Diagram:

angle : - find all
- try to name 1 angle then infer

- keep in mind w/ tri uk., use simultaneous eq.

- roots $b^2 - 4ac$
- reject value!
- proof.

- Area, volume

How many?

1. list few cav.
2. find common
3. infer a rule

$$\frac{n!}{r!\,(n-r)!}$$

n pick r.

Digit / prime / sequence

① list some possible values

② find common

③ find a rule

④ apply, solve for UK

⑤ mind the range.

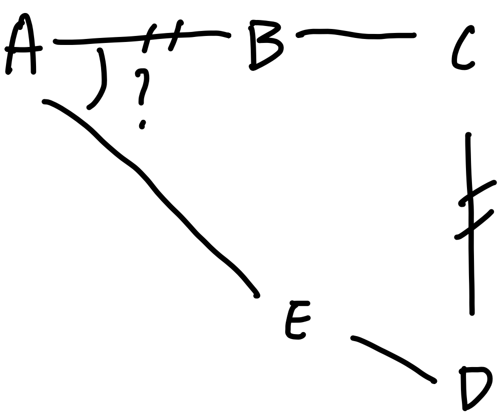
Notes

- Ratio of sides use similar Δ
- Re-sketch figures.
- factorial function

e.g 3 items: A, B, C	e.g 5 items pick 2
no. of combinations = $3! = 3 \times 2 \times 1 = 6$ ABC, ACB, BAC, BCA, CAB, CBA	$\dfrac{5!}{2!\,(5-2)!} = \dfrac{120}{2 \times 6} = 10$ combinations

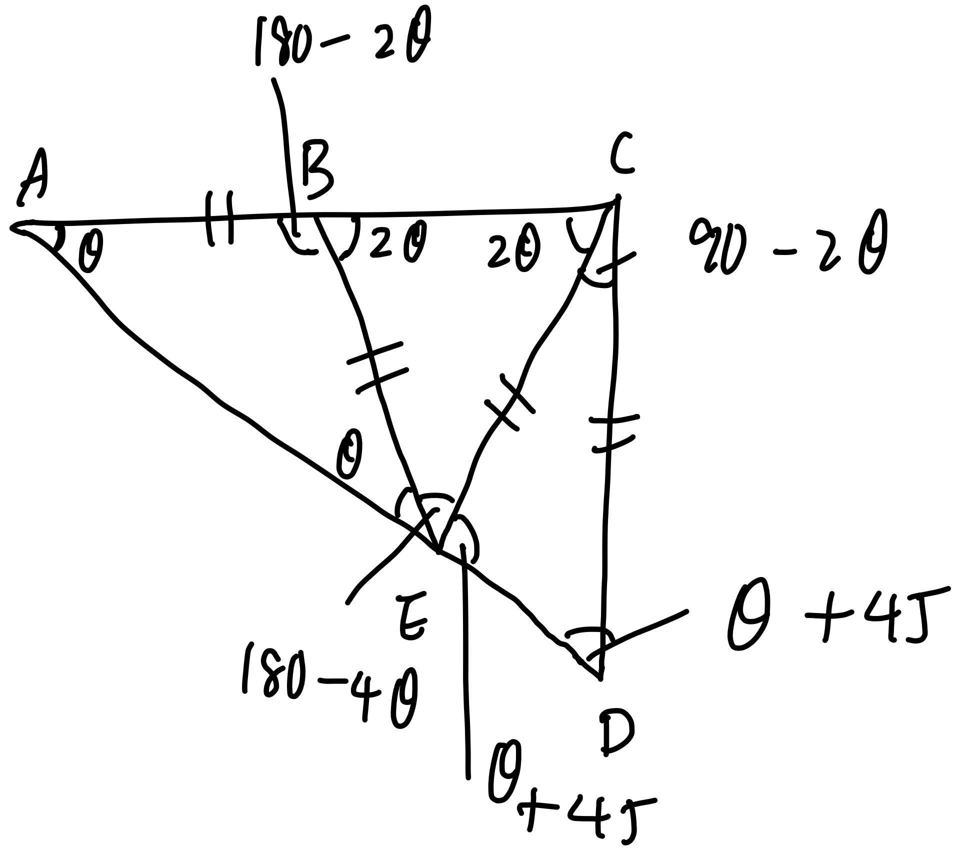
22

(1a)



- 以小推大△
- 找多些 angle

(1b)



$\theta + \cancel{2\theta} + 90 = \cancel{2\theta} + \theta + 45 = 180$

$2\theta = 45$

$\theta = 22.5^\circ$ ~~180~~

21

20

$\boxed{x-1 \mid x \mid x+1}$

A

sum $= 3x$

sum of no.s $= \frac{1}{2}(16 \times 17)$

$= 136$

uncovered

$= 136 - 3x$, is not a multiple

of 3.

26

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

a)

	$x-4$
$x-1$	x

sum $= 3x-5$

b)

$x-4$	
x	$x+1$

sum $= 3x-3$

c)

$x-1$	x
	$x+4$

sum $= 3x+3$

d)

x	$x+1$
$x+4$	

sum $= 3x+5$

a) $$136-(3x-5) = 141-3x$$
$$= 3k-3x$$

b) $136 - (3x-3) = 139 - 3x$

c) $136 - (3x+3) = 133 - 3x$

d) $136 - (3x+5) = 131 - 3x$

$\Rightarrow$ only way a produced

a multiple of 3.

$\Rightarrow$ 4 ways

3a

$\angle DEC = \angle ABC = 90^\circ$ (given)

let $\angle BAC = \theta$

$\angle BAC = \angle ACE = \theta$ (alt $\angle$, $AB \parallel CE$)

$\angle ACE + \angle ECD = 90^\circ$ (given)

$\angle ECD = 90^\circ - \theta$

$\angle ACB + \angle BAC + \angle ABC = 180^\circ$ ($\angle$ sum of Δ)

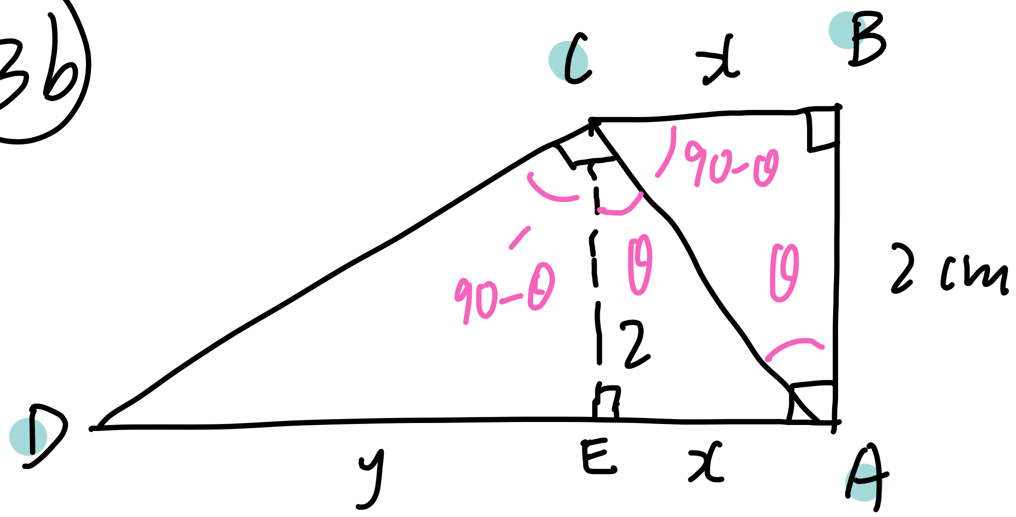
$\angle ACB + \theta + 90^\circ = 180^\circ$

$$\angle ACB = 90^\circ - \theta$$

$$= \angle ECD$$

$$\Rightarrow \triangle ABC \sim \triangle DEC \quad (A.A.)$$

(3b)



$$\frac{1}{2}(x + x+y)(2) = 6$$

$$2x + y = 6 \quad —(1)$$

$$\frac{CB}{CE} = \frac{AB}{DE}$$

$$\frac{x}{2} = \frac{2}{y}$$

$$xy = 4$$

$$x = \frac{4}{y} \quad —(2)$$

$$2\left(\frac{4}{y}\right) + y = 6$$

$$8 + y^2 = 6y$$

$$y^2 - 6y + 8 = 0$$

$1 \quad -4$

$1 \quad -2$

$$(y-4)(y-2) = 0$$

$y = 4$ or 2

$x(4) = 4$	$x(2) = 4$
$x = 1$	$x = 2$

$$P = y + 2x + z + \sqrt{z^2 + y^2}$$

$$4 + 2 + 2 + \sqrt{4+16}$$

$$= 8 + \sqrt{20}$$

$$\Rightarrow 8 + 2\sqrt{5}$$

$$2 + 4 + 2 + \sqrt{4+4}$$

$$\Rightarrow 8 + 2\sqrt{2}$$

20

2a

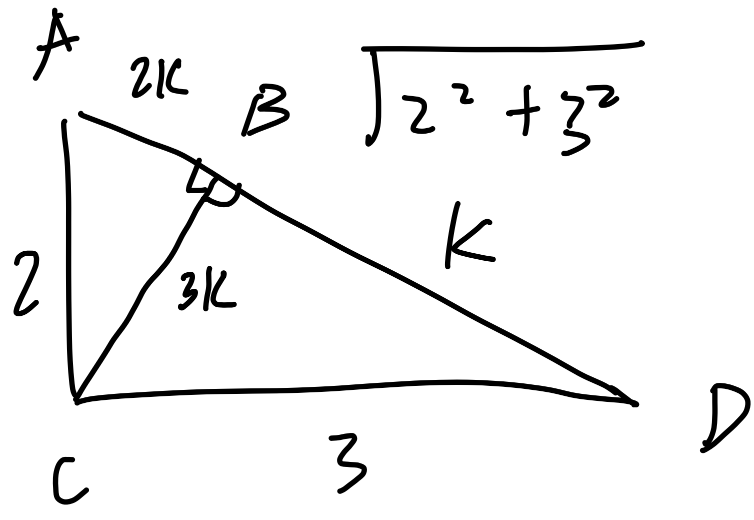
$\triangle ABC \sim CBD$

$$\frac{AC}{CD} = \frac{AB}{BC} = \frac{2}{3}$$

2b

-Ratio
看清
前題

$\sqrt{2^2+3^2} = \sqrt{13}$



$$\frac{2}{3} = \frac{3}{n} \qquad \frac{13}{2}k = \sqrt{13}$$

$$n = \frac{9}{2} \qquad k = \frac{2\sqrt{13}}{13}$$

$$\text{Area } ABC = \frac{1}{2}(3k)(2k)$$

$$= \frac{1}{2}\left(\frac{3\sqrt{13}}{8}\right)\left(\frac{2\sqrt{13}}{8}\right)$$

$$= \frac{1}{2}\left(\frac{3(13)}{32}\right) = \cancel{\frac{39}{32}}$$

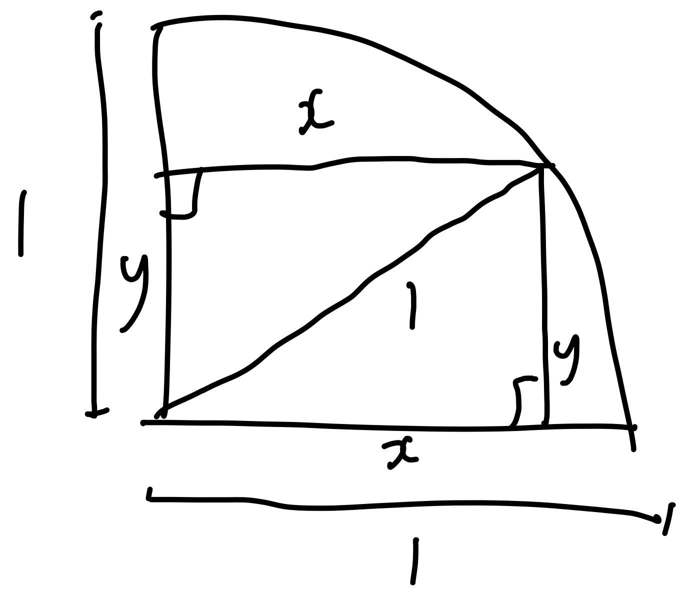
$$\frac{39}{64} \qquad \cancel{\frac{48}{13}} \qquad \frac{12}{13}$$

$$\text{Area } D = 3^2 - \frac{4}{2}(3)(2) + \cancel{\frac{39}{64}}\frac{12}{13}(4)$$

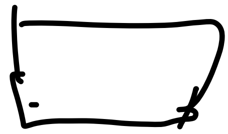
$$= 9/13$$

19

4a



P of



$$x^2+y^2=1 \quad -①$$

$$2x+2y=3 \quad -②$$

$$x=\frac{3-2y}{2} \quad -③$$

$$\left(\frac{3-2y}{2}\right)^2+y^2=1$$

$$9 - 12y + 4y^2 + 4y^2 = 4$$

$$8y^2 - 12y + 5 = 0$$

$$b^2 - 4ac$$

$$= (12)^2 - 4(8)(5)$$

$$= 144 - 160$$

$$= -16$$

$\Rightarrow$ no real sol.

$\Rightarrow$ perimeter $= 3$ is impossible

4b

$$x^2 + y^2 = 1$$

$$2x + 2y = n$$

$$\left(\frac{n-2y}{2}\right)^2 + y^2 = 1$$

$$n^2 - 4ny + 4y^2 + 4y^2 = 4$$

$$8y^2 - 4ny + n^2 - 4 = 0$$

$$b^2 - 4ac \geq 0$$

$$(4n)^2 - 4(8)(n^2 - 4) \geq 0$$

$$16n^2 - 32n^2 + 128 \geq 0$$

$$128 \geq 16n^2$$

$$\begin{array}{r} 16 \\ \times 8 \\ 4 \\ \hline 128 \end{array}$$

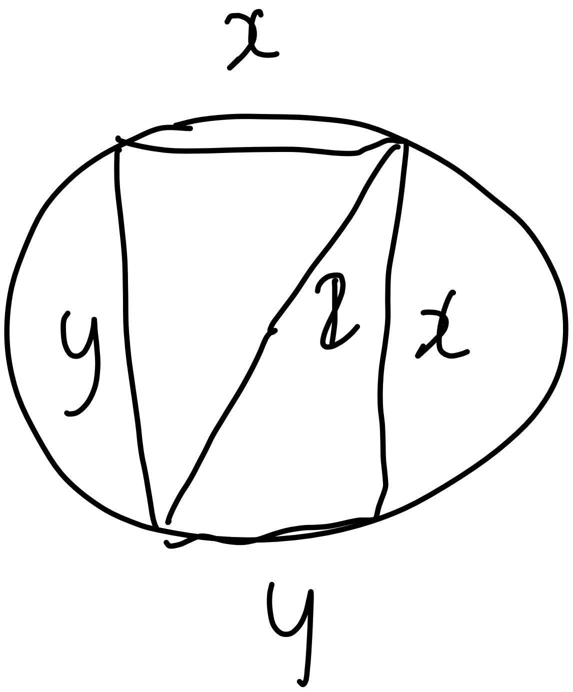
$$\cancel{8}$$

$$n \leq \sqrt{8}$$

$$\leq 2\sqrt{2}$$

sub back ...

4c



if ~~square~~,

$\cancel{2x^2 = 2^2}$

$\cancel{x = \sqrt{2}}$

$$x^2 + y^2 = 2 \quad — (1)$$

$$2x + 2y = \overset{n}{\cancel{4\sqrt{2}}} \quad — (2)$$

$$x = \frac{\overset{n}{\cancel{4\sqrt{2}}} - 2y}{2}$$

$$\left(\frac{4\sqrt{2} - 2y}{2}\right)^2 + y^2 = 2$$

$$32 - 16\sqrt{2}\,y + 4y^2 + 4y^2 = 8$$

$$8y^2 - 16\sqrt{2}\,y + 24 = 0$$

$$(16\sqrt{2})^2 - 4(8)(24)$$

$$8y^2 - 4ny + n^2 - 16 =$$

$$(-4n)^2 - 4(8)(n^2 - 16) \geq 0$$

$$16n^2 - 32n^2 + 512 \geq 0$$

$$512 \geq 16n^2$$

$$n \leq 4\sqrt{2}$$

if square:

$$2x^2 = 4$$

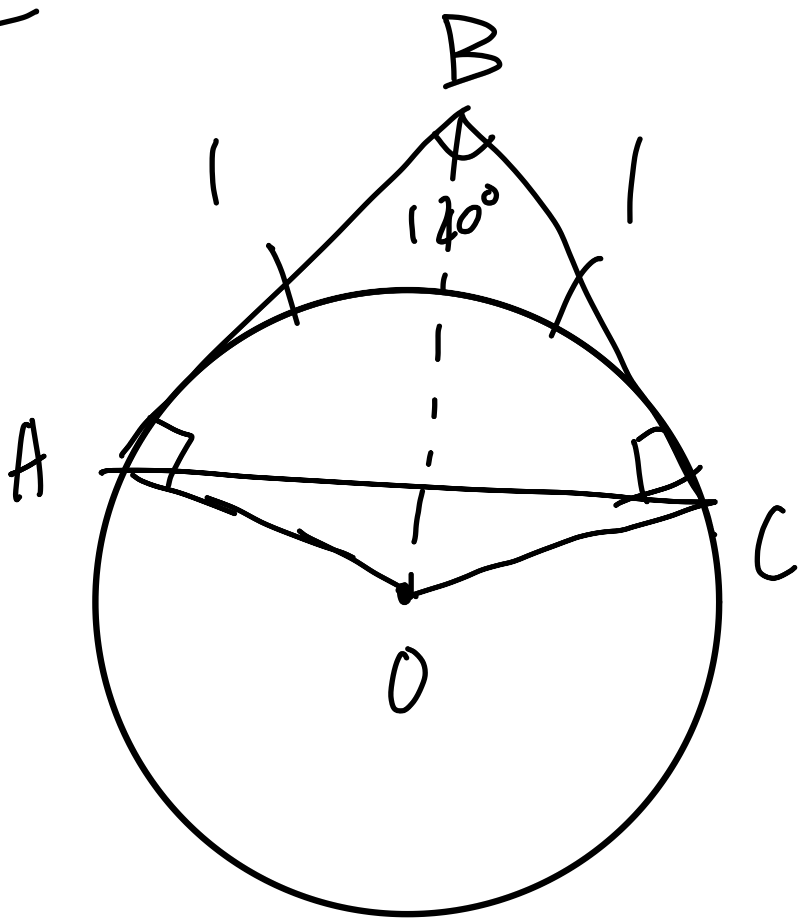
$$x = \sqrt{2}$$

perimeter $= 4\sqrt{2}$

$= n$

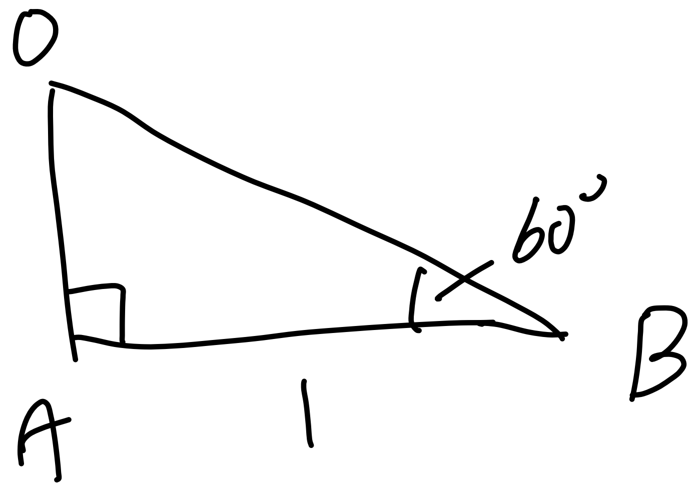
18

(2)



find radius

from ΔABO



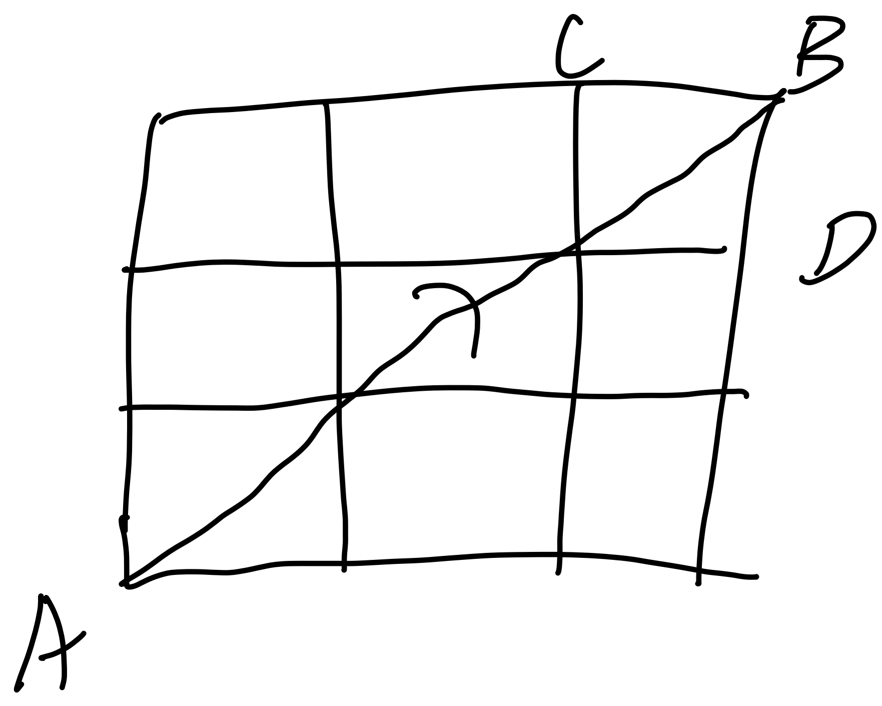
$$\tan 60^\circ = \frac{OA}{1}$$

$$\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}} = OA$$

$$OA = \sqrt{3}$$

3a

$b = c + d$



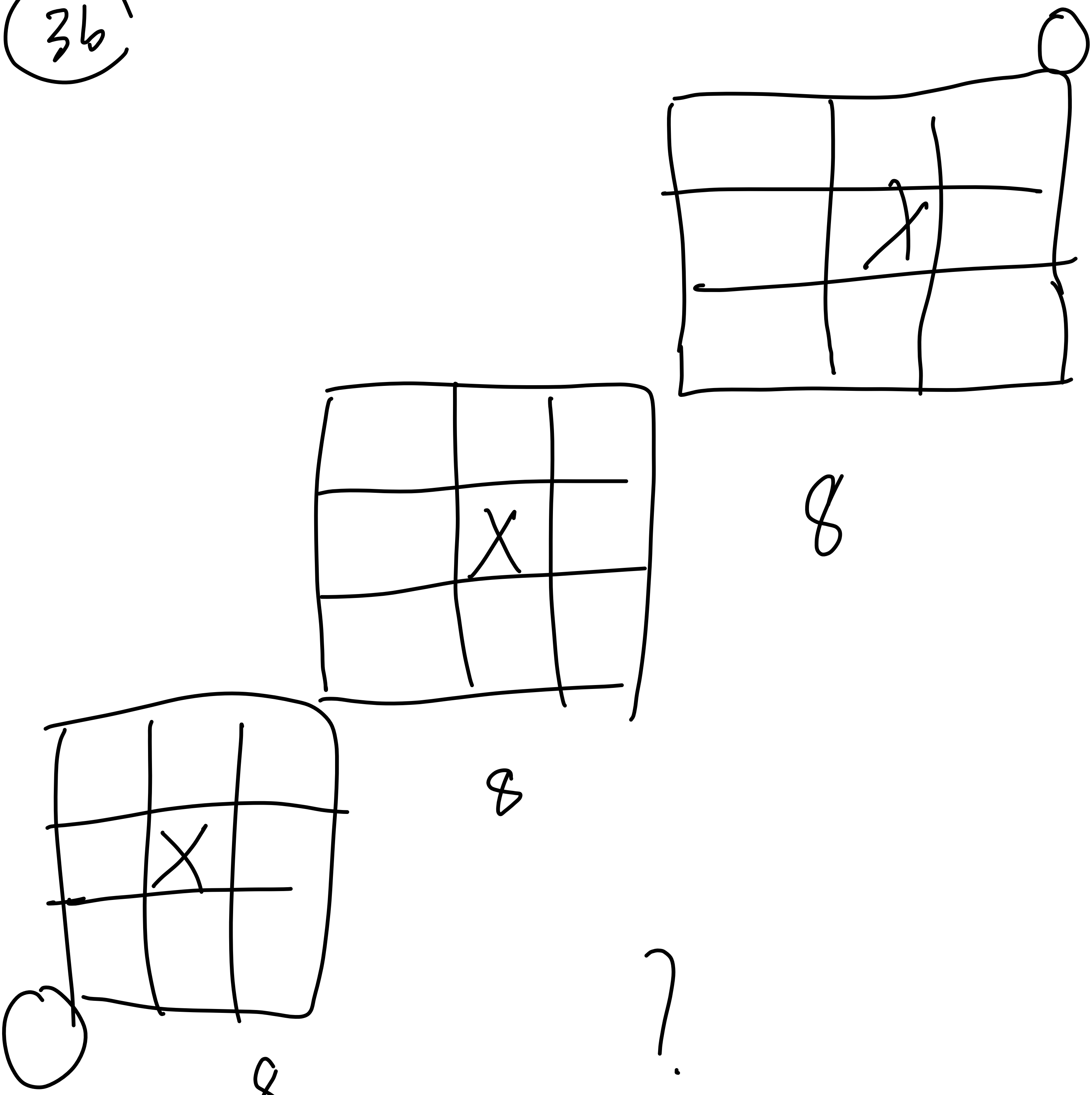
To get to B, sheila can either go to C

and then move to
the right, or go
to D then move
up. Hence

w = move to B +
move to C

$\Rightarrow$ $b = c + d$, as
learned.

36



8

8

8

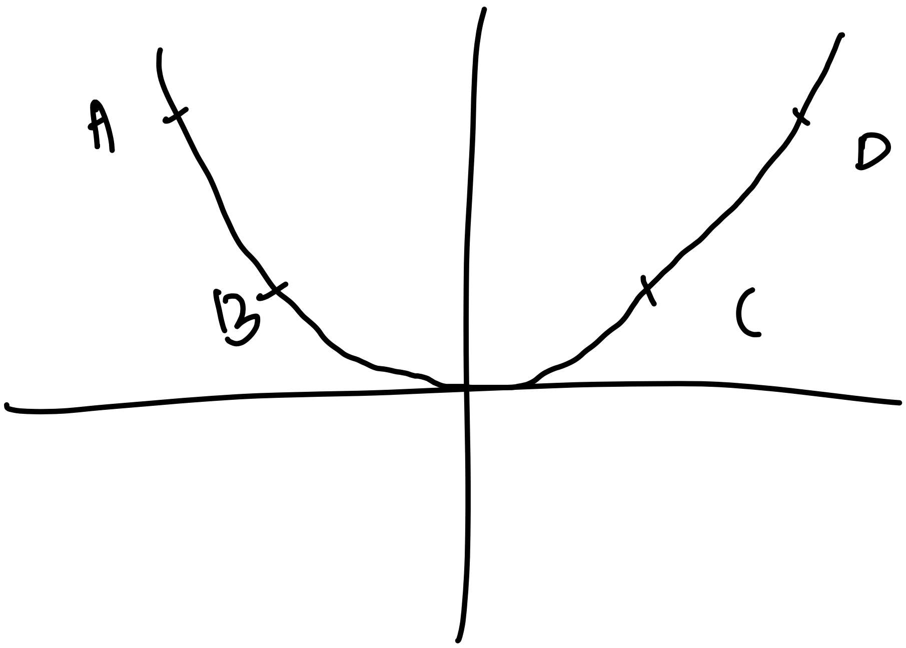
?

$$\frac{8!}{1!(8-1)!}$$

54

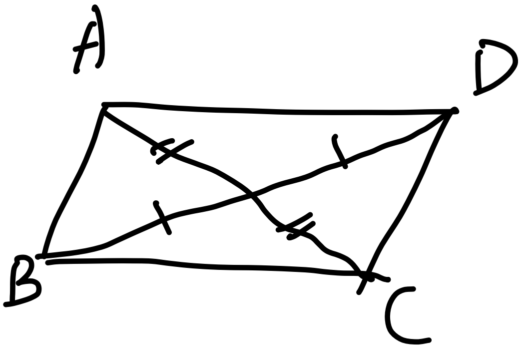
18

(3)



× ||gram

$$\text{let } A(a, a^2)$$
$$B(b, b^2)$$
$$C(c, c^2)$$
$$D(d, d^2)$$



where $a < b < c < d$

-set uk
看 equation
of line
— 要 specific
uk 之間
的大小
關係

$$m_{AB} = \frac{a^2-b^2}{a-b} = a+b$$

$$m_{CD} = \frac{c^2-d^2}{c-d} = c+d$$

$\Leftarrow$ $a+b < c+d$

$\Rightarrow$ $a+b \neq c+d$, and they can never be $\parallel$ to each other.

$m_{AC} = a+c$

$m_{BD} = b+d$

AC can be $\parallel$ to BD.

$\because$ $\parallel$gram has 2 pairs of parallel lines, and the graph can at most have 1 pair of parallel lines

$\therefore$ ABCD $\neq$ //gram.

16

(i)

let ~~the~~ number labelling be as the shown in the diagram.

since $T = a+b+c = a+d+e$

$b+c = d+e$ — (1)

also $T = b+d+g = c+e+g$

$b+d = c+e$ — (2)

$$\Rightarrow \quad 2b+c+d = c+d+2e$$

$$b = e$$

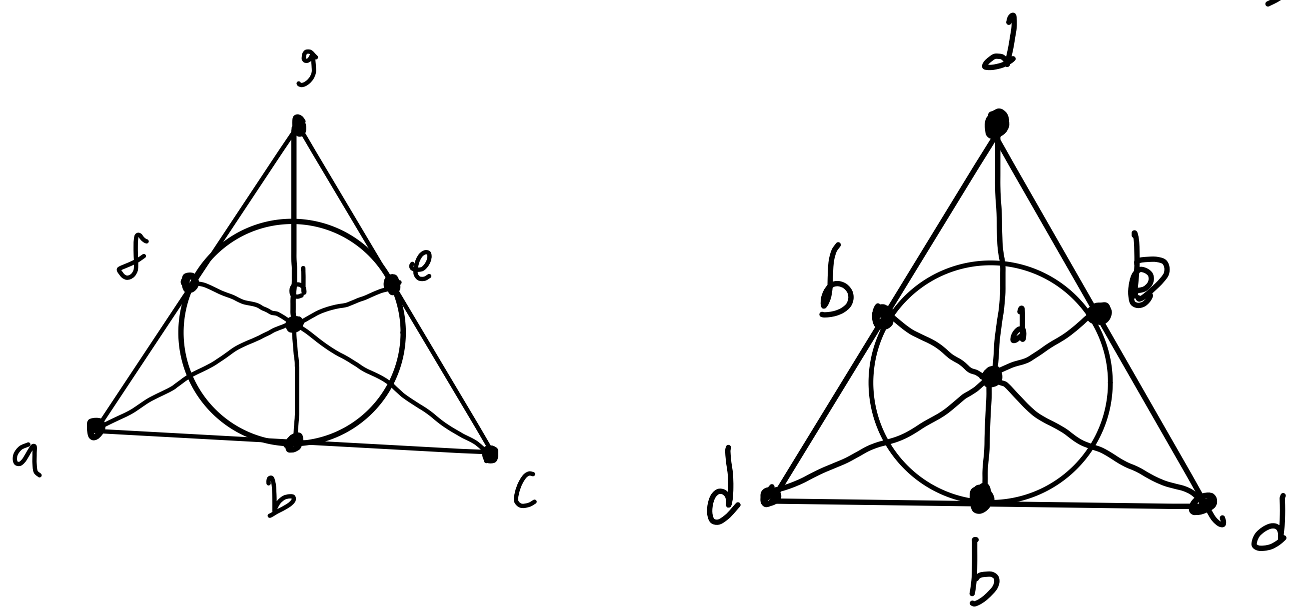
sub $b=e$ into ②

$$e+d = c+e$$

$$c = d$$

Analogously, we can prove that

$$b=f, \quad d=a, \quad d=g$$



considering 3 pt on $\bigcirc$

we have $3b = T$

$$\Rightarrow \quad b = \frac{1}{3}T$$

base of Δ

we have $2d + b = T$

$$2d = \frac{2}{3}T$$

$$\Rightarrow \quad d = \frac{1}{3}T$$

Thus each no. $= \frac{1}{3}T$

16

2

$$\frac{PB}{BQ} = \frac{AC_1}{AC_2}$$

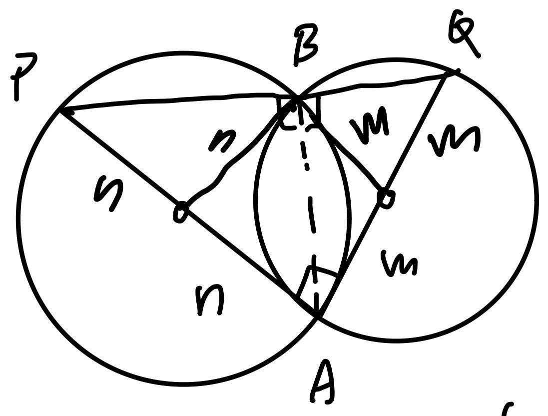
Since PA is tangent to C_2 at A, $\angle PAQ = 90°$

let radius of $C_1 = n$

$C_2 = m$

$$\frac{\text{Area } C_1}{\text{Area } C_2} = \frac{\pi n^2}{\pi m^2}$$

$$= \frac{n^2}{m^2} = \frac{n}{m}$$



C_1 C_2

$$n^2 + n^2 = PB^2$$

$$2n^2 = PB^2$$

$$n^2 = \frac{1}{2} PB^2$$

$$m^2 + m^2 = BQ^2$$

$$m^2 = \frac{1}{2} BQ^2$$

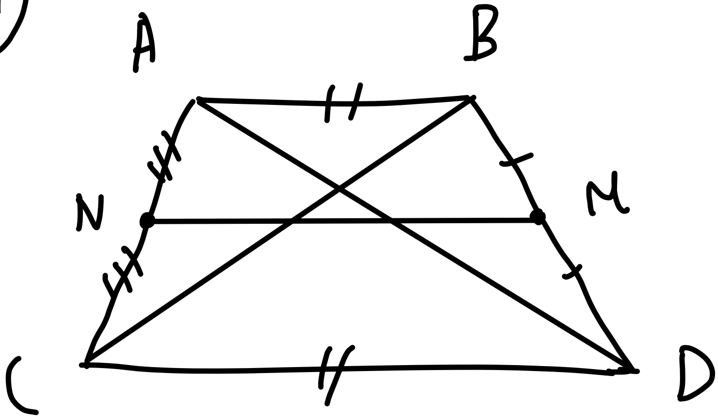
$$\frac{n^2}{m^2} = \frac{\frac{1}{2} PB^2}{\frac{1}{2} BQ^2}$$

$$\frac{n^2}{m^2} = \frac{PB^2}{BQ^2}$$

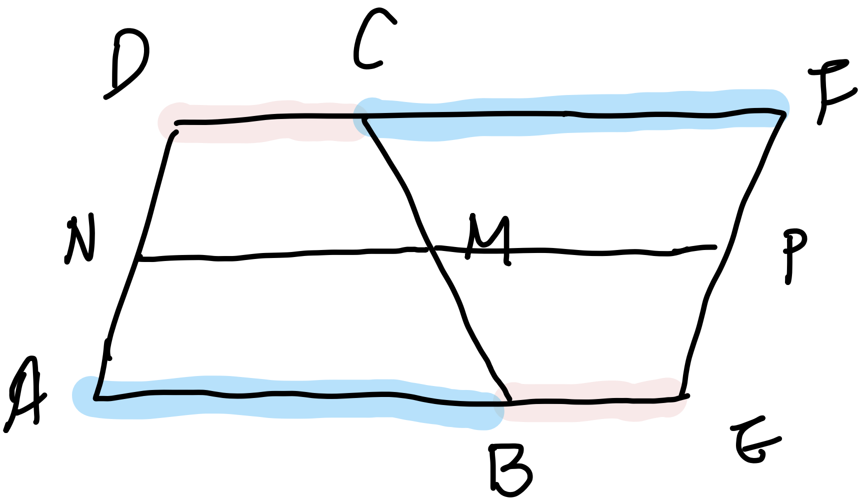
$$\frac{n}{m} = \frac{PB}{BQ}$$

//

(4a)



$2MN = AB + CD$



$BE = DC$

$CF = AB$

$\Rightarrow$ AEFD is // gram.

since AE // DF, and
both have length $AB + CD$,
it follows that EF // AE and

equal length.

Let P be mid of EF

then $AN \parallel EP$ and equal length

so $AEPN$ is also a $\parallel$ gram.

$\Rightarrow NP = AB + CD$

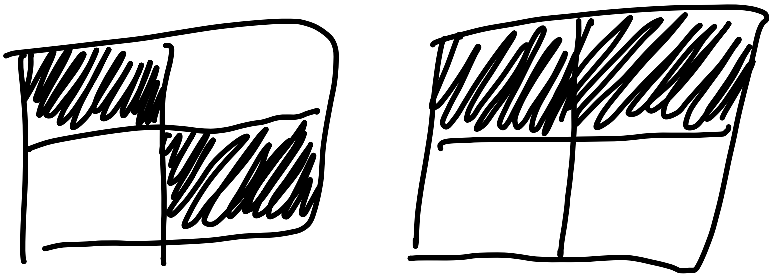
$NM = MP$ due to congruent $\triangle$,
shared mid pt.

$\Rightarrow NP = 2MN$

$\therefore 2MN = AB + CD$

22

2ai



一看清題目!
i+ii+iii

2aii

2B + 2W × 2

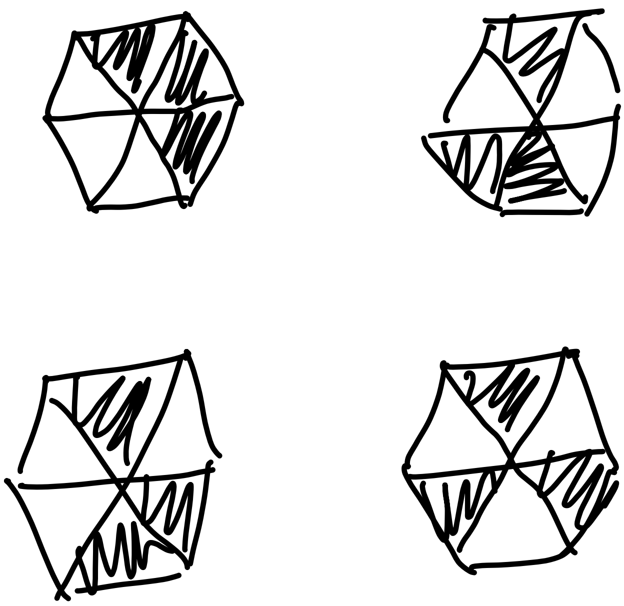
3B + 1 W × 1

3W + 1 B × 1 ⇒ 6 ways

4W × 1

4B × 1

2bi



⇒ 4

2bii

1W 5B ×1

5W 1B ×1

2W 4B ×3

4W 2B ×3

3W 3B ×4 ⇒ 14

3B 3W

6W ×1

6B ×1

20

(1a)

$a = p^2$

$b = pq$

$c = p^3q$

$ab \quad bc \quad ca$

$p^3q \quad p^4q^2$

$\times \quad \Downarrow$

square

$\Leftarrow \quad (p^2q)^2$

(1b)

$\longrightarrow$

2	5	10
3	6	8
1	4	9

$\leftarrow$ square

$\leftarrow$ square

$\leftarrow$ square

10

$1, 2, 3, 2^2, 5, 2\times3, \cancel{7}, 2^3, 3^2, 2\times5$

$2\times5\times10$

$3\times6\times8$

$1\times4\times9$

$2\times3\times6$

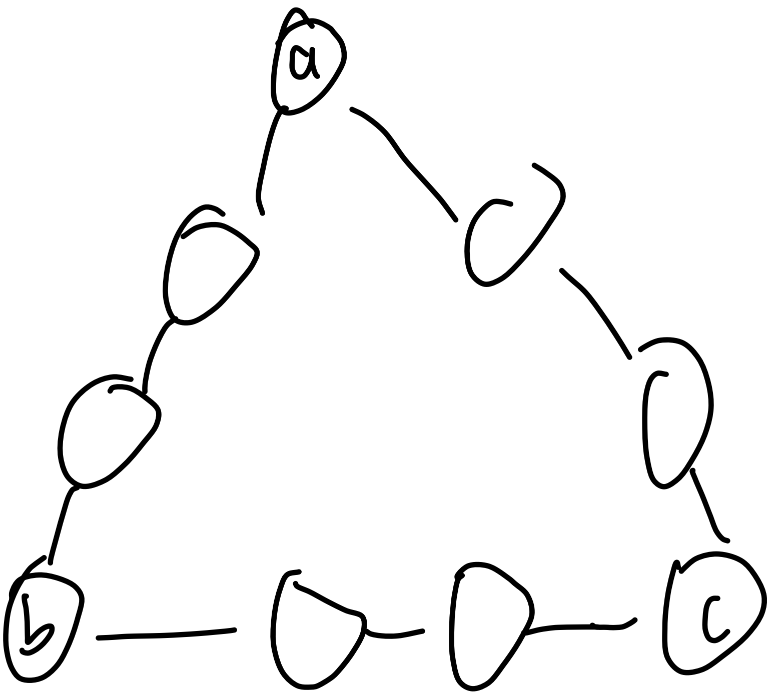
$2\times4\times8$

$2\times4\times6$

17

(1) Total 7 is possible

40 → 7 is possible



sum $1 \to 9 = 45$

10
10
10
10
5

$$\begin{array}{r} 15 \\ 3\overline{)45} \\ \underline{3} \\ 15 \end{array}$$

sum all rows =

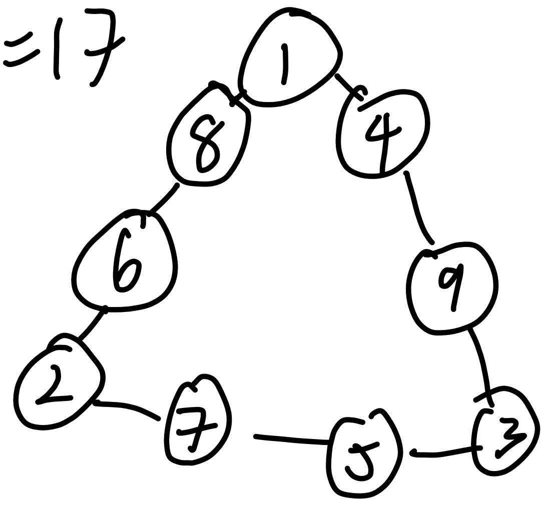
possible combo: possible T > 15

T	40 − T
20	20
try ~~16~~	24
17 ✓	23
~~18~~ ✓	22
19 ✓	21

← impossible to have $a+b+c=3$

$$\boxed{\begin{aligned} a+b+c &\geq 6 \\ a+b+c &\leq 24 \end{aligned}}$$

T=17



$16 \times 3 - 45 = 3$

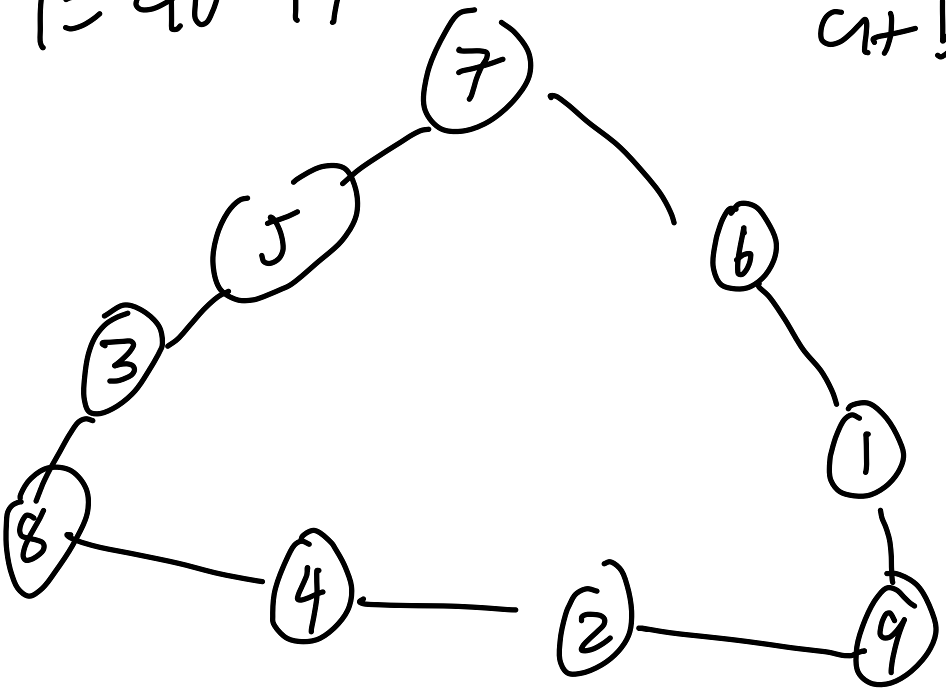
$17 \times 3 - 45 = 6$ ✓

$18 \times 3 - 45 = 9$

$19 \times 3 - 45 = 12$

$T = 40 - 17 = 23$

$a+b+c = 24$ ✓



similarly, the value of $a+b+c$ for

$T = 17$, ~~18~~, 19 and

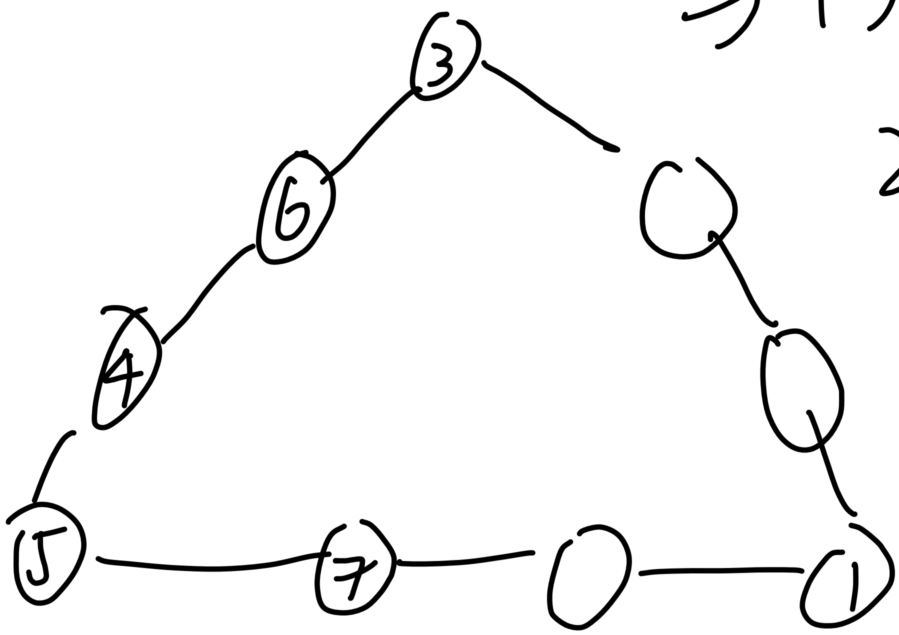
$T = 40-17$, ~~$40-18$~~, $40-19$

is within the range

$6 \leq a+b+c \leq 24$,

and $T = 18$ is

an exception



→ 17, 19, 20, 21, 23 //

② 4 digits, non 0, multiple of 12

12, 24, 36, 48, 60, 2, 4, 6, 8, 0

$\square \quad \square \quad \square \quad \square$

12 12 X

24 24 X

36 36 X

48 48 ✓

72 72 X

84 84 ✓

96 96 X

4 4 8 8

4 8 4 8

4 8 8 4

8 8 4 4

8 4 4 8

8 4 8 4

$\Rightarrow 6$

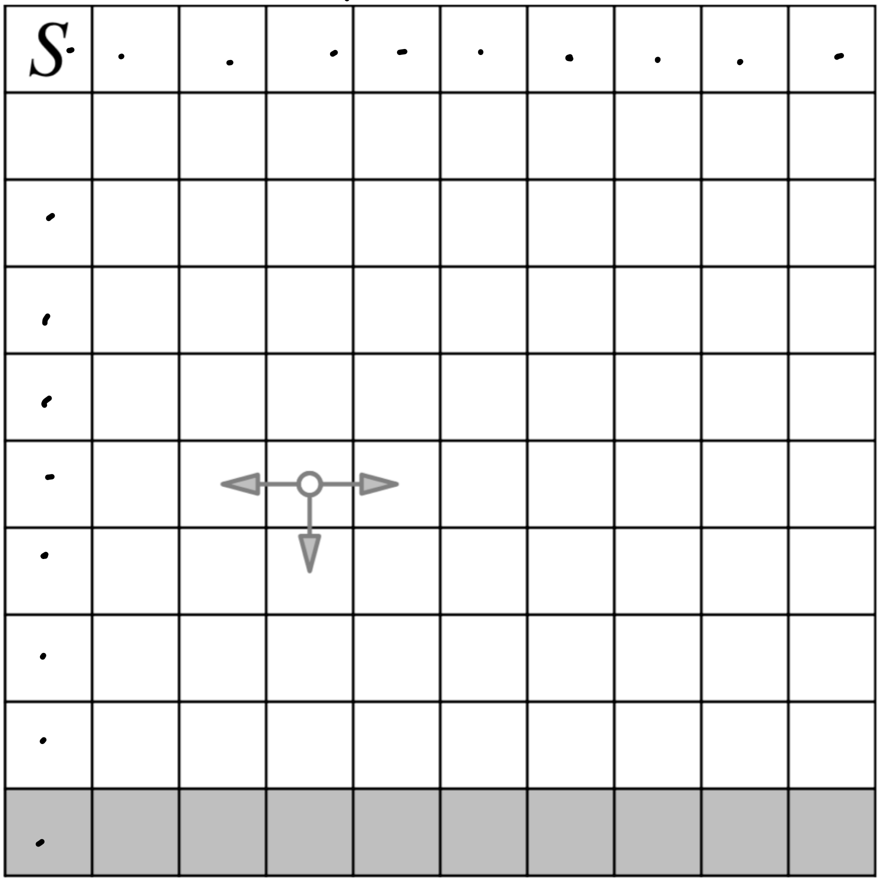
$$\frac{4!}{2!\,(4-2)!}$$

$\frac{2015}{}$ ②

横 10

left
right
down.

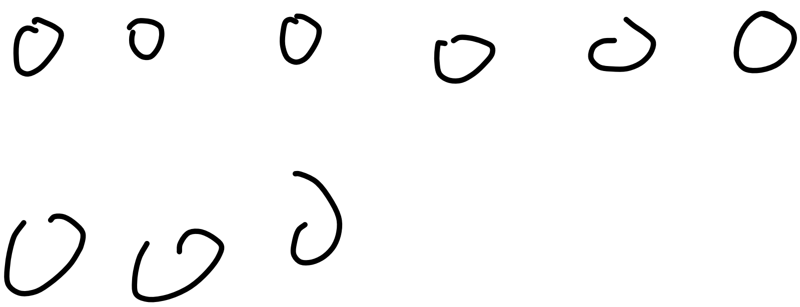
下 9



$10^9 = 1\,000\,000\,000$

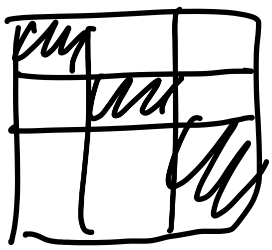
49

6



$$\frac{1}{2}(n^2 - n)$$

$$3^2 - 3$$



a b c

$$\frac{n!}{2!\,(n-2)!}$$

$$= \frac{n \times (n-1) \cancel{\times (n-2)!}}{2! \cancel{(n-2)!}}$$

$$= \frac{1}{2} n(n-1)$$

(46) let m be no. of contestant exclude A and B

$\Rightarrow \frac{1}{2}m(m-1)$

+ 10 game played by A, includes w/ B

$\frac{1}{2}m(m-1)+10=55$

$\Rightarrow m=10$

+ 1 game played by B, w/ A

$\frac{1}{2}m(m-1)+10+1=55$

$\Rightarrow$ ~~X~~ NA

子 played Egt.

5a)

$1, 4, 9, 18, 25, 36$

$n^2 = N$

$0 \leq R \leq 15$

$n = 16k + r$

e.g. $r = 5$

$n^2 = (16k+5)^2$

$= 256k^2 + 160k + 25$

$$\begin{array}{r} 16 \\ 16\overline{)256} \\ \underline{16} \\ 96 \\ \underline{96} \end{array}$$

$$\frac{25}{16} = 9$$

$$n^2 = 16k + r$$

$$n^2 = (16k+1)^2$$

$$R = 1$$

$$n^2 = (16k+2)^2$$

$$R = 4$$

$$n^2 = (16k+3)^2$$

$$R = 9$$

$$n^2 = (16k+4)^2$$

$$R = 0$$

r	R
0	0
1	1
2	4
3	9
4	0
5	9
6	4
7	1

$$\begin{array}{r} 2 \\ 16\overline{)36} \\ \underline{32} \end{array} \qquad \begin{array}{r} 3 \\ 16\overline{)49} \\ \underline{48} \end{array} \qquad \Rightarrow R = 0, 1, 4, 9$$

55)

$$m! + 76 = n^2$$

value for m and n

$$m(m-1)(m-2)\dots(3)(2)(1) + 76 = n^2$$

$$\begin{array}{ccc} & 76 & \\ / & & \backslash \\ 64 & & 81 \quad 100 \end{array}$$

n must be greater than 76

bcz $0! + 76 = 77$

minimum value.

first square $= 81$

$= 76 + 5$

impossible to achieve by $m!$

next square $= 100$

$= 76 + 24$

4

$= 4!$

$\Rightarrow \quad m \geq 4 \;, \quad n \geq 10$

$\Rightarrow$ 76 must be even no. away from $n^2 \Rightarrow n^2$ is even.

$\Leftarrow$ factorial will always be even when $m \geq 4$

$$\begin{array}{r} 24 \\ \times \; 5 \\ \hline 120 \\ - \; 76 \\ \hline 44 \end{array}$$

22

,0x F ✓

... H ✓

4a $an + b$

2017

7
4
1
8
5
2
9
6
3
0

Since the ~~m~~ multiple of 2017 (ie $2017n$) can end up with all numbers $0 \le m \le 9$, so no ~~number~~ matter ~~what~~ no. Hilary chose, the sum of the numbers will adds up to a multiple of 10

4b

5
0
5
0
5
0
⋮

$\underbrace{1, 2, 3, 4, 6, 7, 8, 9}$

every 10 no.s hv 8 values

$$\frac{2020}{10} \times 8 + 2$$

202×18

$= 1616 + 2$

$= 1618$

4c

	1	2	3	4	5	6	7	8	9
×1	1		3				7		9
2	2		6				4		8
3	3		9				1		7
4	4		2				8		6
5	5		5				5		5
6	6		8				2		4
7	7		1				9		3
8	8		4				6		2
9	9		7				3		1
10	0		0				0		0

⇒ ~~Freya~~ can win if the last digit of a is 1, 3, 7, 9.

$$\frac{2020}{10} \times 4 + 1$$

$$= 809$$

$$\begin{array}{r} 202 \\ \times\ \ 4 \\ \hline 808 \end{array}$$

(5) $1 \quad +5 \quad +5$

C

$a \quad -3 \quad -3$

1	6	11	16	21	26
⋮	⋮	⋮	⋮		
a	$(a-3)$	$(a-6)$	$(a-9)$	$(a-12)$	$(a-15)$

1	1, -2, ...
6	6, 3, 0 ...
11	11, 8, 5, 2, -1 ...

if $a = 16$

16, 13, 10, 7, 4, 1, -2 X

bi) $1 \xrightarrow{+5} 6 \xrightarrow{+5} 11 \quad 16 \quad 21 \quad 26$

$a \xrightarrow{-3} a-3 \xrightarrow{-3} a-6$

~~24~~ 21 18 15 12 9 6 3 0

~~26 23 20 17 14 11 8 5~~

$\left.\begin{array}{l} +15 \\ -15 \end{array}\right]$ common

$\Rightarrow$ 3 ~~consecutive~~ Successive terms of F

has common with

5 cosequative terms of H

bi) 100 ~~common~~ common terms are

$C + 15k$, where $k = 0, 1, \ldots 99$.

~~$C = 1, 6, 11$~~

ii) ~~1+15k~~ $c = 1$

$1 + 15(99)$

$= 1 + 1500 - 15$

$= 1486$

iii) 2, 4, 8, 16, 32, 64,

128, 256, 512, 1024

$10^9 = 1{,}000{,}000{,}000$