Quantum tunnelling

[redacted]

Word count: 1322

Introduction:

Quantum tunnelling is a phenomenon in which an atom or a subatomic particle appears on the other side of a barrier that should be impenetrable to the particle due to its insufficient kinetic energy according to the theories of classical mechanics.

i) Wave-particle duality:

Due to Louis De Broglie, every fundamental particle process has both wave-like and particle-like properties [3], thus if light has the dual property, then an electron could also have this property. For the wave nature of matters, the relationship is

$$\lambda = h/p$$

Where 'λ' represents the wavelength, 'h' is the plank's consistent, and 'p' represents the momentum of the particle.

ii) Heisenberg Uncertainty Principle:

An object's velocity and location cannot be reliably determined at the same time [1]. The uncertainty is negligible for greater masses of items but becomes considerable for subatomic particles like electrons. When you try to measure them, they are knocked out in unpredictable ways [2]; in other words, you can only focus on one parameter with great precision at a time [1]. The following is an equation representing the principle:

$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

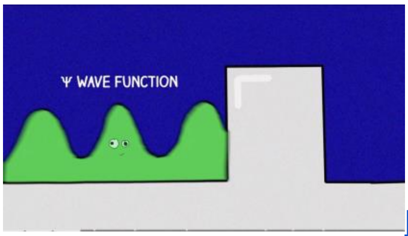
, where ħ is the reduced Planck constant, **Δ**x represents uncertainty in position, and **Δ**p represents uncertainty in momentum. The more accurate the position (smaller the **Δ**x), the less accurate the momentum (larger the **Δ**p), and vice versa.

This principle is a consequence of the wave-particle duality. An elementry particle can either be a wave or a corpuscles, that's why we can no longer treat it as an object to determine its momentum and location.

iii) Wave function:

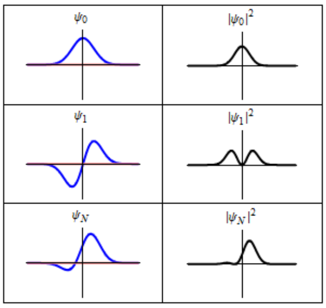
A wave function is a mathematical representation of a particle's quantum state as a function of spin, time, momentum, and location [4]. The probability of finding an electron in a given region within the atom can be described as the suqare of the wave function.

Quantum physics is a probabilistic science. Unlike the ball, the precise location of an electron is indeterminable, as suggested by the Heisenberg Uncertainty Principle. Even though the electron precise position is unknown, its approximate location can be determined. As a result, the likelihood of the election position could be described as a wave function:



[6]

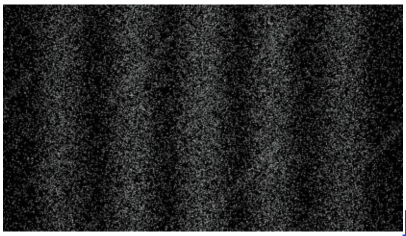
A graph of the wave function (y-axis) with respect to the position (x-axis) could be drawn:



[7]

An electron is more likely to be detected at the x-coordinates represented by a peak than at the x-axis interception point. If an electron is detected in a specific location, the wave function will instantly become a point because it cannot be discovered anywhere else.

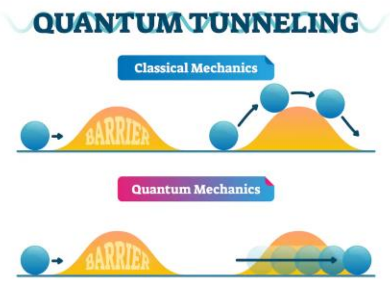
The wave-particle duality phenomenon describes how particles behave both like waves and like particles; the probability of a particle being in a position is a wave, but the particle's actual physical appearance is not. When individual particles shoot at the screen, the result is a dot, but when all the dots are combined, an interference pattern, similar to the waves, appears:


[8]

The brightest part has the highest possibility of an electron appearing, whereas the darker part has a lesser probability of an electron appearing.

iv) The quantum tunnelling effect:

In classical mechanics, the ball requires kinetic energy $>$ potential energy to overcome the physical barrier. However, in quantum tunnelling, the ball may overcome the physical barrier with kinetic energy $<$ potential energy.



[5]

v) Using total internal reflection as a quantum tunnelling analogue:

Light may be refracted or reflected when it crosses the boundaries of materials with different refractive indices. When the angle of incidence $<$ critical angle, the light ray is refracted with little reflection; When the angle of incidence $>$ critical angle, the light ray is reflected totally; When The angle of incidence = critical angle, the refracted light ray leaves at 90° to the normal.

Quantum tunnelling can be demonstrated with a cup of water (The following pictures does not show exactly what quantum tunelling is because in tunnelling, the particle penetrates a potential energy barrier, whereas in frustrated total internal reflection, the light penetrates a material barrier with a different refractive index). [9]


(picture took by me, 02/07/2023)

When the cup is tilted, total internal reflection appeared.


(picture took by me, 02/07/2023)

No fingers, however, could be seen when we looked directly into the cup. This is due to the air having a lower index of refraction than the water. In this case, the probability of finding a photon outside the cup decayed rapidly to zero. In other words, there is a thick enough layer of air that acts as a physical barrier, preventing the photon from passing through the cup.


(picture took by me, 02/07/2023)

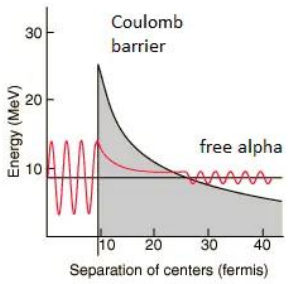
My fingers reappeared when I squeezed the cup hard. This is because the thickness of the physical barrier, which in this case is air, has decreased, allowing photons from my fingers to tunnel through it. The physical barrier is now sufficiently thin, as opposed to the previous image.


(picture took by me, 02/07/2023)

Another method for reducing the air boundary is to wet my finger. Because the water on my finger has a similar index of refraction to the water inside the cup than the air, the photon on my finger could easily tunnel through the cup.

vi) The importance of quantum tunnelling in various physical processes:

Alpha decay is a quantum tunnelling process [10]. The alpha particles have to penetrate a potential barrier to be emitted(escape from the potential well of nucleus). The energy of a potential barrier has a height of 20-25 MeV, while the kinetic energy of an alpha particle ranges between 4-9 MeV. The alpha particle is unable to leave the nucleus typically, but this behaviour can be explained with quantum physics, it has a very slim (yet non-zero) chance of "tunneling" through the wall and showing up on the other side of the wall.



[10]

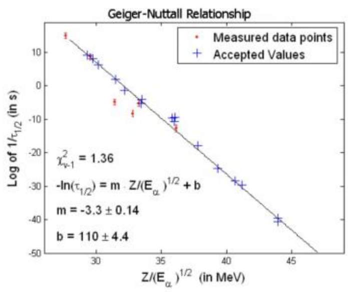
This is an equation for the transmission probability of an alpha particle:

$$TP \approx e^{-2k_2 L}$$

$$k_2 = \sqrt{\frac{2m(v-E)}{\hbar}}$$ [11]

L is the width of the barrier, and K2 is the difference in energy. The higher the energy the alpha particle has, the greater the transmission probability due to the thinner barrier, the lower the energy of the alpha particle, the lower the transmission probability due to the thicker barrier. This means the half-life of the high-energy alpha sources (i.e. radioactive) is shorter compared to the half-life of the low-energy particle (Geiger Nuttall Law, 1911) [12].

There is a linear relationship between the log of half-life and the kinetic energy of the alpha particle:



[13]

Quantum tunneling also keeps the sun shining by using nuclear fusion.
Nuclear fusion generates energy for the sun [14]. Although the sun's core heats up tremendously due to gravity's compression, yet, the temperature is not high enough for nuclear fusion (15 million celcius)

Hydrogen fusion occurs when four hydrogen nuclei (protons) combine to generate one helium nucleus. However, because the protons repel each other due to their like-charge, strong nuclear force is necessary to bring them together.

Wave functions can be used to model protons. One of the proton can tunnel through the barrier to reach other proton and get close enough for the strong nuclear force to take over and attract them together (quantum tunnelling), overcoming the repulsive force and allowing the protons to fuse together [15]. When four protons combine to form helium, energy is released as gamma photons. This is referred to as the proton-proton reaction chain.

The gamma photons produced are the main source of energy for the majority of stars in the universe. The gamma photons are usually produced in the core of the sun and go to the sun's surface. The photons eventually reach the earth, and act as visible light.

Conclusion:

Quantum tunnelling is a quantum phenomenon that explains why particle penetration through a physical barrier has a non-zero probability; the thinner the barrier, the greater the likelihood; and the higher the particle's kinetic energy, the higher the probability. Quantum tunneling can also explain the alpha decay process and how stars shine.

References:

[1] Editors of Encyclopaedia Britannica (2023) "uncertainty principle", britannica.com
Uncertainty principle | Definition & Equation | Britannica

[2] Venkatesh Vaidyanathan (2022) "What is quantum tunneling", scienceabc.com
What Is Quantum Tunneling? » Science ABC

[3] Wave Particle Duality: Definition, History, Theory, Applications (2023) testbook.com
Wave-Particle Duality: Definition, History, Theory, Applications (testbook.com)

[4] Wave Function (no date) www.toppr.com
Wave Function: Definition, Formula, Derivation, and Examples (toppr.com)

[5] U. Satya Sainadh (2019) "We did a breakthrough 'speed test' in quantum tunnelling, and here's why that's exciting", theconversation.com
We did a breakthrough 'speed test' in quantum tunneling, and here's why that's exciting (theconversation.com)

[6] Up and Atom (2019) "What is Quantum Tunneling, Exactly?", youtube.com
What is Quantum Tunneling, Exactly? - YouTube

[7] Understanding wave function graph (2020) physics.stackexchange.com
quantum mechanics - Understanding wave function graph - Physics Stack Exchange

[8] Double slit experiment electron pattern (no date) sciencephoto.com
Double slit experiment electron pattern - Stock Video Clip - K009/7746 - Science Photo Library

[9] The Action Lab (2020) "How to Make a Quantum Tunnel In Real Life", youtube.com
How to Make a Quantum Tunnel In Real Life - YouTube

[10] Theory of Alpha Decay – Quantum Tunneling (no date) nuclear-power.com
Theory of Alpha Decay - Quantum Tunneling | nuclear-power.com

[11] For the Love of Physics (2019) “Gamow's Theory of Alpha Decay AND Geiger Nuttal Law”
Gamow's Theory of Alpha Decay AND Geiger Nuttal Law - YouTube

[12] H. Geiger and J.M. Nuttall (1911) "The ranges of the α particles from various radioactive substances and a relation between range and period of transformation," *Philosophical Magazine*, Series 6, vol. 22, no. 130, pages 613-621.

[13] Lulu Liu (2007) “The Quantum Mechanics of Alpha Decay”, mit.edu
alpha.pdf (mit.edu)

[14] The Science Asylum (2020) “The Sun can’t work without Quantum Tunneling”, youtube.com
https://youtu.be/lQapfUcf4Do

[15] Think English (2021) “Why The Sun Keeps Shining: Fusion and Quantum Tunnelling”, youtube.com
https://youtu.be/94sOAIeZmjU