A FEDERATED KALMAN FILTER DESIGN USING A GAIN FUSION ALGORITHM

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Abstract: As an alternative fusion process of the federated Kalman filter, a gain fusion algorithm is newly proposed in this paper. In this algorithm the optimal covariance and estimate are obtained by using local Kalman gains and estimates. Consequently, this algorithm reduces the amount of communications and avoids the need to calculate inverse covariance matrices in local filters. It is mathematically shown that the suggested algorithm guarantees the global optimality when all local sensors produce equivalent information except their precision. It is prospected that this algorithm may be well suited for implementation of the multisensor navigation systems. *Copyright © 1998 IFAC*

Keywords: Kalman filter, multisensor integration, decentralized systems, gain

1. INTRODUCTION

In recent years interest has been growing in the synergistic use of multiple sensors to increase the capabilities of systems. For these systems to use multiple sensors effectively, some methods are required for integrating the information provided by these sensors into the operation of the system. While in many multisensor systems the information from each sensor serves as a separate input to the system, the actual combination or fusion of information prior to its use in the system has been a particularly active area of research(Luo and Kay, 1989).

Kalman filter has been used in the processing of data in dynamic processes. Its inherent recursive computational procedure is particularly efficient for multisensor systems(Siouris, 1993). In a multisensor system, sensors are understood to be different hardware devices, each with its own data processing. A standard centralized Kalman filter(CKF), however, may result in a high computational load when implemented in a strictly optimal fashion where all measurements are passed centrally to yield a solution and its covariance matrix. Also, there is the issue of the lack of robustness of centralized systems when there is spurious data in any of the sensors.

For overcoming these disadvantages, a decentralized Kalman filter(DKF) has been studied for a lot of years(Hashemipour, *et al.*, 1988; Kerr, 1987; Willsky, *et al.*, 1982). A decentralized Kalman filter is a two-stage data processing technique, with the design divided into a master filter and one or more local filters. In the first stage, the local filters process their own data in parallel to yield the best possible local estimates. In the second stage, the master filter fuses the local estimates, yielding the best global estimate.

Recently, as a specialized case of decentralized Kalman filter, a federated Kalman filter(FKF) technique is introduced by Carlson(1990, 1994). This scheme employs the principle of information sharing among the local Kalman filters to guarantee the global optimality. The list of possible information to be shared is understood to be the process noise and the initial conditions. Even though the FKF can still be categorized as a DKF, the use of the information sharing principle distinguishes it from the existing DKFs.

Considering the positive definiteness of the covariance matrices, the Joseph stabilized version of Kalman filter equation is widely implemented in real applications(Gelb, 1974; Lewis, 1986). This form does not require an inverse of covariance matrix and needs an inverse matrix of smaller dimension determined by the number of measurements, not state variables. In the FKF scheme, or even in other decentralized Kalman filters, however, complex matrix inversions are required because the inverse covariance matrices are included in the mathematical representations of the fusion process. Some alternative representations of the decentralized Kalman filter adopt local Kalman gain matrices yielded by the Joseph stabilized version to combine local information(Brumback and Srinath, 1987; Gardner and Leondes, 1990). In these schemes, however, the global optimality of combined estimate and covariance is not guaranteed while it can be satisfied in the FKF or other DKFs.

In this paper an alternative fusion process of the FKF named as "gain fusion algorithm" is presented. Specifically, the scheme is available when all local sensors produce equivalent information except their precision. In this scheme instead of inverse covariance matrices, Kalman gain matrices of local filters are transferred to the master filter and the global optimal covariance and estimate are produced by using local gains and estimates. This scheme reduces the amount of communications and avoids the need to calculate inverse covariance matrices in local filters.

Section 2 is a brief review of the FKF. In section 3, a newly derived gain fusion algorithm is suggested as an alternative of the FKF fusion algorithm. Its mathematical representations and procedure for guaranteeing the global optimality are suggested and proved. Also, characteristics and advantages of the suggested algorithm are discussed in section 4. In section 5, an example is illustrated to apply the gain fusion algorithm to a satellite tracking system. Conclusion is given in section 5.

2. FEDERATED KALMAN FILTER

The federated Kalman filter(FKF) developed by Carlson(1990), has a lot of advantages over all the other decentralized Kalman filter techniques. First of all, it does not require a dynamic compensation term(Kerr, 1987; Speyer, 1979; Willsky, *et al.*, 1982) for eliminating the correlation between local filters. Also, communications of only filtered estimates and covariances is required while filtered and predicted

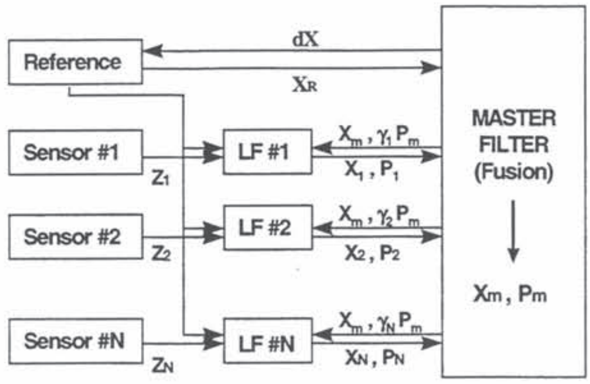

Fig. 1. Federated Kalman filter structure

variables are necessarily required in other decentralized technique(Hashemipour, *et al.*, 1988). In the FKF, a very simple yet effective technique called *information-sharing approach* for handling the correlation between local filters is provided. The concept of the information-sharing approach implemented by the FKF can be summarized as follows(Carlson, 1994).

a) Scale the initial values of local filter covariance and process noise matrices.

b) Perform local time propagation and measurement update process.

c) Combine the updated local information into a global information.

d) Reset local information to the scaled global information.

The FKF employs a two-stage data processing architecture, in which the outputs of sensor-related local filter(LF)s are subsequently combined by a larger master filter(MF), as illustrated in Fig. 1. As indicated, each LF is dedicated to a separate sensor subsystem, and also utilizes data from a common reference system, generally an inertial navigation system(INS). Even though any common reference system is not involved in the whole system, the correlation between local filters still exists when all local filters estimate the same state. In the structure of Fig. 1, a globally optimal estimate and its error covariance are obtained by the following additive equation when the aforementioned information-sharing approach is employed.

$$P_m = \left[P_1^{-1} + P_2^{-1} + \cdots + P_N^{-1}\right]^{-1} \quad (1)$$

$$\hat{x}_m = P_m[P_1^{-1}\hat{x}_1 + P_2^{-1}\hat{x}_2 + \cdots + P_N^{-1}\hat{x}_N] \quad (2)$$

where $P_i^{-1}(i=1,\ldots N)$ and $\hat{x}_i(i=1,\ldots N)$ are local information matrices and local estimates respectively.

3. GAIN FUSION ALGORITHM

In the FKF described in the previous chapter, information including inverse covariance matrices of local filters must be transferred to the master filter. In a large system like an integrated inertial navigation system, local filters generally have a large dimension n. In this case, $n \times n$ variables are communicated between the master filter and each local filter. It needs a heavy burden of communication besides calculation of $n \times n$ inverse matrix. Considering the positive definiteness of the covariance matrices, the Joseph stabilized version of Kalman filter equation that does not require inverse of covariance matrix is appropriate for implementation. In the FKF scheme, or even in other decentralized Kalman filters, however, complex matrix inversions are required because the inverse covariance matrices are included in the mathematical representations of the fusion process.

In this paper an alternative fusion process of the FKF is proposed. Specifically, suggested scheme is available when all local sensors produce equivalent information except their precision. In this scheme instead of inverse covariance matrices, Kalman gain matrices of local filters are transferred to the master filter and the global optimal covariance and estimates are produced by using local gains and estimates. This scheme reduces amount of communications and avoids the needs to calculate the inverse covariance matrices in local filters.

Consider the discrete-time model

$$\begin{aligned} \boldsymbol{x}(k+1) &= \boldsymbol{F}(k)\boldsymbol{x}(k) + \boldsymbol{G}(k)\boldsymbol{w}(k) \\ \boldsymbol{x}(0) &\sim N(\hat{\boldsymbol{x}}(0), \boldsymbol{P}(0)). \end{aligned} \tag{3}$$

It is assumed that linear combinations of the states can be observed through multiple sensor measurements modeled as

$$z_i(k) = \boldsymbol{H}\boldsymbol{x}(k) + \boldsymbol{v}_i(k),\ i = 1, \ldots, N. \tag{4}$$

Also assumed is that all sensors measure the same information but have different noise level. The initial state, process noise and measurement noise are all uncorrelated and satisfy following equations.

$$\begin{gathered} E[\boldsymbol{w}(k)] = \boldsymbol{0}, E[\boldsymbol{w}(k)\boldsymbol{w}^T(j)] = \boldsymbol{Q}\delta_{kj} \\ E[\boldsymbol{v}_i(k)] = \boldsymbol{0}, E[\boldsymbol{v}_i(k)\boldsymbol{v}_i^T(j)] = \boldsymbol{R}_i\delta_{kj} = \gamma_i \boldsymbol{R}\delta_{kj} \\ E[\boldsymbol{v}_i(k)\boldsymbol{w}^T(k)] = \boldsymbol{0}, E[\boldsymbol{x}(0)\boldsymbol{w}^T(k)] = \boldsymbol{0}, E[\boldsymbol{x}(0)\boldsymbol{v}_i^T(k)] = \boldsymbol{0} \end{gathered} \tag{5}$$

where

$$\sum_{i=1}^{N} \frac{1}{\gamma_i} = 1 \tag{6}$$

and it is notified that scale factors of measurement noise covariances can be designed to satisfy above condition.

In a centralized Kalman filter scheme, the global estimate can be computed with one Kalman filter that is based on the global model and processes all measurements. Define a global measurement model as follows.

$$z_a(k) = \left[z_1^T(k) \mid z_2^T(k) \mid \ldots \mid z_N^T(k)\right]^T \tag{7}$$

$$\boldsymbol{H}_a = \left[\boldsymbol{H}^T \mid \boldsymbol{H}^T \mid \ldots \mid \boldsymbol{H}^T\right]^T \tag{8}$$

$$\boldsymbol{R}_a = diag(\boldsymbol{R}_1, \boldsymbol{R}_2, \ldots, \boldsymbol{R}_N). \tag{9}$$

The globally optimal estimate and covariance can be computed from

$$\hat{\boldsymbol{x}}(k+1|k) = \boldsymbol{F}(k)\hat{\boldsymbol{x}}(k|k) \tag{10}$$

$$\boldsymbol{P}(k+1|k) = \boldsymbol{F}(k)\boldsymbol{P}(k|k)\boldsymbol{F}(k)^T + \boldsymbol{G}\boldsymbol{Q}\boldsymbol{G}^T \tag{11}$$

$$\boldsymbol{K}_a(k+1) = \boldsymbol{P}(k+1|k)\boldsymbol{H}_a^T\left[\boldsymbol{H}_a\boldsymbol{P}(k+1|k)\boldsymbol{H}_a^T + \boldsymbol{R}_a\right]^{-1} \tag{12}$$

$$\begin{aligned} &\hat{\boldsymbol{x}}(k+1|k+1) \\ &= \hat{\boldsymbol{x}}(k+1|k) + \boldsymbol{K}_a(k+1)\left[z_a(k+1) - \boldsymbol{H}_a\hat{\boldsymbol{x}}(k+1|k)\right] \end{aligned} \tag{13}$$

$$\begin{aligned} &\boldsymbol{P}(k+1|k+1) \\ &= (\boldsymbol{I} - \boldsymbol{K}_a(k+1)\boldsymbol{H}_a)\boldsymbol{P}(k+1|k)(\boldsymbol{I} - \boldsymbol{K}_a(k+1)\boldsymbol{H}_a)^T \\ &+ \boldsymbol{K}_a(k+1)\boldsymbol{R}_a\boldsymbol{K}_a(k+1)^T. \end{aligned} \tag{14}$$

In a general decentralized Kalman filter scheme, local filters are generally based on the models

$$\boldsymbol{x}_i(k+1) = \boldsymbol{F}(k)\boldsymbol{x}_i(k) + \boldsymbol{G}(k)\boldsymbol{w}(k),\ i = 1, \ldots, N \tag{15}$$

$$z_i(k) = \boldsymbol{H}\boldsymbol{x}(k) + \boldsymbol{v}_i(k),\ i = 1, \ldots, N. \tag{16}$$

Because all local filters estimate same state variables, these models that have the same dynamics are appropriate. Using standard Kalman filter equations, local Kalman gain matrices and estimates can be computed from

$$\boldsymbol{P}_i(k+1|k) = \boldsymbol{F}(k)\boldsymbol{P}_i(k|k)\boldsymbol{F}(k)^T + \boldsymbol{G}\boldsymbol{Q}_i\boldsymbol{G}^T \tag{17}$$

$$\hat{\boldsymbol{x}}_i(k+1|k) = \boldsymbol{F}(k)\hat{\boldsymbol{x}}_i(k|k) \tag{18}$$

$$\boldsymbol{K}_i(k+1) = \boldsymbol{P}_i(k+1|k)\boldsymbol{H}^T\left[\boldsymbol{H}\boldsymbol{P}_i(k+1|k)\boldsymbol{H}^T + \boldsymbol{R}_i\right]^{-1} \tag{19}$$

$$\begin{aligned} &\hat{\boldsymbol{x}}_i(k+1|k+1) \\ &= \hat{\boldsymbol{x}}_i(k+1|k) + \boldsymbol{K}_i(k+1)\left[z_i(k+1) - \boldsymbol{H}\hat{\boldsymbol{x}}_i(k+1|k)\right] \end{aligned} \tag{20}$$

In the gain fusion algorithm, global estimate and covariance may be combined using local Kaman gain matrices and estimates as

$$\hat{x}_m(k+1|k+1) = \sum_{i=1}^{N} \frac{1}{\gamma_i} \hat{x}_i(k+1|k+1) \quad (21)$$

$$P_m(k+1|k+1) = (I - K_m(k+1)H)P_m(k+1|k) \quad (22)$$

where

$$K_m(k+1) = \sum_{i=1}^{N} \frac{1}{\gamma_i} K_i(k+1). \quad (23)$$

Global covariance matrix can be obtained by using an alternative equation that is similar to Joseph form. In this paper, (24) is suggested as a nominal form of covariance fusion.

$$\begin{aligned} &P_m(k+1|k+1) \\ &= (I - K_m(k+1)H)P_m(k+1|k)(I - K_m(k+1)H)^T \quad (24) \\ &+ \sum_{i=1}^{N} \frac{1}{\gamma_i^2} K_i(k+1)R_i K_i(k+1)^T \end{aligned}$$

Theorem 1 describes the procedure of the gain fusion algorithm.

Theorem 1. For the system (3)--(6), suggested global estimate and covariance represented in (21) and (24) are globally optimal under following procedure.

a) A master filter is configured as

$$\begin{aligned} P_m(0) &= P(0) \\ Q_m &= Q \\ \hat{x}_m(0) &= \hat{x}(0) \end{aligned} \quad (25)$$

where subscript m denotes the master filter.

b) Local filters are configured as

$$\begin{aligned} P_i(0) &= \gamma_i P(0) \\ Q_i &= \gamma_i Q \\ \hat{x}_i(0) &= \hat{x}(0), \quad i = 1, \ldots N. \end{aligned} \quad (26)$$

c) Local filters calculate local gains and estimates using (17)-(20).

d) Master filter calculate predicted a global covariance matrix as

$$P_m(k+1|k) = F(k)P_m(k|k)F(k)^T + GQ_mG^T \quad (27)$$

and calculate global estimate and covariances using (21) and (24).

e) Reset local filters as

$$P_i(k+1|k+1) = \gamma_i P_m(k+1|k+1) \quad (28)$$

$$\hat{x}_i(k+1|k+1) = \hat{x}_m(k+1|k+1). \quad (29)$$

Proof of Theorem 1. A mathematical induction is used. At the initial time, the covariance and estimate of master filter is optimal by initialization process of (25), If Theorem 1 holds at k,

$$P_m(k|k) = P(k|k) \quad (30)$$

$$\hat{x}_m(k|k) = \hat{x}(k|k). \quad (31)$$

By (17), (18), (28) and (29), following equations holds.

$$P_m(k+1|k) = P(k+1|k) \quad (32)$$

$$P_i(k+1|k) = \gamma_i P(k+1|k) \quad (33)$$

$$\hat{x}_i(k+1|k) = \hat{x}(k+1|k). \quad (34)$$

Rewriting globally optimal Kalman gain matrix in (12),

$$K_a(k+1) = P(k+1|k)H_a^T \left[H_a P(k+1|k)H_a^T + R_a\right]^{-1} \quad (35)$$

Using Matrix inversion lemma,

$$\begin{aligned} &K_a(k+1) \\ &= P(k+1|k)H_a^T \\ &\bullet \left[R_a^{-1} - R_a^{-1}H_a\left\{H_a^T R_a^{-1} H_a + P^{-1}(k+1|k)\right\}^{-1} H_a^T R_a^{-1}\right] \end{aligned} \quad (36)$$

Using $P^{-1}(k+1|k+1) = P^{-1}(k+1|k) + H_a^T R_a^{-1} H_a$,

$$\begin{aligned} &K_a(k+1) \\ &\quad = P(k+1|k)H_a^T R_a^{-1} \\ &\quad - P(k+1|k)H_a^T R_a^{-1} H_a P(k+1|k+1) H_a^T R_a^{-1} \end{aligned} \quad (37)$$

Using $K_a(k+1) = P(k+1|k+1)H_a^T R_a^{-1}$,

$$\begin{aligned} &K_a(k+1) \\ &= \left[I + P(k+1|k)H_a^T R_a^{-1} H_a\right]^{-1} P(k+1|k)H_a^T R_a^{-1} \\ &= \left[I + P(k+1|k)H^T R^{-1} H\right]^{-1} P(k+1|k) \\ &\bullet \left[H^T R_1^{-1} \;\vdots\; H^T R_2^{-1} \;\vdots\; \ldots \;\vdots\; H^T R_N^{-1}\right] \end{aligned} \quad (38)$$

Considering local gain as

$$K_i(k+1) = \left[I + P_i(k+1|k)H^T R_i^{-1} H\right]^{-1} P_i(k+1|k)H^T R_i^{-1}. \quad (39)$$

Using (33),

$K_i(k+1)$
$$= \left[I + \gamma_i P(k+1|k) H^T (\gamma_i R)^{-1} H\right]^{-1} \gamma_i P(k+1|k) H^T R_i^{-1}$$
$$= \left[I + P(k+1|k) H^T R^{-1} H\right]^{-1} \gamma_i P(k+1|k) H^T R_i^{-1}. \tag{40}$$

Consequently (38) is equal to

$$K_a(k+1) = \left[\frac{K_1(k+1)}{\gamma_1} \;\vdots\; \frac{K_2(k+1)}{\gamma_2} \;\vdots\; \cdots \;\vdots\; \frac{K_N(k+1)}{\gamma_N} \right] \tag{41}$$

and substituting (41) into (14) and using (32), we can conclude that the combined global covariance (24) is equivalent the globally optimal covariance (14) at k+1.

For the globally optimal estimate,

$\hat{x}(k+1|k+1)$
$$= \hat{x}(k+1|k) + K_a(k+1)\left[z_a(k+1) - H_a \hat{x}(k+1|k)\right]$$
$$= \hat{x}(k+1|k) + \sum_{i=1}^{N} \frac{1}{\gamma_i} K_i(k+1)\left[z_i(k+1) - H\hat{x}(k+1|k)\right] \tag{42}$$

Using (20) and (34),

$\hat{x}(k+1|k+1)$
$$= \hat{x}(k+1|k) + \sum_{i=1}^{N} \frac{1}{\gamma_i}\left[\hat{x}_i(k+1|k+1) - \hat{x}_i(k+1|k)\right]$$
$$= \hat{x}(k+1|k) + \sum_{i=1}^{N} \frac{1}{\gamma_i}\left[\hat{x}_i(k+1|k+1) - \hat{x}(k+1|k)\right] \tag{43}$$
$$= \sum_{i=1}^{N} \frac{1}{\gamma_i} \hat{x}_i(k+1|k+1)$$

and we can conclude that the combined global estimate (21) is equivalent the globally optimal estimate (13) at k+1. By the results of the mathematical induction, it can be concluded that Theorem 1 holds. ■

In Theorem 1, the global optimality means that combined estimate and covariance obtained by (21) and (24) are equivalent to those obtained by (13) and (14) in the centralized Kalman filter equations.

4. FEATURES OF THE GAIN FUSION ALGORITHM

While the gain fusion algorithm is similar to the existing FKF algorithm, it has some unique characteristics in its implementation. First of all, a local Kalman gain matrix takes a main role away from a local covariance matrix in the fusion process.

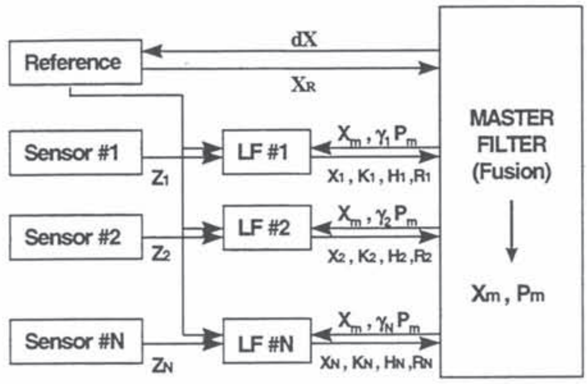

Fig. 2. Gain fusion algorithm structure
: covariance reset

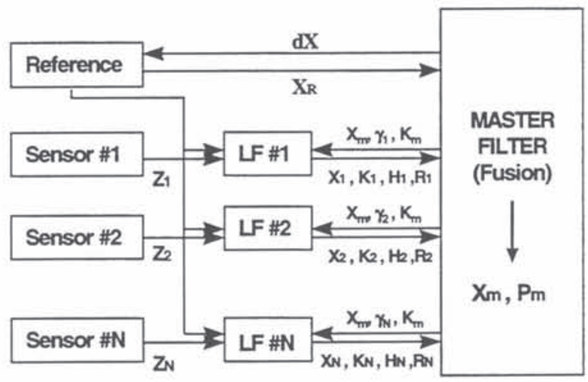

Fig. 3. Gain fusion algorithm structure
: gain reset

For the reset process, two alternative configurations can be considered. One is a covariance reset mode and the other is a gain reset mode. In the covariance reset mode illustrated in Fig. 2, fused covariance matrices multiplied by sharing factors are distributed to local filters in the same way of the FKF. Unless the amount of communication in the reset process, it is simpler implementation than the gain reset mode because no supplementary reset process is required in local filters. In this mode, local filters do not have to update local filter covariance using local Kalman gain.

In the gain reset mode, the fused gain matrix is delivered to local filters together with sharing factors as illustrated in Fig. 3. Due to the size of gain matrix, this mode has an advantage over the other mode. For the global optimality, however, covariance matrices of local filters must be reset to the scaled master filter covariance matrix before the next iteration begins. Therefore a supplement process, in which local covariance matrices are updated using the fused gain matrix, is required in all local filters.

Another favorite feature of the gain fusion algorithm is the reduction of communication burden. Suppose n state variables and m measurements are used in a local filter described in (17)-(20). The dimensions of P, K, H and R are $n\times n$, $n\times m$, $m\times n$ and $m\times m$, respectively. In the gain fusion algorithm, K, H and R are required to be transferred to the master filter while an inverse of P is required in the federated Kalman filter. Hence, in the gain fusion algorithm and FKF, respective total amount of variables required to transferred are $2n\times m + m\times m$ and $n\times n$. These values are obtained not considering state estimates, whose amount of variables are equivalent in those two approaches. Consequently, the gain fusion algorithm has advantages over the FKF in communication of variables if following condition

$$\frac{n}{m} > \frac{n+m}{n-m} \tag{44}$$

is satisfied. In many applications of multisensor navigation systems, (44) generally hold because n is much larger than m due to the state augmentation for inertial sensor errors.

5. EXAMPLE

In this paper, an example is suggested to give the quantitative comparison between a conventional FKF and the gain fusion algorithm. Since these two algorithms show the same results when all local sensors produce equivalent information except their precision, it is important to compare the amount of calculation of these algorithms.

Consider a practical 4$^{\text{th}}$ order satellite tracking system(Rauch, *et al.*, 1965; Watanabe, 1985). Let x_1 be the angular position of the satellite, x_2 the angular velocity of the satellite, x_3 constant acceleration, and x_4 stochastic acceleration, respectively. Further, we have

$$\begin{aligned} \boldsymbol{x}(k+1) &= \begin{bmatrix} 1 & 1 & 0.5 & 0.5 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.606 \end{bmatrix} \boldsymbol{x}(k) + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} w(k) \\ &\equiv \boldsymbol{F}(k)\boldsymbol{x}(k) + \boldsymbol{G}(k)w(k) \end{aligned} \tag{45}$$

where

$$\boldsymbol{x}(0) \sim N(\hat{\boldsymbol{x}}(0), \boldsymbol{P}(0))$$
$$w(k) \sim N(0, Q).$$

Assume that two parallel measurements are available as follows.

$$\begin{aligned} z_i(k) &= \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \boldsymbol{x}(k) + v_i(k) \\ &\equiv \boldsymbol{H}\boldsymbol{x}(k) + v_i(k),\ i = 1,\ldots,2. \end{aligned} \tag{46}$$

where

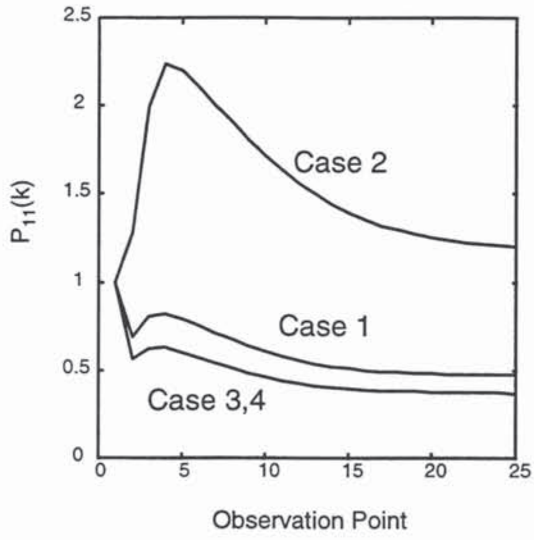

Fig. 4. Simulation results(P_{11})

$$v_i(k) \sim N(0, R_i)$$

Numerical values of initial covariance, process noise and measurement noises are assumed as in Table 1. In this example, measurements 1, 2 are assumed to be equivalent information except their precision. Hence, it satisfies the assumption of Theorem 1 and the gain fusion algorithm can be applied to this system.

Table 1. Numerical values in simulation

error sources	values
$\boldsymbol{P}(0)$	$diag(1,1,1,0.01)$
Q	0.0063
R_1	1
R_2	3

Simulations are performed considering 4 different cases, which can be classified by system conditions and applied filter methods. Simulation conditions according to system conditions and applied filters are represented in Table 2.

Table 2. Classification of simulation

Classification	System	Filter
Case 1	sensor 1	general KF
Case 2	sensor 2	general KF
Case 3	sensor 1,2	FKF
Case 4	sensor 1,2	GFA

Fig. 4 represents the time-plot of the (1,1) element of the error covariance matrix. Fig. 4 shows an illustration of the synergistic use of two sensors to increase the capabilities of systems. Also it represents that the gain fusion algorithm produces the same results obtained by using the optimal federated

Kalman filter. It can be concluded that this example surely confirms the theoretical results suggested in Theorem 1.

Not only the optimality, but also reduction of calculation is an advantage of the gain fusion algorithm. Even though it can produce the optimal results, it needs less computation than the conventional FKF. Table 3 represents the amount of calculation in one cycle using the gain fusion algorithm and the FKF. In Case A, a conventional Kalman filter routine is used to update covariance matrices while the Joseph form is used in Case B. In all cases, it is not required to update local covariance matrices when the gain fusion algorithm is used. This results from the fact that local covariance matrices are not used in the gain fusion process and reset by scaled global covariance matrices.

Table 3. Amount of calculation(FLOPS)

conditions	FKF	GFA
Case A	1836	1266
Case B	2292	1610

The amount of communications is another strong good point of the gain fusion algorithm. Using n=4 and m=1 which satisfy (44), the amount of variables required to transferred are 10 and 16 in the gain fusion algorithm and FKF, respectively. As mentioned in section 4, these values are obtained not considering state estimates, whose amount of variables are equivalent in those two approaches. Consequently, it is shown that this example makes clear the mathematical results suggested in previous sections.

6. CONCLUSION

In this paper a gain-fusion algorithm is newly proposed. This algorithm is really an alternative fusion process of the federated Kalman filter and has some advantages over existing fusion process using information matrices of local filters. In this algorithm, global optimal covariance and estimates are produced by using local Kalman gains and estimates.

Consequently, this algorithm reduces the amount of communications and avoids the need to calculate inverse covariance matrices in local filters. It is mathematically shown that the suggested algorithm guarantees the global optimality when all local sensors produce equivalent information except their precision. Also the global optimality and the efficiency of numerical calculations are shown through an example of a satellite tracking system. Prospected is that this algorithm may be well suited for implementation of the multisensor navigation systems.

ACKNOWLEDGEMENT

Authors wish to acknowledge that this work has been partially supported by Special Project Fund given to Automatic Control Research Center(ACRC), Seoul National University from Agency for Defense Development(ADD).

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