Exercise 3

Monday, September 12, 2022 5:36 PM

1.3.3 Suppose that a client performs an intermixed sequence of (stack) *push* and *pop* operations. The push operations put the integers 0 through 9 in order onto the stack; the pop operations print out the return values. Which of the following sequence(s) could *not* occur?

a. 4 3 2 1 0 9 8 7 6 5
b. 4 6 8 7 5 3 2 9 0 1
c. 2 5 6 7 4 8 9 3 1 0
d. 4 3 2 1 0 5 6 7 8 9
e. 1 2 3 4 5 6 9 8 7 0
f. 0 4 6 5 3 8 1 7 2 9
g. 1 4 7 9 8 6 5 3 0 2
h. 2 1 4 3 6 5 8 7 9 0

output

means the first number being pushed is 0, then 1, then 2 -, and so on

Answer is at the bottom.

Example:

a) 4 3 2 1 0 9 8 7 6 5

We know that pop() will return the numbers backwards, therefore,

when pop() returns the numbers "4 3 2 1 0", we can assure that the first set of push() operations are

Pushes 0, 1, 2, 3, 4 The stack is - [0, 1, 2, 3, 4]

Then, when the pop() is called, the stack will be displayed on the screen in reverse order.

Pop 4, 3, 2, 1, 0

a. 4 3 2 1 0

Next we have "9 8 7 6 5", meaning,

The stack is - [ ] (stack is empty because the numbers are popped)
Pushes 5, 6, 7, 8, 9 The stack is - [5, 6, 7, 8, 9]

Pop 9, 8, 7, 6, 5

a. 4 3 2 1 0 9 8 7 6 5 The stack is - [ ]

Example 2:

b) 4 6 8 7 5 3 2 9 0 1

Looking at this sequence, we can defer the order of the push and pop operations.

pushes	The current stack	pops	output	The current stack
0 1 2 3 4	[0, 1, 2, 3, 4]	4	4	[0, 1, 2, 3]
5 6	[0, 1, 2, 3, 5, 6]	6	4 6	[0, 1, 2, 3, 5]
7 8	[0, 1, 2, 3, 5, 7, 8]	8, 7, 5, 3, 2	4 6 8 7 5 3 2	[0, 1]
9	[0, 1, 9]	9 1 0	4 6 8 7 5 3 2 9 1 0	[ ]
		Sequence:	4 6 8 7 5 3 2 9 0 1	**which could not occur**

Example 3:

c) 2 5 6 7 4 8 9 3 1 0

Looking at this sequence, we can defer the order of the push and pop operations.

pushes	The current stack	pops	output	The current stack
0 1 2	[0, 1, 2]	2	2	[0, 1]
3 4 5	[0, 1, 3, 4, 5]	5	2 5	[0, 1, 3, 4]
6	[0, 1, 3, 4, 6]	6	2 5 6	[0, 1, 3, 4]
7	[0, 1, 3, 4, 7]	7 4	2 5 6 7 4	[0, 1, 3]
8	[0, 1, 3, 8]	8	2 5 6 7 4 8	[0, 1, 3]
9	[0, 1, 3, 9]	9 3 1 0	2 5 6 7 4 8 9 3 1 0	[ ]
		Sequence:	2 5 6 7 4 8 9 3 1 0	

Example 4:

d) 4 3 2 1 0 5 6 7 8 9

Looking at this sequence, we can defer the order of the push and pop operations.

pushes	The current stack	pops	output	The current stack
0 1 2 3 4	[0, 1, 2, 3, 4]	4 3 2 1 0	4 3 2 1 0	[ ]
5	[5]	5	4 3 2 1 0 5	[ ]
6	[6]	6	4 3 2 1 0 5 6	[ ]
7	[7]	7	4 3 2 1 0 5 6 7	[ ]
8	[8]	8	4 3 2 1 0 5 6 7 8	[ ]
9	[9]	9	4 3 2 1 0 5 6 7 8 9	[ ]
		Sequence:	4 3 2 1 0 5 6 7 8 9	

Example 5:

e) 1 2 3 4 5 6 9 8 7 0

Looking at this sequence, we can defer the order of the push and pop operations.

pushes	The current stack	pops	output	The current stack
0 1	[0, 1]	1	1	[0]
2	[0, 2]	2	1 2	[0]
3	[0, 3]	3	1 2 3	[0]
4	[0, 4]	4	1 2 3 4	[0]
5	[0, 5]	5	1 2 3 4 5	[0]
6	[0, 6]	6	1 2 3 4 5 6	[0]
7 8 9	[0, 7, 8, 9]	9 8 7 0	1 2 3 4 5 6 9 8 7 0	[ ]
		Sequence:	1 2 3 4 5 6 9 8 7 0	

Example 6:

f) 0 4 6 5 3 8 1 7 2 9

Looking at this sequence, we can defer the order of the push and pop operations.

pushes	The current stack	pops	output	The current stack
0	[0]	0	0	[ ]
1 2 3 4	[1, 2, 3, 4]	4	0 4	[1, 2, 3]
5 6	[1, 2, 3, 5, 6]	6 5 3	0 4 6 5 3	[1, 2]
7 8	[1, 2, 7, 8]	8	0 4 6 5 3 8	[1, 2, 7]
9	[1, 2, 7, 9]	9 7 2 1	0 4 6 5 3 8 9 7 2 1	[ ]
		Sequence:	0 4 6 5 3 8 1 7 2 9	**which could not occur**

Example 7:

g) 1 4 7 9 8 6 5 3 0 2

Looking at this sequence, we can defer the order of the push and pop operations.

pushes	The current stack	pops	output	The current stack
0 1	[0, 1]	1	1	[0]
2 3 4	[0, 2, 3, 4]	4	1 4	[0, 2, 3]
5 6 7	[0, 2, 3, 5, 6, 7]	7	1 4 7	[0, 2, 3, 5, 6]
8 9	[0, 2, 3, 5, 6, 8, 9]	9 8 6 5 3 2 0	1 4 7 9 8 6 5 3 2 0	[ ]
9	[1, 2, 7, 9]	9 7 2 1	1 4 7 9 8 6 5 3 2 0	[ ]
		Sequence:	1 4 7 9 8 6 5 3 0 2	**which could not occur**

Example 5:

h) 2 1 4 3 6 5 8 7 9 0

Looking at this sequence, we can defer the order of the push and pop operations.

pushes	The current stack	pops	output	The current stack
0 1 2	[0, 1, 2]	2 1	2 1	[0]
3 4	[0, 3, 4]	4 3	2 1 4 3	[0]
5 6	[0, 5, 6]	6 5	2 1 4 3 6 5	[0]
7 8	[0, 7, 8]	8 7	2 1 4 3 6 5 8 7	[0]
9	[0, 9]	9 0	2 1 4 3 6 5 8 7 9 0	[ ]
		Sequence:	2 1 4 3 6 5 8 7 9 0	

Answer: b, f, and g could not occur.