search radius of a search unit, reward value the search unit can get obeys the following expression:

$$reward\ value = w \cdot p \tag{13}$$

  w stands for weight of the ball. One thing to mention is that the weight of a ball may change everyday because of our adjustment. We use the weight presently to calculate reward value. p is the probability in expression (11). Total reward value that a unit can get along a certain route is the sum of reward value from balls in its search range. One thing to mention is that even a ball might be detected by more than one unit. For example, if a ball appears in search range of unit 1, reward value unit 1 can get from it is $w \cdot p_1$. If the ball then appears in search range of unit 2, the value unit 2 can get is $w \cdot p_1 \cdot p_2$. For unit 3 the value obtained is $w \cdot p_1 \cdot p_2 \cdot p_3$, with p_i for the probability of unit i to detect the ball.

  In the next step, tree algorithm and method of iteration are adopted to help find the best route. Firstly, we just put one search unit into the ocean and use tree search algorithm to find a route, along which the unit can get the most reward value. Subtracting the value taken by the first unit, we put another unit in and find a route for the second vessel to get the most reward value. By using tree algorithm on every single unit and iterate, we can get the searching route for every vessel. Noticing that accumulation of local optimum may not be the global optimum, we can further adjust the searching route. Subtracting value on balls which is taken by units 2~N, we can refind a route for unit 1 obeying the principle above. Also, we can do similar adjustment on routes of other units. We just repeat the adjustment until the routes hardly change. Lastly, we regard the final route as the best strategy for searching.

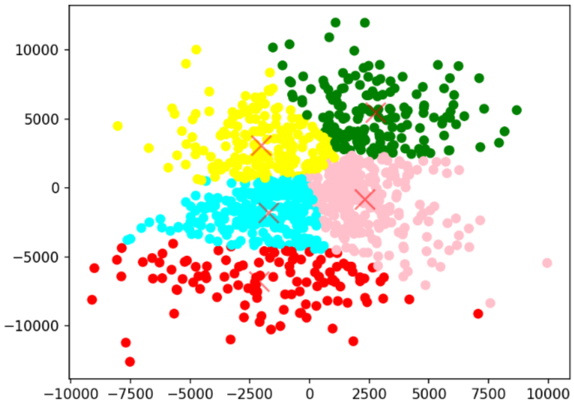

Figure 11: clustering of simulated points

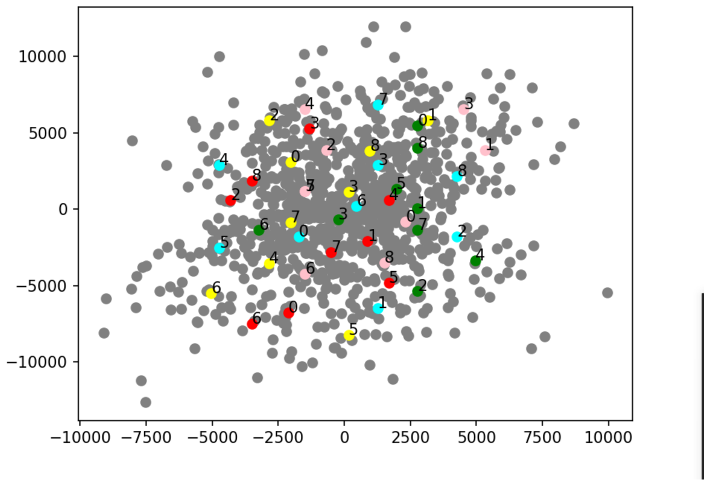

Figure 12: search route with 5 vessels

- Calculate success rate and search time based on emulation

  Firstly, we emulate a motion curve as the true moving trail of the missing submersible and we name it as Ω. The we adopt our simulation and searching strategy to map out the search route. As sonars move along the search route, Ω might be detected by some sonars. Probability for Ω to be detected by a single sonar can be calculated following the expression (11). According to Bayes Theory, the rate for Ω to be found is as follows:

$$r = 1 - \prod_{i=1}^{k} (1 - p_i) \tag{14}$$

k is the number of total times Ω appears in detection range of a sonar. And p_i is the probability in the i-th time Ω being detected by a sonar. When the rate is over 95%, we think a missing boat can be successfully found with high probability. Meanwhile, we can get results of search time and search route from our simulation. Otherwise, we think we fail to find the missing submersible in that day. By repeating the emulation and simulation, we can compute success rate over time.

- Determine the optimal number of rescue vessels
  With reference to the rescue deployment of occurred searching for missing submersibles. We initially set the range of the number of rescue vessels to be 2~8. Then we consider three indexes to evaluate different choice of number. The first one is the total price of all device, which is proportionate to the quantity of rescue vessels. The second is the length of search time. The third is success rate. After that, we adopt Analytical Hierarchy Process(AHP) to give different weight to the indexes.

Table 5: Information about indexes

Index	Weight	Formula to calculate
Purchase cost	**0.102**	$\sum \boldsymbol{N_i p_i}$
Use cost	**0.195**	$\boldsymbol{N} \cdot (\sum_{i=1}^{n} \boldsymbol{m_i (i+1) c_{use}})$
Success rate	**0.703**	**-**

In the table above, $\boldsymbol{N}$ stands for the number of device and $\boldsymbol{p}$ for price. $\boldsymbol{m_i}$ is the probability we fail to find the submersible in the i-th day and $\boldsymbol{c_{use}}$ is the unit cost for using the device. $\boldsymbol{n}$ is the total number of days of our research. Then we turn the maximum of purchase cost and use cost to the minimum so that the more we spend on purchase and use, the less the index will be. Next we normalize all indexes to make their value vary in the interval $[\boldsymbol{0}, \boldsymbol{1}]$. And we consider the situation to search for 14 days with success rate over 75%. The outcome of our assessment is as follows:

Table 6: outcome of our assessment

Number of vessels	Total index
5	**0.630097**
6	**0.713088**
7	**0,892157**
8	**0.729791**

According to the weighed total indexes, we select 7 as the most suitable for number of vessels

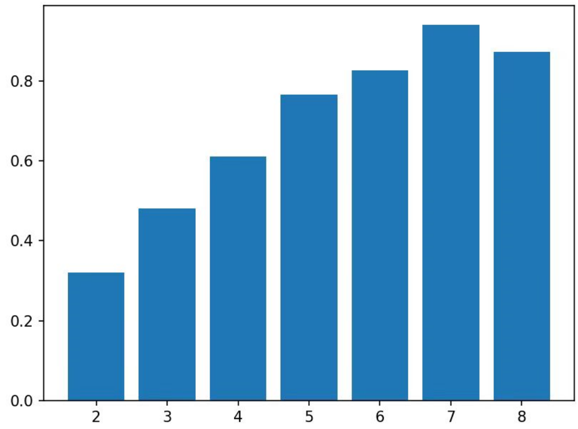

Figure 13: success rate with different number of vessels in 14 days

7 Extrapolation of Our Model

7.1 Searching in Other Sea Area

In order to improve adaptability of our model in various situations, we extrapolate it to the Caribbean Sea. Major changes brought by extrapolation reflect in topography of seabed and motion characteristics of current. According to global topographic data we extract in Model 1, we can simulate the seabed topography of the Caribbean Sea.

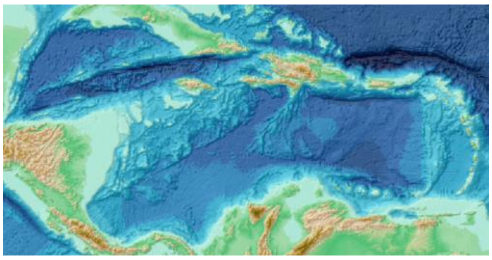

Figure 14: Topography of the Caribbean Sea

With topographic data, we can predict the motion of a missing submersible if it sinks to the bottom of the ocean. Moreover, with current information we can predict possible area where the missing submersible might appear according to Model 3. The prediction figure is as follows:

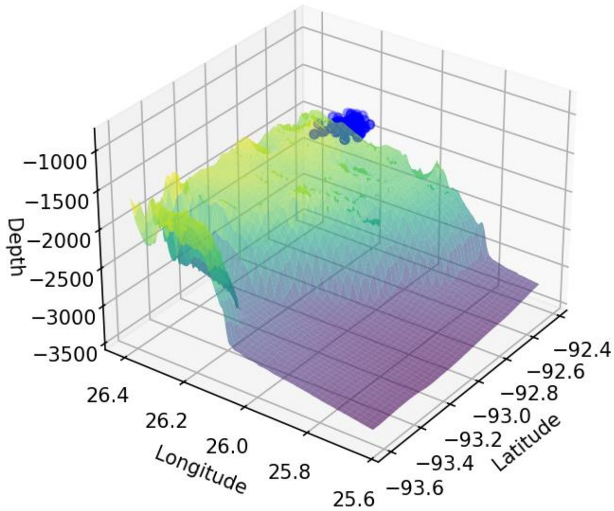

Figure 15: Position simulation in one day after missing

7.2 Case of Multiple Submersibles

Besides extrapolation in other sea area, the host ship might lose contact with a number of submersibles at the same time. To deal with such situation, the following adjustments are applied to our model.

• Simulation of possible area for every single submersible follows the principle in Model 3. One thing more is that we need to operate the procedure for every missing submersible at the same time. The core idea is that we add more than one initial points for Monte-Carlo simulation, while in Model 3 we just put in one initial point for simulation. In total we get a possible area which is the summation of simulated possible area of every submersible.

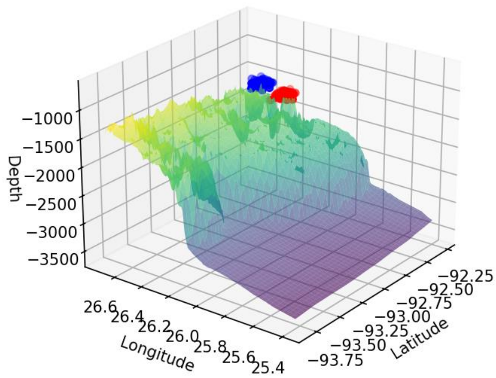

Figure 16: Position simulation of two submersibles in the Caribbean Sea

• Setting of search routes obeys that in Model 3 to a large extent. For each missing submersible, we can calculate the success rate according to our search route. We end our search only when the success rate for every submersible is over 95% to ensure that we may find all of the missing as far as possible.

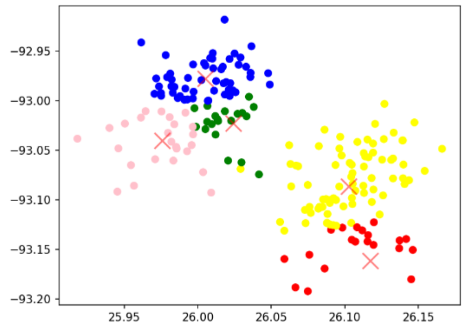

Figure 17: Clustering of simulated points

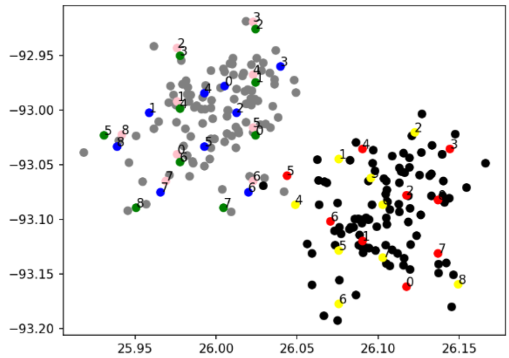

Figure 18: Search route for two submersibles with five rescue vessels

8 Sensitivity Analysis

Our localization model is primarily based on data such as the velocity and oceanic currents obtained from the last communication of the lost submarine. In reality, the information returned by the submarine's last communication may not be accurate before it lost power, so it is necessary to consider the effect of changes in the submarine's speed on the model's positioning. In addition, due to the strong uncertainty of the deep-sea current, we need to simulate it through the current information returned by the submarine, so we also analyze the impact of the error of the current speed on the model.

As shown in Figure 19, with 10m/s as the accurate speed of the ocean current, when the ocean current speed fluctuates 20% upward, the radius fluctuation of the search radius area is about 2%, and the fluctuation of the search area center from the initial position is about 8%. On the whole, the difference of ocean current velocity has little influence on the search radius and the center of the search area, indicating that the model has strong stability and can adapt to the actual situation of uncertain ocean current velocity. However, with the passage of time, the influence of the error of ocean current speed on the model gradually becomes obvious, so in order to better use the model, it is necessary to use multiple rescue vessels to search and rescue the submarine as soon as possible.

At the same time, since the submarines used in our model are mainly sightseeing submarines, whose initial speed is small, after the fluctuation of the initial speed of the submarine, we find that the uncertainty of the initial speed of the submarine has almost no impact on the search radius and the distance from the search area center to the initial position, because the unpowered submarine mainly relies on the ocean current to move. At the same time, as shown in the figure, when its speed is increased from 1m/s to 3m/s and 5m/s, its moving distance has a great change, which indicates that the model can also accurately screen the influence brought by different speeds of the submarine.

In general, our model has good robustness to small range of ocean current velocity fluctuations and submarine velocity fluctuations.

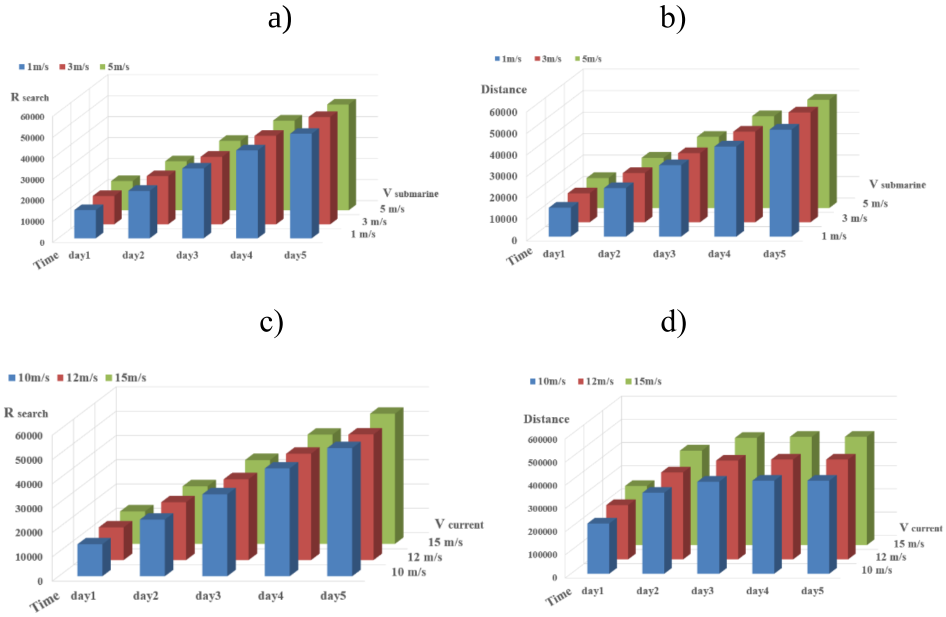

Figure 19: Sensitivity analysis for ocean current velocity and submarine velocity

Figure 19 a-b) represent the sensitivity of the center of the search area and the search radius due to the change of submarine velocity changes; Figure 19 c-d) represent the sensitivity of the center of the search area and the search radius due to the change of ocean current velocity fluctuations.

9 Model Evaluation and Further Discussion

9.1 Strengths

- **Strength1: Application of simulation**

We introduce hydrodynamic simulation algorithms to predict the movement of the submarine in the deep sea, thus making our model as close to reality as possible and making it better practical. At the same time, according to the ocean current data of real sampling points, we simulate the deep sea current through Gaussian distribution to confirm the search efficiency of the model.

- **Strength2: Flexible adjustability**

Our model fully considers the effects of ocean current characteristics, seawater salinity, temperature, topography and other factors. By adjusting these parameters, the model can adapt to various search and rescue environments and has strong extrapolation. At the same time, our search strategy can be flexibly adjusted according to the search scope, which has stronger adaptability than the traditional search method.

- **Strength3: The innovation of the plan**

The position of Monte Carlo simulation point and sonar discovery probability are cleverly combined into search strategy by probabilistic reward model and search tree, and the results of the model are evaluated by double-layer Monte Carlo simulation, which provides an innovative way to solve the problem.

9.2 Weakness

- **Weakness: Changes in the submarine's posture**

In the model, we does not take into account in detail the changes in the attitude of the submarine in the ocean current, which may affect its movement.

- **Weakness: Search and rescue strategies can still be optimized**

Limited time step in a single search and rescue operation, leading to mechanized behavior.

9.3 Further Discussion

In view of the shortcomings of the current models, we give the following optimization ideas:

- Attempting to model or estimate the posture of submarines.
- Replacing discrete search with gradient descent to allow more flexible behavior in search strategy at each time step (distance, direction).

10 Conclusion

In this paper we mainly aim to predict the position of a missing submersible and then come up with searching strategy to find it.

We firstly analyze the motion of a submersible underwater based on fluid dynamics. Then we establish a model to predict the possible position of the submersible using Gaussian random process to simulate the current.

Then we determine how to equip searching device on rescue vessels with consideration of time efficiency and cost control. With analysis we establish the mode to equip each rescue vessel with one sonar, one magnetic sonde and one ROV as a search unit.

With knowledge about searching ability of rescue vessels, we adopt tree search algorithm and rewarding function to determine the best search route based on the simulate searching area. Through emulation of motion of a submersible, we can calculate the probability for the submersible being detected by rescue vessels. Over repeat of the procedure, we get the success rate to be over 80% in four days with help of 7 vessels, over 90% in 7 days. Taking cost and search time into account, we select 7 as the most suitable number of rescue vessels.

Extrapolation of our model from the Ionian Sea to the Caribbean Sea works well, which indicates that our model highly adaptable to other enviromnt.

Memo

Date: February 5, 2024

To: the Greek Government
From: Maritime Cruises Mini-Submarine Co.
Subject: Safety procedures in case of emergency for ocean exploration program
Message:

Our company is planning to take tourists to explore the bottom of the Ionian Sea with the use of submersibles we have manufactured. Before putting the submersibles into practical use, we comprehensively assess the security of our program. In response to safety concerns, we set up safety procedures in case of communication loss and mechanical defect.

In the exploration program, we will bring submersibles to the location for sightseeing with a host ship. Then we will deploy submersibles untethered from the host ship. Once losing contact and mechanical power, submersibles would only be brought away by the power of current. The structure of our submersibles is strong enough to endure seawater pressure in the Ionian Sea. Risks coming with emergencies might be oxygen deficiency or collision with other objects underwater.

We're asked to establish corresponding safety procedures if submersibles lose communication or power. The safety procedures mainly include two parts. One is prediction about possible positions of a missing submersible. The other is determination of searching strategy based on position prediction.

For the first part, we comprehensively consider fluid dynamics, uncertainty of the current velocity and so on. And we set up a physical motion model to predict the real-time position of a missing submersible. With consideration about uncertainty brought by the current velocity, we adopt Monte-Carlo simulation to prediction possible area where the missing submersible might appear over time. Via predicting possible area with real data of a submarine accident, we get the result that the position where they found the submarine is located within the area of our prediction. The average radius of our prediction is much shorter than that of the actual search area, which indicates that our model for prediction is highly precise.

For the second part, we consider the deployment of searching equipment on rescue vessels at first. With reference to documents and existing rescue guide and with tradeoff about costs and usability, we select sonars, magnetic sondes and ROVs as additional equipment for rescue vessels to take. After balancing time efficiency and the costs for purchasing and maintenance, we equip one rescue with one sonar, one magnetic sonde and one ROV to make search efficiency the most. As for the layout of the search route, we utilize the outcome of daily searching to optimize our future arrangement of search route. Meanwhile, tree search algorithm and rewarding function are adopted to compute the best route. In response to possible emergency in the Ionian Sea, we would send out no less than rescue vessels to assist. According to our model and simulation, the success rate with deployment no less than 7 vessels is over 80% in four days and over 90% in seven days. Our submersibles are designed to carry at least four days' need of oxygen.

Conclusively, our prediction of a missing submersible is highly accurate with verification of an occurred submarine accident. Meanwhile, searching strategy given from our model ensures a high probability of success. With oxygen and food supply in our submersibles, tourists are quite likely to be found safe and sound even if an accident happens. The concrete structure of our submersibles and sufficient storage of rescue equipment further ensures the safety of our passengers.

Above all, we think our safety procedure in case of emergency guards the security of tourists to the greatest extent. Therefore, our company sincerely hopes that the Greek government may approve our project for exploration of the deep ocean.

If there's any problem, feel free to contact us.

References

[1] ncei.noaa.gov/access/global-ocean-currents-database/cmportal.html
https://earth.nullschool.net/

[2] https://docs.gmt-china.org/6.1/dataset/earth-relief/

[3] Haisen Li, Yang Zhang, Jianjun Zhu, et al. Current Status and Trends of Underwater Search Technology for Lost Targets[J]. Journal of Harbin Engineering University, 2023, 44(04): 546-553+579.