

APPENDIX

A. Mathematical Model of 6-Node Network

In order to describe the proposed model in detail, reliability assessment model proposed is applied in a 6-node network as Fig.1 shown.



Fig. 1 Fault scenario example of 6-node distribution network

(a) RA stage constraints

Assumed a fault happen on branch 3-4, RA stage constraint can be described as follows (To simplify the description, the following constraints are simplified with the known values, such as installment variables $\{x_{ij}^{i,CB}\}_{\forall ij \in Y}$, $\{x_{ij}^{j,CB}\}_{\forall ij \in Y}$, $\{x_{ij}^{i,MS}\}_{\forall ij \in Y}$, $\{x_{ij}^{j,MS}\}_{\forall ij \in Y}$, $\{x_{ij}^{i,AD}\}_{\forall ij \in Y}$, $\{x_{ij}^{j,AD}\}_{\forall ij \in Y}$, and normal operation variables $\{s_{ij}^{i,NO}\}_{\forall ij \in Y}$, $\{s_{ij}^{j,NO}\}_{\forall ij \in Y}$):

$$l_{ij}^{34,RA} = 0, ij = 23,34,45 \quad (1a)$$

$$l_i^{34,RA} = 1, i = 1,6 \quad (2a)$$

$$l_{ij}^{34,RA} = 0, ij = 23,34,45 \quad (3a)$$

$$-(1 - s_{12}^{1,34,RA})M + l_1^{34,RA} \leq l_{12}^{34,RA} \leq (1 - s_{12}^{1,34,RA})M + l_1^{34,RA} \quad (4a)$$

$$-(1 - s_{12}^{1,34,RA})M + l_2^{34,RA} \leq l_{12}^{34,RA} \leq (1 - s_{12}^{1,34,RA})M + l_2^{34,RA} \quad (5a)$$

$$-(1 - s_{56}^{6,34,RA})M + l_5^{34,RA} \leq l_{56}^{34,RA} \leq (1 - s_{56}^{6,34,RA})M + l_5^{34,RA} \quad (6a)$$

$$-(1 - s_{56}^{6,34,RA})M + l_6^{34,RA} \leq l_{56}^{34,RA} \leq (1 - s_{56}^{6,34,RA})M + l_6^{34,RA} \quad (7a)$$

$$l_{12}^{34,RA} \leq s_{12}^{1,34,RA} \quad (8a)$$

$$l_{56}^{34,RA} \leq s_{56}^{6,34,RA} \quad (9a)$$

$$l_i^{34,RA} = 0, i = 2,3,4 \quad (10a)$$

$$s_{ij}^{i,34,RA} = s_{ij}^{i,NO}, ij = 23,34,45,56 \quad (11a)$$

$$s_{ij}^{j,34,RA} = s_{ij}^{j,NO}, ij = 12,23,34,45 \quad (12a)$$

$$s_{ij}^{j,NO} = 1, ij = 12,23,34 \quad (13a)$$

$$s_{56}^{5,NO} = 1 \quad (14a)$$

$$|s_{12}^{1,34,RA} - 1| + |s_{56}^{6,34,RA} - 1| \leq 1 \quad (15a)$$

$$\sum_{i=2,3,4,5} l_i^{34,RA} = l_5^{34,RA} = l_{12}^{34,RA} + l_{56}^{34,RA} \quad (16a)$$

Analysis of the above constraints shows that the fundamental variables are switch state variables s and $s_{56}^{6,34,RA}$. The rest of the switch state variables s are easily determined at RA stage. The value of $s_{12}^{1,34,RA}$ and $s_{56}^{6,34,RA}$ determine the decision variables of other unknown values. Based on the reliability objective function, l of all load nodes will be maximized. Then l^{RA} of all load nodes can be obtained at RA stage. The calculation result can be seen in Fig. 2.

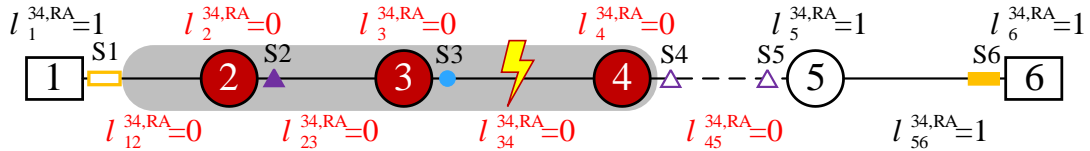


Fig. 2 RA stage of fault scenario for 6-node distribution network

(b) RI stage constraints

RI stage can be described as follows (This section mainly focuses on reliability calculation constraints. Therefore, power flow constraints are not listed in this part):

$$l_{34}^{34,RI} = 0 \quad (1b)$$

$$l_i^{34,RI} = 1, i = 1,6 \quad (2b)$$

$$l_i^{34,RI} = 0, i = 3,4 \quad (3b)$$

$$-(1 - s_{12}^{1,34,RI})M + l_1^{34,RI} \leq l_{12}^{34,RI} \leq (1 - s_{12}^{1,34,RI})M + l_1^{34,RI} \quad (4b)$$

$$-(1-s_{12}^{1,34,RI})M + l_2^{34,RI} \leq l_{12}^{34,RI} \leq (1-s_{12}^{1,34,RI})M + l_2^{34,RI} \quad (5b)$$

$$-(1-s_{23}^{2,34,RI})M + l_2^{34,RI} \leq l_{23}^{34,RI} \leq (1-s_{23}^{2,34,RI})M + l_2^{34,RI} \quad (6b)$$

$$-(1-s_{23}^{2,34,RI})M + l_3^{34,RI} \leq l_{23}^{34,RI} \leq (1-s_{23}^{2,34,RI})M + l_3^{34,RI} \quad (7b)$$

$$-(2-s_{45}^{4,34,RI}-s_{45}^{5,34,RI})M + l_4^{34,RI} \leq l_{45}^{34,RI} \leq (2-s_{45}^{4,34,RI}-s_{45}^{5,34,RI})M + l_4^{34,RI} \quad (8b)$$

$$-(2-s_{45}^{4,34,RI}-s_{45}^{5,34,RI})M + l_5^{34,RI} \leq l_{45}^{34,RI} \leq (2-s_{45}^{4,34,RI}-s_{45}^{5,34,RI})M + l_5^{34,RI} \quad (9b)$$

$$-(1-s_{56}^{6,34,RI})M + l_5^{34,RI} \leq l_{56}^{34,RI} \leq (1-s_{56}^{6,34,RI})M + l_5^{34,RI} \quad (10b)$$

$$-(1-s_{56}^{6,34,RI})M + l_6^{34,RI} \leq l_{56}^{34,RI} \leq (1-s_{56}^{6,34,RI})M + l_6^{34,RI} \quad (11b)$$

$$l_{12}^{34,RI} \leq s_{12}^{1,34,RI} \quad (12b)$$

$$l_{23}^{34,RI} \leq s_{23}^{2,34,RI} \quad (13b)$$

$$l_{45}^{34,RI} \leq s_{45}^{4,34,RI} \quad (14b)$$

$$l_{45}^{34,RI} \leq s_{45}^{5,34,RI} \quad (15b)$$

$$l_{56}^{34,RI} \leq s_{56}^{6,34,RI} \quad (16b)$$

$$l_i^{34,RI}=0, \quad i=3,4 \quad (17b)$$

$$s_{ij}^{i,34,RI} = s_{ij}^{i,34,RA} = s_{ij}^{i,NO} = 1, \quad i=34,56 \quad (18b)$$

$$s_{ij}^{j,34,RI} = s_{ij}^{j,34,RA} = s_{ij}^{j,NO} = 1, \quad ij=12,23,34 \quad (19b)$$

$$\sum_{i=2,3,4,5} l_i^{34,RI} = \sum_{i=2,5} l_i^{34,RI} = l_{12}^{34,RI} + l_{23}^{34,RI} + l_{45}^{34,RI} + l_{56}^{34,RI} \quad (20b)$$

$$l_5^{xy,RA} \leq l_5^{xy,RI} \quad (21b)$$

It is clearly that $s_{12}^{1,34,AF}$, $s_{23}^{2,34,AF}$, $s_{45}^{4,34,AF}$, $s_{45}^{5,34,AF}$ and $s_{56}^{6,34,AF}$ are fundamental variables, which determined other switch state variable s under the RI stage constraints. Then l^{RI} of all load nodes and fictitious power flow variables l can be obtained. The calculation result can be seen in Fig. 3.

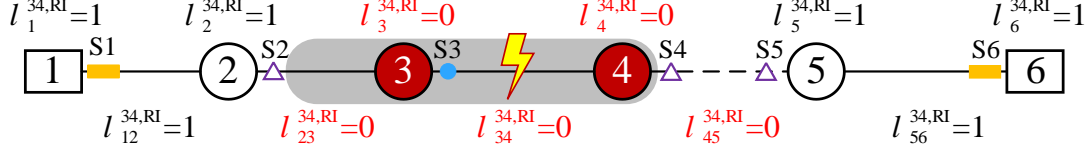


Fig. 3 RI stage of fault scenario for 6-node distribution network

(c) MI stage constraints

MI stage can be described as follows (This section mainly focuses on reliability calculation constraints. Therefore, power flow constraints are not listed in this part):

$$l_{34}^{34,MI} = 0 \quad (1c)$$

$$l_i^{34,MI} = 1, \quad i=1,6 \quad (2c)$$

$$l_4^{34,MI} = 0 \quad (3c)$$

$$-(1-s_{12}^{1,34,MI})M + l_1^{34,MI} \leq l_{12}^{34,MI} \leq (1-s_{12}^{1,34,MI})M + l_1^{34,MI} \quad (4c)$$

$$-(1-s_{12}^{1,34,MI})M + l_2^{34,MI} \leq l_{12}^{34,MI} \leq (1-s_{12}^{1,34,MI})M + l_2^{34,MI} \quad (5c)$$

$$-(1-s_{23}^{2,34,MI})M + l_2^{34,MI} \leq l_{23}^{34,MI} \leq (1-s_{23}^{2,34,MI})M + l_2^{34,MI} \quad (6c)$$

$$-(1-s_{23}^{2,34,MI})M + l_3^{34,MI} \leq l_{23}^{34,MI} \leq (1-s_{23}^{2,34,MI})M + l_3^{34,MI} \quad (7c)$$

$$-(1-s_{34}^{3,34,MI})M + l_3^{34,MI} \leq l_{34}^{34,MI} \leq (1-s_{34}^{3,34,MI})M + l_3^{34,MI} \quad (8c)$$

$$-(2-s_{45}^{4,34,MI}-s_{45}^{5,34,MI})M + l_4^{34,MI} \leq l_{45}^{34,MI} \leq (2-s_{45}^{4,34,MI}-s_{45}^{5,34,MI})M + l_4^{34,MI} \quad (9c)$$

$$-(2-s_{45}^{4,34,MI}-s_{45}^{5,34,MI})M + l_5^{34,MI} \leq l_{45}^{34,MI} \leq (2-s_{45}^{4,34,MI}-s_{45}^{5,34,MI})M + l_5^{34,MI} \quad (10c)$$

$$-(1-s_{56}^{6,34,MI})M + l_5^{34,MI} \leq l_{56}^{34,MI} \leq (1-s_{56}^{6,34,MI})M + l_5^{34,MI} \quad (11c)$$

$$-(1-s_{56}^{6,34,MI})M + l_6^{34,MI} \leq l_{56}^{34,MI} \leq (1-s_{56}^{6,34,MI})M + l_6^{34,MI} \quad (12c)$$

$$l_{12}^{34,MI} \leq s_{12}^{1,34,MI} \quad (13c)$$

$$l_{23}^{34,MI} \leq s_{23}^{2,34,MI} \quad (14c)$$

$$l_{45}^{34,MI} \leq s_{45}^{4,34,MI} \quad (15c)$$

$$l_{45}^{34,MI} \leq s_{45}^{5,34,MI} \quad (16c)$$

$$l_{56}^{34,MI} \leq s_{56}^{6,34,MI} \quad (17c)$$

$$l_4^{34,MI} = 0 \quad (18c)$$

$$l_i^{34,MI} = l_i^{34,MI}, \quad i = 2, 3, 5 \quad (19c)$$

$$s_{56}^{5,34,MI} = 1 \quad (20c)$$

$$s_{ij}^{j,34,MI} = 1, \quad ij = 12, 23, 34 \quad (21c)$$

$$\sum_{i=2,3,4,5} l_i^{34,MI} = \sum_{i=2,3,5} l_i^{34,MI} = l_{12}^{34,MI} + l_{23}^{34,MI} + l_{45}^{34,MI} + l_{56}^{34,MI} \quad (22c)$$

$$l_i^{xy,RI} \leq l_i^{xy,MI}, \quad i = 2, 5 \quad (23c)$$

It is clearly that $s_{12}^{1,34,MI}$, $s_{23}^{2,34,MI}$, $s_{45}^{4,34,MI}$, $s_{45}^{5,34,MI}$ and $s_{56}^{6,34,MI}$ are fundamental variables, which determined other switch state variable s under the MI stage constraints. Then l^{MI} of all load nodes and fictitious power flow variables l can be obtained. The calculation result can be seen in Fig. 4.

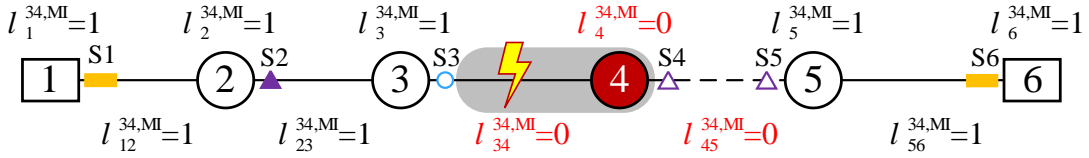


Fig. 4 MI stage of fault scenario for 6-node distribution network

(d) Interruption time of load nodes

Finally, the interruption time of load nodes in 6-node network can be expressed as follows:

$$t_2^{34} = \tau_{34}^{RI} (1 - l_2^{34,RA}) + \tau_{34}^{MI} (1 - l_2^{34,RI}) + \tau_{34}^{RP} (1 - l_2^{34,MI}) = \tau_{34}^{RI} \quad (1d)$$

$$t_3^{34} = \tau_{34}^{RI} (1 - l_3^{34,RA}) + \tau_{34}^{MI} (1 - l_3^{34,RI}) + \tau_{34}^{RP} (1 - l_3^{34,MI}) = \tau_{34}^{RI} + \tau_{34}^{MI} \quad (2d)$$

$$t_4^{34} = \tau_{34}^{RI} (1 - l_4^{34}) + \tau_{34}^{MI} (1 - l_4^{34,AF}) + \tau_{34}^{RP} (1 - l_4^{34,MF}) = \tau_{34}^{RI} + \tau_{34}^{MI} + \tau_{34}^{RP} \quad (3d)$$

$$t_5^{34} = \tau_{34}^{RI} (1 - l_5^{34}) + \tau_{34}^{MI} (1 - l_5^{34,RI}) + \tau_{34}^{RP} (1 - l_5^{34,MI}) = 0 \quad (4d)$$