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An Improved State-Independent Fusion Algorithm Based on the Federated Kalman Filters

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Abstract: In this paper, an improved optimal fusion algorithm based on independent fusion of states is proposed for federal filter, which can mainly solve the problem that the state with poor estimation accuracy pollutes other states during fusion, which causes the estimation accuracy of the federated filter system to decrease and the convergence rate to slow down. This improves the stability of the federated filter, thereby improving the robustness of the federal filter. Meanwhile, for the problem of complex fusion weighting matrix, large amount of calculation and poor stability in the fusion algorithm proposed by Carlson, the improved fusion algorithm, the improved fusion algorithm can reduce the complexity of fusion, reduce the operation time of the filtering system, and improve the stability of the filtering system.

Key Words: Integrated navigation system, fusion algorithm, Federal filtering.

1 Introduction

As the technology of navigation develops, the demand for the performance of navigation and guidance systems becomes increasingly high [1]. Single navigation method has its shortcomings. Strapdown inertial navigation system (SINS) has the advantages of the external environment, but its navigation accuracy decreases with time [2]. Global positioning system (GPS) can obtain the high accuracy position and velocity information continuously, but it's vulnerable to external interference. So the integrated navigation system has been one of the most important navigation schemes [1].

Data fusion process is an important issue in the integrated navigation system [3]. The main disadvantages of the centralized filter architecture are its huge calculation cost and low reliability. The federal filter which is proposed in [4] has the characteristics of high reliability, design flexibility, and being easy to apply to data fusion of navigation. The fusion method is based on Kalman filtering technology [5]. There are four commonly used modes of the federal filter. The fusion-feedback mode can greatly improve the accuracy of the filtering system. Despite the wide application of the federal Kalman filtering and its extensions, it still fails to achieve satisfactory performance due to the following reasons. When faults occur, subsystems can pollute each other which reduces the accuracy of the system. Moreover, the fusion of the federal Kalman filtering is complex. To over these shortcomings, significant achievements have been made in research on information distribution and fault detection.

Different information distribution methods can lead to different filtering structures, which directly affect the fault tolerance and robustness of the federal filter [1]. In recent years, the improvement of the federal filter fault tolerance has focused on the research of dynamic information factor allocation. The literature [6] proposed a kind of dynamic information allocation method based on the F-norm of the estimation error covariance matrix. The literature [1] proposed a method that allows each system state variable to have a different dynamic information allocation method to improve system fault tolerance. The literature [7] proposed an information allocation method based on the ratio of the real-time residual covariance matrix to the prior estimation, which improves the stability of the federal filtering system.

Another part of scholars focused on improving the fault tolerance and enhancing the robustness of the system through fault detection and isolation [2]. The basic fault detection methods are residual chi-square detection and state chi-square detection [8]. State chi-square test "recursor" recursive state values may diverge over time. The literature [9] proposed an integrated fault detection scheme in which the state chi-square test and residual chi-square test are integrated together and applied to the fault detection of the federal Kalman filter. Residual chi-square detection method is not effective for slowly changing fault detection, and the literature [10] proposed a scheme which combines the estimation capability of adaptive Kalman filter and the learning capability of BP neural net.

However, these improvements have no way to improve the pollution of the system by state with poor estimation accuracy which reduce the stability of the filtering system, and these improvements takes a lot of time to perform fault detection in each subsystem, increasing system complexity. Motivated by the mentioned problems, the objective of this paper is to improve the fusion algorithm to improve the independence of state fusion to improve the robustness and stability of the system and reduce the time cost of fusion.

The main contributions of this paper are summarized as follows. Compared with the fusion algorithm in [4], which do not take different estimation accuracy and convergence speed for different states into consideration, the proposed fusion algorithm use diagonal matrix as fusion weighting matrix, which effectively reduce the system pollution caused by poor estimation accuracy. Meanwhile, since the inversion operation is not required in the fusion, the calculation amount of the system is greatly reduced, and the real-time performance of the system is improved. Rounding errors in

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system calculations are also effectively limited. In summary, the robustness and stability of the filtering system have been greatly improved.

The rest of this paper is organized as follows. Section II states the principle of federated filtering; the proof of improved fusion algorithm reported in Section III; Section IV builds integrated navigation model, and simulation results and analysis is reported in Section V.

2 Principle of conventional federated filtering information fusion mode

The federated filtering system is a decentralized filtering system with a parallel two-stage structure. Each subsystem consists of a common state and a proprietary state. Federal filtering uses the principles of variance upper bound to eliminate the correlation between subsystems, so that each subsystem processes the filtering information separately, and then subsystems send the local estimate and the estimation error variance matrix to the main filter for fusion. After that, the main filter distributes the fused information to each sub-filter. The general structure of the federated filter is shown in the figure (see Fig. [1]).

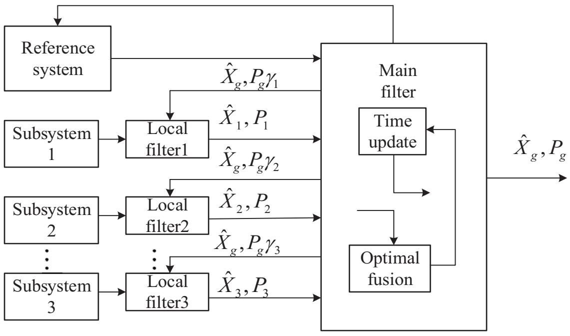

Fig. 1: Traditional federated filter

There are n subsystems in the federated filter. The state equations and measurement equations of the subsystems can be expressed as follows.

$$X^c_{k+1} = \Phi_{k+1,k} X^c_k + \Gamma_{k+1,k} W_k \tag{1}$$

$$Z^i_{k+1} = H^i_{k+1} Z^i_k + V_{k+1} \tag{2}$$

where X^c_k is the common state of the system at time k; Z^i_k is the observation of the i subsystem at time k; $\Phi_{k+1,k}$ is the transfer matrix from time k to time $k+1$; $\Gamma_{k+1,k}$ is the process noise input matrix; W_k is system noise sequence; H^i_{k+1} is the measurement matrix; V_{k+1} is the measurement noise sequence which can be approximatively described as the gauss white noise.

Federal filtering includes information distribution, sub-filter time update and measurement update, and main filter information fusion. The process is as follows:

Part* 1: *Information distribution

The state estimation information P_k^{-1} and the process noise information Q_k^{-1} are distributed as follows:

$$\begin{cases} (P^i_k)^{-1} = \beta_i (P^g_k)^{-1} \\ (Q^i_k)^{-1} = \beta_i (Q^g_k)^{-1}, \sum_{i=1}^{n} \beta_i = 1 \\ \hat{X}^i_k = \hat{X}^g_k \end{cases} \tag{3}$$

Where β_i is the information sharing coefficient.

Part* 2: *Sub-filter time update and measurement update

(1) Time update:

$$\bar{X}^i_{k+1,k} = \Phi_{k+1,k} \hat{X}^i_k \tag{4}$$

$$\bar{P}^i_{k+1,k} = \Phi_{k+1,k} P^i_k \Phi^T_{k+1,k} + \Gamma_{k+1,k} Q^i_k \Gamma^T_{k+1,k} \tag{5}$$

(2) Measurement update:

$$K^i_{k+1} = P^i_{k+1} (H^i_{k+1})^T R_i^{-1} \tag{6}$$

$$\hat{X}^i_{k+1} = \bar{X}^i_{k+1,k} + K^i_{k+1}(Z^i_{k+1} - H^i_{k+1}\bar{X}^i_{k+1,k}) \tag{7}$$

$$P^i_{k+1} = [I - K^i_{k+1} H^i_{K+1}] \bar{P}^i_{k+1,k} \tag{8}$$

Part* 3: *Main filter data fusion

$$P^g_{k+1} = (\sum_{i=1}^{n} (P^i_{k+1})^{-1})^{-1} \tag{9}$$

$$\hat{X}^g_{k+1} = P^g_{k+1} \sum_{i=1}^{n} (P^i_{k+1})^{-1} \hat{X}^i_{k+1} \tag{10}$$

where $\bar{X}_{k+1,k}$ is the prior estimate of the system state at time k; $\hat{X}_k$ is the updated estimate of the system state at time k; $\bar{P}_{k+1,k}$ is the prior error covariance matrix at time k; $\hat{P}_{k+1}$ is the updated error covariance matrix at time k; K_{k+1} is the Kalman Gain.

3 Fusion mode based on diagonal weighting matrix

3.1 Effect of Sub-filter Fusion Algorithm on System Performance

The federated Kalman filtering fusion method proposed by Carlson meets the minimum trace of the error covariance matrix of the global state estimation after fusion.

Covariance matrix of global state estimation:

$$P_g = [P_{11}^{-1} + P_{22}^{-1} + \cdots + P_{nn}^{-1}] \tag{11}$$

where P_g is the error covariance matrix of the global state estimation, and $P_{11} \cdots P_{nn}$ is the error covariance matrix of the state estimation of each filtering subsystem. From Eq. (11), it can be concluded that when the number of subsystems increases, the traditional sub-filter fusion method requires a large number of inversion operations for the state estimation error covariance matrix of the sub-filter system.

Moreover, as the number of fusion calculations increases and affected by the rounding error and accuracy limitations of the computing system, the error covariance matrix of the state estimation of the subsystem may become a non-singular matrix, causing the calculation to fail and the entire system to fail. This kind of risk becomes even greater as the number of subsystems increases.

Therefore, we can conclude that if the sub-filter fusion algorithm can be improved so that the main filter fusion no longer needs to perform the inversion of the error covariance matrix of the sub-filter state estimation, it will greatly reduce the calculation amount and greatly improve the stability of the system. As the number of sub-filter systems increases, the benefits of this improvement become more apparent.

Considering the case where multiple sub-filters are fused, global state estimate after fusion can be expressed as a linear combination of the local state estimates of the sub-filters.

$$\hat{X}_g = W_1\hat{X}_1 + W_2\hat{X}_2 + \cdots + W_n\hat{X}_n \tag{12}$$

where $\hat{X}_g$ is the global state estimation after fusion, $\hat{X}_n$ is the local state estimation of the sub-filter, and W_n is the weighting matrix of the sub-filter fusion.

During the fusion of the main filter of the federal filter, if the weighting matrix is a diagonal matrix, the fusion weighting matrix of sub-filter i can be expressed as follows:

$$W_i = \begin{bmatrix} q_{i1} & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & & 0 & \vdots \\ \vdots & & q_{il} & & \vdots \\ \vdots & 0 & & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & q_{in} \end{bmatrix} \tag{13}$$

According to Eq. (13), for any number l $(1 \le l \le n)$, the l-dimensional state estimator of the global state estimate after the main system fusion of the federal system is expressed as follows:

$$q_f = q_{1l} + q_{2l} + \cdots + q_{nl} \tag{14}$$

$$X_{gl} = \frac{q_{1l}}{q_f} X_{1l} + \frac{q_{2l}}{q_f} X_{2l} \cdots + \frac{q_{nl}}{q_f} X_{nl} \tag{15}$$

It can be concluded from Eq. (14) and Eq. (15) that when the fusion weighting matrix is a diagonal matrix, the state estimator of any dimension after fusion is only related to the state estimator of the corresponding dimension in each sub-filter.

In the fusion, if the fusion weighting matrix is a non-diagonal matrix, the weighting matrix of the sub-filter i can be expressed as follows:

$$W_i = \begin{bmatrix} q_{i1} & 0 & \cdots & \cdots & 0 \\ 0 & \ddots & & q_{l-1,l+1} & \vdots \\ \vdots & & q_{il} & & \vdots \\ \vdots & q_{x,y} & & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & q_{in} \end{bmatrix} \tag{16}$$

There is at least one non-diagonal element in the non-diagonal matrix that is not zero, and $q_{x,y}$ is a non-zero number. $q_{x,y}$ is the value of the x dimensional state estimation of the sub-filter i to the y dimensional global state estimation.

We can conclude from Eq. (16) that when the fusion weighting matrix is a non-diagonal matrix, the state estimator of the sub-filter will affect the state estimators of different dimensions in the global state estimation after fusion. However, when the fusion weighting matrix is a diagonal matrix, the state estimator of the sub-filter will not affect the state estimators of other dimensions of the global state estimation.

3.2 Improved diagonal fusion weighting matrix

The sub-filter fusion weighting matrix is the part that can be improved for fusion algorithms. In the federal Kalman filter proposed by Carlson, the fusion weighting matrix of the sub-filter i is as follows:

$$W_i = \frac{P_{ii}^{-1}}{P_g^{-1}} \tag{17}$$

It can't be determined from Eq. (17). In the federal filter proposed by Carlson, through the simulation experiment of SINS/ODO/GPS integrated navigation, which can make whether the fusion weighting matrix of the odometer and the GPS subsystem is a diagonal matrix or not clear. According to the statistics of the simulation results, in the 85,000 filtering update cycles, all the weighting matrices are non-diagonal matrices.

When the federal Kalman filtering method is applied to integrated navigation systems, the common state quantities are often in different orders of magnitude, and the different state quantities of the subsystems have different estimation accuracy and convergence velocity. Therefore, the mutual influence between state quantities will reduce the robustness of the federated filtering system and the stability of the entire system in the face of strong interference. A highly disturbed state may pollute a state with high estimation accuracy, further affecting the global stability of the system.

On the contrary, while the fusion weighting matrix is a diagonal matrix, when the sub-filters are fused, the different state quantities do not affect each other. In the case of a large interference in the state quantities of one or more subsystems, the Robustness and system stability.

In the case where the weighting matrix is a diagonal matrix, the error covariance matrix of the global state estimation after the federation system fusion is simplified compared with the federal filtering method proposed by Carlson. The covariance matrix of the global estimate after fusion is as follows:

$$P_g = E\left[(X - \hat{X}_g)(X - \hat{X}_g)^T\right] \tag{18}$$

Since global estimates meets unbiased principle, thus

$$W_1 + W_2 + \cdots + W_n = I \tag{19}$$

Putting Equation (11) into Equation (18), we can get global estimation error is shown in Eq. (20):

$$P_g = E\left[\left[\tilde{X}\right]\left[\tilde{X}\right]^T\right] \tag{20}$$

where

$$\tilde{X} = \begin{pmatrix} W_1 + W_2 \\ + \cdots + W_n \end{pmatrix} X - \begin{pmatrix} W_1\hat{X}_1 + W_2\hat{X}_2 \\ + \cdots + W_n\hat{X}_n \end{pmatrix} \tag{21}$$

By merging similar terms, we can get

$$P_g = E\left[\begin{bmatrix} W_1(X - \hat{X}_1) + \cdots \\ + W_n(X - \hat{X}_n) \end{bmatrix}\begin{bmatrix} W_1(X - \hat{X}_1) + \cdots \\ + W_n(X - \hat{X}_n) \end{bmatrix}^T\right] \tag{22}$$

In the integrated navigation system, the state estimates of the sub-filters are not correlated, so the following conclusions are drawn

$$P_{Wij} = E\left[\left[W_i(X - \hat{X}_i)\right]\left[W_j(X - \hat{X}_j)\right]^T\right] \tag{23}$$

$$P_{Wij} = \begin{cases} 0 & i \ne j \\ W_i P_{ii} W_i^T & i = j \end{cases} \tag{24}$$

P_{Wij} in Eq. (23) and Eq. (24) is the error covariance matrix of the weighted local estimated state during the fusion process. Eq. (24) indicates that when the state quantities of different sub-filters are not correlated, the error variance matrix of the local state estimation.

By taking Eq. (24) into Eq. (22), we can obtain Eq. (25).

$$P_g = W_1 P_{11} W_1^T + \cdots + W_n P_{nn} W_n^T \tag{25}$$

From Eq. (25), it can be seen that if the weighting matrix can be determined in advance and the error covariance matrix of the global state estimation can be calculated by Eq. (25), a large number of inversion operations and division operations can be avoided, which not only reduces time cost but also improves system operation The stability.

If the fusion weighting matrix is a diagonal matrix, and the weights are assigned according to the estimation accuracy and convergence velocity of each sub-filter state estimator, a diagonal fusion weighting matrix is obtained, and then the main filter is fused according to Eq. (25). The entire process is obtained this simplifies and makes the system more stable.

3.3 Optimal design of diagonal fusion matrix

In the federated filter of multiple sub-filters, search for the optimal diagonal weighting matrix, so that for the diagonal fusion weighting matrix, the state estimator of each dimension of each sub-filter is fused according to the optimal weight, and The global state estimate after fusion is also optimal to some extent.

The error of the i-dimensional state estimator is as follows:

$$\tilde{X}_i = q_{1i}\hat{X}_{1i} + q_{2i}\hat{X}_{2i} + \cdots + q_{ni}\hat{X}_{ni} - X_i \tag{26}$$

$$P_{gii} = E\left[\left[\tilde{X}_i\right]\left[\tilde{X}_i\right]^T\right] \tag{27}$$

where $\tilde{X}_i$ represents the estimation error of the i-th state estimator $\hat{X}_{ni}$ represents the estimated value of the i-th state quantity of the n-th sub-filter, and X_i represents the true value of the i-th state quantity. q_{ni} represents the weighted value of the i-th state quantity of the n-th sub-filter. P_{gii} represents the variance of the i-th dimension of the global estimate after fusion.

Since the global estimation is unbiased, we can get

$$q_{1i} + q_{2i} + \cdots + q_{ni} = 1 \tag{28}$$

Combining Eq. (27) and Eq. (28), we can get

$$\begin{aligned} P_{gii} &= E(\left[q_{1i}(\hat{X}_{1i} - X_i) + \cdots q_{ni}(\hat{X}_{ni} - X_i)\right] \\ &\quad \left[q_{1i}(\hat{X}_{1i} - X_i) + \cdots q_{ni}(\hat{X}_{ni} - X_i)\right]^T) \\ &= E[q_{1i}^2(\hat{X}_{1i} - X_i)(\hat{X}_{1i} - X_i)^T + \cdots + \\ &\quad q_{xi}q_{yi}(\hat{X}_{xi} - X_i)(\hat{X}_{yi} - X_i)^T + \cdots + \\ &\quad q_{ni}^2(\hat{X}_{ni} - X_i)(\hat{X}_{ni} - X_i)^T] \end{aligned} \tag{29}$$

Since the state estimates of different sub-filters are different, we can get from Eq. (29)

$$E[q_{xi}q_{yi}(\hat{X}_{xi} - X_i)(\hat{X}_{yi} - X_i)^T] = 0 \tag{30}$$

Putting Eq. (30) into Eq. (29), we get

$$\begin{aligned} P_{gii} = E[&q_{1i}^2(\hat{X}_{1i} - X_i)(\hat{X}_{1i} - X_i)^T + \cdots + \\ &q_{ni}^2(\hat{X}_{ni} - X_i)(\hat{X}_{ni} - X_i)^T] \end{aligned} \tag{31}$$

To get the optimal weighted diagonal matrix for each state estimator to fusion. Eq. (31) uses the Lagrangian method to obtain the optimal fusion weight value of the i-th state quantity of the sub-filter m.

The constructed Lagrangian equation is as follows:

$$\begin{aligned} La = &q_{1i}{}^2 P_{1ii} + q_{2i}{}^2 P_{2ii} + \cdots + q_{ni}{}^2 P_{nii} \\ &+ \lambda(q_{1i} + q_{2i} + \cdots + q_{ni} - 1) \end{aligned} \tag{32}$$

where

$$P_{nii} = E[(\hat{X}_{ni} - X_i)(\hat{X}_{ni} - X_i)^T] \tag{33}$$

The Lagrange equation finds the partial derivative of the state of the sub-filter and makes its result zero. The Eq. (34) can be obtained as following:

$$\begin{aligned} 2q_{1i}P_{1ii} + \lambda &= 0 \\ 2q_{2i}P_{2ii} + \lambda &= 0 \\ &\vdots \\ 2q_{ni}P_{nii} + \lambda &= 0 \\ q_{1i} + q_{2i} + \cdots + q_{ni} - 1 &= 0 \end{aligned} \tag{34}$$

By solving Eq. (34), The optimal fusion weighting value for the i-th dimension of the sub-filter m can be obtained as

$$q_{mi} = \frac{1/P_{mii}}{1/P_{1ii} + 1/P_{1ii} + \cdots + 1/P_{1ii}} \tag{35}$$

In Eq. (35), q_{mi} is the value on the i-th row and i-th diagonal fusion matrix of the sub-filter m. Therefore, the estimated fusion matrix of each sub-filter state can be obtained from the error covariance matrix of the sub-filter state estimation. The fusion weighting matrix of sub-filter m is as follows:

$$W_m = diag\left(q_{m1}, \cdots, q_{mi}, \cdots, q_{mn}\right) \tag{36}$$

This greatly simplifies the calculation process and avoids inversion and division operations. Federated filter system is more stable and more robust to sub-filter interference problems.

4 SINS/GPS/ODO Federated filtering based on diagonal optimal information fusion

4.1 Improved federation system architecture

The federated Kalman filter based on the decentralized filtering theory firstly decentralizes the subsystem information and then fuses it together. In the fusion-feedback mode, each sub-filter is updated and calculated by the feedback information of the main filter, and then fused in the main filter. This mode has higher accuracy.

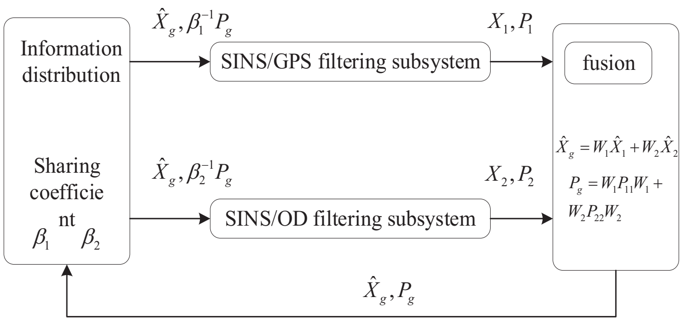

Fig. 2: Improved federation system

The fused state estimation and its error covariance matrix are allocated to each sub-filter through information allocation. The sub-filter constructs a fusion weighting matrix according to Eq. (35) and Eq. (36), and sends the state estimation and error covariance matrix to the main filter for fusion according to Equation (25).

4.2 SINS/GPS/OD Integrated Navigation System Model

(a) Integrated Navigation System State Equation

Select the local Cartesian coordinate system for the navigation coordinate system, SINS as the reference system, and attitude, velocity, displacement, gyroscope, and accelerometer errors as state variables. The total number of dimensions is 15, and the state equation of the system is as follows:

$$\dot{X}(t) = F(t)X(t) + G(t)W(t) \tag{37}$$

The common state variables are as follows:

$$\begin{aligned} X = [&\delta\psi\ \delta\varphi\ \delta\theta\ \delta v_E\ \delta v_N\ \delta v_U \\ &\delta L\ \delta\lambda\ \delta h\ \varepsilon_x\ \varepsilon_y\ \varepsilon_z\ \nabla_x\ \nabla_y\ \nabla_z] \end{aligned} \tag{38}$$

In Eq. (38): δψ, δφ, δθ is the attitude angle error; δv_E, δv_N, δv_U is the velocity error of the vehicle in the east, north, and sky directions; δL, δλ, δh is the latitude, longitude, and altitude errors; ε_x, ε_y, ε_z is the gyro constant drifts, and $\nabla_x, \nabla_y, \nabla_z$ is the accelerometer white Gaussian noise.

(b) SINS/GPS subsystem measurement equation

Select the difference between the output position and velocity of the SINS and the GPS as the filter observation measurement, then the measurement equation of the SINS/GPS subsystem can be expressed as

$$Z_1(t) = \begin{bmatrix} Z_v(t) \\ Z_p(t) \end{bmatrix} = \begin{bmatrix} H_v(t) \\ H_p(t) \end{bmatrix} X(t) + V_1 \tag{39}$$

where

$$\begin{aligned} H_v(t) &= [0_{3\times3}\ I_{3\times3}\ 0_{3\times9}] \\ H_p(t) &= [0_{3\times6}\ I_{3\times3}\ 0_{3\times6}] \end{aligned} \tag{40}$$

V_1 represents the noise variance matrix of the vehicle receiving GPS acceptance velocity and position information.

(c) SINS/OD subsystem measurement equation

Due to factors such as road instability, tire friction, etc., the actual mileage represented by each pulse of the odometer has an error, that is, the odometer scale factor has a certain error, recorded as δK, and at the same time as the exclusive state of the SINS / OD subsystem.

Select the difference between the velocity calculated by SINS and the velocity estimated by odometer as the observation. In this subsystem, the difference between the velocity calculated by SINS in the navigation coordinate system and the OD output velocity converted to the navigation coordinate system is used as a measurement. The state information of the subsystem is estimated in real time through the Kalman filter method. The observation matrix of this subsystem is determined by the observation calculation. The observation calculation formula is shown below:

$$\begin{aligned} \Delta v^n &= v^n + \delta v^n - [I - (\varphi\times)]C_b^n(1+\delta K)v_D^b \\ &= \delta v^n + (\varphi\times)v^n - C_b^n \delta v_D^b \end{aligned} \tag{41}$$

where v^n is the velocity calculated by SINS, δv^n is the error between the velocity calculated by SINS and the actual velocity, φ × is the antisymmetric matrix, C_b^n is the direction cosine matrix of the navigation calculation system, and v_D^b is the velocity in the carrier coordinate system calculated by the odometer.

Then the observation equation is as follows:

$$Z_2(t) = H_v^{'} X(t) + V_2 \tag{42}$$

where

$$H_v^{'} = \left[0_{3\times3}\ I_{3\times3}\ (\varphi\times)v^n\ 0_{3\times6}\ -C_b^n \delta v_D^b\right] \tag{43}$$

5 Simulation and analysis

5.1 Simulation comparison experiment Ⅰ

This simulation experiment aims to compare the time cost of the two fusion methods to show better real-time and practicality of the improved fusion method. Carrier vehicle driving simulation experiments are performed. The trajectories of the simulation experiments include motion states such as uniform acceleration, left turn, right turn, and climbing. Suppose that the gyro random constant drift is 0.002(°)/h, and the accelerometer random constant drift is 10μg. SINS system output frequency is 400Hz. Simulation travel time is about 850s, 34,0000 status update cycles

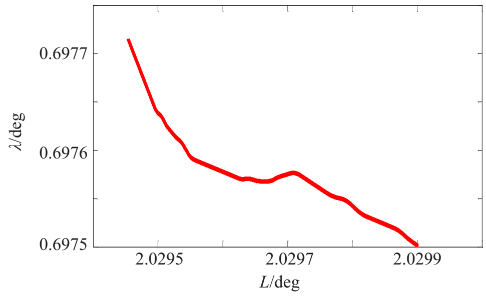

Fig. 3: Vehicle trajectory

By calculating the time cost of fusion of the simulation experiment for many times and take the average, the following table can be obtained

Table 1: Time cost of the two fusion modes

Mode	time
The running time of traditional fusion	181.387558
The running time of improved fusion	155.405515
Program of traditional fusion running time	8.166430×10^{-5}
Program of improved fusion running time	2.815069×10^{-5}

According to the Table 1, we get the conclusions that the time cost of the whole program of the improved fusion mode is less than the traditional one, and as the number of subsystems increases, so does the time cost reduction. The program of improved fusion process cost much less time than the traditional one, and ss vehicle travel time increases, the saved time cost is also increasing. This undoubtedly increases the real-time nature of the federated filtering system.

5.2 Simulation comparison experiment Ⅱ

This simulation experiment aims to compare the error between the simulated route of two fusion modes and the actual route in the case of large GPS and OD errors. We use the Euclidean distance to represent this difference. This simulation selects the conditions where GPS latitude error increased to 13m, GPS longitude error increased to 10m, and speed error increased to 0.15 m/s

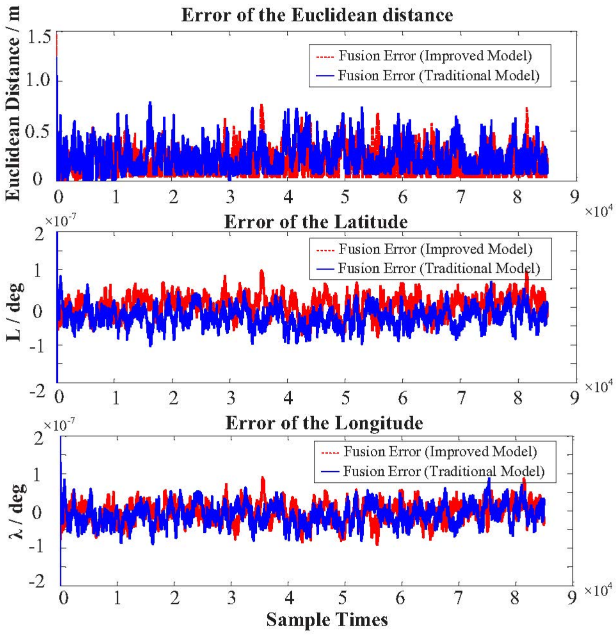

Fig .4: Contrast under larger error

Fig. [4] shows that when the GPS and odometer errors are relatively large, the route error simulated by the improved fusion method is smaller than the route error simulated by the traditional fusion method proposed by Carlson. After fusion by the improved method, the effect of error convergence is better.

In order to more intuitively compare the errors of the two fusion methods in the case of large errors, Table 2 lists the root mean square error (RMSE) and mean absolute percentage error (MAPE) of the simulated driving trajectory under the two fusion methods as follows:

Table 2: contrast of the two fusion algorithms

Mode	value
The RMSE of the improved fusion	0.2150
The RMSE of traditional fusion	0.2738
The MAPE of the improved fusion	1.6358e×10^{-7}
The MAPE of the traditional fusion	2.1528e×10^{-7}

5.3 Simulation comparison experiment Ⅲ

This simulation aims to compare the errors between the simulated route and the actual route of the two fusion methods in the case of a sudden change. Based on the simulation experiment II, the GPS error is increased when the experiment is 100-120s, so that the position error is 90m and the speed error is 0.2m/s. The error is simulated and the following figure is obtained.

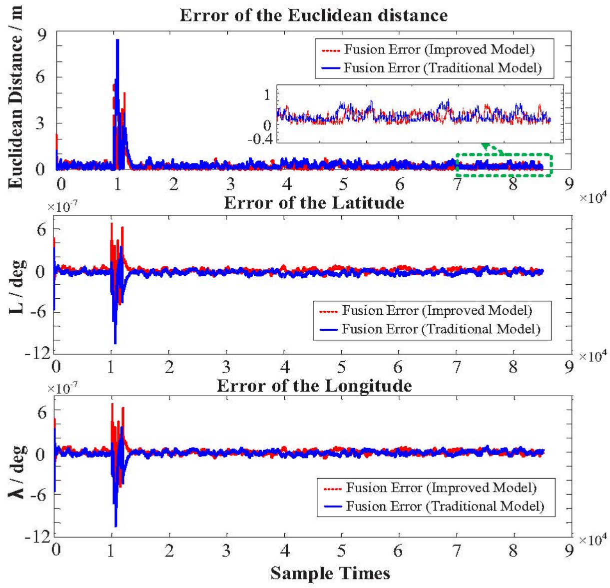

Fig. 5: contrast under a sudden change

Fig. [5] shows the Euclidean distance error, latitude error, and longitude error of the simulated route. Obviously, the filtering system with improved fusion algorithm is more robust. Table 3 lists the root mean square error (RMSE) and mean absolute percentage error (MAPE) of the simulated driving trajectory after the mutation under the two fusion algorithms as follows:

Table 3: contrast of the two fusion algorithms under mutation

Mode	value
The RMSE of the improved fusion	0.2191
The RMSE of traditional fusion	0.2870
The MAPE of the improved fusion	1.6571e×10^{-7}
The MAPE of the traditional fusion	2.1850e×10^{-7}

According to Fig. [5] and Table 3, it can be concluded that in the face of the mutation, the improved fusion method is more robust and more stable. Since each state is fused separately in the improved algorithm, there is no problem of mutual pollution of states, so the fusion effect after the mutation is better than traditional fusion method proposed by Carlson.

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