section 1.1

function : $f(x) = x^2$

f = function = sets of rules applied

x = domain, which is the subject

x^2 = codomain, which is the output

notations : $[2, 5]$ ← all no. from 2 to 5

$(2, 5)$ ← all no. between 2 and 5

$[2,5) =$ 

$(2,5] =$ 

$$\tan(90°) = \frac{\sin(90°)}{\cos(90°)} = \frac{1}{0}$$

domain is undefined.

$$f(x) = \frac{\log_{10}(x+8)\sqrt{26-2x}}{(x-2)(x+19)}$$

f

domain: ① $26-2x \geq 0$
$x \geq 13$

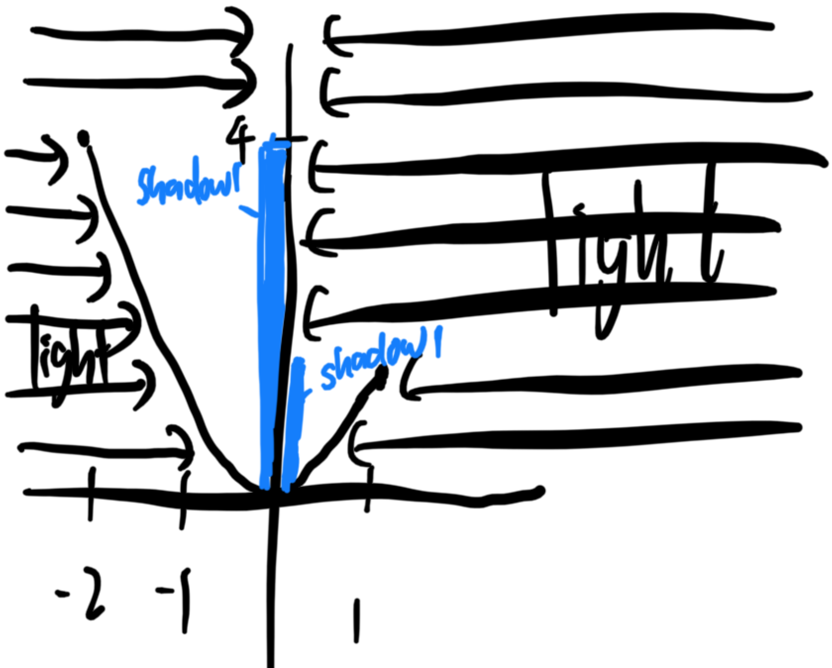
③ $x-2 \neq 0$
$x \neq 2$

② $x+8 > 0$
$x > -8$

④ $x+19 \neq 0$
$x \neq -19$

$\Rightarrow [-8, 13] \setminus \{2\}$

backslash = not including



$f(x) = x^2$
range of F:

$S_1 \cup S_2$

$[0, 4]$

The vertical

Take a pt w/ coordinate

line test

graph of a function?

$(x, f(x))$

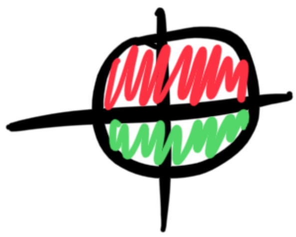
↑ domain

↑ test

$x^2+y^2=9$

ANS{ NO, because intersects twice.

make it into a graph with one function

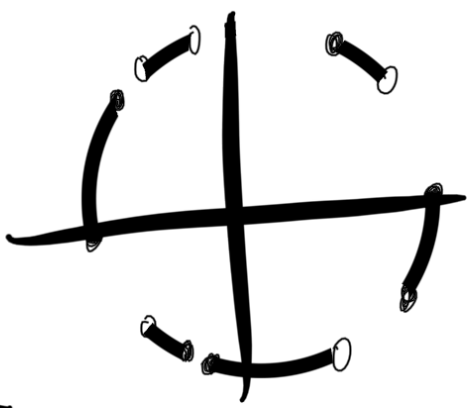


$y=\sqrt{9-x^2}$

$y=-\sqrt{9-x^2}$

both w/ domain $[-3, 3]$

OR



Section 1.2

inverse function f : ① for any y in the range of f, there's exactly 1 number x such that $f(x)=y$.

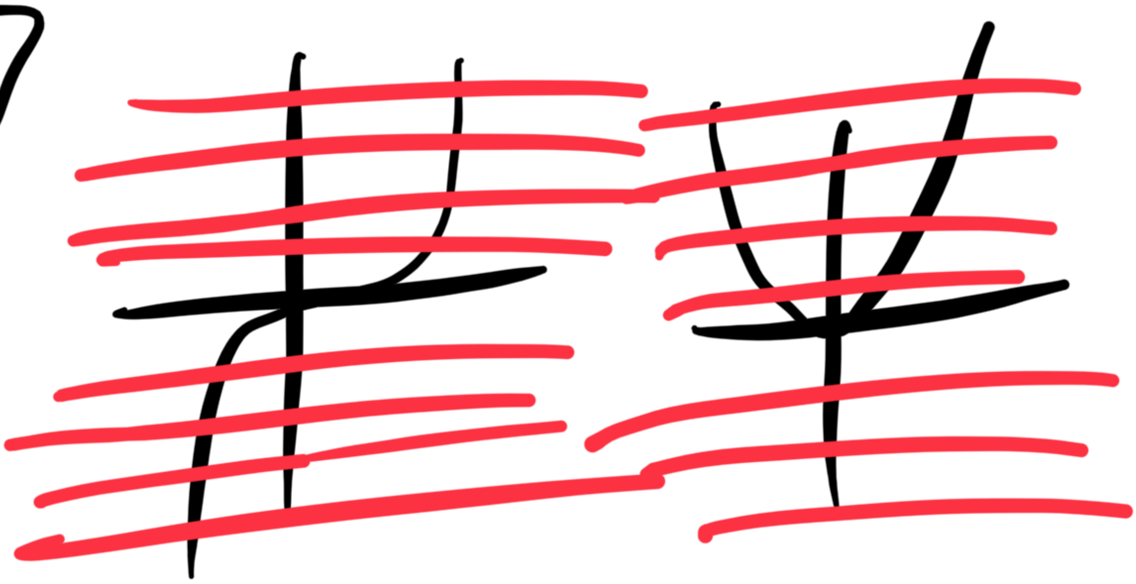
② the domain of f^{-1} is the same as the range of f

③ same range

④ $f(x)=y$, $f^{-1}(y)=x$

f

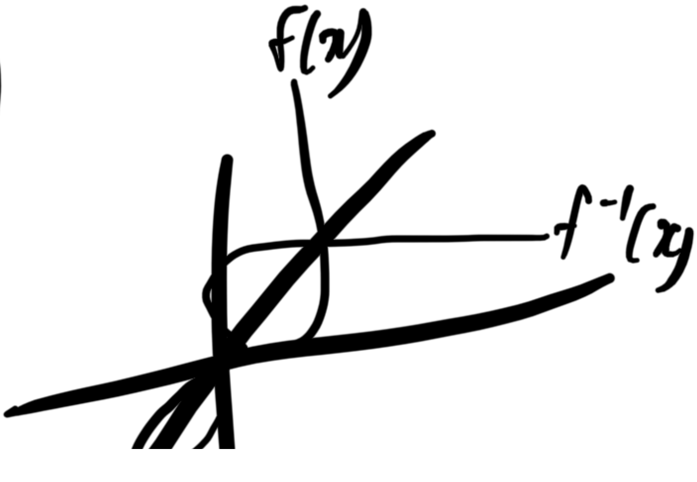
$\boxed{\text{horizontal line test}}$



↑ has an inverse

f doesn't have inverse.

$\boxed{\text{finding the inverse}}$



$f^{-1}(f(x))$ may not equal x, only when x is in the restricted domain.

range f = domain f^{-1}

range f^{-1} = domain f

$f^{-1}(f(x))$ ✓ only when x is in restricted domain

1.3 composite functions

e.g.1

$f = g \circ h \circ j$; $g(x) = 2^x$, $h(x) = 5x^4$, $j(x) = 2x-1$

$f(x) = g(h(j(x)))$

$= g(h(2x-1)) = g(5(2x-1)^4) = 2^{5(2x-1)^4}$

e.g.2 $f(x) = \dfrac{1}{\tan(5\log_2(x+3))}$ $\Rightarrow f = m \circ k \circ j \circ h \circ g$

let $g(x) = x+3$, $h(x) = \log_2(x)$, $j(x) = 5x$,

$k(x) = \tan(x)$, $m(x) = \frac{1}{x}$

$\Rightarrow f(x) = m(k(j(h(g(x)))))$

1.4

→ odd function and even function, or neither

neither: zero function

odd: $f(-x) = -f(x)$

even: $f(-x) = f(x)$

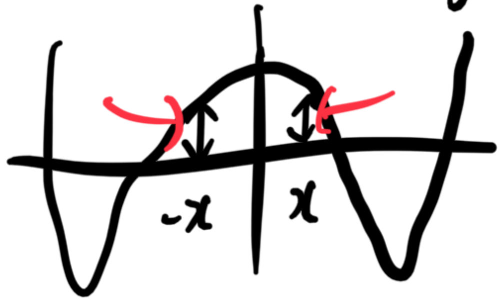
e.g. $f(x) = x^3$, where

e.g. $f(x) = x^2$, where $x = -3$

$x=-3$

$f(-3) = (-3)^3 = -27$ has 180° point symmetry about the origin

$f(-3) = (-3)^2 = 9$ has mirror symmetry about y-axis



e.g $f(x) = \log_5 (2x^6 - 6x^2 + 3)$

Calculate $f(-x)$ → $f(x)$, even

→ $-f(x)$, odd

→ non, either

$$\Rightarrow f(-x) = \log_5 (2(-x)^6 - 6(-x)^2 + 3)$$
$$= \log_5 (2x^6 - 6x^2 + 3)$$
= original function $f(x)$

⇒ even

1.6 Common functions and graphs

① polynomials

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

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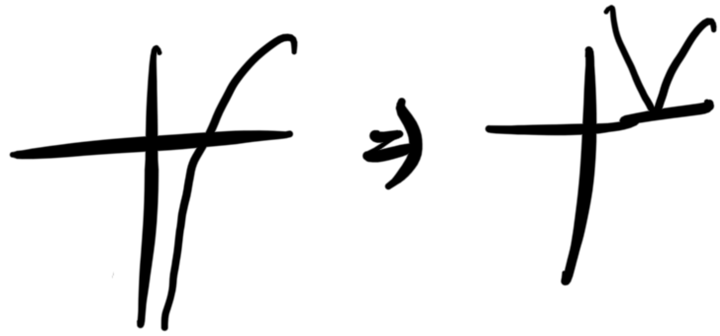
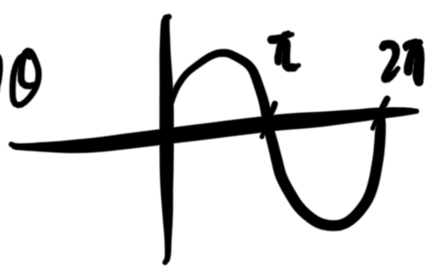
inverse of a function: reflect by line $x = y$

mod. of a function: reflect everything below the x-axis up to above x-axis

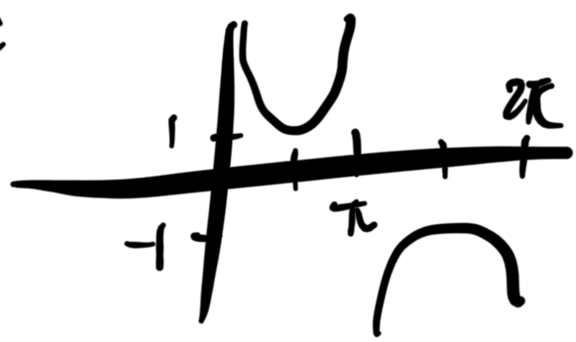
e.g $y = \log_2(x) \Rightarrow y = |\log_2(x)|$



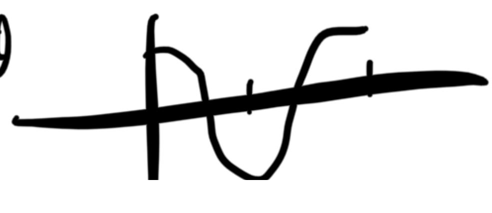
$\sin\theta$



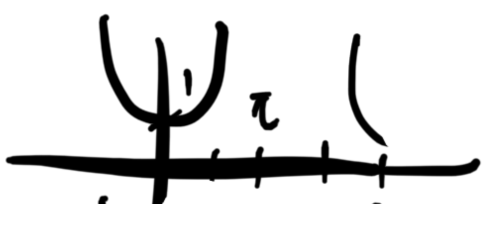
cosec



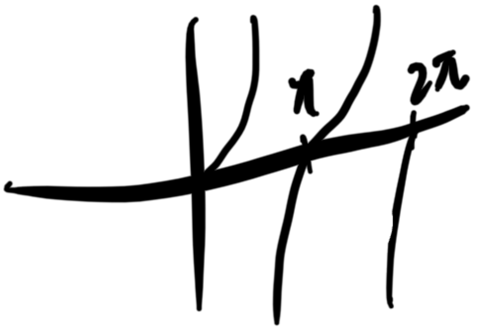
$\cos\theta$



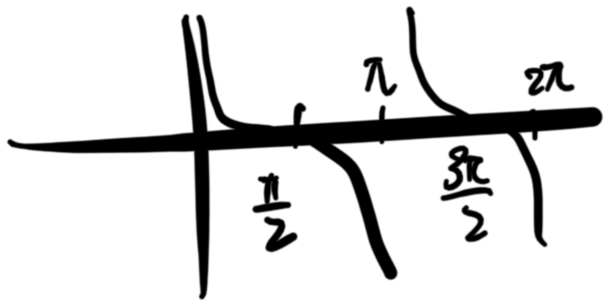
sec



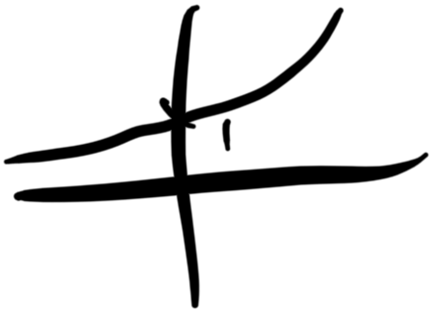
tanθ



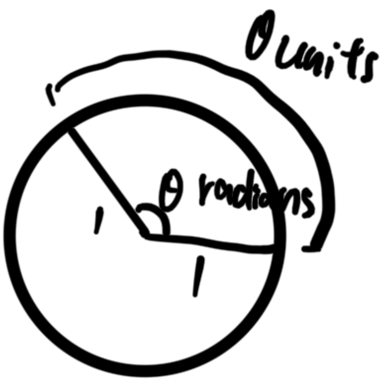
cot



$y = a^x$



$y = \log_a x$



The circumference of a circle of radius 1 unit is 2π units

$\Rightarrow$ The arc length of a wedge of this circle = angle of the wedge

angle in radians $= \frac{\pi}{180} \times$ angle in degrees

$\sin(x)$, $\tan(x)$, $\cot(x)$, $\csc(x)$] odd functions

$\cos(x)$, $\sec(x)$] even functions

2.4 Trig identities

$$\tan x = \frac{\sin x}{\cos x} \xrightarrow{1 \div} \cot(x) = \frac{\cos x}{\sin x}$$

$$\cos^2\theta + \sin^2\theta = 1 \xrightarrow{\div \cos^2} 1 + \tan^2\theta = \sec^2\theta$$

$\downarrow \div \sin^2$

$$\cot^2\theta + 1 = \operatorname{cosec}^2\theta$$

co = complementary

→ 2 ∠s add up to $\frac{\pi}{2}$

→ trig function (x)

= co-trig function $(\frac{\pi}{2} - x)$

= co-trig function $(\frac{\pi}{2} - x)$

↳ $\sin x = \cos\cancel{x}(\frac{\pi}{2} - x)$

$\tan x = \cot(\frac{\pi}{2} - x)$

$\sec(x) = \csc(\frac{\pi}{2} - x)$

} reverse as well.

---

10 :

$$\sin\theta = \frac{AB}{AC}$$

$A = 90° - C$

$$\cos(90° - C) = \frac{AB}{AC} = \sin\theta$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

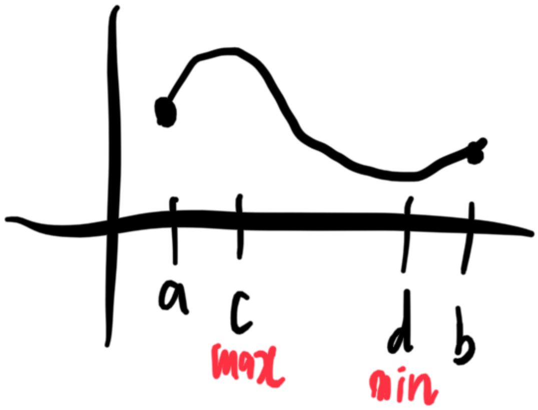
5.1 Continuity

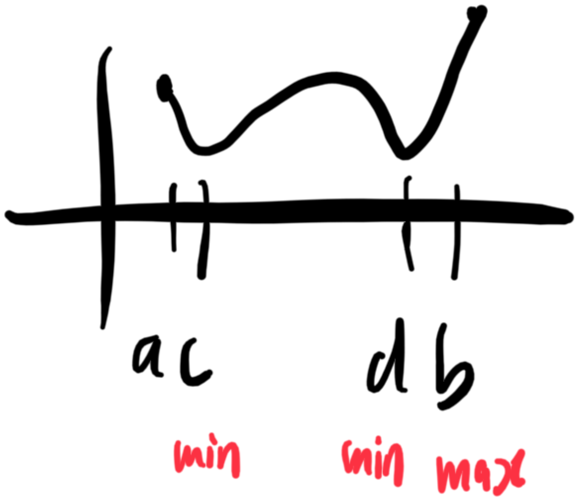
$$\lim_{x \to a} f(x) = f(a)$$

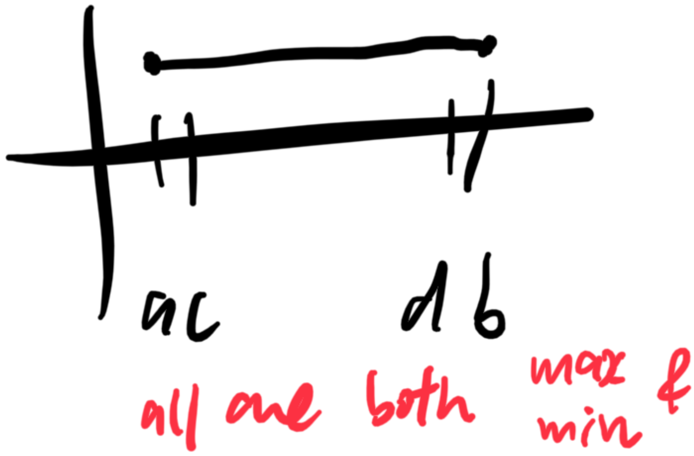
Max-min Theorem: If f is continuous on $[a, b]$, then f has at least one maximum and one minimum on $[a, b]$

mostly ~~the~~ one continuous function !!!

e.g

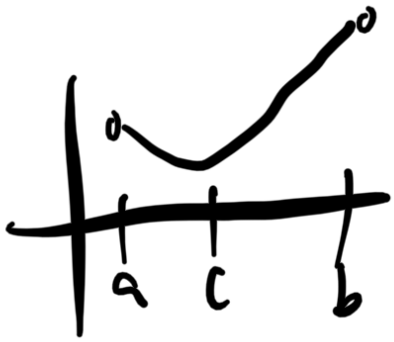






or else

↳ has asymptote
⇒ no max min
as the graph keeps going



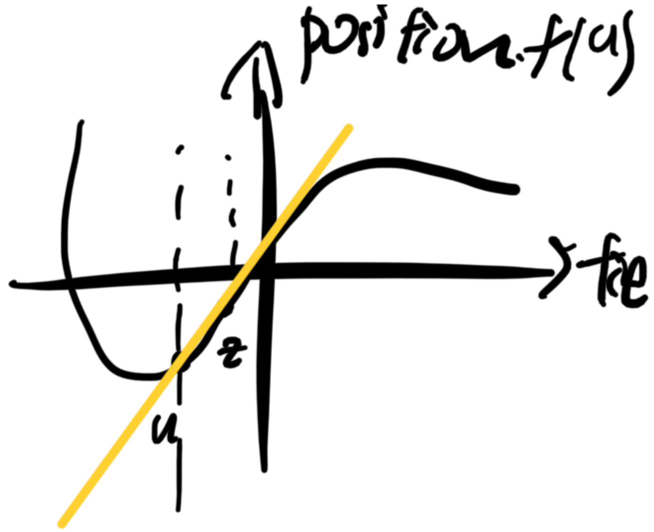
↳ min at c
but what's the max?
b is undefined, and can't really figure out a value that is the nearest to b.

↳ has min and max
⇒ can only use the theorem when the func is continuous on the closed interval

$$\lim_{z \to u} \frac{f(z) - f(u)}{z - u}$$

↳ let $h = z - u$

$$\lim_{h \to 0} \frac{f(z) - f(u)}{h}$$



$$\boxed{f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}}$$

f is differentiable at x

reasons for fail to differentiate

① $y = |x|$

has a sharp corner

② x isn't in the domain of f.

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if $f(x) = x^2$, find $f'(x)$

$$f'(x) = \lim \frac{f(x+h) - f(x)}{\ }$$

$$\lim_{h\to 0} \frac{\quad}{h}$$

$$= \lim_{h\to 0} \frac{(x+h)^2 - x^2}{h}$$

$$= \lim_{h\to 0} \frac{x^2+2xh+h^2-x^2}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2}{h}$$

$$= \lim_{h\to 0} (2x+h) = 2x$$

$$\Rightarrow f'(x) = 2x$$