计量经济学作业

19 经管三班 王泽源 19317278

2.15.

a. 因为 Y_i 是 i.i.d. 随机变量，服从 $N(10,4)$分布:

$\Pr(9.6 \le \bar{Y} \le 10.4)$
$= \Pr(\bar{Y} \le 10.4) - \Pr(\bar{Y} \le 9.6)$
$= \Pr(\bar{Y} \le 10.4) - [1 - \Pr(\bar{Y} \le 10.4)]$
$= 2\Pr(\bar{Y} \le 10.4) - 1$
$= 2\Phi(10.4) - 1$

由于:

$\Phi(10.4)$

$= \Phi_0(\frac{10.4-10}{\sigma_{\bar{Y}}})$

$= \Phi_0(\frac{10.4-10}{\frac{\sigma_Y}{\sqrt{n}}})$

$= \Phi_0(\frac{0.4}{\frac{2}{\sqrt{n}}})$

$= \Phi_0(\frac{\sqrt{n}}{5})$

可得:

$\Pr(9.6 \le \bar{Y} \le 10.4) = 2\Phi_0\left(\frac{\sqrt{n}}{5}\right) - 1$

因此:

①$n = 20$

$\Pr(9.6 \le \bar{Y} \le 10.4) = 2\Phi_0\left(\frac{2\sqrt{5}}{5}\right) - 1$

$\approx 2\Phi_0(0.89) - 1$
$= 2 \times 0.8133 - 1$
$= 0.6266$

②$n = 100$

$\Pr(9.6 \le \bar{Y} \le 10.4) = 2\Phi_0\left(\frac{\sqrt{100}}{5}\right) - 1$

$\approx 2\Phi_0(2) - 1$
$= 2 \times 0.9772 - 1$
$= 0.9544$

③$n=1000$

$\Pr(9.6 \le \bar{Y} \le 10.4) = 2\Phi_0\left(\frac{10\sqrt{10}}{5}\right) - 1$

$\approx 2\Phi_0(6.32) - 1$
≈ 1

b.$\Pr(10 - c \le \bar{Y} \le 10 + c)$
$= 2\Pr(\bar{Y} \le 10 + c) - 1$

$= 2\Phi_0\left(\frac{c}{\frac{2}{\sqrt{n}}}\right) - 1$

$= 2\Phi_0\left(\frac{c}{2}\sqrt{n}\right) - 1$

由于 n 逐渐增大，$\frac{c}{2}\sqrt{n}$也逐渐增大，$\Phi_0\left(\frac{c}{2}\sqrt{n}\right)$作为概率分布函数，单调递增，

且$\lim_{x\to\infty} \Phi_0(x) = 1$

故$\Pr(10 - c \le \bar{Y} \le 10 + c) = 2\Phi_0\left(\frac{c}{2}\sqrt{n}\right) - 1$越来越接近于 1.0

c.对任意常数$c > 0$：

$\lim_{n\to\infty} \Pr(10 - c \le \bar{Y} \le 10 + c)$

$=\lim_{n\to\infty} 2\Phi_0\left(\frac{c}{2}\sqrt{n}\right) - 1$

$= \lim_{x\to\infty} 2\Phi_0(x) - 1$

$= 2 - 1 = 1$
依定义可知$\bar{Y}$依概率收敛到μ_Y=10

2.17.
a.依据中心极限定理，在一般条件下，当n较大时正态分布能较好地近似$\bar{Y}$的分布，本题中取$n = 100, n = 400$，故可以使用中心极限定理
由于Y_i为$p = 0.4$的 i.i.d.伯努利随机变量，故

$\mu_Y = 0.4, \sigma_Y = \sqrt{0.4 \times (1 - 0.4)^2 + (1 - 0.4) \times 0.4^2} = \frac{\sqrt{6}}{5}$
①$n = 100, \Pr(\bar{Y} \ge 0.43)$
$\Pr(\bar{Y} \ge 0.43)$
=$1 - \Pr(\bar{Y} \le 0.43)$

$= 1 - \Phi_0\left(\frac{0.43 - 0.4}{\frac{\frac{\sqrt{6}}{5}}{\sqrt{n}}}\right)$

$= 1 - \Phi_0(\frac{\sqrt{6}}{4})$
$\approx 1 - \Phi_0(0.61)$
$= 1 - 0.7291$
$= 0.2709$

②$n = 400, \Pr(\bar{Y} \le 0.37)$
$\Pr(\bar{Y} \le 0.37)$

$$= \Phi_0(\frac{0.37 - 0.4}{\frac{\frac{\sqrt{6}}{5}}{\sqrt{n}}})$$

$= \Phi_0(-\frac{\sqrt{6}}{2})$
$\approx \Phi_0(-1.22)$
$= 0.1112$

③$\Pr(0.39 \le \bar{Y} \le 0.41)$
$= \Pr(\bar{Y} \le 0.41) - \Pr(\bar{Y} \le 0.39)$
$= \Pr(\bar{Y} \le 0.41) - [1 - \Pr(\bar{Y} \le 0.41)]$
$= 2\Pr(\bar{Y} \le 0.41) - 1$

$$= 2\Phi_0\left(\frac{0.41 - 0.4}{\frac{\frac{\sqrt{6}}{5}}{\sqrt{n}}}\right) - 1$$

$$= 2\Phi_0\left(\frac{\sqrt{n}}{20\sqrt{6}}\right) - 1 \ge 0.95$$

即$\Phi_0\left(\frac{\sqrt{n}}{20\sqrt{6}}\right) \ge 0.975$

查表可得$\Phi_0(1.96) = 0.975$

故$\frac{\sqrt{n}}{20\sqrt{6}} \ge 1.96$

$n \ge 9219.84$
因n为整数，故需要$n \ge 9220$, 才能保证$\Pr(0.39 \le \bar{Y} \le 0.41) \ge 0.95$