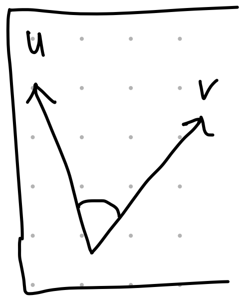
# Dot product

**$u \cdot v$**

$$u \cdot v = \begin{cases} \|u\|\|v\| \cos[u, v], & \text{if } u \neq 0 \text{ and } v \neq 0 \\ 0 & \text{if } u = 0 \text{ or } v = 0 \end{cases}$$




**rules about dot product**

$u \cdot v > 0 \iff 0 < [u, v] < \pi/2$

$u \cdot v < 0 \iff \pi/2 < [u, v] \leq \pi$

$u \cdot v = 0 \iff u \perp v$, i.e., $[u, v] = \pi/2$ or $u = 0$ or $v = 0$

**unit vector**

A vector whose length is 1.

Use normalization to create it.

$$n = \frac{1}{\|v\|} v$$

$n$: normalized vector

$\|v\|$: length of vector.

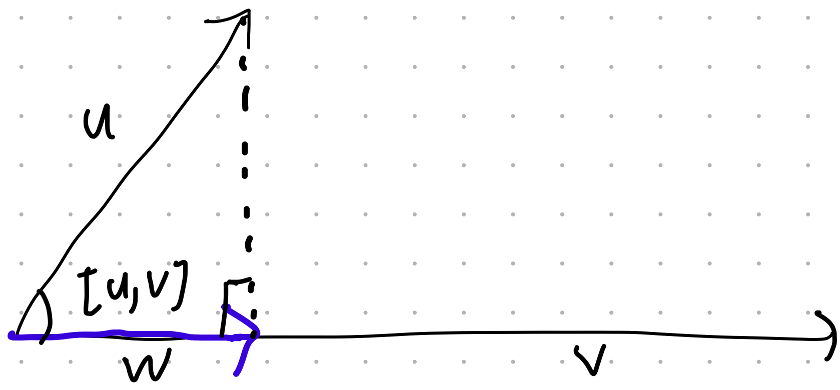
$v$: non-zero vector

prove $n$ is a unit vector, ie. $\|n\| = 1$

let $\lambda = \frac{1}{\|v\|}$

$\|n\| = \|\lambda v\| = |\lambda| \|v\| = \frac{1}{\|v\|} \|v\| = 1$

projection




vector u projected onto vector v, creates a new vector w.

$$\cos[u,v] = \frac{\|w\|}{\|u\|}$$

$$\|w\| = \|u\| \cos[u,v]$$

if $\|v\| = 1$

$\Rightarrow$ projected vector : $w = \|w\| v$

$$= \|u\| \cos[u,v]\, v$$

prove

$\|w\| v = w$

manguitude w in the direction of vector v.

rn, $\|v\| \neq 1$,

$$w = \|w\|\, v \frac{1}{\|v\|} = \|u\| \cos[u,v]\, v \frac{1}{\|v\|}$$

$$w = \frac{\|u\| \|v\| \cos[u,v]}{\|v\|^2} v$$

general orthogonal projection formula.

$$\boxed{w = \frac{u \cdot v}{\|v\|^2} v}$$

Dot product rule

i) $u \cdot v = v \cdot u$

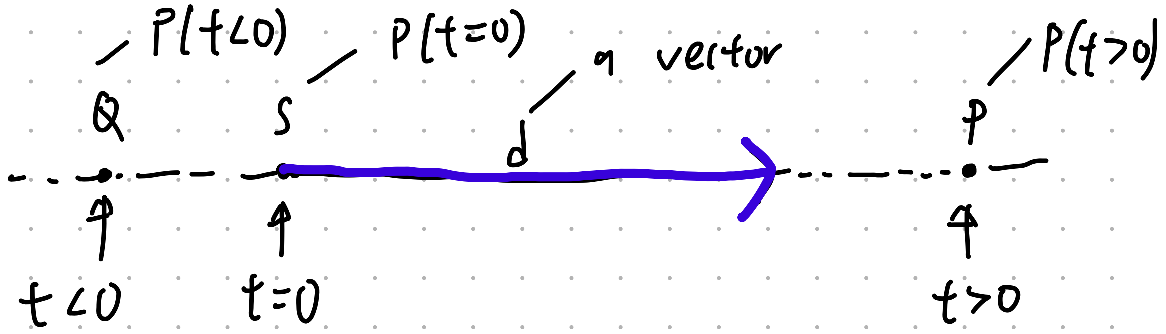
ii) $k(u \cdot v) = (ku) \cdot v$

iii) $v \cdot (u + w) = v \cdot u + v \cdot w$

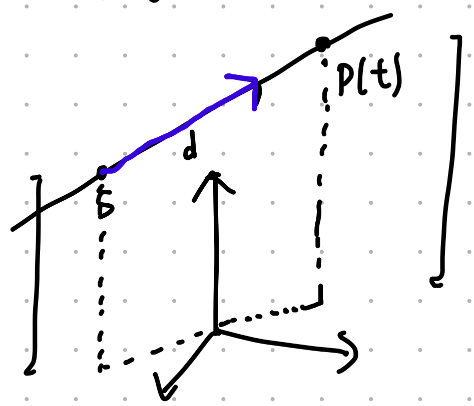
iv) $v \cdot v = \|v\|^2 \geq 0$, whith equality only when $v = 0$

# Lines and planes

2D




3D




diff d from grount.

A Parameterized line

$$P(t) = S + td$$

- $P(t)$: all points
- $S$: starting pt
- $t$: scalar, tells you how far down the line
- $d$: vector of line

2-D pt. $S(S_x, S_y)$, $d(d_x, d_y)$

line can be expressed in terms of its scalar components of the vectors and points as:

$$P(t) = S + td \iff \begin{cases} P_x(t) = S_x + td_x \\ P_y(t) = S_y + td_y \end{cases}$$

2D implicit line

$$e(P) = n \cdot (P - S)$$

all points on the line

normal vector

starting point

$$e(P) = n \cdot (P - S) = 0$$ , when